Pearson Edexcel

Mark Scheme (Results)

Summer 2023

Pearson Edexcel AEA
In Mathematics (9811) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question	Scheme		Marks	AOs
1(a)	$\cos 405° = \cos 45° = \frac{1}{\sqrt{2}}$ oe	Correct value	**B1**	2
			(1)	
(b) Way 1	$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ with $\theta = A$ or $2A$ (where A = 101.25°)	Applies double angle identity to get cos$A$ or its square in terms of cos2A or cos2A in terms of cos 4A	**M1**	1
	$\Rightarrow \cos 2A = \pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}$	Proceeds to find expression for cos 2A, allow if sign not considered.	**M1**	1
	As 2A is in third quadrant $\cos 2A = -\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}$	Correct value for cos 2A (must have the negative and reject the positive)	**A1**	2
	So $\cos A = \pm\sqrt{\frac{1}{2}(1+\cos 2A)} = \pm\sqrt{\frac{1}{2}\left(1\pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}\right)}$	Complete method to get the required cosine (ie second use of double angle formula)	**M1**	3
	A is obtuse so $\cos A = -\sqrt{\frac{1}{2}\left(1-\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}\right)}$ $= -\frac{1}{2}\sqrt{\left(2-\sqrt{\left(2+\sqrt{2}\right)}\right)}$	Correct answer, accept any reasonably simplified answer in the form specified E.g. $-\sqrt{\frac{1}{2}-\frac{1}{4}\sqrt{2+\sqrt{2}}}$	**A1**	3
			(5)	
			(Total 6 marks)	
(b) Way 2	$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$	Applies double angle identity to get sin$A$ or its square in terms of cos2A	**M1**	1
	$\cos 101.25° = \cos(90°+11.25°) = -\sin 11.25°$ and $\sin 11.25° = \pm\sqrt{\frac{1}{2}(1-\cos 22.5°)}$	Proceeds to find expression for cos 2A, allow if sign not considered.	**M1**	1
	As 11.25 is acute $\Rightarrow \sin 11.25° = \sqrt{\frac{1}{2}(1-\cos 22.5°)}$	Correct value for cos 2A (must have the negative and reject the positive)	**A1**	2
	$\cos 22.5° = (\pm)\sqrt{\frac{1}{2}(1+\cos 45°)} \Rightarrow \sin 11.25° = ...$	Complete method to get the required sine or cosine(ie use of double angle formula)	**M1**	3
	$\cos 101.25° = -\sin 101.25° = -\sqrt{\frac{1}{2}\left(1-\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}\right)}$	Correct answer, accept any reasonably simplified answer in the form specified.	**A1**	3

	$= -\frac{1}{2}\sqrt{\left(2-\sqrt{\left(2+\sqrt{2}\right)}\right)}$	E.g. $-\sqrt{\frac{1}{2}-\frac{1}{4}\sqrt{2+\sqrt{2}}}$		
			(5)	

(b) Other variations exist but all should mark in a similar way:
M1 applies a double angle identity.
M1 reaches intermediary value up to sign error using an appropriate starting angle.
A1 correct intermediary value using an appropriate angle.
M1 reaches value through second use of identity.
A1 Correct and in a simplified form.

(b) Way 3	$\cos 4A = 2\cos^2 2A - 1 = 2\left(2\cos^2 A - 1\right)^2 - 1$ (where A = 101.25°)	Applies double angle identity to get cos4A in terms of $\cos^2 A$	**M1**	1
	$\Rightarrow 8\cos^4 A - 8\cos^2 A + 1 - \frac{1}{\sqrt{2}} = 0$ $\Rightarrow \cos^2 A = \ldots$	Proceeds to form and solve a quadratic in $\cos^2 A$ with their answer to (a).	**M1**	1
	$\cos^2 A = \frac{2 \pm \sqrt{2+\sqrt{2}}}{4}$ but $\cos^2 A < \cos^2 \frac{\pi}{4} \Rightarrow \cos^2 A = \frac{2-\sqrt{2+\sqrt{2}}}{4}$	Correct value for $\cos^2 A$ (must have the "negative root" and reject the "positive root" solution). Need not be simplified.	**A1**	2
	$\Rightarrow \cos A = \pm\sqrt{\frac{2 \pm \sqrt{2+\sqrt{2}}}{4}}$	Takes the square root of their (positive) value for $\cos^2 A$	**M1**	3
	$\cos 101.25° = -\sqrt{\frac{1}{2}\left(1-\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{2}}{2}\right)}\right)}$ $= -\frac{1}{2}\sqrt{\left(2-\sqrt{\left(2+\sqrt{2}\right)}\right)}$	Correct answer, accept any reasonably simplified answer in the form specified. E.g. $-\sqrt{\frac{1}{2}-\frac{1}{4}\sqrt{2+\sqrt{2}}}$	**A1**	3

