Mark Scheme Summer 2009

AEA

AEA Mathematics (9801)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

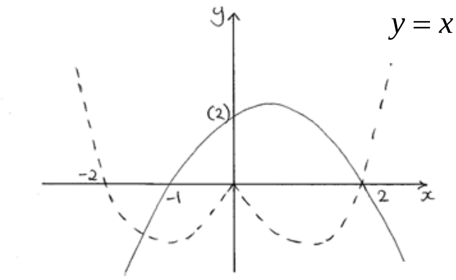
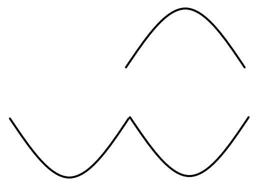
http://www.edexcel.com/Aboutus/contact-us/

Summer 2009

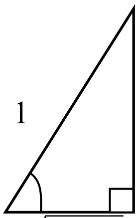
Publications Code UA021532

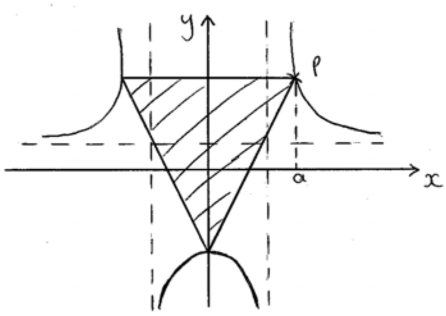
Contents

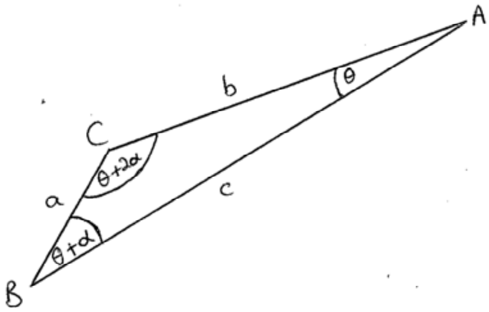
June 2009
9801 Advanced Extension Award Mathematics
Mark Scheme

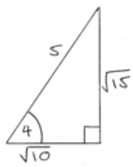
Question Number	Scheme	Marks	Notes
Q1 (a)	 $y = x^2 - 2\|x\|$  -2, -1, 2, (0,2) $y = (x+1)(2-x)$	B1 B1 B1 (3)	Don't insist on labels
(b)	One intersection at $x = 2$	B1	
	Second at $(x+1)(2-x) = x(x+2)$	M1	Attempt correct equation Must be $x + 2$ on RHS
	$(0 =)\ 2x^2 + x - 2$	A1	Correct 3TQ
	$x = \dfrac{-1 \pm \sqrt{1+16}}{4}$, since root is in (-2, -1) $x = \dfrac{-1 - \sqrt{17}}{4}$	M1 A1 cso (5) **[8]**	Solving Must choose -

Question Number	Scheme	Marks	Notes
Q2 (a)	$y = x^{\sin x}$ so when $x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2}^{1} = \frac{\pi}{2}$	B1	
	$\ln y = \sin x \ln x$	M1	Use of logs (o.e)
	$\frac{1}{y}\frac{dy}{dx} = \cos x \ln x + \frac{\sin x}{x}$	M1 A1	Use of product rule
	$\left[\frac{dy}{dx} = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)\right]$		Some correct sub in their y' $\left.\frac{dy}{dx}\right\|_{x=\pi/2}$
	at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ gradient $= \frac{\pi}{2}\left(0 + \frac{1}{\pi/2}\right) = 1$	M1	
	$\therefore$ Equation of tangent is $\underline{y = x}$	A1 cso (6)	
(b)	If it touches again then $y = x \Rightarrow \sin x = 1$	M1	Method $\rightarrow \sin x = 1$
	$\Rightarrow x = \frac{\pi}{2} + 2n\pi$	A1	May be listed…
	Gradient at $\left(\frac{\pi}{2} + 2n\pi\right)$ is $\left(\frac{\pi}{2} + 2n\pi\right)\left[0 + \frac{1}{\frac{\pi}{2} + 2n\pi}\right] = 1$	A1 (3)	Check points satisfy $m = 1$ plus comment
	$\therefore$ at points $\left(\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right)$ $y = x$ is a tangent.	**[9]**	

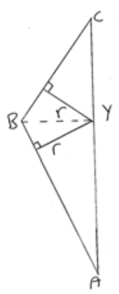
Question Number	Scheme	Marks	Notes
Q3 (a)	$\sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta \qquad = \frac{1}{\sqrt{3}}\cos\theta$	M1	Use of sin (A - B)
	$\frac{1}{\sqrt{3}}\cos\theta \qquad = \sin\theta$ (o.e)	M1	Use of sin $\frac{\pi}{3}$, cos $\frac{\pi}{3}$ and collect terms
	$\Rightarrow \qquad \tan\theta = \frac{1}{\sqrt{3}}$	A1	$\tan\theta = \frac{1}{\sqrt{3}}$ oe.
	$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$	A1, B1 $\sqrt{\ }$	
		(5)	
(b)	$\sin\left[\arcsin(1-2x)\right] = \sin\left[\frac{\pi}{3} - \arcsin x\right]$		
	$\sin[\arcsin(1-2x)] = \sin\frac{\pi}{3}\cos[\arcsin x] - \cos\frac{\pi}{3}\sin(\arcsin x)$	M1	Use of sin $(A \pm B)$
	$1 - 2x \quad = \frac{\sqrt{3}}{2}\sqrt{1-x^2} - \frac{1}{2}x$	M1,B1	B1 for cos[arcsin$x$] $=\sqrt{1-x^2}$
	 $[2 - 3x \quad = \sqrt{3}\sqrt{1-x^2}\ ]$		
	$4 - 12x + 9x^2 = 3 - 3x^2$	M1	Simplify to quadratic in x
	$12x^2 - 12x + 1$ (=0)	A1	correct 3TQ
	$x = \frac{12 \pm \sqrt{144-48}}{24}$	M1	Attempt to solve if at least one previous M scored in (b)
	$x = \frac{3 \pm \sqrt{6}}{6}$		
	$\because \qquad 0 < x < 0.5 \qquad x = \frac{3-\sqrt{6}}{6}$ (o.e.)	A1	Must choose '_'
		(7) **[12]**	

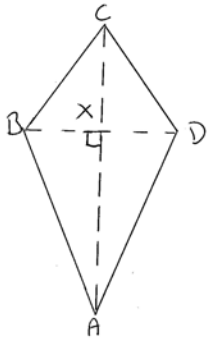
Question Number	Scheme	Marks	Notes
Q4 (a)	$f''(x) = \frac{vu^1 - uv^1}{v^2}$	M1	Use of Quotient rule
	$f'(k) = 0 \Rightarrow u(k) = 0 \quad \therefore \quad f''(k) = \frac{vu^1 - 0}{v^2}$	M1	Sub $u(k) = 0$
	$\therefore \quad f''(k) = \frac{u^1(k)}{v(k)} \quad (*) \quad$ (accept $\frac{u^1}{v}$)	A1 cso (3)	Insist on k not x
(b) (i)	A (0, -3)	B1 (1)	Accept $y = -3$
(ii)	Asymptotes $\quad x = 1, x = -1$	B1	Both
	and $y = 2$	B1 (2)	
	 Area, $T = \frac{1}{2} \times 2a \times (b+3)$	M1	Any correct exp. for T in terms of a and b or complete 2nd line
	$T = a\left[\frac{2a^2+3}{a^2-1} + 3\right] = \frac{5a^3}{a^2-1} \quad (*)$	A1 cso (2)	
(iv)	$\frac{dT}{da} = \frac{(a^2-1)15a^2 - 5a^3 2a}{(a^2-1)^2}$	M1	Use of quotient rule to find $\frac{dT}{da}$
	$= \frac{5a^2(3a^2-3-2a^2)}{(a^2-1)^2} \quad = \quad \frac{5a^2(a^2-3)}{(a^2-1)^2}$	M1	Solving $\frac{dy}{dx}$=0 --> a =… or a^2 =...
	$\frac{dT}{da} = 0 \quad \Rightarrow a^2 = 3$ or $a = \sqrt{3}$ $\quad$ (or $a = 0$ but $a > 0$)	A1 (S+)	Condone $a = \pm\sqrt{3}$
	$\frac{dT}{da} = \frac{5a^4 - 15a^2}{(a^2-1)^2}$ compare $\frac{u}{v}$ $\therefore \left.\frac{d^2T}{da^2}\right\vert_{a=\sqrt{3}} = \left.\frac{20a^3 - 30a}{(a^2-1)^2}\right\vert_{a=\sqrt{3}}$	M1	Full method e.g. $T''(\sqrt{3})$ attempted
	$T''(\sqrt{3}) = \frac{60\sqrt{3} - 30\sqrt{3}}{4} = \left(\frac{15\sqrt{3}}{2}\right) > 0 \quad \therefore$ min	A1	Full accuracy + comment
	$\therefore$ Minimum area $= \frac{5\sqrt{3} \times 3}{3-1} \quad = \quad \frac{15\sqrt{3}}{2}$	B1 (6)	Must come from $T(\sqrt{3})$ not $T''(\sqrt{3})$
		[14]	
	N.B $\frac{d^2T}{da^2} = \frac{10a(a^2+3)}{(a^2-1)^3}$ or $\frac{10a(a^4+2a^2-3)}{(a^2-1)^4}$		Suggest S1 > 12 S2 for S+ and 13 or 14.
	ALT for (iv) $\quad$ Attempt $\frac{d^2T}{da^2} = \ldots$	M1	No value of a needed.
	Correct $\frac{d^2T}{da^2}$ and comment.	A1	Fully correct and full comment.

Question Number	Scheme	Marks	Notes
Q5 (a) (i)	 $\theta+(\theta+\alpha)+(\theta+2\alpha)=180$	M1	Equate $S_3=180$
	$3\theta+3\alpha = 180$		
	$\therefore \hat{B}=(\theta+\alpha) = 60°$	A1	Show $\hat{B} = 60^{o}$
	Area $=\frac{1}{2}ac\sin(\theta+\alpha)$	M1	Use of $\frac{1}{2}ac\sin B$
	$=\frac{1}{2}ac\frac{\sqrt{3}}{2} \quad = \underline{\frac{ac\sqrt{3}}{4}} \quad (*)$	A1 (4)	
(ii)	<u>Sine Rule</u> $\frac{b}{\sin(\theta+\alpha)}=\frac{a}{\sin A}$ <u>OR</u> $\frac{1}{2}bc\sin A=\frac{ac\sqrt{3}}{4}$	M1	Correct use of sine rule or $\frac{1}{2}bc\sin A$ and (a)
	$\therefore \quad b = 2\times\frac{5}{\sqrt{15}}\times\frac{\sqrt{3}}{2}=\underline{\sqrt{5}}$	A1 (2)	
(iii)	<u>Cosine Rule</u> $b^2=a^2+c^2-2ac\cos(\theta+\alpha)$	M1	Use of cos rule where all terms are known, except c.
	$5 = 4+c^2-2\times2\times c\ \frac{1}{2}$		
	$0 = c^2-2c-1$ OR $c^2-2\sqrt{2}+1=0$	M1	Sub & simplify -> 3TQ
	$c = \frac{2\pm\sqrt{4+4}}{2}$	M1	Solving
	$c = \underline{1+\sqrt{2}}$ OR $\underline{(3+2\sqrt{2})^{1/2}}$	A1 (4)	
(b)	$S_n=\frac{n}{2}[2\times143+2(n-1)] \quad =\{n(142+n)\}$	M1	For use of S_n needn't be simplified.
	Sum of internal angles $=180(n-2)$	B1	
	$n(142+n)=180(n-2) \Rightarrow \quad 0=n^2-38n+360$	A1	Correct 3TQ.
	$0=(n-19)^2-19^2+360$ $n-19=\pm1 \quad (n=20 \text{ or } 18)$	M1	Attempt to solve relevant 3TQ
	Internal angles all <180 $\quad u_{20}=143+19\times2>180$ $u_{18}=143+17\times2<180$		] S+
	$\therefore n \underline{=18}$	A1 (5) **[15]**	

Question Number	Scheme	Marks	Notes
	<u>ALT for c</u> If get $\sin C = \dfrac{\sqrt{15}+\sqrt{30}}{10}$ or method to find this	M1	
	$\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$ use of	M1	
	→ $c = 1 + \sqrt{2}$	M1 A1	
	If get $\cos C = \dfrac{\sqrt{45}-\sqrt{10}}{10}$ or method to find this	M1	
	Then $c^2 = a^2 + b^2 - 2ab\cos C$ use of	M1	
	→ $c^2 = 3 + 2\sqrt{2}$ ⇒ $c = (3 + 2\sqrt{2})^{\frac{1}{2}}$	M1 A1	
	Look out for similar variations of cosine rule with cos A 		
	<u>Pythagoras</u> height = a sin 60 + Pythagoras once	M1	
	2^{nd} Pythagoras	M1	
	a cos 60 = 1 + other bit	M1	
	$1 + \sqrt{2}$	A1	

Question Number	Scheme	Marks	Notes
Q6 (a)	P is $\underline{(\sqrt{3}, \ln 2)}$	B1	Score anywhere.
	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\tan t}{2\cos t}$	M1 A1	M1 attempt $\frac{dy}{dx}$ A1 correct
	When $t = \frac{\pi}{3}$ $\quad m = \sqrt{3}$	A1	
	Equation of tangent at P is: $y - \ln 2 = \sqrt{3}(x - \sqrt{3})$	M1	Attempt tangent at P. √ their P and m
	A is where $y = 0$ $\therefore$ $-\frac{\ln 2}{\sqrt{3}} + \sqrt{3} = x \Rightarrow (x =)\frac{\sqrt{3}}{3}(3 - \ln 2)$	A1 cso (6)	Allow $\frac{3-\ln 2}{\sqrt{3}}$
(b)	Area under curve = $\int_{t=0}^{t=\pi/3} y\mathrm{d}x = \int_{(0)}^{(\pi/3)} \ln\sec t . 2\cos t \mathrm{d}t$	M1	Attempt $\int y\dot{x}\mathrm{d}t$ √ $\dot{x}$ condone missing 2
	= [2 sin t ln sec t] - $\int$ 2 sin t tan t dt	M1 A1	Attempt parts. Both parts correct.
	= [ ] - $\int 2\frac{(1-\cos^2 t)}{\cos t}dt$	M1	Use of $s^2 = 1 - c^2$
	= [ ] - 2 $\int$ sec t dt + 2 $\int$ cos t dt	M1	Split
	= [2 sin t ln sec t] - $2\ln\lvert\sec t + \tan t\rvert$ $+ 2\sin t$	A1, A1	Accept $\cos t \tan t$
	= $\sqrt{3}$ ln 2 - (2 ln [2 + $\sqrt{3}$] – 0) + (2 $\frac{\sqrt{3}}{2}$ – 0)	M1	Use of correct limits on all 3 integrals
	$\left[= \sqrt{3}(\ln 2 + 1) - 2\ln(2 + \sqrt{3})\right]$		
	Area of Δ $= \frac{1}{2}\left[\sqrt{3} - \frac{\sqrt{3}}{3}(3 - \ln 2)\right]\ln 2$ $\left\{= \frac{\sqrt{3}}{6}(\ln 2)^2\right\}$	B1	Any correct expression.
	Area of R = are under curve – area of Δ	M1	Strategy must be $\int$ or area
	$= \sqrt{3}(\ln 2 + 1) - 2\ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$ $(*)$	A1 cso (11) **[17]**	
	<u>ALT</u> Area = $-\frac{1}{2}$ $\int$ ln (1 - $\frac{x^2}{4}$)dx o.e.	M1	Condone missing $-\frac{1}{2}$
	$= \left[-\frac{1}{2}x\ln(1 - \frac{x^2}{4})\right] + \int \frac{-x^2}{4 - x^2}\mathrm{d}x$	M1 A1	Parts correct
	= [ ] + $\int$ 1 dx - $\int \frac{4}{4 - x^2}\mathrm{d}x$	M1	Split
	= [ ] + $x - \int\left(\frac{1}{2 - x} + \frac{1}{2 + x}\right)\mathrm{d}x$	M1	Partial Fractions
	$= \left[-\frac{1}{2}x\ln\left(1 - \frac{x^2}{4}\right)\right] + x + \ln\left(\frac{2 - x}{2 + x}\right)$	A1, A1 o.e.	
	Then use of limits etc as before.		

Question Number	Scheme	Marks	Notes
Q7 (a)	$\overrightarrow{BA} = \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$ Attempt both	M1	Allow $\pm$
	$\overrightarrow{BA} . \overrightarrow{BC} = -10 = 5\sqrt{2} \times 2\sqrt{5} \cos(A\hat{B}C)$ Use of .	M1	Use of . to form equation for cos $A\hat{B}C$
	$\therefore$ $\cos A\hat{B}C = -\frac{1}{\sqrt{10}}$ o.e.	A1 cso	
		(3)	
(b)	Area of K = 2 Area of Δ ABC		
	$= 2 \times \frac{1}{2} \times 5\sqrt{2} \times 2\sqrt{5} \sin(A\hat{B}C)$	M1	Use of $\frac{1}{2}ab\sin C \times 2$
	$\sin(A\hat{B}C) = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}$	M1	Attempt $\sin A\hat{B}C$ $\sqrt{}$ their (a)
	$\therefore$ Area $= 5\sqrt{2} \times 2\sqrt{5} \times \frac{3}{\sqrt{10}}$ $= \underline{30}$	A1	
		(3)	
(c)	 Identify $r \perp^r$ to BC and $r \perp^r$ to AB	B1	
	Area $= 2 \times$ [Area of BYC + Area of BYA]	M1	Method $\rightarrow$ equation in r
	$30 = 2 \times \left[\frac{1}{2}.2\sqrt{5}r + \frac{1}{2}.5\sqrt{2}r\right]$	A1	Correct equation in r
	$r = \frac{30}{2\sqrt{5} + 5\sqrt{2}} = 30\frac{(5\sqrt{2} - 2\sqrt{5})}{50 - 20}$	M1	Attempt r = with rational denom.
	$r = \underline{5\sqrt{2} - 2\sqrt{5}}$	A1	
		(5)	

Question Number	Scheme	Marks	Notes
(d)	 $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 4 \\ -5 \end{pmatrix}$	M1	Attempt $\overrightarrow{AC}$
	$\overrightarrow{BX} = \overrightarrow{BA} + t\overrightarrow{AC} = \begin{pmatrix} -5+7t \\ 4t \\ 5-5t \end{pmatrix}$	M1	Expression for $\overrightarrow{BX}$ in terms of t
	But $\overrightarrow{BX} \perp^r \overrightarrow{AC}$ $\therefore$ $\begin{pmatrix} -5+7t \\ 4t \\ 5-5t \end{pmatrix} \begin{pmatrix} 7 \\ 4 \\ -5 \end{pmatrix} = 0$	M1	Use of … • … = 0
	$-35 + 49t + 16t - 25 + 25t = 0$	M1	Linear equation in t based on •⁻
	$90t = 60$		
	$t = \frac{2}{3}$	A1	
	$\overrightarrow{OD} = \overrightarrow{OB} + 2\overrightarrow{BX} = \begin{pmatrix} 4 \\ 4/3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} -5+14/3 \\ 8/3 \\ 5-10/3 \end{pmatrix}$	M1	Method for $\overrightarrow{OD}$ in terms of known vectors
	$= \begin{pmatrix} 10/3 \\ 20/3 \\ 16/3 \end{pmatrix}$	A1	
		(7) **[18]**	
	<u>Marks for Style Clarity and Presentation (up to max of 7)</u>		
S1 or S2	For a fully correct (or nearly fully correct) solution that is neat and succinct in question 2 to question 7		
T1	For a good attempt at the whole paper. Progress in all questions. Pick best 3 S1/S2 scores to form total.		

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code UA021532 Summer 2009

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH