Paper Reference 9MA0/02
Pearson Edexcel
Level 3 GCE

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

Time: 2 hours

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Diagram Booklet
Answer Booklet

X69602A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 16 questions in this Question Paper.
The total mark for this paper is 100

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Refer to the diagram for Question 1 in the Diagram Booklet.

It shows a sketch of the graph with equation

$y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(Total for Question 1 is 4 marks)

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2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

There are blank axes on pages 34 – 45 in the Answer Booklet if you wish to use them.

(2 marks)

(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2 marks)

(Total for Question 2 is 4 marks)

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2 marks)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2 marks)

(Total for Question 3 is 4 marks)

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(Total for Question 4 is 3 marks)

5. Refer to the table for Question 5 in the Diagram Booklet.

It shows corresponding values of x and y for

$y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3 marks)

(continued on the next page)

5. continued.

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10}\, dx$

(ii) $\int_3^9 \log_3 18x\, dx$

(3 marks)

(Total for Question 5 is 6 marks)

6. Refer to the diagram for Question 6 in the Diagram Booklet.

It shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in the diagram, is a local maximum point on the curve.

Using calculus and the sketch in the diagram,

(a) find the x coordinate of P, giving your answer to 3 significant figures.
(4 marks)

(continued on the next page)

6. continued.

The curve crosses the x–axis at $x = \alpha$, as shown in the diagram.

Given that, to 3 decimal places, $f(4) = 4 \cdot 274$ and $f(5) = -1 \cdot 212$

(b) explain why α must lie in the interval $[4, 5]$

(1 mark)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton–Raphson method once to $f(x)$ to obtain a second approximation to α

Show your method and give your answer to 3 significant figures.

(2 marks)

(Total for Question 6 is 7 marks)

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4 marks)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1 mark)

(Total for Question 7 is 5 marks)

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Refer to the diagram for Question 8 in the Diagram Booklet.

It shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in the diagram, is bounded by the curve and the x–axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(Total for Question 8 is 6 marks)

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9. **Refer to Diagram 1 and Diagram 2 for Question 9 in the Diagram Booklet.**
Diagram 1 shows a sketch of a Ferris wheel.

The height above the ground, H metres, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)^\circ|$$

where A, b and α are constants.

Diagram 2 shows a sketch of the graph of H against t, for one revolution of the wheel.

(continued on the next page)

9. continued.

Given that

- the maximum height of the passenger above the ground is 50 metres
- the passenger is 1 metre above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4 marks)

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)^{\circ}| + d$$

where d is a positive constant, would be a more appropriate model.

(1 mark)

(Total for Question 9 is 5 marks)

10. The function f is defined by

$$f(x) = \frac{8x + 5}{2x + 3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2 marks)

(b) Show that

$$f(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2 marks)

(continued on the next page)

10. continued.

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leq x \leq 4$$

(c) State the range of g^{-1}

(1 mark)

(d) Find the range of fg^{-1}

(3 marks)

(Total for Question 10 is 8 marks)

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$

(Total for Question 11 is 4 marks)

12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3 marks)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k

(3 marks)

(Total for Question 12 is 6 marks)

13. Relative to a fixed origin O

- the point A has position vector $4\underline{i} - 3\underline{j} + 5\underline{k}$
- the point B has position vector $4\underline{j} + 6\underline{k}$
- the point C has position vector $-16\underline{i} + p\underline{j} + 10\underline{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p

(3 marks)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $|\overrightarrow{OD}|$, writing your answer as a fully simplified surd.

(3 marks)

(Total for Question 13 is 6 marks)

14. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3 marks)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,m^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geq 0 \qquad t \geq k$$

where k is a constant.

(continued on the next page)

14. continued.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,m^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t - 1)}{(t + 1)}$$

(5 marks)

(continued on the next page)

14. continued.

The scientist noticed that

- **there was a TIME DELAY between the chemicals being mixed and oxygen being produced**
- **there was a LIMIT to the total volume of oxygen produced**

Deduce from the model

(c) (i) the TIME DELAY giving your answer in minutes,

(ii) the LIMIT giving your answer in m^3

(2 marks)

(Total for Question 14 is 10 marks)

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15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3 marks)

(continued on the next page)

15. continued.

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2 marks)

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1 - \sqrt{3})$$

where k is a constant to be found.

(5 marks)

(Total for Question 15 is 10 marks)

16. Refer to the diagram for Question 16 in the Diagram Booklet.

It shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad y = 2\sec^2 t + 3 \qquad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The line L is the normal to C at the point P where

$$t = \frac{\pi}{4}$$

(a) Using parametric differentiation, show that an equation for L is

$$y = -\frac{1}{2}x + \frac{17}{2}$$

(5 marks)

(continued on the next page)

16. continued.

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x - 1)^2 + 5$$

(2 marks)

The straight line with equation

$y = -\frac{1}{2}x + k$ where k is a constant

intersects C at two distinct points.

(c) Find the range of possible values for k

(5 marks)

(Total for Question 16 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER