Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Award

Tuesday 25 June 2024

Afternoon (Time: 3 hours)

Paper reference **9811/01**

Advanced Extension Award
Mathematics

You must have:
Mathematical Formulae and Statistical Tables
An insert for questions 5, 6 and 7 (enclosed)

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may not be used.**
- You must **show all your working.**
- Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}, \sqrt{2}, 3\sqrt{2}$.

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for **each** question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P72907A

F:1/1/1/

P 7 2 9 0 7 A 0 1 3 2

Pearson

1. In the binomial expansion of

$$(1-8x)^p \qquad\qquad |x| < \frac{1}{8}$$

where p is a positive constant,

- the sum of the coefficient of x and the coefficient of x^2 is equal to the coefficient of x^3
- the coefficient of x^2 is positive

Determine the value of p.

(7)

Question 1 continued

(Total for Question 1 is 7 marks)

2.

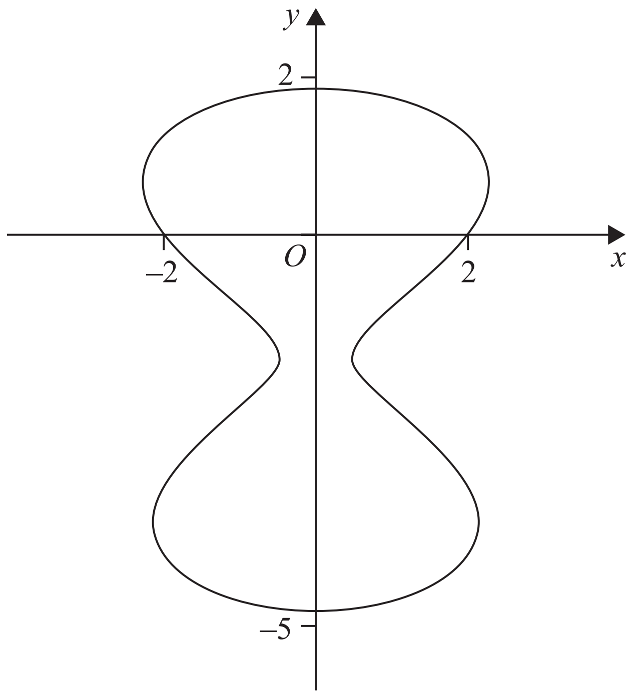

Figure 1

Figure 1 shows the curve defined by the equation

$$y^2 + 3y - 6\sin y = 4 - x^2$$

The point $P(x, y)$ lies on the curve.

The distance from the origin, O, to P is D.

(a) Write down an equation for D^2 in terms of y only.

(1)

(b) Hence determine the minimum value of D giving your answer in simplest form.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 7 2 9 0 7 A 0 4 3 2

Question 2 continued

(Total for Question 2 is 6 marks)

3. (i) Determine the value of k such that

$$\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan k$$

(3)

(ii) (a) Prove that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(3)

Given that $a = \cos 20°$

(b) write down, in terms of a, an expression for $\cos 40°$

(1)

(c) determine, in terms of a, a simplified expression for $\cos 80°$

(2)

(d) Use part (a) to show that

$$4a^3 - 3a = \frac{1}{2}$$

(1)

(e) Hence, or otherwise, show that

$$\cos 20° \cos 40° \cos 80° = \frac{1}{8}$$

(4)

Question 3 continued

Question 3 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 14 marks)

4. (a) Use the substitution $x = \sqrt{3}\tan u$ to show that

$$\int \frac{1}{3 + x^2}\,\mathrm{d}x = p\arctan\left(px\right) + c$$

where p is a real constant to be determined and c is an arbitrary constant.

(5)

(b) Use the substitution $x = \dfrac{3u + 3}{u - 3}$ to determine the exact value of I where

$$I = \int_{-3}^{1} \frac{\ln(3 - x)}{3 + x^2}\,\mathrm{d}x$$

giving your answer in simplest form.

(10)
(+S1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 16 marks)

P 7 2 9 0 7 A 0 1 3 3 2

5.

