Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In A level Further Mathematics
Paper 9FM0/4D

Summer 2019
Publications Code 9FM0_4D_1906_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- CAO – correct answer only
- CSO - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (CAO.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1 (a)</td><td>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th></tr>
<tr><td>A</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td></td><td>$5-\theta$</td><td></td><td>$1+\theta$</td></tr>
<tr><td>D</td><td></td><td>θ</td><td></td><td>$19-\theta$</td></tr>
</table>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th></tr>
<tr><td>A</td><td>23</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>2</td><td>40</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>12</td><td>6</td></tr>
<tr><td>D</td><td></td><td>5</td><td></td><td>14</td></tr>
</table>
</td><td>1M1
1A1</td><td>2.1
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th></th><th></th><th>15</th><th>14</th><th>16</th><th>18</th></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>0</td><td>1</td><td>−7</td></tr>
<tr><td>−5</td><td>B</td><td>X</td><td>X</td><td>5</td><td>−1</td></tr>
<tr><td>−8</td><td>C</td><td>4</td><td>7</td><td>X</td><td>X</td></tr>
<tr><td>−1</td><td>D</td><td>1</td><td>X</td><td>1</td><td>X</td></tr>
</table>
</td><td>1M1
1A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th></tr>
<tr><td>A</td><td>$23-\theta$</td><td></td><td></td><td>θ</td></tr>
<tr><td>B</td><td>$2+\theta$</td><td>$40-\theta$</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td></td><td></td></tr>
<tr><td>D</td><td></td><td>$5+\theta$</td><td></td><td>$14-\theta$</td></tr>
</table>
<table>
<tr><th></th><th>P</th><th>Q</th><th>R</th><th>S</th></tr>
<tr><td>A</td><td>9</td><td></td><td></td><td>14</td></tr>
<tr><td>B</td><td>16</td><td>26</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>12</td><td>6</td></tr>
<tr><td>D</td><td></td><td>19</td><td></td><td></td></tr>
</table>
Entering cell is AS and exiting cell is DS</td><td>2M1
2A1</td><td>1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th></th><th></th><th>15</th><th>14</th><th>9</th><th>11</th></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>0</td><td>8</td><td>X</td></tr>
<tr><td>−5</td><td>B</td><td>X</td><td>X</td><td>12</td><td>6</td></tr>
<tr><td>−1</td><td>C</td><td>−3</td><td>0</td><td>X</td><td>X</td></tr>
<tr><td>−1</td><td>D</td><td>1</td><td>X</td><td>8</td><td>7</td></tr>
</table>
A negative II so solution is not optimal</td><td>1M1
1A1
2A1</td><td>2.1
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>(£)1086</td><td>1B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(10 marks)</td></tr>
</table>

Notes:
(a) 1M1: A valid route, only one empty square used, θ's balance. **Note:** If not entering in DQ, allow this mark for a valid route and entry in CP or DP only. **1A1:** CAO **(b) 1M1**: Finding (exactly) 8 shadow costs and (exactly) 9 improvement indices for their improved solution **1A1:** Shadow costs [Alt: A(15), B(10), C(7), D(14), P(0), Q(-1), R(1), S(3)] and II CAO. Please check top table for Shadow Costs. **2M1:** A valid route, their most negative II chosen, only one empty square used, θ's balance **2A1:** CAO – including the deduction of all entering and exiting cells **(c) 1M1:** finding all 8 shadow costs **and** all 9 negative improvement indices **or** sufficient number of shadow costs for at least 1 negative II found (May just see SC: A, C, P and S and II: PC). This mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution **1A1:** CAO negative II from correct working **2A1:** CSO for (a), (b), (c) including the correct reasoning that the solution is not optimal because there is a negative II. Do not allow for 'some IIs are not positive' o.e. [Alt shadow costs: A(15), B(10), C(14), D(14), P(0), Q(-1), R(-6), S(-4)]. **(d) 1B1:** CAO ignore lack of or incorrect units.

Question	Scheme	Marks	AOs
2(a)	Subtracting each entry from a value ≥ 203 e.g. $\begin{bmatrix} 100 & 106 & 129 & 123 \\ 2 & 48 & 58 & 48 \\ 92 & 123 & 126 & 111 \\ 0 & 15 & 66 & 19 \end{bmatrix}$	1B1	1.1b
	Reduce rows $\begin{bmatrix} 0 & 6 & 29 & 23 \\ 0 & 46 & 56 & 46 \\ 0 & 31 & 34 & 19 \\ 0 & 15 & 66 & 19 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 40 & 27 & 27 \\ 0 & 25 & 5 & 0 \\ 0 & 9 & 37 & 0 \end{bmatrix}$	1M1 1A1ft	2.1 1.1b
	followed by $\begin{bmatrix} 5 & 0 & 0 & 9 \\ 0 & 35 & 22 & 27 \\ 0 & 20 & 0 & 0 \\ 0 & 4 & 32 & 0 \end{bmatrix}$	2M1 2A1ft	2.1 1.1b
	T – 2, H – 1, J – 3, M – 4	3A1	2.2a
		(6)	
(b)	(£)559	1B1	1.1b
		(1)	
			(7 marks)

Notes:

(a) 1B1: CAO

1M1: simplifying the initial matrix by reducing rows and then columns. Allow no more than a single error in row reduction together with no more than a single error in column reduction. May combine the two stages of converting from maximum to a minimum problem and row reduction which is acceptable.

1A1ft: CAO following on from their earlier conversion to maximising. If 1B1 awarded, the result of row and column reduction must be as in the main scheme. If 1B0 awarded, then row and column reduction must ft correctly from their attempt to convert to maximisation problem.

2M1: develops an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

2A1ft: Improved solution following on from their previous table

3A1: CSO Correct allocation. Must have gained all previous marks in the question.

(b) 1B1: CAO – solution of original problem (ignore units lack of or incorrect units)

SC: Minimising

After row reduction $\begin{bmatrix} 29 & 23 & 0 & 6 \\ 56 & 10 & 0 & 10 \\ 34 & 3 & 0 & 15 \\ 66 & 51 & 0 & 47 \end{bmatrix}$ and then after column reduction $\begin{bmatrix} 0 & 20 & 0 & 0 \\ 27 & 7 & 0 & 4 \\ 5 & 0 & 0 & 9 \\ 37 & 48 & 0 & 41 \end{bmatrix}$

After augmentation $\begin{bmatrix} 0 & 20 & 4 & 0 \\ 23 & 3 & 0 & 0 \\ 5 & 0 & 4 & 9 \\ 33 & 44 & 0 & 37 \end{bmatrix}$ or $\begin{bmatrix} 0 & 24 & 4 & 0 \\ 23 & 7 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 33 & 48 & 0 & 37 \end{bmatrix}$

Scores B0 M1A0 M1A1(ft)A0 B0 (So 3 marks max)

1B0: No Minimisation

1M1: simplifying the initial matrix by reducing rows and then columns – all values ‘correct’

1A0: Must be maximising.

2M1: develops an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

2A1ft: Improved solution following on from their previous table

3A0: CSO

1B0: Must be maximising

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td>Dijkstra’s algorithm cannot be used on a network with negative weights</td><td>1B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)(i)</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>Action</th><th>Destination</th><th>Value</th></tr>
<tr><td>0</td><td>G</td><td>GT</td><td>T</td><td>28*</td></tr>
<tr><td></td><td>H</td><td>HT</td><td>T</td><td>19*</td></tr>
<tr><td></td><td>I</td><td>IT</td><td>T</td><td>-24*</td></tr>
<tr><td></td><td>J</td><td>JT</td><td>T</td><td>17*</td></tr>
<tr><td>1</td><td>D</td><td>DG
DH</td><td>G
H</td><td>$-24+28=4*$
18 + 19 = 37</td></tr>
<tr><td></td><td>E</td><td>EH
EI</td><td>H
I</td><td>24 + 19 = 43
27 – 24 = 3*</td></tr>
<tr><td></td><td>F</td><td>FG
FI
FJ</td><td>G
I
J</td><td>$-25+28=3$
$14-24=-10$
$-30+17=-13*$</td></tr>
<tr><td>2</td><td>A</td><td>AD
AE</td><td>D
E</td><td>32 + 4 = 36
29 + 3 = 32*</td></tr>
<tr><td></td><td>B</td><td>BD
BE
BF</td><td>D
E
F</td><td>20 + 4 = 24
19 + 3 = 22
$-17-13=-30*$</td></tr>
<tr><td></td><td>C</td><td>CF</td><td>F</td><td>13 – 13 = 0*</td></tr>
<tr><td>3</td><td>S</td><td>SA
SB
SC</td><td>A
B
C</td><td>13 + 32 = 45
x – 30
24 + 0 = 24</td></tr>
</table>

$x-30<24$
(0≤) x < 54
</td><td>1B1

1M1
1A1
2A1

2M1
3A1ft
4A1

3M1
5A1ft

4dM1
6A1</td><td>3.1a

3.1a
1.1b
1.1b

1.1b
1.1b
1.1b

1.1b
1.1b

3.1a
2.2a</td></tr>
<tr><td>(ii)</td><td>Route: S – B – F – J – T</td><td>1B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(12)</td><td></td></tr>
<tr><td colspan="4" align="right">(13 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a) 1B1: CAO – Do not accept ‘Dijkstra’s cannot be used on a directed network’
Condone any reference to negative (weights) or ‘negative edges’. Also condone ‘cannot be used with positive and negative arcs.’ But NOT answers which include incorrect statements.</td></tr>
</table>

Throughout (b):
• Condone lack of destination column and/or reversed stage numbers throughout
• Only penalise incorrect result in value – ie ignore working values
• Penalise absence of state or action column with first two A marks earned only
• Penalise empty/errors in stage column with first A mark earned only
• Penalise occurrence of single errors in state/action/destination with the relevant A mark - once only.
• Interchanged state and destination columns penalise with first two A marks

M marks - must bring earlier optimal results into calculations at least once per stage

1B1: Stage 0 correct

1M1: Stage 1 completed with 3 states and at least 7 rows. Bod if something in each cell
1A1: any two states in Stage 1 correct
2A1: CAO all 3 states correct in Stage 1 (must be 7 rows and no extra rows)

2M1: Stage 2 completed with 3 states and at least 6 rows. Bod if something in each cell
3A1ft: CAO any 2 states correct in Stage 2 on the follow through
4A1: CAO all 3 states in Stage 1 (no extra rows)

3M1: Stage 3 completed with 1 state and at least 3 rows. Bod if something in each cell
5A1ft: CAO for Stage 3 following through their optimal values (no extra rows)

4dM1: Dependent on scoring 3rd method mark and at least one of the first two method marks. Award for forming a correct inequality using their SB and the least of their SA and their SC. Allow $\leq$ for this mark. Must follow from stage 3 in their table. So M0 if no *x* appears in stage 3. This M mark could be implied, however by ‘$x < 54$’ if stage 3 is correct.
6A1: CSO correct deduction of range of possible values of *x***.** Strict inequality required for this mark.
1dB1: Correct route. Dependent on 3rd method mark.

<u>SC in part b:</u>
Minimax/Maximin approach
Could score a maximum of B1 M1A0A0 M1A0A0 M1A0 M0A0 (max 4 marks) for the following:
1M1: Stage 1 completed with 3 states and at least 7 rows. Bod if something in each cell
2M1: Stage 2 completed with 3 states and at least 6 rows. Bod if something in each cell
3M1: Stage 3 completed with 1 state and at least 3 rows. Bod if something in each cell
NB must bring earlier optimal results into calculations at least once per stage for each M mark.

Question	Scheme	Marks	AOs
4(a)	Row minima: $-2, -4, -1$ max is -1	1M1	1.2
	Column maxima: 3, 4, 0 min is 0		
	Row maximin $(-1) \neq$ Column minimax (0) so not stable	1A1	2.4
		(2)	
(b)	As the value of V must be non-negative the coefficients of the three inequalities involving V must be non-negative so add at least 4 to each value $\left[\text{e.g. adding 5 gives } \begin{pmatrix} 8 & 3 & 5 \\ 1 & 9 & 3 \\ 6 & 7 & 4 \end{pmatrix}\right]$	1B1	2.3
	Furthermore, as V is the minimum that A can expect to win, the constraints should be $V \leq \ldots$	2B1	2.3
	e.g. 'adding 5' $V \leq 8p_1 + p_2 + 6p_3$ $V \leq 3p_1 + 9p_2 + 7p_3$ $V \leq 5p_1 + 3p_2 + 4p_3$ e.g. 'adding 4' $V \leq 7p_1 + 5p_3$ $V \leq 2p_1 + 8p_2 + 6p_3$ $V \leq 4p_1 + 2p_2 + 3p_3$	3B1	1.1b
		(3)	
(c)	'adding 5' (see tableau below)	1B1 1M1 1A1	1.2 3.3 1.1b
		(3)	
(d)	(ii) Substitute p values to obtain $V \leq 4.6$ Value of the game to player A = $4.6 - 5 = -0.4$	1M1 1A1	3.4 2.2a
		(2)	
(e)	Player B's choices are options Y and Z only	1B1	3.4
	Either $2q = 0.4$ or $-2q + 1(1 - q) = 0.4$ (where q is the probability Player B plays their option Y and $(1 - q)$ is the corresponding probability for option Z)	1M1	3.1a
	$q = 0.2$	1A1	1.1b
	Player B should never play their option X, they should play their option Y with probability 0.2 and option Z with probability 0.8	2A1ft	3.2a
		(4)	
			(14 marks)

Tableau for (c), 'adding 5':

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
r	1	-8	-1	-6	1	0	0	0	0
s	1	-3	-9	-7	0	1	0	0	0
t	1	-5	-3	-4	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	-1	0	0	0	0	0	0	0	0

Notes:

(a) 1M1: attempt at row minima and column maxima – condone one error

1A1: correct reasoning that the game is not stable (accept "$-1 \neq 0$" + statement) – dependent on correct row minima and column maxima

(b) 1B1: Indicates that coefficients are incorrect because *V* must be non-negative. Must convey both underlined aspects. Condone 'positive' for 'non-negative'

2B1: Indicates that inequality signs are the wrong way because *V* is the minimum (so expected winnings are $\geq V$). Must convey both aspects but give bod.

3B1: CAO (all three inequalities). Give this mark for correct equations with slack variables provided 2B1 has been awarded.

(c) 1B1: All row and column labels correct for Simplex tableau

1M1: Setting up the Simplex model - any two of my 'r', 's' or 't' rows correct. Or a completely correct answer with either one column or one row missing – condone lack of basic variable column. Should follow from **changed** constraints of the correct form from b). So, constraints must have been of the form $V \leq ap_1 + bp_2 + cp_3$ ($a, b, c \geq 0$) o.e.

1A1: CAO on numerical values.

Note: The B mark is for labelling the simplex tableau correctly; the M and A marks are for values only.

(d) 1M1: substitutes their *p* values into all three expressions for the upper bound of *V*. Condone use of an equals sign here. But not '$V \geq \ldots$'

May see one of:

- $V \leq 3(0.6) - 4(0) + 0.4 = 2.2$ $\{= \frac{11}{5}\}$; $V \leq -2(0.6) - 4(0) + 2(0.4) = -0.4$ $\{= -\frac{2}{5},\}$; $V \leq -2(0) - (0.4) = -0.4$ $\{= -\frac{2}{5},\}$
- $V \leq 8(0.6) + (0) + 6(0.4) = 7.2$ $\{= \frac{36}{5}\}$; $V \leq 3(0.6) + 9(0) + 7(0.4) = 4.6$ $\{= \frac{23}{5},\}$; $V \leq 5(0.6) + 3(0) + 4(0.4) = 4.6$ $\{= \frac{23}{5},\}$
- $V \leq 7(0.6) + 5(0.4) = 6.2$ $\{= \frac{31}{5}\}$; $V \leq 2(0.6) + 8(0) + 6(0.4) = 3.6$ $\{= \frac{18}{5},\}$; $V \leq 4(0.6) + 2(0) + 3(0.4) = 3.6$ $\{= \frac{18}{5},\}$

1A1: CAO for the value of the game to player A

(e) 1B1: CAO – uses the model to determine that Player B only plays Y and Z

1M1: A *correct* equation for B (where value of game to B = – 1 x **their** value of game to A)

1A1: CAO for *q* (the probability that B plays Y)

2A1ft: Correct optimal strategy in context (not just in terms of *q*) following through their *q*

Alternative for (e)

1B1: CAO – uses the model to determine that Player B only plays Y and Z

1M1: Formulates two correct expressions for the expected value of the game to B and finds the intersection: $2q = -2q + 1(1 - q)$ $(= 1 - 3q)$

1A1: As above

2A1ft: As above

Question	Scheme	Marks	AOs
5(a)	$u_n - \frac{1}{2}(u_{n-1} + u_{n-2}) = 6 \Rightarrow 2u_n - (u_{n-1} + u_{n-2}) = 12$	1M1	2.1
	$2u_n - u_{n-1} - u_{n-2} = 12$	1A1	2.2a
		(2)	
(b)	aux equation $2m^2 - m - 1 = 0 \Rightarrow m = 1, m = -\frac{1}{2}$	1B1	2.1
	$u_n = A + B\left(-\frac{1}{2}\right)^n$	2B1	1.1b
	particular solution try $u_n = \lambda n$ $\therefore 2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12 \Rightarrow \lambda[= 4]$	1M1	1.1b
	$u_n = A + B\left(-\frac{1}{2}\right)^n + 4n$	1A1	1.1b
	$u_1 = 2 \Rightarrow 2A - B = -4$	2M1	1.1b
	$u_2 = 8 \Rightarrow 4A + B = 0$	3M1	1.1b
	$A = -\frac{2}{3}, B = \frac{8}{3} \Rightarrow u_n = -\frac{2}{3} + \frac{8}{3}\left(-\frac{1}{2}\right)^n + 4n$	2A1	2.2a
		(7)	
(c)	$u_n \to 4n \quad (k = 4)$ As $n \to \infty, \left(-\frac{1}{2}\right)^n \to 0$ and $4n$ is considerably greater than $-\frac{2}{3}$	1M1 1A1ft	2.1 1.1b
		(2)	
			(11 marks)

Notes:

(a) 1M1: attempt to write given information in terms of a recurrence equation (allow sign errors and errors in notation)

1A1*: CAO (NB equation is provided in the question so must see a correct unsimplified recurrence relation which is simplified with no errors to the required form for this mark).

(b) 1B1: CAO for auxiliary equation and corresponding solutions (this mark can be implied by correct complementary function)

2B1: complementary function CAO. Condone lack of '$u_n =$'.

1M1: substitutes $u_n = \lambda n$ into their 2nd-order recurrence relation and solves to obtain $\lambda = \ldots$

Note: $2\lambda n - \lambda(n-1) - \lambda(n-2) = 12$ o.e. $\Rightarrow \lambda = \cdots$ can earn this mark

Note: Do not condone sign errors or errors in coefficients e.g.

$2\lambda n - \lambda(n+1) - \lambda(n-2) = 12$ is M0

$2\lambda n - 2\lambda(n-1) - \lambda(n-2) = 12)$ is M0

1A1: Correct general solution. Condone lack of '$u_n =$'.

2M1: Forms one equation in A and B. General Solution must be of the form.

$u_n = A + B\left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$

3dM1: Forms a second equation in A and B. Dependent on the previous method mark.

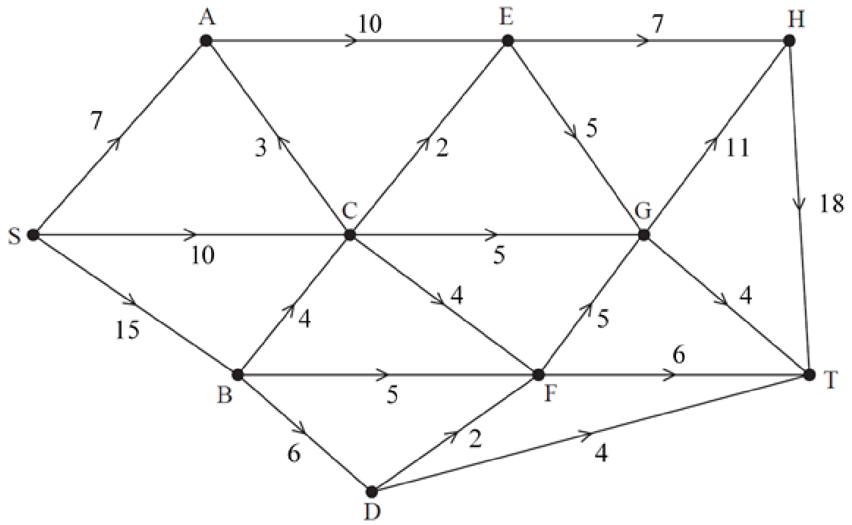
2A1: Particular solution CAO. Do not condone lack of '$u_n = \ldots$'

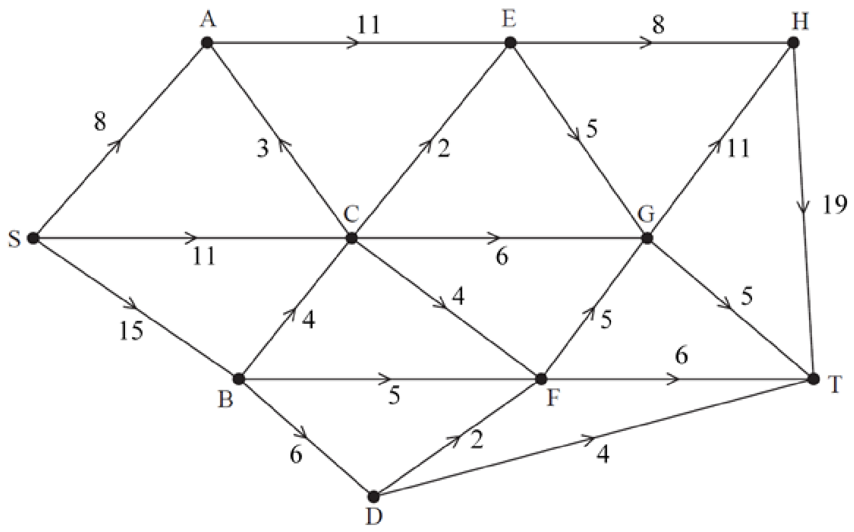
(c) M1: Obtains correct limit (ft their particular solution which must be of the correct form i.e. $u_n = A + B\left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$)

A1ft: Provides correct reasoning including comments relating to both of the following:

- $n \to \infty, \left(-\frac{1}{2}\right)^n \to 0$
- '$-\frac{2}{3}$' becomes negligible (compared to $4n$ as $n \to \infty$)

Note: Condone '$-\frac{2}{3}$ becomes insignificant'

Question	Scheme	Marks	AOs
6 (a)(i)	$C_1 = 9 + 11 + 4 + 5 + 7 = 36$	1B1	1.1b
(ii)	$C_2 = 8 + 5 - 2 - 3 + 11 + 4 + 5 + 2 + 4 = 34$	2B1	1.1b
		(2)	
(b)	If AE and CE were both full to capacity then 12 + 4 = 16 litres per second would flow though E but the maximum capacity of the two arcs out of E (EH and EG) is only 8 + 5 = 13 so AE and CE cannot both be full to capacity	1B1	2.4
		(1)	
(c)	The minimum flow through the arcs AE, CE, CG, FG, FT and DT (which provides a cut for the network) is 10 + 2 + 5 + 5 + 6 + 4 = 32 so a minimum of 32 must be flowing through the system so 31 is not possible.	1B1	2.4
		(1)	
(d)	Attempt to find a flow of 32 using the answer to (c)	1M1	3.4
	E.g. Minimum flow of 32 	1A1	1.1b
	Attempt to augment minimum flow and recognise that from (a) the maximum flow is less than or equal to 34	1B1	2.1

	E.g. Maximum flow of 34 	2B1	1.1b
		(4)	
			(8 marks)

Notes:

(a)(i) 1B1: CAO

(ii) 1B1: CAO

(b) 1B1: Calculates maximum capacity entering E and compares with maximum capacity leaving E. Concludes that maximum capacity into E exceeds maximum capacity out of E.

Condone statements such as ‘flow into E = 12 + 4 < 13 which is the flow out of E’.

(c) 1B1: valid reason why the flow in the network cannot be 31 litres per second.

Note: If smallest of C_1 and C_2 in a) is less than 31 then DO NOT allow this mark for deducing that ‘31 > {smallest of answers from a} hence, flow of 31 is not possible’.

(d) 1M1: Award this mark for either:

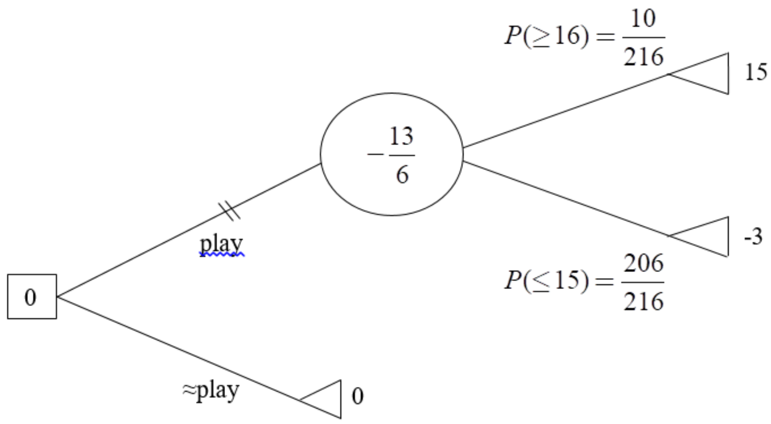
- Indicates ‘minimum flow > 31 so minimum flow could be 32’, OR
- Attempts to find flow of 32 in which: the sum of flows along arcs from S is 32 **and** $7 \leq$ flow along SA ≤ 9, $8 \leq$ flow along SC ≤ 11 **and** $13 \leq$ flow along SB ≤ 17, OR
- Attempts to find flow of 32 in which: the sum of flows along arcs into0 T is 32 **and** flow along DT = 4, $6 \leq$ flow along FT ≤ 8, $4 \leq$ flow along GT ≤ 4 **and** $17 \leq$ flow along HT ≤ 20
- Identifies both ‘min flow = 32’ **AND** ‘max flow = 34’

Note: Only need consider arcs incident to S for the second bullet point above, or arcs incident to T for the third bullet point. Flows along other arcs may be incorrect or missing.

1A1: CAO for consistent flow of 32. One number per arc. Check for consistency at each node.

1B1: States that maximum flow must be 34 and makes some reference to (smallest cut in) part a).

1B1: CAO for consistent flow of 34. One number per arc. Check for consistency at each node.

Question	Scheme	Marks	AOs
7(a)(i)		1M1 1B1 1A1 2M1 2A1	3.3 1.1b 1.1b 3.4 1.1b
(ii)	Optimum EMV is (£)0 and Aisha should not play the game	1B1ft	3.2a
		(6)	
(b)	Expected utility is $\frac{10}{216}\left(1 - e^{\frac{-(x+3)}{500}}\right)$	1M1 1A1	3.4 2.2a
		(2)	
(c)	If Aisha doesn't play she will have 3 $\Rightarrow 1 - e^{-\frac{3}{500}}$	1B1	3.1a
	For the prize to be worthwhile $\frac{10}{216}\left(1 - e^{\frac{-(x+3)}{500}}\right) > 1 - e^{\frac{-3}{500}}$	1M1	1.1b
	Correct order of operations and use of logs to find x	2dM1	1.1b
	$x > 66.178...$ so (minimum prize amount should be) £66.18	1A1	3.2a
		(4)	
			(12 marks)

Notes:

(a)(i) 1M1: tree diagram with at least three end pay-offs, one (rectangular) decision node and one (circular) chance node

1B1: Correct probability for rolling 16 or more (accept equivalent fractions)

1A1: Correct structure of tree diagram with each arc labelled correctly. The following must be seen for this mark:

- Probabilities on branches leading from chance node. Probabilities may not be correct but must sum to 1
- 'Play' and 'Not Play' (o.e.) labelled correctly. (If one branch is labelled correctly then condone lack of label on the other branch).
- Pay offs: 15, –3, 0 placed at tree ends (do not condone '3' for '–3')

2M1: Correct chance node. ft from their probabilities: $15p - 3(1 - p)$ where p is their probability for rolling 16 or more. If given as a decimal allow for correct (or truncated) to 2dp

2A1: CAO (must be exact) for chance and decision node including double line through inferior option

Note must see 0 at the decision node for this mark.

(ii) 1B1ft: correct optimal EMV (clearly indicated) **and** analysis in context.

So: If *their* EMV for playing game < 0 then 'Optimum EMV = *0*, together with corresponding conclusion: 'Aisha should play' o.e. would earn B1.

Whereas, if *their* EMV for playing game > 0 then 'Optimum EMV = *their* EMV of playing game' together with corresponding conclusion: 'Aisha should not play' o.e. would earn B1.

BUT If their EMV for playing game does not ft from their probabilities then B0.

(b) 1M1: $p\left(1-e^{\frac{-(x+3)}{500}}\right)$ with their p from (a)

Note: isw after a correct expression for M mark. May see:

$p\left(1-e^{\frac{-(x+3)}{500}}\right)+(1-p)\left(1-e^{\frac{-(-3+3)}{500}}\right)$ o.e.

1A1: CAO for expected utility. Must be of the correct form as is specified in the question. So:

$\frac{10}{216}\left(1-e^{\frac{-(x+3)}{500}}\right)$ but allow fractions equivalent to $\frac{10}{216}$

(c) 1B1: CAO. May be in decimal form: 0.00598…. May be embedded in inequality/equation.

Note: Look out for missing minus sign in the power if stated exactly.

Note: Do <u>not</u> award this mark if seen in only part (b).

1M1: Sets up an inequality or equation with their expected utility of playing the game (from part b).

2dM1: Solving for x (dependent on previous M mark). Requires correct order of operations and log work.

For example,

May see: $e^{\frac{-(x+3)}{500}} \square 1-'p'\left(1-e^{\frac{-(3)}{500}}\right)$ o.e. [Isolates exponential term]

Or $e^{-\frac{(x+3)}{500}} \square 0.870 \ldots$

Followed by: $\frac{(x+3)}{500} \square -ln\left(1-'p'\left(1-e^{\frac{-(3)}{500}}\right)\right)$ o.e. [Applies logs correctly]

Or $\frac{(x+3)}{500} \square$ 0.138…

Where □ is any inequality or equals sign.

Note: May not get to $x = \ldots$

Note: Can be implied by correct answer

1A1: CAO with units

Note: For this mark, condone '£66.18' or 'x = £66.18' or '$x \geq$ £66.18', but not 'x > £66.18'.

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