Mark Scheme (Results Day)

Summer 2025

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 02 Pure Mathematics 2

Modified Version

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2025
Question Paper Log Number P74091A
Publications Code 9MA0_02MV_2506_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	For either $x=4$ or $y=10$	M1	1.1b
	$(4, 10)$	A1	1.1b
		(2)	
(b)	$-x+4+10=3x \Rightarrow x=...$	M1	1.1b
	$x=\frac{7}{2}$	A1	2.1
		(2)	
			(4 marks)

Notes:

(a)

M1: One correct coordinate. May be seen embedded as $(4,10)$ or e.g. $4,10$

A1: $(4,10)$ but may be given $x=4\ \ y=10$ Condone $4,10$

(b)

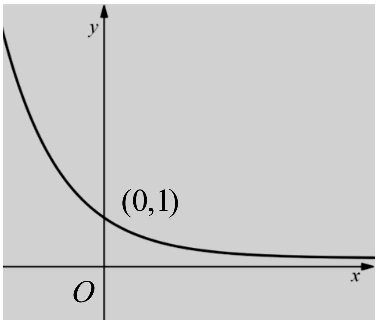
M1: For an attempt to solve $-x+4+10=3x \Rightarrow x=...$ or the equivalent equation e.g. $-x+4=3x-10 \Rightarrow x=...$, $x-4=10-3x \Rightarrow x=...$ etc.

A1: $x=\frac{7}{2}$ oe only. There must be no extra solutions.

Alternative by squaring:

$$|x-4|+10=3x \Rightarrow |x-4|=3x-10 \Rightarrow x^2-8x+16=(3x-10)^2=9x^2-60x+100$$

$$\Rightarrow 8x^2-52x+84=0 \Rightarrow x=\frac{7}{2} \text{ only}$$

Question	Scheme		Marks	AOs
2 (a)		Correct shape or correct intercept – see notes	B1	1.2
		Fully correct – see notes	B1	1.1b
			(2)	

Notes:

Note that B0B1 is not possible in part (a)

(a) **Axes do not need to be labelled. No sketch is no marks.**

B1: Correct shape or correct intercept.

Shape: A negative exponential curve in quadrants 1 and 2 only, passing through one point on the positive y-axis. Must "level out" in quadrant 1 but not necessarily asymptotic to the x-axis and allow if the curve bends up slightly for $x > 0$ but do not allow a clear "U" shape. It must not clearly "stop" on the x-axis to the right of the y-axis.

OR

Intercept: The intercept can be marked as 1 or (0, 1) or $y = 1$ or (1, 0) as long as it is in the correct place. May also be seen away from the sketch but must be seen as (0, 1) or possibly these coordinates in a table but it must correspond to the sketch. If there is any ambiguity, the sketch takes precedence.

B1: Fully correct.

Shape: A negative exponential curve in quadrants 1 and 2 only, passing through one point on the positive y-axis. The curve must appear to be asymptotic to the x-axis **and it must level out at least half way below the intercept**.

Allow if the curve bends up slightly for $x > 0$ but do not allow a clear "U" shape. The curve must not bend back on itself on the rhs of the y-axis. There must be no suggestion that the curve approaches another horizontal asymptote other than the x-axis e.g. a horizontal dotted line that the curve approaches.

AND

Intercept: As above

(2)

(b)	Sets $\frac{1}{32}\sqrt{2} = 2^{-\frac{9}{2}}$ or $\frac{1}{2^{\frac{9}{2}}}$	B1	1.1b
	$\Rightarrow x = \frac{3}{2}$	B1	1.1b
		(2)	
			(4 marks)

(b)

B1: Writes $\frac{1}{32}\sqrt{2} = 2^{-\frac{9}{2}}$ or $\frac{1}{2^{\frac{9}{2}}}$ which may be implied.

B1: $\frac{3}{2}$ or exact equivalent

Correct answer only scores both marks.

Question	Scheme	Marks	AOs
3	Uses the given sequence formula correctly at least once $a_2 = 4k+3$, $a_3 = k(4k+3)+3$	M1	1.1b
	$\sum_{n=1}^{3} a_n = 12 \Rightarrow 4+4k+3+k(4k+3)+3=12$	dM1	3.1a
	$4k^2+7k-2=0$ $(4k-1)(k+2)=0$	ddM1	1.1b
	$k=-2$ only	A1	2.2a
		(4)	
			(4 marks)

Notes:

M1: Uses the given sequence formula at least once

dM1: Dependent upon having used the sequence formula at least once, scored for setting $4+\text{their } a_2+\text{their } a_3=12$ where a_2 and a_3 are in terms of k.

ddM1: Dependent upon both previous marks. It is for solving a 3 term quadratic equation in k by any correct method including a calculator.

A1: cso $k=-2$ only. If $k=\frac{1}{4}$ is written down it must be rejected.

