Paper Reference 9FM0/02
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
Paper 2: Core Pure Mathematics 2

Thursday 6 June 2019 – Afternoon

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green)
Calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

V61178A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams − there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are **8** questions in this Question Paper.

The total mark for this paper is **75**

The marks for EACH question are shown in brackets − use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k

(5 marks)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5 marks)

(Total for Question 1 is 10 marks)

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2. **The roots of the equation**

$$x^3 - 2x^2 + 4x - 5 = 0$$

are p, q and r

Without solving the equation, find the value of

(i) $\dfrac{2}{p} + \dfrac{2}{q} + \dfrac{2}{r}$

(ii) $(p - 4)(q - 4)(r - 4)$

(iii) $p^3 + q^3 + r^3$

(Total for Question 2 is 8 marks)

3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4 marks)

(continued on the next page)

Turn over

3. continued.

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval $[0, 3]$

(2 marks)

(Total for Question 3 is 6 marks)

4. The infinite series C and S are defined by

$$C = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \ldots$$

$$S = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \ldots$$

(continued on the next page)

4. continued.

Given that the series C and S are both convergent,

(a) show that

$$C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}}$$

(4 marks)

(b) Hence show that

$$S = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

(4 marks)

(Total for Question 4 is 8 marks)

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5. An engineer is investigating the motion of a sprung diving board at a swimming pool.

Let E be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.

(continued on the next page)

5. continued.

A diver jumps from the diving board.

The vertical displacement, h cm, of the end of the diving board above E is modelled by the differential equation

$$4\frac{d^2h}{dt^2} + 4\frac{dh}{dt} + 37h = 0$$

where t seconds is the time after the diver jumps.

(a) Find a general solution of the differential equation.

(2 marks)

(continued on the next page)

5. continued.

When $t = 0$, the end of the diving board is 20 cm below E and is moving upwards with a speed of 55 cm s^{-1}

(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above E

(8 marks)

(c) Comment on the suitability of the model for large values of t

(2 marks)

(Total for Question 5 is 12 marks)

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6. In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin.

The point A represents the complex number $6 + 2i$

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + iy$, where x and y are real and exact.

(6 marks)

(continued on the next page)

6. continued.

The points D, E and F are the midpoints of the sides of triangle ABC

(b) Find the exact area of triangle DEF

(3 marks)

(Total for Question 6 is 9 marks)

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7. $$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix}$$

where k is a constant

(a) Find the values of k for which the matrix $\mathbf{M}$ has an inverse.
(2 marks)

(continued on the next page)

7. continued.

(b) Find, in terms of p, the coordinates of the point where the following planes intersect

$$2x - y + z = p$$
$$3x - 6y + 4z = 1$$
$$3x + 2y - z = 0$$

(5 marks)

(continued on the next page)

Turn over

7. continued.

(c) (i) Find the value of q for which the set of simultaneous equations

$$\begin{aligned} 2x - y + z &= 1 \\ 3x - 5y + 4z &= q \\ 3x + 2y - z &= 0 \end{aligned}$$

can be solved.

(ii) For this value of q, interpret the solution of the set of simultaneous equations geometrically.

(4 marks)

(Total for Question 7 is 11 marks)

8. Refer to the diagrams for Question 8 in the Diagram Book.

Diagram 1 shows the central vertical cross section ABCD of a paddling pool that has a circular horizontal cross section.

Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

(continued on the next page)

8. continued.

Using these measurements, the curve BD is modelled by the equation

$$y = \ln(3\cdot6x - k) \qquad 1 \leq x \leq 1\cdot18$$

as shown in Diagram 2

(a) Find the value of k

(1 mark)

(continued on the next page)

8. continued.

(b) Find the depth of the paddling pool according to this model.
(2 marks)

The pool is being filled with water from a tap.

(c) Find, in terms of h, the volume of water in the pool when the pool is filled to a depth of h metres
(5 marks)

(continued on the next page)

8. continued.

Given that the pool is being filled at a constant rate of 15 litres every minute,

(d) find, in cm h^{-1}, the rate at which the water level is rising in the pool when the depth of the water is $0 \cdot 2$ metres.
(3 marks)

(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER