Paper Reference 9FM0/01

Pearson Edexcel

Level 3 GCE

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

Time: 1 hour 30 minutes

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Q71776A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are **10** questions in this Question Paper.

The total mark for this paper is **75**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. $f(z) = z^3 + az + 52$ where a is a real constant

Given that $2 - 3i$ is a root of the equation $f(z) = 0$

(a) write down the other complex root.
(1 mark)

(b) Hence

(i) solve completely $f(z) = 0$

(ii) determine the value of a

(4 marks)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.
(1 mark)

(Total for Question 1 is 6 marks)

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2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0$$

Give your answers in the form $q\ln 2$ where q is rational and in simplest form.

(Total for Question 2 is 4 marks)

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3. (a) Determine the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = e^{2x} \cos^2 x$$

giving your answer in the form $y = f(x)$

(3 marks)

Given that $y = 3$ when $x = 0$

(b) determine the smallest positive value of x for which $y = 0$

(3 marks)

(Total for Question 3 is 6 marks)

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4. (a) Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right)$$

(4 marks)

(b) Hence find the exact value of

$$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35}$$

Give your answer in the form $a\ln\left(\frac{b}{c}\right)$ where a, b and c are integers to be determined.

(3 marks)

(Total for Question 4 is 7 marks)

5. $\mathbf{M} = \begin{pmatrix} a & 2 & -3 \\ 2 & 3 & 0 \\ 4 & a & 2 \end{pmatrix}$ where a is a constant

(a) Show that $\mathbf{M}$ is non-singular for all values of a

(2 marks)

(b) Determine, in terms of a, $\mathbf{M}^{-1}$

(4 marks)

(Total for Question 5 is 6 marks)

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6. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x + 1)(x^2 + 4)}$$

(3 marks)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x + 1)(x^2 + 4)}\,dx = \ln\left(a\sqrt{2}\right) + b\pi$$

where a and b are constants to be determined.

(4 marks)

(Total for Question 6 is 7 marks)

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7. Given that $z = a + bi$ is a complex number where a and b are real constants,

(a) show that zz^* is a real number.

(2 marks)

Given that

- $zz^* = 18$
- $\frac{z}{z^*} = \frac{7}{9} + \frac{4\sqrt{2}}{9}i$

(b) determine the possible complex numbers z

(5 marks)

(Total for Question 7 is 7 marks)

8. (a) Given

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad n \in \mathbb{N}$$

show that

$$32\cos^6\theta \equiv \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

(5 marks)

(continued on the next page)

8. continued.

Refer to the diagrams for Question 8 in the Diagram Booklet.

Diagram 1 shows a solid paperweight with a flat base.

Diagram 2 shows the curve with equation

$$y = H\cos^3\left(\frac{x}{4}\right) \qquad -4 \le x \le 4$$

where H is a positive constant and x is in radians.

The region R, shown shaded in Diagram 2, is bounded by the curve, the line with equation $x = -4$, the line with equation $x = 4$ and the x–axis.

The paperweight is modelled by the solid of revolution formed when R is rotated $180°$ about the x–axis.

(continued on the next page)

8. continued.

Given that the maximum height of the paperweight is 2 cm,

(b) write down the value of H

(1 mark)

(c) Using algebraic integration and the result in part (a), determine, in cm^3, the volume of the paperweight, according to the model.
Give your answer to 2 decimal places.

[Solutions based entirely on calculator technology are not acceptable.]

(5 marks)

(d) State a limitation of the model.

(1 mark)

(Total for Question 8 is 12 marks)

9. (i) (a) Explain why $\int_0^\infty \cosh x \, dx$ is an improper integral.

(1 mark)

(b) Show that $\int_0^\infty \cosh x \, dx$ is divergent.

(3 marks)

(ii) $4 \sinh x = p \cosh x$ where p is a real constant

Given that this equation has real solutions, determine the range of possible values for p

(2 marks)

(Total for Question 9 is 6 marks)

10. Refer to the diagram for Question 10 in the Diagram Booklet.

The motion of a pendulum, shown in the diagram, is modelled by the differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

where θ is the angle, in radians, that the pendulum makes with the downward vertical, t seconds after it begins to move.

(a) (i) Show that a particular solution of the differential equation is

$$\theta = \frac{1}{12}t\sin 3t$$

(4 marks)

(ii) Hence, find the general solution of the differential equation.

(4 marks)

(continued on the next page)

10. continued.

Initially, the pendulum

- **makes an angle of $\frac{\pi}{3}$ radians with the downward vertical**
- **is at rest**

Given that, 10 seconds after it begins to move, the pendulum makes an angle of α radians with the downward vertical,

(b) determine, according to the model, the value of α to 3 significant figures.

(4 marks)

(continued on the next page)

10. continued.

Given that the true value of α is $0\cdot62$

(c) evaluate the model.

(1 mark)

The differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

models the motion of the pendulum as moving with forced harmonic motion.

(d) Refine the differential equation so that the motion of the pendulum is simple harmonic motion.

(1 mark)

(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
