Mark Scheme (Results)

Summer 2013

AEA Mathematics (9801/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

AEA June 2013 Mark Scheme – Final

Question	Scheme	Marks	Notes
1. (a)	$\dfrac{n(n-1)}{2!}\left(\dfrac{12n}{5}\right)^2 = \dfrac{n(n-1)(n-2)}{3!}\left(\dfrac{12n}{5}\right)^3$	M1	For attempting suitable equation. Ignore *x*s but must use binomial.
	$3\times 5 = n(n-2)\times 12$ or $4n^2 - 8n - 5 = 0$ (o.e.)	A1	Correct 3TQ in *n* May be other factors
	$(2n+1)(2n-5)=0$	dM1	Dep on 1st M1
	$n = -\frac{1}{2}, \frac{5}{2}$	A1	Both & no others unless revoked later
		(4)	
(b)	$n = -\frac{1}{2}$ in $\left\lvert\dfrac{12nx}{5}\right\rvert < 1$ gives $\lvert x\rvert < \dfrac{5}{6}$ and $n = \frac{5}{2}$ in $\left\lvert\dfrac{12nx}{5}\right\rvert$ gives $\lvert x\rvert < \dfrac{1}{6}$	M1	Attempt both cases Just check $n = -\frac{1}{2}$ SC B1
	So should choose $n = -\frac{1}{2}$	A1 (2)	
	May sub $x = \frac{1}{2}$ and get $\lvert n\rvert < \frac{5}{6}$ for M1 and A1 for stating $n = -\frac{1}{2}$	**(6)**	

Question	Scheme	Marks	Notes
2. (a)	sin(90 – x) = sin90cos$x$ – cos90sin$x$ = 1.cos$x$ – 0.sin$x$ = cos$x$	B1 (1)	One intermediate line
(b)	$2\sin(\theta+17)\cos(\theta+17)=\cos(\theta+8)\Rightarrow\sin\left[2(\theta+17)\right]=\cos(\theta+8)$	M1	Use of sin2A = …
	$2\theta+34=90-(\theta+8)$	dM1	Use of (a) – not trigθ
	$3\theta=82-34=48$ so $\underline{\boldsymbol{\theta=16}}$	A1	
	$2\theta+34=180-\left[90-(\theta+8)\right]$ <u>or</u> $2\theta+34=\left[90-(\theta+8)\right]+360$	M1	2^{nd} eqn for θ
	$\theta=98-34$ <u>or</u> $\underline{\boldsymbol{\theta=64}}$	A1	
	$3\theta=48+460$ $\underline{\boldsymbol{\theta=136}}$	A1	
	$\underline{\boldsymbol{\theta=256}}$	A1 (7)	
NB	sin(2θ+34) – sin(82 – θ) gives 2cos[(θ+116)/2]sin[(3θ–48)/2] Then: θ/2 + 58 = 90 gets M1 and e.g. 3θ/2 – 24 = 0 gets M1	**(8)**	

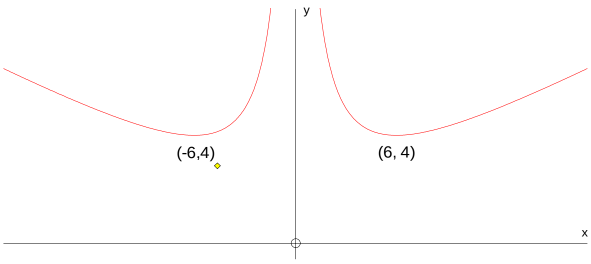
Question	Scheme	Marks	Notes
3. (a)	$-7+2\lambda=7+10\mu$ and $1-3\lambda=-6-\mu$ (o.e.)	M1	Form suitable eqns
	$\Rightarrow 14\mu=-14$ **$\underline{\mu = -1, (\lambda = 2)}$**	M1A1	M1 for eqn in 1 var
	Check in 3rd equation: $7=p-4\mu$ **$\underline{p=3}$**	A1	Check in 3rd, $p = \ldots$
	Position vector of C is $\begin{pmatrix}-3\\7\\-5\end{pmatrix}$	A1 (5)	Accept as coordinates
(b)	$\mu=-2 \Rightarrow 7-2\times10=-13,\ 3-2\times-4=11$ and $-6-2\times-1=-4$	B1 (1)	See $\mu = -2$ & ans
(c)	$\overrightarrow{CA}=\begin{pmatrix}-4\\0\\6\end{pmatrix}$ and $\overrightarrow{CB}=\begin{pmatrix}-10\\4\\1\end{pmatrix}$ giving $\overrightarrow{CA}\bullet\overrightarrow{CB}=40+0+6=46$	M1	Attempts a suitable scalar product. Allow 1 sign slip Allow $\pm$
	$\cos(ACB)=\dfrac{46}{\sqrt{52}\sqrt{117}},=\dfrac{46}{2\sqrt{13}\times3\sqrt{13}}$ $=\dfrac{23}{39}$ (o.e.)	dM1 A1 (3)	Allow $\pm$ A1 for an exact fraction (no surds)
(d)	Form Rhombus. Let $\overrightarrow{CM}=\frac{1}{2}\overrightarrow{CA}$ then $\overrightarrow{CD}=\overrightarrow{CB}+3\overrightarrow{CM}$	M1	Attempt suitable rhombus or unit vectors
	$\overrightarrow{CD}=\begin{pmatrix}-16\\4\\10\end{pmatrix}$ $\underline{\text{or}}$ $\overrightarrow{OD}=\begin{pmatrix}-19\\11\\5\end{pmatrix}$	A1	
	$\mathbf{r}=\overrightarrow{OC}+t\overrightarrow{CD}$, $\mathbf{r}=\begin{pmatrix}-3\\7\\-5\end{pmatrix}+t\begin{pmatrix}-8\\2\\5\end{pmatrix}$ (o.e.)	dM1 A1 (4) **(13)**	Dep. On 1st M1. For attempt equation of line

Question	Scheme	Marks	Notes
4. (a)	$a_1 = 1,\ a_2 = 3,\ a_3 = 7,\ a_4 = 15,\ a_5 = 31,\ a_6 = 63$	B1 (1)	
(b)	Sub: $a_{r+1} = 2^{r+1} - 1;\quad 2a_r + 1 = \underline{2(2^r - 1) + 1} = 2^{r+1} - 1$	B1cso (1)	Correct demonstration in *r*
(c)	$\sum a_r = \sum 2^r - \sum 1 = \sum 2^r - n$	B1	For $\sum 1 = n$
	$\sum 2^r = \frac{2\left(2^n - 1\right)}{2-1}$, therefore $\sum a_r = 2(2^n - 1) - n$ (o.e.)	M1 A1 (3)	Use of GP formula Any correct expres' A1 needs – *n* too.
(d)	$a_{r+1} = 2a_r + 1 \Rightarrow \underline{a_{r+1} > 2a_r} \rightarrow \frac{1}{a_{r+1}} < \frac{1}{2} \times \frac{1}{a_r}$	B1cso (1)	Or equiv in words
(e)	$\frac{1}{a_4} < \frac{\frac{1}{2}}{a_3}$ and $\frac{1}{a_5} < \frac{\frac{1}{2}}{a_4} < \frac{\left(\frac{1}{2}\right)^2}{a_3}$	M1	Use of (d) to get any 2 inequality for 4th and 5th terms
	So: $\sum_{r=1}^{5} \frac{1}{a_r} < 1 + \frac{1}{3} + \frac{1}{7} + \frac{\left(\frac{1}{2}\right)}{7} + \frac{\left(\frac{1}{2}\right)^2 \text{ or } \frac{1}{4}}{7}$	A1cso (2)	All 3 inequalities & no incorrect work
(f)	Lower limit = $1 + \frac{1}{3} + \frac{1}{7} = \frac{31}{21}$	B1cso	
	Identify GP $a = \frac{1}{7},\quad r = \frac{1}{2}$	M1	Correct *r* <u>or</u> *a*
	Use $S_\infty = \frac{\frac{1}{7}}{1 - \frac{1}{2}} \quad \left(= \frac{2}{7} \right)$	dM1 A1	Attempt sum |*r*|<1 Correct expression or sum
	Upper limit = $1 + \frac{1}{3} + \frac{2}{7} = \frac{34}{21}$	A1cso (5) **(13)**	

Question	Scheme	Marks	Notes
5. (a)	Differentiate: $uv = v\int u\,dx + u\int v\,dx$	M1 A1	Attempt to diff Correct prod. rule
	$\div$ uv leading to $1 = \frac{\int u\,dx}{u} + \frac{\int v\,dx}{v}$ **(*)**	A1cso (3)	
(b)	$\frac{\int v\,dx}{v} = \cos^2 x$	B1 (1)	S+ for $1 - c^2 = s^2$
(c)	Diff. $u\sin^2 x = \int u\,dx$ gives $u = \frac{du}{dx}\sin^2 x + u2\sin x\cos x$	M1	Multiply by u and differentiate Or quotient rule
	$\frac{du}{dx}\sin^2 x = u(1-2\sin x\cos x)$ $\therefore \frac{1}{u}\frac{du}{dx} = \frac{1-2\sin x\cos x}{\sin^2 x}$	dM1 A1cso (3)	Collect u terms
(d)	Separate variables: $\int \frac{1}{u}du = \int\left(\frac{1-2\sin x\cos x}{\sin^2 x}\right)dx$	M1	Separation of vars. Condone missing integral signs.
	RHS $= \int\left(\text{cosec}^2 x - 2\cot x\right)dx$	M1	Prepares RHS
	Integrate: $\ln u = -\cot x, -2\ln\sin x \quad +c$	A1,A1	$+c$ on 2nd A1
	$\ln\left(u\sin^2 x\right) = -\cot x \quad (+c)$	M1	Collect ln terms or remove ln
	$u = Ae^{-\cot x}\text{cosec}^2 x$	A1cso (6)	No incorrect work
(c)	$y = e^{\tan x} \Rightarrow \frac{dy}{dx} = e^{\tan x}\sec^2 x$ or $e^{\tan x}\frac{d}{dx}(\tan x)$	M1	For differentiation
	Hence $v = Be^{\tan x}\sec^2 x$	A1 (2) **(15)**	Condone A not B but S-

Question	Scheme	Marks	Notes
6. (a)	$\left[f(x)-\lambda g(x)\right]^2=\left[f(x)\right]^2-2\lambda f(x)g(x)+\lambda^2\left[g(x)\right]^2$	M1	Attempt to multiply
S+ for area comment	Integrate dx throughout with inequality	A1cso	No incorrect work
		(2)	
(b)	Treat as quadratic in λ and attempt to use discriminant	M1	Δ & identify a, b, c
	Clear reason for use of $b^2-4ac\le 0$ (or < 0) e.g. "no real roots"	M1	Reason for ≤ 0
	Giving: $\left[\int f(x)g(x)\,dx\right]^2\le\left[\int\left[f(x)\right]^2 dx\right]\times\left[\int\left[g(x)\right]^2 dx\right]$ (o.e.)	A1cso	Condone 4s
		(3)	
(c)	g(x) = $\left(1+x^3\right)^{\frac{1}{2}}$ and f(x) = 1		
	Then $[E]^2\le\left[\int\left(1+x^3\right)dx\right]\times\left[\int 1^2\,dx\right]$	M1	
	$\int_{-1}^{2}\left(1+x^3\right)dx=\left[x+\frac{x^4}{4}\right]_{-1}^{2}=,(2+4)-(-1+\frac{1}{4})=\frac{27}{4}$	M1, A1	Integration 6.75 (o.e.)
	So $E^2\le\frac{81}{4}$ i.e. $E\le\frac{9}{2}$	A1cso	
		(4)	
(d)	$\int x^2\left(1+x^3\right)^{\frac{1}{4}}dx=\frac{4}{15}\left(1+x^3\right)^{\frac{5}{4}}$	M1 A1	k(..) and 5/4 power All correct
	$\left\{\left[\frac{4}{15}\left(1+x^3\right)^{\frac{5}{4}}\right]_{-1}^{2}=\right\}\frac{4}{15}\left[(9)^{\frac{5}{4}}-0\right]=\frac{4}{15}\times 9\sqrt{3}=\frac{12\sqrt{3}}{5}$	A1cso	Must see one of the expr' between {..} and the answer
		(3)	
(e)	Let E = required integral.		
	f(x) = $\left(1+x^3\right)^{\frac{1}{4}}$ and g(x) = x^2	B1	Suitable f and g
	Then $[(d)]^2\le E\times\int_{-1}^{2}x^4\,dx$	M1	Suitable inequality for E
	$\int_{-1}^{2}x^4\,dx=\left[\frac{x^5}{5}\right]_{-1}^{2}=\frac{32}{5}--\frac{1}{5}=\frac{33}{5}$	M1	Allow slip e.g $\frac{16}{5}--\frac{1}{5}$ or $\frac{32}{5}-\frac{1}{5}$
	So $\frac{144\times 3}{25}\le E\times\frac{33}{5}\to E\ge\frac{144}{55}$	A1cso	
		(4) **(16)**	

Awarding of S and T marks		
Questions	**Marks**	
2, 3, 4	S1	For a fully correct solution that is succinct or includes an S+ point
5, 6, 7	S2	For a fully correct solution that is succinct and includes some S+ points
5, 6, 7	S1	For a fully correct solution that is succinct but does not mention any S+ points
5, 6, 7	S1	For a fully correct solution that is slightly laboured but includes an S+ point
5, 6, 7	S1	For a score of n -1 but solution is otherwise succinct or contains an S+ point
Maximum S score is 6		
ALL	T1	For at least half marks on all questions

Question	Scheme	Marks	Notes
7. (a)	$\text{f}'(x)=\frac{1}{3}-12x^{-2}$ $\text{f}'(x)=0 \Rightarrow x^2=36$ So A (6, 4) and $B\,(-6,-4)$ [1st A1 for $\pm$ 6 or (6, 4)]	M1 M1 A1A1 (4)	Some correct diff f '(x) =0 to give x^2=.. 2nd A1 is cso
(b)	k = 6 (Allow $k = \pm$ 6)	B1ft (1)	
(c) S+ for B1 comment	Grad of normal = $\frac{1}{3}$, so gradient of tangent must be -3 So $-3=\frac{1}{3}-12x^{-2}$ $\left[\text{f}'(x)=-3 \text{ or } \frac{-1}{\text{f}'(x)}=\frac{1}{3}\right]$ $x^2=\frac{36}{10}$ so $(\alpha=)\frac{6}{\sqrt{10}}$ or $\frac{3}{5}\sqrt{10}$ or $3\sqrt{\frac{2}{5}}$	B1M1 dM1 dM1 A1 (5)	M1 for perp. rule Form a suitable eqn using their f'(x) Solving suitable eqn $p\sqrt{q}$ where p or q is an integer
(d)	y coord: $\beta=\frac{\sqrt{10}}{5}+\frac{12\sqrt{10}}{6}=2.2\sqrt{10}$ or $\frac{11}{5}\sqrt{10}$ Equation of normal is: $y-\beta=\frac{1}{3}(x-\alpha)$ i.e. $y=\frac{1}{3}x+2\sqrt{10}$ (o.e.)	M1 M1 A1 (3)	Attempt y coord ft their α and β Must be values and m=$\frac{1}{3}$
(e)	 Shape (6, 4); (-6, 4) Asymptotes $x=0$, $y=\pm\frac{1}{3}x$	B1 B1ft B1B1 (4)	Both branches Follow through their A and B -1 each omission $y=\left\lvert\frac{x}{3}\right\rvert$ is OK
(f) S+ for comment S+ for comment on m > ...	If intersect then line = curve gives: $(3m-1)x^2+3x-36=0$ Discinimant <0 gives: $9<4\times(3m-1)(-36)$ Solving : $48m<15$, so $m<\frac{5}{16}$ From sketch : $-\frac{5}{16}<m<\frac{5}{16}$	M1 M1 M1 A1 A1 (5)	Attempt line = curve $\longrightarrow$3TQ Correct use of discr leading to ineq in m Solving to m < k A1 for k=$\frac{5}{16}$ (o.e.) Both [Allow M1M1M1 for MR of l for 1]
ALT (f)	Tangent at $\left(\delta,\frac{\delta}{3}+\frac{12}{\delta}\right)$ goes through (0, 1), gradient = m = f '(δ) Leads to equation: $\frac{1}{3}-\frac{12}{\delta^2}=\frac{\frac{\delta}{3}+\frac{12}{\delta}-1}{\delta}$ $\frac{\delta^2-36}{3\delta^2}=\frac{\delta^2+36-3\delta}{3\delta^2}\Rightarrow 3\delta=72$ or $\delta=24$ $m=\frac{1}{3}-\frac{12}{\delta^2}=\frac{5}{16}$ etc	M1 M1 **(22)**	Use of limiting case: gradient of chord = gradient of tangent (= gradient of line) Solve for δ Then as above

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