

AEA Mathematics



Sample Assessment Materials

Pearson Edexcel Advanced Extension Award in Mathematics

First teaching from September 2017

First certification from 2019

Issue 1

Edexcel, BTEC and LCCI qualifications

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Introduction

The Pearson Edexcel Advanced Extension Award in Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson Edexcel.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet '*Mathematical formulae and statistical tables*' should be used for this assessment and can be downloaded from our website, qualifications.pearson.com.

General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks: Unconditional accuracy marks, independent of M marks
- **S** marks: For mathematical style and clarity in the presentation of solutions.
- Marks should not be subdivided.

2. Abbreviations

These are some of the traditional marking abbreviations that may appear in the mark schemes.

- | | |
|--|---|
| • bod benefit of doubt | • awrt answers which round to |
| • ft follow through | • SC: special case |
| • $\sqrt{}$ this symbol is used for correct ft | • o.e. or equivalent (and appropriate) |
| • cao correct answer only | • d... dependent or dep |
| • cs0 correct solution only. There must be no errors in this part of the question to obtain this mark | • indep independent |
| • isw ignore subsequent working | • dp decimal places |
| | • sf significant figures |

- * The answer is printed on the paper or agreed answer given dependent on gaining the first mark
- □ or d... The second mark is

3. All M marks are follow through.

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0 , should never be awarded A marks.

The style and clarity of presentation marks will be awarded as indicated in the mark scheme. These are denoted as S marks and will be awarded for clear, elegant and succinct complete solutions to questions. There will be clear indicators in the mark scheme as to where S marks will be awarded.

An example of indicators of how S marks will be seen and awarded in the mark scheme is shown in the table below.

Question	Scheme		Marks	AOs
4(a)	(Parallel to plane): $1.2W \cos \theta = 0.6W$	Resolves parallel to plane with correct value of $\sin \alpha$ used. (S+)	M1	AO3
	$\theta = \frac{\pi}{3}$	Accept 60° (S-)	A1	AO3
S1	S1 mark: Award S1 for a clear and concise solution that is either: - fully correct and does not include an S- point. or - that scores 11/12 and includes an S+ point but not an S- point.			

- For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
- Ignore wrong working or incorrect statements following a correct answer.
- Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Pearson Edexcel Award		Centre Number	Candidate Number
Sample Assessment Material for first teaching September 2017			
Time: 3 hours		Paper Reference 9811/11	
Advanced Extension Award Mathematics			
You must have: Mathematical Formulae and Statistical Tables An insert for Question 6 and 7			Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may not be used.**
- You must **show all your working**.
- Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$.

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Answer ALL questions. Write your answers in the spaces provided.

1. (a) For $|y| < 1$, write down the binomial series expansion of $(1 - y)^{-2}$ in ascending powers of y up to and including the term in y^3 (1)

- (b) Show that when it is convergent, the series

$$1 + \frac{2x}{x+3} + \frac{3x^2}{(x+3)^2} + \dots + \frac{rx^{r-1}}{(x+3)^{r-1}} + \dots$$

can be written in the form $(1 + ax)^n$, where a and n are constants to be found. (4)

- (c) Find the set of values of x for which the series in part (b) is convergent. (3)

Question 1 continued

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(Total for Question 1 is 8 marks)

2. (a) On separate diagrams, sketch the curves with the following equations. On each sketch you should label the exact coordinates of the points where the curve meets the coordinate axes.

(i) $y = 8 + 2x - x^2$

(ii) $y = 8 + 2|x| - x^2$

(iii) $y = 8 + x + |x| - x^2$

(7)

- (b) Find the values of x for which

$$|8 + x + |x| - x^2| = 8 + 2|x| - x^2$$

(4)

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Question 2 continued

Lined area for writing the answer to Question 2.

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Question 2 continued

(Total for Question 2 is 11 marks)

3.

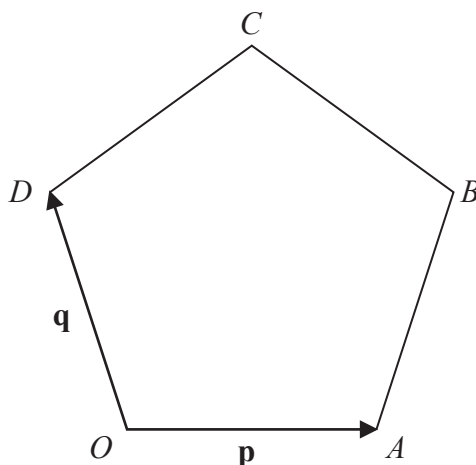


Figure 1

Figure 1 shows a regular pentagon $OABCD$. The vectors $\mathbf{p}$ and $\mathbf{q}$ are defined by $\mathbf{p} = \overrightarrow{OA}$ and $\mathbf{q} = \overrightarrow{OD}$ respectively.

Let k be the number such that $\overrightarrow{DB} = k\overrightarrow{OA}$.

(a) Write down $\overrightarrow{AC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and k as appropriate.

(1)

(b) Show that $\overrightarrow{CD} = -\mathbf{p} - \frac{1}{k}\mathbf{q}$

(3)

(c) Hence find the value of k

(4)

(+S1)

By considering triangle DBC , or otherwise,

(d) find the exact value of $\sin 54^\circ$

(3)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 12 marks)

4.

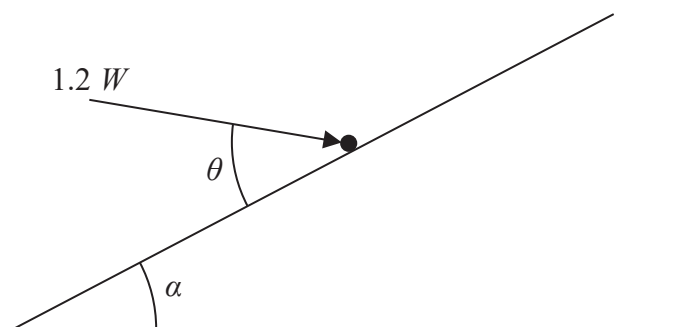


Figure 2

A particle of weight W lies on a rough plane. The plane is inclined to the horizontal at an angle α where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The particle is held in equilibrium by a force of magnitude $1.2W$. The force makes an angle θ with the plane, where $0 < \theta < \pi$, and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 2.

- (a) Find the value of θ for which there is no frictional force acting on the particle.

(2)

The minimum value of θ for the particle to remain in equilibrium is β

- (b) Show that

$$\beta = \arccos\left(\frac{\sqrt{5}}{3}\right) - \arctan\left(\frac{1}{2}\right)$$

(5)

- (c) Find the range of values of θ for which the particle remains in equilibrium with the frictional force acting up the plane.

(5)

(+S1)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 13 marks)

5.

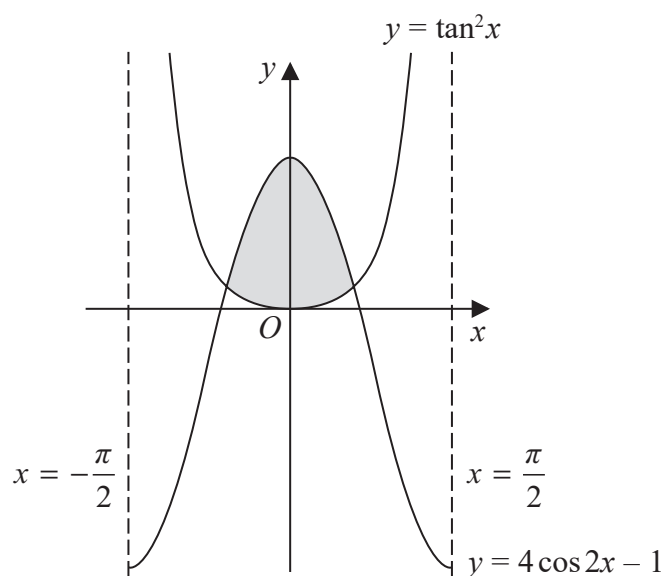


Figure 3

Show that the area of the finite region between the curves $y = \tan^2 x$ and $y = 4 \cos 2x - 1$ in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$, shown shaded in Figure 3, is given by

$$2\sqrt{2\sqrt{3}} - 2\sqrt{2\sqrt{3}} - 3$$

(12)

(+S1)

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Question 5 continued

Question 5 continued

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6. (i) Eden, who is confused about the laws of logarithms, states that

$$(\log_5 p)^2 = \log_5(p^2)$$

$$\text{and } \log_5(q - p) = \log_5 q - \log_5 p$$

However, there is a value of p and a value of q for which both statements are correct.

Determine these values.

(7)

- (ii)(a) Let $r \in \mathbb{R}^+$, $r \neq 1$. Prove that

$$\log_r A = \log_{r^2} B \Rightarrow A^2 = B$$

(2)

- (b) Solve

$$\log_4(3x^3 + 26x^2 + 40x) = 2 + \log_2(x + 2)$$

(7)

(+S2)

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[illegible]

Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

Question 6 continued

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(Total for Question 6 is 18 marks)

7.

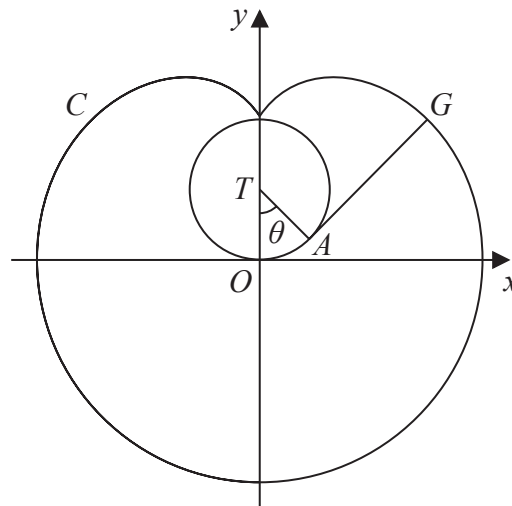


Figure 4

A circular tower of radius 1 metre stands in a large horizontal field of grass. A goat is attached to one end of a rope and the other end of the rope is attached to a fixed point O at the base of the tower. The goat cannot enter the tower.

Taking the point O as the origin $(0, 0)$, the centre of the base of the tower is at the point $T(0, 1)$, where the unit of length is the metre.

The rope has length π metres and you may ignore the size of the goat.

The curve C shown in Figure 4 represents the edge of the region that the goat can reach.

(a) Write down the equation of C for $y < 0$

(1)

When the goat is at the point $G(x, y)$, with $x > 0$ and $y > 0$, as shown in Figure 4, the rope lies along OAG where OA is an arc of the circle with angle $OTA = \theta$ radians and AG is a tangent to the circle at A .

(b) With the aid of a suitable diagram show that

$$\begin{aligned} x &= \sin \theta + (\pi - \theta) \cos \theta \\ y &= 1 - \cos \theta + (\pi - \theta) \sin \theta \end{aligned}$$

(5)

Question 7 continued

- (c) By considering $\int y \frac{dx}{d\theta} d\theta$, show that the area, in the first quadrant, between C , the positive x -axis and the positive y -axis can be expressed in the form

$$\int_0^\pi u \sin u du + \int_0^\pi u^2 \sin^2 u du + \int_0^\pi u \sin u \cos u du \quad (5)$$

- (d) Show that $\int_0^\pi u^2 \sin^2 u du = \frac{\pi^3}{6} + \int_0^\pi u \sin u \cos u du \quad (4)$

- (e) Hence find the area of grass that can be reached by the goat. (8)

(+S2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 25 marks)

FOR STYLE AND CLARITY OF PRESENTATION: 7 MARKS
TOTAL FOR PAPER IS 100 MARKS

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Pearson Edexcel Award

Sample Assessment Material for first teaching September 2017

Paper Reference **9811/11**

Advanced Extension Award Mathematics

Insert of questions 6 and 7

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6. (i) Eden, who is confused about the laws of logarithms, states that

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$$\text{and } \log_5(q - p) = \log_5 q - \log_5 p$$

However, there is a value of p and a value of q for which both statements are correct.

Determine these values.

(7)

- (ii)(a) Let $r \in \mathbb{R}^+$, $r \neq 1$. Prove that

$$\log_r A = \log_{r^2} B \Rightarrow A^2 = B$$

(2)

- (b) Solve

$$\log_4(3x^3 + 26x^2 + 40x) = 2 + \log_2(x + 2)$$

(7)

(+S2)

(Total for Question 6 is 18 marks)

7.

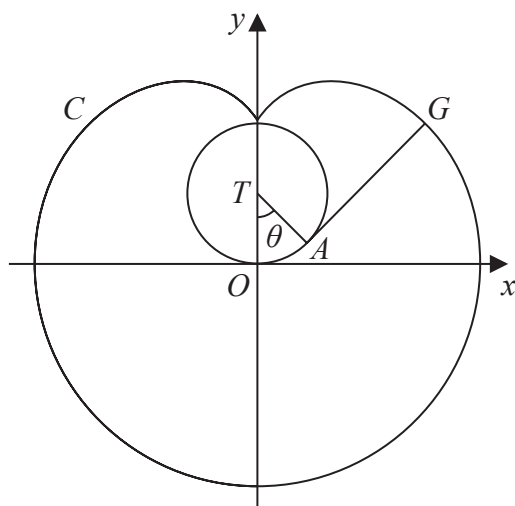


Figure 4

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(b) With the aid of a suitable diagram show that

$$\begin{aligned} x &= \sin \theta + (\pi - \theta) \cos \theta \\ y &= 1 - \cos \theta + (\pi - \theta) \sin \theta \end{aligned}$$

(5)

Question 7 continued

- (c) By considering $\int y \frac{dx}{d\theta} d\theta$, show that the area, in the first quadrant, between C , the positive x -axis and the positive y -axis can be expressed in the form

$$\int_0^\pi u \sin u du + \int_0^\pi u^2 \sin^2 u du + \int_0^\pi u \sin u \cos u du \quad (5)$$

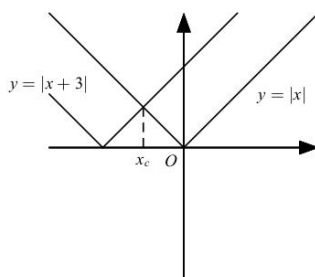
- (d) Show that $\int_0^\pi u^2 \sin^2 u du = \frac{\pi^3}{6} + \int_0^\pi u \sin u \cos u du \quad (4)$

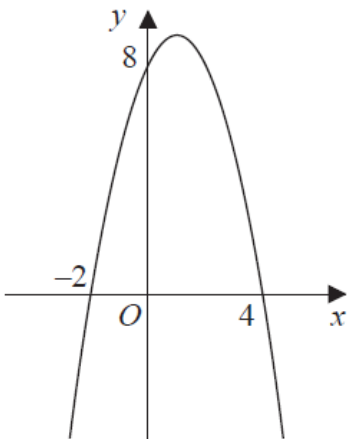
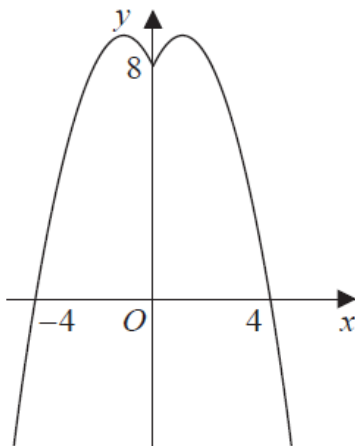
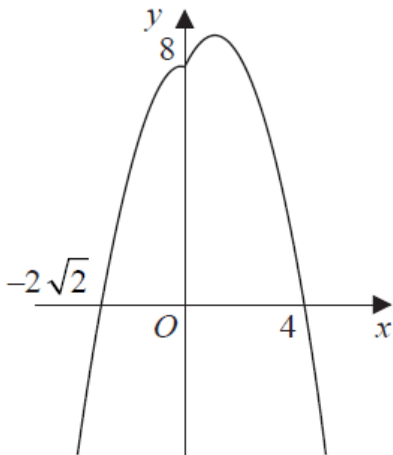
- (e) Hence find the area of grass that can be reached by the goat. (8)

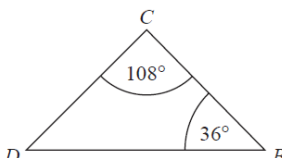
(+S2)

(Total for Question 7 is 25 marks)

AEA Mark scheme

Question	Scheme		Marks	AOs
1(a)	$(1-y)^{-2} = 1 + 2y + 3y^2 + 4y^3 + \dots$		B1	AO1
			(1)	
(b)	$S = 1 + 2\left(\frac{x}{x+3}\right) + 3\left(\frac{x}{x+3}\right)^2 + \dots$	Identifies $y = \frac{x}{x+3}$	M1	AO1
	$\Rightarrow S = 1 + 2y + 3y^2 + \dots = \left(1 - \frac{x}{x+3}\right)^{-2}$	Uses (a) to form correct expression.	A1	AO2
	$= \left(\frac{x+3-x}{x+3}\right)^{-2} = \left(\frac{x+3}{3}\right)^2$	Combines terms and reciprocates.	M1	AO2
	$= \left(1 + \frac{1}{3}x\right)^2$ (so $a = \frac{1}{3}$ and $n = 2$)	Correct answer.	A1	AO2
			(4)	
(c)	Need $\left \frac{x}{x+3}\right < 1$ or $ x < x+3 $	Correct condition.	B1	AO1
	 Critical value is when $x + 3 = -x$, so $x = \dots$	Any suitable strategy for critical value.	M1	AO3
	So convergent on $\left\{x \in \mathbb{R} : x > -\frac{3}{2}\right\}$ Accept equivalent set notation forms, e.g. $\left\{x : x \in \mathbb{R}, x > -\frac{3}{2}\right\}$ or $\left\{x : x > -\frac{3}{2}\right\}$ or $\left(-\frac{3}{2}, \infty\right]$			A1
			(3)	
(8 marks)				

Question	Scheme		Marks	AOs
2(a)	(i)		(ii)	
	(i)	Correct shape and (0, 8) Crossing x-axis at -2 and 4	B1 B1	AO1 AO1
(ii)	Symmetrical shape with 2 maxima and crosses at ±4	B1	AO1	
	Correct shape at (0, 8) - not curved as a minimum	B1	AO3	
(iii)		Correct for x > 0 and (4, 0) marked	B1	AO1
		Correct for x ≤ 0 with -2√2 marked and zero gradient at (0, 8)	B1	AO3
		Clear “kink” at (0, 8)	B1	AO3
			(7)	
(b)	Considers x ≥ 0 : 0 ≤ x ≤ 4 Correct inequality with both end points included.		B1	AO3
	Considers x < 0 : attempts -(8 - x²) = 8 - 2x - x² ⇒ x = ...		M1	AO3
	x < 0, 2x² + 2x - 16 = 0 ⇒ x = (-1 - √33)/2 only (withhold A if second root given).		M1 A1	AO3 AO3
			(4)	
(11 marks)				

Question	Scheme	Marks	AOs	
3(a)	$\overrightarrow{AC} = k\mathbf{q}$	B1	AO1	
		(1)		
(b)	$\overrightarrow{BO} = \overrightarrow{BD} + \overrightarrow{DO} = -k\mathbf{p} - \mathbf{q}$ or other suitable vector equation leading to $\overrightarrow{BO}$ used.	M1	AO1	
	$\overrightarrow{BO} = k\overrightarrow{CD}$ or $\overrightarrow{CD} = \frac{1}{k}\overrightarrow{BO}$ or equivalent used (or implied by use).	B1	AO3	
	$\overrightarrow{CD} = \frac{1}{k}(-k\mathbf{p} - \mathbf{q}) = -\mathbf{p} - \frac{1}{k}\mathbf{q}^*$	A1*	AO2	
		(3)		
(c)	Use of $\mathbf{p} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{q}$ or equivalent vector equation used.	B1	AO3	
	$\Rightarrow \mathbf{p} + \overrightarrow{AC} + \overrightarrow{CD} = \mathbf{q}$ $\Rightarrow \mathbf{p} + k\mathbf{q} - \mathbf{p} - \frac{1}{k}\mathbf{q} = \mathbf{q} \Rightarrow (k^2 - k - 1)\mathbf{q} = \mathbf{0}$	M1	AO3	
	$\Rightarrow k^2 - k - 1 = 0 \Rightarrow k = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$	M1	AO3	
	(So as $k > 0$, we must have) $k = \frac{1 + \sqrt{5}}{2}$ (S+)	A1	AO2	
		(4)		
(d)	 Use the symmetry of the shape to deduce relevant angle e.g. $\angle BCD = 180^\circ - 72^\circ = 108^\circ$. (May use another triangle within the shape.)	B1	AO1	
	Hence $\sin 54^\circ = \frac{\frac{1}{2}DB}{BC} = \frac{\frac{1}{2}kOA}{OA} = \frac{1}{2}k$	Correct work in their triangle to reduce to an expression in k	M1	AO3
	$= \frac{1 + \sqrt{5}}{4}$		A1	AO3
			(3)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - 11 marks with correct notation for vectors in the main. or - that scores 10+ and includes the S+ point but may have some poor notation.	(1)	AO2	
(11 + 1 marks)				
Notes:				
(c) S+ for clear consideration of which root is needed.				

Question	Scheme		Marks	AOs
4(a)	(Parallel to plane): $1.2W \cos \theta = 0.6W$	Resolves parallel to plane with correct value of $\sin \alpha$ used. (S+)	M1	AO3
	$\theta = \frac{\pi}{3}$	Accept 60° (S-)	A1	AO3
			(2)	
(b)	(Parallel to plane): $F = 1.2W \cos \theta - 0.6W$	M resolves in both directions. A for both correct. (S+)	M1	AO3
	(Perpendicular to plane): $R = 0.8W + 1.2W \sin \theta$		A1	AO3
	Minimum value of θ when $F = \mu R$ $\Rightarrow 1.2 \cos \theta - 0.6 = 0.4 + 0.6 \sin \theta$	Uses $F = \mu R$ to form equation in θ only.	M1	AO3
	$\Rightarrow 2 \cos \theta - \sin \theta = \frac{5}{3} \Rightarrow \sqrt{5} \cos(\theta + \gamma) = \frac{5}{3}$	Applies $R \cos(\theta + \alpha)$	M1	AO2
	$\Rightarrow \theta = \arccos\left(\frac{\sqrt{5}}{3}\right) - \arctan\left(\frac{1}{2}\right)^*$		A1*	AO2
			(5)	
(c)	(Parallel to plane): $F = 0.6W - 1.2W \cos \theta$ (Perpendicular to plane): $R = 0.8W + 1.2W \sin \theta$	Resolves both directions.	M1	AO3
	$F = \mu R \Rightarrow 0.6W - 1.2W \cos \theta = 0.4W + 0.6W \sin \theta$	Correct equation using $F = \mu R$	A1	AO3
	$\Rightarrow 2 \cos \theta + \sin \theta = \frac{1}{3} \Rightarrow \sqrt{5} \cos(\theta - \gamma) = \frac{1}{3}$ OR $\frac{6t}{1+t^2} + \frac{6(1-t^2)}{1+t^2} = 1 \Rightarrow 7t^2 - 6t - 5 = 0$	Correct strategy for finding θ	M1	AO3
	Greatest value of $\theta = \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right)$ OR $\theta = 2 \arctan\left(\frac{3+2\sqrt{11}}{7}\right)$	Either form for θ	A1	AO3

Question	Scheme		Marks	AOs
4(c) <i>continued</i>	So range is $\frac{\pi}{3} < \theta \leq \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right) \text{ (oe)}$	Allow 'least value' of θ is $\frac{\pi}{3}$ ($\frac{\pi}{3} \leq \theta$) i.e lower strict inequality not required for this mark but incorrect inequality is. (S-)	A1	AO2
			(5)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - fully correct and does not include an S- point. or - that scores 11/12 and includes an S+ point but not an S- point.		(1)	AO2
(12 + 1 marks)				
Notes:				
(a)+(b)	S+ for good reasoning for the angle, good use of diagrams etc. S- for giving answer in degrees.			

Question	Scheme		Marks	AOs
5	$\tan^2 x = 4 \cos 2x - 1$ $\Rightarrow \sec^2 x - 1 = 4(2 \cos^2 x - 1) - 1$ OR $\frac{\sin^2 x}{\cos^2 x} = 4(2 \cos^2 x - 1) - 1$	Reduces RHS to single angle and adapts $\tan^2 x$ suitably.	M1	AO1
	$\Rightarrow 1 - \cos^2 x = 8 \cos^4 x - 5 \cos^2 x$	Achieve equation in just $\cos x$	M1	AO1
	$\Rightarrow 8 \cos^4 x - 4 \cos^2 x - 1 = 0$	Correct quartic.	A1	AO1
	$\Rightarrow \cos^2 x = \frac{4 \pm \sqrt{16 - 4 \times 8 \times -1}}{16} = \dots$	Attempts roots.	M1	AO3
	Intersection point α satisfies $\cos^2 \alpha = \frac{1}{4}(1 + \sqrt{3})$	Correct root identified. (S+)	A1	AO3
	So $\sin^2 \alpha = 1 - \frac{1}{4}(1 + \sqrt{3}) = \frac{1}{4}(3 - \sqrt{3})$	Correct expressions for $\sin^2 \alpha$ or $\sin \alpha$ and $\tan^2 \alpha$ or $\tan \alpha$ seen anywhere in solution.	B1	AO2
	And $\tan^2 \alpha = \frac{3 - \sqrt{3}}{1 + \sqrt{3}} = 2\sqrt{3} - 3$		B1	AO2
	Area = $2 \times \int_0^\alpha (4 \cos 2x - 1 - \tan^2 x) dx$	Correct strategy for the area, use of 0 and α with $2\times$, or with $-\alpha$ and α . May integrate separately and combine later. (S+)(S-)	M1	AO3
	$= 2 \times \int_0^\alpha (4 \cos 2x - 1 - (\sec^2 x - 1)) dx$	Uses suitable identity to set up integral (ignore $2\times$ and limits here).	M1	AO3
	$= 2 \times [2 \sin(2x) - \tan x]_0^\alpha$	Correct integral (ignore limits etc.)	A1	AO3
	$= 2 \times (4 \sin \alpha \cos \alpha - \tan \alpha)$ $= 2 \times \left(4 \times \frac{1}{2} \sqrt{3 - \sqrt{3}} \times \frac{1}{2} \sqrt{1 + \sqrt{3}} - \sqrt{2\sqrt{3} - 3} \right)$	Uses the expression in $\cos \alpha$, $\sin \alpha$ and $\tan \alpha$ in their integral.	M1	AO3

Question	Scheme		Marks	AOs
5 <i>continued</i>	$= 2\sqrt{2\sqrt{3}} - 2\sqrt{2\sqrt{3} - 3} *$	Completes to given answer.	A1*	AO2
			(12)	
S1	S1 mark: Award S1 for a clear and concise solution that is either - fully correct with no S− point. or - that scores 11/12 and includes an S+ point and no S−.		(1)	AO2
(12+1 marks)				
Notes:				
S+	for showing consideration and rejection of the negative root for $\cos^2 x$			
S+	for good use of symmetry and/or odd and even function nature of the trig functions.			
S−	if the strategy is cumbersome – e.g. Separate integrals, no use of symmetry etc.			

Question	Scheme		Marks	AOs
6(i)	$(\log_5 p)^2 = \log_5(p^2) \Rightarrow (\log_5 p)^2 = 2 \log_5 p$	Applies power rule.	M1	AO1
	$\Rightarrow \log_5 p (\log_5 p - 2) = 0 \Rightarrow \log_5 p = 0 \therefore p = 1$		A1	AO2
	or $\Rightarrow \log_5 p = 2 \therefore p = 25$		A1	AO2
	$\log_5(q - p) = \log_5 q - \log_5 p$ $\Rightarrow \log_5(q - p) = \log_5\left(\frac{q}{p}\right)$	Applies correct rule.	M1	AO1
	$\Rightarrow q - p = \frac{q}{p}$		A1	AO2
	$(p = 1 \Rightarrow q - 1 = q \text{ is impossible})$ so $q = \frac{p^2}{p-1}$	Makes q the subject, no need to see $p = 1$ rejected for M (S+)	M1	AO3
	So $p = 25$ and $q = \frac{625}{24}$ (only)		A1	AO3
			(7)	
(ii)(a)	$\log_r(A) = \log_{r^2}(B) \Rightarrow (r^2)^{\log_r(A)} = (r^2)^{\log_{r^2}(B)}$	Adopts a correct strategy, raise both sides to base r^2 or use change of base.	M1	AO2
	$\Rightarrow r^{2\log_r(A)} = B \Rightarrow r^{\log_r(A^2)} = B \Rightarrow A^2 = B^*$	Achieves result correctly.	A1*	AO2
			(2)	
(ii)(b)	$2 = \log_2(4)$ or $2 = \log_4(16)$	Seen anywhere.	B1	AO1
	$\therefore \log_4(3x^3 + 26x^2 + 40x) = \log_2(4(x+2))$ (oe)	Combines to just two log terms.	M1	AO3
	$\therefore 3x^3 + 26x^2 + 40x = 16(x+2)^2$	Applies the result of (ii)(a)	M1	AO3
	e.g. $x(3x+20)(x+2) = 16(x+2)^2$ or $3x^3 + 10x^2 - 24x - 64 = 0$ leading to $(x+2)(3x^2 + kx \pm 32) = 0$	Identifying a linear factor.	M1	AO3
	$(x+2)(3x^2 + 4x - 32) = 0$	Correct quadratic reached (no need for $(x+2)$ for A1) (S+)	A1	AO3

Question	Scheme		Marks	AOs
6(ii)(b)	$(x + 2)(3x - 8)(x + 4) = 0$	Factorises	M1	AO3
<i>continued</i>	But equation not defined for $(x = -2 \text{ and}) x = -4$ $\left(\text{but equation works for } x = \frac{8}{3} \right)$. So $x = \frac{8}{3}$ is only solution. (S+)		A1	AO3
S2			(7)	
	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for either - a fully correct solution that is succinct but does not mention any S+ points. or - a fully correct solution that is slightly laboured but includes an S+ point. or - part (i) or (ii) fully correct and contains an S+ point.		(2)	AO2
(16 + 2 marks)				
Notes:				
(i) S+ for giving reason $p \neq 1$				
(ii) S+ for dealing correctly with $(x + 2)$ and not just “cancelling” – if statement $x \neq -2$ is given, cancelling is fine. S+ for correct reasoning for the only valid root with check that equation does work for the given value.				

Question	Scheme		Marks	AOs
7(a)	$x^2 + y^2 = \pi^2$ or equivalent.		B1	AO1
			(1)	
(b)	$OA = \theta$ seen or implied.		B1	AO1
	$AG = \pi - \theta$ seen or implied.		B1	AO1
	Clear method for x or y e.g. Let X be the point vertically below G such that angle $GXA = 90^\circ$ OR identifies $x = \sin \theta + GA \cos \theta$ or $y = GA \sin \theta + 1 - \cos \theta$ (S+)		M1	AO2
	So $x = TA \sin \theta + AX = \sin \theta + (\pi - \theta) \cos \theta^*$		A1*	AO2
	and $y = 1 - TA \cos \theta + GX = 1 - \cos \theta + (\pi - \theta) \sin \theta^*$		A1*	AO2
			(5)	
(c)	$\frac{dx}{d\theta} = \cos \theta - (\pi - \theta) \sin \theta - \cos \theta$ (allow one slip)		M1	AO1
	So Area $\int_{x=0}^{x=\pi} y \frac{dx}{d\theta} d\theta = \int_{\pi}^0 [1 - \cos \theta + (\pi - \theta) \sin \theta] [-(\pi - \theta) \sin \theta] d\theta$ (ignore limits)		A1	AO1
	Identifies suitable substitution: Let $u = \pi - \theta$		M1	AO3
	Applies $\cos(\pi - \theta) = -\cos \theta$ and $\sin(\pi - \theta) = \sin \theta$ to their integral.		M1	AO3
	Simplifies to given answer, with limits correctly derived. Area $= -\int_0^{\pi} (-u \sin u - u \cos u \sin u - u^2 \sin^2 u) du \rightarrow \text{ans}^*$ (S+)		A1*	AO2
			(5)	
(d)	$\int_0^{\pi} u^2 \sin^2 u du = \int_0^{\pi} \frac{u^2}{2} du - \int_0^{\pi} \frac{u^2}{2} \cos 2u du$	Use of $\sin^2 u$ in terms of $\cos 2u$	M1	AO2
	$= \frac{\pi^3}{6}, -\left\{ \left[\frac{u^2}{4} \sin 2u \right]_0^{\pi} - \int_0^{\pi} u \frac{\sin 2u}{2} du \right\}$	achieves $\frac{\pi^3}{6}$	A1cso	AO2
		Use integration by parts on second integral.	M1	AO2
	$= \frac{\pi^3}{6} + \int_0^{\pi} u \sin u \cos u du^*$	Shows [...] = 0 and simplifies to answer.	A1*	AO2
			(4)	

Question	Scheme		Marks	AOs
7(d) <i>continued</i>	Alternative			
	$\int \sin^2 u \, du = \int \frac{1}{2} - \frac{1}{2} \cos 2u \, du = \frac{1}{2}u \pm \frac{1}{4} \sin 2u$	Use of $\sin^2 u$ in terms of $\cos 2u$ and integrates to use in parts.	M1	AO2
	$\int_0^\pi u^2 \sin^2 u \, du = \left[u^2 \left(\frac{1}{2}u - \frac{1}{4} \sin 2u \right) \right]_0^\pi - \int_0^\pi 2u \left(\frac{1}{2}u - \frac{1}{4} \sin 2u \right) du$			
	$= \frac{\pi^3}{2} - \int_0^\pi u^2 \, du + \int_0^\pi \frac{1}{2} \sin 2u \, du$	achieves $\frac{\pi^3}{6}$	A1cso	AO2
	$= \frac{\pi^3}{6}$	Having attempted parts directly.	M1	AO2
	$= \frac{\pi^3}{6} + \int_0^\pi u \sin u \cos u \, du *$	Correct work and simplifies to answer.	A1*	AO2
			(4)	
(e)	$\int_0^\pi u \sin u \, du = [-u \cos u]_0^\pi + \int_0^\pi \cos u \, du$	Use of parts on this integral (ignore limits).	M1	AO2
	$= \pi$		A1	AO1
	$\int_0^\pi u \sin u \cos u \, du = \int_0^\pi u \frac{\sin 2u}{2} du = \left[-u \frac{\cos 2u}{4} \right]_0^\pi + \int_0^\pi \frac{\cos 2u}{4} du$		M1	AO2
	$= -\frac{\pi}{4}$	M1 – use of parts A1 – correct result	A1	AO1
	Area between curve and +ve axes $= \frac{\pi^3}{6} + \pi - 2 \times \frac{\pi}{4} = \frac{\pi^3}{6} + \frac{\pi}{2}$		A1	AO2
	Total area available = – tower + semicircle	Suitable strategy. (S+)	M1	AO3
	Area of semicircle $= \frac{\pi^3}{2}$ <u>or</u> area of tower's base $= \pi$		B1	AO2
	So area reachable is $\frac{5\pi^3}{6}$ <u>m²</u>		A1	AO3
			(8)	

Question	Scheme	Marks	AOs
7(e) <i>continued</i> S2	Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below). Award S1 for either - a fully correct solution that is succinct but does not mention any S+ points. or - a fully correct solution that is slightly laboured but includes an S+ point. or - a score of >20 but solution is otherwise succinct or contains an S+ point.	(2)	AO2
(23 + 2 marks)			
Notes:			
(b) S+ for a clear and well expressed explanation or clear diagram with accompanying work. (c) S+ for clear demonstration of the change of limits. (e) S+ for a concise strategy using symmetry.			

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