Paper Reference 9FM0/4C
Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

Time: 1 hour 30 minutes

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Answer Booklet

Diagram Booklet

Y72114A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Unless otherwise indicated, whenever a value of g is required, take $g = 9{\cdot}8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 8 questions in this Question Paper. The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Three particles of masses $2m$, $3m$ and km are placed at the points with coordinates $(3a, 2a)$, $(a, -4a)$ and $(-3a, 4a)$ respectively.

The centre of mass of the three particles lies at the point with coordinates $(\bar{x}, \bar{y})$

(a) (i) Find $\bar{x}$ in terms of a and k

(ii) Find $\bar{y}$ in terms of a and k

(4 marks)

(continued on the next page)

1. continued.

Given that the distance of the centre of mass of the three particles from the point $(0, 0)$ is $\frac{1}{3}a$

(b) find the possible values of k

(2 marks)

(Total for Question 1 is 6 marks)

2. A cyclist and her cycle have a combined mass of $60\ \text{kg}$

The cyclist is moving along a straight horizontal road and is working at a constant rate of 200 watts.

When she has travelled a distance x metres, her speed is $v\ \text{m s}^{-1}$ and the magnitude of the resistance to motion is $3v^2$ newtons.

(a) Show that $\dfrac{dv}{dx} = \dfrac{200 - 3v^3}{60v^2}$

(4 marks)

(continued on the next page)

2. continued.

The distance travelled by the cyclist as her speed increases from $2\,\mathrm{m\,s^{-1}}$ to $4\,\mathrm{m\,s^{-1}}$ is D metres.

(b) Find the exact value of D

(3 marks)

(Total for Question 2 is 7 marks)

3. Refer to the diagram for Question 3 in the Diagram Booklet.

Nine uniform rods are joined together to form the rigid framework **ABCDEFA**, with **AB = BC = DF = 3a**, **BF = CD = DE = 4a** and **AF = FE = CF = 5a**, as shown in the diagram in the Diagram Booklet.

All nine rods lie in the same plane.

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3. continued.

The mass per unit length of each of the rods **BF**, **CF** and **DF** is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from **AC**

(4 marks)

(continued on the next page)

3. continued.

The mass of the framework is M

A particle of mass kM is attached to the framework at E to form a loaded framework.

When the loaded framework is freely suspended from F, it hangs in equilibrium with CE horizontal.

(b) Find the exact value of k

(3 marks)

(Total for Question 3 is 7 marks)

4. Refer to the diagram for Question 4 in the Diagram Booklet.

A small smooth ring R of mass m is threaded onto a light inextensible string.

One end of the string is attached to a fixed point A and the other end of the string is attached to the fixed point B such that B is vertically above A and $AB = 6a$

(continued on the next page)

4. continued.

The ring moves with constant angular speed ω in a horizontal circle with centre A

The string is taut and BR makes a constant angle θ with the downward vertical, as shown in the diagram in the Diagram Booklet.

The ring is modelled as a particle.

(continued on the next page)

4. continued.

Given that $\tan\theta = \frac{8}{15}$

(a) find, in terms of m and g, the magnitude of the tension in the string,

(3 marks)

(b) find ω in terms of a and g

(5 marks)

(Total for Question 4 is 8 marks)

5. Refer to the diagram for Question 5 in the Diagram Booklet.

The uniform plane lamina shown in the diagram in the Diagram Booklet is formed from two squares, $ABCO$ and $ODEF$, and a sector ODC of a circle with centre O

Both squares have sides of length $3a$ and AO is perpendicular to OF

The radius of the sector is $3a$

(continued on the next page)

5. continued.

[In part (a) you may use, without proof, any of the centre of mass formulae given in the Formulae Booklet.]

(a) Show that the distance of the centre of mass of the sector ODC from OC is $\frac{4a}{\pi}$

(3 marks)

(b) Find the distance of the centre of mass of the lamina from FC

(4 marks)

(continued on the next page)

5. continued.

The lamina is freely suspended from F and hangs in equilibrium with FC at an angle θ° to the downward vertical.

(c) Find the value of θ

(4 marks)

(Total for Question 5 is 11 marks)

6. Refer to the diagram for Question 6 in the Diagram Booklet.

The shaded region shown in the diagram in the Diagram Booklet is bounded by the x–axis, the line with equation $x = 9$ and the line with equation $y = \frac{1}{3}x$

This shaded region is rotated through $360°$ about the x–axis to form a solid of revolution.

This solid of revolution is used to model a solid right circular cone of height 9 cm and base radius 3 cm

(continued on the next page)

6. continued.

The cone is non−uniform and the mass per unit volume of the cone at the point (x, y, z) is $\lambda x \, \text{kg} \, \text{cm}^{-3}$, where $0 \leq x \leq 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex.

(6 marks)

(continued on the next page)

6. continued.

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius 3 cm, so that the centres of the two plane surfaces coincide.

The weight of the cone is W newtons and the weight of the hemisphere is kW newtons.

(continued on the next page)

6. continued.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of k

(4 marks)

(Total for Question 6 is 10 marks)

7. Refer to the diagram for Question 7 in the Diagram Booklet.

A package P of mass m is attached to one end of a string of length $\frac{2a}{5}$
The other end of the string is attached to a fixed point O
The package hangs at rest vertically below O with the string taut and is then projected horizontally with speed u, as shown in the diagram in the Diagram Booklet.

(continued on the next page)

7. continued.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T

The package is modelled as a particle and the string as being light and inextensible.

(a) Show that

$$T = 3mg\cos\theta - 2mg + \frac{5mu^2}{2a}$$

(6 marks)

(continued on the next page)

7. continued.

Given that P moves in a complete vertical circle with centre O

(b) find, in terms of a and g, the minimum possible value of u

(2 marks)

Given that $u = 2\sqrt{ag}$

(c) find, in terms of g, the magnitude of the acceleration of P at the instant when OP is horizontal.

(3 marks)

(continued on the next page)

7. continued.

(d) Apart from including air resistance, suggest one way in which the model could be refined to make it more realistic.

(1 mark)

(Total for Question 7 is 12 marks)

8. **Throughout this question, use $g = 10\,m\,s^{-2}$**

A light elastic string has natural length $1 \cdot 25$ metres and modulus of elasticity 25 newtons.

A particle P of mass $0 \cdot 5$ kg is attached to one end of the string. The other end of the string is attached to a fixed point A
Particle P hangs freely in equilibrium with P vertically below A

(continued on the next page)

8. continued.

The particle is then pulled vertically down to a point B and released from rest.

(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\frac{\pi}{\sqrt{10}}$ seconds.

(6 marks)

(continued on the next page)

8. continued.

The maximum kinetic energy of P during the subsequent motion is $2 \cdot 5$ joules.

(b) Show that $AB = 2$ metres

(3 marks)

(continued on the next page)

8. continued.

The particle returns to B for the first time T seconds after it was released from rest at B

(c) Find the value of T

(5 marks)

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
