Mark Scheme (Results)

Summer 2024

Pearson Advanced Extension Award
In Mathematics (9811)
Paper 01

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General Marking Guidance

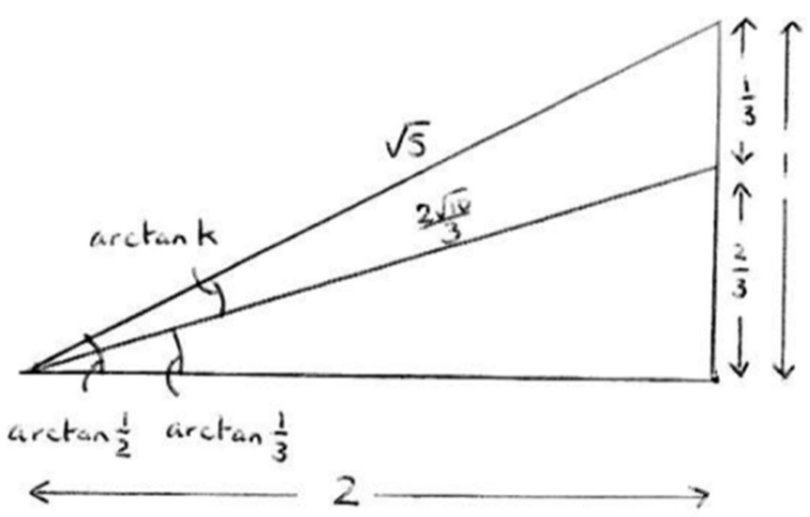
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question	Scheme		Marks	AOs
1	$(1-8x)^p = (1+) - 8px + \frac{p(p-1)}{2}(-8x)^2 + \frac{p(p-1)(p-2)}{6}(-8x)^3 + \ldots$	Correct numerical coefficients of relevant terms, may be seen in an expansion or listed separately. Need not be simplified.	**B1**	1
	$\Rightarrow -8p + \frac{64p(p-1)}{2} = -\frac{8^3 p(p-1)(p-2)}{6}$	Equates the relevant coefficients. If the x's are left in and never recovers just coefficients, M0.	**M1**	2
	$\Rightarrow -3 + 12(p-1) = -32(p^2 - 3p + 2)$	Cancels or factors to achieve a quadratic in p (need not be expanded).	**dM1**	1
	$\Rightarrow 32p^2 - 84p + 49 = 0$	Correct expanded and gathered quadratic.	**A1**	2
	$\Rightarrow (4p-7)(8p-7) = 0 \Rightarrow p = \ldots$	Solves their 3 term quadratic	**ddM1**	1
	Need $p(p-1) > 0$ so ($p < 0$ or) $p > 1$	Correct consideration of the x^2 coefficient to disambiguate (a selection must be made). Must have had at least two roots to choose from.	**M1**	3
	May be by substitution to see which gives a positive, or consideration of ranges. Note $p > 0$ stated is M0. If they have incorrect roots allow for due consideration given to attempt to select appropriate root(s).			
	$p = \frac{7}{4}$	Correct answer only.	**A1**	2
			(7)	
			(Total 7 marks)	

Question	Scheme		Marks	AOs
2(a)	Distance from curve to O is given by $D^2=(x-0)^2+(y-0)^2=x^2+y^2=4-3y+6\sin y$	Correct formula. Allow with $D^2=\ldots$ or $x^2+y^2=\ldots$	**B1**	1
			(1)	
(b)	Need to minimise: $\dfrac{\mathrm{d}}{\mathrm{d}y}D^2\left(=-3+6\cos y\right)=0$ Or e.g. $\dfrac{\mathrm{d}D}{\mathrm{d}y}\left(=\dfrac{1}{2}\dfrac{-3+6\cos y}{\sqrt{4-3y+6\sin y}}\right)=0$	Realises they need to minimise and applies a correct process to do so. The differentiation need not be correct for this mark	**M1**	3
	$\Rightarrow \cos y=\dfrac{1}{2}\Rightarrow y=\pm\dfrac{\pi}{3}$	At least one of $y=\pm\dfrac{\pi}{3}$ from a correct derivative.	**A1**	1
	$\Rightarrow D^2=4-3\left(\pm\dfrac{\pi}{3}\right)+6\sin\left(\pm\dfrac{\pi}{3}\right)=\ldots$	Proceeds to find a value for D or D^2 for either root of form $k\pi\in(-5,2),k\neq 0$.	**dM1**	2
	Note: If the derivative was not fully correct, allow the final two marks below for work from a correct numerator set to zero, ie $-3+6\cos y=0$			
	From graph, minimum is clearly in lower half plane. (Alt: $\dfrac{\mathrm{d}^2}{\mathrm{d}y^2}D^2=-6\sin y>0$ for the negative root only.) Hence $D^2=4+\pi-6\dfrac{\sqrt{3}}{2}$	Selects correct solution for the minimum with minimal reason, getting at least as far as D^2. Reason may be comparison of relative values and choosing smaller (accept if both found and correct one clearly indicated as answer).	**A1**	3
	So minimum distance is $D=\sqrt{4+\pi-3\sqrt{3}}$	Correct simplified value only for minimum distance. (Allow if no reason given.)	**A1**	2
			(5)	
			(Total 6 marks)	

Question	Scheme		Marks	AOs
Note: mark the mathematics throughout, do not penalise minor notation indiscretions if the intention is clear throughout.				
3(i)	$k=\tan\left(\arctan\frac{1}{2}-\arctan\frac{1}{3}\right)$	Takes tangents of both sides.	**M1**	2
	$=\dfrac{\tan\arctan\frac{1}{2}-\tan\arctan\frac{1}{3}}{1+\tan\arctan\frac{1}{2}\tan\arctan\frac{1}{3}}$	Applies the correct compound angle formula – signs must be correct.	**M1**	1
	$\dfrac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{2}\frac{1}{3}}=\dfrac{\frac{3-2}{6}}{\frac{6+1}{6}}=\frac{1}{7}$	Simplifies to the correct fraction	**A1**	2
			(3)	
3(i) Alt	Geometric Approach: where $\theta=\arctan k$ $\cos\theta=\dfrac{\sqrt{5}^2+\left(\frac{2\sqrt{10}}{3}\right)^2-\left(\frac{1}{3}\right)^2}{2\sqrt{5}\times\frac{2\sqrt{10}}{3}}=..\left(=\frac{7\sqrt{2}}{10}\right)$	Identifies suitable triangles and applies the cosine rule to find $\cos\theta$	**M1**	2
	$\Rightarrow k=\tan\theta=\dfrac{\sqrt{1-\left(7\sqrt{2}/10\right)^2}}{7\sqrt{2}/10}=...\left(\frac{\sqrt{2}}{7\sqrt{2}}=\frac{1}{7}\right)$	Proceeds to use Pythagoras to find $k=\tan\theta$	**M1**	1
	$\Rightarrow k=\frac{1}{7}$	Simplifies to the correct fraction.	**A1**	2
			(3)	

Helpful Diagram for the geometric approach:



Question	Scheme		Marks	AOs
3(ii)(a)	$(\cos 3A =)\cos(2A+A) = \cos 2A\cos A - \sin 2A\sin A$	Applies the correct compound angle formula	**M1**	1
	$= (2\cos^2 A - 1)\cos A - 2\sin A\cos A\sin A$	Uses correct double angles formulae to reach an expression in single angle only.	**M1**	2
	$= 2\cos^3 A - \cos A - 2(1-\cos^2 A)\cos A$ $= 4\cos^3 A - 3\cos A$ *	Eliminates sin A and completes proof correctly.	**A1***	2
			(3)	
(ii)(b)	$[\cos 40° = \cos(2\times 20°) =]\, 2a^2 - 1$ Alt: $[\cos 40° = \cos(60° - 20°) =]\, \frac{1}{2}a + \frac{\sqrt{3}}{2}\sqrt{1-a^2}$	Correct expression	**B1**	1
(ii)(c)	$\cos 80° = \cos(2\times 40°) = 2(2a^2-1)^2 - 1$ Alt: $\cos 80° = \cos(60° + 20°) = \frac{1}{2}a \pm \frac{\sqrt{3}}{2}\sin 20°$	Applies the double angle formula for cos a second time with their (b)	**M1**	1
	$= 8a^4 - 8a^2 + 1$ Alt $\frac{1}{2}a - \frac{\sqrt{3}}{2}\sqrt{1-a^2}$	Simplified to correct expression.	**A1**	2
			(3)	
(ii)(d)	$(A = 20° \Rightarrow)\, 4a^3 - 3a = \cos 60° = \frac{1}{2}$ *	Uses A = 20º and achieves the result.	**B1***	1
			(1)	
(ii)(e)	$\cos 20°\cos 40°\cos 80° = a(2a^2-1)(8a^4-8a^2+1)$ $(= 16a^7 - 24a^5 + 10a^3 - a)$	Uses the expressions from (b) and (c) to form an expression in a only provided their expressions are not just multiples of a.	**M1**	1
	$= (2a^3 - a)(6a^2 + a - 8a^2 + 1)$ Or e.g. $(4a^3-3a)^2 = 16a^6 - 24a^4 + 9a^2$ $\Rightarrow .. = a\left((4a^3-3a)^2 + a^2 - 1\right)$	Uses the expression from (d) to reduce power at least once or attempts to use (d) to compare. Allow if attempted on a partial expression.	**M1**	3
	$= \left(\frac{1}{2}\left(3a + \frac{1}{2}\right) - a\right)(a - 2a^2 + 1)$ $= \frac{1}{4}(2a+1)(a - 2a^2 + 1)$ $= \frac{1}{4}(-4a^3 + 3a + 1)$	Continues to reduce powers of a to get to a cubic provided their expressions in (b) and (c) were correct up to sign error. Other steps are possible.	**M1**	2

