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Mark Scheme (Results)

Summer 2025

Pearson Advanced Extension Award
In Mathematics (9811)
Paper 01

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NOTES ON MARKING PRINCIPLES

1 Types of mark

M marks: method marks
A marks: accuracy marks
B marks: unconditional accuracy marks (independent of M marks)

2 Abbreviations

cao – correct answer only
isw – ignore subsequent working
oe – or equivalent (and appropriate)
indep - independent
ft – follow through
SC: special case
dep – dependent

3 No working

If no working is shown then correct answers normally score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.

4 With working

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks
If there is no answer on the answer line then check the working for an obvious answer.
Any case of suspected misread loses A (and B) marks on that part, but can gain the M marks. Discuss each of these situations with your Team Leader.
If there is a choice of methods shown, then no marks should be awarded, unless the answer on the answer line makes clear the method that has been used.

5 Follow through marks

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.
Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

6 Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct
It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.
Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

7 Parts of questions

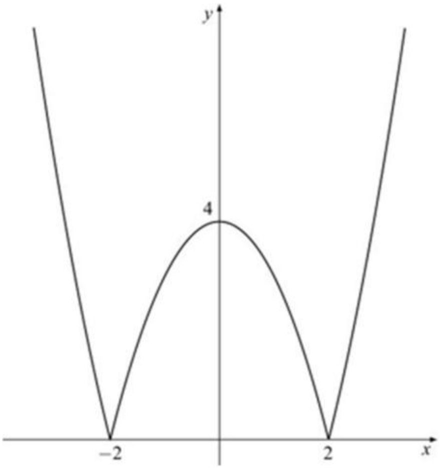
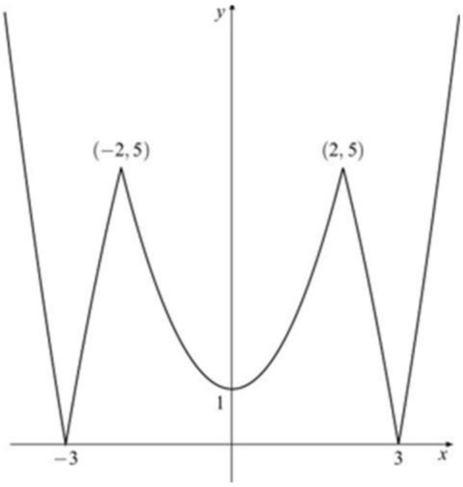
Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

8 Use of ranges for answers

If an answer is within a range this is inclusive, unless otherwise stated.

Question	Scheme		Marks
1	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\pi}{4}(1+x)^{-\frac{1}{2}}$ o.e. e.g. $\left(\frac{4}{\pi^2}u^2 = 1+x \Rightarrow\right) \frac{8}{\pi^2}u\,\mathrm{d}u = \mathrm{d}x$	Correct derivative of *u*. Accept any equivalent.	**B1**
	$(I =) \int \sec^2\left(\frac{\pi}{2}\sqrt{1+x}\right)\mathrm{d}x = \frac{8}{\pi^2}\int u\sec^2 u\,\mathrm{d}u$	Full method of substitution. Limits not required. d*u* or integral symbol may be implied by integration.	**M1 ;** **A1**
	$\int u\sec^2 u\,\mathrm{d}u = u\tan u - \int \tan u\,\mathrm{d}u$	Attempts integration by parts on $\int u \sec^2 u\,\mathrm{d}u$ in the correct direction (allow constant multiples of each term) **;** correct.	**M1 ;** **A1**
	$\int \tan u\,\mathrm{d}u = \ln\lvert\sec u\rvert$ or $= -\ln\lvert\cos u\rvert$	Integral of tan *u* seen separately or as part of working. Modulus may be missing.	**M1**
	$\int u\sec^2 u\,\mathrm{d}u = u\tan u - \ln\lvert\sec u\rvert$ o.e.	Fully correct integral for $\int u\sec^2 u\,\mathrm{d}u$ Modulus may be missing. The two elements may be seen at separate points in their work.	**A1**
	$x = \frac{5}{4} \Rightarrow u = \frac{3\pi}{4}$, $x = \frac{21}{4} \Rightarrow u = \frac{5\pi}{4}$ $\Rightarrow \int_{u_1}^{u_2} \ldots\mathrm{d}u = \left(\mathrm{f}(u_2) - \mathrm{f}(u_1)\right)$	Correct new limits applied correctly to their integral or returns to *x* correctly for their integral and applies the given limits.	**B1**
	$\left([I]_{5/4}^{21/4} = \frac{8}{\pi^2}\right)\left[\left(\frac{5\pi}{4}\tan\frac{5\pi}{4} - \ln\left\lvert\sec\frac{5\pi}{4}\right\rvert\right) - \left(\frac{3\pi}{4}\tan\frac{3\pi}{4} - \ln\left\lvert\sec\frac{3\pi}{4}\right\rvert\right)\right]$ $= \left(\frac{8}{\pi^2}\right)\left[\left(\frac{5\pi}{4} - \ln\left\lvert -\sqrt{2}\right\rvert\right) - \left(\frac{3\pi}{4}(-1) - \ln\left\lvert -\sqrt{2}\right\rvert\right)\right] = k$		**dddM1**
	Evaluates trig terms correctly and simplifies to single term *k*. If their limits are incorrect, they must have negative values so that the modulus signs are required – it cannot simplify the problem. Must be working in radians in their integration. Using e.g. incorrect sec(5π/4) = √2 simplifies the problem and does not score this mark. Must deal with the ln(negative) terms **correctly**. May be using moduli or bringing a power of 2 inside the ln terms. Note they may combine ln terms and realise that ln1 = 0		
	$= \frac{16}{\pi}$	cso	**A1**
			(10)
			Total 10 marks

Note: if substitution $u = \sqrt{1+x}$ or similar is used instead of the given one, then all the marks may be awarded as the work is equivalent.

