Paper Reference 9FM0/4D
Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green)
Calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X72118A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return the Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams in the Diagram Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 8 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There are two copies of each diagram in case you need them.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to Diagram 1 in the Diagram Booklet.

Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4

Each task must be assigned to just one worker and each worker must do only one task.

The cost of assigning each worker to each task is shown in Diagram 1 in the Diagram Booklet.

The total cost is to be minimised.

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the total cost.

You must make your method clear and show the table after each stage.

(5 marks)

(b) State the minimum total cost.

(1 mark)

(Total for Question 1 is 6 marks)

2. The general solution of the second order recurrence relation

$$u_{n+2} + k_1 u_{n+1} + k_2 u_n = 0 \quad n \geq 0$$

is given by

$$u_n = (A + Bn)(-3)^n$$

where A and B are arbitrary non–zero constants.

(a) Find the value of k_1 and the value of k_2
(2 marks)

Given that $u_0 = u_1 = 1$

(b) find the value of A and the value of B
(2 marks)

(Total for Question 2 is 4 marks)

3. **Refer to Diagram 2 in the Diagram Booklet.**

It shows the transport options, usual travel times, possible delay times and corresponding probabilities of delay for a journey.

All times are in minutes.

(a) **Draw a decision tree to model the transport options and the possible outcomes.**

(5 marks)

(b) **State the minimum expected travel time and the corresponding transport option indicated by the decision tree.**

(2 marks)

(Total for Question 3 is 7 marks)

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4. Refer to Diagram 3 in the Diagram Booklet.

It shows a capacitated, directed network of pipes.

The uncircled number on each arc represents the capacity of the corresponding pipe.

The numbers in circles represent an initial flow.

(a) List the saturated arcs.

(1 mark)

(b) State the value of the initial flow.

(1 mark)

(c) Explain why arc FT cannot be full to capacity.

(1 mark)

(d) State the capacity of cut C_1 and the capacity of cut C_2

(2 marks)

(continued on the next page)

4. continued.

(e) By inspection find one flow–augmenting route to increase the flow by three units.
You must state your route.
(1 mark)

(f) Prove that, once the flow–augmenting route found in part (e) has been applied, the flow is maximal.
(3 marks)

(Total for Question 4 is 9 marks)

5. A standard transportation problem is described in the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where $i \in \{A, B, C, D\}$

$j \in \{R, S, T\}$ and $x_{ij} \geq 0$

$$\text{Minimise } P = 23x_{AR} + 17x_{AS} + 24x_{AT} + 15x_{BR} + 29x_{BS} + 32x_{BT} + 25x_{CR} + 25x_{CS} + 27x_{CT} + 19x_{DR} + 20x_{DS} + 25x_{DT}$$

subject to

$$\sum x_{Aj} \leq 34$$
$$\sum x_{Bj} \leq 27$$
$$\sum x_{Cj} \leq 41$$
$$\sum x_{Dj} \leq 18$$
$$\sum x_{iR} \geq 44$$
$$\sum x_{iS} \geq 37$$
$$\sum x_{iT} \geq k$$

(continued on the next page)

5. continued.

Given that the problem is balanced,

(a) state the value of k

(1 mark)

(b) Explain precisely what the constraint $\sum x_{iR} \geq 44$ means in the transportation problem.

(2 marks)

(c) Use the north–west corner method to obtain the cost of an initial solution to this transportation problem.

(2 marks)

(continued on the next page)

5. continued.

(d) Perform one iteration of the stepping–stone method to obtain an improved solution.
You must make your method clear by showing the route and the

- shadow costs
- improvement indices
- entering cell and exiting cell.

(4 marks)

(Total for Question 5 is 9 marks)

6. Refer to Diagram 4 and Diagram 5 in the Diagram Booklet.
Bernie makes garden sheds.
He can build up to four sheds each month.

If he builds more than two sheds in any one month, he must hire an additional worker at a cost of £250 for that month.

In any month in which sheds are made, the overhead costs are £35 for each shed made that month.

A maximum of three sheds can be held in storage at the end of any one month, at a cost of £80 per shed per month.

Sheds must be delivered at the end of the month.

The order schedule for sheds is shown in Diagram 4 in the Diagram Booklet.

(continued on the next page)

6. continued.

There are no sheds in storage at the beginning of January and Bernie plans to have no sheds left in storage after the May delivery.

Use dynamic programming to determine the production schedule that minimises the costs given on the previous page.
Complete the working in Diagram 5 provided in the Diagram Booklet and state the minimum cost.

(Total for Question 6 is 14 marks)

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7. Refer to Diagram 6 in the Diagram Booklet.

A two person zero–sum game is represented by the pay–off matrix for player A shown in Diagram 6 in the Diagram Booklet.

It is given that k is an integer.

(a) Show that Q is the play–safe option for player A regardless of the value of k

(2 marks)

Given that Z is the play–safe option for player B,

(b) determine the range of possible values of k

You must make your working clear.

(2 marks)

(c) Explain why player B should never play option X

You must make your reasoning clear.

(2 marks)

(continued on the next page)

7. continued.

Player A intends to make a random choice between options Q, R and S, choosing option Q with probability p_1, option R with probability p_2 and option S with probability p_3

Player A wants to find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

Given that $k > -4$, player A formulates the following objective function for the corresponding linear program.

Maximise $P = V$, where $V =$ the value of the original game $+ 4$

(d) (i) Formulate the constraints of the linear programming problem for player A
You should write the constraints as equations.

(ii) Write down an initial Simplex tableau, making your variables clear.

(7 marks)

(continued on the next page)

7. continued.

The Simplex algorithm is used to solve the linear programming problem.

It is given that in the final Simplex tableau the optimal value of $p_1 = \frac{7}{37}$, the optimal value of $p_2 = \frac{17}{37}$ and all the slack variables are zero.

(e) Determine the value of k, making your method clear.

(4 marks)

(Total for Question 7 is 17 marks)

8. The owner of a new company models the number of customers that the company will have at the end of each month.

The owner assumes that

- a constant proportion, p (where $0 < p < 1$), of the previous month's customers will be retained for the next month
- a constant number of new customers, k, will be added each month.

Let u_n (where $n \geq 1$) represent the number of customers that the company will have at the end of n months.

The company has 5000 customers at the end of the first month.

(a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n, determine an expression for u_n in terms of n, p and k

(6 marks)

(continued on the next page)

8. continued.

The owner believes that 95% of the previous month's customers will be retained each month and that there will be $10\,000$ new customers each month.

According to the model, the company will first have at least $135\,000$ customers by the end of the mth month.

(b) Using logarithms, determine the value of m

(3 marks)

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
