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Candidate surname		Other names
Centre Number	Candidate Number	
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**Pearson Edexcel Level 3 GCE**

**Wednesday 13 October 2021 – Afternoon**

Time 2 hours	Paper reference	<b>9MA0/02</b>
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**Mathematics**

**Advanced**

**PAPER 2: Pure Mathematics 2**

<b>You must have:</b> Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks
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**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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A:1/1/1/1/1/

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

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Question 1 continued

(Total for Question 1 is 4 marks)

2. The functions  $f$  and  $g$  are defined by

$$f(x) = 7 - 2x^2 \quad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \quad x \in \mathbb{R} \quad x \neq \frac{1}{5}$$

(a) State the range of  $f$

(1)

(b) Find  $gf(1.8)$

(2)

(c) Find  $g^{-1}(x)$

(2)

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Question 2 continued

(Total for Question 2 is 5 marks)

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

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Question 3 continued

(Total for Question 3 is 3 marks)

4. Given that  $\theta$  is small and measured in radians, use the small angle approximations to show that

$$4 \sin \frac{\theta}{2} + 3 \cos^2 \theta \approx a + b\theta + c\theta^2$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

(3)

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Question 4 continued

(Total for Question 4 is 3 marks)

5. The curve  $C$  has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \quad x \in \mathbb{R}$$

(a) Find

(i)  $\frac{dy}{dx}$

(ii)  $\frac{d^2y}{dx^2}$

(3)

(b) (i) Verify that  $C$  has a stationary point at  $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

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Question 5 continued

(Total for Question 5 is 7 marks)

6.

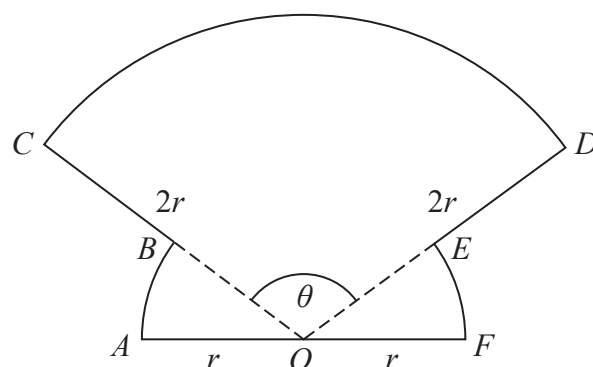


Figure 1

The shape  $OABCDEFO$  shown in Figure 1 is a design for a logo.

In the design

- $OAB$  is a sector of a circle centre  $O$  and radius  $r$
- sector  $OFE$  is congruent to sector  $OAB$
- $ODC$  is a sector of a circle centre  $O$  and radius  $2r$
- $AOF$  is a straight line

Given that the size of angle  $COD$  is  $\theta$  radians,

(a) write down, in terms of  $\theta$ , the size of angle  $AOB$

(1)

(b) Show that the area of the logo is

$$\frac{1}{2} r^2 (3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of  $r$ ,  $\theta$  and  $\pi$ .

(2)

Question 6 continued



Question 6 continued

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Question 6 continued

(Total for Question 6 is 5 marks)

7.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

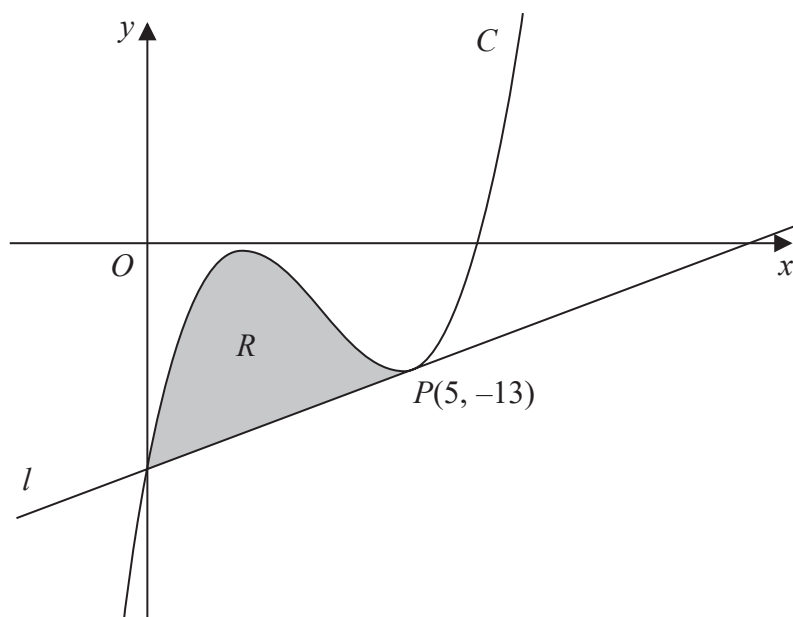


Figure 2

Figure 2 shows a sketch of part of the curve  $C$  with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point  $P(5, -13)$  lies on  $C$ The line  $l$  is the tangent to  $C$  at  $P$ 

- (a) Use differentiation to find the equation of  $l$ , giving your answer in the form  $y = mx + c$  where  $m$  and  $c$  are integers to be found. (4)

- (b) Hence verify that  $l$  meets  $C$  again on the  $y$ -axis. (1)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve  $C$  and the line  $l$ .

- (c) Use algebraic integration to find the exact area of  $R$ . (4)



Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

8. The curve  $C$  has equation

$$px^3 + qxy + 3y^2 = 26$$

where  $p$  and  $q$  are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

(4)

Given that

- the point  $P(-1, -4)$  lies on  $C$
- the normal to  $C$  at  $P$  has equation  $19x + 26y + 123 = 0$

(b) find the value of  $p$  and the value of  $q$ .

(5)

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Question 8 continued



Question 8 continued

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Question 8 continued

(Total for Question 8 is 9 marks)

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{9}{28}$$

(3)

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Question 9 continued

(Total for Question 9 is 3 marks)

10. The time,  $T$  seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where  $l$  metres is the length of the pendulum and  $a$  and  $b$  are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b \log_{10} l + \log_{10} a \quad (2)$$

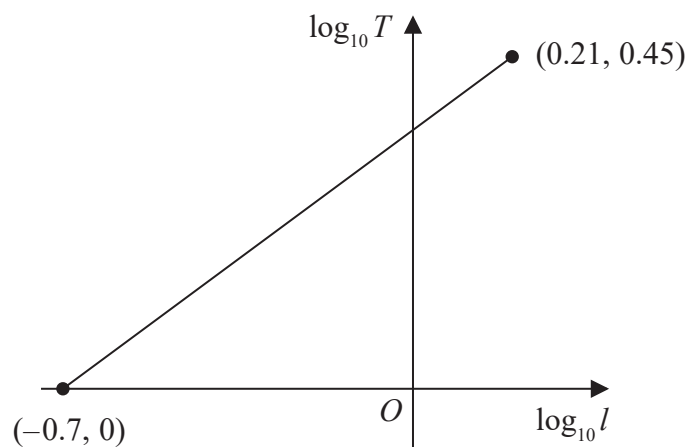


Figure 3

A student carried out an experiment to find the values of the constants  $a$  and  $b$ .

The student recorded the value of  $T$  for different values of  $l$ .

Figure 3 shows the linear relationship between  $\log_{10} l$  and  $\log_{10} T$  for the student's data. The straight line passes through the points  $(-0.7, 0)$  and  $(0.21, 0.45)$

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of  $a$  and the value of  $b$ , each to 3 significant figures. (3)

(c) With reference to the model, interpret the value of the constant  $a$ . (1)

Question 10 continued



Question 10 continued

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Question 10 continued

(Total for Question 10 is 6 marks)

11.

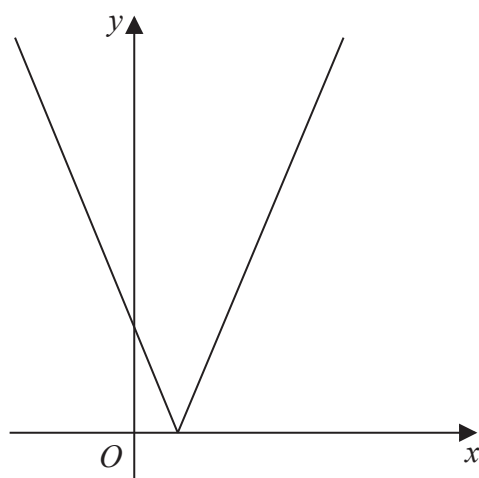


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where  $k$  is a positive constant.

(a) Sketch the graph with equation  $y = f(x)$  where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of  $k$ , the set of values of  $x$  for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of  $k$ , the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

Question 11 continued



Question 11 continued

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Question 11 continued

(Total for Question 11 is 10 marks)

12. (a) Use the substitution  $u = 1 + \sqrt{x}$  to show that

$$\int_0^{16} \frac{x}{1 + \sqrt{x}} \, dx = \int_p^q \frac{2(u-1)^3}{u} \, du$$

where  $p$  and  $q$  are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1 + \sqrt{x}} \, dx = A - B \ln 5$$

where  $A$  and  $B$  are constants to be found.

(4)

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Question 12 continued



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Question 12 continued

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(Total for Question 12 is 7 marks)



13. The curve  $C$  has parametric equations

$$x = \sin 2\theta \quad y = \operatorname{cosec}^3 \theta \quad 0 < \theta < \frac{\pi}{2}$$

- (a) Find an expression for  $\frac{dy}{dx}$  in terms of  $\theta$  (3)
- (b) Hence find the exact value of the gradient of the tangent to  $C$  at the point where  $y = 8$  (3)

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Question 13 continued

(Total for Question 13 is 6 marks)

14.

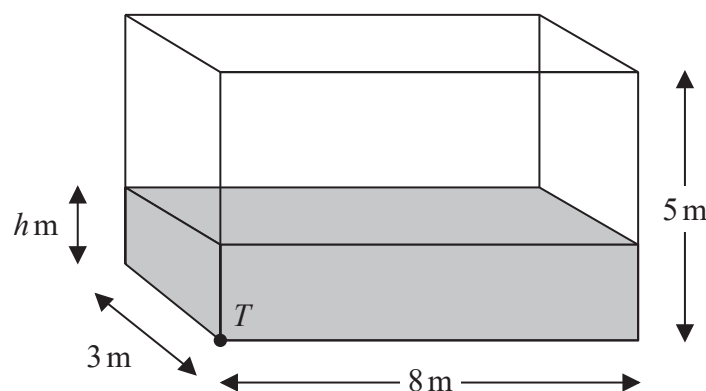


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point  $T$  at the bottom of the tank, as shown in Figure 5.

At time  $t$  minutes after the tap has been opened

- the depth of water in the tank is  $h$  metres
- water is flowing into the tank at a constant rate of  $0.48 \text{ m}^3$  per minute
- water is modelled as leaving the tank through the tap at a rate of  $0.1h \text{ m}^3$  per minute

(a) Show that, according to the model,

$$1200 \frac{dh}{dt} = 24 - 5h \quad (4)$$

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where  $A$ ,  $B$  and  $k$  are constants to be found. (6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer. (2)





Question 14 continued



Question 14 continued

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Question 14 continued

(Total for Question 14 is 12 marks)

15. (a) Express  $2\cos\theta - \sin\theta$  in the form  $R\cos(\theta + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$

Give the exact value of  $R$  and the value of  $\alpha$  in radians to 3 decimal places.

(3)

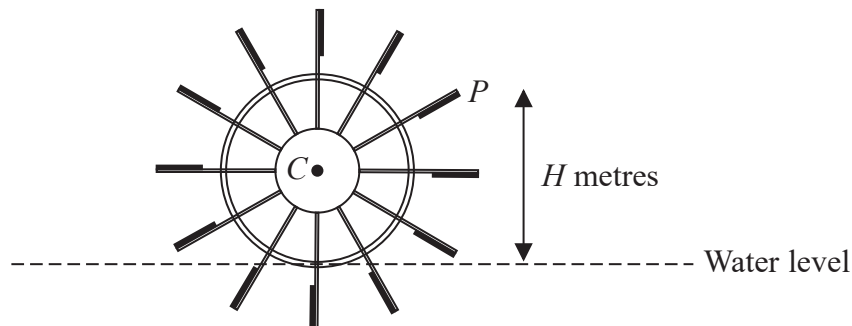


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point  $C$ .

The point  $P$  is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height,  $H$  metres, of  $P$  above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where  $t$  is the time in seconds after the wheel starts rotating.

Using the model, find

- (b) (i) the maximum height of  $P$  above the water level,  
(ii) the value of  $t$  when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel,  $P$  is below the water level for a total of  $T$  seconds.

According to the model,

- (c) find the value of  $T$  giving your answer to 3 significant figures.

*(Solutions based entirely on calculator technology are not acceptable.)*

(4)

In reality, the water level may not be of constant height.

- (d) Explain how the equation of the model should be refined to take this into account.

(1)

Question 15 continued



Question 15 continued

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Question 15 continued

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Question 15 continued

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(Total for Question 15 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