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="4">2(a)(i)</td><td rowspan="4"></td><td>Correct “W” type shape, symmetric about y axis, entirely above x-axis. Curvature should be correct in each section but condone “almost straight” lines in all but the inverted U shape.. Do not be concerned about cusps.</td><td>B1</td></tr>
<tr><td>Inverted U (condone inverted V) turning point on y-axis at $y = 4$. Coordinates not required.</td><td>B1</td></tr>
<tr><td>Correct x intercepts. Coordinates not required. Condone additional x intercepts.</td><td>B1</td></tr>
<tr><td></td><td>(3)</td></tr>
<tr><td rowspan="5">(ii)</td><td rowspan="5"></td><td>Correct shape, symmetric with portions in correct places. Curvature should be correct in each section but condone “almost straight” lines in all but the U shape. Do not be concerned about cusps.</td><td>B1</td></tr>
<tr><td>Correct y intercept at a U (condone V) minimum point on the y axis. Coordinates not required.</td><td>B1</td></tr>
<tr><td>Correct x intercepts. Coordinates not required. Condone additional x intercepts.</td><td>B1</td></tr>
<tr><td>Coordinates of the other two vertices correct and clearly labelled on the diagram. May be seen using –2, 2 and 5 clearly marked on the axes.</td><td>B1</td></tr>
<tr><td></td><td>(4)</td></tr>
<tr><td rowspan="5">(b)</td><td>$-\left(x^2-4\right)+5=\frac{1}{2}x^2 \to x=...$
or $\left(x^2-4\right)-5=\frac{1}{2}x^2 \to x=...$</td><td>Attempts to solve one of the relevant quadratic equations using the correct section of the graph without modulus.
May be seen as $9=\frac{3}{2}x^2$ or $9=\frac{1}{2}x^2$</td><td>M1</td></tr>
<tr><td>Two of $x=\pm\sqrt{6}$ and/or $x=\pm 3\sqrt{2}$</td><td>Any two correct answers.
Accept $\sqrt{18}$ for $3\sqrt{2}$ for this mark.</td><td>A1</td></tr>
<tr><td>$-\left(x^2-4\right)+5=\frac{1}{2}x^2 \to x=...$
and $\left(x^2-4\right)-5=\frac{1}{2}x^2 \to x=...$</td><td>Attempts to solve both of the relevant quadratic equations using the correct section of the graph without modulus.</td><td>dM1
(A1 on EPEN)</td></tr>
<tr><td>$x=\pm\sqrt{6}$ and $x=\pm 3\sqrt{2}$</td><td>All four correct answers and no others.
Must now have $3\sqrt{2}$ and not just $\sqrt{18}$
Do not ISW so e.g. if they discard any of the correct roots, or use an inequality, they score A0.</td><td>A1</td></tr>
<tr><td></td><td></td><td>(4)</td></tr>
<tr><td colspan="4" align="right">Total 11 marks</td></tr>
</table>

Question	Scheme		Marks
3	$\arcsin 2x = \arccos x - \frac{\pi}{6}$		
	$\Rightarrow 2x = \sin(\arccos x)\cos\frac{\pi}{6} - \cos(\arccos x)\sin\frac{\pi}{6}$ **or** $\Rightarrow x = \cos(\arcsin 2x)\cos\frac{\pi}{6} - \sin(\arcsin 2x)\sin\frac{\pi}{6}$	Takes either sin or cos of both sides and attempts correct compound angle formula condoning sign errors. Other arrangements may be possible.	**M1**
	(1) $2x = \sqrt{1-x^2}\frac{\sqrt{3}}{2} - \frac{x}{2}$ **or** (2) $x = \frac{\sqrt{3}}{2}\sqrt{1-(2x)^2} - \frac{2x}{2}$ Other equations are possible if starting with e.g. $\frac{\pi}{6} = \arccos x - \arcsin 2x$ then they may get (3) $\frac{1}{2} = \sqrt{1-x^2}\sqrt{1-(2x)^2} - 2x^2$ **or** (4) $\frac{\sqrt{3}}{2} = x\sqrt{1-(2x)^2} + 2x\sqrt{1-x^2}$	Inverses sin and arcsin or cos and arccos uses Pythagoras to achieve an equation without trig terms.	**M1**
		Any correct equation in x only with trig terms removed.	**A1**
	e.g. $4x + x = \sqrt{3}\sqrt{1-x^2}$ $\Rightarrow (25x^2 = 3 - 3x^2 \Rightarrow)\ x = ...$	Full method to find a real value for x. Minor slips in solving the equation are condoned, but they should not materially simplify the equation.	**ddM1**
	$x = \frac{\sqrt{21}}{14}$	cao	**A1**
			(5)
			(Total 5 marks)

Note all equations give the same solution but less efficient routes take a bit more algebra, e.g.

(3) $\left(2x^2 + \frac{1}{2}\right)^2 = (1-x^2)(1-4x^2) \Rightarrow 4x^4 + 2x^2 + \frac{1}{4} = 1 - 5x^2 + 4x^4 \Rightarrow 7x^2 = \frac{3}{4}$ etc or even

(4) $\left(\sqrt{3} - 2x\sqrt{1-4x^2}\right)^2 = 16x^2(1-x^2) \Rightarrow 3 - 4x\sqrt{3}\sqrt{1-4x^2} + 4x^2(1-4x^2) = 16x^2(1-x^2)$

$\Rightarrow (3 - 12x^2)^2 = 48x^2(1-4x^2) \Rightarrow 9 - 72x^2 + 144x^4 = 48x^2 - 192x^4 \Rightarrow 112x^4 - 40x^2 + 3 = 0 \Rightarrow x^2 = ...$

However, this generates extra solutions which would need to be rejected.

(5) $\arcsin\left(\frac{5}{\sqrt{3}}x\right) = \arccos(x) \Rightarrow \left(\frac{5}{\sqrt{3}}x\right)^2 + x^2 = 1 \Rightarrow x^2 = ...$

Question	Scheme		Marks
3 Alt	Let e.g. $y = \arcsin 2x = \arccos x - \frac{\pi}{6}$		
	$\Rightarrow x = \cos\left(y + \frac{\pi}{6}\right) = \cos y \cos\frac{\pi}{6} - \sin y \sin\frac{\pi}{6}$	Sets y = arccos$x - \frac{\pi}{6}$, takes cos of both sides and attempts compound angle formula.	**M1**
	$\left(x = \frac{1}{2}\sin y\right)$ so	Sets y = arcsin2x, takes sin of both sides and eliminates x to create an equation in y only with trig terms evaluated.	**M1**
	$\Rightarrow \frac{1}{2}\sin y = \frac{\sqrt{3}}{2}\cos y - \frac{1}{2}\sin y$	Any correct equation in y only, with trig terms evaluated.	**A1**
	e.g. $\tan y = \frac{\sqrt{3}}{2} \rightarrow \sin y = \frac{\sqrt{3}}{\sqrt{7}}$ $x = \frac{1}{2}\sin y = \frac{\sqrt{3}}{2\sqrt{7}}$	Full method to find a real value for x. Minor slips in solving the equation are condoned, but they should not materially simplify the equation.	**ddM1**
	$x = \frac{\sqrt{21}}{14}$	cao	**A1**
			(5)
			(Total 5 marks)

Question	Scheme		Marks
4(a)	Suppose there is such a rational number a and integer b such that $\sqrt{5}=a+b\sqrt{7}$	Sets up the proof. Need not be as formal. Allow b to be rational but not a to be an integer.	**B1**
	Then $5=a^2+2ab\sqrt{7}+7b^2$ **Alt 1:** $b\sqrt{7}=\sqrt{5}-a \Rightarrow 7b^2=5-2a\sqrt{5}+a^2$	Squares the equation. Condone a slip e.g. missing square but not a missing term.	**M1**
	If a = 0 then 5 = $7b^2$, so e.g. $b^2 \neq 1$, and as 5 is prime this cannot happen or e.g. $b^2 < 1$ so b is not an integer. If b = 0 then $\sqrt{5}=a$, which is rational, a contradiction. (b = 0 not required in Alt 1)	a = 0 and b = 0 dealt with at some point. Condone $b=\dfrac{\sqrt{5}}{\sqrt{7}}$ is not an integer. (S+)	**B1**
	So, (a,$b \neq 0$) and we can write e.g. $\sqrt{7}=\dfrac{5-a^2-7b^2}{2ab}$ **or** $\sqrt{5}=\dfrac{5+a^2-7b^2}{2a}$ **or** $2ab\sqrt{7}=5-a^2-7b^2$ **or** $2a\sqrt{5}=5+a^2-7b^2$	Makes e.g. $\sqrt{7}$ or $2ab\sqrt{7}$ the subject of the equation (see below). In Alt 1, makes $\sqrt{5}$ or $2a\sqrt{5}$ the subject. Allow instead an argument that 5, a^2 and $7b^2$ are rational etc. directly from $5=a^2+2ab\sqrt{7}+7b^2$	**M1**
	e.g. But $\dfrac{5-a^2-7b^2}{2ab}$ is rational as a and b are rational, which contradicts that $\sqrt{7}$ is irrational. Hence there are no such numbers (or other conclusion).	Correct work and conclusion drawn, referencing contradiction. A0 if there are incorrect statements in the body of their work e.g. "$2ab$ is an integer".	**A1**
	Note: The final A is possible if the second B is not scored (zeros cases omitted) but the first B must have been scored, so 11011 is possible. (4/5 is likely)		
			(5)
(b)	Suppose $\sqrt{5}+\sqrt{7}=a$ (where a is rational), then $\sqrt{5}=a+(-1)\sqrt{7}$	Sets up and rearranges an appropriate equation using the result of (a). e.g. b = –1 so $\sqrt{5}--\sqrt{7}=a$	**M1**
	Since a is rational and b is an integer, by e.g. the result of (a) this is a contradiction, hence $\sqrt{5}+\sqrt{7}$ is irrational.	Uses the result of (a) to draw the conclusion including all aspects of the main scheme.	**A1**
			(2)
			(Total 7 + 1 marks)
	Award S1 for: • a fully correct solution that is succinct and includes the S+ point.		**S1**
			(1)
S+ Good explanations of zero condition(s).			

Notes:
They may write $a = p/q$ where p, q are (coprime) integers (and q is non-zero) which should not affect the way the proof works.

They may write $2ab\sqrt{7} = 5 - a^2 - 7b^2$ for second M1. They may then proceed as e.g.

The RHS is rational. Since ab is rational (either a or b is 0 or) $\sqrt{7}$ is rational → contradiction for A1. Then deal with the zero cases as in the main scheme for B1.

If they reach $\sqrt{35}$ (from $a = \frac{-b}{\sqrt{5}\sqrt{7}} \to a^2 = 5 + 7b - 2b\sqrt{35}$ – which scores first M1) they must prove that $\sqrt{35}$ is irrational for the second M, alongside a similar rational/irrational argument as above. There is no need to prove e.g. sum of rational and irrational numbers is irrational.

Alternative phrasing in (b):
Let $b = -1$ then rearrange to reach $a = \sqrt{5} + \sqrt{7}$. Since a is irrational if b is an integer then $\sqrt{5} + \sqrt{7}$ is irrational.

Proof that $\sqrt{35}$ is rational
Assume that $\sqrt{35}$ is rational, so that $\sqrt{35} = \frac{p}{q}$ for p, q coprime integers (or in simplest form) and q non-zero.

Then $35q^2 = p^2$, hence p^2 is a multiple of 35 (or 5 and 7). Hence p is a multiple of 35 (or 5 and 7). So, we can write $p = 35n$, for integer n, so that $35q^2 = (35n)^2 \Rightarrow q^2 = 35n^2$, hence q^2 is a multiple of 35 (or 5 and 7). Hence q is a multiple of 35 (or 5 and 7). So, we can write $q = 35m$ which means that we have a contradiction since p and q are not coprime. Hence $\sqrt{35}$ is irrational.
(Or, then $\sqrt{35} = \frac{p}{q} = \frac{35n}{35m} = \frac{n}{m}$ which means that we have a contradiction since $\frac{p}{q}$ is not in simplest form.)

Question	Scheme		Marks
5(a)	e.g. $\frac{\csc^2 x}{10}+a+\frac{\cot^2 x}{5}+\frac{\sec^2 x}{10}+\frac{\tan^2 x}{5}=1$	States or implies sum of probabilities is 1.	**B1**
	$\frac{1}{10\sin^2 x}+a+\frac{\cos^2 x}{5\sin^2 x}+\frac{1}{10\cos^2 x}+\frac{\sin^2 x}{5\cos^2 x}=1$ or e.g. $\csc^2 x+10a+2\csc^2 x-2+\sec^2 x+2\sec^2 x-2=10$	Uses trig identities to create an equation in two trig functions only. Condone the usual sign errors.	**M1**
	$\frac{\cos^2 x+2\cos^4 x+\sin^2 x+2\sin^4 x}{10\sin^2 x\cos^2 x}=1-a$ or e.g. $\frac{\cos^2 x+\sin^2 x}{\sin^2 x\cos^2 x}=\frac{14-10a}{3}$	Combines the trig terms (which must now be sin*x* and cos*x*) to a single fraction or multiplies through. May have also replaced $4\sin^2 x\cos^2 x$ with $\sin^2 2x$	**M1**
	$1+2\left(\cos^2 x+\sin^2 x\right)\left(\cos^2 x-2\sin^2 x\cos^2 x+\sin^2 x\right)=\frac{5}{2}(1-a)\sin^2 2x$ $\Rightarrow 1+2\left(1-2\sin^2 x\cos^2 x\right)=\frac{5}{2}(1-a)\sin^2 2x$ or e.g. $\frac{1}{\sin^2 x\cos^2 x}=\frac{14-10a}{3}$		**M1**
	Replaces the quartic terms with quadratic terms using a valid method and replaces any $\sin^2 x+\cos^2 x$ with 1 to achieve an equation in $\sin^2 x\cos^2 x$ only (or mixed $\sin^2 x\cos^2 x$ and $\sin^2 2x$). Note: $\sin^4 x+\cos^4 x=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$		
	$1+2-\sin^2 2x=\frac{5}{2}(1-a)\sin^2 2x$	Use double angle formula to get to an equation in $\sin^2 2x$ only. Dependent on the previous M mark.	**dM1**
	$6=\left(2+5(1-a)\right)\sin^2 2x$	Gathers the $\sin^2 2x$ (or the $\sin^2 x\cos^2 x$) terms on one side and factorises or e.g. $14-10a=\frac{12}{\sin^2 2x}$	**M1**
	$\Rightarrow \sin^2 2x=\frac{6}{7-5a}$ *	Completes correctly. Condone occasional notational slips such as missing arguments.	**A1***
			(7)
(b)	$\sin^2 2x \leqslant 1 \Rightarrow$ e.g. $6 \leqslant 7-5a$	Uses the maximum value of $\sin^2 2x$ to form an inequality/equation in *a* which may come from e.g. max value of *a* is when $\sin 2x=\pm 1$	**M1**
	$\Rightarrow (0 \leqslant)\ a \leqslant \frac{1}{5}$	Correct condition or maximum value for *a*. The lower limit is not required. Condone $a=1/5$ but not $a<1/5$ A0 if another interval given e.g. $a>13/5$	**A1**
			(2)
(c)	Possibilities for $X_1\times X_2=16$ are 4^2 or 2×8 or 8×2	Correct possible ways to form 16 identified or implied. Must include both ways round and 4^2 only once.	**B1**

	Scheme	Notes	Marks
	$P\left(X_1 X_2 = 16\right) = a^2 + 2\times\frac{1}{10}\text{cosec}^2 x \times \frac{1}{10}\sec^2 x$	Attempts the probability as the sum of $(P(X=4))^2$ and $2\times P(X=2)\times P(X=8)$, but allow if the "2×" is missing or if $(P(X=4))^2$ is doubled.	**M1**
	$= a^2 + \frac{1}{50(\sin x\cos x)^2} = a^2 + \frac{2}{25\sin^2 2x}$	Writes a probability in terms of $\sin^2 2x$ (or equivalent if working in reverse) so the a^2 may be missing **;** Correct expression for the **sum** of the probabilities which may be unsimplified.	**M1;** **A1**
	$\Rightarrow a - \frac{1}{25} = a^2 + \frac{2(7-5a)}{25\times 6}$	Uses the given probability and the given answer to part (a) to get an equation in a only which may be missing the a^2	**M1**
	$\Rightarrow 75a^2 - 80a + 10 = 0$ o.e.	Correct quadratic equation e.g. $15a^2 - 16a + 2 = 0$ (=0 may be implied).	**A1**
	$\Rightarrow a = \frac{16 \pm \sqrt{16^2 - 4\times 15\times 2}}{30} = \frac{8\pm\sqrt{34}}{15}$	Correct attempt to solve their 3TQ using the formula or by completing the square. If their equation will factorise, allow factorisation to be used. Dependent on all previous method marks.	**dddM1**
	$a \square \frac{1}{5}$ so $\Rightarrow a = \frac{8-\sqrt{34}}{15}$ only	Requires (minimal) justification for final mark (S+ for good reasoning) Condone occasional notational slips such as missing arguments or invisible brackets which are recovered.	**A1**
	Also $a - \frac{1}{25} > 0$ as $a = \frac{8-\sqrt{34}}{15} > \frac{8-6}{15} = \frac{2}{15}$		**S+**
			(8)
	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a fully correct solution that may be laboured but includes an S+ point • a succinct solution that scores 15+ marks that includes at least one S+ point.		**S2**
			(Total 17 + 2 marks)

S+ for good reasoning for rejecting larger root e.g., $\frac{8+\sqrt{34}}{15} > \frac{8}{15}$ $\left(\text{or } \frac{13}{15}\right) > \frac{1}{5}$ shown.

S+ for good reasoning for accepting the smaller root e.g. $\frac{8-\sqrt{34}}{15} < \frac{8-5}{15} = \frac{3}{15} = \frac{1}{5}$

or e.g. $\frac{8-\sqrt{34}}{15} > \frac{8-6}{15} = \frac{2}{15} > \frac{1}{25}$ so $a - \frac{1}{25} > 0$ $(\text{or} \square 0)$ hence valid.

Notes:

Condone use of $\sin^2 2x = 2\sin^2 x\cos^2 x$ and trig identities with sign errors for the M marks only.

Condone the use of e.g. $s^2 x$ or s^2 for all intermediate marks, provided $s^2 2x$ is clearly identified as different. For the final mark of part (a) they must write it in the form given.

If they do not sum the probabilities, they may score maximum 100101000.

Question	Scheme		Marks
6(a)	$\left\|\overrightarrow{AB}\right\| = \sqrt{14^2 + 2^2 + 5^2} = ...$ **and** $\left\|\overrightarrow{BC}\right\| = \sqrt{2^2 + 10^2 + 11^2} = ...$	Attempts at least the lengths or squared lengths of *AB* and *BC*.	**M1**
	$\left\|\overrightarrow{AB}\right\| = \sqrt{225} = 15$ and $\left\|\overrightarrow{BC}\right\| = \sqrt{225} = 15$ $\left\|\overrightarrow{AB}\right\| = \left\|\overrightarrow{BC}\right\|$ Hence $(ABC$ is$)$ isosceles.	Correct lengths and conclusion. May use squared lengths. Condone sign errors in finding the vectors. Ignore any attempt at $\left\|\overrightarrow{AC}\right\|$.	**A1***
			(2)
(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \frac{5}{6}\overrightarrow{AC} = ...$ or $\overrightarrow{OC} - \frac{1}{6}\overrightarrow{CA} = ...$	Correct method to find *D* (follow through their lengths). May use $\left\|\lambda(-12\mathbf{i} + 12\mathbf{j} + 6\mathbf{k})\right\| = "15"$ condone poor squaring.	**M1**
	$\overrightarrow{OD} = 15\mathbf{j} + 9\mathbf{k}$	Correct position vector. ISW after correct position vector throughout the question.	**A1**
			(2)
(c)	Let the midpoint be *M* then $\overrightarrow{OM} = 0.5\left(\overrightarrow{OD} + \overrightarrow{OB}\right)$ or e.g. $\overrightarrow{OM} = \overrightarrow{OB} + 0.5\left(\overrightarrow{OD} - \overrightarrow{OB}\right)$	Method for midpoint.	**M1**
	$= -2\mathbf{i} + 11\mathbf{j} + 4\mathbf{k}$	Correct position vector.	**A1**
			(2)
(d)	e.g. $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = "-2\mathbf{i} + 11\mathbf{j} + 4\mathbf{k}" - (10\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$ or e.g. $\overrightarrow{AM} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BD} = "-14\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}" + ("2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}")$ **;** $\Rightarrow \overrightarrow{AM} = -12\mathbf{i} + 6\mathbf{j}$	Attempts $\pm\overrightarrow{AM}$ with their *M* **;** $\pm\overrightarrow{AM}$ correct	**M1 ;** **A1 (M1 on EPEN)**
	(Triangle *ABD* is isosceles, so *X* lies on the line *AM*, so is given by) $\overrightarrow{OX} = \overrightarrow{OA} + m\overrightarrow{AM}$	Forms an equation for $\overrightarrow{OX}$ using $\overrightarrow{AM}$ that is not just the given answer (unless there is sufficient justification).	**M1**
	$\overrightarrow{OX} = 10\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + m(-12\mathbf{i} + 6\mathbf{j})$ Letting $p = 6m$ (S+) gives $\overrightarrow{OX} = 10\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + p(-2\mathbf{i} + \mathbf{j})$*	Achieves correct answer including $\overrightarrow{OX} =$ following a correct intermediate equation for $\overrightarrow{OX}$ (may be implied if they show clearly $\overrightarrow{OX} = \overrightarrow{OA} + m\overrightarrow{AM}$ and deal with the change in direction vector), not just the answer given. (S+ for good explanation of *p*)	**A1***
			(4)
(e)	Midpoint *AC* is *N* $4\mathbf{i} + 11\mathbf{j} + 7\mathbf{k}$ **or** direction of angle bisector at *B* is $\left(\overrightarrow{BN} =\right) \frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{BC}\right) = 8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$	Correct midpoint of *AC* **or** direction vector for bisector.	**B1**
	(Since point *X* is on angle bisector *ABC* it	Forms a second vector equation in terms of *q* using a	**M1 ;**

Question	Scheme	Notes	Marks
	satisfies) $\overrightarrow{OX} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k} + q(8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$ or e.g. $\overrightarrow{OX} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k} + q(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\overrightarrow{OX} = 4\mathbf{i} + 11\mathbf{j} + 7\mathbf{k} + q(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	direction vector found using an acceptable method **;** Correct equation including $\overrightarrow{OX}$ = (only penalise absence of $\overrightarrow{OX}$ once)	**A1**
			(3)
(f)	Comparing **k** components gives $4 = -1 + 8q$ so $q = \frac{5}{8}$ (or e.g. $4 = -1 + 2q$ so $q = \frac{5}{2}$)	Sets the **k** component from their $\overrightarrow{OX}$ from (e) equal to 4 and solves for q **;** For the A1, follow through for their second equation provided the direction vector is a scalar multiple of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ (they may have e.g. $q = \frac{5}{2}$ or e.g. $q = -\frac{3}{2}$ if using N)	**M1 ;** **A1**
			(2)
(g)(i)	So, position vector for centre is $\overrightarrow{OX} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k} + \frac{5}{8}(8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$	Attempts the coordinates using their q in their $\overrightarrow{OX}$. (May find using $p = \frac{9}{2}$ and the given $\overrightarrow{OX}$ in part (d))	**M1**
	$= \mathbf{i} + \frac{19}{2}\mathbf{j} + 4\mathbf{k}$	Correct centre.	**A1**
			(2)
(ii)	e.g. Midpoint, N, AC on circle so need $\overrightarrow{XN}$ ... **Alt:** N is midpoint of AC, then $AN = 9$ and $\lvert\overrightarrow{AX}\rvert = \ldots$ (or $\lvert\overrightarrow{CX}\rvert = \ldots$)	Strategy that can be used to solve the problem – e.g. Identifies or implies a correct point on the circle and uses it in an attempt to find the radius, or uses a right triangle with one side length deduced. There are alternatives.	**M1**
	$\overrightarrow{XN} = (4\mathbf{i} + 11\mathbf{j} + 7\mathbf{k}) - \left(\mathbf{i} + \frac{19}{2}\mathbf{j} + 4\mathbf{k}\right) = 3\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$ **or** $AN = 9$ and $\lvert\overrightarrow{AX}\rvert = \sqrt{81 + \frac{81}{4} + 0} = \frac{9\sqrt{5}}{2}$	Correct vector for radius or correct lengths of both required sides (or the squares of the side lengths).	**A1**
	$\lvert\overrightarrow{XN}\rvert = \sqrt{9 + \frac{9}{4} + 9} = \ldots$ **or** $r^2 = AX^2 - AN^2 = \frac{5}{4} \times 81 - 81 = \ldots$	Completes the method. Dependent on the previous method mark.	**dM1**
	$r = \frac{9}{2}$	Correct answer.	**A1**
			(4)
	Award S2 for a solution scoring 20+ marks that is succinct and includes at least two S+ points (see notes on the next page).		**S2**

	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 18+ marks that may be laboured but includes at least two S+ points • a succinct solution that scores 18+ marks that includes at least one S+ point.	
		(Total 21+2 marks)

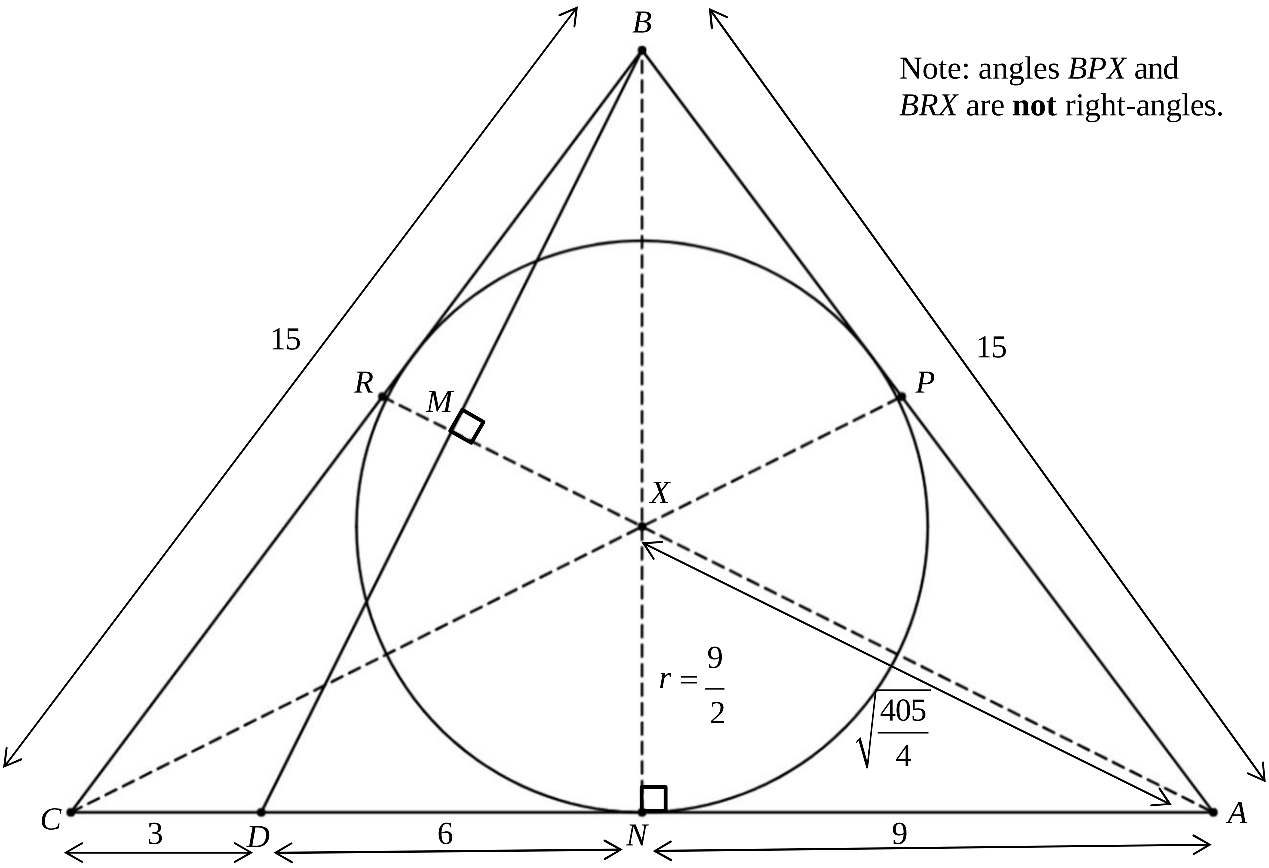
Notes: Only penalise use of coordinates instead of a position vector on the first occurrence.

Allow different vector notation throughout. Do not condone e.g. $-2\mathbf{i}\ \ 11\mathbf{j}\ \ 4\mathbf{k}$ embedded in a column vector for the A mark but only penalise for the first occurrence.

If the addition/subtraction of two vectors is not shown then allow this to be implied by two correct components.

S+ for good vector notation used (e.g. consistency, arrows used correctly).
S+ for use of suitable sketches to investigate the problem.
S+ marks for good explanations for changing to *p* in part (d).

Useful Diagram and Vectors:



$\overrightarrow{OA} = 10\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$ $\quad$ $\overrightarrow{OB} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k}$ $\quad$ $\overrightarrow{OC} = -2\mathbf{i} + 17\mathbf{j} + 10\mathbf{k}$

$\overrightarrow{AB} = -14\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ $\quad$ $\overrightarrow{AC} = -12\mathbf{i} + 12\mathbf{j} + 6\mathbf{k}$ $\quad$ $\overrightarrow{BC} = 2\mathbf{i} + 10\mathbf{j} + 11\mathbf{k}$

$\overrightarrow{OD} = 15\mathbf{j} + 9\mathbf{k}$ $\quad$ $\overrightarrow{BD} = 4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$

$\overrightarrow{BM} = 2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ $\quad$ $\overrightarrow{OM} = -2\mathbf{i} + 11\mathbf{j} + 4\mathbf{k}$ $\quad$ $\overrightarrow{AM} = -12\mathbf{i} + 6\mathbf{j}$

$\overrightarrow{AN} = -6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ $\quad$ $\overrightarrow{ON} = 4\mathbf{i} + 11\mathbf{j} + 7\mathbf{k}$ $\quad$ $\overrightarrow{BN} = 8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$

(d) $\overrightarrow{OX} = 10\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} + p\left(-2\mathbf{i} + \mathbf{j}\right)$

(e) $\overrightarrow{OX} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k} + q\left(8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}\right)$ or e.g. $\overrightarrow{OX} = -4\mathbf{i} + 7\mathbf{j} - \mathbf{k} + q\left(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right)$

(g)(i) $\overrightarrow{OX} = \mathbf{i} + \frac{19}{2}\mathbf{j} + 4\mathbf{k}$ $\quad$ (g)(ii) $\overrightarrow{AX} = -9\mathbf{i} + \frac{9}{2}\mathbf{j}$ $\quad$ $\overrightarrow{XN} = 3\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$

Question	Scheme		Marks
7(a)	$\ln u = x\ln y$	Takes ln and applies power law correctly.	**B1**
	$\dfrac{1}{u}\dfrac{\mathrm{d}u}{\mathrm{d}x} = \ln y + \dfrac{x}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	Differentiates implicitly and attempts the product rule. Allow one slip for M1.	**M1** **A1**
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = u\left(\ln y + \dfrac{x}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$	Makes $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ the subject.	**M1**
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = y^x\left(\ln y + \dfrac{x}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ o.e.	Correct in terms of y and x. Any equivalent correct expression seen in (a) or at the start of (b).	**A1**
			(5)
(b)	$x^3\ln y \to kx^2\ln y \pm x^3 \dfrac{\mathrm{d}y}{\mathrm{d}x}$	Differentiates the $x^3\ln y$ implicitly with the product rule attempted. Allow sign slips due to the subtraction.	**M1**
	$x^3\ln y \to 3x^2\ln y + \dfrac{x^3}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	Correct differentiation. May have divided by 3 which is acceptable.	**A1**
	So $3y^x\left(\ln y + \dfrac{x}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) - 3x^2\ln y - \dfrac{x^3}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	Uses (a) and makes dy/dx the subject. Must be 2 terms in dy/dx	**M1**
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3y\left(x^2 - y^x\right)\ln y}{3xy^x - x^3}$	Any equivalent correct expression. Condone fractions embedded within the fraction.	**A1**
			(4)
(c)	Note if roles of numerator and denominator are mixed up (so finds "a" from numerator etc) then penalise the first B mark in each part but allow any other marks to be earned.		
	(a is where tangents are vertical, so need) $3xy^x - x^3 = 0$	Sets denominator of their dy/dx = 0.	**B1**
	(So as $x \neq 0$ S+), so $y^x = \dfrac{x^2}{3}$ and so $x^2 - x^3\ln y = 0$	Identifies y^x or $3y^x$ and attempts to solve with original equation $3y^x - x^3\ln y = 0$ (S+ explanation x non-zero)	**M1**
	Also $x\ln y = \ln\dfrac{x^2}{3} \Rightarrow x^2 - x^2\ln\dfrac{x^2}{3} = 0$	Eliminates the ln y using correct log work to get an equation in x only. **;** Correct equation.	**M1 ;** **A1**
	$x \neq 0 \Rightarrow \ln\dfrac{x^2}{3} = 1 \Rightarrow \dfrac{x^2}{3} = \mathrm{e}$	Eliminates the ln from the equation. (S+ for justification of x non-zero if not earlier)	**M1**
	So $a = (\pm)\sqrt{3\mathrm{e}}$	Correct value, may be called x or a, ignore reference to ±.	**A1**
			(6)
(d)	(b, c are where tangents are horizontal, and) ($y \neq 1$, $\ln y \neq 0$ S+) so need $x^2 - y^x = 0$	Sets numerator of their dy/dx = 0. dy/dx must be an algebraic fraction for this mark. (S+ for explaining non-zeroes.)	**B1**

	So $y^x = x^2$ thus $3x^2 - x^3 \ln y = 0$	Identifies y^x and attempts to solve with original equation $3y^x - x^3 \ln y = 0$	**M1**
	Also $x \ln y = \ln x^2 \Rightarrow 3x^2 - x^2 \ln x^2 = 0$	Eliminates the ln y using correct log work to get an equation in x only.	**M1**
	$x \neq 0 \Rightarrow \ln x^2 = 3 \Rightarrow x^2 = \mathrm{e}^3 \left(\text{or} \Rightarrow x = \pm \mathrm{e}^{\frac{3}{2}}\right)$	Correct value for x or x^2 (S+)	**A1**
	$x \ln y = \ln x^2 = \ln \mathrm{e}^3 = 3 \Rightarrow y = \mathrm{e}^{3/x} \ (= \ldots)$	Full method to find an expression or value for y	**M1**
	$b = \mathrm{e}^{-3/\sqrt{\mathrm{e}^3}}$, $c = \mathrm{e}^{3/\sqrt{\mathrm{e}^3}}$ o.e. (e.g. $b = \mathrm{e}^{-3\mathrm{e}^{-\frac{3}{2}}}$, $c = \mathrm{e}^{3\mathrm{e}^{-\frac{3}{2}}}$)	One mark for either, unlabelled **;** second mark both correct, labelled or identified	**A1 ;** **A1**
			(7)
	Award S2 for a solution scoring 20+ marks that is succinct and includes at least two S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 20+ marks that may be laboured but includes an S+ point		**S2**
			(Total 22+2 marks)

Note: You will need to look carefully to see if their work is an attempt at (c) or (d).
Whether the M marks in parts (c) and (d) are available will depend on the complexity of their denominator and numerator respectively. They cannot score the relevant M marks if they are missing the terms that need to be dealt with: e.g. (d) M2 cannot be scored if they don't need to eliminate ln y because the equation is trivial.
If they have an extra factor / missing a factor in their answer to (b) but their work in (c) and/or (d) is otherwise correct, then allow full marks in both parts.
S+ good explanation(s) of non-zero values for cancelling factors in (c)
S+ good explanation(s) of non-zero values for cancelling factors in (d)
S+ for proceeding directly to x^2 in part (d).
S+ for elegant approaches to finding the y values for (d).

Alt for (b) using $u = y^x$

M1: Sets $3u - x^2 \ln u = 0$ and differentiate implicitly with respect to x using the chain and product rule to the form $\lambda \frac{\mathrm{d}u}{\mathrm{d}x} \pm \mu x \ln u \pm \gamma \frac{x^2}{u}\frac{\mathrm{d}u}{\mathrm{d}x} = 0$ o.e.

A1: Correct e.g. $3\frac{\mathrm{d}u}{\mathrm{d}x} - 2x \ln u - \frac{x^2}{u}\frac{\mathrm{d}u}{\mathrm{d}x} = 0$ o.e.

M1: Substitutes their $\frac{\mathrm{d}u}{\mathrm{d}x}$ from (a) and makes $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. Requires two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms.

A1: Correct e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y \ln y^x}{3y^x - x^2} - \frac{y}{x}\ln y$ o.e.