EDEXCEL FOUNDATION

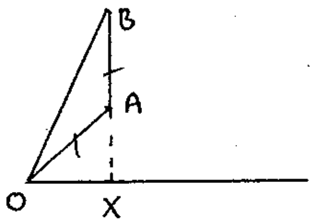
Stewart house 32 Russell Square London WC1B 5DN

June 2003

Advanced Extension Award

General Certificate of Education

Subject: **AEA** | Paper: **9801**

Question Number	Scheme	Marks
1.		
	Δ OAB is Isos (since $\lvert OA \rvert = \lvert AB \rvert = 1$)	B1
	$\angle OAB$ is $\frac{3\pi}{4}$ (since $\lvert OX \rvert = \lvert AX \rvert$ and $\therefore A\hat{O}X = \frac{\pi}{4}$)	B1
	$\therefore \angle BOA$ is $\frac{\pi}{8}$ or $\angle BOX$ is $\frac{3\pi}{8}$	B1
	$\tan\frac{3\pi}{8} = \frac{BX}{OX} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}, = \underline{\underline{\sqrt{2}+1}}$ (*)	M1, A1cso (5)
2. (See ALT)	$2\sin^2\theta - \tan\theta = 2\sin 2\theta - 2$ $\sin\theta\sec\theta = \tan\theta$	M1
	$2\tan^2\theta - \tan\theta\sec^2\theta = 2\frac{\sin 2\theta}{\cos^2\theta} - 2\sec^2\theta$ $\div \cos^2\theta$	M1
	$2\tan^2\theta - \tan\theta(1+\tan^2\theta) = 4\tan\theta - 2(1+\tan^2\theta)$ use of $\sec^2\theta = 1+\tan^2\theta$ & $\sin 2\theta$	M1
	$t = \tan\theta \Rightarrow \quad 0 = t^3 - 4t^2 + 5t - 2$ correct tidied cubic in $\tan\theta$	A1
	$0 = (t-1)(t^2-3t+2)$ 1st factor	M1
	$0 = (t-1)(t-1)(t-2)$ All factors	A1
	$\therefore \quad \underline{\underline{\tan\theta = 1 \text{ or } \tan\theta = 2}}$	A1, A1 (8)
	Style, Clarity & Presentation	
	(a) **S marks** For a novel or neat solution to any of questions 2-7. Apply once per question in upto 3 questions.	
	S2 if solution is fully correct in principle, elegance and accuracy	
	S1 if principle is sound but a minor algebraic or numerical slip occurs.	S6 (S2×3)
	(b) **T mark** For a good and largely accurate attempt at the whole paper.	T1 (7)

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3.	$\frac{dy}{dx} = \frac{2t}{3t^2} = \frac{2}{3t}$		M1
	@ P. $t = 2$, $\therefore$ gradient of tangent $= \frac{1}{3}$		A1
	Equation of tangent is: $y - 4 = \frac{1}{3}(x - 8)$ or $3y = x + 4$		M1 (3)
	Meets C @ Q i.e. $3t^2 = t^3 + 4$	curve = eqn	M1
	$0 = (t-2)(t^2 - t - 2)$ $0 = (t-2)(t-2)(t+1)$	Full method to solve	M1
	$\therefore$ @ Q $\underline{t = -1}$		A1 (3)
	$\int y\,dx = \int t^2 \frac{dx}{dt}\,dt = \int t^2 . 3t^2 dt = \int 3t^4 dt$	suitable $\int$	M1
	Area $= \int_{t=-1}^{t=2} 3t^4 dt = \left[3\frac{t^5}{5}\right]_{-1}^{2} = \left(\frac{96}{5}\right) - \left(-\frac{3}{5}\right)$	Correct $\int$ & use of lim	M1
	$= 19.8$		A1
	Required area is Trapezium $-$ 19.8.		
	Area of trapezium $= \frac{1}{2}(1+4) \times 9 = 22.5$		B1
	$\therefore$ required area $= 22.5 - 19.8 = \underline{\underline{2.7}}$		A1 (5)
			(11)

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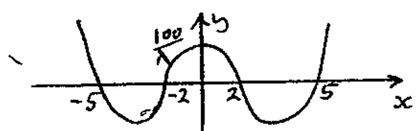
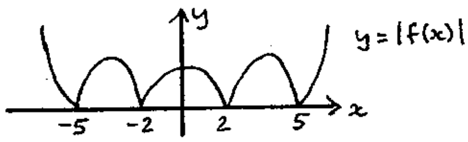
4. (a)	$1-3x \equiv (Ax+B)(1-x)^2 + C(1+3x^2)(1-x) + D(1+3x^2)$	M1
	$x=1 \Rightarrow -2 \equiv 4D$ $\therefore$ $\underline{D=-\frac{1}{2}}$	B1
	$x=0 \Rightarrow 1 = B+C+D$ or $B+C=3/2$ ① Coeff x^3: $0 = A-3C$ or $A=3C$ ② Coeff x : $-3 = A-2B-C$ ③ Sufficient suitable equations	M1
	② into ③ $\Rightarrow$ $-\frac{3}{2} = C-B$, solving with ① $\Rightarrow$ $\underline{C=0, A=0, B=\frac{3}{2}}$	A2/1/0 (5)
(b)	$f(x) \equiv \frac{3}{2}(1+3x^2)^{-1} - \frac{1}{2}(1-x)^{-2}$	
	$= \frac{3}{2}\left[1-3x^2+\frac{(-1)(-2)}{2!}9x^4\ldots\right] - \frac{1}{2}\left[1+2x+\frac{(-2)(-3)}{2!}x^2+\ldots x^3+\ldots x^4\right]$	M1 Use of Bin
	$= \left(\frac{3}{2} - \frac{9}{2}x^2 + \frac{27}{2}x^4\ldots\right) - \left(\frac{1}{2} + x + \frac{3}{2}x^2 + 2x^3 + \frac{5}{2}x^4\ldots\right)$	M1 Collecting Terms
	$= \underline{\underline{1 - x - 6x^2 - 2x^3 + 11x^4\ldots}}$	A2/1/0 (-1eeoo) (4)
(c)	$f'(x) = -1-12x\ldots$ $\therefore$ $f(0)=1$, $f'(0)=-1$ f(0), f'(0)	M1
	$\therefore$ equation of tangent is $\underline{y=1-x}$	A1 (2) (11)
5 (a)	 Shape & symmetry ± 5, ± 2 $\frac{100}{\lambda}$	B1 B1 B1 (3)
(b)	$f'(x) = \frac{1}{\lambda}\left[(x^2-4).2x + (x^2-25).2x\right]$ f'	M1
	$f'(x)=0 \Rightarrow 2x(2x^2-29)=0$, $\therefore$ $x = (0$ or$) \pm\sqrt{\frac{29}{2}}$ f'=0	M1, A1
	From diagram range is $f \geqslant f\left(\sqrt{\frac{29}{2}}\right)$, i.e. $\underline{\underline{f \geqslant -\frac{110.25}{\lambda} \text{ or } -\frac{441}{4\lambda}}}$	M1, A1 (5)
(c)		
	$\underline{\underline{k=2}}$ if $2 > \frac{110.25}{\lambda}$, i.e. $\underline{\underline{\lambda \geqslant 56}}$	M1, A1
	$\underline{\underline{k=4}}$ if $4 = \frac{110.25}{\lambda}$, $\lambda \notin \mathbb{Z}^+$ $\underline{\text{No Sol}^n}$	B1
	$\underline{\underline{k=6}}$ if $\frac{100}{\lambda} < 6 < \frac{110.25}{\lambda}$; i.e. $\frac{100}{6} < \lambda < \frac{110.25}{6}$ i.e. $\underline{\underline{\lambda = 17, 18}}$	M1; A1, A1
	$\underline{\underline{k=7}}$ if $\frac{100}{\lambda} = 7$, $\lambda \notin \mathbb{Z}^+$ $\underline{\text{No solution}}$	B1
	$\underline{\underline{k=8}}$ if $8 < \frac{100}{\lambda}$, i.e. $\lambda < \frac{100}{8}$ i.e. $\underline{\underline{0 < \lambda \leqslant 12}}$	M1, A1 (9) (17)

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6 (a)	$(\text{LHS})^2 = (2+\sqrt{3}) - 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} + (2-\sqrt{3})$ Attempt $(\text{LHS})^2$	M1
	$= 4 - 2\sqrt{4-3}$	A1
	$= 2 = (\text{RHS})^2$	A1 cso. (3)
	[$\because 2+\sqrt{3} > 2-\sqrt{3}$ we know LHS > 0]	
(b)	$\log_{1/8}\sqrt{2} = \dfrac{\log_2\sqrt{2}}{\log_2 1/8}, = \dfrac{\log_2 2^{1/2}}{\log_2 2^{-3}}, = \dfrac{1/2}{-3} = \underline{\underline{-\dfrac{1}{6}}}$ ⊛ Change of base; $2^{1/2}, 2^{-3}$	M1, M1, A1 cso (3)
ALT (b)	$\log_{1/8}\sqrt{2} = x \Rightarrow \sqrt{2} = \left(\frac{1}{8}\right)^x$	M1
	i.e. $2^{1/2} = 8^{-x} = (2^3)^{-x} = 2^{-3x}$ cf powers of 2	M1
	$\therefore \frac{1}{2} = -3x$ i.e. $\underline{x = -\frac{1}{6}}$	A1 cso.
(c) See ALT.	$\sqrt{a+\sqrt{15}} - \sqrt{a-\sqrt{15}} = \left(\frac{1}{n}\right)^{-\frac{1}{2}} = \sqrt{n}$ Out of logs $\sqrt{n}$	M1 A1
	Squaring $\Rightarrow a+\sqrt{15} - 2\sqrt{a^2-15} + a - \sqrt{15} = n$	M1
	i.e. $2(a - \sqrt{a^2-15}) = n$	A1
	So we require $a^2 - 15 = b^2$	M1
	i.e. $a^2 - b^2 = 15$	
	or $(a-b)(a+b) = 15$ Use of 'DOTS'	M1
	But $15 = 3\times5$ or 1×15	M1
	$3\times5 \Rightarrow \left.\begin{matrix} a-b=3 \\ a+b=5 \end{matrix}\right\} \therefore \underline{\underline{a=4}}$	A1
	Solving	M1
	$1\times15 \Rightarrow \left.\begin{matrix} a-b=1 \\ a+b=15 \end{matrix}\right\} \therefore \underline{\underline{a=8}}$	A1
	$a = 4, \quad n = 2(4 - \sqrt{4^2-15}) = 6$	
	$a = 8, \quad n = 2(8 - \sqrt{64-15}) = 2$ $\therefore$ Pairs are $\underline{a=4, n=6}$, $\underline{a=8, n=2}$	M1 A1 A1 (13)
		(19)

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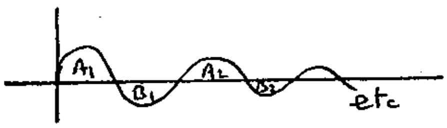
7 (a)	$e^{-x}\sin x = 0 \Rightarrow \sin x = 0,\ x = 0, n\pi$ ≥ 1 sol^n to $\sin x = 0$	M1
	x coordinates of P, Q, R are $\underline{P = \pi,\ Q = 2\pi,\ R = 3\pi}$	A1 (2)
(b)	$I = \left[-e^{-x}\cos x\right] + \int e^{-x}(-\cos x)\,dx$ Attempt Parts	M1 A1
	$= \left[-e^{-x}\cos x\right] - \left\{\left[e^{-x}\sin x\right] - \int -e^{-x}\sin x\,dx\right\}$ Second Parts Attempt	M1
	Recognise I	M1
	$I = -e^{-x}(\cos x + \sin x) - I$	
	i.e. $\underline{I = -\frac{1}{2}e^{-x}(\cos x + \sin x) + \text{constant}}$ ⊛	A1 cso (5)
(c)	$A_1 = \int_0^{\pi}$, $A_2 = \int_{2\pi}^{3\pi}$ ...... $A_n = \int_{(2n-2)\pi}^{(2n-1)\pi}$ $\int_{\text{EVEN}\,\pi}^{\text{ODD}\,\pi}$	M1 A1 A1
	$A_n = \left[-\frac{1}{2}e^{-x}(\cos x + \sin x)\right]_{(2n-2)\pi}^{(2n-1)\pi}$ Apply Limits	M1
	$= -\frac{1}{2}(-1+0)e^{-(2n-1)\pi} - -\frac{1}{2}(1+0)e^{-(2n-2)\pi} = \underline{\underline{\frac{1}{2}e^{\pi}e^{-2n\pi}, + \frac{1}{2}e^{2\pi}e^{-2n\pi}}}$	A1, A1 (6)
(d)	$A_1 = \frac{1}{2}(e^{\pi} + e^{2\pi})e^{-2\pi}$, $A_2 = \frac{1}{2}(e^{\pi} + e^{2\pi})e^{-4\pi}$ ... Identify GP	M1
	$a = \frac{1}{2}(1 + e^{-\pi})$ suitable a	A1
	$r = e^{-2\pi}$ " r	A1
	$S_\infty = \frac{a}{1-r} = \frac{1}{2}\frac{(1+e^{-\pi})}{1-e^{-2\pi}} = \frac{1(1+e^{-\pi})}{2(1-e^{-\pi})(1+e^{-\pi})}$ Use of S_∞	M1
	i.e. $S_\infty = \frac{1}{2(1-e^{-\pi})} = \underline{\frac{e^{\pi}}{2(e^{\pi}-1)}}$	A1 cso. (5)
(e)	 Let $S_A = \sum_1^{\infty} A_n$; $S_B = \sum_1^{\infty} \lvert B_n \rvert$	
	Given integral $\Rightarrow \frac{1}{2} = S_A - S_B$	M1 (Use of $\int = \frac{1}{2}$)
	We require S where $S = S_A + S_B$	M1
	Adding $\Rightarrow S + \frac{1}{2} = 2S_A$ i.e. $S = 2S_A - \frac{1}{2}$	M1 (S in terms of S_A)
	i.e. $S = \frac{2e^{\pi}}{2(e^{\pi}-1)} - \frac{1(e^{\pi}-1)}{2(e^{\pi}-1)} = \underline{\underline{\frac{e^{\pi}+1}{2(e^{\pi}-1)}}}$ o.e.	A1. (4)
		(22)

GBA. 11/02/03

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ALTERNATIVE SOLUTIONS

Question Number	Scheme		Marks
2.	$\frac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta - 1) = 2(\sin 2\theta - 1)$	$\sec\theta = \frac{1}{\cos\theta}$ + attempt factors	M1
	$\tan\theta(\sin 2\theta - 1) = 2(\sin 2\theta - 1)$	$(\sin 2\theta - 1)$ factor	M1
	$(\tan\theta - 2)(\sin 2\theta - 1) = 0$	Two factors both correct in $\tan\theta$ or $\sin 2\theta$	M1 A1
	$\therefore \sin 2\theta = 1 \Rightarrow \theta = 45, 225$		M1, A1
	$\therefore \underline{\tan\theta = 1, \text{ or } \tan\theta = 2}$		A1, A1 (8)
6 (c) TRIAL	1^{st} 5 marks as per scheme $\rightarrow$ seek $a^2 - 15 = b^2$		(5)
	Trial: $a = 4 \Rightarrow a^2 - 15 = 1$ $a = 5 \Rightarrow a^2 - 15 = 10$ $a = 6 \Rightarrow a^2 - 15 = 21$ $a = 7 \Rightarrow \quad = 34$ $a = 8 \Rightarrow \quad = 49$	Testing at least 3 values of $a \geqslant 4$ $a = 4$ $a = 8$	M1 A1 A1
	If $a = 8 \Rightarrow n = 2$ $a > 8 \Rightarrow n < 2$	Argument for $n < 2$, $n > 8$	B1
	$n \neq 1$ since if $n = 1$ $a - \sqrt{a^2 - 15} = 0.5$ and $(a - 0.5)^2 = a^2 - 15 \Rightarrow a = 15.25 \notin \mathbb{Z}$	Argument for $n = 1$	B1
		Possible pairs as in scheme	(3)
SQUARING	1^{st} 4 marks as per scheme $\rightarrow$ $2a - 2\sqrt{a^2 - 15} = n$ *		(4)
	Squaring $\rightarrow$ $a = \frac{n^2 + 60}{4n}$		M1
	$n = 2 \Rightarrow (a = 8)$ $n = 6 \Rightarrow (a = 4)$ $n = 10 \Rightarrow a = 4$ $n = 30 \Rightarrow a = 8$	Trying $n \geqslant 2$ for at least 3 values of n $n = 2$ $n = 6$	M1 A1 A1
		Full argument to show all cases considered	B2
		Checking all answers in *	M1
		$\left.\begin{array}{l} n = 2, a = 8 \\ n = 6, a = 4 \end{array}\right\}$ only valid answers	A2/1/0 −1 eeoo extra case.