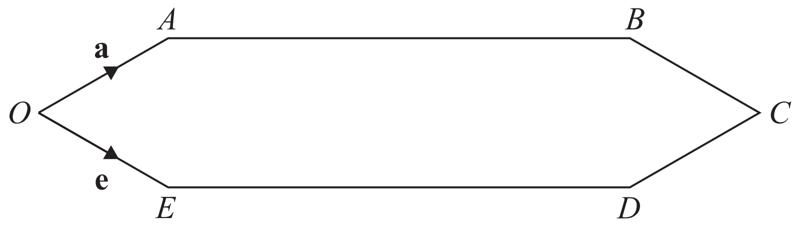

Figure 2

Figure 2 shows a sketch of a hexagon *OABCDE* where

- the interior angle at O and at C are each 60°
- the interior angle at each of the other vertices is 150°
- $OA = OE = BC = CD$
- $AB = ED = 3 \times OA$

Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OE} = \mathbf{e}$

determine as simplified expressions in terms of **a** and **e**

(a) $\overrightarrow{AB}$

(3)

(b) $\overrightarrow{OD}$

(2)

The point R divides AB internally in the ratio $1:2$

(c) Determine $\overrightarrow{RC}$ as a simplified expression in terms of **a** and **e**

(2)

The line through the points R and C meets the line through the points O and D at the point X.

(d) Determine $\overrightarrow{OX}$ in the form $\lambda\mathbf{a} + \mu\mathbf{e}$, where λ and μ are real values in simplest form.

(6)

(+S2)

Question 5 continued

Question 5 continued

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 15 marks)

6.

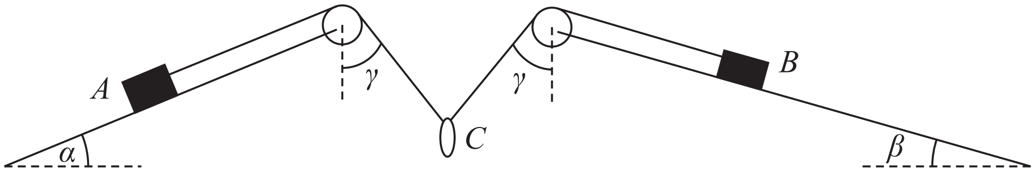

Figure 3

Figure 3 shows a block A with mass $4m$ and a block B with mass $5m$.
Block A is at rest on a rough plane inclined at an angle α to the horizontal.
Block B is at rest on a rough plane inclined at an angle β to the horizontal.

The blocks are connected by a light inextensible string which passes over a small smooth pulley at the top of each plane.

A small smooth ring C, of mass $8m$, is threaded on the string between the pulleys so that A, B and C all lie in the same vertical plane.

The part of the string between A and its pulley lies along a line of greatest slope of the plane of angle α.

The part of the string between B and its pulley lies along a line of greatest slope of the plane of angle β.

The angle between the vertical and the string between each pulley and the ring C is γ.

The two blocks, A and B, are modelled as particles.

Given that

- $\tan\alpha = \dfrac{5}{12}$ and $\tan\beta = \dfrac{7}{24}$ and $\tan\gamma = \dfrac{3}{4}$
- the coefficient of friction, μ, is the same between each block and its plane
- one of the blocks is on the point of sliding up its plane
- the tension in the string is T

(a) determine, in terms of m and g, an expression for T,

(3)

(b) draw a diagram showing the forces on block A, clearly labelling each of the forces acting on the block,

(3)

(c) determine the value of μ, giving a justification for your answer.

(10)
(+S2)

Question 6 continued

Question 6 continued

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 18 marks)

7.

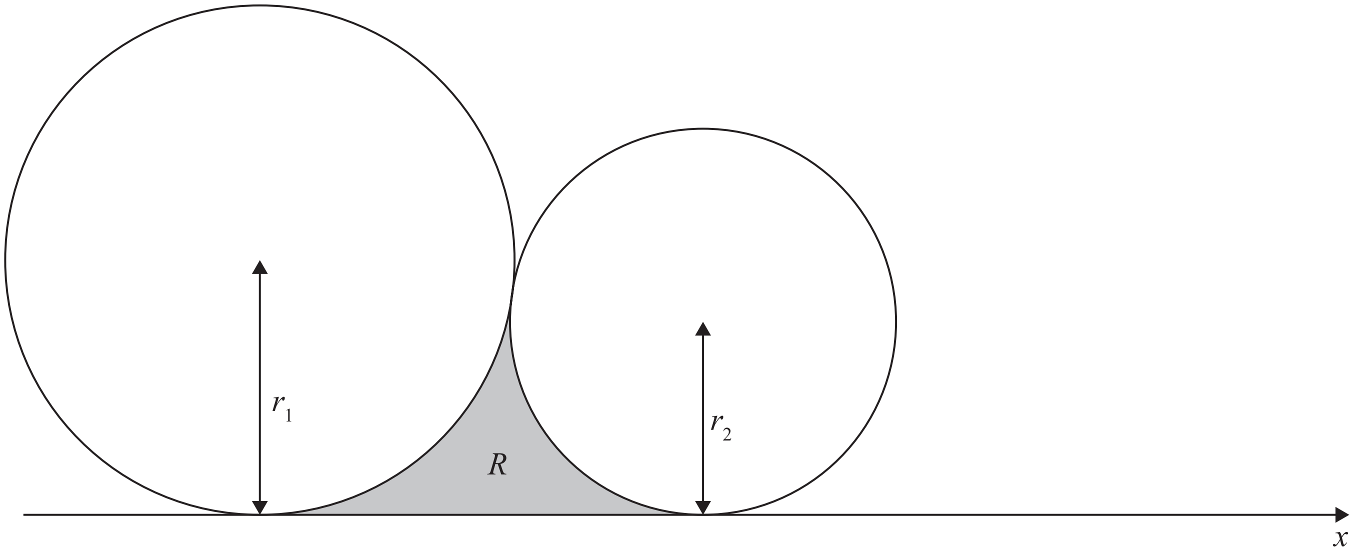

Figure 4

Figure 4 shows a circle with radius r_1 and a circle with radius r_2
The circles touch externally at a single point above the x-axis.
Both circles also have the x-axis as a tangent.

(a) Show that the horizontal distance between the centres of the circles, d, is given by

$$d^2 = 4r_1 r_2$$

(2)

The finite region R, shown shaded in Figure 4, is bounded by the x-axis and minor arcs of the two circles.

Given that $r_1 \geqslant r_2$

(b) show that the area of R is given by

$$(r_1 + r_2)\sqrt{r_1 r_2} - \frac{1}{2}\left(r_1^2 - r_2^2\right)\theta - \frac{1}{2}\pi r_2^2$$

where $\cos\theta = \dfrac{r_1 - r_2}{r_1 + r_2}$

(5)

Question 7 continues on the next page.

Question 7 continued

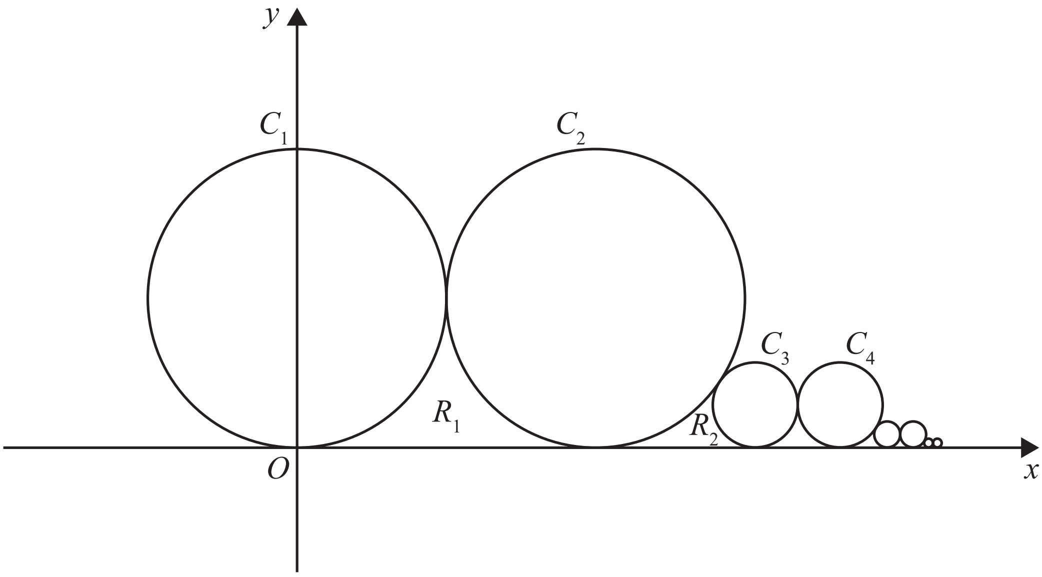

Figure 5

A sequence of circles, C_1, C_2, C_3, ... with radii r_1, r_2, r_3, ... respectively, is constructed such that

- each circle is tangential to and above the x-axis
- the first circle, C_1, has centre (0, 1)
- each successive circle touches the preceding one externally at a single point
- the horizontal distances between the centres of successive circles form a geometric sequence with first term 2 and common ratio $\frac{1}{\sqrt{3}}$

The first few circles in the sequence are shown in Figure 5.

(c) (i) Determine the value of r_3

(ii) Show that, for $n \geqslant 1$, $r_{n+2} = kr_n$ where k is a constant to be determined.

(iii) Hence show that, for $n \geqslant 1$, $r_{2n} = r_{2n-1}$

(9)

The region bounded between C_n, C_{n+1} and the x-axis is R_n

The total area, A, bounded above the x-axis and under all the circles is the sum of the areas of all these regions.

(d) Determine the value of A, giving the answer in simplest form.

(6)
(+S2)

P 7 2 9 0 7 A 0 2 7 3 2

Turn over

Question 7 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

(Total for Question 7 is 24 marks)

TOTAL FOR PAPER IS 100 MARKS

Pearson Edexcel Award

Tuesday 25 June 2024

Afternoon (Time: 3 hours)

Paper reference **9811/01**

Advanced Extension Award Mathematics

Insert for questions 5, 6 and 7

Do not write on this insert.

Turn over

P72907A

F:1/1/1/

P 7 2 9 0 7 A

Pearson

5.

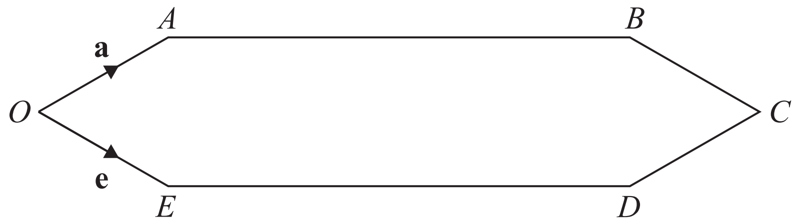

Figure 2

Figure 2 shows a sketch of a hexagon *OABCDE* where

- the interior angle at *O* and at *C* are each 60°
- the interior angle at each of the other vertices is 150°
- $OA = OE = BC = CD$
- $AB = ED = 3 \times OA$

Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OE} = \mathbf{e}$

determine as simplified expressions in terms of **a** and **e**

(a) $\overrightarrow{AB}$

(3)

(b) $\overrightarrow{OD}$

(2)

The point *R* divides *AB* internally in the ratio $1:2$

(c) Determine $\overrightarrow{RC}$ as a simplified expression in terms of **a** and **e**

(2)

The line through the points *R* and *C* meets the line through the points *O* and *D* at the point *X*.

(d) Determine $\overrightarrow{OX}$ in the form $\lambda\mathbf{a} + \mu\mathbf{e}$, where λ and μ are real values in simplest form.

(6)
(+S2)

(Total for Question 5 is 15 marks)

6.

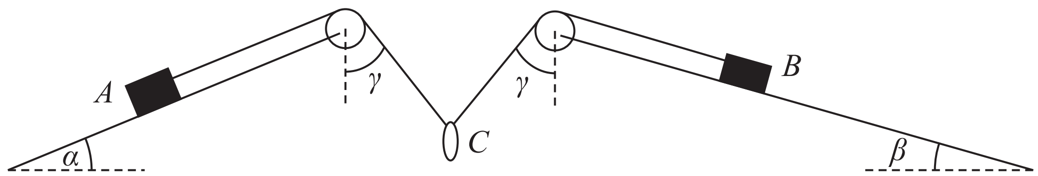

Figure 3

Figure 3 shows a block A with mass $4m$ and a block B with mass $5m$.
Block A is at rest on a rough plane inclined at an angle α to the horizontal.
Block B is at rest on a rough plane inclined at an angle β to the horizontal.

The blocks are connected by a light inextensible string which passes over a small smooth pulley at the top of each plane.

A small smooth ring C, of mass $8m$, is threaded on the string between the pulleys so that A, B and C all lie in the same vertical plane.

The part of the string between A and its pulley lies along a line of greatest slope of the plane of angle α.

The part of the string between B and its pulley lies along a line of greatest slope of the plane of angle β.

The angle between the vertical and the string between each pulley and the ring C is γ.

The two blocks, A and B, are modelled as particles.

Given that

- $\tan\alpha = \dfrac{5}{12}$ and $\tan\beta = \dfrac{7}{24}$ and $\tan\gamma = \dfrac{3}{4}$
- the coefficient of friction, μ, is the same between each block and its plane
- one of the blocks is on the point of sliding up its plane
- the tension in the string is T

(a) determine, in terms of m and g, an expression for T,

(3)

(b) draw a diagram showing the forces on block A, clearly labelling each of the forces acting on the block,

(3)

(c) determine the value of μ, giving a justification for your answer.

(10)
(+S2)

(Total for Question 6 is 18 marks)

7.

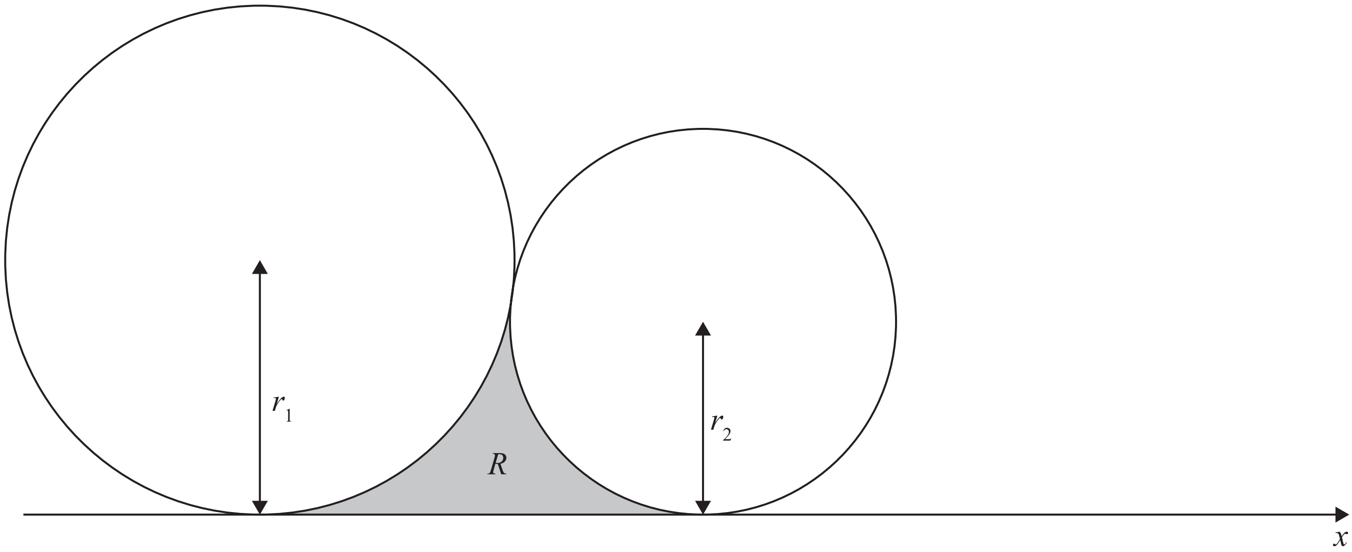

Figure 4

Figure 4 shows a circle with radius r_1 and a circle with radius r_2

The circles touch externally at a single point above the x-axis.

Both circles also have the x-axis as a tangent.

(a) Show that the horizontal distance between the centres of the circles, d, is given by

$$d^2 = 4r_1 r_2$$

(2)

The finite region R, shown shaded in Figure 4, is bounded by the x-axis and minor arcs of the two circles.

Given that $r_1 \geqslant r_2$

(b) show that the area of R is given by

$$(r_1 + r_2)\sqrt{r_1 r_2} - \frac{1}{2}\left(r_1^2 - r_2^2\right)\theta - \frac{1}{2}\pi r_2^2$$

where $\cos\theta = \dfrac{r_1 - r_2}{r_1 + r_2}$

(5)

Question 7 continues on the next page.

Question 7 continued

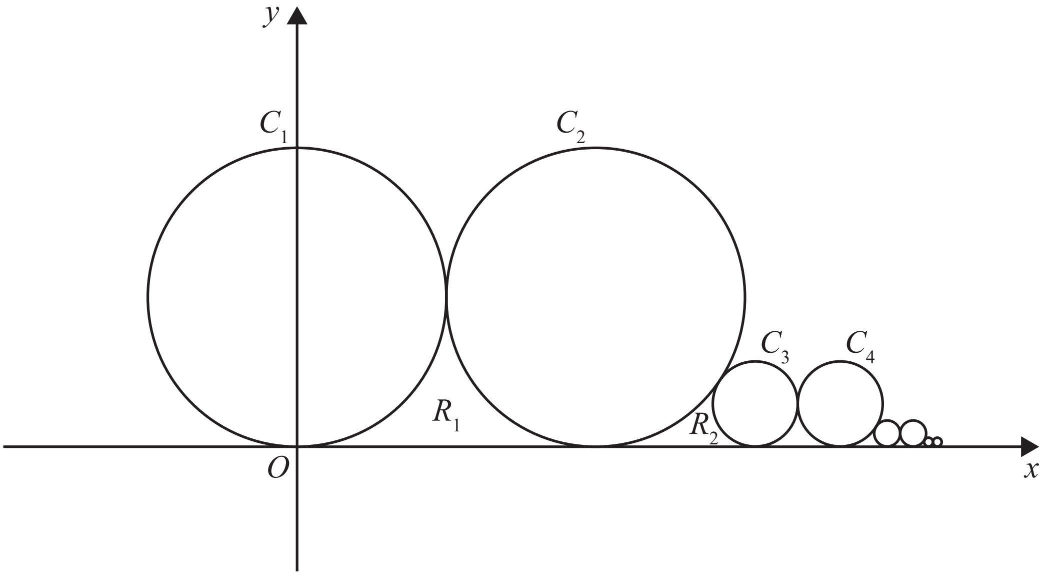

Figure 5

A sequence of circles, $C_1, C_2, C_3, \ldots$ with radii $r_1, r_2, r_3, \ldots$ respectively, is constructed such that

- each circle is tangential to and above the x-axis
- the first circle, C_1, has centre $(0, 1)$
- each successive circle touches the preceding one externally at a single point
- the horizontal distances between the centres of successive circles form a geometric sequence with first term 2 and common ratio $\frac{1}{\sqrt{3}}$

The first few circles in the sequence are shown in Figure 5.

(c) (i) Determine the value of r_3

(ii) Show that, for $n \geqslant 1$, $r_{n+2} = kr_n$ where k is a constant to be determined.

(iii) Hence show that, for $n \geqslant 1$, $r_{2n} = r_{2n-1}$

(9)

The region bounded between C_n, C_{n+1} and the x-axis is R_n

The total area, A, bounded above the x-axis and under all the circles is the sum of the areas of all these regions.

(d) Determine the value of A, giving the answer in simplest form.

(6)
(+S2)

(Total for Question 7 is 24 marks)

BLANK PAGE

BLANK PAGE

BLANK PAGE