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Candidate surname

Other names

Centre Number

Candidate Number

## Pearson Edexcel Level 3 GCE

Wednesday 6 October 2021 – Afternoon

Time 2 hours

Paper  
reference

**9MA0/01**

# Mathematics

Advanced

**PAPER 1: Pure Mathematics 1**

### You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1.

$$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that  $(x - 1)$  is a factor of  $f(x)$ , find the value of the constant  $a$ .

You must make your method clear.

(3)

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Question 1 continued

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(Total for Question 1 is 3 marks)



2. Given that

$$f(x) = x^2 - 4x + 5 \quad x \in \mathbb{R}$$

(a) express  $f(x)$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers to be found.

(2)

The curve with equation  $y = f(x)$

- meets the  $y$ -axis at the point  $P$
- has a minimum turning point at the point  $Q$

(b) Write down

- the coordinates of  $P$
- the coordinates of  $Q$

(2)

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Question 2 continued

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(Total for Question 2 is 4 marks)



3. The sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_{n+1} = k - \frac{24}{u_n} \quad u_1 = 2$$

where  $k$  is an integer.

Given that  $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of  $k$ , giving a reason for your answer.

(2)

(c) Find the value of  $u_3$

(1)

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Question 3 continued

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(Total for Question 3 is 6 marks)



4. The curve with equation  $y = f(x)$  where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at  $x = \alpha$

(a) Show that  $\alpha$  is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0 \tag{4}$$

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for  $\alpha$ .

Starting with  $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of  $x_2$

(ii) the value of  $x_4$

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that  $\alpha$  is 0.341 to 3 decimal places.

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 9 marks)



5.

**In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

- (a) show that the profit for Year 3 will be £23 328 (1)
  
- (b) find the first year when the yearly profit will exceed £65 000 (3)
  
- (c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000 (2)

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Question 5 continued

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(Total for Question 5 is 6 marks)



6.

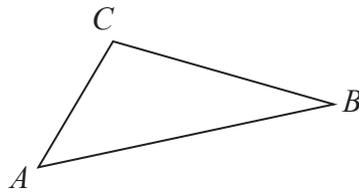


Figure 1

Figure 1 shows a sketch of triangle  $ABC$ .

Given that

- $\vec{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\vec{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find  $\vec{AC}$

(2)

(b) show that  $\cos ABC = \frac{9}{10}$

(3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 5 marks)



7. The circle  $C$  has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

- (i) the coordinates of the centre of  $C$ ,
- (ii) the exact radius of  $C$ , giving your answer as a simplified surd.

(4)

The line  $l$  has equation  $y = 3x + k$  where  $k$  is a constant.

Given that  $l$  is a tangent to  $C$ ,

- (b) find the possible values of  $k$ , giving your answers as simplified surds.

(5)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 9 marks)



8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria,  $N$ , in the **first** population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where  $A$  and  $k$  are positive constants and  $t$  is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria,  $M$ , in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \quad t \geq 0$$

where  $k$  has the value found in part (a) and  $t$  is the time in hours from the start of the study.

Given that  $T$  hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of  $T$ .

(3)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 9 marks)



9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that  $f(x)$  can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where  $A$ ,  $B$  and  $C$  are constants

(a) (i) find the value of  $B$  and the value of  $C$

(ii) show that  $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of  $x$

$$f(x) = p + qx + rx^2 + \dots$$

where  $p$ ,  $q$  and  $r$  are simplified fractions to be found.

(ii) Find the range of values of  $x$  for which this expansion is valid.

(7)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 11 marks)



10.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that  $1 + \cos 2\theta + \sin 2\theta \neq 0$  prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta$$

(4)

(b) Hence solve, for  $0 < x < 180^\circ$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 8 marks)



11.

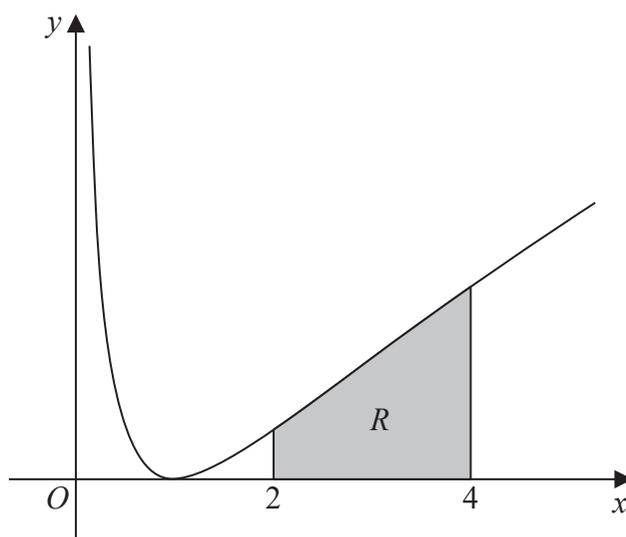


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve, the line with equation  $x = 2$ , the  $x$ -axis and the line with equation  $x = 4$

The table below shows corresponding values of  $x$  and  $y$ , with the values of  $y$  given to 4 decimal places.

$x$	2	2.5	3	3.5	4
$y$	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of  $y$  in the table, to obtain an estimate for the area of  $R$ , giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of  $R$ , giving your answer in the form

$$y = a(\ln 2)^2 + b \ln 2 + c$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

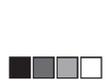
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Question 11 continued

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Question 11 continued

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Question 11 continued

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(Total for Question 11 is 8 marks)



12.

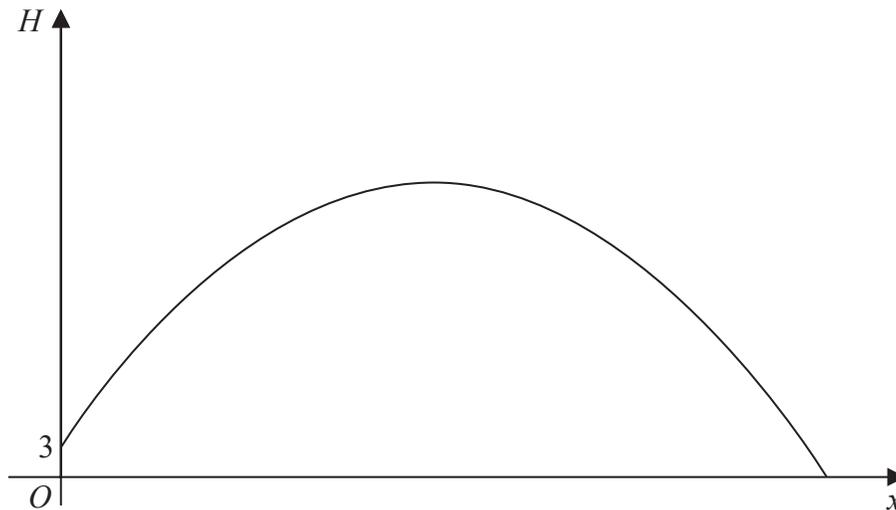


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height,  $H$  metres, of the ball above the ground has been plotted against the horizontal distance travelled,  $x$  metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that  $H$  is modelled as a **quadratic** function in  $x$

- (a) find  $H$  in terms of  $x$  (5)
- (b) Hence find, according to the model,
- (i) the maximum vertical height of the ball above the ground,
- (ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre. (3)
- (c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model. (1)

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Question 12 continued

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Question 12 continued

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Question 12 continued

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(Total for Question 12 is 9 marks)



13. A curve  $C$  has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \quad y = \frac{4t}{t^2 + 1} \quad t \in \mathbb{R}$$

Show that all points on  $C$  satisfy

$$(x - 3)^2 + y^2 = 4 \quad (3)$$

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Question 13 continued

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(Total for Question 13 is 3 marks)



14. Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \quad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \quad x > 0$$

where  $A$  is a constant to be found.

(4)

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Question 14 continued

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Question 14 continued

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Question 14 continued

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(Total for Question 14 is 4 marks)



15. (i) Use proof by exhaustion to show that for  $n \in \mathbb{N}$ ,  $n \leq 4$

$$(n + 1)^3 > 3^n \quad (2)$$

(ii) Given that  $m^3 + 5$  is odd, use proof by contradiction to show, using algebra, that  $m$  is even.

(4)

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Question 15 continued

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Question 15 continued

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Question 15 continued

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Question 15 continued

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(Total for Question 15 is 6 marks)

**TOTAL FOR PAPER IS 100 MARKS**