	$=\frac{1}{4}\left(1-\frac{1}{2}\right)=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$ *	Correct completion of proof.	**A1**	2
			(4)	
			Total 14 marks	
(ii)(e) Alt	$\cos 20°\cos 40°\cos 80° = \frac{1}{2}\cos 20°\left(\cos 120° + \cos 40°\right)$	Applies appropriate product to sum formula	**M1**	1
	$=\frac{1}{2}a\left(-\frac{1}{2}+2a^2-1\right)$	Replaces cos120º correctly and uses result of (b) to get expression in *a*.	**M1**	2
	$=\frac{1}{4}a\left(4a^2-3\right)$ $=\frac{1}{4}\left(4a^3-3a\right)$	Combines terms and identifies the expression from (d) in their expression.	**M1**	3
	$=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$ *	Correct completion of proof.	**A1**	2
			(4)	

Other alternatives are possible but will be uncommon.

(ii)(a) by De Moivre: M1: $(\cos 3A + \mathrm{i}\sin 3A) = \left(\cos A + \mathrm{i}\sin A\right)^3 = c^3(+3\mathrm{i}c^2 s) + 3\mathrm{i}^2 cs^2(+\mathrm{i}^3 s^3)$

M1: $\cos 3A = \mathrm{Re}\left(\cos 3A + \mathrm{i}\sin 3A\right) = \mathrm{Re}\left(\cos A + \mathrm{i}\sin A\right)^3 = \cos^3 A - 3\cos A\sin^2 A$

A1: As scheme.

(ii)(e) M1 for setting up suitable equation using their results as scheme, M1 beginning the process of working towards the answer, M1: continuing the process to a point from which the answer is deducible, A1: Fully correct proof. E.g.

$$\cos 20°\cos 40°\cos 80° = a\left(\frac{1}{2}a - \frac{\sqrt{3}}{2}\sqrt{1-a^2}\right)\left(\frac{1}{2}a + \frac{\sqrt{3}}{2}\sqrt{1-a^2}\right) \quad \text{(M1)}$$

$$= a\left(\frac{1}{4}a^2 - \frac{3}{4}\left(1-a^2\right)\right) \quad \text{(M1)}$$

$$= \frac{1}{4}\left(4a^3 - 3a\right) \quad \text{(M1)} \quad = \frac{1}{8} \quad \text{A1*}$$

Question	Scheme		Marks	AOs
4(a)	$x=\sqrt{3}\tan u \Rightarrow \frac{dx}{du}=\sqrt{3}\sec^2 u$	Any correct derivative expression for the substitution.	B1	1
	$\int \frac{1}{3+x^2}dx=\int \frac{1}{3+3\tan^2 u}\times\sqrt{3}\sec^2 u\,du$	Complete substitution for u made (including replacing dx)	M1	2
	$=\frac{\sqrt{3}}{3}\int \frac{\sec^2 u}{(1+\tan^2 u)}du=\frac{\sqrt{3}}{3}\int \frac{\sec^2 u}{\sec^2 u}du$ $=\frac{\sqrt{3}}{3}\int 1\,du=\frac{\sqrt{3}}{3}u\ (+c)$	Applies Pythagorean identity and simplifies and integrates (allow without $+c$)	M1	2
	$x=\sqrt{3}\tan u \Rightarrow u=\arctan\frac{x}{\sqrt{3}}$	Correct u in terms of x	B1	2
	$\int \frac{1}{3+x^2}dx=\frac{\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}x\right)+c$	Correct answer (including +c) from fully correct working. (Allow with $\frac{1}{\sqrt{3}}$) Allow $\tan^{-1}$ notation.	A1	1
			(5)	
(b)	$x=\frac{3u+3}{u-3}\Rightarrow\frac{dx}{du}=\frac{3(u-3)-(3u+3)\times1}{(u-3)^2}=\frac{-12}{(u-3)^2}$ $\left(\text{or } x=3+\frac{12}{u-3}\Rightarrow...\right)$	Attempts quotient rule or other appropriate method ; correct derivative, suitably simplified numerator.	M1 A1	1 1
	$x=-3\Rightarrow u=1,\ x=1\Rightarrow u=-3$	Correct change of limits seen (S+)	B1	2
	$\int\frac{\ln(3-x)}{3+x^2}dx=\int\frac{\ln\left(3-\frac{3u+3}{u-3}\right)}{3+\left(\frac{3u+3}{u-3}\right)^2}\times\frac{-12}{(u-3)^2}du$	Complete substitution, including replacing dx, condone small slips e.g with signs, but terms should be correctly placed.	M1	2
	$=-12\int\frac{\ln\left(\frac{12}{3-u}\right)}{3(u-3)^2+(3u+3)^2}du=-12\int\frac{\ln12-\ln(3-u)}{12u^2+36}du$	Simplifies to achieve a Quadratic denominator and splits the log term – but $\ln(-12)$ is M0.	M1	3
	$I=\int_{-3}^{1}\frac{\ln(3-x)}{3+x^2}dx=-\int_{1}^{-3}\frac{\ln12-\ln(3-u)}{u^2+3}du$	Fully correct with logs split, u^2+3 clear in denominator and correct limits attached	A1	3
	$=\int_{-3}^{1}\frac{\ln12-\ln(3-u)}{u^2+3}du$	Correct process of changing order of limits and negating seen at some stage of working. (May be on just the I integral.) (S+)	M1	3
	$=\int_{-3}^{1}\frac{\ln12}{u^2+3}du-I\Rightarrow 2I=\ln12\left[\frac{\sqrt{3}}{3}\arctan\frac{u\sqrt{3}}{3}\right]_{-3}^{1}$	Recognises I in the integral, makes the subject and applies their result of (a)	M1	3
	$\Rightarrow I=\frac{\sqrt{3}\ln12}{6}\left(\arctan\frac{\sqrt{3}}{3}-\arctan\left(-\sqrt{3}\right)\right)$	Applies limits. Depends on previous M.	dM1	3

	$= \frac{\sqrt{3}\ln 12}{6}\left(\frac{\pi}{6}+\frac{\pi}{3}\right) = \frac{\pi\sqrt{3}\ln 12}{12}$	Correct answer.	**A1**	2
			(10)	
	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 13+ marks that may be laboured but includes an S+ point		**S1**	2
			(1)	
			Total 15+1 marks	

S+ For clear demonstration of the change of limits.
S+ For good explanation/demonstration of reversal of limits.

Question	Scheme		Marks	AOs
5(a)	$\overrightarrow{AB} = k(\mathbf{a}+\mathbf{e})$	Recognises **a**+**e** is parallel to $\overrightarrow{AB}$	**B1**	2
	$\lvert\mathbf{a}+\mathbf{e}\rvert = 2\lvert\mathbf{a}\rvert\cos 30° = \lvert\mathbf{a}\rvert\sqrt{3}$, $\lvert\overrightarrow{AB}\rvert = 3\lvert\mathbf{a}\rvert \Rightarrow k = ...$	Correct method for comparing modulus/working out length of **a** + **e**	**M1**	3
	$\Rightarrow \overrightarrow{AB} = \sqrt{3}(\mathbf{a}+\mathbf{e})$	Correct answer (oe)	**A1**	3
			(3)	
(b)	$\overrightarrow{OD} = \overrightarrow{OE} + \overrightarrow{ED} = \overrightarrow{OE} + \overrightarrow{AB} = ...$	A correct strategy to find $\overrightarrow{OD}$	**M1**	1
	$\overrightarrow{OD} = \mathbf{e} + \sqrt{3}(\mathbf{a}+\mathbf{e}) = \sqrt{3}\mathbf{a} + \left(\sqrt{3}+1\right)\mathbf{e}$	Either form acceptable (oe).	**A1**	1
			(2)	
(c)	$\overrightarrow{RC} = \overrightarrow{RB} + \overrightarrow{BC} = \frac{2}{3}\overrightarrow{AB} + \overrightarrow{OE} = ...$	For any correct strategy to find $\overrightarrow{RC}$	**M1**	3
	$= \frac{2}{3}\sqrt{3}(\mathbf{a}+\mathbf{e}) + \mathbf{e} = \frac{2}{3}\sqrt{3}\mathbf{a} + \left(\frac{2}{3}\sqrt{3}+1\right)\mathbf{e}$	A correct expression	**A1**	3
			(2)	
(d)	$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AR} + p\overrightarrow{RC}$ $= \mathbf{a} + \frac{\sqrt{3}}{3}(\mathbf{a}+\mathbf{e}) + \frac{2p}{3}\sqrt{3}\mathbf{a} + p\left(\frac{2}{3}\sqrt{3}+1\right)\mathbf{e}$	For any complete, valid method to obtain $\overrightarrow{OX}$ in terms **a**, **e** and a scalar using the collinearity of *R*, *C* and *X*.	**M1** **A1**	3 3
	$\overrightarrow{OX} = q\overrightarrow{OD} = q\sqrt{3}\mathbf{a} + q\left(\sqrt{3}+1\right)\mathbf{e}$	For a second correct expression for $\overrightarrow{OX}$ in terms of **a**, **e** and a scalar using their $\overrightarrow{OD}$	**B1ft**	1
	Note: allow the first three marks if the same variable is used for both equations, but the next M will not be available unless they recover to two variables.			
	$\Rightarrow \begin{cases} 1 + \frac{\sqrt{3}}{3} + \frac{2p\sqrt{3}}{3} = q\sqrt{3} \\ \frac{\sqrt{3}}{3} + \frac{2p}{3}\sqrt{3} + p = q\left(\sqrt{3}+1\right) \end{cases}$ $(\Rightarrow p-1=q)$ $\Rightarrow p = ...$ or $q = ...$	Equates components of the two equations and solves. (S+)	**M1**	3
	$p = \sqrt{3}+4$ or $q = \sqrt{3}+3$	A correct value for either of the two unknowns that were used.	**A1**	3
	$\overrightarrow{OX} = 3\left(\sqrt{3}+1\right)\mathbf{a} + 2\left(2\sqrt{3}+3\right)\mathbf{e}$	Correct answer. Accept equivalent simplified in correct form.	**A1**	3
			(6)	
	Award S2 for a solution scoring 13 marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points		**S2**	2

	• a solution scoring 11+ marks that may be laboured but includes an S+ point			
				(Total 13+2 marks)

Note:
S+ for consistent, correct vector notation used throughout the answer. May use underlining or over arrows, but vectors should be easily distinguishable from scalars.
S+ for elegant approaches to finding the relevant vectors for individual parts.
S+ for efficient solution to the simultaneous equations.

Note use of **i** and **j** vector components may be used but would not be a succinct solution.

Alternative approaches using components in **i** and **j** directions may be seen. They will need to return to **a** and **e** to access the marks in parts worth 2 marks.

Approach via gradients may be seen in (d).
M1: Attempting equation of line $\overrightarrow{RX}$
A1: Correct equation of line $\overrightarrow{RX}$
B1: Correct equation of line $\overrightarrow{OX}$
M1: Solve to find coordinates of X
A1: Correct coordinates with respect to their system.
A1: Correct final expression.

E.g. Using M as midpoint of AE and $\overrightarrow{OM} = \mathbf{i}$, $\overrightarrow{MA} = \mathbf{j}$

M1: $\overrightarrow{RC} = \left(\frac{4\sqrt{3}}{3}+1\right)\mathbf{i}-\mathbf{j}$ so $m_{\overrightarrow{RC}} = \frac{-3}{3+4\sqrt{3}}$, and $\overrightarrow{OR} = \left(\frac{2\sqrt{3}}{3}+1\right)\mathbf{i}+\mathbf{j}$ giving equation

$$y-"1"="\frac{-3}{3+4\sqrt{3}}"\left(x-"\left(1+\frac{2\sqrt{3}}{3}\right)"\right)$$

A1: $y-1=\frac{-3}{3+4\sqrt{3}}\left(x-\left(1+\frac{2\sqrt{3}}{3}\right)\right)$

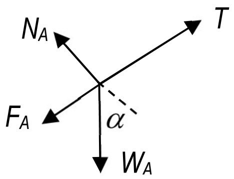
B1: $\overrightarrow{OD} = \left(2\sqrt{3}+1\right)\mathbf{i}-\mathbf{j}$ so $m_{\overrightarrow{OD}} = \frac{-1}{1+2\sqrt{3}} \Rightarrow y = \frac{-1}{1+2\sqrt{3}}x$

M1: $\frac{-1}{1+2\sqrt{3}}x = 1+\frac{-3}{3+4\sqrt{3}}\left(x-\left(1+\frac{2\sqrt{3}}{3}\right)\right) \Rightarrow x = \ldots, y = \ldots$ (lots of work!)

A1: $x = 7\sqrt{3}+9, y = -3-\sqrt{3}$

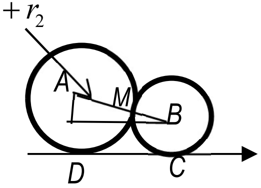
A1: $\overrightarrow{OX}\left[=\left(7\sqrt{3}+9\right)\mathbf{i}-\left(3+\sqrt{3}\right)\mathbf{j}=\left(7\sqrt{3}+9\right)\frac{\mathbf{a}+\mathbf{e}}{2}-\left(3+\sqrt{3}\right)\frac{\mathbf{a}-\mathbf{e}}{2}\right]=\left(3+3\sqrt{3}\right)\mathbf{a}+\left(6+4\sqrt{3}\right)\mathbf{e}$

But other choices of coordinate system are possible!

Question	Scheme		Marks	AOs
6(a)	In equilibrium $\Rightarrow 8mg = 2\times T\cos\gamma$	Attempts to equate upward and downward forces for C (allow if sin used) (must include both forces).	**M1**	3
	$\tan\gamma = \dfrac{3}{4} \Rightarrow \cos\gamma = \dfrac{4}{5} \Rightarrow T = ...$	Uses $\tan\gamma = \dfrac{3}{4}$ to find $\cos\gamma$ (or $\sin\gamma$) (may be implied) and proceeds to obtain an expression for T	**dM1**	2
	$\Rightarrow T = 5mg$		**A1**	1
			(3)	
(b)	 (S+) where W_A is the weight of the block, N_A is the normal reaction, T is the tension in the string and F_A is the friction between the slope and block.	At least forces N, T and W drawn correctly.	**M1**	3
		All four drawn correctly. Must have correct direction.	**A1**	3
		Forces labelled/defined (angle not required).	**A1 (S+)**	2
	Accept for labels letters N (or R), T F, W or the expression in mg, e.g. $4mg$ for weight, their $5mg$ for T and $4mg\cos\alpha$ for N. S+ for formal definitions. Allow M1A0A1 if W split into components (and labelled). F must be directed down the plane for first A. Final A0 if friction is labelled μR. Accept rotated diagrams. Note: if (b) is given in text mark the diagram drawn, or if no (b) is given score for work on Figure 3.			
			(3)	
(c)	Note: If B is considered first, swap the order of A and B in this part (ie first 5 marks can be awarded for the equivalent work on B, next three for A). Note: Use of $F = ma$ can score second and third Ms but will require $a = 0$ to be used to recover further marks.			
	For block A: $\begin{cases} \nearrow & T = F_A + 4mg\sin\alpha \\ \nwarrow & N_A = 4mg\cos\alpha \end{cases}$	Resolves forces for one of the blocks (likely block A, block B has 5 in place of 4), both directions. Allow if sin and cos are mixed up.	**M1**	3
	Alt: $\begin{cases} \rightarrow & T\cos\alpha = F_A\cos\alpha + N_A\sin\alpha \\ \uparrow & N_A\cos\alpha + T\sin\alpha = 4mg + F_A\sin\alpha \end{cases}$	Correct equations with T or follow through their (a). May evaluate the trig terms	**A1ft**	1
	$\Rightarrow F_A = "5mg" - 4mg\sin\alpha$ Alt: $F\left(\sin^2\alpha + \cos^2\alpha\right) + 4mg\sin\alpha = T\left(\cos^2\alpha + \sin^2\alpha\right)$	Uses (a) and finds expression for F_A (or μN or μ) (needs to eliminate N in the alt)	**M1**	2
	Max friction is μN_A $\Rightarrow 5mg - 4mg\sin\alpha \leqslant \mu\times 4mg\cos\alpha$	Considers maximal friction. May use equality or inequality. May have been introduced earlier.	**M1**	3
	$\Rightarrow \mu \geqslant \dfrac{5}{4\cos\alpha} - \tan\alpha = \dfrac{5}{4}\times\dfrac{13}{12} - \dfrac{5}{12} = \dfrac{45}{48} = \dfrac{15}{16}$	Correct bounding value for μ for block A. (oe)	**A1**	3

	For block B by similar reasoning we obtain $\Rightarrow \mu \geqslant \frac{5}{5\cos\beta} - \tan\beta = \frac{25}{\underline{24}} - \frac{7}{24} = \frac{18}{24} = \frac{3}{4}$	Uses symmetry or restarts on block B to find the bounding value for block B Correct value for cos β **used** in their equation. Correct value found.	**M1** **(S+)** **<u>B1</u>** **A1**	2 1 3
	Since one block is on point of moving but system in equilibrium, need only one friction at max, so larger of the two.	Correct reason (S+)	**M1**	2
	$\Rightarrow \mu = \frac{15}{16}$	Must be simplified.	**A1**	2
			(10)	
	Alt for last 5 marks by verification.			
	Check if B is sliding for A at maximum friction. We have For block B: $\begin{cases} \nwarrow & T = F_B + 5mg\sin\beta \\ \nearrow & N_B = 5mg\cos\beta \end{cases}$ $\Rightarrow F_B = "5mg" - 5mg\sin\beta,\ \mu N_B = "\frac{15}{16}" \times 5mg\cos\beta$	Considers block B and proceeds by symmetry or restarting to find the friction force required for equilibrium and μN.	**M1**	2
	$F_B = "5mg" - 5mg \times \frac{7}{25},\ \mu N_B = "\frac{15}{16}"5mg \times \frac{24}{25}$	Correct trig ratios used.	**B1**	1
	$F_B = \frac{18}{5}mg,\ \mu N_B = \frac{9}{2}mg$	Correct values for F and μN	**A1**	3
	Since $\frac{18}{5}\left(< \frac{20}{5} = 4\right) < \frac{9}{2}$ maximum friction is not needed for block B hence it is in equilibrium, so not sliding, thus A is the block on the point of sliding.	Correct reason (S+)	**M1**	2
	$\Rightarrow \mu = \frac{15}{16}$	Must be simplified.	**A1**	2
	Award S2 for a solution scoring 14+ marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 13+ marks that may be laboured but includes multiple S+ points • A succinct solution that scores 13+ marks that includes at least one S+ point		**S2**	2
				(Total 16+2 marks)

S+ For good diagram/definitions/labelling.
S+ For good reasoning using the similarity of the situation (ie not restarting completely for second block).
S+ Good explanation of the inequality/reason for choice of μ or other good reasoning.
S+ Formal definitions of the forces given.

Question	Scheme		Marks	AOs
7(a)	r_1+r_2  Distance between centres is r_1+r_2 So $d^2=(r_1+r_2)^2-(r_1-r_2)^2$	Recognition of the distance between centres as sum of radii and uses Pythagoras on appropriate triangle.	**M1 (S+)**	1
	$\Rightarrow d^2=r_1^2+2r_1r_2+r_2^2-r_1^2+2r_1r_2-r_2^2=4r_1r_2$ *	Expands and completes proof.	**A1***	2
			(2)	
(b)	Let the angle subtended at A be θ. Then $\cos\theta=\dfrac{adj}{hyp}=\dfrac{r_1-r_2}{r_1+r_2}$	Correctly shows cosine of the appropriate angle - may use another appropriate angle, so accept equivalents.	**B1**	2
	The area of sectors is $\dfrac{1}{2}r_1^2\theta+\dfrac{1}{2}r_2^2(\pi-\theta)$	Sets suitable angle and attempts area of both sectors. (Allow with different angles for M.) Correct areas, separate or added.	**M1**	1
	Trapezium has area $\dfrac{1}{2}(r_1+r_2)d=(r_1+r_2)\sqrt{r_1r_2}$	Attempts the area of the trapezium using (a). May use rectangle and triangle.	**M1**	1
	Area bounded is given by Area of trapezium $ABCD$ − sectors AMD and BMC	For a correct overall strategy to find the area evidenced.	**M1**	3
	$=(r_1+r_2)\sqrt{r_1r_2}-\dfrac{1}{2}(r_1^2-r_2^2)\theta-\dfrac{1}{2}\pi r_2^2$ *	Achieves correct expression. Depends on all three M's but not the B.	**A1***	2
			(5)	
(c)(i)	$d^2=4r_2r_3\Rightarrow\left(2\times\dfrac{1}{\sqrt{3}}\right)^2=4\times1\times r_3\Rightarrow r_3=\ldots$ Alt: $\dfrac{2\sqrt{r_2r_3}}{2\sqrt{r_1r_2}}=\dfrac{1}{\sqrt{3}}\Rightarrow r_3=\ldots$	Uses the result of (a) and the second term of the GS for distances to form and solve an equation for r_3	**M1**	3
	$r_3=\dfrac{1}{3}$	Correct value	**A1**	3
(ii)	Horizontal distance between centres of C_{m+1} and C_m is $d=2\left(\dfrac{1}{\sqrt{3}}\right)^{m-1}$ Alt: $\dfrac{2\sqrt{r_{n+2}r_{n+1}}}{2\sqrt{r_{n+1}r_n}}=\dfrac{1}{\sqrt{3}}$	Attempts the distance between centres of two general circles using the G.S. Allow if power out by 1. Allow M's for equivalent work if indexing is out by 1. Alt: uses ratio of successive terms of the GS of distances.	**M1**	1
	$r_{n+2}=\dfrac{"d^2"}{4r_{n+1}}=\dfrac{4/3^n}{4r_{n+1}}$ Alt: $\Rightarrow\dfrac{r_{n+2}}{r_n}=\dfrac{1}{3}$	Uses result of (a) with their distance to find an expression for r_{n+2} in terms of r_{n+1} Alt: Squares and cancels	**M1**	3

	$= \dfrac{1}{3^n\left(\dfrac{4/3^{n-1}}{4r_n}\right)} = \ldots;\ = \dfrac{r_n}{3}$ Alt: $\Rightarrow r_{n+2} = \dfrac{1}{3} r_n$	Repeats process to find expression in terms of r_n ; Correct answer. Alt: Rearranges to get r_{n+2} in terms of r_n	**M1;** **A1**	3 2
(iii)	The r_{2n-1} form a GS with a=1, $r = \dfrac{1}{3}$, so $r_{2n-1} = \left(\dfrac{1}{3}\right)^{n-1}$	Recognises alternating terms form a G.S and identifies the closed form.	**M1**	3
	So $r_{2n} = \dfrac{4/3^{2n-2}}{4r_{2n-1}} = \dfrac{1}{3^{2n-2}} \times 3^{n-1};\ = \dfrac{1}{3^{n-1}} = r_{2n-1}$ *	Substitutes into the formula and find r_{2n} ; completes proof.	**M1;** **A1**	3 2
	Alt: The r_{2n} also form a GS with $a = r_2 = 1$, $r = \dfrac{1}{3}$, so $r_{2n} = \left(\dfrac{1}{3}\right)^{n-1}$ hence $r_{2n} = r_{2n-1}$	Reasons the other alternating terms also form a series ; correct reason and conclusion.		
			(9)	
(d)	For R_{2n-1} radii of adjacent circles are equal $\left(=3^{1-n}\right)$ and for R_{2n} radii of adjacent circles differ by ratio $\dfrac{1}{3}$ $\left(3^{1-n} \to \dfrac{3^{1-n}}{3} = 3^{-n}\right)$	Considers the odd and even cases for R_n and identifies the same ratio of radii. May be implicit in working. May be deduced by considering first few terms.	**M1** **(S+)**	3
	(For "odd" case $\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$ and) For "even" case $\cos\theta \left(= \dfrac{3^{1-n} - 3^{-n}}{3^{1-n} + 3^{-n}} = \dfrac{3-1}{3+1}\right) = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$	Correct value for $\cos\theta$ or θ deduced for the even alternate regions. Allow from the $n = 1$ case.	**B1**	2
	Area $R_{2n-1} = \left(3^{1-n} + 3^{1-n}\right)\sqrt{3^{2(1-n)}} - 0 - \dfrac{1}{2}\pi 3^{2(1-n)}$ Area $R_{2n} = \left(3^{1-n} + \dfrac{1}{3}3^{1-n}\right)\sqrt{\dfrac{1}{3}3^{2(1-n)}} - \dfrac{1}{2}\left(3^{2(1-n)} - \dfrac{1}{9}3^{2(1-n)}\right)\theta - \dfrac{1}{2}\pi\dfrac{3^{2(1-n)}}{3^2}$		**M1**	3
	Attempts the general expression for the two types of area, either separately or as a sum of adjacent pairs. May have θ or their value or an arccos expression. Alternatively, works out the first term for each GS and extrapolates the series. Allow with $\dfrac{1}{3}$ or $\dfrac{1}{\sqrt{3}}$ for the ratio.			
	$\left(\text{Area } R_{2n-1} = \dfrac{9}{9^n}\left(2 - \dfrac{1}{2}\pi\right) \text{ and Area } R_{2n} = \dfrac{9}{9^n}\left(\dfrac{4}{3}\sqrt{\dfrac{1}{3}} - \dfrac{4\pi}{27} - \dfrac{1}{18}\pi\right)\right)$			

	So area is given by sum to infinity of GS $\frac{1}{1-\frac{1}{9}}\times\ldots$	Recognises the sum to infinity of a G.S. is required and applies it with suitable ratio (allow 1/3) to at least one set of areas (or to both combined)	**M1**	3
	$\text{Area} = \frac{1}{1-9^{-1}}\left(2-\frac{1}{2}\pi+\frac{4}{3}\sqrt{\frac{1}{3}}-\frac{4\pi}{27}-\frac{1}{18}\pi\right)$	Full method, both sets of areas considered, or combined, with angle attempted and substituted, to reach an expression for the area.	**dM1**	3
	$\text{Area} = \frac{9}{8}\left(2+\frac{4\sqrt{3}}{9}-\frac{19}{27}\pi\right) = \frac{9}{4}+\frac{\sqrt{3}}{2}-\frac{19}{24}\pi$	Correct answer, accept simplified equivalents.	**A1**	3
			(6)	
	Award S2 for a solution scoring 20+ marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 18+ marks that may be laboured but includes at least 2 S+ points • A succinct solution that scores 18+ marks that includes at least one S+ point.		**S2**	2 2
				(Total 22+2 marks)

Notes:
S+ for use of suitable sketches to investigate the problem.
S+ marks for good explanations at the point indicated.
S+ for well communicated solutions, or efficient solutions identified.

c(iii) Alt Induction	Suppose $r_{2k} = r_{2k-1}$ for $k \geqslant 1$ then $\frac{4r_{2k+2}r_{2k+1}}{4r_{2k+1}r_{2k}} = \frac{1}{3} \Rightarrow \frac{r_{2k+2}r_{2k+1}}{r_{2k+1}r_{2k-1}} = \frac{1}{3}$	Sets up assumption statement and applies a suitable formula with the assumption used. There may be variations but must be a suitable expression used.	**M1**	3
	$\Rightarrow r_{2(k+1)} = \frac{1}{3}r_{2k-1} \Rightarrow r_{2k+2} = \frac{1}{3}\times 3r_{2k-1+2} = r_{2k+1}$	Completes the inductive step using the result from (c)(ii) (oe other appropriate steps)	**M1**	3
	So True for $n = k$ implies true for $n = k + 1$, and also true for $n = 1$ as $r_2 = 1 = r_1$, hence true for all n	Checks base case correctly and gives conclusion with inductive statement clear.	**A1**	2
			(3)	

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