Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

Time: 1 hour 30 minutes

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Answer Booklet

Diagram Booklet

Y65497A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 9 questions in this Question Paper. The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. An ellipse has equation

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \text{ and eccentricity } e_1$$

A hyperbola has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ and eccentricity } e_2$$

Given that

$$e_1 \times e_2 = 1$$

(a) show that

$$a^2 = 3b^2$$

(4 marks)

(continued on the next page)

Turn over

1. continued.

Given also that the coordinates of the foci of the ellipse are the same as the coordinates of the foci of the hyperbola,

(b) determine the equation of the hyperbola.

(3 marks)

(Total for Question 1 is 7 marks)

2. During 2029, the number of hours of daylight per day in London, H, is modelled by the equation

$$H = 0\cdot3\sin\left(\frac{x}{60}\right) - 4\cos\left(\frac{x}{60}\right) + 11\cdot5$$
$$0 \leq x < 365$$

where x is the number of days after 1st January 2029 and the angle is in radians.

(continued on the next page)

2. continued.

(a) Show that, according to the model, the number of hours of daylight in London on the 31st January 2029 will be $8 \cdot 13$ to 3 significant figures.

(1 mark)

(continued on the next page)

2. continued.

(b) Use the substitution

$t = \tan\left(\frac{x}{120}\right)$ to show that H

can be written as

$$H = \frac{at^2 + bt + c}{1 + t^2}$$

where a, b and c are constants

to be determined.

(2 marks)

(continued on the next page)

2. continued.

(c) Hence determine, according to the model, the date of the first day of 2029 when there will be at least 12 hours of daylight in London.

(4 marks)

(Total for Question 2 is 7 marks)

Turn over

3. With respect to a fixed origin O, the points A and B have coordinates $(2, 2, -1)$ and $(4, 2p, 1)$ respectively, where p is a constant.

For each of the following, determine the possible values of p for which,

(a) OB makes an angle of $45°$ with the positive x-axis

(3 marks)

(b) $\overrightarrow{OA} \times \overrightarrow{OB}$ is parallel to $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$

(3 marks)

(continued on the next page)

Turn over

3. continued.

(c) the area of triangle OAB is $3\sqrt{2}$

(3 marks)

(Total for Question 3 is 9 marks)

4. The velocity $v\,\mathrm{ms}^{-1}$, of a raindrop, t seconds after it falls from a cloud, is modelled by the differential equation

$$\frac{dv}{dt} = -0\cdot 1v^2 + 10 \qquad t \geq 0$$

Initially the raindrop is at rest.

(a) Use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the velocity of the raindrop 1 second after it falls from the cloud.

(5 marks)

(continued on the next page)

4. continued.

Given that the initial acceleration of the raindrop is found to be smaller than is suggested by the current model,

(b) refine the model by changing the value of one constant.
(1 mark)

(Total for Question 4 is 6 marks)

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5. The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \frac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3 marks)

(continued on the next page)

5. continued.

The point $Q\left(12t, \frac{3}{t}\right)$ also lies on H

(b) Find the equation of the tangent to H at the point Q

(2 marks)

The tangent at P and the tangent at Q meet at the point R

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4 marks)

(Total for Question 5 is 9 marks)

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6. The points P, Q and R have position vectors $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ respectively.

(a) Determine a vector equation of the plane that passes through the points P, Q and R, giving your answer in the form $\underline{r} = \underline{a} + \lambda\underline{b} + \mu\underline{c}$, where λ and μ are scalar parameters.
(2 marks)

(continued on the next page)

6. continued.

(b) Determine the coordinates of the point of intersection of the plane with the x–axis.

(4 marks)

(Total for Question 6 is 6 marks)

7. **Refer to the diagram for Question 7 in the Diagram Booklet.**

It shows a sketch of the curve with equation $y = |x^2 - 8|$ and a sketch of the straight line with equation $y = mx + c$, where m and c are positive constants.

The equation

$$|x^2 - 8| = mx + c$$

has exactly 3 roots, as shown in the diagram in the Diagram Booklet.

(continued on the next page)

7. continued.

(a) Show that

$$m^2 - 4c + 32 = 0$$

(2 marks)

Given that $c = 3m$

(b) determine the value of m and the value of c

(3 marks)

(continued on the next page)

7. continued.

(c) Hence solve

$$|x^2 - 8| \geq mx + c$$

(3 marks)

(Total for Question 7 is 8 marks)

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8. The Taylor series expansion of $f(x)$ about $x = a$ is given by

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots + \frac{(x-a)^r}{r!}f^{(r)}(a) + \ldots$$

(i) (a) Use differentiation to determine the Taylor series expansion of $\ln x$, in ascending powers of $(x-1)$, up to and including the term in $(x-1)^2$

(4 marks)

(continued on the next page)

Turn over

8. (i) continued.

(b) Hence prove that

$$\lim_{x \to 1}\left(\frac{\ln x}{x-1}\right) = 1$$

(2 marks)

(continued on the next page)

8. continued.

(ii) Use L'Hospital's rule to determine

$$\lim_{x \to 0}\left(\frac{1}{(x+3)\tan(6x)\operatorname{cosec}(2x)}\right)$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4 marks)

(Total for Question 8 is 10 marks)

Turn over

9. A particle P moves along a straight line.

At time t minutes, the displacement, x metres, of P from a fixed point O on the line is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t\frac{dx}{dt} + 2x + 16t^2 x = 4t^3 \sin 2t \quad (\mathrm{I})$$

(continued on the next page)

9. continued.

(a) Show that the transformation $x = ty$ transforms equation (I) into the equation

$$\frac{d^2y}{dt^2} + 16y = 4\sin 2t$$

(5 marks)

(b) Hence find a general solution for the displacement of P from O at time t minutes.

(8 marks)

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER