Unit 19: Further Mathematics in Construction and the Built Environment

Unit code: M/600/0234
QCF Level 3: BTEC Nationals
Credit value: 10
Guided learning hours: 60

Aim and purpose

This unit aims to enable learners to develop skills in applying transposition of formulae, arithmetical methods, calculus and statistics to construction engineering problems.

Unit introduction

Civil engineers and building services engineers need to use appropriate mathematical techniques and formulae in their work to determine important physical properties. These include finding the centre of gravity of an irregular shaped pre-cast cladding unit so that it can be safely lifted, or calculating the root mean square value of an alternating electric current.

This unit gives learners an opportunity to study relevant aspects of 'pure' mathematics and to explore how complex practical problems can be solved. Learners will be able to solve applied mathematical problems involving statistical data, structural properties for beams and columns, complex linear, angular, area and volume measurements, trigonometric identities, rates of change and decay, differentiation of maxima and minima, numerical integration, and complex areas or volumes by definite and indefinite integration.

Although complex calculations can be carried out using specialist software, a clear understanding of the underpinning mathematical techniques is essential so that the results of calculations can be manually evaluated and validated as a part of the design process.

Successful completion of this unit will give learners a sound basis for further study within the construction, civil engineering or building services engineering sectors at Higher National or degree level.

Learning outcomes

On completion of this unit a learner should:

1 Be able to transpose formulae to simplify and solve mathematical problems

2 Be able to apply arithmetical techniques to solve engineering problems

3 Be able to use calculus to solve practical engineering problems

4 Be able to produce solutions to engineering problems using statistical methods.

Unit content

1 Be able to transpose formulae to simplify and solve mathematical problems

Transposition techniques: rearrangement of formulae to determine new subjects; use of given and rearranged formulae to evaluate data

Mathematical formulae to transpose: trigonometric expressions; binomial theorem applied to errors

2 Be able to apply arithmetical techniques to solve engineering problems

Arithmetical techniques: formulae for irregular areas and volumes (Trapezoidal Rule, Mid-Ordinate Rule; Simpson's Rule); arithmetical calculation of various properties of sections

Engineering-related problems: determination of irregular areas and volumes; determination of properties of sections (eg cross-sectional area, location of centroid, neutral axis, moment of inertia, section modulus, radius of gyration); application to simple, regular and irregular shapes

3 Be able to use calculus to solve practical engineering problems

Differential calculus: basic differentiation techniques applied to algebraic, trigonometric and logarithmic functions; products and quotients; function of a function; second order derivatives; the location of stationary values

Integral calculus: indefinite and definite integration techniques applied to algebraic, trigonometric and exponential functions

Practical engineering problems: solution of problems involving maxima and minima; solution of problems involving centroids, moments of inertia, areas under curves and volumes of revolution

4 Be able to produce solutions to engineering problems using statistical methods

Statistical methods: presentation of data (histograms, frequency graphs, cumulative frequency graphs); sampling distributions (normal distribution tables; confidence limits; significance testing; large samples only)

Construction engineering problems: measures of central tendency (mean, mode, median); measures of dispersion (range, variance, standard deviation, quartiles, deciles and percentiles); grouped and ungrouped data

Assessment and grading criteria

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all the learning outcomes for the unit. The assessment criteria for a pass grade describe the level of achievement required to pass this unit.

Assessment and grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P1 apply transposition techniques to mathematical formulae and expressions [IE1, CT5, RL4, SM2]	**M1** transpose and evaluate complex formulae for use in determining properties of sections	**D1** independently solve industry-related problems using appropriate mathematical techniques
P2 use arithmetical techniques to determine values for properties of sections, including irregular areas and volumes [IE1, CT5, RL4, SM2]		
P3 use differential calculus techniques to solve algebraic, trigonometric and logarithmic expressions [IE1, CT5, RL4, SM2]	**M2** use first and second order differentials for the solution of industry-related problems	**D2** independently apply differential calculus to the determination of maxima and minima in industry-related problems.
P4 use integral calculus to solve simple algebraic, trigonometric and exponential expressions [IE1, CT5, RL4, SM2]	**M3** apply the rules of integral calculus to determine solutions for complex industry-related problems	
P5 use statistical methods to produce accurate and appropriate solutions to construction engineering problems. [IE1, CT5, RL4, SM2]	**M4** apply statistical methods to analyse engineering data and make realistic assessments of this data.	

