Mark Scheme (Results)

Summer 2014

Pearson Edexcel Advanced Extension Award in Mathematics
(9801/01)

Edexcel and BTEC Qualifications

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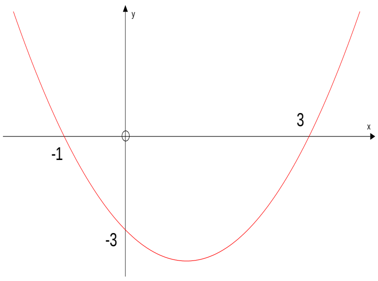
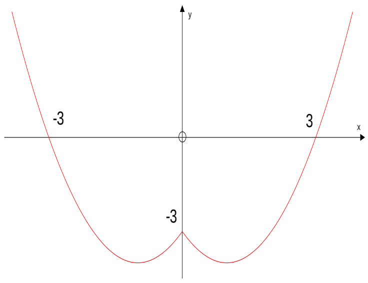
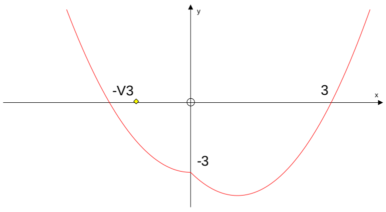
Summer 2014
Publications Code UA039428

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Qu	Scheme	Mark	Notes
1. (a)	$y = \ln(2x-5) \Rightarrow \quad e^y = 2x-5$	M1	1st stage to f^{-1}- use of e
	So $\quad f^{-1}(x) = \dfrac{e^x + 5}{2}$	A1 (2)	Correct inverse
(b)	$g(x) = f^{-1}fg(x) = \dfrac{e^{\ln\left(\frac{x+10}{x-2}\right)} + 5}{2}$	M1	Attempt to <u>use</u> a suitable strategy to find g(x)
	$= \dfrac{\frac{x+10}{x-2} + 5}{2} = \dfrac{x+10+5x-10}{2(x-2)}$	A1	Deal with $e^{\ln}$ and obtain a correct expression
	$= \dfrac{3x}{x-2} \qquad (x > 2)$	A1 (3)	Correct simplified expression.
ALT(b)	$fg(x) = \ln(2g - 5) = \ln\left(\frac{x+10}{x-2}\right)$ or $2g - 5 = \frac{x+10}{x-2}$	M1	Allow $2g \pm 5$
	$2g = \frac{x+10}{x-2} + 5$ or $\frac{x+10+5(x-2)}{x-2}$, $\Rightarrow g = \frac{3x}{x-2}$	A1,A1 **[5]**	1st A1 for $2g$ = ... 2nd A0 for $\frac{6x}{2x-4}$

Qu	Scheme	Mark	Notes
2.	sin*x* (3sin*x* + 2) = 3cos*x* (3sin*x* + 2)	M1	Factorize both sides
	$0 = (3\sin x + 2)(3\cos x - \sin x)$	M1	Finds a 2nd factor(o.e.)
	$3\cos x - \sin x = 0 \Rightarrow \quad \underline{\tan x = 3}$	A1	For tan*x* = 3 (Dep on at least one M)
	$3\sin x + 2 = 0 \Rightarrow \quad \sin x = \frac{-2}{3}$ or e.g. $3\tan x + 2\sqrt{1+\tan^2 x} = 0$	A1	For sin*x* =… or eqn in tan*x*
	[Therefore $\cos^2(x) = \frac{5}{9}$] or $\tan x = \pm\frac{2}{\sqrt{5}}$	M1	Attempt to find tan*x*
	(By considering size of *x* [or quadrant]) $\tan x = -\frac{2}{\sqrt{5}}$	A1	Must have -
		[6]	

Qu	Scheme	Mark	Notes
3. (a) **(i)**		(i) B1	Correct shape and (0,-3)
		B1	Crossing x-axis at -1 and 3
(ii)		(ii) B1	Symmetrical shape with 2 minima and $x = \pm 3$
		B1	Correct shape at (0, - 3)
(iii)		(iii) B1	Correct for $x > 0$ and (3, 0) marked
		B1	Correct for $x \le 0$ and $-\sqrt{3}$ Zero gradient at (0, - 3)
		B1 (7)	Clear "kink" @ (0, -3)
(b)	$x > 0$: $x^2 - 2x - 3 = 2x \Rightarrow x^2 - 4x - 3 = 0$ so $x = \underline{2 + \sqrt{7}}$	M1A1	Method for positive root(A0 for > 1 root)
	$x < 0$: $x^2 - 2x - 3 = -2x \Rightarrow x^2 - 3 = 0$ so $x = \underline{-\sqrt{3}}$	M1A1 (4)	Negative root (A0 for > 1 root)
		[11]	

Qu	Scheme	Mark	Notes
4. (a)	**You must see <u>general</u> terms used for M marks in (a)**		
	rth term $=\dfrac{(-\frac{1}{2})(-\frac{3}{2})...(-\frac{1}{2}-r+1)}{r!}(-x)^r$	M1	Sub. $n=-\frac{1}{2}$ and "x" $=-x$. Condone $-x^r$
	$=\dfrac{(-1)^r}{r!}\times\dfrac{(1.3.5...(2r-1))}{(2.2.2...2)}\times(-1)^r\ x^r$	M1 M1	Remove – signs Simplify numerator
	$=(1)\times\dfrac{1.2.3.4.5...(2r-1)(2r)}{r!2^r\times 2^r r!}x^r$	M1	Insert 2^r and r!
	So sum is $\sum_{r=0}^{\infty}\binom{2r}{r}\left(\dfrac{x}{4}\right)^r$	A1cso (5)	S+ for comment about $r=0$ case
(b)	$(9-4x^2)^{-\frac{1}{2}}=\dfrac{1}{3}\left(1-\dfrac{4x^2}{9}\right)^{-\frac{1}{2}}=\dfrac{1}{3}\sum_{r=0}^{\infty}\binom{2r}{r}\left(\dfrac{\not{4}x^2}{9\times\not{4}}\right)^r$	M1 A1cso	Adjust to form $k(...)^{-1/2}$
	So **q = 2r + 1**	A1 (3)	(M1A0A1 is possible)
(c)	$\dfrac{d}{dx}\left(\dfrac{x^{2r}}{3^{2r+1}}\right)=\dfrac{2r\times x^{2r-1}}{3^2\times 3^{2r-1}}=\dfrac{2r}{9}\times\left(\dfrac{x}{3}\right)^{2r-1}$	M1 dM1	Identify differentiation Chain rule(allow 1 slip)
	So sum = $\dfrac{d}{dx}(9-4x^2)^{-\frac{1}{2}}=-\dfrac{1}{2}(9-4x^2)^{-\frac{3}{2}}\times(-8x)=\underline{\dfrac{4x}{(9-4x^2)^{\frac{3}{2}}}}$	A1 (3)	(S+ for dealing with $r=0$)
(d)	Require $\dfrac{x^{2r-1}}{3^{2r-1}}=\dfrac{\sqrt{5}}{5^r}=\dfrac{1}{(\sqrt{5})^{2r-1}}$ so $x=\dfrac{3}{\sqrt{5}}$	M1	Attempt a suitable substitution for x
	Sum = $4\times\dfrac{3}{\sqrt{5}}\times\dfrac{1}{\left(9-4\times\frac{9}{5}\right)^{\frac{3}{2}}}=4\times\dfrac{3}{\sqrt{5}}\times\dfrac{5\sqrt{5}}{27}=\underline{\dfrac{20}{9}}$	A1 (2)	
		[13]	

