Paper Reference 9MA0/02
Pearson Edexcel
Level 3 GCE

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

Wednesday 13 October 2021 – Afternoon

Time: 2 hours plus your additional time allowance

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X68732A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are 15 questions in this Question Paper.

The total mark for this paper is 100

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

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1. In an arithmetic series

- **the first term is 16**
- **the 21st term is 24**

(a) Find the common difference of the series.

(2 marks)

(b) Hence find the sum of the first 500 terms of the series.

(2 marks)

(Total for Question 1 is 4 marks)

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2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x - 1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1 mark)

(b) Find $gf(1 \cdot 8)$

(2 marks)

(c) Find $g^{-1}(x)$

(2 marks)

(Total for Question 2 is 5 marks)

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3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(Total for Question 3 is 3 marks)

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4. Given that θ is small and measured in radians, use the small angle approximations to show that

$$4\sin\frac{\theta}{2} + 3\cos^2\theta \approx a + b\theta + c\theta^2$$

where a, b and c are integers to be found.

(Total for Question 4 is 3 marks)

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5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \quad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3 marks)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4 marks)

(Total for Question 5 is 7 marks)

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6. Refer to the diagram for Question 6 in the Diagram Booklet.

The shape $OABCDEFO$ shown in the diagram is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ, the size of angle AOB

(1 mark)

(continued on the next page)

6. continued.

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2 marks)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π

(2 marks)

(Total for Question 6 is 5 marks)

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7. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Refer to the diagram for Question 7 in the Diagram Booklet.

It shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line L is the tangent to C at P

(a) **Use differentiation to find the equation of L, giving your answer in the form $y = mx + c$ where m and c are integers to be found.**

(4 marks)

(continued on the next page)

7. continued.

(b) Hence verify that L meets C again on the $y-$axis.

(1 mark)

The finite region R, shown shaded in the diagram, is bounded by the curve C and the line L

(c) Use algebraic integration to find the exact area of R

(4 marks)

(Total for Question 7 is 9 marks)

8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4 marks)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation

$$19x + 26y + 123 = 0$$

(b) find the value of p and the value of q

(5 marks)

(Total for Question 8 is 9 marks)

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9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(Total for Question 9 is 3 marks)

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10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = aL^b$$

where L metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b \log_{10} L + \log_{10} a$$

(2 marks)

Refer to the diagram for Question 10(b) in the Diagram Booklet.

A student carried out an experiment to find the values of the constants a and b

The student recorded the value of T for different values of L

(continued on the next page)

10. continued.

The diagram shows the linear relationship between $\log_{10} L$ and $\log_{10} T$ for the student's data.

The straight line passes through the points $(-0{\cdot}7, 0)$ and $(0{\cdot}21, 0{\cdot}45)$

Using this information,

(b) find a complete equation for the model in the form

$$T = aL^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3 marks)

(c) With reference to the model, interpret the value of the constant a

(1 mark)

(Total for Question 10 is 6 marks)

11. Refer to the diagram for Question 11 in the Diagram Booklet.

It shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

There are blank axes on pages 34 – 45 in the Answer Booklet if you wish to use them.

(4 marks)

(continued on the next page)

11. continued.

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4 marks)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2 marks)

(Total for Question 11 is 10 marks)

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12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = \int_p^q \frac{2(u-1)^3}{u}\,du$$

where p and q are constants to be found.

(3 marks)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,dx = A - B \ln 5$$

where A and B are constants to be found.

(4 marks)

(Total for Question 12 is 7 marks)

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13. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \text{cosec}^3\theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3 marks)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3 marks)

(Total for Question 13 is 6 marks)

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14. Refer to Diagram 1, Diagram 2 and Diagram 3 for Question 14 in the Diagram Booklet.

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 metres by 3 metres and the height of the tank is 5 metres.

There is a tap at a point T at the bottom of the tank, as shown in Diagram 1
Diagram 2 shows the front view of the tank and Diagram 3 shows the side view of the tank.

(continued on the next page)

14. continued.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of $0{\cdot}48\ m^3$ per minute
- water is modelled as leaving the tank through the tap at a rate of $0{\cdot}1h\ m^3$ per minute

(a) Show that, according to the model,

$$1200\frac{dh}{dt} = 24 - 5h$$

(4 marks)

Given that when the tap was opened, the depth of water in the tank was 2 metres,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where A, B and k are constants to be found.

(6 marks)

(continued on the next page)

14. continued.

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2 marks)

(Total for Question 14 is 12 marks)

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15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3 marks)

Refer to the diagram for Question 15 in the Diagram Booklet.

It shows the cross–section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C

The point P is at the end of one of the paddles of the wheel, as shown in the diagram.

The water level is assumed to be horizontal and of constant height.

(continued on the next page)

15. continued.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0{\cdot}5t) - 2\sin(0{\cdot}5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3 marks)

(continued on the next page)

15. continued.

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(Solutions based entirely on calculator technology are not acceptable.)

(4 marks)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1 mark)

(Total for Question 15 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