Must be $k=\ldots$ not $x=\ldots$

Note this is quite common:

$$a_2 = 4k+3,\ a_3 = k(4k+3)$$

$$\sum_{n=1}^{3} a_n = 12 \Rightarrow 4+4k+3+k(4k+3)=12 \Rightarrow 4k^2+7k-5=0$$

$$\Rightarrow k=\frac{-7\pm\sqrt{129}}{8}\ (=0.544,\ -2.29)$$

And scores M1dM1ddM1A0

Question	Scheme	Marks	AOs
4(a)	Attempts $\mathrm{f}(5)=-18$ and $\mathrm{f}(6)=(+)\,4$	M1	1.1b
	States change of sign, function continuous so root between	A1	2.1
		(2)	
(b)	$p=\ln 2$ (so $\mathrm{f}'(x)=2^x\ln 2-10$)	B1	1.2
		(1)	
(c)	Attempts $x_1=6-\dfrac{4}{64\ln 2\text{-}10}$	M1	1.1b
	Awrt 5.88	A1	1.1b
		(2)	
(d)	Sets $\mathrm{f}'(x)=2^x\ln 2-10=0\Rightarrow 2^x=\dfrac{10}{\ln 2}\Rightarrow x=...$	M1	1.1b
	$\Rightarrow x=$ awrt 3.85	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts both $\mathrm{f}(5)=-18$ and $\mathrm{f}(6)=(+)\,4$ with at least one correct.

A1: Completes the argument.
Requires
- both values correct
- gives full reason including "change of sign" (o.e) and "continuity"
- a minimal conclusion, e.g. root, tick, α lies in the interval $[5,6]$

Accept equivalent statements for sign change e.g. $\mathrm{f}(6)>0$, $\mathrm{f}(5)<0$ e.g. $\mathrm{f}(5)\times\mathrm{f}(6)<0$, $\mathrm{f}(5)<0<\mathrm{f}(6)$, “one negative one positive”, “there is a change of sign”

A minimum is “change of sign and continuous” but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. “because *x* is continuous” or “because the interval is continuous”

(b)

B1: $p=\ln 2$ or $\mathrm{f}'(x)=2^x\ln 2-10$ Condone $p=\log 2$ unless $p=\log_{10}2$ is implied by subsequent work.

(c)

M1: Attempts $x_1=6-\dfrac{\mathrm{f}(6)}{\mathrm{f}'(6)}$ following through on their $\mathrm{f}(6)$ and $\mathrm{f}'(6)$
May be implied by their value if no working is shown.

A1: Awrt 5.88

(d)

M1: Sets $p \times 2^x - 10 = 0$ with their positive numerical value of p (which may be 1) and uses correct processing to find a value for x e.g. $2^x = \frac{10}{p} \Rightarrow x = \log_2 \frac{10}{p}$ or $2^x = \frac{10}{p} \Rightarrow x = \frac{\log \frac{10}{p}}{\log 2}$ or $2^x = \frac{10}{p} \Rightarrow x = \frac{\ln \frac{10}{p}}{\ln 2}$
A1: awrt 3.85

Question	Scheme	Marks	AOs
5 (a)	States or uses $h = 0.25$	B1	1.1a
	$\frac{\dots}{2}\{1+0.1054+2\times(0.9394+0.7788+0.5698+0.3679+0.2096)\}$	M1	1.1b
	$= 0.855$	A1	1.1b
		(3)	
(b)(i)	$\int_{-1.5}^{1.5} e^{-x^2}\, dx = 2\times(a) = 1.71$	B1ft	2.2a
(ii)	$\int_0^{1.5}\left(e^{-x^2}+7\right) dx = (a)+[7x]_0^{1.5} = (a)+7\times 1.5$ $= 0.855+10.5$ $= 11.355$	M1 A1ft	3.1a 1.1b
		(3)	
			(6 marks)

Notes:

(a)

B1: States or uses $h = 0.25$

M1: A full attempt at the trapezium rule with their h.
All y terms must be present but condone copying slips.
Do not condone missing brackets unless implied by subsequent work.

A1: 0.855 only (not awrt)
(Note that the calculator answer is 0.856)

(b)(i) **Attempts to use the trapezium rule again score no marks.**

B1ft: For $2\times$ their answer to part (a). Correct method must be seen.

(b)(ii)

M1: Awarded for their answer to part (a) + 7×1.5 or their answer to part (a) + 10.5

A1ft: Correct working followed by awrt 11.4 but ft on their 0.855

Question	Scheme	Marks	AOs
6(a)	$\dfrac{4}{(2+3x)^2}=4\times2^{-2}\times\left(1+\dfrac{3}{2}x\right)^{-2}$	M1	3.1a
	$\left(1+\dfrac{3}{2}x\right)^{-2}=$ $1+(-2)\times\left(\dfrac{3x}{2}\right)+\dfrac{(-2)\times(-3)}{2!}\left(\dfrac{3x}{2}\right)^2+\dfrac{(-2)\times(-3)\times(-4)}{3!}\left(\dfrac{3x}{2}\right)^3$	M1 A1	1.1b 1.1b
	$\dfrac{4}{(2+3x)^2}=1-3x+\dfrac{27}{4}x^2-\dfrac{27}{2}x^3$	A1	1.1b
		(4)	
(b)	$\lvert x \rvert<\dfrac{2}{3}$	B1	2.2a
		(1)	
			(5 marks)

Notes:

(a)

M1: Attempts to take out a factor of 2 from the bracket to obtain $k\left(1+\dfrac{3}{2}x\right)^{-2}$.

Condone slips/errors when taking out the factor of 2 as long as they reach the form $k\left(1+\dfrac{3}{2}x\right)^{-2}$.

M1: For an attempt on the binomial expansion of $(1+ax)^{-2}$.

Look for correct coefficients with correct powers of x for term 3 or term 4

$$\frac{(-2)\times(-3)}{2!}(ax)^2 \text{ or } \frac{(-2)\times(-3)\times(-4)}{3!}(ax)^3$$

where a could be 1 but must be consistent with a in their $(1+ax)^{-2}$.

Condone missing brackets around the "ax"

A1: Correct (may be unsimplified) expansion of $\left(1+\dfrac{3}{2}x\right)^{-2}$

A1: $1-3x+\dfrac{27}{4}x^2-\dfrac{27}{2}x^3$

Allow as a list but not e.g. $1+-3x+\dfrac{27}{4}x^2+-\dfrac{27}{2}x^3$

(b)

B1: $\lvert x \rvert<\dfrac{2}{3}$ oe e.g. $-\dfrac{2}{3}<x<\dfrac{2}{3}$ or $\left(-\dfrac{2}{3},\dfrac{2}{3}\right)$

Question	Scheme	Marks	AOs
7(a)	$\mathrm{gf}(\mathrm{e}^2) = \mathrm{g}(3) = \dfrac{4\times 3+3}{2\times 3+1}$	M1	1.1b
	$= \dfrac{15}{7}$	A1	1.1b
		(2)	
(b)	$\mathrm{g}'(x) = \dfrac{4(2x+1)-2(4x+3)}{(2x+1)^2}$	M1	3.1a
	$\mathrm{g}'(x) = \dfrac{-2}{(2x+1)^2}$	A1	1.1b
	Requires • Correct $\mathrm{g}'(x)$ • Statement $\mathrm{g}'(x) < 0$ oe • Conclusion e.g. "proven", "true", "decreasing"	A1	2.1
		(3)	
(c)	Achieves either boundary $\dfrac{3}{2}\ln 2$ or $\dfrac{3}{2}\ln 3$	M1	1.1b
	$\dfrac{3}{2}\ln 2 < \mathrm{fg} < \dfrac{3}{2}\ln 3$ or e.g. $\dfrac{3}{2}\ln 2 < y < \dfrac{3}{2}\ln 3$	A1	2.5
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts to apply the operations in the correct order. E.g. substitutes $\dfrac{3}{2}\ln \mathrm{e}^2$ or 3 into g

A1: Achieves $\dfrac{15}{7}$ or exact equivalent.

(b)

M1: Attempts the quotient rule (or product rule) to obtain the correct form or divides to reach the form $A + \dfrac{B}{2x+1}$ where A and B are positive constants.

A1: $\mathrm{g}'(x) = \dfrac{-2}{(2x+1)^2}$ OR $\mathrm{g}(x) = 2 + \dfrac{1}{(2x+1)}$

A1: See scheme. For the alternative method look for

- Correct $\mathrm{g}(x) = 2 + \dfrac{1}{(2x+1)}$
- Correct statement. As x increases (or $x \to \infty$) $\dfrac{1}{(2x+1)}$ decreases (or $\to 0$) so $\mathrm{g}(x)$ is a decreasing function

(c)

M1: Achieves either exact boundary $\dfrac{3}{2}\ln 2$ or $\dfrac{3}{2}\ln 3$

A1: Correct range written with correct notation and with strict inequalities

Question	Scheme	Marks	AOs
8	$\int \sqrt{2x-7}\,\mathrm{d}x = \dfrac{(2x-7)^{\frac{3}{2}}}{3}$	M1 A1	1.1b 1.1b
	Deduces **one** of; top limit is 16, bottom limit is $\frac{7}{2}$ or area rectangle = 80	B1	2.2a
	Deduces **all three** of ; top limit is 16, bottom limit is $\frac{7}{2}$ and area rectangle = 80	B1	2.2a
	Full strategy to find area R $= 80 - \left[\dfrac{(2x-7)^{\frac{3}{2}}}{3}\right]_{\frac{7}{2}}^{16}$	M1	3.1a
	Area $R = 80 - \dfrac{125}{3} = \dfrac{115}{3}$	A1	1.1b
		(6)	
			(6 marks)

Notes:

M1: Attempts to integrate $\sqrt{2x-7}$ to achieve $k(2x-7)^{\frac{3}{2}}$

May be implied if a substitution is used e.g. $u = 2x-7 \Rightarrow \int \sqrt{2x-7}\,\mathrm{d}x = ku^{\frac{3}{2}}$

A1: $\int \sqrt{2x-7}\,\mathrm{d}x = \dfrac{(2x-7)^{\frac{3}{2}}}{3}$ or e.g. $\frac{1}{3}u^{\frac{3}{2}}$ where $u = 2x-7$ which can be left unsimplified

B1: Deduces correct top limit, bottom limit or area of rectangle

B1: Deduces correct top limit, bottom limit and area of rectangle

M1: Fully correct strategy to find the area of R e.g. a correct unsimplified expression for area. Follow through their integration but limits etc. must be correct.

A1: $\dfrac{115}{3}$ or exact equivalent. Allow $38.\dot{3}$ or $38\frac{1}{3}$ but not 38.3

Alt method (not on specification but may be seen):

Attempts $\int_0^5 x\,\mathrm{d}y = \int_0^5 \dfrac{y^2+7}{2}\,\mathrm{d}y$ scores M1 A1 B1

$$= \left[\frac{y^3}{6} + \frac{7}{2}y\right]_0^5 = \frac{125}{6} + \frac{35}{2} = \frac{115}{3} \quad \text{B1 M1 A1}$$

Question	Scheme	Marks	AOs
9(a)	Attempts $\pm\left((2p+3)^2-(2p+1)^2\right)$ or equivalent	M1	3.1a
	$=8p+8$ o.e.	A1	1.1b
	Requires • correct expression • minimal conclusion "hence multiple of 8", "proven", "true" etc.	A1	2.1
		(3)	
(b)	Provides a counter example. E.g $4^2-2^2=12$ which is not a multiple of 8	B1	2.4
		(1)	
			(4 marks)

Notes:

(a)

M1: Awarded for the key step in writing down an expression of the form

$$\pm\left((2p+3)^2-(2p+1)^2\right) \text{ oe e.g. } \pm\left((2p+1)^2-(2p-1)^2\right)$$

A1: Correct simplified expression for their $\pm\left((2p+3)^2-(2p+1)^2\right)$

Note that $\pm\left((2p+1)^2-(2p-1)^2\right)=\pm 8p$

A1: Rigorous proof showing all key steps with explanations

(b)

B1: Shows that the statement is not true for consecutive even numbers

Note that a proof such as $(2p+2)^2-(2p)^2=8p+4$ would need fully correct work and an explanation why $8p+4$ is not a multiple of 8.

Question	Scheme	Marks	AOs
10(a)	$f'(x) = 4e^{-x^2} + (4x-k) \times -2xe^{-x^2}$	M1 A1	1.1b 1.1b
	$f'(1) = 4e^{-1} + (4-k) \times -2e^{-1} = 0$	M1	2.1
	$f'(1) = 4e^{-1} - 8e^{-1} + 2ke^{-1} = 0 \Rightarrow -4e^{-1} + 2ke^{-1} = 0 \Rightarrow k = 2*$	A1*	1.1b
		(4)	
(b)	Turning points are where $f'(x) = e^{-x^2}\{-8x^2 + 4x + 4\} = -4e^{-x^2}(x-1)(2x+1) = 0$ And states that $e^{-x^2} \neq 0$ so only other point is $x = -\frac{1}{2}$	B1	2.1
		(1)	
(c)	Attempts $f(1)$ or $f\left(-\frac{1}{2}\right)$ with $k = 2$	M1	2.1
	One correct "end", either $p < 2e^{-1}$ or $p > -4e^{-\frac{1}{4}}$	A1	1.1b
	$-4e^{-\frac{1}{4}} < p < 2e^{-1}, \ p \neq 0$	A1	2.5
		(3)	
			(8 marks)

Notes:

(a)

M1: Correct attempt to differentiate using the product rule (or quotient rule).

Award for $f'(x) = \alpha e^{-x^2} + \beta x(4x-k)e^{-x^2}$ or $f'(x) = \dfrac{\alpha e^{x^2} + \beta x(4x-k)e^{x^2}}{\left(e^{x^2}\right)^2}$

If they expand first it is for $xe^{-x^2} \to \alpha e^{-x^2} + \beta x^2 e^{-x^2}$

A1: Correct differentiation which may be unsimplified.

M1: Sets $f'(1) = 0$ to obtain an equation in k (may just use the numerator for quotient rule).

A1*: Shows that $k = 2$ with sufficient working shown.

(b)

B1: Shows that $x = -\frac{1}{2}$ is the only other turning point. This requires

- A **correct** $f'(x) = (4x-2) \times -2xe^{-x^2} + 4e^{-x^2}$ using $k = 2$ (may just use the numerator for the quotient rule)
- Some statement alluding to the fact that $e^{-x^2} \neq 0$
- A correct factorisation/solution of the quadratic factor
- A minimal conclusion

Attempts to just verify that $f'(-\frac{1}{2}) = 0$ score B0

(c)

M1: For the key step in finding one of the limits.

A1: One correct end. Allow any equivalent expressions and allow decimals here:

NB $-4e^{-\frac{1}{4}} = -3.1152...,\ 2e^{-1} = 0.73575...$

Condone use of non-strict inequalities for this mark.

A1: For subtle use of inequalities in realising that $p = 0$ is not included due to the nature of the curve.

$-4e^{-\frac{1}{4}} < p < 2e^{-1}, p \neq 0$ or equivalent such as $-4e^{-\frac{1}{4}} < p < 0, 0 < p < 2e^{-1}$

Allow $-4e^{-\frac{1}{4}} < p < 0$ or $0 < p < 2e^{-1}$ and condone $-4e^{-\frac{1}{4}} < p < 0$ and $0 < p < 2e^{-1}$

Allow $-4e^{-\frac{1}{4}} < p < 0 \cup 0 < p < 2e^{-1}$ but not $-4e^{-\frac{1}{4}} < p < 0 \cap 0 < p < 2e^{-1}$

Question	Scheme	Marks	AOs
11	$\dfrac{\sin(\theta+h)-\sin\theta}{h}$	B1	2.1
	$\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	M1	1.1b
		A1	1.1b
	(As $h\to 0$), $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)\to 0\times\sin\theta+1\times\cos\theta$	M1	2.1
	so $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

Throughout the question allow the use of $h=\delta\theta$ if used consistently

B1: Gives the correct fraction such as $\dfrac{\sin(\theta+h)-\sin\theta}{\theta+h-\theta}$ or $\dfrac{\sin\theta-\sin(\theta+h)}{-h}$ or $\dfrac{\sin(\theta+h)-\sin(\theta-h)}{2h}$ or $\dfrac{\sin(\theta-h)-\sin\theta}{\theta-h-\theta}$. Condone invisible brackets.

May be implied by $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$

M1: Uses the compound angle formula for $\sin(\theta\pm h)$ to give $\sin\theta\cos h\pm\cos\theta\sin h$

A1: Achieves $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent (may be implied by further work).

Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**

Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)=\sin\theta\times 0+\cos\theta\times 1$

Accept as a minimum $\sin\theta\left(\dfrac{\cos h-1}{h}\right)+\cos\theta\left(\dfrac{\sin h}{h}\right)=\cos\theta$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}$ to $0\times\sin\theta+\cos\theta$ (or e.g. $0+\cos\theta$) then this can be implied and score dM1

$\dfrac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}=\cos\theta$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.

e.g. $\lim\limits_{h\to 0}(\sin\theta\times 0+\cos\theta\times 1)$

Alternatively, condone use of the small angle approximations such that

$$\frac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}\to\frac{-\frac{h^2}{2}\sin\theta+h\cos\theta}{h}=-\frac{h}{2}\sin\theta+\cos\theta$$ and replaces $\frac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\cos\theta$ with no errors seen. (cso)

We need to see $h\to 0$ at some point in their solution e.g.

- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\lim_{h\to 0}\left(\sin\theta\left(\frac{\cos h-1}{h}\right)+\cos\theta\left(\frac{\sin h}{h}\right)\right)=\cos\theta$
- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\lim_{h\to 0}\left(-\frac{h}{2}\sin\theta+\cos\theta\right)=0\times\sin\theta+\cos\theta=\cos\theta$ (using small angle approximations)
- $\left(\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=...\right)=\frac{\sin\theta(\cos h-1)+\cos\theta\sin h}{h}=\sin\theta\times 0+1\times\cos\theta=\cos\theta$ as $h\to 0$

Condone $\mathrm{f}'(\theta)$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ in place of $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\sin\theta\left(\frac{\cos h-1}{h}\right)+\cos\theta\left(\frac{\sin h}{h}\right)=\sin\theta\times 0+\cos\theta\times 1=\cos\theta$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h}=1$ and $\frac{\cos h-1}{h}=0$ and attempting to apply these (without seeing e.g. $h\to 0$ at some point in their solution)

If they work in another variable (e.g. x) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin\theta(\cos h-1)$ e.g. $\frac{\sin\theta\cos h-1+\cos\theta\sin h}{h}$ but accept terms written as e.g. $\sin\theta\frac{\cos h-1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Question	Scheme	Marks	AOs
12(a)	Attempts $\overrightarrow{OM}$ and $\overrightarrow{ON}$ and subtracts to find $\pm\overrightarrow{MN}$ E.g. $\overrightarrow{OM} = 4\mathbf{i} + \frac{3}{2}\mathbf{j}$ $\overrightarrow{ON} = 3\mathbf{j} + \mathbf{k}$ $\left(\overrightarrow{MN}\right) = (3\mathbf{j} + \mathbf{k}) - \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) = -4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}$	M1	1.1b
	Uses the given information to find either $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$ Either $\overrightarrow{MP} = \frac{3}{4}\overrightarrow{MN} = \frac{3}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ Or $\overrightarrow{PN} = \frac{1}{4}\overrightarrow{MN} = \frac{1}{4}\left(-4\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}\right) = -\mathbf{i} + \frac{3}{8}\mathbf{j} + \frac{1}{4}\mathbf{k}$	M1 A1	3.1a 1.1b
	$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \left(4\mathbf{i} + \frac{3}{2}\mathbf{j}\right) + \left(-3\mathbf{i} + \frac{9}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ * or e.g. $\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP} = (3\mathbf{j} + \mathbf{k}) + \left(\mathbf{i} - \frac{3}{8}\mathbf{j} - \frac{1}{4}\mathbf{k}\right) = \mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ *	A1*	2.1
		(4)	
(b)	$\overrightarrow{OQ} = \frac{8}{7}\left(\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \ldots$	M1	3.1a
	Coordinates $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$	A1	3.2a
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts to find $\overrightarrow{MN}$ (ignore labelling for this mark)

M1: For the key step in using $\overrightarrow{MP} = 3\overrightarrow{PN}$ in a correct attempt to find $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$

A1: Correct $\pm\overrightarrow{MP}$ or $\pm\overrightarrow{PN}$

A1*: Completes proof showing all key steps to obtain $\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}$ or e.g. $\begin{pmatrix} 1 \\ \frac{21}{8} \\ \frac{3}{4} \end{pmatrix}$ but not $\begin{pmatrix} \mathbf{i} \\ \frac{21}{8}\mathbf{j} \\ \frac{3}{4}\mathbf{k} \end{pmatrix}$

(b)

M1: Attempts $\overrightarrow{OQ} = \frac{8}{7}\left(\mathbf{i} + \frac{21}{8}\mathbf{j} + \frac{3}{4}\mathbf{k}\right) = \ldots$ May be implied by a correct vector or point.

A1: Deduces that $Q = \left(\frac{8}{7}, 3, \frac{6}{7}\right)$.

Do not allow as a vector unless the correct coordinates are seen first then isw.

Question	Scheme	Marks	AOs
13(a)	$\dfrac{3\sin 2\theta}{8\sin\theta}=\dfrac{2+2\cos 2\theta}{3\sin 2\theta}$ o.e. e.g. $(3\sin 2\theta)^2=8\sin\theta(2+2\cos 2\theta)$	M1	3.1a
	$\dfrac{3\times 2\sin\theta\cos\theta}{8\sin\theta}=\dfrac{2+2(2\cos^2\theta-1)}{3\times 2\sin\theta\cos\theta}=\dfrac{4\cos^2\theta}{3\times 2\sin\theta\cos\theta}$ or e.g. $(3\times 2\sin\theta\cos\theta)^2=8\sin\theta\left(2+2\left(2\cos^2\theta-1\right)\right)=32\sin\theta\cos^2\theta$	dM1 A1	2.1 1.1b
	$\dfrac{6\sin\theta\cos\theta}{8\sin\theta}=\dfrac{4\cos^2\theta}{6\sin\theta\cos\theta}\Rightarrow\dfrac{6\cancel{\cos\theta}}{8}=\dfrac{4\cancel{\cos\theta}}{6\sin\theta}\Rightarrow\sin\theta=\dfrac{32}{36}=\dfrac{8}{9}$ * or e.g. $36\sin^2\theta\cos^2\theta=32\sin\theta\cos^2\theta\Rightarrow 36\sin\theta=32\Rightarrow\sin\theta=\dfrac{8}{9}$ *	A1*	2.1
		(4)	

M1: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

Allow any valid attempt E.g. $a_3=a_1\times\left(\dfrac{a_2}{a_1}\right)^2$, $a_2{}^2=a_1\times a_3$

dM1: For using $\sin 2\theta=2\sin\theta\cos\theta$ and $\cos 2\theta=2\cos^2\theta-1$ to eliminate the constant term.

A1: For any correct equation in single angles without a constant term e.g.

$$\frac{6\sin\theta\cos\theta}{8\sin\theta}=\frac{4\cos^2\theta}{6\sin\theta\cos\theta}\text{ or }(3\times 2\sin\theta\cos\theta)^2=32\sin\theta\cos^2\theta$$

A1*: Proceeds to the given answer with accurate and sufficient work.
There is no requirement to justify why $\sin\theta\neq 0$ or $\cos\theta\neq 0$

	(a) Alternative:		
	$\dfrac{3\sin 2\theta}{8\sin\theta}=\dfrac{2+2\cos 2\theta}{3\sin 2\theta}$ o.e. e.g. $(3\sin 2\theta)^2=8\sin\theta(2+2\cos 2\theta)$	M1	3.1a
	$\dfrac{3\times 2\sin\theta\cos\theta}{8\sin\theta}=\dfrac{2+2(1-2\sin^2\theta)}{3\times 2\sin\theta\cos\theta}$ $\Rightarrow 36\sin\theta\cos^2\theta=32-32\sin^2\theta$ $\Rightarrow 36\sin\theta\left(1-\sin^2\theta\right)=32-32\sin^2\theta$	dM1 A1	2.1 1.1b
	$\Rightarrow 36\sin^3\theta-32\sin^2\theta-36\sin\theta+32=0$ $36\sin^3\theta-32\sin^2\theta-36\sin\theta+32=0\Rightarrow\sin\theta=\dfrac{8}{9}$ *	A1*	2.1

M1: As above

dM1: For using $\sin 2\theta=2\sin\theta\cos\theta$, $\cos 2\theta=1-2\sin^2\theta$ and $\cos^2\theta=1-\sin^2\theta$ and proceeding to a cubic (or quartic) equation in just $\sin\theta$

A1: Correct equation in $\sin\theta$ e.g. $36\sin\theta\left(1-\sin^2\theta\right)=32-32\sin^2\theta$

A1*: Solves their cubic (or quartic) by any means (including a calculator) to obtain $\sin\theta=\dfrac{8}{9}$

If the other roots (1 and −1) are found, they must be rejected.

Mark (b) and (c) together.

