Paper Reference 9FM0/3C
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
Paper 3C: Further Mechanics 1

Thursday 20 June 2019 – Morning

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green), calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

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Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams − there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9{\cdot}8\ ms^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 7 questions in this question paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

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1. Refer to the diagram for Question 1 in the Diagram Book.

It represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls.
The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leq 3$, from W_1

(continued on the next page)

1. continued.

At time $t = 0$, the particle is projected from O towards W_1 with speed $u \text{ m s}^{-1}$ in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

(continued on the next page)

1. continued.

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that

$$T = \frac{45 - 5d}{4u}$$

(6 marks)

(continued on the next page)

1. continued.

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u

You must give a reason for your answer.

(2 marks)

(Total for Question 1 is 8 marks)

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2. Refer to the diagram for Question 2 in the Diagram Book.

It represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC

A small ball is projected along the floor towards AB with speed $6\ m\,s^{-1}$ on a path that makes an angle α with AB, where $\tan\alpha = \frac{4}{3}$

(continued on the next page)

2. continued.

The ball hits AB and then hits BC

Immediately after hitting AB, the ball is moving at an angle β to AB, where

$$\tan\beta = \frac{1}{3}$$

The coefficient of restitution between the ball and AB is e

The coefficient of restitution between the ball and BC is $\frac{1}{2}$

(continued on the next page)

2. continued.

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$

(5 marks)

(b) find the speed of the ball immediately after it hits BC

(4 marks)

(c) Suggest two ways in which the model could be refined to make it more realistic.

(2 marks)

(Total for Question 2 is 11 marks)

3. A particle P, of mass $0 \cdot 5$ kg, is moving with velocity $(4\underline{i} + 4\underline{j})$ m s^{-1} when it receives an impulse $\underline{I}$ of magnitude $2 \cdot 5$ Ns

As a result of the impulse, the direction of motion of P is deflected through an angle of $45°$

Given that $\underline{I} = (\lambda\underline{i} + \mu\underline{j})$ Ns, find all the possible pairs of values of λ and μ

(Total for Question 3 is 9 marks)

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4. A car of mass $600\ \text{kg}$ pulls a trailer of mass $150\ \text{kg}$ along a straight horizontal road.

The trailer is connected to the car by a light inextensible towbar, which is parallel to the direction of motion of the car.

The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 newtons.

At the instant when the speed of the car is $v\ \text{m s}^{-1}$, the resistance to the motion of the car is modelled as a force of magnitude $(200 + \lambda v)$ newtons, where λ is a constant.

(continued on the next page)

4. continued.

When the engine of the car is working at a constant rate of $15\ \text{kW}$, the car is moving at a constant speed of $25\ \text{m s}^{-1}$

(a) Show that $\lambda = 8$

(4 marks)

Later on, the car is pulling the trailer up a straight road inclined at an angle θ to the horizontal, where

$$\sin\theta = \frac{1}{15}$$

(continued on the next page)

4. continued.

The resistance to the motion of the trailer from non–gravitational forces is modelled as a constant force of magnitude 200 newtons at all times. At the instant when the speed of the car is v ms^{-1}, the resistance to the motion of the car from non–gravitational forces is modelled as a force of magnitude $(200 + 8v)$ newtons.

The engine of the car is again working at a constant rate of 15 kW

(continued on the next page)

4. continued.

When $v = 10$, the towbar breaks.

The trailer comes to instantaneous rest after moving a distance d metres up the road from the point where the towbar broke.

(b) Find the acceleration of the car immediately after the towbar breaks.

(4 marks)

(c) Use the work–energy principle to find the value of d

(4 marks)

(Total for Question 4 is 12 marks)

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane.

The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$

Immediately after the collision P and Q are moving in opposite directions.

(continued on the next page)

5. continued.

The coefficient of restitution between P and Q is e

(a) Find the range of possible values of e, justifying your answer.

(8 marks)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e

(3 marks)

(Total for Question 5 is 11 marks)

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6. **[In this question $\underline{\mathbf{i}}$ and $\underline{\mathbf{j}}$ are perpendicular unit vectors in a horizontal plane.]**

A smooth uniform sphere A has mass $0{\cdot}2$ kg and another smooth uniform sphere B, with the same radius as A, has mass $0{\cdot}4$ kg

The spheres are moving on a smooth horizontal surface when they collide obliquely.

(continued on the next page)

6. continued.

Immediately before the collision, the velocity of **A** is $(3\underline{i} + 2\underline{j})\ \mathrm{m\,s^{-1}}$ and the velocity of **B** is $(-4\underline{i} - \underline{j})\ \mathrm{m\,s^{-1}}$

At the instant of collision, the line joining the centres of the spheres is parallel to $\underline{i}$

(continued on the next page)

6. continued.

The coefficient of restitution between the spheres is $\frac{3}{7}$

(a) Find the velocity of **A** immediately after the collision.

(7 marks)

(b) Find the magnitude of the impulse received by **A** in the collision.

(2 marks)

(continued on the next page)

6. continued.

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of **A** is deflected as a result of the collision.

(3 marks)

(Total for Question 6 is 12 marks)

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7. A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$

The point B is vertically below O such that $OB = \frac{1}{2}a$

(continued on the next page)

7. continued.

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B

(a) Show that

$$k = \frac{4}{3}$$

(3 marks)

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A

(3 marks)

(continued on the next page)

7. continued.

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B

(6 marks)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