PLTS: This summary references where applicable, in the square brackets, the elements of the personal, learning and thinking skills applicable in the pass criteria. It identifies opportunities for learners to demonstrate effective application of the referenced elements of the skills.

Key			
	IE – independent enquirers	RL – reflective learners	SM – self-managers
	CT – creative thinkers	TW – team workers	EP – effective participators

Essential guidance for tutors

Delivery

Tutors can use a wide range of techniques to deliver this unit. Lectures, discussions, seminar presentations, site visits, supervised practicals, research using the internet and/or library resources and use of personal and/or industrial experience are all suitable. Delivery should stimulate, motivate, educate and enthuse learners. Visiting expert speakers could add to the relevance of the subject.

Learning outcome 1 forms the basis for all the following learning outcomes and should be delivered first. Learning outcomes 2 and 3 reflect the development of different mathematical skills and techniques and would be best introduced and developed separately, but with clear links drawn between their 'pure' and 'applied' aspects. Learning outcome 4 regarding statistical methods is essentially a stand-alone element and could be delivered at any point during the programme.

Teaching and learning strategies designed to support delivery of this unit should involve theory, worked examples and, most importantly, practice. There needs to be time and resources available for differentiated support. Within the scheme of work time should be allowed for regular workshops and/or tutorials, as some learners may find this unit particularly challenging.

Evidence generated should be thoroughly worked calculations, graphical solutions and other mathematical exercises. It is acceptable for early examples to be 'pure' mathematics but the intention is that the majority of evidence should be linked to learning in other areas of learners' programme of study.

In mathematics, possibly more so than in any other area, assessment drives delivery. It is, therefore, reasonable to consider delivery methods in terms of how the unit is to be assessed. Practice is, as noted above, key and learners must be given many opportunities to practise the relevant techniques. The use of formative tests and coursework will help learners to see where they may be going wrong and the tutor should provide feedback but should not grade the formative work. This process then becomes part of the delivery.

Formal formative assessment techniques, such as short time-controlled assessments or case studies/projects, are suggested so learners can demonstrate their level of ability. It is suggested that every attempt should be made to provide industry-based scenarios, relevant to learners' vocational aims, for use in unit delivery and formative assessments.

The statistical elements of the unit can be used as a link to the industrial environment by the careful choice of scenarios. For example, there is a great deal of statistical data relating to health, safety and welfare issues freely available from various government and industry bodies. Alternatively, the testing of materials such as concrete, bricks and metals will yield useful data to support delivery of statistical analysis.

It is anticipated that this unit will be delivered after completion of *Unit 3: Mathematics in Construction and the Built Environment*.

Group activities are allocated, but tutors will need to ensure that individual learners have equal experiential and assessment opportunities.

Health, safety and welfare issues are paramount and should be reinforced through close supervision of all workshops and activity areas, and risk assessments must be undertaken before practical activities are taken. Centres are advised to read the *Delivery approach* section in the specification, and *Annexe H: Provision and Use of Work Equipment Regulations 1998 (PUWER).*

Outline learning plan

The outline learning plan has been included in this unit as guidance and can be used in conjunction with the programme of suggested assignments.

The outline learning plan demonstrates one way in planning the delivery and assessment of this unit.

Topic and suggested assignments/activities and/assessment
Introduction Whole-class, tutor-led input on the rules of transposition. Use of construction formulae only to enhance relevance Learners to practise examples of transposition as formative assessment. Tutors to assess work (but not for summative assessment purposes) and correct errors, suggest improvements and praise as appropriate. This cycle to be repeated until the learner is ready for summative assessment
Whole-class, tutor-led input on the principles and uses of arithmetical techniques. Use of construction examples only to enhance relevance Learners to practise examples of arithmetical techniques as formative assessment. Tutors to assess work (but not for summative assessment purposes), correct errors, suggest improvements and praise as appropriate. This cycle to be repeated until the learner is ready for a summative assessment
Assignment 1: Transposition and Arithmetical Techniques
Whole-class, tutor-led input on the principles and uses of calculus. Use of construction examples only to enhance relevance Learners to practise examples of calculus as formative assessment. Tutors to assess work (but not for summative assessment purposes), correct errors, suggest improvements and praise as appropriate. This cycle to be repeated until the learner is ready for summative assessment
Assignment 2: Calculus
Whole-class, tutor-led input on the principles and uses of statistical techniques Use of construction examples only to enhance relevance Learners to practise examples of statistical techniques as formative assessment. Tutors to assess work (but not for summative assessment purposes) ,correct errors, suggest improvements and praise as appropriate. This cycle to be repeated until the learner is ready to perform a summative assessment
Assignment 3: Statistics
Review of unit and assignment feedback

Assessment

Evidence for this unit can be gathered from short time-controlled phase tests, tutor-provided practical construction scenarios, case studies, practical work or traditional example-based methods.

There are many suitable forms of assessment that could be used. Some example assessment approaches are suggested below. However, these are not intended to be prescriptive or restrictive, and are provided as an illustration of the alternative forms of assessment evidence that would be acceptable.

Some criteria can be assessed directly by the tutor during practical activities. If this approach is used, suitable evidence would be observation records or witness statements. Guidance on the use of these is provided on the Edexcel website.

The complexity of the techniques covered in this unit would imply that learners need regular incremental assessment as they progress through the unit.

The structure of the unit suggests that the grading criteria could be addressed fully by using three assignments.

The first assignment would cover P1, P2, M1 and D1. P1 is essentially 'pure' mathematics and provides an introduction to M1 which also builds on activities covered in P2. D1 would allow the most able learners to appraise and apply earlier techniques to real industrial applications.

The second assignment would cover the calculus element of the unit comprising P3, P4, M2, M3 and D2. The third assignment, for P5 and M4, would cover the statistical part of the unit including industrial application.

To achieve a pass grade learners must meet the five pass criteria listed in the grading grid.

For P1, learners must apply transposition techniques to mathematical formulae and expressions. They must transpose and simplify a variety of expressions and solve by substitution. Formulae to be solved include linear, quadratic and cubic expressions, binomial expansions for errors, logarithms, and fractional powers. Work should have a logical structure, using correct mathematical conventions and appropriate units where required.

For P2, learners must use arithmetical techniques to determine values for a range of properties of sections, including irregular areas and volumes. They must find the cross-sectional area, volumes and position of the centroid for a variety of symmetrical and non-symmetrical composite shapes, using a range of arithmetic and trigonometrical formulae. Their work should have a logical structure, use correct mathematical conventions and the correct units should be stated for the final answer.

For P3, learners need to use differential calculus to solve algebraic, trigonometric and logarithmic expressions. They must demonstrate a clear understanding that differentiation arises out of considering the gradient of a line on a graph, and that this represents the rate of change of the function of the line. Learners should be able to find the gradient of a line or curve at any point for the following functions using differentiation: ax^n, sine ax, cosine ax, $\log_e x$ and e^{ax}. They will also be able to differentiate using 'function of a function' ie differentiation by substitution. Their work should have a logical structure and use correct mathematical conventions.

For P4, learners need to use integral calculus to solve simple algebraic, trigonometric and exponential expressions. They must demonstrate a clear understanding that integration arises out of considering a thin strip of area summated between limits along the x-axis of graph. With this knowledge learners must determine indefinite and definite integrals of functions involving ax^n, sine ax, cosine ax, 1/x, and e^{ax}. Work should have an understandable structure and use correct mathematical conventions.

For P5, learners need to use a range of simple statistical methods to produce accurate and appropriate solutions to construction engineering problems. They must demonstrate a satisfactory understanding and knowledge of basic data handling and statistical manipulation techniques. Learners should be able to interpret the results and draw relevant conclusions.

To achieve a merit grade learners must meet all of the pass grade criteria **and** the four merit grade criteria.

For M1, learners must transpose and evaluate complex formulae, with limited tutor support. Examples could include the moment of inertia, section modulus or radius of gyration of given, standard symmetrical (about y-y axis only) composite cross-sections. Their work should have a clear, understandable and well-presented structure. Learners should apply the correct mathematical conventions and use the correct units throughout.

For M2, learners must use first and second order differentials for the solution of industry-related problems. With limited tutor support, they should be able to explain the notation for second order derivatives and find the second order derivative by applying the basic rules of differential calculus to the simplified result of a first order differentiation. They should also be able to solve first order differential equations given specific boundary conditions, and determine the turning point positions using second order differentiation methods. Their work should have a clear, understandable and well-presented structure. Learners should apply the correct mathematical conventions and use the correct units throughout.

For M3, learners needs to apply the rules of integral calculus to determine solutions for a range of complex industry-related problems. They need to solve, with limited tutor support, integration problems involving the position of centroids of areas/volumes and the volumes of revolution. These problems should be specified

clearly in written or diagrammatic form to enable learners to grasp the functions to be integrated. Their work should have a clear, understandable and well-presented structure. Learners should apply the correct mathematical conventions and use the correct units throughout.

For M4, learners must apply statistical methods to analyse engineering data and make realistic assessments of this data. This should be done with minimal tutor support and be based on given industrial situations. Access to secondary research data will be sufficient as learners are not expected to be undertaking primary research at this level.

To achieve a distinction grade learners must meet all of the pass and merit grade criteria and the two distinction grade criteria.

For D1, learners must be able to, independently, solve industry-related problems using appropriate algebraic, trigonometric and logarithmic functions. Alternative methods of solution should be carried out where appropriate. Their work should have a neat, efficient, logical and clear structure. Learners should apply the correct mathematical conventions and use the correct units throughout.

For D2, learners must be able to, independently, apply differential calculus to the determination of maxima or minima in industry-related problems. They must develop, from the industry-based case study particular formulae for the required area or volume. Using this mathematical model learners must be able to apply the principles of differentiation, learned from previous theory, to find maximum and minimum values of areas/volumes. Learners needs to demonstrate awareness of the correct solution where more than one solution is possible ie two or more roots. Work should have a neat, efficient, logical and clear structure. Learners should apply the correct mathematical conventions and use the correct units where applicable.

Programme of suggested assignments

The following table shows a programme of suggested assignments that cover the pass, merit and distinction criteria in the grading grid. This is for guidance and it is recommended that centres either write their own assignments or adapt any Edexcel assignments to meet local needs and resources.

Criteria covered	Assignment title	Scenario	Assessment method
P1, P2, M1, D1	Transposition and Arithmetical Techniques	You have been contracted as a mathematical consultant to solve and confirm a number of mathematical problems/solutions for projects on a major contract for the government.	Successful transposition of construction-related formulae, and application of formulae to determine a range of irregular areas and volumes and properties of sections.
P3, P4, M2, M3, D2	Calculus	You have been contracted as a mathematical consultant to solve and confirm a number of mathematical problems/solutions for projects on a major contract for the government.	Calculations, supported by text, diagrams, graphs, tables and charts as appropriate.
P5, M4	Statistics	You have been contracted as a mathematical consultant to solve and confirm a number of mathematical problems/solutions for projects on a major contract for the government.	Calculations, supported by text, diagrams, graphs, tables and charts as appropriate.

Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications

This unit forms part of the BTEC Construction and the Built Environment sector suite. This unit has particular links with the following unit titles in the Construction and the Built Environment suite:

Level 1	Level 2	Level 3
	Use of Science and Mathematics in Construction	Mathematics in Construction and the Built Environment

The learning outcomes in this unit are closely linked with similar units at Higher National and degree level.

There are no specific links with National Occupational Standards at Level 3.

Essential resources

The resource requirements are limited. The application of mathematical techniques requires little in the way of resources other than scientific calculators and simple drawing equipment. Both of these are implicit requirements for many other units and, therefore, no extra resources are required for this unit other than a range of realistic and feasible project materials appropriate to the application of a range of mathematical methods.

Employer engagement and vocational contexts

Support to enable centres to initiate and establish links to industry, and to networks arranging visits to industry and from property practitioners is given below:

- Learning and Skills Network – www.vocationallearning.org.uk
- National Education and Business Partnership Network – www.nebpn.org
- The Royal Institution of Chartered Surveyors – www.rics.org
- Work Experience/Workplace learning frameworks – Centre for Education and Industry (CEI University of Warwick) – www.warwick.ac.uk/wie/cei/

Indicative reading for learners

Textbooks

Bird J and May A – *Technician Mathematics: Level 3, 2nd Edition* (Pearson, 1994) ISBN 0582234247

Websites

www.aaamath.com	Maths lessons
www.bbc.co.uk/education/megamaths/tables.html	BBC site
www.coolmath.com	Cool Maths
www.easymaths.org	Easy Maths Tutorials
www.figurethis.org	Figure This!
www.funbrain.com/numbers.html	FunBrain
www.mathcats.com	Math Cats
www.themathleague.com	The Math League

Delivery of personal, learning and thinking skills (PLTS)

The following table identifies the PLTS opportunities that have been included within the assessment criteria of this unit:

Skill	When learners are ...
Independent enquirers	identifying questions to answer and problems to resolve, as they: • apply the rules of transposition to simple mathematical formulae and expressions • use arithmetical techniques to determine values for a range of properties of sections, including irregular areas and volumes • use differential calculus techniques to solve a range of algebraic, trigonometric and logarithmic expressions • use integral calculus to solve a range of simple algebraic, trigonometric and exponential expressions • use a range of simple statistical methods to produce accurate and appropriate solutions to construction engineering problems
Creative thinkers	trying out alternatives or new solutions and following ideas through, as they: • apply the rules of transposition to simple mathematical formulae and expressions • use arithmetical techniques to determine values for a range of properties of sections, including irregular areas and volumes • use differential calculus techniques to solve a range of algebraic, trigonometric and logarithmic expressions • use integral calculus to solve a range of simple algebraic, trigonometric and exponential expressions • use a range of simple statistical methods to produce accurate and appropriate solutions to construction engineering problems
Reflective learners	inviting feedback and dealing positively with praise, setbacks and criticism, as they: • apply the rules of transposition to simple mathematical formulae and expressions • use arithmetical techniques to determine values for a range of properties of sections, including irregular areas and volumes • use differential calculus techniques to solve a range of algebraic, trigonometric and logarithmic expressions • use integral calculus to solve a range of simple algebraic, trigonometric and exponential expressions • use a range of simple statistical methods to produce accurate and appropriate solutions to construction engineering problems

Skill	When learners are ...
Self-managers	working towards goals and showing initiative, commitment and perseverance, as they: • apply the rules of transposition to simple mathematical formulae and expressions • use arithmetical techniques to determine values for a range of properties of sections, including irregular areas and volumes • use differential calculus techniques to solve a range of algebraic, trigonometric and logarithmic expressions • use integral calculus to solve a range of simple algebraic, trigonometric and exponential expressions • use a range of simple statistical methods to produce accurate and appropriate solutions to construction engineering problems.

Functional Skills – Level 2

Skill	When learners are ...
Mathematics	
Understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations	transposing formulae and using arithmetical techniques, calculus and statistical techniques to solve construction engineering problems.
Identify the situation or problem and the mathematical methods needed to tackle it	
Select and apply a range of skills to find solutions	
Use appropriate checking procedures and evaluate their effectiveness at each stage	
Interpret and communicate solutions to practical problems in familiar and unfamiliar routine contexts and situations	
Draw conclusions and provide mathematical justifications	