Paper Reference 9FM0/4D
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

Paper 4D: Decision Mathematics 2

Tuesday 25 June 2019 – Morning

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green)
Calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

X61186A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

Write your answers for this paper in the Answer Book provided.

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Do NOT return the Question Paper with the Answer Book.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 7 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read EACH question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book provided.

1. Refer to Diagram 1 and Diagram 2 in the Diagram Book.

Diagram 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, **A**, **B**, **C** and **D**, to each of four demand points, **P**, **Q**, **R** and **S**

It also shows the stock held at each supply point and the stock required at each demand point.

A minimum cost solution is required.

Diagram 2 shows an initial solution given by the north–west corner method.

(a) Taking **DQ** as the entering cell, use the stepping–stone method to find an improved solution.

Make your method clear.

(2 marks)

(continued on the next page)

1. continued.

(b) Perform one further iteration of the stepping–stone method to obtain an improved solution.

You must make your method clear by stating the

- shadow costs
- improvement indices
- route
- entering cell and exiting cell.

(4 marks)

(c) Determine whether the solution obtained from this second iteration is optimal, giving a reason for your answer.

(3 marks)

(d) State the cost of the solution found in (b)

(1 mark)

(Total for Question 1 is 10 marks)

2. Refer to Diagram 3 in the Diagram Book.

Four workers, Ted (T), Harold (H), James (J) and Margaret (M), are to be assigned to four tasks, 1, 2, 3 and 4

Each worker must be assigned to just one task and each task must be done by just one worker.

The profit, in pounds, resulting from allocating each worker to each task, is shown in Diagram 3

The profit is to be maximised.

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit.

You must make your method clear and show the table after each stage.

(6 marks)

(b) Determine the resulting total profit.

(1 mark)

(Total for Question 2 is 7 marks)

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3. Refer to Diagram 4 in the Diagram Book.

In Diagram 4 the weight of arc SB is denoted by x where $x \geq 0$

(a) Explain why Dijkstra's algorithm cannot be used on the directed network in the diagram.
(1 mark)

It is given that the minimum weight route from S to T passes through B

(b) Use dynamic programming to find

(i) the range of possible values of x

(ii) the minimum weight route from S to T
(12 marks)

(Total for Question 3 is 13 marks)

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4. Refer to Diagram 5 in the Diagram Book.

A two person zero–sum game is represented by the pay–off matrix for player **A** shown in Diagram 5

(a) Verify that there is no stable solution to this game.
(2 marks)

(continued on the next page)

4. continued.

Player A intends to make a random choice between options P, Q and R, choosing option P with probability p_1, option Q with probability p_2 and option R with probability p_3

Player A wants to find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.
Player A formulates the following linear programming problem for the game, writing the constraints as inequalities.

Maximise $P = V$

subject to

$$V \geq 3p_1 - 4p_2 + p_3$$
$$V \geq -2p_1 + 4p_2 + 2p_3$$
$$V \geq -2p_2 - p_3$$
$$p_1 + p_2 + p_3 \leq 1$$
$$p_1 \geq 0,\ p_2 \geq 0,\ p_3 \geq 0,\ V \geq 0$$

(continued on the next page)

4. continued.

(b) Correct the errors made by player A in the linear programming formulation of the game, giving reasons for your answer.
(3 marks)

(c) Write down an initial Simplex tableau for the corrected linear programming problem.
(3 marks)

The Simplex algorithm is used to solve the corrected linear programming problem.

The optimal values are $p_1 = 0 \cdot 6$, $p_2 = 0$ and $p_3 = 0 \cdot 4$

(d) Calculate the value of the game to player A
(2 marks)

(continued on the next page)

4. continued.

(e) Determine the optimal strategy for player B, making your reasoning clear.

(4 marks)

(Total for Question 4 is 14 marks)

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5. An increasing sequence $\{u_n\}$ for $n \in \mathbb{N}$ is such that the difference between the nth term of $\{u_n\}$ and the mean of the previous two terms of $\{u_n\}$ is always 6

(a) Show that, for $n \geq 3$

$$2u_n - u_{n-1} - u_{n-2} = 12$$

(2 marks)

Given that $u_1 = 2$ and $u_2 = 8$

(b) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n

(7 marks)

(c) Show that as $n \to \infty$, $u_n \to kn$ where k is a constant to be determined.

You must give reasons for your answer.

(2 marks)

(Total for Question 5 is 11 marks)

6. Refer to Diagram 6, Diagram 7 and Diagram 8 in the Diagram Book.

Diagram 6 shows a capacitated, directed network. The network represents a system of pipes through which fluid flows from a source, S, to a sink, T

The numbers (l, u) on each arc represent, in litres per second, the lower capacity, l, and the upper capacity, u, of the corresponding pipe.

Two cuts C_1 and C_2 are shown.

(a) Find the capacity of

(i) cut C_1

(ii) cut C_2

(2 marks)

(continued on the next page)

6. continued.

(b) Explain why the arcs **AE** and **CE** cannot be at their upper capacities simultaneously.

(1 mark)

(c) Explain why a flow of 31 litres per second through the system is not possible.

(1 mark)

(d) Hence determine a minimum feasible flow and a maximum feasible flow through the system. You must draw these feasible flows on Diagram 7 and Diagram 8 in the Diagram Book and give reasons to justify your answer.

You should not apply the labelling procedure to find these flows.

(4 marks)

(Total for Question 6 is 8 marks)

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7. Aisha is deciding whether or not to play a game.

The game involves rolling three fair six–sided dice, which have faces numbered from 1 to 6

If the total score on the three dice is 16 or more then she wins a prize.
If the total score is 15 or less then she loses and will have to pay the person running the game £3

(a) Given that the prize is £15

(i) draw a decision tree to model Aisha's possible decisions and the possible outcomes

(ii) determine Aisha's optimal EMV and state the optimal strategy indicated by the decision tree.

(6 marks)

(continued on the next page)

7. continued.

The utility function of the game to Aisha is $u(m) = 1 - e^{-\frac{m}{500}}$ where m is the amount of money that Aisha has available.

Given that Aisha has exactly £3 and that the prize is now £x

(b) find the expected utility to Aisha of playing the game in the form

$$\frac{a}{b}\left(1 - e^{-\frac{(x+c)}{500}}\right)$$

where a, b and c are integers to be found.

(2 marks)

Aisha decides to use the expected utilities to determine whether she should play the game or not.

(c) Find the minimum prize for which Aisha would consider playing the game.

(4 marks)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER