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Other names

Pearson Edexcel
Level 3 GCE

Centre Number

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Candidate Number

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Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

Monday 14 May 2018 – Afternoon

Time: 1 hour 40 minutes

Paper Reference

8FM0/01

You must have:

Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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- (2)

- $$2x + y - 3z = -4$$

$$4x - 2y + z = 9$$

$$3x + 5y - 2z = 5$$

(2)

- (1)

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Question 1 continued

Lined area for writing the answer to Question 1.

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Question 1 continued

Lined area for writing the answer to Question 1.

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Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



2. The cubic equation

$$z^3 - 3z^2 + z + 5 = 0$$

has roots α , β and γ .

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p , q and r are integers to be found.

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Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 5 marks)



3. (a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 1 - i| \leq 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leq \arg(z - 2) \leq \frac{3\pi}{4}\right\} \quad (5)$$

The complex number w satisfies

$$|w - 1 - i| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

(b) Find, in simplest form, the exact value of $|w|^2$

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Question 3 continued

Lined area for writing the answer to Question 3 continued.

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Question 3 continued

Handwriting practice area with 30 horizontal lines.

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Question 3 continued

Lined area for writing the answer to Question 3.

(Total for Question 3 is 9 marks)



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4. Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface. The water pipes are modelled as straight line segments. One water pipe, W , is buried beneath a particular road. With respect to a fixed origin O , the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and W passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$. The units are in metres.

(a) Use the model to calculate the acute angle between W and the road surface.

(5)

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to W from C .

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect C to W .

(6)



Question 4 continued

Lined area for writing the answer to Question 4.

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Question 4 continued

Lined area for writing the answer to Question 4.

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Question 4 continued

Handwriting practice area with 25 horizontal lines.

(Total for Question 4 is 11 marks)



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Question 5 continued

Lined area for writing the answer to Question 5.

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Question 5 continued

Lined area for writing the answer to Question 5.

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Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 10 marks)



6. (a) Use the standard results for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$ to show that

$$\sum_{r=1}^n (3r-2)^2 = \frac{1}{2}n[6n^2 - 3n - 1]$$

for all positive integers n .

(5)

(b) Hence find any values of n for which

$$\sum_{r=5}^n (3r-2)^2 + 103 \sum_{r=1}^{28} r \cos\left(\frac{r\pi}{2}\right) = 3n^3$$

(5)

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Question 6 continued

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Question 6 continued

Lined area for writing the answer to Question 6.

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Question 6 continued

Lined area for writing the answer to Question 6.

(Total for Question 6 is 10 marks)



7.

$$f(z) = z^3 + z^2 + pz + q$$

where p and q are real constants.

The equation $f(z) = 0$ has roots z_1, z_2 and z_3

When plotted on an Argand diagram, the points representing z_1, z_2 and z_3 form the vertices of a triangle of area 35

Given that $z_1 = 3$, find the values of p and q .

(7)

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Question 7 continued

Handwriting practice area with 25 horizontal lines.

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Question 7 continued

Handwriting practice area with 20 horizontal lines.

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Question 7 continued

Handwriting practice area with 28 horizontal lines.

(Total for Question 7 is 7 marks)



8. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^n = \begin{pmatrix} 4n+1 & -8n \\ 2n & 1-4n \end{pmatrix} \quad (6)$$

(ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)

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Question 8 continued

Handwriting practice area with 20 horizontal lines.

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Question 8 continued

Lined area for writing the answer to Question 8.

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Question 8 continued

Handwriting practice area with 24 horizontal lines.

(Total for Question 8 is 12 marks)



9.

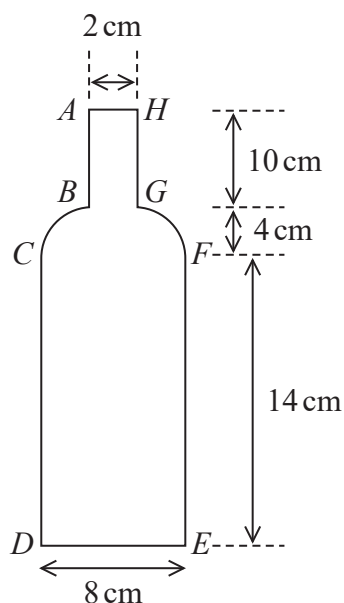


Figure 1

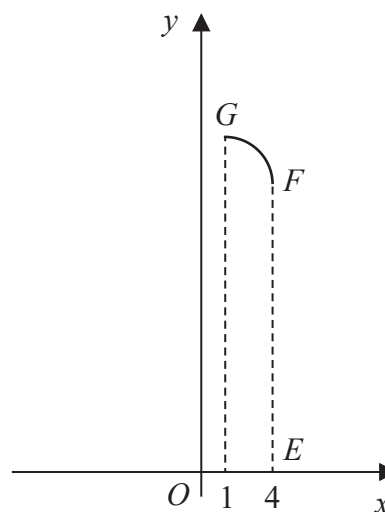


Figure 2

A mathematics student is modelling the profile of a glass bottle of water. Figure 1 shows a sketch of a central vertical cross-section $ABCDEFGHA$ of the bottle with the measurements taken by the student.

The horizontal cross-section between CF and DE is a circle of diameter 8 cm and the horizontal cross-section between BG and AH is a circle of diameter 2 cm.

The student thinks that the curve GF could be modelled as a curve with equation

$$y = ax^2 + b \quad 1 \leq x \leq 4$$

where a and b are constants and O is the fixed origin, as shown in Figure 2.

(a) Find the value of a and the value of b according to the model.

(2)

(b) Use the model to find the volume of water that the bottle can contain.

(7)

(c) State a limitation of the model.

(1)

The label on the bottle states that the bottle holds approximately 750 cm^3 of water.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)



Question 9 continued

Handwriting practice area with 20 horizontal lines.

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Question 9 continued

Handwriting practice area with 20 horizontal lines.

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Question 9 continued

Handwriting practice area with 20 horizontal lines.

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Question 9 continued

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(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 80 MARKS