(b) Way 4	$\cos 101.25° = \cos(405° - 303.75°)$ $= \cos 405° \cos 303.75° + \sin 405° \sin 303.75°$ $= \frac{\sqrt{2}}{2}\left(\cos 303.75° + \sin 303.75°\right)$	Applies compound angle identity in an appropriate direction with some attempt to find one of the "cos303.75°" or "sin303.75°" terms- you may need to look where it leads - and use result of (a) (may be seen later).	**M1**	1
	$\text{e.g} \cos 303.75° = \pm\sqrt{\frac{1}{2}\left(1 + \cos 607.5°\right)}$ $= \pm\sqrt{\frac{1}{2}\left(1 \pm \sqrt{\frac{1}{2}\left(1 + \cos 1215°\right)}\right)}$ $= \pm\sqrt{\frac{1}{2}\left(1 \pm \sqrt{\frac{1}{2}\left(1 + \cos 135°\right)}\right)}$ $= \pm\sqrt{\frac{1}{2}\left(1 \pm \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}\right)}$	A correct full process to find the value for cos303.75° or sin303.75°	**M1**	1
	$\cos 303.75° = \sqrt{\frac{1}{2}\left(1 - \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}\right)}$ or $\sin 303.75° = -\sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}\right)}$	Correct value for sin for cos expression in their original identity using appropriate signs (i.e, considers quadrants or similar for each angle when square rooting).	**A1**	2
	$\text{e.g.} \sin 303.75° = \sqrt{1 - \cos 303.75°} = ...$ $\Rightarrow \cos 101.25° = ...$ $\left(= \frac{\sqrt{2}}{2}\left(\sqrt{\frac{1}{2}\left(1 - \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}\right)} - \sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}\right)}\right)\right)$	Proceeds to find the other value required and substitutes back into original expression.	**M1**	3
	$\cos 101.25° = -\sqrt{\frac{1}{2}\left(1 - \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right)}\right)}$ $= -\frac{1}{2}\sqrt{\left(2 - \sqrt{\left(2 + \sqrt{2}\right)}\right)}$	Correct answer, accept any reasonably simplified answer in the form specified. E.g. $-\sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2 + \sqrt{2}}}$	**A1**	3

Question	Scheme		Marks	AOs
2(a)	For $p=7$ we have $n=2p+1=15=(5\times 3)$, so is not prime.	Identifies factors/not prime for the case *p=7*	**B1**	2
			(1)	
(b)	We suspect it is not correct so look for a counter example. Observe $6\times ..9=..4$ which is 1 less than a multiple of 5 so any prime ending in 9 will do	Looks for a suitable counterexample. Most likely by at least one trial with a prime *p*.	**M1**	2
	e.g. $6\times 19+1=115$ which is a multiple of 5, so not prime.	Correct example and explanation with conclusion.	**A1**	1
			(2)	
(c)	Assume there are only finitely many primes, $p_1, p_2, \ldots, p_n$ say.	Sets up an assumption with all primes listed or indexed.	**M1**	2
	Now let $N=p_1p_2...p_n+1$	Correct expression for *N* set up. Allow if 1 is one of the primes for this mark.	**A1**	1
	E.g. Now *N* cannot be prime as it is bigger than any prime in the list, so it must have a prime divisor p_i for some *i*. OR We deduce *N* must be a prime number that is not in the list as none of the prime factors can divide *N*...	A suitable strategy to derive a contradiction, considering the primality of *N*. There are numerous variations here.	**M1**	3
	But p_i divides $p_1p_2...p_n$ but not 1, so cannot divide *N* *OR* ... since dividing *N* by p_i will always give remainder 1, hence is not a factor.	Carries out full necessary work to derive the contradiction. Must give evidence, just stating *N* is not divisible by any of the primes with no justification is M0.	**M1**	2
	This is a contradiction and hence the original assumption is false. We conclude there are infinitely many primes.	For for a full argument and deduces a contradiction and concludes the result required. All work must have correct, A0 if 1 was in the list of primes.	**A1**	2
			(5)	
	Award S1 for a good style of proof in (c) which is fully correct, well reasoned and concise. E.g. must clearly define all primes before forming the product, give algebraic, not just verbal, reasoning.		**S1**	2
			(Total 8+1 marks)	

Note:
Variations on the proof are possible.
The second M should include reference to *N* not being prime or considering the cases (i) *N* is prime (ii) *N* is not prime.

Question	Scheme		Marks	AOs
3(a)	Need $\left(\frac{5}{\sqrt{3}}\sin t\right)^2+\left(5(1-\cos t)\right)^2=\left(\frac{5\sqrt{2}}{2}\right)^2$ $\Rightarrow \frac{25}{3}(1-\cos^2 t)+25(1-\cos t)^2=\frac{25}{2}$	Solves circle and parametric equations simultaneously/uses distance formula at point B and applies $\sin^2 t=1-\cos^2 t$ to get equation in $\cos t$ only	M1	1
	$\Rightarrow 4\cos^2 t-12\cos t+5=0$ $\Rightarrow (2\cos t-5)(2\cos t-1)=0\Rightarrow \cos t=...$	Gathers terms and solve a three term quadratic in cos t via correct method.	M1	1
	$\cos t=\frac{1}{2}\Rightarrow t=\frac{\pi}{3}\left(,\frac{5\pi}{3}\right)$	Correct value of t, accept if both are given or just $\frac{\pi}{3}$ (as $x>0$ for this value)	A1	2
			(3)	
(b)	For sector, angle $OBB_x=\arctan\frac{5/2}{5/2}=\frac{\pi}{4}$, so angle in sector OBB_x is $\pi-2\times\frac{\pi}{4}=\frac{\pi}{2}$	Identifies the angle of the sector required. Alternatively, may set up the integral of circle equation.	M1	2
	Area sector = $\frac{1}{2}\times\frac{\pi}{2}\times\left(\frac{5\sqrt{2}}{2}\right)^2$ (or with $\frac{\pi}{4}$ if only working in 1st quadrant)	Applies sector or segment formula with their angle which may be $\frac{\pi}{3}$ but not π or 2π. Alt, carries out integral of circle with limits.	M1	1
	Area under curve from O to B is given by $\int_{t=0}^{t=\frac{\pi}{3}} y\,dx=\int_{t=0}^{t=\frac{\pi}{3}} 5(1-\cos t)\frac{5}{\sqrt{3}}\cos t\,dt$	Correct method for area under curve, either half or all of it considered. Limits not needed.	M1 ↕	1
	$=\frac{25}{\sqrt{3}}\int_0^{\frac{\pi}{3}}\cos t-\frac{1}{2}(1+\cos 2t)\,dt=\frac{25}{\sqrt{3}}\left[\sin t-\frac{1}{2}\left(t+\frac{1}{2}\sin 2t\right)\right]_0^{\frac{\pi}{3}}$		dM1	3
		Applies a correct method of integration (limits not needed).		
	$=\frac{25}{\sqrt{3}}\left[\frac{\sqrt{3}}{2}-\frac{1}{2}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)\right]$	A correct, not necessarily fully simplified (trig terms evaluated) answer for the area under the curve.	A1	3
	E.g. Area = Area of sector OAB + 2 ×(Area triangle OBB_x −Area under curve from 0 to $\frac{\pi}{3}$) Or equivalent method.	A correct identifiable overall strategy. E.g. as shown (B_x is point on x axis below B) May use segment and difference between rectangle and area under curve. May integrate circle and take difference.	M1	3
	Area $=\frac{25\pi}{8}+2\left(\frac{1}{2}\times\frac{5}{2}\times\frac{5}{2}-\frac{25}{\sqrt{3}}\left(\frac{3\sqrt{3}}{8}-\frac{\pi}{6}\right)\right)=25\left(\frac{\pi}{8}+\frac{\pi}{3\sqrt{3}}-\frac{1}{2}\right)$ oe		A1	3
			(7)	
			(Total 10 marks)	

Alt via Cartesian (a)	$\sin^2 t+\cos^2 t=1\Rightarrow\left(\frac{x\sqrt{3}}{5}\right)^2+\left(1-\frac{y}{5}\right)^2=1$ $\Rightarrow 3x^2+(5-y)^2=25, x^2+y^2=\frac{25}{2}$ $\Rightarrow 3\left(\frac{25}{2}-y^2\right)+(5-y)^2=25$	Uses $\sin^2 t+\cos^2 t=1$ to find Cartesian equation for curve C and solves simultaneously with the circle equation to find a an equation in either x or y only.	**M1**	1
	$\Rightarrow 4y^2+20y-75=0$ $\Rightarrow(2y-5)(2y+15)=0\Rightarrow y=...$	Solves for x or y from their equation.	**M1**	1
	$\Rightarrow\cos t=\frac{1}{2}\Rightarrow t=\frac{\pi}{3}\left(,\frac{5\pi}{3}\right)$	Correct value of t, accept if both are given or just $\frac{\pi}{3}$ (as $x>0$ for this value)	**A1**	2
			(3)	
(b)	Area under circle is given by $\int_0^{\frac{5}{2}} y\,dx=\int_0^{\frac{5}{2}}\sqrt{\frac{25}{2}-x^2}\,dx$	Attempts to set up the area under the circle using Cartesian equations, reaching suitable form. May use lower limit $-\frac{5}{2}$	**M1**	2
	$\sqrt{2}x=5\sin u\Rightarrow\sqrt{2}dx=5\cos u$ $\Rightarrow\int_0^{\frac{\pi}{4}}\sqrt{\frac{25}{2}\left(1-\sin^2 u\right)}\frac{5\cos u}{\sqrt{2}}du=\frac{25}{2}\int_0^{\frac{\pi}{4}}\cos^2 u\,du=$ $=\frac{25}{4}\int_0^{\frac{\pi}{4}}1+\cos 2u\,du=\frac{25}{4}\left[u+\frac{1}{2}\sin 2u\right]_0^{\frac{\pi}{4}}=...\left(=\frac{25\pi}{16}+\frac{25}{8}\right)$	Correct method for integration carried out, likely by substitution (others possible).	**M1**	1
	Area under curve from O to B is given by $\int_0^{\frac{5}{2}} y\,dx=\int_0^{\frac{5}{2}}5-\sqrt{25-3x^2}\,dx$	Correct method for area under curve, either half or all of it considered. Limits not needed. Via Cartesian equations, must reached a suitable integrable expression.	**M1** ↕	1
	$\left(5\sin v=\sqrt{3}x\Rightarrow 5\cos v\,dv=\sqrt{3}dx\right)$ $=[5x]_0^{\frac{5}{2}}-\frac{25}{\sqrt{3}}\int_0^{\frac{\pi}{3}}\cos^2 v\,dv=\frac{25}{2}-\frac{25}{2\sqrt{3}}\int_0^{\frac{\pi}{3}}1+\cos 2v\,dv=\frac{25}{2}-\frac{25}{2\sqrt{3}}\left[v+\frac{1}{2}\sin 2v\right]_0^{\frac{\pi}{3}}=...$		**dM1**	3
		Applies a correct method of integration (limits not needed)		
	$=\frac{25}{2}-\frac{25}{2\sqrt{3}}\left[\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right]$	A correct, not necessarily fully simplified but with trig terms evaluated, answer for the area under the curve.	**A1**	3
	E.g. Area = Area under circle − area under curve Area $=2\left(\frac{25\pi}{16}+\frac{25}{8}-\frac{25}{2}+\frac{25}{2\sqrt{3}}\left[\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right]\right)$	A correct identifiable overall strategy, in this case the difference of areas under circle and curve.	**M1**	3
	$=25\left(\frac{\pi}{8}+\frac{\pi}{3\sqrt{3}}-\frac{1}{2}\right)$ oe		**A1**	3
			(7)	
			(Total 10 marks)	

Question	Scheme		Marks	AOs
4(a)	$A \approx \frac{1}{2} \times \frac{1}{4}\left(1 + 16 + 2\left(16^{1/4} + 16^{1/2} + 16^{3/4}\right)\right) = \ldots$	Correct structure and value for *h* used.	**M1**	1
	$= \frac{1}{8}\left(17 + 2\left(2+4+8\right)\right) = \frac{45}{8}$	Correct and simplified.	**A1**	1
			(2)	
(b)	$A \approx \frac{1}{2n}\left(1+16+2\left(16^{1/n} + 16^{2/n} + 16^{3/n} + \ldots + 16^{(n-1)/n}\right)\right)$	Correct structure, including *h*. Allow slips in powers. Fully correct expression. May use $\sum$ notation.	**M1** **A1**	1 2
			(2)	
(c)	$\int 16^x \, dx = \lim_{n\to\infty} \frac{1}{2n}\left(17 + 2\left(16^{1/n} + 16^{2/n} + 16^{3/n} + \ldots + 16^{(n-1)/n}\right)\right)$	Expresses as limit of their sum	**M1**	2
	$= \lim_{n\to\infty} \frac{15}{2n} + \lim_{n\to\infty} \frac{1}{n}\left(1 + 16^{1/n} + 16^{2/n} + \ldots + 16^{(n-1)/n}\right)$	Splits their sum to extract the desired terms (S+ if limit explained)	**M1 (S+)**	3
	$= \lim_{n\to\infty} \frac{1}{n}\left(1 + 16^{1/n} + 16^{2/n} + \ldots + 16^{(n-1)/n}\right)$ *	Applies limit and reaches correct answer.	**A1***	3
			(3)	
(d)	$\int 16^x \, dx = k16^x$	Integral is a non-unit multiple of 16^x	**M1**	1
	$\int 16^x \, dx = \frac{16^x}{\ln 16}$	Correct integral	**A1**	1
	$\int_0^1 16^x \, dx = \left[\frac{16^x}{\ln 16}\right]_0^1 = \frac{16-1}{\ln 16} = \frac{15}{\ln 16}$	Correct simplified answer - must be seen in (d).	**A1**	2
			(3)	
(e)	$1 + 16^{1/n} + 16^{2/n} + \ldots + 16^{(n-1)/n} = 1 + 16^{1/n} + \left(16^{1/n}\right)^2 + \ldots + \left(16^{1/n}\right)^{n-1}$ is a geometric sequence with $a = 1$ and $r = 16^{1/n}$, so $\ldots = \frac{1 \times \left(\left(16^{1/n}\right)^n - 1\right)}{16^{1/n} - 1} = \ldots$	Recognises geometric sequence and applies sum	**M1**	3
	$= \frac{15}{16^{1/n} - 1}$		**A1**	3
	So $\frac{15}{\ln 16} = \lim_{n\to\infty} \frac{1}{n}\left(\frac{15}{16^{1/n} - 1}\right) = 15 \lim_{n\to\infty}\left(\frac{1/n}{16^{1/n} - 1}\right)$	Equates answers with sum applied, and $1/n$ placed appropriately	**M1**	3
	And letting $x = 1/n$, then $x \to 0$ as $n \to \infty$, then as the limit exists so (by continuity) we have $\frac{1}{\ln 16} = \lim_{x\to 0} \frac{x}{16^x - 1}$	Change variable. (S+ good explanation, or mention of continuity)	**M1 (S+)**	2
	Hence $\lim_{x\to 0} \frac{16^x - 1}{x} = \ln 16$ oe such as $4\ln 2$	Takes reciprocal and deduces correct limit.	**A1**	3

		(5)	
	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • A succinct solution that scores 13+ marks that includes at least one S+ point.	**S1**	2
(Total 15+1 marks)			
S+ See notes in scheme. (c) for good demonstration of the limit tending to 0 (ie not just disappearing without comment).			

Question	Scheme		Marks	AOs
5(a)	F and $H \cap G$ independent $\Rightarrow \text{P}(F \cap G \cap H) = \text{P}(F)\text{P}(G \cap H)$	Applies the condition on independence correctly	**M1**	1
	$\text{P}(H \mid G) = \frac{1}{3} \Rightarrow \text{P}(G \cap H) = \text{P}(G) \times \frac{1}{3}$	Applies the conditional probability correctly.	**M1**	1
	Hence $\text{P}(F \cap G \cap H) = \frac{3}{7} \times \frac{1}{3}\text{P}(G) = \frac{1}{7}\text{P}(G)$ *	Draws both facts together to deduce the given result	**A1***	2
			(3)	
(b)	(Need to show $\text{P}(X' \cap Y) = \text{P}(X')\text{P}(Y)$) (Since $X' \cap Y$ and $X \cap Y$ are mutually exclusive) $\text{P}(X' \cap Y) = \text{P}(Y) - \text{P}(X \cap Y)$	Sets or applies up correct relation using mutual exclusivity of $X' \cap Y$ and $X \cap Y$ (S+)	**M1**	3
	$= \text{P}(Y) - \left(\text{P}(X)\text{P}(Y)\right)$ (since X and Y are independent)	**Uses** independence of X and Y (must be applied, not just stated). May be score before 1st M.	**M1**	3
	$= \text{P}(Y)\left(1 - \text{P}(X)\right) = \text{P}(X')\text{P}(Y)$ hence X' and Y are independent.*	Factorises and uses complement property, and concludes independent.	**A1***	2
			(3)	
(c)	By (a) and (b) $\text{P}(F' \cap G \cap H) = \text{P}(F')\text{P}(G \cap H) = \left(1 - \text{P}(F)\right)\frac{1}{3}\text{P}(G)$	Applies the result of (b) together with the similar work shown in (a)	**M1**	2
	$= \frac{4}{7} \times \frac{1}{3}\text{P}(G) = \frac{4}{21}\text{P}(G)$ or $k = \frac{4}{21}$	Correct value of k seen in formula or identified.	**A1**	1
			(2)	
(d)	$\text{P}(F \cap G \cap H) = \frac{a}{n}$ and $\text{P}(F' \cap G \cap H) = \frac{c}{n}$ $\Rightarrow \frac{c}{n} = \frac{4}{21}\left(7\frac{a}{n}\right)$ Or $\text{P}(F \mid G \cap H) = \text{P}(F) \Rightarrow \frac{a}{a+c} = \frac{3}{7} \Rightarrow c = \ldots$	Uses either the results from (a) and (c) or the fact that F is independent of $H \cap G$ to set up an equation in a and c (S+ if explaining independence)	**M1**	3
	$\Rightarrow c = \frac{4}{3}a$ *	Correct result.	**A1***	2
	If the n's are missing in probability statements allow M1A0, but if just sets are referred to and there is no incorrect work, allow M1A1.			
			(2)	
(e)	$\text{P}(F \mid H) = \frac{1}{5} \Rightarrow \frac{a+d}{a+c+d+16} = \frac{1}{5}$ $\Rightarrow 5(a+d) = a + \frac{4}{3}a + d + 16$	Uses the given fact and result of (b) to set up and equation in a and d only.	**M1**	3
	$\Rightarrow 4d = \frac{7a}{3} + 16 - 5a \Rightarrow d = 4 - \frac{2}{3}a$	Correct answer for d in terms of a.	**A1**	3

	As $d \geqslant 0$ so the maximum value of a is 6	Correct value. (S+)	**A1**	2
			(3)	
(f)	Since c must be an integer, a must be a multiple of 3 less than or equal to 6. Hence a is 0, 3 or 6	Uses the given constraint to work out possible values for a. Allow if seen in (d). (S+)	**M1**	2
	$P(H \mid G) = \frac{1}{3} \Rightarrow \frac{a+c}{a+c+b+5} = \frac{1}{3} \Rightarrow 2(a+c) = b+5$	Sets up an equation in order to find b	**M1**	3
	$\Rightarrow b = \frac{14}{3}a - 5 \quad (b = 9 \text{ or } b = 23)$	Correct equation for b in terms of a or correct value for either a = 3 or 6.	**A1**	3
	So $P(F) = \frac{1}{n}\left(22 + a + \frac{14}{3}a - 5 + 4 - \frac{2}{3}a\right) = \frac{21+5a}{n}$	Find P(F) in terms of a or for a = 3 or 6	**M1**	3
	$\Rightarrow n = \frac{7}{3}(21+5a)$	Uses P(F) to find n in terms of a or with a value for a	**A1**	3
	n = 84 or n = 119 (a = 3 or 6 respectively)	Correct two values and no others.	**A1**	3
			(6)	
	Award S2 for a solution scoring 17+ marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 16+ marks that may be laboured but includes an S+ point		**S2**	2
(Total 19+2 marks)				
Note: S+ marks for good explanations at the point indicated.				
5(a) Alt using Venn diagram	F and $H \cap G$ independent $\Rightarrow P(F \cap G \cap H) = P(F)P(G \cap H) \Rightarrow \frac{a}{n} = P(F) \times \frac{a+c}{n}$	Applies the condition on independence correctly. May use $(a + b + d + 22)/n$ for P(F).	**M1**	1
	$P(H \mid G) = \frac{1}{3} \Rightarrow \frac{a+c}{a+b+c+5} = \frac{1}{3}$	Applies the conditional probability correctly.	**M1**	1
	Hence $P(F \cap G \cap H) = \frac{a}{n} = \frac{3}{7} \times \frac{a+b+c+5}{3n} = \frac{1}{7}P(G)$*	Draws both facts together to deduce the given result with no incorrect step shown.	**A1***	2
			(3)	

Question	Scheme		Marks	AOs
6(a)	E.g. by similar triangles full cone would have height 8*r*, so volume of truncated cone is $\frac{1}{3}\pi(2r)^2(8r)-\frac{1}{3}\pi r^2(4r)$	Correct strategy for finding the volume of truncated cone. (S+ good reason)	**M1 (S+)**	3
	$=\frac{1}{3}\pi r^3\left(4\times 8-4\right)=\frac{28}{3}\pi r^3$	Correct volume for truncated cone.	**A1**	2
	Total volume is $\frac{28}{3}\pi r^3+\frac{1}{2}\frac{4\pi}{3}R^3$	Sums their volume for truncated cone with volume for hemisphere.	**M1**	1
	So $\frac{28}{3}\pi r^3+\frac{1}{2}\frac{4\pi}{3}R^3=2100\pi \Rightarrow R^3=...$	Depends on both Ms. Sets expression equal to 2100π and solves for R^3	**ddM1**	3
	$R^3=\frac{3}{2}\left(2100-\frac{28}{3}r^3\right)=3150-14r^3$ *	Correct expression reached with a suitable intermediate line.	**A1***	2
			(5)	
(b)	$kR^2\frac{\mathrm{d}R}{\mathrm{d}r}=lr^2$ or $\frac{\mathrm{d}R}{\mathrm{d}r}=kr^2\left(3150-14r^3\right)^{-\frac{2}{3}}$	Differentiates implicitly or explicitly to an expression of the correct form.	**M1**	3
	$3R^2\frac{\mathrm{d}R}{\mathrm{d}r}=-42r^2$ or $\frac{\mathrm{d}R}{\mathrm{d}r}=-14r^2\left(3150-14r^3\right)^{-\frac{2}{3}}$	Any correct expression.	**A1**	1
			(2)	
(c)	$A=2\pi R^2+\pi R^2-\pi r^2+...$	Finds area of external part of hemisphere visible – ie removing the intersection of cone and sphere. (S+ good reasoning)	**M1 (S+)**	3
	$+...\pi 2r\sqrt{(8r)^2+(2r)^2}-\pi r\sqrt{(4r)^2+r^2}$	Correct attempt at area of curved surface of truncated cone.	**M1**	3
	$=3\pi R^2-\pi r^2+2\pi r\sqrt{64r^2+4r^2}-\pi r\sqrt{16r^2+r^2}$	Correct total area	**A1**	3
	Note if the not visible base is included, allow M1M1A0 (and M1A0 below). If it is later removed with explanation all the marks can be recovered, but if no explanation is given both A's will be withheld.			
	$=3\pi R^2-\pi r^2+4\pi r^2\sqrt{17}-\pi r^2\sqrt{17}$	Simplifies the expressions for the SA of the frustum to the correct form.	**M1**	2
	$=\left(3\sqrt{17}-1\right)\pi r^2+3\pi R^2$ *	Correct answer reached with no errors seen.	**A1***	3
			(5)	
(d)	$\frac{\mathrm{d}A}{\mathrm{d}r}=2\left(3\sqrt{17}-1\right)\pi r+6\pi R\frac{\mathrm{d}R}{\mathrm{d}r}$ $=2\left(3\sqrt{17}-1\right)\pi r+6\pi\left(\frac{-42r^2}{3R}\right)$	Differentiates implicitly and uses their answer to (b) to replace the $\frac{\mathrm{d}R}{\mathrm{d}r}$ or differentiates explicitly and replaces $3150-14r^3$ by R^3	**M1**	3

	Alt $\frac{dA}{dr}=2\left(3\sqrt{17}-1\right)\pi r+3\pi\frac{2}{3}\left(3150-14r^3\right)^{-\frac{1}{3}}.-42r^2=2\left(3\sqrt{17}-1\right)\pi r-84\pi\frac{r^2}{\left(R^3\right)^{\frac{1}{3}}}$			
	$\frac{dA}{dr}=2\left(3\sqrt{17}-1\right)\pi r-\frac{84\pi r^2}{R}$	Correct answer	**A1**	3
			(2)	
(e)	(i) r coordinate is where $R = 0$ so $\sqrt[3]{\frac{3150}{14}}=\sqrt[3]{225}$	For $\sqrt[3]{225}$	**B1**	2
	(ii) If $r > \sqrt[3]{225}$ then R would be negative (S+ if further explains that this doesn't make sense in the context of the question, cannot have negative radius etc).	Correct explanation	**B1** **(S+)**	2
			(2)	
(f)	A is SP so $\frac{dA}{dr}=0\Rightarrow 2\pi\left(3\sqrt{17}-1\right)r=\frac{84\pi r^2}{R}\Rightarrow ...$	Sets their derivative equal to zero and attempts to solve	**M1**	1
	$\Rightarrow R\left(3\sqrt{17}-1\right)=42r\Rightarrow R^3\left(3\sqrt{17}-1\right)^3=(42r)^3$ $\Rightarrow\left(3150-14r^3\right)\left(3\sqrt{17}-1\right)^3=(42r)^3$	Cubes and substitutes for R^3 Correct equation in just r (allow with γ and δ as long as these were correctly found in (d).	**M1** **A1**	3 3
	$\Rightarrow r^3=\frac{3150\left(3\sqrt{17}-1\right)^3}{42^3+14\left(3\sqrt{17}-1\right)^3}$	Makes r or r^3 the subject	**M1**	3
	$\Rightarrow r=\sqrt[3]{\frac{225\left(3\sqrt{17}-1\right)^3}{3\times 42^2+\left(3\sqrt{17}-1\right)^3}}$ (so $p = 225$ and $q = 42$)	Correct answer.	**A1**	3
			(5)	
	Award S2 for a solution scoring 19+ marks that is succinct and includes some S+ points (see notes below). Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • a solution scoring 19+ marks that may be laboured but includes an S+ point • A succinct solution that scores 17+ marks that includes at least one S+ point.		**S2**	2 2
				(Total 21+2 marks)

Notes: S+ marks for good explanations at the point indicated.

S+ for use of implicit rather than explicit differentiation as it is as the question leads and is more succinct.				
Alternative for 7(f) first two marks.				
(f)	$\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow 2\left(3\sqrt{17}-1\right)\pi r-\frac{84\pi r^2}{\left(3150-14r^3\right)^{\frac{1}{3}}}=0\Rightarrow ...$	Sets their derivative in terms of just r to zero and attempts to solve/	**M1**	1
	$\Rightarrow\left(3150-14r^3\right)^{\frac{1}{3}}\left(3\sqrt{17}-1\right)=42r$ $\Rightarrow\left(3150-14r^3\right)\left(3\sqrt{17}-1\right)^3=(42r)^3$	Rearranges and cubes to equation without fractional indices. Correct equation	**M1** **A1**	3 3

Question	Scheme		Marks	AOs
7(a)	$a_{n+1} = p + \frac{q}{a_n}$, $q \neq 0$			
	Need $a_{n+1} = a_n$ so $a = p + \frac{q}{a}$ oe such as	Attempts to set up an equation where $a_{n+1} = a_n$ oe. Allow with a_n or a_1	**M1**	1
	$q = a^2 - ap$	Correct equation. Must be in terms of a	**A1**	2
			(2)	
(b)	Need $a_{n+2} = a_n$ so $a = p + \cfrac{q}{p + \frac{q}{a}}$ $\left(= p + \frac{qa}{pa+q} = \frac{ap^2 + pq + aq}{ap+q} \right)$	Uses period to set up an equation in a, p and q. Allow with e.g. a_1 instead of a must must be same index on both sides. Need not be simplified. Alternatively sets up simultaneous equations such as $b = p + \frac{q}{a}$ and $a = p + \frac{q}{b}$	**M1**	1
	$\Rightarrow a^2 p + aq = ap^2 + pq + aq \Rightarrow p(a^2 - ap - q) = 0$ or $a\left(p + \frac{q}{a}\right) - b\left(p + \frac{q}{b}\right) = 0 \Rightarrow p(a-b) = 0$	For a correct one-line =0 equation in p, with p factored out, or equivalent suitable correct working to establish the constraint.	**A1**	1
	But $a^2 - ap - q = 0$ gives constant sequence, not period 2, so need $p = 0$	Deduces $p = 0$ required. Must give p=0 only, but need not give full explanation, S+ if explanation given.	**B1 (S+)**	2
			(3)	
(c)	Have $p = 0$, so need $a = \frac{q}{a}$ to give constant (ie not order 2), so $a^2 = q$	Realise need for order 1 sequence, may be implied by attempt to find one, or by a correct sequence being given. (S+ for explanation)	**M1 (S+)**	3
	E.g. $a_{n+1} = \frac{4}{a_n}$ with $a = 2$	Correct sequence with both recurrence relation and either first term or a given or with terms listed.	**A1**	2
			(2)	
(d)	Need $a_{n+4} = a_n$, and have $a_{n+4} = \frac{a_{n+2}(p^2+q) + pq}{a_{n+2}p + q} = \frac{\left(\frac{a_n(p^2+q)+pq}{a_n p + q}\right)(p^2+q) + pq}{\left(\frac{a_n(p^2+q)+pq}{a_n p + q}\right)p + q}$	Sets up an equation for a_{n+4} in terms of a_n, or with a for both. M for attempt, A if correct.	**M1** **A1**	1 1
	$\Rightarrow a\left(\frac{a(p^2+q)+pq}{ap+q}\right)p + aq = \left(\frac{a(p^2+q)+pq}{ap+q}\right)(p^2+q) + pq$ $\Rightarrow a\left(a(p^2+q)+pq\right)p + aq(ap+q)$ $= \left(a(p^2+q)+pq\right)(p^2+q) + pq(ap+q)$	Multiplies through to a single line equation.	**M1**	2

	$\Rightarrow a^2p(p^2+q)+ap^2q+a^2pq+aq^2$ $= a(p^2+q)^2+pq(p^2+q)+ap^2q+pq^2$ $\Rightarrow (p^2+q)(a^2p-ap^2-aq-pq)+q(a^2p+aq-pq)=0$ $\Rightarrow p^2(a^2p-ap^2-aq-pq)+q(2a^2p-ap^2-2pq)=0$ $\Rightarrow (p^2+2q)(a^2p-ap^2-pq)-p^2aq+qap^2=0$ $\Rightarrow p(p^2+2q)(a^2-ap-q)=0$	Expands or otherwise and factorises to extract at least one of the order 1 and order 2 conditions from the equation. Correct equation with both factors extracted or cancelled.	**M1** **A1**	3 3
	Allow if e.g. cancelling *p* rather than factorising for the method.			
	But $p = 0$ gives order 2 equation from (b) and $a^2-ap-q=0$ gives order 1 equation from (a), so cannot have these	Explains why these two factors are not possible for order 4 (even if not actually factored or cancelled)	**B1**	2
	Hence for period exactly 4, we need $q=-\frac{p^2}{2}$		**A1**	3
			(7)	
	Award S1 for: • a fully correct solution that is succinct but does not mention any S+ points • A succinct solution that scores 12+ marks that includes at least one S+ point.		**S1**	2
			(Total 14+1 marks)	

Note: S+ marks for good explanations at the point indicated.
S+ for referencing $a \neq 0$ *or* $a_n \neq 0$ oe at an appropriate point where it is needed.

NB Some may first find a_{n+3} before finding a_{n+4} in (d). For reference

$$a_{n+1} = p+\frac{q}{a_n};a_{n+2} = p+\frac{qa_n}{pa_n+q};a_{n+3} = p+\frac{q(pa_n+q)}{(p^2+q)a_n+pq} = \frac{p(p^2+2q)a_n+p^2q+q^2}{(p^2+q)a_n+pq}$$

For reference, fully expanded equation is $ap^4-a^2p^3+p^3q+2ap^2q-2a^2pq+2pq^2=0$

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