Paper Reference 9FM0/4C
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

Paper 4C: Further Mechanics 2

Tuesday 25 June 2019 – Morning

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green), calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

X61185A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9{\cdot}8\ \mathrm{m\,s^{-2}}$, and give your answer to either 2 significant figures or 3 significant figures.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 7 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1. Refer to the diagram for Question 1 in the Diagram Book.

A hemispherical shell of radius a is fixed with its rim uppermost and horizontal.
A small bead, B, is moving with constant angular speed, ω, in a horizontal circle on the smooth inner surface of the shell.
The centre of the path of B is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in the diagram.

Find the magnitude of ω, giving your answer in terms of a and g

(Total for Question 1 is 6 marks)

---

2. A particle, P, of mass $0 \cdot 4$ kg is moving along the positive x–axis, in the positive x direction under the action of a single force.

At time t seconds, $t > 0$, P is x metres from the origin O and the speed of P is v m s^{-1}

The force is acting in the direction of x increasing and has magnitude $\frac{k}{v}$ newtons, where k is a constant.

At $x = 3$, $v = 2$ and at $x = 6$, $v = 2 \cdot 5$

(a) Show that

$$v^3 = \frac{61x + 9}{24}$$

(6 marks)

(continued on the next page)

2. continued.

The time taken for the speed of P to increase from $2\,m\,s^{-1}$ to $2{\cdot}5\,m\,s^{-1}$ is T seconds.

(b) Use algebraic integration to show that

$$T = \frac{81}{61}$$

(4 marks)

(Total for Question 2 is 10 marks)

3. **Numerical (calculator) integration is not acceptable in this question.**

Refer to the diagram for Question 3 in the Diagram Book.

The shaded region OAB in the diagram is bounded by the x–axis, the line with equation $x = 4$ and the curve with equation $y = \frac{1}{4}(x - 2)^3 + 2$

The point A has coordinates $(4, 4)$ and the point B has coordinates $(4, 0)$

A uniform lamina L has the shape of OAB

The unit of length on both axes is one centimetre.

The centre of mass of L is at the point with coordinates $(\bar{x}, \bar{y})$

Given that the area of L is $8\ cm^2$,

(a) show that $\bar{y} = \frac{8}{7}$

(4 marks)

(continued on the next page)

3. continued.

The lamina is freely suspended from A and hangs in equilibrium with AB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ

(7 marks)

(Total for Question 3 is 11 marks)

---

4. A flagpole, AB, is 4 metres long.

The flagpole is modelled as a non–uniform rod so that, at a distance x metres from A, the mass per unit length of the flagpole, $m\,kg\,m^{-1}$, is given by $m = 18 - 3x$

(a) Show that the mass of the flagpole is $48\ kg$

(3 marks)

Refer to the diagram for Question 4(b) in the Diagram Book.

The end A of the flagpole is fixed to a point on a vertical wall.

A cable has one end attached to the midpoint of the flagpole and the other end attached to a point on the wall that is vertically above A

The cable is perpendicular to the flagpole.

The flagpole and the cable lie in the same vertical plane that is perpendicular to the wall.

A small ball of mass $4\ kg$ is attached to the flagpole at B

The cable holds the flagpole and ball in equilibrium, with the flagpole at $45°$ to the wall, as shown in the diagram.

(continued on the next page)

4. continued.

The tension in the cable is T newtons.

The cable is modelled as a light inextensible string and the ball is modelled as a particle.

(b) Using the model, find the value of T

(8 marks)

(c) Give a reason why the answer to part (b) is not likely to be the true value of T

(1 mark)

(Total for Question 4 is 12 marks)

---

5. Refer to the diagram for Question 5 in the Diagram Book.

The region R, shown shaded in the diagram, is bounded by part of the curve with equation $y^2 = 2x$, the line with equation $y = 2$ and the y–axis.

The unit of length on both axes is one centimetre.

A uniform solid, S, is formed by rotating R through $360°$ about the y–axis.

Given that the volume of S is $\frac{8}{5}\pi\,\text{cm}^3$,

(a) show that the centre of mass of S is $\frac{1}{3}\,\text{cm}$ from its plane face.

(4 marks)

(continued on the next page)

5. continued.

Refer to the diagram for Question 5(b) in the Diagram Book.

A uniform solid cylinder, C, has base radius 2 cm and height 4 cm

The cylinder C is attached to S so that the plane face of S coincides with a plane face of C, to form the paperweight P, shown in the diagram.

The density of the material used to make S is three times the density of the material used to make C

The plane face of P rests in equilibrium on a desk lid that is inclined at an angle θ° to the horizontal. The lid is sufficiently rough to prevent P from slipping.

Given that P is on the point of toppling,

(b) find the value of θ

(7 marks)

(Total for Question 5 is 11 marks)

6. The points A and B lie on a smooth horizontal surface with $AB = 4 \cdot 5$ metres

A light elastic string has natural length $1 \cdot 5$ metres and modulus of elasticity 15 newtons.
One end of the string is attached to A and the other end of the string is attached to B
A particle, P, of mass $0 \cdot 2$ kg, is attached to the stretched string so that APB is a straight line and $AP = 1 \cdot 5$ metres.
The particle rests in equilibrium on the surface.

The particle is now moved directly towards A and is held on the surface so APB is a straight line with $AP = 1$ metre.

The particle is released from rest.

(a) Prove that P moves with simple harmonic motion.
(5 marks)

(continued on the next page)

6. continued.

(b) Find

(i) the maximum speed of P during the motion,

(ii) the maximum acceleration of P during the motion.

(3 marks)

(c) Find the total time, in each complete oscillation of P, for which the speed of P is greater than $5\,m\,s^{-1}$

(5 marks)

(Total for Question 6 is 13 marks)

---

7. A particle, P, of mass m is attached to one end of a light rod of length L

The other end of the rod is attached to a fixed point O so that the rod is free to rotate in a vertical plane about O

The particle is held with the rod horizontal and is then projected vertically downwards with speed u

The particle first comes to instantaneous rest at the point A

(a) Explain why the acceleration of P at A is perpendicular to OA

(1 mark)

(continued on the next page)

7. continued.

At the instant when P is at the point A the acceleration of P is in a direction making an angle θ with the horizontal.

Given that

$$u^2 = \frac{2gL}{3},$$

(b) find

(i) the magnitude of the acceleration of P at the point A,

(ii) the size of θ

(6 marks)

(c) Find, in terms of m and g, the magnitude of the tension in the rod at the instant when P is at its lowest point.

(5 marks)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER