Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Award

Tuesday 27 June 2023

Afternoon (Time: 3 hours)

Paper reference **9811/01**

Advanced Extension Award Mathematics

You must have:
Mathematical Formulae and Statistical Tables
An insert for questions 5, 6 and 7 (enclosed)

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- **Calculators may not be used.**
- You must **show all your working.**
- Answers should be given in as simple a form as possible, e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$.

Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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P 7 1 3 8 0 A 0 1 3 6
Pearson

1. (a) Write down the exact value of $\cos 405°$

(1)

(b) Hence, using a double angle identity for cosine, or otherwise, determine the exact value of $\cos 101.25°$, giving your answer in the form

$$a\sqrt{b + c\sqrt{2 + \sqrt{2}}}$$

where a, b and c are rational numbers.

(5)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. A student is attempting to prove that there are infinitely many prime numbers.

The student's attempt to prove this is in the box below.

> Assume there are only finitely many prime numbers, then there is a biggest prime number, p.
>
> Let $n = 2p + 1$. Then n is bigger than p and since $2p + 1$ is not divisible by p, n is a prime number.
>
> Hence n is a prime number bigger than p, contradicting the initial assumption. So we conclude there are infinitely many prime numbers.

(a) Use $p = 7$ to show that the following claim made in the student's proof is **not** true:

since $2p + 1$ is not divisible by p, n is a prime number.

(1)

The student changes their proof to use $n = 6p + 1$ instead of $n = 2p + 1$

(b) Show, by counter example, that this does not correct the student's proof.

(2)

(c) Write out a correct proof by contradiction to show that there are infinitely many prime numbers.

(5)
(+S1)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3.

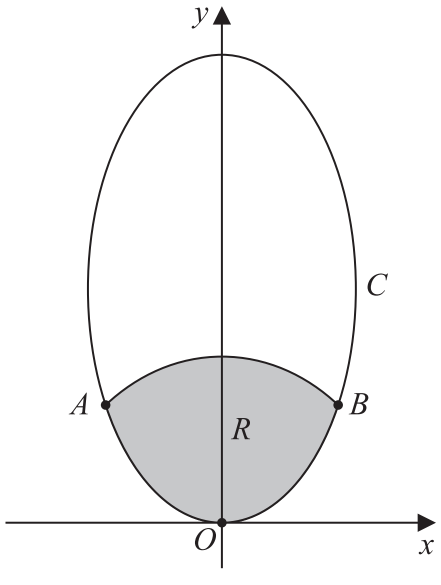

Figure 1

Figure 1 shows the curve C given by the parametric equations

$$x = \frac{5}{\sqrt{3}}\sin t \qquad y = 5(1 - \cos t) \qquad 0 \leqslant t \leqslant 2\pi$$

The circle with centre at the origin O and with radius $\frac{5\sqrt{2}}{2}$ meets the curve C at the points A and B as shown in Figure 1.

(a) Determine the value of t at the point B.

(3)

The region R, shown shaded in Figure 1, is bounded by the curve C and the circle.

(b) Determine the area of the region R.

(7)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. (a) Use the trapezium rule with 4 strips to find an approximate value for

$$\int_0^1 16^x \, dx$$

(2)

(b) Use the trapezium rule with n strips to write down an expression that would give an approximate value for

$$\int_0^1 16^x \, dx$$

(2)

(c) Hence show that

$$\int_0^1 16^x \, dx = \lim_{n \to \infty}\left(\frac{1}{n}\left(1 + 16^{\frac{1}{n}} + \ldots + 16^{\frac{n-1}{n}}\right)\right)$$

(3)

(d) Use integration to determine the exact value of

$$\int_0^1 16^x \, dx$$

(3)

Given that the limit exists,

(e) use part (c) and the answer to part (d) to determine the exact value of

$$\lim_{x \to 0} \frac{16^x - 1}{x}$$

(5)
(+S1)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 16 marks)

5.

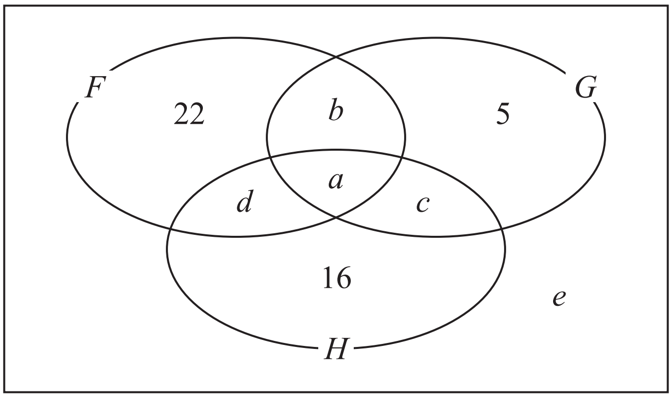

Figure 2

Figure 2 shows a partially completed Venn diagram of sports that a year group of students enjoy, where a, b, c, d and e are non-negative integers.

The diagram shows how many students enjoy a combination of football (F), golf (G) and hockey (H) or none of these sports.

There are n students in the year group.

It is known that

- $\text{P}(F) = \frac{3}{7}$
- $\text{P}(H \mid G) = \frac{1}{3}$
- F is independent of $H \cap G$

(a) Show that $\text{P}(F \cap H \cap G) = \frac{1}{7}\text{P}(G)$

(3)

(b) Prove that if two events X and Y are independent, then X' and Y are also independent.

(3)

(c) Hence find the value k such that $\text{P}(F' \cap H \cap G) = k\text{P}(G)$

(2)

(d) Show that $c = \frac{4}{3}a$

(2)

Given further that $\text{P}(F \mid H) = \frac{1}{5}$

(e) find an expression for d in terms of a, and hence deduce the maximum possible value of a.

(3)

(f) Determine the possible values of n.

(6)
(+S2)

Question 5 continued

Question 5 continued

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 21 marks)

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6. *In this question you may assume the following formulae for the volume and curved surface area of a cone of base radius r and height h and of a sphere of radius r.*

***Cone**: volume* $V = \frac{1}{3}\pi r^2 h$ *and curved surface area* $S = \pi r\sqrt{h^2 + r^2}$

***Sphere**: volume* $V = \frac{4\pi}{3}r^3$ *and curved surface area* $S = 4\pi r^2$

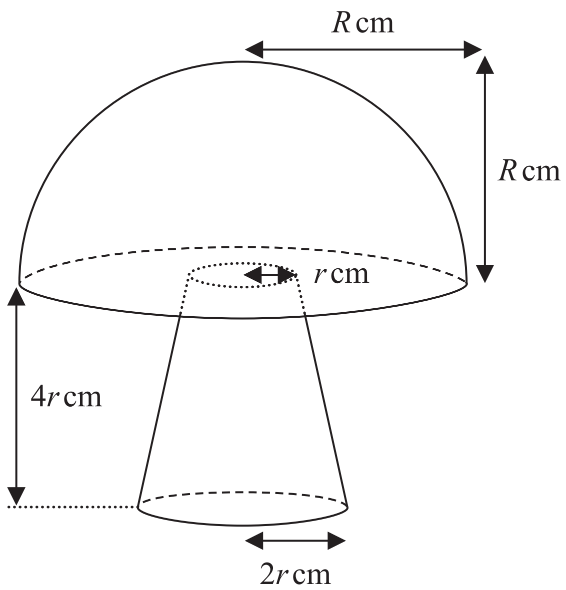

Figure 3

Figure 3 shows the design for a garden ornament.

The ornament is made of a hemisphere on top of a truncated cone.

The truncated cone has base radius $2r$ cm, top radius r cm and height $4r$ cm.

The hemisphere has radius R cm.

Given that the volume of the ornament is $2100\pi\,\text{cm}^3$

(a) show that

$$R^3 = 3150 - 14r^3$$

(5)

(b) Find an expression involving $\frac{dR}{dr}$ in terms of r and/or R.

(2)

The base of the truncated cone of the ornament is fixed to the ground.

(c) Show that the visible surface area of the ornament, $A\,\text{cm}^2$, is given by

$$A = \left(3\sqrt{17} - 1\right)\pi r^2 + 3\pi R^2$$

(5)

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Question 6 continued

(d) Hence show that

$$\frac{dA}{dr} = \gamma\pi r - \frac{\delta\pi r^2}{R}$$

where γ and δ are real numbers to be determined.

(2)

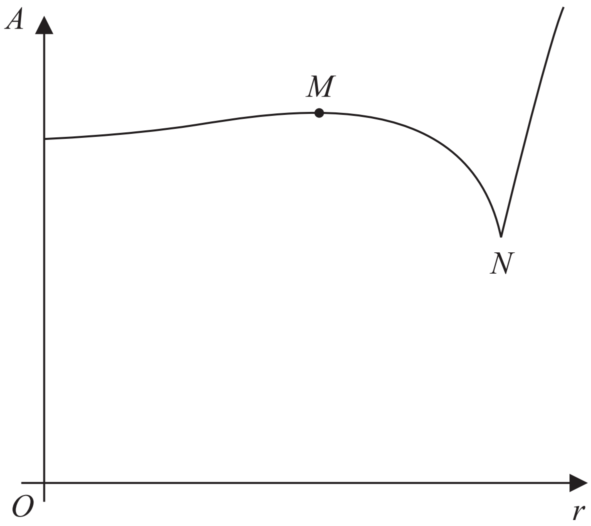

Figure 4

Figure 4 shows a sketch of A against r, for $r \geqslant 0$

There is a local minimum at $r = 0$ and a local maximum at the point M. The overall minimum point is at the point N, where the gradient of the curve is undefined.

(e) (i) Determine the r coordinate of the point N.

(ii) Explain why, for the ornament, r must be less than this value.

(2)

(f) Show that the r coordinate of the point M is

$$\sqrt[3]{\frac{p\left(3\sqrt{17}-1\right)^3}{3q^2+\left(3\sqrt{17}-1\right)^3}}$$

where p and q are integers to be determined.

(5)
(+S2)

Question 6 continued

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Question 6 continued

Question 6 continued

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 23 marks)

7. A sequence of non-zero real numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = p + \frac{q}{a_n} \qquad n \in \mathbb{N}$$

where p and q are real numbers with $q \neq 0$

It is known that

- one of the terms of this sequence is a
- the sequence is periodic

(a) Determine an equation for q, in terms of p and a, such that the sequence is constant (of period/order one).

(2)

(b) Determine the value of p that is necessary for the sequence to be of period/order 2.

(3)

(c) Give an example of a sequence that satisfies the condition in part (b), but is **not** of period/order 2.

(2)

(d) Determine an equation for q, in terms of p only, such that the sequence has period/order 4.

(7)
(+S1)

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Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 100 MARKS

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Advanced Extension Award Mathematics

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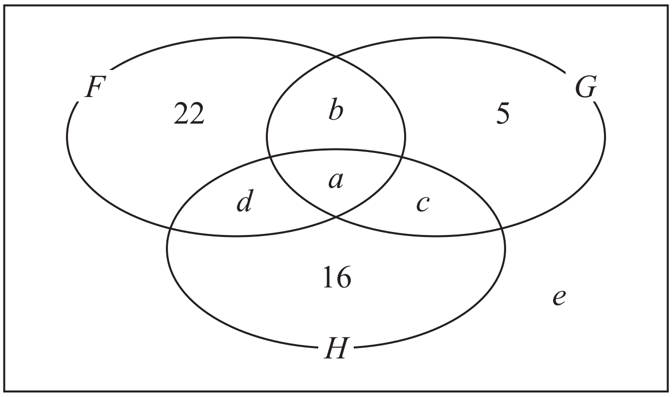

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(6)
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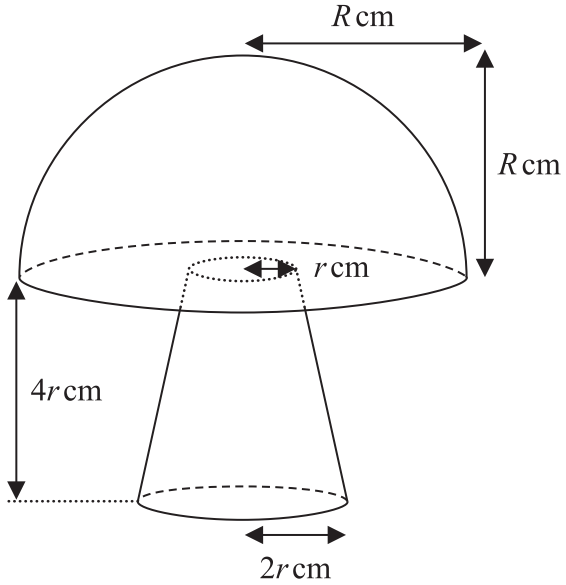

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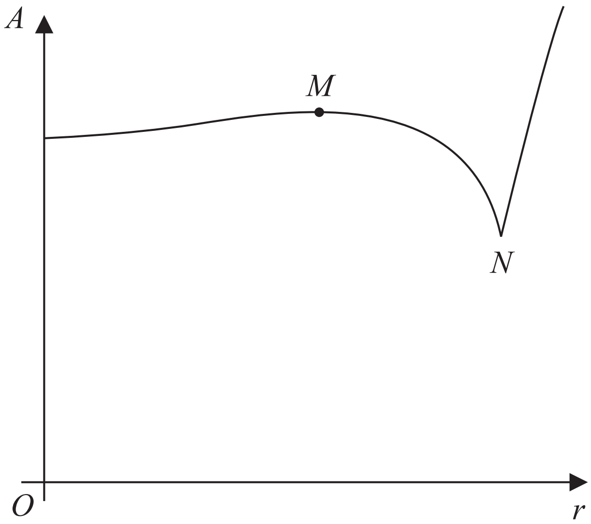

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(7)

(+S1)

(Total for Question 7 is 15 marks)

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