Qu	Scheme	Mark	Notes
5. (a)	$\overrightarrow{AB}=\begin{pmatrix}7\\5\\-5\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}4\\2\\4\end{pmatrix}$, $\overrightarrow{AD}=\begin{pmatrix}8\\-2\\2\end{pmatrix}$,	M1	Attempt at least one and condone ±
	$\overrightarrow{BC}=\begin{pmatrix}-3\\-3\\9\end{pmatrix}$, $\overrightarrow{BD}=\begin{pmatrix}1\\-7\\7\end{pmatrix}$, $\overrightarrow{CD}=\begin{pmatrix}4\\-4\\-2\end{pmatrix}$	A2 (3)	All correct (-1 e.e.o.o.)
(b)	$\lvert\overrightarrow{AB}\rvert=\sqrt{99}$, $\lvert\overrightarrow{AC}\rvert=6$, $\lvert\overrightarrow{AD}\rvert=\sqrt{72}$, $\lvert\overrightarrow{BC}\rvert=\sqrt{99}$, $\lvert\overrightarrow{BD}\rvert=\sqrt{99}$, $\lvert\overrightarrow{CD}\rvert=6$	M1	Attempt at least 3 lengths
(i)	($\overrightarrow{AC}\perp\overrightarrow{CD}$) so length of base = <u>**6**</u>	A1	(S+ for clear reason)
(ii)	Need $\overrightarrow{AB}\bullet\overrightarrow{AD}$ or $\overrightarrow{BD}\bullet\overrightarrow{AD}$	M1	Identify a suitable pair
	$\cos\theta=\dfrac{\frac{1}{2}\lvert\overrightarrow{AD}\rvert}{\lvert\overrightarrow{AB}\rvert}=\dfrac{3\sqrt{2}}{3\sqrt{11}}$ or $\cos\theta=\dfrac{36}{\sqrt{99}\times\sqrt{72}}$	M1	Finding an expression for $\cos\theta$ using trigonometry or **.** prod.
	$\cos\theta=\dfrac{\sqrt{2}}{\sqrt{11}}$ (o.e.)	A1	
(iii)	Pythagoras: $h^2+\dfrac{"72"}{4}="99"$, so $\underline{h=9}$ May use $\lvert\overrightarrow{BM}\rvert$	M1A1	M1 ft their 72 and 99 (full method)
(iv)	Position vector = **a** + $\overrightarrow{CD}$ (o.e.), $=\begin{pmatrix}-2\\3\\-1\end{pmatrix}+"\begin{pmatrix}4\\-4\\-2\end{pmatrix}"=\underline{\begin{pmatrix}2\\-1\\-3\end{pmatrix}}$	M1 A1 (9)	Suitable expression using known vectors ft their *CD*
(c)	Let *M* be midpoint of *AD*. Eq'n of *BM* is $\mathbf{r}=\begin{pmatrix}5\\8\\-6\end{pmatrix}+t\begin{pmatrix}-3\\-6\\6\end{pmatrix}$	M1	Attempt equation of *BM* or other line containing the other vertex. **Or** can award for just vector *BM*
	When t = 1 **r** gives *OM*, so use t = 2 to get other vertex	M1	Full method e.g. **a** + $\overrightarrow{BD}$
	$=\underline{\begin{pmatrix}-1\\-4\\6\end{pmatrix}}$	A1 (3)	Allow $\begin{pmatrix}1\\4\\-6\end{pmatrix}$ M1M0A0
		[15]	

Qu	Scheme	Mark	Notes
6. (i)	$u = x - h \quad \Rightarrow I = \pi\int_a^b [r + f(u)]^2 \, du$	M1	Select and use a suitable substitution. Change limits and function
	$I = \pi\int_a^b \left(r^2 + 2rf(u) + [f(u)]^2\right) du$	dM1	Expand bracket
	$= \pi r^2(b-a) + 2\pi rA + V$	A1cso (3)	Split and integrate
(ii) (a)	$x = 0 \Rightarrow l = 4 + \frac{2}{\sqrt{3}}$	B1	
	$\sqrt{3}\cos x + \sin x = 0 \Rightarrow \tan x = -\sqrt{3}$ (o.e.)	M1	Method for finding asymptotes
	So $m = -\frac{\pi}{3}$ and $n = \frac{2\pi}{3}$	A1 A1 (4)	
(b)	$\sqrt{3}\cos x + \sin x = 2\cos[x - \frac{\pi}{6}]$	M1A1	Use of $R\cos(x \pm a)$ o.e.
	So $y = 4 + \sec\left(x - \frac{\pi}{6}\right)$ or $4 + \text{cosec}\left(x + \frac{\pi}{3}\right)$	B1 A1 (4)	$r = 4$ [$\Rightarrow$ by 4 +sec(...)] sec part o.e.
(c)	Using (a) $h = \frac{\pi}{6}$, $a = 0$, $b = \frac{\pi}{6}$ f(x)= sec(x) and r = 4	M1	Identify connection. **Or** give at end if fully correct
	$\int \sec x \, dx = \ln(\sec x + \tan x)$ or $\ln\left[\tan\left(\frac{1}{2}x + \frac{\pi}{4}\right)\right]$	B1	(May be in $x - h$)
	So $A = \left[\ln(\sec x + \tan x)\right]_0^{\frac{\pi}{6}} = \left(\ln\left[\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right]\right) - (\ln[1+0]) = \underline{\ln\sqrt{3}}$	M1 A1	M1 for use of appropriate limits
	$\int \sec^2 x \, dx = \tan x$	B1	(May be in $x - h$)
	So $V = \pi[\tan x]_0^{\frac{\pi}{6}} = \frac{\pi}{\sqrt{3}}$	M1 A1	Must have $[\tan x]_0^{\frac{\pi}{6}}$ or $\left[\tan(x - \frac{\pi}{6})\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ i.e. compatible with int.
	Volume = $16\pi \times \frac{\pi}{6} + 8\pi A + V$	M1	Can give for correctly getting $\frac{8\pi^2}{3}$
	$= \frac{8\pi^2}{3} + 8\pi\ln\sqrt{3} + \frac{\pi}{\sqrt{3}}$ (o.e)	A1 (9)	Dep. on previous 3 Ms
		[20]	

Qu	Scheme	Mark	Notes
7.(a)	$x^2 + y^2 = \pi^2$	B1 (1)	o.e.
(b)	$OA = \theta$, $AG = \pi - \theta$	B1,B1	1st B1 may be implied by *GA* and diagram
	Let *X* be the point vertically below *G* such that angle *GXA*= 90^o Or $x = \sin\theta + GA\cos\theta$ or $y = GA\sin\theta + 1 - \cos\theta$	M1	Clear method for *x* or *y*
	So $x = TA\sin\theta + AX = \sin\theta + (\pi - \theta)\cos\theta$	A1cso	
	and $y = 1 - TA\cos\theta + GX = 1 - \cos\theta + (\pi - \theta)\sin\theta$	A1cso (5)	
(c)	Area = $\int_{x=0}^{x=\pi} y\frac{dx}{d\theta}\,d\theta$, $\frac{dx}{dy} = \cos\theta - (\pi - \theta)\sin\theta - \cos\theta$	,M1	For d*x*/dθ Allow 1 slip
	Area $= \int_{\pi}^{0}\left[1 - \cos\theta + (\pi - \theta)\sin\theta\right]\left[-(\pi - \theta)\sin\theta\right]\,d\theta$	A1	Ignore limits
	Let $u = \pi - \theta$, $\cos(\pi - \theta) = -\cos\theta$ and $\sin(\pi - \theta) = \sin\theta$	M1,M1	Suitable sub
	Area = $-\int_0^{\pi}\left(-u\sin u - u\cos u\sin u - u^2\sin^2 u\right)\,du \rightarrow$ ans	A1cso (5)	Simplify to printed answer. Limits correctly derived
(d)	$\int_0^{\pi} u^2\sin^2 u\,du = \int_0^{\pi}\frac{u^2}{2}\,du - \int_0^{\pi}\frac{u^2}{2}\cos 2u\,du$	M1	Use of $\sin^2 x$ in terms of $\cos 2x$
	$= \frac{\pi^3}{6}, -\left\{\left[\frac{u^2}{4}\sin 2u\right]_0^{\pi} - \int_0^{\pi} u\frac{\sin 2u}{2}\,du\right\}$	A1cso ,{M1}	For $\frac{\pi^3}{6}$ M1 for int. by parts
	$= \frac{\pi^3}{6} + \int_0^{\pi} u\sin u\cos u\,du$	A1cso (4)	Show [..]=0 and simplify to ans.
(e)	$\int_0^{\pi} u\sin u\,du = \left[-u\cos u\right]_0^{\pi} + \int_0^{\pi}\cos u\,du$	M1	Use of parts to integrate (Ignore limits for Ms)
	$= \pi$	A1	
	$\int_0^{\pi} u\sin u\cos u\,du = \int_0^{\pi} u\frac{\sin 2u}{2}\,du = \left[-u\frac{\cos 2u}{4}\right]_0^{\pi} + \int_0^{\pi}\frac{\cos 2u}{4}\,du$	M1	
	$= -\frac{\pi}{4}$	A1	
	Area between curve and +ve axes $= \frac{\pi^3}{6} + \pi - 2\times\frac{\pi}{4} = \underline{\frac{\pi^3}{6} + \frac{\pi}{2}}$	A1	
	Total area available = $2\left[\frac{\pi^3}{6} + \frac{\pi}{2}\right]$ – tower + semicircle	M1	Suitable strategy
	Area of semicircle = $\frac{\pi^3}{2}$ or area of tower's base = π	B1	For either
	So area reachable is $\underline{\frac{5\pi^3}{6}}$	A1 (8)	
		[23]	

Awarding of S and T marks		
Questions	**Marks**	
3, 4	S1	For a fully correct solution that is succinct or includes an S+ point
5, 6, 7	S2	For a fully correct solution that is succinct and includes some S+ points
5, 6, 7	S1	For a fully correct solution that is succinct but does not mention any S+ points
5, 6, 7	S1	For a fully correct solution that is slightly laboured but includes an S+ point
5, 6, 7	S1	For a score of n -1 but solution is otherwise succinct or contains an S+ point
Maximum S score is 6		
ALL	T1	For at least half marks on all questions

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