(b)	$r = \dfrac{3\sin 2\theta}{8\sin\theta} = \dfrac{6\cos\theta}{8} = \dfrac{6\cos(1.0949...)}{8} = 0.344$	M1	3.1a
	$r =$ awrt 0.344 so $\lvert r \rvert < 1$ so S_∞ exists	A1	2.4
		(2)	
(c)	$"a" = 8\sin\theta = \dfrac{64}{9}$	B1	2.2a
	Attempts $\dfrac{a}{1-r} - \dfrac{a\left(1-r^{10}\right)}{1-r} = \dfrac{ar^{10}}{1-r} = \dfrac{\frac{64}{9}\times"0.344"^{10}}{1-"0.344"}$	M1	1.1b
	$S_\infty - S_{10} =$ awrt 2.5×10^{-4}	A1	1.1b
		(3)	
			(9 marks)

(b)

M1: Attempts to find the value of r with $\sin\theta = \dfrac{8}{9}$

Alternatively states that $r = \dfrac{3}{4}\cos\theta$ o.e and states $r < 1$

A1: Achieves $r =$ awrt 0.344 and states that as $\lvert r \rvert < 1$ S_∞ exists

In the alternative gives a reason e.g. as $\lvert r \rvert < 1$ S_∞ exists

Some values for reference:

$$r = \frac{\sqrt{17}}{12}(=0.34359...),\ a_1 = \frac{64}{9}(=7.111...),\ a_2 = \frac{16\sqrt{17}}{27}(=2.443...),\ a_3 = \frac{68}{81}(=0.8395...)$$

(c)

B1: Deduces the value of a to be $\dfrac{64}{9}$ but condone awrt 7.11

M1: Attempts $S_\infty - S_{10}$ using correct formulae and with their values of a and r

A1: awrt 2.5×10^{-4}

Question	Scheme	Marks	AOs
14	Deduces that $A=2$ or $b=\frac{\pi}{6}$	B1	3.4
	Deduces that $A=2$ and $b=\frac{\pi}{6}$	B1	3.4
	$\frac{\mathrm{d}H}{\mathrm{d}t}=A\cos(bt)\Rightarrow H=\frac{A}{b}\sin(bt)+c$	M1	3.1a
	Uses $H_{MAX}=\frac{30}{\pi}\Rightarrow c=...$	dM1	3.1b
	$H=\frac{12}{\pi}\sin\left(\frac{\pi}{6}t\right)+\frac{18}{\pi}$	A1	3.3
		(5)	
			(5 marks)

Notes:

Condone the use of e.g. *a* for *A* or *B* for *b* as long as the intention is clear.

B1: Uses the equation of the given model to deduce that $A=2$ or $b=\frac{\pi}{6}$ o.e.

This is implied by a statement $\frac{\mathrm{d}H}{\mathrm{d}t}=2\cos(...t)$ or $\frac{\mathrm{d}H}{\mathrm{d}t}=...\cos\left(\frac{\pi}{6}t\right)$

B1: Uses the equation of the given model to deduce that $A=2$ and $b=\frac{\pi}{6}$ o.e.

This is implied by a statement $\frac{\mathrm{d}H}{\mathrm{d}t}=2\cos\left(\frac{\pi}{6}t\right)$

M1: Integrates to obtain $H=...\sin(bt)(+c)$ oe with or without the + c.

dM1: Attempts to use $H_{MAX}=\frac{30}{\pi}$ with $\sin(bt)=1$ and a numeric b with an equation of the form $H=k\sin(bt)+c$ and obtains a value for c.

A1: $H=\frac{12}{\pi}\sin\left(\frac{\pi}{6}t\right)+\frac{18}{\pi}$

Question	Scheme	Marks	AOs
15(a)	Substitutes $t=0, x=4 \Rightarrow 4=\frac{k\times 2}{5} \Rightarrow k=...$	M1	3.1b
	$x=\frac{10(3t+2)}{4t+5}$	A1	3.3
		(2)	
(b)	7500	B1ft	3.4
		(1)	
(c)	$\frac{4}{6x-x^2}=\frac{A}{x}+\frac{B}{6-x} \Rightarrow 4=A(6-x)+Bx \Rightarrow A=...,B=...$	M1	1.1b
	$A=\frac{2}{3}$ or $B=\frac{2}{3}$	A1	1.1b
	$\frac{4}{6x-x^2}=\frac{2}{3x}+\frac{2}{3(6-x)}$ o.e. e.g. $\frac{2}{3x}+\frac{2}{18-3x}$	A1	1.1b
		(3)	

(a)

M1: Substitutes $x = 4$ and $t = 0$ into the given equation and solves to find a value for k.

A1: Correct **equation** $x=\frac{10(3t+2)}{4t+5}$ oe e.g. $x=\frac{30t+20}{4t+5}$ but not just for $k = 10$

(b)

B1ft: 7500 oe e.g. 7.5 thousand. Follow through their k so allow for $0.75k\times 1000$

(c)

M1: Correct method of partial fractions leading to values for A and B

A1: Correct A or B

A1: $\frac{4}{6x-x^2}=\frac{2}{3x}+\frac{2}{3(6-x)}$ Correct **partial fractions** not just correct values but award if correct fractions are seen subsequently.

(d)	$\int \frac{4}{6x-x^2}\,dx = \int dt \Rightarrow \frac{2}{3}\ln x - \frac{2}{3}\ln(6-x) = t(+c)$	M1 A1ft	3.1a 1.1b
	$t=0, x=4 \Rightarrow c = \left(\frac{2}{3}\ln 2\right)$	M1	3.4
	$\frac{2}{3}\ln x - \frac{2}{3}\ln(6-x) = t + \frac{2}{3}\ln 2 \Rightarrow \ln\frac{x}{2(6-x)} = \frac{3}{2}t$ $\Rightarrow x + 2xe^{\frac{3}{2}t} = 12e^{\frac{3}{2}t}$	M1	2.1
	$x = \dfrac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$ *	A1*	1.1b
		(5)	
(e)	6000	B1	3.4
		(1)	
			(12 marks)

(d)

M1: Correct attempt at integration using lns. Look for $\alpha\ln x \pm \beta\ln(6-x) = t(+c)$

A1ft: Correct integration ft on their values for A and B. No requirement for + c here.

Note that $\frac{2}{3}\ln 3x - \frac{2}{3}\ln 3(6-x) = t(+c)$ is also correct.

dM1: Correct method to find "c" using $t = 0$ and $x = 4$

dM1: **Depends on the <u>first</u> method mark only.**

For the key steps in making x the subject of the equation.

- Uses correct ln work and proceeds to a form $\ln f(x) = at$
- Uses e as the inverse function to ln and cross multiplies (o.e) to get both x terms on the same side of the equation
- Requires a constant of integration that may or may not have been evaluated.

A1*: Correct processing leading to the given answer $x = \dfrac{12e^{\frac{3}{2}t}}{1+2e^{\frac{3}{2}t}}$

Note that 2nd and 3rd method marks may be awarded in the reverse order e.g.

$$\frac{2}{3}\ln x - \frac{2}{3}\ln(6-x) = t + c \Rightarrow \frac{2}{3}\ln\frac{x}{6-x} = t + c \Rightarrow \ln\frac{x}{6-x} = \frac{3t}{2} + k$$

$$\Rightarrow \frac{x}{6-x} = Ae^{\frac{3t}{2}} \Rightarrow x = 6Ae^{\frac{3t}{2}} - xAe^{\frac{3t}{2}} \Rightarrow x = \frac{6Ae^{\frac{3t}{2}}}{1+Ae^{\frac{3t}{2}}}$$

$$4 = \frac{6A}{1+A} \Rightarrow A = 2 \Rightarrow x = \frac{12e^{\frac{3t}{2}}}{1+2e^{\frac{3t}{2}}}*$$

(e)

B1: 6000 oe e.g. 6 thousand

Question	Scheme	Marks	AOs
16(a)	$4=2+4\cos t \Rightarrow \cos t=\frac{1}{2} \Rightarrow t=-\frac{\pi}{3} \Rightarrow y=...$	M1	2.1
	$y=3\times\left(-\frac{\pi}{3}\right)+2\sin\left(-\frac{\pi}{3}\right)=-\pi-\sqrt{3}$	A1	1.1b
		(2)	
(b)	Attempts $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3+2\cos t}{-4\sin t}=-\frac{3}{2}$ $\Rightarrow 6\sin t-2\cos t=3$	M1 A1	2.1 1.1b
	E.g. $6\sin t-2\cos t=R\sin(t-\alpha)$ $\Rightarrow R=\sqrt{6^2+2^2}=\sqrt{40},\ \tan\alpha=\frac{1}{3}\Rightarrow \alpha=0.322...$ or $18.4°$	M1 A1	3.1a 1.1b
	$\sqrt{40}\sin(t-0.322)=3\Rightarrow t=0.322+\sin^{-1}\left(\frac{3}{\sqrt{40}}\right)$	dM1	2.1
	$t=$ awrt 0.816	A1	1.1b
		(6)	
			(8 marks)

Notes:

(a)

M1: For the key step in finding y when $t=-\frac{\pi}{3}$

A1: $-\pi-\sqrt{3}$

(b)

M1: For setting up an equation in t using all of the information given.
The key steps required to score this mark are

- Finding $\frac{dy}{dx}$ by dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$
- Setting $\frac{dy}{dx}=-\frac{3}{2}$ and proceeding to an equation of the form $a\sin t+b\cos t=c$

A1: $6\sin t-2\cos t=3$ or exact equivalent

M1: Chooses a suitable method to solve the equation.
Look for a correct overall method condoning slips.

A1: Correct values of "R" and "α"

dM1: Full and complete method to find t using correct order of operations.
It is dependent upon <u>all</u> previous M marks.

A1: For awrt 0.816 and no other values.

Alternative for the final 4 marks:

$$6\sin t-2\cos t=3\Rightarrow 6\sin t=3+2\cos t\Rightarrow 36\sin^2 t=9+12\cos t+4\cos^2 t$$

$$\Rightarrow 36-36\cos^2 t=9+12\cos t+4\cos^2 t\Rightarrow 40\cos^2 t+12\cos t-27=0$$

$$\Rightarrow \cos t=\frac{-3\pm 3\sqrt{31}}{20}\left(0.685...,\ -0.985...\right)\Rightarrow t=0.816$$

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom