Paper Reference 9MA0/01
Pearson Edexcel
Level 3 GCE

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

Wednesday 6 October 2021 – Afternoon

Time: 2 hours plus your additional time allowance

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X68731A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **15** questions in this Question Paper.

The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. $f(x) = ax^3 + 10x^2 - 3ax - 4$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a

You must make your method clear.

(Total for Question 1 is 3 marks)

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2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2 marks)

The curve with equation $y = f(x)$

- meets the y–axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2 marks)

(Total for Question 2 is 4 marks)

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3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3 marks)

(b) Find the value of k, giving a reason for your answer.

(2 marks)

(c) Find the value of u_3

(1 mark)

(Total for Question 3 is 6 marks)

4. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4 marks)

(continued on the next page)

4. continued.

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\,2} - 2x_n^{\,3})$$

is used to find an approximate value for α

Starting with $x_1 = 0 \cdot 3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3 marks)

(continued on the next page)

4. continued.

Using a suitable interval and a suitable function that should be stated,

(c) show that α is $0\cdot341$ to 3 decimal places.
(2 marks)

(Total for Question 4 is 9 marks)

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5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328
(1 mark)

(b) find the first year when the yearly profit will exceed £65 000
(3 marks)

(continued on the next page)

5. continued.

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2 marks)

(Total for Question 5 is 6 marks)

6. Refer to the diagram for Question 6 in the Diagram Booklet.

It shows a sketch of triangle ABC

Given that

- $\overrightarrow{AB} = -3\underline{i} - 4\underline{j} - 5\underline{k}$
- $\overrightarrow{BC} = \underline{i} + \underline{j} + 4\underline{k}$

(a) find $\overrightarrow{AC}$

(2 marks)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3 marks)

(Total for Question 6 is 5 marks)

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7. **The circle C has equation**

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4 marks)

The line L has equation $y = 3x + k$ where k is a constant.
Given that L is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.
(5 marks)

(Total for Question 7 is 9 marks)

8. A scientist is studying the growth of two different populations of bacteria.
The number of bacteria, N, in the FIRST population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.
(4 marks)

(continued on the next page)

8. continued.

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study.
Give your answer to 2 significant figures.
(2 marks)

(continued on the next page)

8. continued.

The number of bacteria, M, in the SECOND population is modelled by the equation

$$M = 500e^{1 \cdot 4kt} \quad t \geq 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T

(3 marks)

(Total for Question 8 is 9 marks)

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4 marks)

(continued on the next page)

9. continued.

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7 marks)

(Total for Question 9 is 11 marks)

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4 marks)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4 marks)

(Total for Question 10 is 8 marks)

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11. Refer to the diagram for Question 11 in the Diagram Booklet.

It shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \quad x > 0$$

The finite region R, shown shaded in the diagram, is bounded by the curve, the line with equation $x = 2$, the x–axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	y
2	0·4805
2·5	0·8396
3	1·2069
3·5	1·5694
4	1·9218

(continued on the next page)

11. continued.

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.
(3 marks)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.
(5 marks)

(Total for Question 11 is 8 marks)

12. Refer to the diagram for Question 12 in the Diagram Booklet.

It is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

(continued on the next page)

12. continued.

Given that the ball

- **is hit from a point on the top of a platform of vertical height 3 metres above the ground**
- **reaches its maximum vertical height after travelling a horizontal distance of 90 metres**
- **is at a vertical height of 27 metres above the ground after travelling a horizontal distance of 120 metres**

Given also that H is modelled as a QUADRATIC function in x

(a) find H in terms of x

(5 marks)

(continued on the next page)

12. continued.

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3 marks)

(c) The possible effects of wind or air resistance are two limitations of the model.

Give one other limitation of this model.

(1 mark)

(Total for Question 12 is 9 marks)

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \quad y = \frac{4t}{t^2 + 1} \quad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(Total for Question 13 is 3 marks)

14. Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(Total for Question 14 is 4 marks)

15. (i) **Use proof by exhaustion to show that for**

$n \in \mathbb{N}, n \leq 4$

$$(n + 1)^3 > 3^n$$

(2 marks)

(ii) **Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.**

(4 marks)

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER is 100 MARKS

END OF PAPER
