Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced
Paper 3A: Further Pure Mathematics 1

Thursday 13 June 2019 – Afternoon

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green), calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Answer Book

X61179A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A Booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 8 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1. Use Simpson's rule with **4** intervals to estimate

$$\int_{0 \cdot 4}^{2} e^{x^2} \, dx$$

(5 marks)

(Total for Question 1 is 5 marks)

2. Given that k is a real non–zero constant and that

$$y = x^3 \sin kx$$

use Leibnitz's theorem to show that

$$\frac{d^5y}{dx^5} = (k^2x^2 + A)k^3x \cos kx + B(k^2x^2 + C)k^2 \sin kx$$

where A, B and C are integers to be determined.

(4 marks)

(Total for Question 2 is 4 marks)

3.
$$\frac{dy}{dx} = x - y^2 \qquad \text{(I)}$$

(a) Show that

$$\frac{d^5y}{dx^5} = ay\frac{d^4y}{dx^4} + b\frac{dy}{dx}\frac{d^3y}{dx^3} + c\left(\frac{d^2y}{dx^2}\right)^2$$

where a, b and c are integers to be determined.

(4 marks)

(b) Hence find a series solution, in ascending powers of x as far as the term in x^5, of the differential equation (I), given that $y = 1$ at $x = 0$

(5 marks)

(Total for Question 3 is 9 marks)

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4. The parabola C has equation

$$y^2 = 16x$$

The distinct points $P(p^2, 4p)$ and $Q(q^2, 4q)$ lie on C, where $p \neq 0, q \neq 0$

The tangent to C at P and the tangent to C at Q meet at the point $R(-28, 6)$

Show that the area of triangle PQR is 1331

(Total for Question 4 is 8 marks)

5.

$$I = \int \frac{1}{4\cos x - 3\sin x}\,dx \qquad 0 < x < \frac{\pi}{4}$$

Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$I = \frac{1}{5}\ln\left(\frac{2 + \tan\left(\frac{x}{2}\right)}{1 - 2\tan\left(\frac{x}{2}\right)}\right) + k$$

where k is an arbitrary constant.

(Total for Question 5 is 8 marks)

6. The concentration of a drug in the bloodstream of a patient, t hours after the drug has been administered, where $t \leq 6$, is modelled by the differential equation

$$t^2 \frac{d^2C}{dt^2} - 5t \frac{dC}{dt} + 8C = t^3 \qquad \text{(I)}$$

where C is measured in micrograms per litre.

(a) Show that the transformation $t = e^x$ transforms equation (I) into the equation

$$\frac{d^2C}{dx^2} - 6 \frac{dC}{dx} + 8C = e^{3x} \qquad \text{(II)}$$

(5 marks)

(b) Hence find the general solution for the concentration C at time t hours.

(7 marks)

(continued on the next page)

6. continued.

Given that when $t = 6$, $C = 0$ and $\frac{dC}{dt} = -36$

(c) find the maximum concentration of the drug in the bloodstream of the patient.

(5 marks)

(Total for Question 6 is 17 marks)

7. With respect to a fixed origin O, the points A, B and C have coordinates $(3, 4, 5)$, $(10, -1, 5)$ and $(4, 7, -9)$ respectively.

The plane Π has equation $4x - 8y + z = 2$

The line segment AB meets the plane Π at the point P and the line segment BC meets the plane Π at the point Q

(a) Show that, to 3 significant figures, the area of quadrilateral $APQC$ is $38 \cdot 5$
(6 marks)

(continued on the next page)

7. continued.

The point D has coordinates $(k, 4, -1)$, where k is a constant.

Given that the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ form three edges of a parallelepiped of volume 226

(b) find the possible values of the constant k

(4 marks)

(Total for Question 7 is 10 marks)

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8. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

The line L_1 is the tangent to H at the point $P(4\cosh\theta, 3\sinh\theta)$

The line L_1 meets the x–axis at the point A

The line L_2 is the tangent to H at the point $(4, 0)$

The lines L_1 and L_2 meet at the point B and the midpoint of AB is the point M

(a) Show that, as θ varies, a Cartesian equation for the locus of M is

$$y^2 = \frac{9(4 - x)}{4x} \qquad p < x < q$$

where p and q are values to be determined.
(11 marks)

(continued on the next page)

8. continued.

Let S be the focus of H that lies on the positive x–axis.

(b) Show that the distance from M to S is greater than 1

(3 marks)

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
