

**Paper Reference 9FM0/02**  
**Pearson Edexcel**  
**Level 3 GCE**

**Further Mathematics**  
**Advanced**  
**PAPER 2: Core Pure Mathematics 2**

**Time: 1 hour 30 minutes**

**YOU MUST HAVE**

**Mathematical Formulae and Statistical  
Tables (Green), calculator**

**YOU WILL BE GIVEN**

**Diagram Booklet**  
**Answer Booklet**

**Y71801A**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## **INSTRUCTIONS**

**In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.**

**Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.**

**Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.**

**Do NOT write on the Question Paper.**

**You should show sufficient working to make your methods clear. Answers without working may not gain full credit.**

**Inexact answers should be given to three significant figures unless otherwise stated.**

**Turn over**

## **INFORMATION**

**A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.**

**There are 9 questions in this Question Paper. The total mark for this paper is 75**

**The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.**

## **ADVICE**

**Read each question carefully before you start to answer it.**

**Try to answer every question.**

**Check your answers if you have time at the end.**

1. Refer to the information for Question 1 in the Diagram Booklet. A student was asked to answer the following:

For the complex numbers  $z_1 = 3 - 3i$  and  $z_2 = \sqrt{3} + i$ , find the value of  $\arg\left(\frac{z_1}{z_2}\right)$

The student's attempt is shown in the Diagram Booklet.

The student made errors in line 1 and line 3

(continued on the next page)

Turn over

**1. continued.**

**Correct the error that the student  
made in**

**(a) (i) line 1**

**(ii) line 3**

**(2 marks)**

**(b) Write down the correct value**

**of  $\arg\left(\frac{z_1}{z_2}\right)$**

**(1 mark)**

**(Total for Question 1 is 3 marks)**

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**Turn over**

- 2. In this question you must show all stages of your working.**

**A college offers only three courses:  
Construction, Design and Hospitality.**

**Each student enrolls on just one of  
these courses.**

**In 2019, there was a total of  
1110 students at this college.**

**(continued on the next page)**

**Turn over**

**2. continued.**

**There were 370 more students enrolled on Construction than Hospitality.**

**In 2020 the number of students enrolled on**

- Construction INCREASED by 1·25%**
- Design INCREASED by 2·5%**
- Hospitality DECREASED by 2%**

**(continued on the next page)**

**Turn over**



**2. continued.**

**In 2020, the total number of students at the college increased by  $0.27\%$  to 2 significant figures.**

- (a) (i) Define, for each course, a variable for the number of students enrolled on that course in 2019**
- (ii) Using your variables from part (a)(i), write down THREE equations that model this situation.**

**(4 marks)**

**(continued on the next page)**

**Turn over**

**2. continued.**

- (b) By forming and solving a matrix equation, determine how many students were enrolled on each of the three courses in 2019**  
**(4 marks)**

**(Total for Question 2 is 8 marks)**

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3.  $M = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$  where  $a$  is a constant

(a) Prove by mathematical induction that, for  $n \in \mathbb{N}$

$$M^n = \begin{pmatrix} 3^n & \frac{a}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}$$

(6 marks)

(continued on the next page)

Turn over

3. continued.

Triangle **T** has vertices **A**, **B** and **C**

Triangle **T** is transformed to triangle **T'** by the transformation represented by  $M^n$  where  $n \in \mathbb{N}$

Given that

- triangle **T** has an area of  $5 \text{ cm}^2$
- triangle **T'** has an area of  $1215 \text{ cm}^2$
- vertex **A**(2, −2) is transformed to vertex **A'**(123, −2)

(continued on the next page)

Turn over

**3. continued.**

**(b) determine**

**(i) the value of  $n$**

**(ii) the value of  $a$**

**(5 marks)**

**(Total for Question 3 is 11 marks)**

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**Turn over**

4. (i) Given that

$$z_1 = 6e^{\frac{\pi}{3}i} \text{ and } z_2 = 6\sqrt{3}e^{\frac{5\pi}{6}i}$$

show that

$$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$$

(3 marks)

(continued on the next page)

Turn over

**4. continued.**

**(ii) Given that**

$$\arg(z - 5) = \frac{2\pi}{3}$$

**determine the least value of  $|z|$   
as  $z$  varies.**

**(3 marks)**

**(Total for Question 4 is 6 marks)**

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**Turn over**

5. (a) Given that

$$y = \arcsin x \quad -1 \leq x \leq 1$$

show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

(3 marks)

(continued on the next page)

Turn over



5. continued.

(b)  $f(x) = \arcsin(e^x)$        $x \leq 0$

Prove that  $f(x)$  has no stationary points.

(3 marks)

(Total for Question 5 is 6 marks)

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Turn over

6. The cubic equation

$$4x^3 + px^2 - 14x + q = 0$$

where  $p$  and  $q$  are real positive constants, has roots  $\alpha$ ,  $\beta$  and  $\gamma$

Given that

$$\alpha^2 + \beta^2 + \gamma^2 = 16$$

(a) show that  $p = 12$

(3 marks)

(continued on the next page)

Turn over

**6. continued.**

**Given that**

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{14}{3}$$

**(b) determine the value of  $q$**

**(3 marks)**

**Without solving the cubic equation,**

**(c) determine the value of**

$$(\alpha - 1)(\beta - 1)(\gamma - 1)$$

**(4 marks)**

**(Total for Question 6 is 10 marks)**

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**Turn over**

7. Refer to the diagram for Question 7 in the Diagram Booklet.

It shows a sketch of the curve **C** with equation

$$r = 1 + \tan \theta \qquad 0 \leq \theta < \frac{\pi}{3}$$

The diagram also shows the tangent to **C** at the point **A**

This tangent is perpendicular to the initial line.

(continued on the next page)

**7. continued.**

**(a) Use differentiation to prove that  
the polar coordinates of **A** are**

$$\left(2, \frac{\pi}{4}\right)$$

**(4 marks)**

**(continued on the next page)**

**7. continued.**

**The finite region  $R$ , shown shaded in the diagram, is bounded by  $C$ , the tangent at  $A$  and the initial line.**

- (b) Use calculus to show that the exact area of  $R$  is  $\frac{1}{2}(1 - \ln 2)$  (6 marks)**

**(Total for Question 7 is 10 marks)**

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**Turn over**

8. Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

$$\underline{r}_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

(continued on the next page)

Turn over

8. continued.

and the equation for the flight path of the second bird is

$$\underline{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters and  $a$  is a constant.

(continued on the next page)



**8. continued.**

**In the model, the angle between the birds' flight paths is  $120^\circ$**

**(a) Determine the value of  $a$   
(4 marks)**

**(b) Verify that, according to the model, there is a common point on the flight paths of the two birds and find the coordinates of this common point.  
(5 marks)**

**(continued on the next page)**

**Turn over**

**8. continued.**

**The position of the nest is modelled as being at this common point.**

**The tree containing the nest is in a park.**

**The ground level of the park is modelled by the plane with equation**

$$2x - 3y + z = 2$$

**(continued on the next page)**

**Turn over**

**8. continued.**

**(c) Hence determine the shortest distance from the nest to the ground level of the park.**

**(3 marks)**

**(d) By considering the model, comment on whether your answer to part (c) is reliable, giving a reason for your answer.**

**(1 mark)**

**(Total for Question 8 is 13 marks)**

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**Turn over**

9.  $y = \cosh^n x \quad n \geq 5$

(a) (i) Show that

$$\frac{d^2 y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$$

(4 marks)

(ii) Determine an expression for

$$\frac{d^4 y}{dx^4}$$

(2 marks)

(continued on the next page)

Turn over

**9. continued.**

- (b) Hence determine the first three non-zero terms of the Maclaurin series for  $y$ , giving each coefficient in simplest form. (2 marks)**

**(Total for Question 9 is 8 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**

**END OF PAPER**

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