Paper Reference 9FM0/3D
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X72094A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return the Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams in the Diagram Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 7 questions in this Question Paper.
The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There are two copies of each diagram in case you need them.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

---

1. A gardener needs the eleven lengths of string shown below.

All lengths are in metres.

4·3	6·1	5·1	4·7	2·5	5·9
3·4	1·7	2·1	0·4	1·3	

She cuts the lengths from balls of string.
Each ball contains 10 metres of string.

(a) Calculate a lower bound for the number of balls of string the gardener needs.
You must make your method clear.
(2 marks)

(b) Use the first–fit bin packing algorithm to determine how the lengths could be cut from the balls of string.
(3 marks)

(Total for Question 1 is 5 marks)

2. Refer to Diagram 1 and Diagram 2 in the Diagram Booklet.

[The total weight of the network is 299]

Diagram 1 represents a network of cycle tracks between 10 landmarks, **A**, **B**, **C**, **D**, **E**, **F**, **G**, **H**, **J** and **K**

The number on each edge represents the length, in kilometres, of the corresponding track.

One day, Blanche wishes to cycle from **A** to **K**
She wishes to minimise the distance she travels.

(a) (i) Use Dijkstra's algorithm and Diagram 2 in the Diagram Booklet to find the shortest path from **A** to **K**

(ii) State the length of the shortest path from **A** to **K**

(6 marks)

(continued on the next page)

2. continued.

The cycle tracks between the landmarks now need to be inspected.

Blanche must travel along each track at least once.

She wishes to minimise the length of her inspection route.

Blanche will start her inspection route at **D** and finish at **E**

(b) (i) State the edges that will need to be traversed twice.

(ii) Find the length of Blanche's route.

(2 marks)

(continued on the next page)

2. continued.

It is now decided to start the inspection route at A and finish at K

Blanche must minimise the length of her route and travel along each track at least once.

(c) By considering the pairings of all relevant nodes, find the length of Blanche's new route. You must make your method and working clear. (5 marks)

(Total for Question 2 is 13 marks)

---

3. Refer to Diagram 3, Diagram 4, Diagram 5 and Diagram 6 in the Diagram Booklet.

The initial distance matrix (Diagram 3) shows the lengths, in metres, of the corridors connecting six classrooms, A, B, C, D, E and F, in a school. For safety reasons, some of the corridors are one–way only.

(a) By adding the arcs from vertex A, along with their weights, complete the drawing of this network on Diagram 4 in the Diagram Booklet. (2 marks)

(continued on the next page)

3. continued.

Floyd's algorithm is to be used to find the complete network of shortest distances between the six classrooms.

The distance matrix after TWO iterations of Floyd's algorithm is shown in Diagram 5 in the Diagram Booklet.

(b) Perform the next two iterations of Floyd's algorithm that follow from Diagram 5 You should show the distance matrix after each iteration using the tables in the Answer Booklet.
(4 marks)

(continued on the next page)

3. continued.

Refer to Diagram 6 in the Diagram Booklet.
It shows the final distance matrix after completion of Floyd's algorithm.

Yinka must visit each classroom.
He will start and finish at E and wishes to minimise the total distance travelled.

(c) (i) Use the nearest neighbour algorithm, starting at E, to find two Hamiltonian cycles in the completed network of shortest distances.

(ii) Find the length of each of the two cycles.

(iii) State, with a reason, which of the two cycles provides the better upper bound for the length of Yinka's route.

(4 marks)

(Total for Question 3 is 10 marks)

---

4. Refer to Diagram 7 in the Diagram Booklet.

A linear programming problem in x, y and z is to be solved using the big–M method.

The initial tableau is shown in Diagram 7

(a) Using the information in Diagram 7, formulate the linear programming problem.

You should

- **list each of the constraints as an inequality**
- **state the two possible objectives**

(4 marks)

(b) Obtain the most efficient pivot for a first iteration of the big–M method.

You must give reasons for your answer.

(2 marks)

(Total for Question 4 is 6 marks)

---

5. Refer to Diagram 8, Diagram 9, Diagram 10, Diagram 11, Diagram 12 and Diagram 13 in the Diagram Booklet.

The network shown in Diagram 8 in the Diagram Booklet shows the activities that need to be completed for a project.
Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets.
The early event times are shown in the diagram.

(a) Complete Diagram 9 in the Diagram Booklet to show the immediately preceding activities for each activity.
(2 marks)

(continued on the next page)

5. continued.

It is given that $4 < x \leq m$

(b) State the largest possible integer value of m

(1 mark)

(c) (i) Complete Diagram 10 in the Diagram Booklet to show the late event times.

(ii) State the activities that must be critical.

(3 marks)

(d) Calculate the total float for activity G

(1 mark)

(continued on the next page)

5. continued.

The resource histogram shown in Diagram 11 in the Diagram Booklet shows the number of workers required when each activity starts at its earliest possible time.

The histogram also shows which activities happen at each time.

(e) Complete Diagram 12 in the Diagram Booklet to show the number of workers required for each activity of the project.

(2 marks)

(f) Draw a Gantt chart on Diagram 13 in the Diagram Booklet to represent the activity network.

(5 marks)

(Total for Question 5 is 14 marks)

---

6. The following algorithm determines the number of comparisons made when Prim's algorithm is applied to K_n

Step 1	Start
Step 2	Input the value of n
Step 3	Let $a = 1$
Step 4	Let $b = n - 2$
Step 5	Let $c = b$
Step 6	Let $a = a + 1$
Step 7	Let $b = b - 1$
Step 8	Let $c = c + (a \times b) + (a - 1)$
Step 9	If $b > 0$ go to Step 6
Step 10	Output c
Step 11	Stop

(a) For K_5, complete the table in the Answer Booklet to show the results obtained at each step of the algorithm.
(3 marks)

(continued on the next page)

6. continued.

Refer to Diagram 14 in the Diagram Booklet.

The weights of the ten arcs in the diagram are shown below.

17	21	24	14	23
13	15	19	28	20

(b) (i) Starting at the left–hand end of the above list, sort the list into ascending order using bubble sort.

You need only write down the state of the list at the end of each pass.

(ii) Find the total number of comparisons performed during the sort.

(5 marks)

(continued on the next page)

6. continued.

(c) Find the maximum total number of comparisons required to sort the weights of the 10 arcs of K_5 into ascending order using bubble sort.

(1 mark)

It is given that the maximum total number of comparisons required to sort the weights of the arcs of K_n into ascending order using bubble sort is

$$\lambda n(n-1)(n+1)(n-2)$$

where λ is a constant.

(d) Determine the maximum total number of comparisons required to sort the weights of the arcs of K_{50} into ascending order using bubble sort.

You must make your method and working clear.

(3 marks)

(Total for Question 6 is 12 marks)

7. Refer to Diagram 15 and Diagram 16 in the Diagram Booklet.

Diagram 15 shows the constraints of a linear programming problem in x and y where R is the feasible region.

The objective is to maximise $P = x + ky$, where k is a positive constant.

The optimal vertex of R is to be found using the Simplex algorithm.

(a) Set up an initial tableau in the Answer Booklet for solving this linear programming problem using the Simplex algorithm.
(5 marks)

(continued on the next page)

7. continued.

After two iterations of the Simplex algorithm a possible tableau T is shown in Diagram 16 in the Diagram Booklet.

(b) State the value of each variable after the second iteration.
(1 mark)

(continued on the next page)

7. continued.

It is given that T does not give an optimal solution to the linear programming problem.

After a third iteration of the Simplex algorithm the resulting tableau does give an optimal solution to the problem.

(c) Perform the third iteration of the Simplex algorithm using the table in the Answer Booklet and hence determine the range of possible values for P

You should state the row operations you use and make your method and working clear.

(9 marks)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
