Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 28 : Decision Mathematics 2

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2023
Publications Code 8FM0_28_2306_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Add an additional dummy column with equal values (e.g. 0) to create a square array	B1	3.5c
	Input a suitable large number (e.g. 100 but > 44) in cells BQ and ES	B1	1.1b
		(2)	
(b)	e.g. $\begin{pmatrix} & P & Q & R & S & X \\ A & 38 & 39 & 37 & 37 & 0 \\ B & 39 & 100 & 39 & 40 & 0 \\ C & 41 & 44 & 40 & 42 & 0 \\ D & 40 & 41 & 39 & 38 & 0 \\ E & 36 & 39 & 41 & 100 & 0 \end{pmatrix}$	B1	1.1b
	Reducing (rows and) columns gives $\begin{pmatrix} & P & Q & R & S & X \\ A & 2 & 0 & 0 & 0 & 0 \\ B & 3 & 61 & 2 & 3 & 0 \\ C & 5 & 5 & 3 & 5 & 0 \\ D & 4 & 2 & 2 & 1 & 0 \\ E & 0 & 0 & 4 & 63 & 0 \end{pmatrix}$ Three lines required to cover the zeros hence solution is not optimal	M1	1.1b
	$\begin{pmatrix} & P & Q & R & S & X \\ A & 2 & 0 & 0 & 0 & 1 \\ B & 2 & 60 & 1 & 2 & 0 \\ C & 4 & 4 & 2 & 4 & 0 \\ D & 3 & 1 & 1 & 0 & 0 \\ E & 0 & 0 & 4 & 63 & 1 \end{pmatrix}$ Four lines required to cover the zeros hence solution is not optimal	M1	1.1b
	e.g. $\begin{pmatrix} & P & Q & R & S & X \\ A & 2 & 0 & 0 & 0 & 2 \\ B & 1 & 59 & 0 & 1 & 0 \\ C & 3 & 3 & 1 & 3 & 0 \\ D & 3 & 1 & 1 & 0 & 1 \\ E & 0 & 0 & 4 & 63 & 2 \end{pmatrix}$ or $\begin{pmatrix} & P & Q & R & S & X \\ A & 2 & 0 & 0 & 1 & 2 \\ B & 1 & 59 & 0 & 2 & 0 \\ C & 3 & 3 & 1 & 4 & 0 \\ D & 2 & 0 & 0 & 0 & 0 \\ E & 0 & 0 & 4 & 64 & 2 \end{pmatrix}$	M1	1.1b
	Five lines required to cover the zeros hence solution is optimal	B1	2.4
	Allocation: A to Q, B to R, D to S, and E to P (C does no task)	A1	2.2a
		(6)	
(c)	152 (mins)	B1	1.1b
		(1)	
			(9 marks)

Notes for Question 1

(a)

B1: Explain the need to add a dummy column (to create a square array) – must mention that it is a column being added (but ignore any mention of the values being added to this column)

B1: Input a large value in cells BQ and ES (just saying 'input a large value in the empty (oe) cells' is sufficient – we do not need to know at this stage what is meant by 'large')

(b)

B1: Mark awarded when both steps complete (addition of extra column and large values (> 44) in cells BQ and ES)

M1: Simplifying the initial matrix by reducing (rows and) columns – allow at most 2 slips

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 4 lines needed to 5 lines needed (so getting to the optimal table)

B1: Dependent on two augmentations taking place (3 to 4 lines and then 4 to 5 lines). Either a correct statement(s) regarding the minimum number of lines to cover the zeros at each stage **or** a general statement that covers all augmentations.

In the first case, at each stage, they must state the number of lines (not just shown on the diagram), state whether it is optimal or not (so must use the word 'optimal') and mention 'zeros' at least once.

In the second case, they must state that until 5 lines cover the zeros then the solution is not optimal (or equivalent e.g. if there are 5 lines covering the zeros then the solution is optimal) – in this case they must show the lines.

Accept a hybrid of the two e.g. at each stage they could say whether it requires five lines or not but they would still have to mention 'zeros' at least once and make it clear at each augmentation whether it is optimal or not.

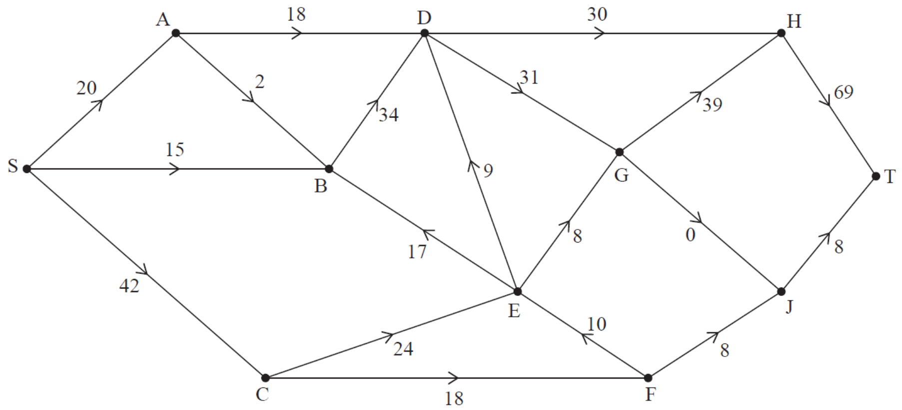
To award this mark we must see mention at least one mention of 'zeros' and the word 'optimal' being used.

A1: cso on final table + deduction of the correct allocation

Special case in (b) – if dashes are not replaced with larger values (e.g. values $\leqslant 44$ or leaving dashes in place) then only the **M** marks are available in **(b)**

(c)

B1: cao

Question	Scheme	Marks	AOs
2(a)	72	B1	1.1b
		(1)	
(b)	Value of cut = 25 + 15 + 27 + 15 + 10 = 92	B1	1.1b
		(1)	
(c)	e.g. SCEFJGHT – 3; SABDHT – 2 SCEFJGHT – 3; SCEBDHT – 1; SABDHT – 1 SABEFJGHT – 3; SABDHT – 2	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(d)	e.g. (based on the first example in **(c)**) 	B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem Identification of cut through AD, BD, ED, EG, GJ and JT Value of flow = 77 Therefore it follows that flow is optimal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	

(9 marks)

Notes for Question 2

(a)

B1: cao (72)

(b)

B1: cao (92)

(c)

M1: One correct flow augmenting route found from S to T (so any routes that contain SB, BA, AD, CF, ED, EG, DG or JT are incorrect routes) - a 'correct' route is one in which the flow through the system can be increased

A1: Two correct routes (ignoring numerical value of the flow for this mark)

A1: cso – increasing the flow by 5 (and no more) **–** so at least two routes with corresponding correct values stated

(d)

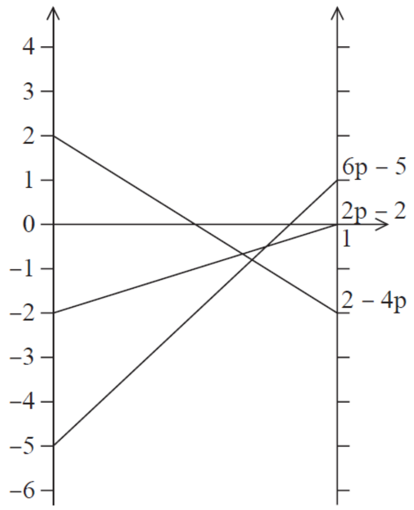
B1: cao – if there are two numbers on each arc neither of which is circled then **B0**, if there are two numbers on each arc, one circled and one not, then consider the circled numbers only as the maximum flow pattern. Do not accept a blank arc as a zero

(e)

M1: Construct an argument based on max-flow min-cut theorem (e.g. attempt to find a cut (but not the one through SA, SB, CE, FE, FJ) through saturated arcs – must contain source on one side and sink on the other) – allow cut shown on the Diagram 1 in the answer book – this mark is dependent on an attempt at part (d) (so values on all but two arcs)

A1: Use appropriate process of finding a minimum cut – cut (AD, BD, ED, EG, GJ, JT) and the value of the flow through the network stated correctly (77)

A1: Correct deduction that the flow is maximal – must use all four words 'maximum', 'flow', 'minimum' and 'cut' (allow abbreviations for maximum and minimum) – dep on first A mark and the B mark in (d)

Question	Scheme	Marks	AOs
3(a)(i)	Row minima: –2, –1, 3, 0 max is 3 Column maxima: 3, 5 min is 3 Row(maximin) = Col(minimax) which are both 3 therefore game is stable	M1 A1	1.1b 2.4
(a)(ii)	Value of the game to player *B* is –3	A1	2.2a
		(3)	
(b)	$\begin{pmatrix} -2 & 1 & 0 \\ 2 & -5 & -2 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	If *A* plays option Q, *B*'s gains are $-2p + 2(1 - p) = 2 - 4p$ If *A* plays option R, *B*'s gains are $p + (-5)(1 - p) = -5 + 6p$ If *A* plays option T, *B*'s gains are $(-2)(1 - p) = -2 + 2p$  $2 - 4p = -5 + 6p \Rightarrow p = 7/10$ *B* should play option X with probability 7/10 and option Y with probability 3/10	M1 A1 M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b 1.1b 3.2a
		(6)	
(d)	**(i)** $6\ 7/10\ -5 = -4/5 \Rightarrow$ value of the game to player *A* is 4/5 **(ii)** Player *A* should never play option T **(iii)** If *A* plays their option Q with probability *q* and their option R with probability $1 - q$ then $2q + (-1)(1-q) = \frac{4}{5}$ $q = \frac{3}{5} \Rightarrow$ *A* should play option Q with probability 3/5 and option R with probability 2/5 (play S and T never)	B1 B1 M1 A1	3.1a 2.2a 3.1b 3.2a
		(4)	
			(14 marks)

Notes for Question 3

(a)

M1: finding row minimums and column maximums – condone one error

A1: row maximin 3 = col minimax 3 (so stable) – dependent on all 6 correct values – as a minimum must see the two 3's explicitly being considered

A1: cao (–3) – 'lose 3' is **A0**

(b)

B1: cao (must be the reduced game so with option S removed)

(c)

In (c) if the candidate includes option S (and so has four expressions and lines) then this can score the first 3 marks only in this part

M1: setting up three expressions in terms of p

A1: all three expressions correctly simplified

M1: at least two lines correctly drawn for their expressions – if values on at least one vertical axis not given then lines must be in the right position relative to each other

A1: completely correct graph with clear indication of the correct intersection points at the ends where $p = 0$ and $p = 1$ (so if no scaling on the vertical axis assume that 1 line = 1 unit (unless other suitable scaling is clear), and lines must not extend past $p < 0$ and/or $p > 1$)

A1: using the graph (with 3 lines) to obtain the correct probability expressions leading to the correct value of p

A1ft: interpret their value of p in the context of the question – must refer to 'play' and the associated probabilities (but do not need to explicitly use the word 'probability')

(d)(i)

B1: cao ($\frac{4}{5}$ or 0.8)

(d)(ii)

B1: cao (option T)

(d)(iii)

M1: Setting up a linear equation with their V(A) from **(d)(i)** wither either of the two correct expressions $2q+(-1)(1-q)$ **or** $-2q+5(1-q)$

or setting up the correct equation $-2q+5(1-q)=2q+(-1)(1-q)$ (or $2q-5(1-q)=-2q+(1-q)$)

A1: interpret the correct value of q in the context of the question – must refer to 'play' and the associated probabilities (but do not need to explicitly use the word 'probability')

Question	Scheme	Marks	AOs
4(a)	(aux equation $m-\frac{3}{2}=0\Rightarrow$) complementary function is $A(1.5)^n$	B1	2.1
	Particular solution try $u_n=\alpha n^2+\beta n+\gamma$ and substitute into recurrence relation	M1	1.1b
	$2\alpha n^2+(4\alpha+2\beta)n+(2\alpha+2\beta+2\gamma)=(3\alpha-4)n^2+3\beta n+(3\gamma-8)$ and by comparing quadratic, linear and constant terms gives $2\alpha=3\alpha-4$ $4\alpha+2\beta=3\beta$ $2\alpha+2\beta+2\gamma=3\gamma-8$	dM1	1.1b
	$\alpha=4,\beta=16,\gamma=48$	A1	1.1b
	$(u_n=)A(1.5)^n+4n^2+16n+48$		
	$u_0=k\Rightarrow A+48=k$	ddM1	3.4
	$(u_n=)(k-48)(1.5)^n+4n^2+16n+48$	A1	1.1b
		(6)	
(b)	$(k-48)(1.5)^{10}+4(10)^2+16(10)+48>5000$	M1	1.1b
	$k>124.163...\Rightarrow k=125$	A1	2.2a
		(2)	

(8 marks)

Notes for Question 4

(a)

B1: cao (or equivalent e.g. $A(1.5)^{n-1}$)

M1: correct form for particular solution e.g. $\alpha n^2+\beta n+\gamma,\ \alpha(n-1)^2+\beta(n-1)+\gamma,$ etc. (so anything that is equivalent to a three term quadratic in n with unknown coefficients and constant term) **together** with a valid substitution into recurrence relation (so not substituting u_n on both sides of the recurrence relation)

dM1: compares coefficients and setting up all three equations in α,β,γ - dependent on previous **M** mark

A1: cao for the values of α,β,γ

ddM1: use correct initial condition correctly to form a linear equation in their *A* and *k* – dependent on the two previous **M** marks

A1: correct particular solution (in terms of *k*) – need not see $u_n=$ but if seen then it must be correct (and therefore $u_{n+1}=$ is **A0**)

(b)

M1: dependent on all **M** marks in **(a)** – substituting n = 10 and setting the particular solution > 5 000 (or equal to)

A1: cao (125) from correct working so must have had a correct expression for u_n in **(a)** (so dependent on at least the first 5 marks in **(a)**)

Additional guidance:

Those candidates who re-write $u_{n+1} = \frac{3}{2}u_n - 2n^2 - 4$ as $u_n = \frac{3}{2}u_{n-1} - 2(n-1)^2 - 4$ can score full marks. Their solution will usually begin: CF is $A(1.5)^n$ then a PS of the form $\alpha n^2 + \beta n + \gamma$ leading to $\alpha n^2 + \beta n + \gamma = \left(\frac{3}{2}\alpha - 2\right)n^2 + \left(-3\alpha + \frac{3}{2}\beta + 4\right)n + \left(\frac{3}{2}\alpha - \frac{3}{2}\beta + \frac{3}{2}\gamma - 6\right)$ and then $\frac{3}{2}\alpha - 2 = \alpha \Rightarrow \alpha = 4,\ -3\alpha + \frac{3}{2}\beta + 4 = \beta \Rightarrow \beta = 16,\ \frac{3}{2}\alpha - \frac{3}{2}\beta + \frac{3}{2}\gamma - 6 = \gamma \Rightarrow \gamma = 48$ and then their general solution should be as in the main scheme (although for the final mark in (a) do look out for those who call the left-hand side u_{n+1})

It is common for candidates to re-write $u_{n+1} = \frac{3}{2}u_n - 2n^2 - 4$ as $u_n = \frac{3}{2}u_{n-1} - 2n^2 - 4$ which is incorrect. This can score all **B** and **M** marks in both parts only.

Slightly less common is to have a CF of the form $A(1.5)^{n-1}$ and a PS of the form $\alpha(n-1)^2 + \beta(n-1) + \gamma$ this leads to $u_{n+1} = A(1.5)^{n-1} + 4(n-1)^2 + 32(n-1) + 96$

Now to use the initial condition correctly $u_0 = k \Rightarrow u_1 = \frac{3}{2}k - 4$ (oe) and this leads to

$A = \frac{9}{4}k - 108$

So, $u_{n+1} = \left(\frac{9}{4}k - 108\right)(1.5)^{n-1} + 4(n-1)^2 + 32(n-1) + 96$ which when re-written in terms of u_n gives the form as in the main mark scheme

Of course, any CF of the form $A(1.5)^{n\pm k_1}$ and any PS of the form $a(n \pm k_2)^2 + b(n \pm k_3) + c$ where k_1, k_2, k_3 are constants will work. So, award the **M** marks for the correct methods as illustrated in the notes in mark scheme and the first **A** mark in (a) for three correct coefficients of their PS and the second **A** mark in (a) for a fully correct expression

Pearson Education Limited. Registered company number 872828

with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom