Examiners' Report

Summer 2016

Pearson Edexcel GCE in Further Pure Mathematics 3 (6669/01)

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GCE Mathematics Further Pure 3

Specification 6669/01

Introduction

This paper proved a good test of student knowledge and understanding. There were many accessible marks available to students who were confident with topics such as matrices, differentiation, integration, coordinate systems and vectors.

Report on individual questions

Question 1

The overwhelming majority of students knew that they had to solve the equation det **A** = 0, with only a very small number unaware of the definition of "singular". Most students processed the determinant conventionally, usually by using row 1, although the Rule of Sarrus was seen quite regularly. Sign and algebraic errors were rarely seen and most students obtained the correct values for k and the full four marks.

Question 2

It was very rare to see errors with the differentiation of y or with the use of the arc length formula. However, some basic algebraic errors were seen when students squared and added one to their derivative. A significant number of students failed to realise that the expression obtained was a perfect square and so many did not remove the square root. This led to failed attempts at integrations using substitutions or by parts. Those that achieved the correct integrand in a simple form almost always proceeded to integrate correctly and use the given limits to obtain the correct exact value.

Question 3

Q03(a) proved to be quite a challenging task for many. Misconceptions such as $\operatorname{arcoth} x = \dfrac{1}{\operatorname{artanh} x}$ or $\dfrac{\operatorname{arcosh} x}{\operatorname{arsinh} x}$ were often seen. Those who proceeded to $\coth y = x$ invariably made progress. Implicit differentiation of $\coth y = x$ was the most successful method although it was common to see the minus sign missing from $-\operatorname{cosech}^2 y \dfrac{dy}{dx} = 1$. Writing $x = \dfrac{\cosh y}{\sinh y}$ or $\tanh y = \dfrac{1}{x}$ before differentiation were also successful routes for many. Use of $y = \operatorname{artanh} \dfrac{1}{x}$ was also seen. Most differentiations were carried out correctly although sign errors with the appropriate hyperbolic identities were occasionally made. Students who introduced exponential or logarithmic forms rarely made much progress.

A small number of students elected to integrate $\dfrac{1}{1-x^2}$ which had mixed results, although the few who used the substitution $x = \coth u$ produced an elegant solution.

Q03(b) proved more successful for most students, although it was occasionally not attempted by those who struggled with Q03(a). A small number of students proceeded with $y = \operatorname{arcoth} x$ instead of $y = (\operatorname{arcoth} x)^2$. A correct first derivative was widely seen, usually followed by correct use of the quotient or product rule to obtain the second derivative. Most then substituted into the differential equation and produced the required answer although some algebraic errors were seen. Many attempts at reforming the obtained equation containing the second derivative instead of substituting were successful. A small number multiplied their equation in the first derivative by $1-x^2$ before the second differentiation, which produced the given answer more easily.

Question 3 continued

Attempts starting with $y^{\frac{1}{2}}$ = arcoth x were rare.

Students should try and set out their work carefully and clearly distinguish between hyperbolic and trigonometric functions (eg cosh y rather than cos y) and do not mix the x and y variables.

Question 4

A well–answered question by the majority with full marks commonly awarded.

Q04(a) was the most likely to cause problems and a small number of students failed to realise the need to complete the square and sometimes tried to factorise the quadratic expression. The negative coefficient of x^2 was handled wrongly by some, with $(x-1)^2 - 16$ instead of $16-(x-1)^2$ seen, leading to an arcosh expression after integration. A small number had the alternative $16-(1-x)^2$ but tended to make a sign error when integrating. Those who obtained the correct integrand invariably proceeded correctly. Although it was a standard integral that most dealt with directly, substitutions of $u = x+1$ and $4\sin\theta = x-1$ were quite common and were usually used successfully.

Q04(ii)(a) was well answered by almost all students. Only a very small number had any errors in their proof, usually from a sign error when combining a subtraction of fractions. The given answer was occasionally miscopied.

Good scoring was also seen in Q04(ii)(b), with only a few making errors with the substitution (usually producing a numerator of $2u^2$ in the integrand). An arctan term was produced by most although logarithmic expressions or attempts with incorrect partial fractions were occasionally seen. Some students lost the “2” during integration. The most common error was to fail to replace u with e^x in the final line of the answer.

Question 5

The work required in Q05(a) was done well by the vast majority. A few students chose inefficient methods or did not simplify expressions when they could and were slightly more prone to error. Those who chose to differentiate parametrically made light work of obtaining the required gradient. Implicit differentiation was also seen but encouragingly, explicit differentiation was very rare. Only a small number failed to apply the correct perpendicular gradient and straight line methods. The $y = mx + c$ approach was seen on occasion. Only a few students arrived at the given answer with no working after their straight line equation. A gradient in terms of *x* and *y* with substitution later on in the working was also infrequently seen.

It was also rare to not be awarding marks in Q05(b). The correct eccentricity formula was almost always used and the correct positive directrix obtained. A few incorrect values for x arose from careless errors evaluating 4 divided by $\frac{5}{4}$. Two common mistakes were seen with the rest of the question; the answer was occasionally given as a single fraction instead of in the form specified and the error $3y = 25 - \frac{32}{5}\sqrt{2}$ leading to $3y = \frac{25}{3} - \frac{32}{5}\sqrt{2}$ was also seen.

Question 6

It was unusual to see an incorrect method in both Q06(a) and Q06(b), with $\mathbf{M}x = \lambda x$ rather than $(\mathrm{M} - \lambda\mathbf{I})x = 0$ preferred by most. Occasionally *x* was not fully substituted with the eigenvector or only substituted after the equations had been obtained. The correct values for λ, *p* and *q* were widely obtained but errors were fairly common.

The method in Q06(c) was correctly applied and it was rare to see a student produce an incorrect eigenvector if they had been correct in Q06(a) and Q06(b). An eigenvector of 0 was occasionally offered as a solution.

The final part was better received than equivalent questions in previous series. Almost all knew that **P** required a matrix of the eigenvectors although some transcription errors were seen. Those who understood the topic then produced a consistent matrix **D** although a small number then went on to unnecessarily obtain **D** again by multiplication. Those who could only obtain **D** by performing $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$ rarely arrived at the correct diagonal matrix.

Question 7

Q07(a) proved very challenging and was omitted by many students. Many attempted integration by parts and only a few who had done so later realised that a different approach was required here. Those who appreciated the need for trigonometric identities were usually able to make some progress, although only the most able students were able to produce a convincing error free proof. A wide range of successful strategies were seen but expanding $\sin(n-2)x$ or $\sin((n-2)x+2x)$ were popular approaches. Students familiar with the factor formulae tended to be particularly successful.

Q07(b) was successfully answered by a large number of students. Almost all knew to use the reduction formula twice and it was very rare to see sign or other errors with the integrations. The evaluation of I_1 was occasionally missing or included attempts to write I_1 in terms of "I_{-1}" or "I_0". Direct attempts at I_3 were rare and produced mixed results. The final mark was sometimes lost when students incorrectly factored out the $\frac{1}{12}$.

Question 8

In Q08(a), most students attempted the cross product of two appropriate vectors although the correct normal was not always achieved. The majority proceeded to calculate an appropriate scalar product but a small number did not explicitly show its evaluation. The full $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ was unnecessarily used by some, but usually correctly. A small number failed to give a conclusion.

Q08(b) was similarly well answered, with many correct solutions seen. A few students incorrectly interpreted the information in the formulae book and produced $\mathbf{r}.\mathbf{n} = -\mathbf{a}.\mathbf{n}$ and so $7x+5y-9z=8$. A small number of Cartesian equations of lines were seen.

A variety of correct approaches were taken with Q08(c). By far the most successful was to obtain the direction of the line using a vector product of the normals followed by identification of a point. Those who set x or y or $z = 0$ invariably found a correct point on the planes and proceeded to a correct answer. Attempts using simultaneous equations, parameters or sometimes a hybrid of the two were also widely seen but were more prone to confusion and algebraic errors. The technique of finding two points on the line and subtracting for the direction was less common. The small number of those who pursued this approach by substituting the parametric form of Π_2 into Π_1 to obtain an equation in λ and μ were very prone to errors. Incorrect bracketing or the absence of the "= 0" in the final equation cost a small number of students the final mark.

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