Paper Reference 8FM0−01
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

Monday 13 May 2019 − Afternoon

Time: 1 hour 40 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green), calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

Q58333A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **10** questions in this Question Paper.

The total mark for this paper is **80**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix $\mathbf{M}$ is non–singular.

(2 marks)

The transformation T of the plane is represented by the matrix $\mathbf{M}$

The triangle R is transformed to the triangle S by the transformation T

Given that the area of S is 63 square units,

(b) find the area of R

(2 marks)

(c) Show that the line $y = 2x$ is invariant under the transformation T

(2 marks)

(Total for Question 1 is 6 marks)

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2. **The cubic equation**

$$2x^3 + 6x^2 - 3x + 12 = 0$$

has roots α, β and γ

Without solving the equation, find the cubic equation whose roots are $(\alpha + 3)$, $(\beta + 3)$ and $(\gamma + 3)$, giving your answer in the form $pw^3 + qw^2 + rw + s = 0$, where p, q, r and s are integers to be found.

(Total for Question 2 is 5 marks)

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3. **Prove by mathematical induction that, for $n \in \mathbb{N}$**

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(Total for Question 3 is 6 marks)

4. The line L has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\underline{r} \cdot (\underline{i} - 2\underline{j} + \underline{k}) = -7$$

Determine whether the line L intersects Π at a single point, or lies in Π, or is parallel to Π without intersecting it.

(Total for Question 4 is 5 marks)

5. Refer to the diagram for Question 5 in the Diagram Book.

The complex numbers $z_1 = -2$, $z_2 = -1 + 2i$ and $z_3 = 1 + i$ are plotted on an Argand diagram for the complex plane with $z = x + iy$, as shown in the Diagram Book.

(a) Explain why z_1, z_2 and z_3 cannot all be roots of a quartic polynomial equation with real coefficients.

(2 marks)

(b) Show that

$$\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4}$$

(3 marks)

(c) Hence show that

$$\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

(2 marks)

(continued on the next page)

5. continued.

(d) Shade, on the diagram, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leq |z - 1 - i|$$

(2 marks)

(Total for Question 5 is 9 marks)

6. An art display consists of an arrangement of n marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams.
The mass of each subsequent marble is 3 grams more than the mass of the previous one, so that the rth marble has mass $(7 + 3r)$ grams.

(a) Show that the mean mass, in grams, of the marbles in the display is given by

$$\frac{1}{2}(3n + 17)$$

(3 marks)

(continued on the next page)

6. continued.

Given that there are 85 marbles in the display,

(b) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

(6 marks)

(Total for Question 6 is 9 marks)

7.

$$f(z) = z^3 - 8z^2 + pz - 24$$

where p is a real constant.

Given that the equation $f(z) = 0$ has distinct roots

α, β and $\left(\alpha + \dfrac{12}{\alpha} - \beta\right)$

(a) solve completely the equation $f(z) = 0$

(6 marks)

(b) Hence find the value of p

(2 marks)

(Total for Question 7 is 8 marks)

8. **A gas company maintains a straight pipeline that passes under a mountain.**

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) **Find a vector equation for the line PQ, giving your answer in the form $\underline{r} = \underline{a} + \lambda\underline{b}$, where λ is a scalar parameter.**

(2 marks)

(continued on the next page)

8. continued.

The equation of the plane modelling the side of the mountain is

$$2x + 3y - 5z = 300$$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7 marks)

(continued on the next page)

8. **continued.**

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ

(c) **Determine whether the company should build the new accessway.**

(2 marks)

(d) **Suggest one limitation of the model.**

(1 mark)

(Total for Question 8 is 12 marks)

9.

$$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x–axis and the line $x = 8$ is rotated through θ radians about the x–axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(Total for Question 9 is 8 marks)

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10. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year.
A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0{\cdot}15 \\ 0{\cdot}08 & 0{\cdot}82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \quad n = 0, 1, 2, \ldots$$

where a is a constant, and J_n and A_n are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

(a) Interpret the meaning of the constant a in the context of the model.
(1 mark)

(continued on the next page)

10. continued.

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

(b) (i) Find, in terms of a

$$\begin{pmatrix} a & 0\cdot15 \\ 0\cdot08 & 0\cdot82 \end{pmatrix}^{-1}$$

(3 marks)

(ii) Hence, or otherwise, find the value of a

(3 marks)

(iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

(2 marks)

(continued on the next page)

10. continued.

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short–term suitability of this model.

(1 mark)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2 marks)

(Total for Question 10 is 12 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

END OF PAPER
