# Pearson Edexcel 

# Examiners' Report Principal Examiner Feedback 

November 2021

Pearson Edexcel GCE
In Mathematics (9MAO)
Paper 01 Pure Mathematics 1

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In October 2019 we informed customers that all papers from summer 2020 onwards will enhance student experience when sitting examinations.

The improvements to papers will focus on:

- ensuring early questions are accessible to all and then steadily ramp in demand to encourage engagement and help build students’ confidence through the papers
- dividing questions into parts so students are clear where marks can be achieved and can manage their focus and exam timings accordingly
- using clear, concise language to better enable all students to access the questions and understand the type of response expected.

The October 2021 paper was the second to showcase these changes within an examination series. Early questions did prove to be very accessible with the prepared candidate scoring high marks in questions 1 to 6 .

## Question 1.

This was a very familiar and accessible question, and hence proved to be a suitable first question. A significant majority of candidates gave a fully correct response to this question, with the most common and successful approach being via the use of the factor theorem. Very few candidates tried to evaluate $f(-1)$. Following on from correct use of $f(1)=0$, there were occasional errors in solving the resulting simple linear equation. One (perhaps unexpected) error seen several times was to simplify $a(1)^{3}$ as $a^{3}$.
Algebraic division was attempted by some candidates, but the working needed here was much more complicated and so often led to errors (mainly with signs). This does highlight the importance of choosing the best method to a problem, as well as carrying it out accurately.

## Question 2

This focussed on the ability of a student to complete the square and proved to be as accessible as the first question. The vast majority of responses seen were fully correct. In part (a) most candidates were able to score full marks. The form $(x-2)^{2}+\ldots$. was almost always achieved, although occasionally an incorrect $(x+2)^{2}+\ldots$ was offered. The most common error was having +9 instead of +1 . In response to part (b) both $P$ and $Q$ were often stated correctly with some candidates able to score the follow through mark for their $Q$ from their value of ' $b$ ' from part (a). Occasionally a mark was lost if they had $P=5$ instead of $y=5$ when $x=0$. Very few sketches were seen, which if correctly labelled would have also scored full marks in (b). It was relatively common in (b)(ii) to see differentiation used, rather than the completed square expression.

## Question 3

This proved to be a little more demanding than the first two questions but over $40 \%$ of candidates still scored all 6 marks.

Errors seen by candidates are summarised below.
In part (a) several candidates did not simplify $u_{2}$ sufficiently, giving $u_{2}=k-24 / 2$ which then lead to a more complicated expression for $u_{3}$ which they had difficulty simplifying. Some candidates did not use the given expression, but rather used $u_{1}+u_{2}+u_{3}=0$.

In part (b) candidates usually found both roots of the given quadratic either by factorisation or by use of the quadratic formula, with a small number by completing the square. A significant minority failed to gain the A mark here as they did not state that 6 was an integer or that $40 / 3$ was not an integer, with some excluding $40 / 3$ on the grounds that it was irrational.

Part (c) was invariably answered correctly by those who had solved the quadratic in part (b). although there were some candidates who gave a choice of answers.

## Question 4

Aspects of this question proved to be more demanding than expected, but overall good marks were scored by candidates with most achieving over half marks in this question. Part(a) Almost all candidates identified the need to differentiate and set their expression equal to zero. Although the derivative was often accurately found - the chain rule aspect of the log expression was omitted by some so that the expression $4 x-4$ was not seen as the numerator of the required fraction. Some numerators seen included $2 x-2,2 x$ 4 or most often 1 . Manipulating their equation into the desired form was also an issue for some, with sign errors causing problems, whilst others could not properly convert their equation involving fractions into one not involving fractions.
Part (b) A significant majority of candidates obtained both required answers to this part. Occasionally only $x_{2}$ and $x_{3}$ were found. There were just a few occasions where a candidate had misread the $1 / 7$ in the iterative formula as $1 / 2$. If the working was shown for this situation, then a method mark could still have been awarded.
Part (c) Whilst many candidates realised that a sign change of a function between 0.3405 and 0.3415 was required, it was also common to see an attempt at further iteration. This method was not acceptable as a proof of the location of the root. A few chose to substitute into an inappropriate function such as $\mathrm{f}(x)$, or the iterative formula, and some failed to specify a function at all. For those who did use (typically) the expression from (a), most selected correct values to substitute in and identify the change of sign, finishing with a clear conclusion. It was very common, however, for the final mark to be lost in this part because there was no mention that the chosen function was continuous.

## Question 5

This was yet another good source of marks with the modal mark being 6 out of 6 , achieved by over $35 \%$ of the candidature.

Almost every candidate gained the B1 mark in part (a) with a fairly even split between those who used $20000 \times 1.08^{2}$ and those who first calculated the profit after Year $1=$ 21600 and then Year $2=23328$ showing each step of their working to gain the mark.

Part (b) was more problematic for many candidates. There was a very good understanding of the inequality/equation to be formed, and the use of logs was almost always very good. However, a significant minority of candidates failed to gain the final A mark giving an answer of 15.3 or 16 instead of Year 17.

Part (c) proved to be very well answered with the vast majority using the sum formula correctly. Errors included using $n=19$ instead of 20 in the formula or finding the 20th term instead of the sum. A few candidates listed all the terms before adding them which often led to accuracy errors.

## Question 6

This was another question where the modal mark was full marks, this time achieved by over $40 \%$ of the candidates
Part(a) A very significant majority of candidates obtained vector AC correctly. Only a few subtracted rather than adding the given vectors $\boldsymbol{A B}$ and $\boldsymbol{B C}$.
Part (b) The vast majority of candidates made some progress in this part, realising that the lengths of the vectors would be required to solve this problem. A small minority then simply stopped, having done no more than finding the lengths of the sides of the triangle $A B C$, with some assuming that the triangle was right angled. However most then went on to use the cosine rule to try to show the required result. In almost all cases a correct cosine rule was stated and used, so that following on from a previous accuracy error, the method marks could be awarded. Fully correct calculations were necessary in order to gain the final accuracy mark and very many candidates managed to do this and so gained full marks for this question.
A few candidates, presumably those who had studied further maths, used the scalar product. This method would be perfectly acceptable, as long as the vectors are used the correct way around. Many using this method showed that $\cos A B C=-\frac{9}{10}$ but failed to explain why the answer should be positive.

## Question 7

The first few marks in this question were very accessible but the final part of the question proved to be a useful discriminator especially around grade C and below. Part(a) The majority of candidates achieved full marks in this part. Sign errors were responsible for most errors, although failure to halve was also seen with either or both of $\quad(x \pm 10)^{2}$ and $(y \pm 4)^{2}$ being seen. Having achieved $(x-5)^{2}+(y+2)^{2}=18$, it rare to see the centre or radius not found correctly.
Part (b) This part was found to be discriminating with complete solutions seen from only the most competent of candidates. The most common approach was to substitute $y$ $=3 x+k$ into the circle equation to find a quadratic in $x$ (with coefficients in terms of $k$ ) for the points of intersection. Errors mostly arose when $(3 x+k+2)^{2}$ was expanded incorrectly or the $2 x+6 k x$ terms not being gathered together. Many solutions then stopped after the substitution and expansion step, as candidates did not seem to appreciate what to do with an equation of this complexity in two variables.
Application of $b^{2}=4 a c$ to find equal roots was used on many occasions but errors frequently occurred. Only a very few solutions involved attempts via $b^{2}-4 a c>0$. It was not always obvious to see how a student had solved their quadratic (formula not quoted and values for $a, b$ and $c$ not seen) but a limited number of correct solutions were seen. Using a calculator quadratic solver was frequent and acceptable.
Several attempts started by using implicit differentiation on the circle equation. Although this is a possible method, it did not often go beyond using $\mathrm{d} y / \mathrm{d} x=3$ to get $x+3 y+1=0$. Another method seen several times was to find the equation of the normal ( i.e. the radius ) to try to make progress towards finding values of $k$, but this method also seldom went further than the equation of the line.

## Question 8

This was the first significant modelling question on this paper. As with other series, candidates find aspects of modelling difficult. The barriers to success here were parts (b) and (c).

In part (a) many candidates made progress and achieved a complete equation for the model with $k$ given exactly as $\ln (2) / 5$ or rounded to 0.139 . A few students rounded more accurately giving $k$ as 0.1386 . Some rounded $k$ to 0.14 and lost the accuracy mark. A fair number of students who failed to find the correct value of ' $A$ ' evaluated $e^{0}$ as e.

In part (b) many students used an incorrect method to find the rate of increase in the number of bacteria. Some substituted $t=8$ into their expression for $N$, which just found the number of bacteria after 8 hours; others then went on to divide this number by 8 or even 1000 to try and find the rate of increase. Those who realised that the expression for $N$ needed to be differentiated to find the rate of increase often scored at least the method mark.

For part (c) the majority of candidates who obtained the correct equation of the model in (a) were able to use both models given to set up an appropriate equation in $t$ or $T$ to gain the first method mark. Processing the resulting equation using logs to obtain a linear equation in $t$ or $T$ posed more of a challenge and many made no further progress. A large proportion who did attempt to make progress often used incorrect log work: some multiplied the logs rather than add them; others did not find the $\log$ of all the
components, usually the ' 2 '. Some students combined the exponential functions incorrectly by dividing the powers rather than subtracting them. Those scoring all three marks here were usually the most able candidates.

## Question 9

This question was answered with varying degrees of success. Around 28\% of the candidates failed to score any marks here at all, which was very suprising. There were however, many well formed and accurate solutions by careful candidates.

In part (a) most candidates managed to write down a correct identity and use an appropriate method to find values for $B$ and $C$. To prove $A=0$ was generally done via equating coefficients of $x^{2}$ or by setting $x=0$ or $x=1$. A few candidates wrote down three equations for the coefficients and constants, but rarely was a correct set of solutions gained from this method. It was also not uncommon to see the RHS expanded either with or without the terms involving $A$ in an attempt to prove that $A=0$. Errors included having an incorrect identity from the start which resulted in no marks.

In part (b)(i) it was rare to see fully correct expansions. Good candidates often lost the final mark due to an error in adding the required terms. A common error was forgetting to use their value for $C$ and only adding the expansion for $(1-2 x)^{-1}$ rather than $C(1-2 x)^{-1}$. A few were unable to extract $2^{-1}$ correctly from $(5 x+2)^{-2}$ (usually having this as 2 ). Other errors seen involved an expansion of $\quad(1+2 / 5 x)^{-2}$ or else $(1+5 / 2 x)^{2}$ rather than $(1+5 / 2 x)^{-2}$.

Very few correct answers were seen for part (b)(ii) and this was very often not attempted. Those who did write a correct range sometimes also gave a second range which therefore did not gain the B mark here.

## Question 10

This was found to be another discriminating question. Those who were familiar with the topic made good progress yet there were many other attempts that scored no marks at all.
Part(a) There were a significant number of fully correct solutions for this part, although some candidates did not achieve the stage where both the numerator and denominators of their fraction were factorised correctly with a common factor of $(\sin \theta+\cos \theta)$. Another difficulty encountered by some, was how to use the different forms of the identity for $\cos 2 \theta$ in order to deal with, and remove, the presence of both " 1 's " in the fraction. Very few notational errors were seen, which was a pleasing improvement. Part (b) A significant number of candidates used the result of (a) correctly and started their solution with $\tan 2 x=3 \sin 2 x$. However, it was also the case that $\tan x=3 \sin 2 x$ was seen several times as well. Of those candidates who started correctly with $\tan 2 x=$ $3 \sin 2 x$, the significant majority simply cancelled out the resulting factor of $\sin 2 x$ and in so doing neglected to consider the possibility that $\sin 2 x=0$ and in so doing the solution $x$ $=90^{\circ}$ was omitted from the fully correct list of three answers. Many candidates however, did state that $\cos 2 x=1 / 3$ and went on to get $35.3^{\circ}$ and $144.7^{\circ}$ and so only one mark was lost.

## Question 11

The modal mark here was 3 , most likely due to the accessibility of part (a). Marks in part (b) were only really awarded to the highest scoring candidates on this exam.

Most candidates successfully answered part (a) applying the Trapezium Rule using all five values of $y$ from the given table. Some students, who found the correct estimate of the area of $R$, failed to round their answer as requested, or occasionally rounded incorrectly, and lost the accuracy mark. Errors, although rare, were usually as a result of an incorrect strip width.

In part (b) many candidates failed to recognise that integration by parts was needed to make progress. Several students, who made progress and correctly applied 'integration by parts' twice, unfortunately lost the final accuracy mark due to sign or coefficient errors that had crept in. Most who integrated appropriately applied the correct limits and applied $\ln (4)=2 \ln (2)$. Errors were many and varied here and included

- Candidates who believed that $\int(\ln x)^{2} \mathrm{~d} x=\frac{1}{3}(\ln x)^{3}$
- Candidates who believed that $(\ln x)^{2}=2 \ln x$

Candidates who used the substitution $u=\ln x$ seemed to more easily recognise $\int u^{2} \mathrm{e}^{u} \mathrm{~d} u$ as integration by parts twice.

## Question12.

This was the second modelling question on this paper. It proved to be a significant discriminator between candidates, with a significant minority making little or no progress. It certainly seems to be a topic that students find hard to access.
Part (a) The most common approach which led to either partial or complete success was to start with $H=a x^{2}+b x+c$ and use $(0,3)$ to find that $c=3$. Many candidates then used $(120,27)$ to find an equation connecting $a$ and $b$, but sometimes no more progress was made. Of those who did make further progress, the most common approach was to use differentiation and the knowledge that the gradient was zero when $x=90$ to get a second equation in $a$ and $b$. Candidates using this approach often went on to get (a) fully correct and then also to gain most if not all of the marks in (b) as well. It was only rarely that there was a misunderstanding about $H=0$ rather than $H=3$ when $x=0$.
There were other methods used in (a) and any of these could have led to success if applied correctly. For example, using the fact that the quadratic has a turning point when $x=90$ and so $H=a(x-90)^{2}+c$ was used correctly by some candidates.
Alternatively, using additional implied points such as $(60,27)$ or $(180,3)$ due to the symmetry of quadratic curves, was used appropriately by candidates.
Part (b) Both parts were done well by candidates who had achieved a correct answer to (a).

Part (b) (i) There were only a very few who omitted the units here. A wrong answer to (a) would have had made it difficult to gain the accuracy mark here.

Part (b)(ii) A wrong answer to (a) could have scored a method mark here if the candidate's quadratic had been solved correctly.

Part (c) A wide range of comments were quoted here- some of those that did not gain the B1. Many of these focussed on what happened to the ball after hitting the ground, or
picked out a relatively trivial aspect such as the height of the tee being inaccurate. Others mentioned the weight of the ball, or that it depended on the force of the contact. There were many acceptable comments. These should have focussed on why one of the four modelling assumptions referred to in the question may not have been valid. Acceptable responses included the fact that the ball is not a particle, the ground may not be horizontal, the path may not be a quadratic (parabola) or it might not travel in a vertical plane (Spin was mentioned in this context).

## Question 13

A solution gaining all three marks was relatively rare to see, and there were many very poor demonstrations of algebraic manipulation seen. Many candidates managed to score the first M mark but usually were unable to make further progress. The attempts were split between candidates who attempted to substitute into the LHS of the Cartesian equation to give an equation/expression in $t$ and those who attempted to eliminate $t$, usually via $t^{2}=(5-x) /(x-1)$, to give an equation involving $x$ and $y$ only. The errors then ranged from incorrect squaring of a bracket to incorrect manipulation of fractions with very few unable to complete the proof

## Question 14

This question proved equally challenging and very few fully correct solutions were seen. The modal mark achieved here was 2 , scored by over $40 \%$ of the candidates, most often for a fully correct attempt at $\mathrm{d} y / \mathrm{d} x$.
Most solutions seen used the quotient rule although incorrect application of the rule often meant that no marks could be awarded. The most common errors were subtracting the wrong way round on the numerator, or else adding the two expressions on the numerator. Differentiation of $\sqrt{ } X$ suprisingly caused a few problems as well. Product rule expansions seldom got past the first expression.
The ability to manipulate the ensuing algebraic expression was often lacking, with fractional terms within a fraction, the main cause for failure.
Alternative methods which might have simplified the algebra were seen, but only rarely. Only a handful spotted that a difference of two squares method applied to the numerator would have made this a very simple problem.

## Question 15

Candidates have not been too successful in answering questions on the topic of 'Proof' within this new specification, and this question proved to be no different.
Part (i) The meaning of "proof by exhaustion" was not fully understood by all. Many candidates failing to consider all four of the possible cases mentioned. A common response was to test just $n=4$ and $n=5$ and conclude that it was true for $n=4$ but not $n$ $=5$ so it was proven for $n \leq 4$.
Some candidates who did consider all values of $n=1,2,3$ and 4 , left some expressions unevaluated. Occasionally the eight required values were found but there was no evidence that they were being compared. It was essential that a proper conclusion is made at the end of a proof, and despite having convincing working and reasons, some
candidates lost the final accuracy mark here because they did not have a conclusion saying that the required result was indeed true.

Part (ii) Very many candidates did score the first mark here for saying "Suppose that $m$ is odd". However, a common error was to state "let $m^{3}+5$ be odd" instead leading to 0 marks. It was common to see $2 p+1$ or $2 k+1$ being used to represent an odd number and there were many occasions where $(2 p+1)^{3}+5$ was seen in a candidate's working. In some cases there were algebraic errors in attempting to expand the cube of ( $2 p+1$ ), but most candidates did achieve the required four term cubic $8 p^{3}+12 p^{2}+6 p+6$. When this cubic was correct, those candidates very often went on to gain the next two accuracy marks by proving, by factorisation, that $m^{3}+5$ was even, and then stating that there is a contradiction (to the assumption that $m$ is odd) and so $m$ must be even.

