

**Paper Reference 9FM0/02**  
**Pearson Edexcel**  
**Level 3 GCE**

**Further Mathematics**  
**Advanced**  
**Paper 2: Core Pure Mathematics 2**

**Thursday 6 June 2019 – Afternoon**

**Time: 1 hour 30 minutes plus your  
additional time allowance.**

**MATERIALS REQUIRED FOR  
EXAMINATION**

**Mathematical Formulae and Statistical  
Tables (Green)**  
**Calculator**

**ITEMS INCLUDED WITH QUESTION  
PAPERS**

**Diagram Book**  
**Answer Book**

**Y61178A**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## **INSTRUCTIONS**

**In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.**

**Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.**

**Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.**

**Do NOT write on the Question Paper.**

**You should show sufficient working to make your methods clear. Answers without working may not gain full credit.**

**Answers should be given to three significant figures unless otherwise stated.**

**Turn over**

## **INFORMATION**

**A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.**

**There are 8 questions in this Question Paper.**

**The total mark for this paper is 75**

**The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.**

**ADVICE**

**Read each question carefully before you start to answer it.**

**Try to answer every question.**

**Check your answers if you have time at the end.**

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**Turn over**

**6**

**Answer ALL questions.**

**Write your answers in the  
Answer Book.**

**Turn over**

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad -k < x < k$$

stating the value of the constant  $k$   
(5 marks)

(b) Hence, or otherwise, solve the  
equation

$$2x = \tanh(\ln \sqrt{2-3x})$$

(5 marks)

(Total for Question 1 is 10 marks)

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Turn over

2. The roots of the equation

$$x^3 - 2x^2 + 4x - 5 = 0$$

are  $p$ ,  $q$  and  $r$

Without solving the equation, find the value of

(i)  $\frac{2}{p} + \frac{2}{q} + \frac{2}{r}$

(ii)  $(p - 4)(q - 4)(r - 4)$

(iii)  $p^3 + q^3 + r^3$

(Total for Question 2 is 8 marks)

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Turn over

3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

- (a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A \sinh^{-1}(Bx) + c$$

where **C** is an arbitrary constant and **A** and **B** are constants to be found.

(4 marks)

(continued on the next page)

Turn over

**3. continued.**

- (b) Hence find, in exact form in terms of natural logarithms, the mean value of  $f(x)$  over the interval  $[0, 3]$**
- (2 marks)**

**(Total for Question 3 is 6 marks)**

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**Turn over**

4. The infinite series **C** and **S** are defined by

$$\begin{aligned} \mathbf{C} = & \cos \theta + \frac{1}{2} \cos 5\theta + \frac{1}{4} \cos 9\theta \\ & + \frac{1}{8} \cos 13\theta + \dots \end{aligned}$$

$$\begin{aligned} \mathbf{S} = & \sin \theta + \frac{1}{2} \sin 5\theta + \frac{1}{4} \sin 9\theta \\ & + \frac{1}{8} \sin 13\theta + \dots \end{aligned}$$

(continued on the next page)

Turn over

4. continued.

Given that the series **C** and **S** are both convergent,

(a) show that

$$\mathbf{C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}}}$$

(4 marks)

(b) Hence show that

$$\mathbf{S = \frac{4 \sin \theta + 2 \sin 3\theta}{5 - 4 \cos 4\theta}}$$

(4 marks)

(Total for Question 4 is 8 marks)

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Turn over

5. An engineer is investigating the motion of a sprung diving board at a swimming pool.

Let  $E$  be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.

(continued on the next page)

**5. continued.**

**A diver jumps from the diving board.**

**The vertical displacement,  $h$  cm, of the end of the diving board above**

**$E$  is modelled by the differential equation**

$$4\frac{d^2h}{dt^2} + 4\frac{dh}{dt} + 37h = 0$$

**where  $t$  seconds is the time after the diver jumps.**

**(a) Find a general solution of the differential equation.**

**(2 marks)**

**(continued on the next page)**

**Turn over**

**5. continued.**

**When  $t = 0$ , the end of the diving board is 20 cm below E and is moving upwards with a speed of  $55 \text{ cm s}^{-1}$**

**(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above E**

**(8 marks)**

**(c) Comment on the suitability of the model for large values of  $t$**

**(2 marks)**

**(Total for Question 5 is 12 marks)**

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**Turn over**

6. In an Argand diagram, the points **A**, **B** and **C** are the vertices of an equilateral triangle with its centre at the origin.

The point **A** represents the complex number  $6 + 2i$

- (a) Find the complex numbers represented by the points **B** and **C**, giving your answers in the form  $x + iy$ , where  $x$  and  $y$  are real and exact.

(6 marks)

(continued on the next page)

**6. continued.**

**The points D, E and F are the  
midpoints of the sides of  
triangle ABC**

**(b) Find the exact area of  
triangle DEF  
(3 marks)**

**(Total for Question 6 is 9 marks)**

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**Turn over**

7. 
$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix}$$

where  $k$  is a constant

- (a) Find the values of  $k$  for which the matrix  $\mathbf{M}$  has an inverse.  
(2 marks)

(continued on the next page)

Turn over

**7. continued.**

**(b) Find, in terms of  $p$ , the coordinates of the point where the following planes intersect**

$$2x - y + z = p$$

$$3x - 6y + 4z = 1$$

$$3x + 2y - z = 0$$

**(5 marks)**

**(continued on the next page)**

**Turn over**

**7. continued.**

- (c) (i) Find the value of  $q$  for which the set of simultaneous equations**

$$2x - y + z = 1$$

$$3x - 5y + 4z = q$$

$$3x + 2y - z = 0$$

**can be solved.**

- (ii) For this value of  $q$ , interpret the solution of the set of simultaneous equations geometrically.**

**(4 marks)**

**(Total for Question 7 is 11 marks)**

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**Turn over**

8. Refer to the diagrams for Question 8 in the Diagram Book.

Diagram 1 shows the central vertical cross section **ABCD** of a paddling pool that has a circular horizontal cross section.

Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

(continued on the next page)

8. continued.

Using these measurements, the curve **BD** is modelled by the equation

$$y = \ln(3 \cdot 6x - k) \quad 1 \leq x \leq 1.18$$

as shown in Diagram 2

(a) Find the value of **k**  
(1 mark)

(continued on the next page)

Turn over

**8. continued.**

- (b) Find the depth of the paddling pool according to this model.  
(2 marks)**

**The pool is being filled with water from a tap.**

- (c) Find, in terms of  $h$ , the volume of water in the pool when the pool is filled to a depth of  $h$  metres  
(5 marks)**

**(continued on the next page)**

**Turn over**

**8. continued.**

**Given that the pool is being filled  
at a constant rate of 15 litres every  
minute,**

**(d) find, in  $\text{cm h}^{-1}$ , the rate at which  
the water level is rising in the  
pool when the depth of the water  
is 0.2 metres.**

**(3 marks)**

**(Total for Question 8 is 11 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**

**END OF PAPER**

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