Paper Reference 9MA0/01

Pearson Edexcel Level 3 GCE

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

Time: 2 hours

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Answer Booklet

Diagram Booklet

V69601A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **16** questions in this Question Paper. The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

You may be provided with a model for Question 15

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x), x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$

(1 mark)

(b) $y = |f(x)|$

(1 mark)

(continued on the next page)

Turn over

1. continued.

(c) $y = 3f(x - 2) + 2$

(2 marks)

(Total for Question 1 is 4 marks)

2. $f(x) = (x - 4)(x^2 - 3x + k) - 42$

where k is a constant

Given that $(x + 2)$ is a factor of $f(x)$,

find the value of k

(Total for Question 2 is 3 marks)

Turn over

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3 marks)

(continued on the next page)

3. continued.

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2 marks)

(Total for Question 3 is 5 marks)

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4. (a) Express $\lim_{\delta x \to 0} \sum_{x=2\cdot 1}^{6\cdot 3} \frac{2}{x}\delta x$ as an integral.

(1 mark)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2\cdot 1}^{6\cdot 3} \frac{2}{x}\delta x = \ln k$$

where k is a constant to be found.

(2 marks)

(Total for Question 4 is 3 marks)

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leq t < 25$$

where a and b are constants.

(continued on the next page)

5. continued.

Given that

- **the height of the tree was $2 \cdot 60$ metres, exactly 2 years after being planted**
- **the height of the tree was $5 \cdot 10$ metres, exactly 10 years after being planted**

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.
(4 marks)

(continued on the next page)

5. continued.

Given that the height of the tree was 7 metres, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.
(2 marks)

(Total for Question 5 is 6 marks)

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6. Refer to the diagram for Question 6 in the Diagram Booklet.

It shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(continued on the next page)

6. continued.

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1 mark)

(continued on the next page)

6. continued.

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.
(2 marks)

(c) Find the equation of C
You may leave your answer in factorised form.
(3 marks)

(Total for Question 6 is 6 marks)

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7. (i) Given that p and q are integers such that

pq is even

use algebra to prove by contradiction that at least one of p or q is even.

(3 marks)

(continued on the next page)

7. continued.

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2 marks)

(Total for Question 7 is 5 marks)

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8. Refer to the diagram for Question 8 in the Diagram Booklet.

A car stops at two sets of traffic lights.

The diagram shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

(continued on the next page)

8. continued.

The speed of the car is modelled by the equation

$$v = (10 - 0{\cdot}4t)\ln(t + 1) \qquad 0 \leq t \leq T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1 mark)

(continued on the next page)

8. continued.

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t+1)} - 1$$

(4 marks)

(continued on the next page)

8. continued.

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3 marks)

(Total for Question 8 is 8 marks)

9. Refer to the diagram for Question 9 in the Diagram Booklet.

It shows a sketch of a parallelogram PQRS

Given that

- $\overrightarrow{PQ} = 2\underline{i} + 3\underline{j} - 4\underline{k}$
- $\overrightarrow{QR} = 5\underline{i} - 2\underline{k}$

(a) show that parallelogram PQRS is a rhombus.

(2 marks)

(continued on the next page)

9. continued.

(b) Find the exact area of the rhombus PQRS

(4 marks)

(Total for Question 9 is 6 marks)

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220e^{0 \cdot 05t}$$

where t is the number of years from the start of the study.

(continued on the next page)

10. continued.

According to the model,

(a) find the number of bees at the start of the study,
(1 mark)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a RATE of approximately 18 thousand per year.
(3 marks)

(continued on the next page)

10. continued.

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800e^{-0 \cdot 05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(continued on the next page)

10. continued.

(c) Find the value of T to 2 decimal places.

(4 marks)

(Total for Question 10 is 8 marks)

11. Refer to the diagram for Question 11 in the Diagram Booklet.

It shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(continued on the next page)

11. continued.

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2 marks)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5 marks)

(Total for Question 11 is 7 marks)

12. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(Total for Question 12 is 5 marks)

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13. (i) In an arithmetic series, the first term is a and the common difference is d

Show that

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

(3 marks)

(continued on the next page)

Turn over

13. continued.

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9·20 in week 2, £8·40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

(continued on the next page)

13. (ii) continued.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2 marks)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1 mark)

(continued on the next page)

Turn over

13. (ii) continued.

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.
(1 mark)

(Total for Question 13 is 7 marks)

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(continued on the next page)

14. continued.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4 marks)

(continued on the next page)

14. continued.

(b) **Hence or otherwise solve, for**

$$0 \leq \theta < 180°$$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4 marks)

(Total for Question 14 is 8 marks)

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15. Refer to Diagram 1 and Diagram 2 for Question 15 in the Diagram Booklet. You may be provided with a model.

A company makes toys for children.

Diagram 1 and the model shows the design for a solid toy that looks like a piece of cheese.

(continued on the next page)

15. continued.

The toy is modelled so that

- **face ABC is a sector of a circle with radius r cm and centre A as shown by Diagram 2**
- **angle BAC = $0 \cdot 8$ radians**
- **faces ABC and DEF are congruent**
- **edges AD, CF and BE are perpendicular to faces ABC and DEF**
- **edges AD, CF and BE have length h cm**

(continued on the next page)

15. continued.

Given that the volume of the toy is $240\,cm^3$

(a) show that the surface area of the toy, $S\,cm^2$, is given by

$$S = 0 \cdot 8r^2 + \frac{1680}{r}$$

making your method clear.

(4 marks)

(continued on the next page)

15. continued.

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.
(4 marks)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.
(2 marks)

(Total for Question 15 is 10 marks)

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16. Refer to the diagram for Question 16 in the Diagram Booklet.

It shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t$$
$$y = 2\sin 2t + 3\sin t$$
$$0 \leq t \leq \frac{\pi}{2}$$

The region R, shown shaded in the diagram, is bounded by C, the x–axis and the line with equation $x = 4$

(continued on the next page)

Turn over

16. continued.

(a) Show that the area of R is given by

$$\int_0^a (8 - 8\cos 4t + 48\sin^2 t\cos t)\,dt$$

where a is a constant to be found.

(5 marks)

(continued on the next page)

16. continued.

(b) Hence, using algebraic integration, find the exact area of R

(4 marks)

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
