Paper Reference 8FM0/01
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced Subsidiary
PAPER 1: Core Pure Mathematics

Time: 1 hour 40 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

Q68730A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 8 questions in this Question Paper. The total mark for this paper is 80

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1.

$$A = \begin{pmatrix} 4 & -1 \\ 7 & 2 \\ -5 & 8 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 6 & 5 \end{pmatrix}$$

$$C = \begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}$$

(continued on the next page)

1. continued.

Given that $\mathbf{I}$ is the 3×3 identity matrix,

(a) (i) show that there is an integer k for which

$$AB - 3C + kI = 0$$

stating the value of k

(ii) explain why there can be no constant m such that

$$BA - 3C + mI = 0$$

(4 marks)

(continued on the next page)

1. continued.

(b) (i) Show how the matrix **C** can be used to solve the simultaneous equations

$$\begin{aligned} -5x + 2y + z &= -14 \\ 4x + 3y + 8z &= 3 \\ -6x + 11y + 2z &= 7 \end{aligned}$$

(ii) Hence use your calculator to solve these equations.

(3 marks)

(Total for Question 1 is 7 marks)

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2. (a) Express the complex number $w = 4\sqrt{3} - 4i$ in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$

(4 marks)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by

$$\arg(z + 10i) = \frac{\pi}{3}$$

There are blank axes on pages $34 - 45$ in the Answer Booklet if you wish to use them.

(3 marks)

(c) Hence determine the minimum distance of w from the locus $\arg(z + 10i) = \frac{\pi}{3}$

(3 marks)

(Total for Question 2 is 10 marks)

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3. **With respect to the RIGHT−HAND RULE, a rotation through $\theta°$ anticlockwise about the y–axis is represented by the matrix**

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

The point P has coordinates (8, 3, 2)

The point Q is the image of P under the transformation reflection in the plane $y = 0$

(a) Write down the coordinates of Q

(1 mark)

(continued on the next page)

3. continued.

The point R is the image of P under the transformation rotation through $120°$ anticlockwise about the y–axis, with respect to the RIGHT–HAND RULE.

(b) Determine the exact coordinates of R

(2 marks)

(c) Hence find $|\overrightarrow{PR}|$ giving your answer as a simplified surd.

(2 marks)

(d) Show that $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular.

(1 mark)

(e) Hence determine the exact area of triangle PQR, giving your answer as a surd in simplest form.

(2 marks)

(Total for Question 3 is 8 marks)

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4. The roots of the quartic equation

$$3x^4 + 5x^3 - 7x + 6 = 0$$

are α, β, γ and δ

Making your method clear and without solving the equation, determine the exact value of

(i) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

(3 marks)

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma} + \dfrac{2}{\delta}$

(3 marks)

(iii) $(3 - \alpha)(3 - \beta)(3 - \gamma)(3 - \delta)$

(3 marks)

(Total for Question 4 is 9 marks)

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5. (a) Use the standard summation formulae to show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n}(3r^2 - 17r - 25) = n(n^2 - An - B)$$

where A and B are integers to be determined.

(4 marks)

(b) Explain why, for $k \in \mathbb{N}$,

$$\sum_{r=1}^{3k} r\tan(60r)^\circ = -k\sqrt{3}$$

(2 marks)

Using the results from part (a) and part (b) and showing all your working,

(c) determine any value of n that satisfies

$$\sum_{r=5}^{n}(3r^2 - 17r - 25) = 15\left[\sum_{r=6}^{3n} r\tan(60r)^\circ\right]^2$$

(6 marks)

(Total for Question 5 is 12 marks)

6. The surface of a horizontal tennis court is modelled as part of a horizontal plane, with the origin on the ground at the centre of the court, and

- $\underline{i}$ and $\underline{j}$ are unit vectors directed across the width and length of the court respectively
- $\underline{k}$ is a unit vector directed vertically upwards
- units are metres

After being hit, a tennis ball, modelled as a particle, moves along the path with equation

$$\underline{r} = (-4 \cdot 1 + 9\lambda - 2 \cdot 3\lambda^2)\underline{i} + (-10 \cdot 25 + 15\lambda)\underline{j} + (0 \cdot 84 + 0 \cdot 8\lambda - \lambda^2)\underline{k}$$

where λ is a scalar parameter with $\lambda \geq 0$

(continued on the next page)

6. continued.

Assuming that the tennis ball continues on this path until it hits the ground,

(a) find the value of λ at the point where the ball hits the ground.
(2 marks)

The direction in which the tennis ball is moving at a general point on its path is given by

$$(9 - 4\cdot6\lambda)\underline{i} + 15\underline{j} + (0\cdot8 - 2\lambda)\,\underline{k}$$

(b) Write down the direction in which the tennis ball is moving as it hits the ground.
(1 mark)

(c) Hence find the acute angle at which the tennis ball hits the ground, giving your answer in degrees to one decimal place.
(4 marks)

(continued on the next page)

6. continued.

The net of the tennis court lies in the plane $\underline{r} \cdot \underline{j} = 0$

(d) Find the position of the tennis ball at the point where it is in the same plane as the net.
(3 marks)

The maximum height above the court of the top of the net is $0 \cdot 9$ metres.

Modelling the top of the net as a horizontal straight line,

(e) state whether the tennis ball will pass over the net according to the model, giving a reason for your answer.
(1 mark)

(continued on the next page)

6. continued.

With reference to the model,

(f) decide whether the tennis ball will actually pass over the net, giving a reason for your answer.
(2 marks)

(Total for Question 6 is 13 marks)

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7. **Prove by mathematical induction that, for $n \in \mathbb{N}$**

$$\begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}^n = \begin{pmatrix} 1-6n & 9n \\ -4n & 1+6n \end{pmatrix}$$

(Total for Question 7 is 6 marks)

8. Refer to Diagram 1 and Diagram 2 for Question 8 in the Diagram Booklet.

Diagram 1 shows a front view sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross–section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Diagram 2, through $360°$ about the x–axis.

(a) State the value of a that should be used when setting up the model.
(1 mark)

(continued on the next page)

8. continued.

Two possible equations are suggested for the curve in the model.

Model **A** $\quad y = a - 2\sin\left(\frac{45}{2}x\right)^{\circ}$

Model **B** $\quad y = a + \frac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7 marks)

(continued on the next page)

8. continued.

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1 mark)

(d) Using algebraic integration, find the volume for the vase predicted by Model B

You must make your method clear.

(5 marks)

(continued on the next page)

8. continued.

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

(1 mark)

(Total for Question 8 is 15 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

END OF PAPER
