Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE
In Further Pure Mathematics FP2 (6668/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websitesat www.edexcel.comor www.btec.co.uk.Alternatively, you can get in touch with us using the details on our contact us page atwww.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2018
Publications Code 6668_01_1806_MS
All the material in this publication is copyright
© Pearson Education Ltd 2018

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$,leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$,leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$,leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

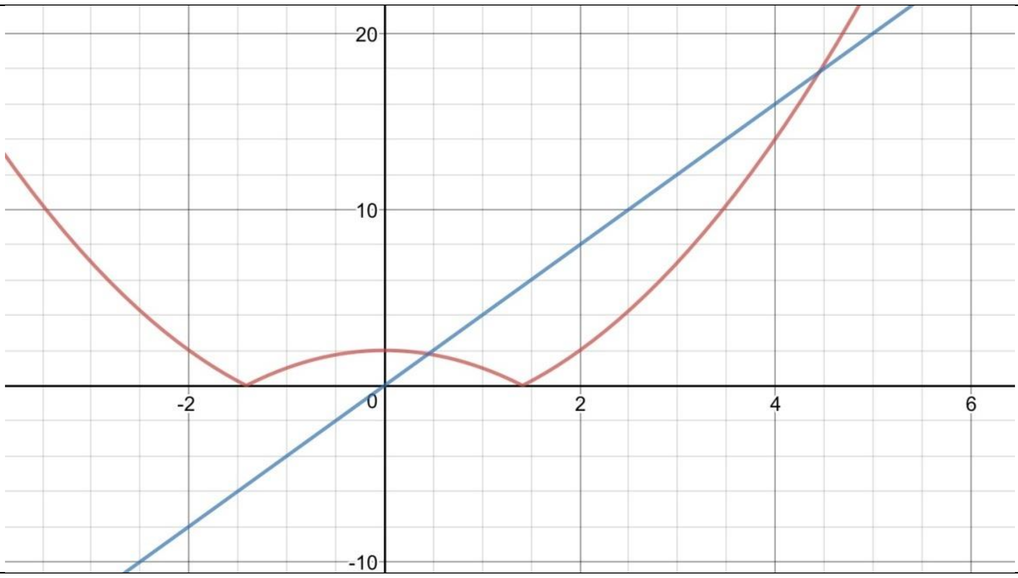
Question Number	Scheme	Notes	Marks
	Mark (a) and (b) together – ignore labels		
1(a)	$\frac{1}{(r+3)(r+4)} \equiv \frac{1}{(r+3)} - \frac{1}{(r+4)}$	Cao No working needed – ignore any shown	B1
			(1)
(b)	$r=1$: $\frac{1}{4}-\frac{1}{5}$		
	$r=2$: $\frac{1}{5}-\frac{1}{6}$		
	$...r=n-1$: $\frac{1}{(n+2)}-\frac{1}{(n+3)}$		
	$r=n$: $\frac{1}{(n+3)}-\frac{1}{(n+4)}$	First 2 and last term or first and last 2 terms required. Must start at $r=1$ (First term complete, 2nd and last may be partial or last term complete 1st and penultimate partial.)	M1
	$\sum_{r=1}^{n}\frac{1}{(r+2)(r+3)} = \frac{1}{4}-\frac{1}{(n+4)}$	Cancel terms.	M1A1
	$\sum_{r=1}^{n}\frac{1}{(r+2)(r+3)} = \frac{n}{4(n+4)}$	Find common denominator, dep on second M mark Cso (All M marks required)	dM1 A1cso
	$(a=4)$	Need not be shown explicitly	(5)
NB: 1	All marks can be awarded if work done with values 1,2,...*r* and then *r* replaced with *n*; if no replacement made, deduct final A mark.		
2	$\frac{1}{4}-\frac{1}{(n+4)}$ with **NO** other working gets M0M1A1M1A0 max		
(c)	$\sum_{r=15}^{30}\frac{1}{(r+3)(r+4)} = \frac{30}{4(30+4)} - \frac{14}{4(14+4)}$	Accept $n=30$ and $n=14$ only in their answer to (b) Must be subtracted	M1
	$=\frac{4}{153}$ oe (exact)	Exact answer $\frac{4}{153}$ implies method provided no incorrect work seen in (c).	A1
			(2)
ALT	Use the method of differences again, starting at $r=15$ and ending at $r=30$	Complete method	M1
	$=\frac{4}{153}$ oe (exact)	Correct answer	A1
			Total 8

Question Number	Scheme	Notes	Marks
2	$z=x+\mathrm{i}y$ and $w=u+\mathrm{i}v$ used. Candidates may use any suitable letters.		
	$z=x\Rightarrow w=\dfrac{1-\mathrm{i}x}{x}$	Replaces at least one z with x ie indicate that $y=0$ (may be done later)	M1
	$w=\dfrac{1}{x}-\mathrm{i}$ or $w=\dfrac{1-\mathrm{i}x}{x}$ oe	Reach this statement somewhere	A1
	$u+\mathrm{i}v=\dfrac{1}{x}-\mathrm{i}$	$w=u+\mathrm{i}v$ and equating real or imaginary parts to obtain either u or v in terms of x or just a (real) number	M1
	$v=-1$ oe $\left(u=\dfrac{1}{x}\right.$ need not be shown$\left.\right)$	$v=-1$ or $v+1=0$ oe ie equation of the line	A1
NB	If $x+\mathrm{i}y$ has been used for z and then also for w allow M1A1M1A0 max.		
			(4)
ALT 1	$z=\dfrac{1}{w+\mathrm{i}}=\dfrac{1}{u+\mathrm{i}v+\mathrm{i}}=\dfrac{u-\mathrm{i}(v+1)}{u^2+(v+1)^2}$	Multiplies numerator and denominator by complex conjugate.	M1
		$\dfrac{u-\mathrm{i}(v+1)}{u^2+(v+1)^2}$	A1
	$(y=0\Rightarrow)\dfrac{(v+1)}{u^2+(v+1)^2}=0\Rightarrow v+1=0$	Uses $y=0$ and equates real or imaginary parts to obtain either u or v in terms of x or just a number	M1
		$v=-1$ or $v+1=0$ oe	A1
NB 1	If $x+\mathrm{i}y$ has been used for z and then also for w allow M1A1M1A0 max.		
2.	M1A0M1A1 is possible		(4)
ALT 2	$\lvert z+\mathrm{i}\rvert=\lvert z-\mathrm{i}\rvert$		
	$\left\lvert\dfrac{1}{w+\mathrm{i}}+\mathrm{i}\right\rvert=\left\lvert\dfrac{1}{w+\mathrm{i}}-\mathrm{i}\right\rvert$	M1: Use of real line and attempt to substitute A1: Correct substitution	M1 A1
	$\left\lvert\dfrac{1+w\mathrm{i}-1}{w+\mathrm{i}}\right\rvert=\left\lvert\dfrac{1-w\mathrm{i}+1}{w+\mathrm{i}}\right\rvert$		
	$\lvert w\mathrm{i}\rvert=\lvert 2-w\mathrm{i}\rvert$	Common denominator and equate numerators	M1
	$\lvert w\rvert=\lvert w+2\mathrm{i}\rvert$	Equation of the line – any form accepted	A1
			(4)
ALT 3	$z=\dfrac{1}{w+\mathrm{i}}$		
	z lies on real axis $\Rightarrow\dfrac{1}{w+\mathrm{i}}$ is real	Re-arrange equation and state that $\dfrac{1}{w+\mathrm{i}}$ is real	M1
	$\Rightarrow w+\mathrm{i}$ is real	Deduce that $w+\mathrm{i}$ is real	A1
	$w=u+iv,\quad u+\mathrm{i}(v+1)$ is real	Replace w with $u+\mathrm{i}v$ (any letters inc $x+\mathrm{i}y$ allowed here)	M1

	$v+1=0$	Deduce equation of the line	A1
			(4)
ALT 4	Choose any 2 points on the real axis in the *z*-plane:		
	$z=a: w_a = \dfrac{1-\mathrm{i}a}{a}$	Any one point	M1
	$z=b: w_a = \dfrac{1-\mathrm{i}b}{b}$	Any two points	A1
	$w_a = \dfrac{1}{a}-\mathrm{i} \quad w_b = \dfrac{1}{b}-\mathrm{i}$	Simplify both	M1
	$v=-1$ oe	Any letter (inc *y*) allowed here	A1
NB	The work can be done using arguments to find the equation. If seen, send to review.		
			Total 4

Question Number	Scheme	Notes	Marks
3(a)	$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) \quad =\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$ $\sin\frac{\pi}{12}=\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}-\frac{1}{2}\frac{\sqrt{2}}{2}$	Correct expansion for sine, including surd values for all 4 trig functions. $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ accepted	M1
(i)	$\sin\frac{\pi}{12}=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ **	Completion to **given answer**: No errors seen, cso	A1cso
	$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$ $\cos\frac{\pi}{12}=\frac{1}{2}\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$	Correct expansion for cosine, including surd values for all 4 trig functions. $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ accepted OR other complete method eg using $\sin^2\theta+\cos^2\theta=1$	M1 NB A1 on e-PEN
(ii)	$\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$ **	Completion to **given answer**: No errors seen, cso	A1cso
			(4)
(b)	**Allow all marks using EXACT calculator values for the trig functions. Decimal answers qualify for M marks only.**		
	$z^4=4\left(\cos\left(2k\pi+\frac{\pi}{3}\right)+\mathrm{i}\sin\left(2k\pi+\frac{\pi}{3}\right)\right)$ OR $z^4=4\mathrm{e}^{\mathrm{i}\left(2k\pi+\frac{\pi}{3}\right)}$	Use a valid method to generate at least 2 roots (eg use of $2k\pi$ or rotate through $\frac{\pi}{2}$, multiply by I, symmetry)	M1
	$z=4^{\frac{1}{4}}\left(\cos\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)+\mathrm{i}\sin\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)\right)$ OR $z=4^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{k\pi}{2}+\frac{\pi}{12}\right)}$	Application of de Moivre's theorem resulting in at least 1 root being found. ($4\to\sqrt{2}$ and arg divided by 4) $4^{\frac{1}{4}}$ or $\sqrt{2}$ accepted	M1
	$(k=0\to)$ $z=\sqrt{2}\left(\cos\frac{\pi}{12}+\mathrm{i}\sin\frac{\pi}{12}\right)$ or $\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{12}\right)}$ or $\frac{\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)$ or $\frac{1+\sqrt{3}}{2}+\mathrm{i}\frac{-1+\sqrt{3}}{2}$ oe	Any correct root (this is the most likely one if only one found) Can be in any exact form ($4^{\frac{1}{4}}$ or $\sqrt{2}$ oe) Can be unsimplified using results from (a) ie $\frac{\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)$ with $\frac{\sqrt{2}}{4}$ or $\frac{1}{2\sqrt{2}}$ or $4^{-\frac{3}{4}}$ oe Or simplified/calculator values ie $\frac{1+\sqrt{3}}{2}+\mathrm{i}\frac{-1+\sqrt{3}}{2}$	B1

	$\left\{(k=1\rightarrow)z=\sqrt{2}\left(\cos\frac{7\pi}{12}+\mathrm{i}\sin\frac{7\pi}{12}\right)\text{ or }\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{7\pi}{12}\right)}\right\}$ $=\frac{-\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)+\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)\text{ or }\frac{1}{2}\left(1-\sqrt{3}\right)+\frac{\mathrm{i}}{2}\left(1+\sqrt{3}\right)$ $\left\{(k=2\rightarrow)z=\sqrt{2}\left(\cos\frac{13\pi}{12}+\mathrm{i}\sin\frac{13\pi}{12}\right)\text{ or }\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{13\pi}{12}\right)}\right\}$ $=\frac{-\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)-\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)\text{ or }-\frac{1}{2}\left(1+\sqrt{3}\right)+\frac{\mathrm{i}}{2}\left(1-\sqrt{3}\right)$ $\left\{(k=3\rightarrow)z=\sqrt{2}\left(\cos\frac{19\pi}{12}+\mathrm{i}\sin\frac{19\pi}{12}\right)\text{ or }\sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{19\pi}{12}\right)}\right\}$ $=\frac{\sqrt{2}}{4}\left(\sqrt{6}-\sqrt{2}\right)-\frac{\mathrm{i}\sqrt{2}}{4}\left(\sqrt{6}+\sqrt{2}\right)\text{ or }\frac{1}{2}\left(-1+\sqrt{3}\right)-\frac{\mathrm{i}}{2}\left(1+\sqrt{3}\right)$	
	Two correct roots in form *a* + i*b* unsimplified or calculator values, must be exact surd form	A1
	All 4 correct roots in form *a* + i*b* unsimplified or calculator values must be exact surd form.	A1
		(5)
		Total 9

Question Number	Scheme	Notes	Marks
4.	$\left\|x^2 - 2\right\| > 4x$		
			
	Note: Candidates may include a sketch such as the one shown at some point in their working. Please be aware that this sketch without algebra used to find the critical values merits **0 marks**. Marks may be only be awarded for the algebra used		
NB	**First 4 marks are available with =, > or < used**		
	$x^2 - 2 = 4x$ ①	Form 3 TQ **and** attempt to solve - may be implied by correct value(s) (allow decimals 4.449 ,-0.449..)	M1
	$x = 2 \pm \sqrt{6}$ or $2 + \sqrt{6}$	Correct exact values or value (NB: Corresponding 3TQ must have been seen)	A1
	$x^2 - 2 = -4x$ ②	Form 3 TQ **and** attempt to solve - may be implied by correct value(s) (allow decimals -4.449 ,0.449..))	M1
	$x = -2 \pm \sqrt{6}$ or $x = -2 + \sqrt{6}$	Correct exact values or value (NB: Corresponding 3TQ must have been seen)	A1
	$x >$ larger root of ① or $x <$ larger root of ②	Forms at least one of the required inequalities using their exact values Must be a strict inequality Depends on **either** previous M mark	dM1
	One of $x < -2 + \sqrt{6}$ or $x > 2 + \sqrt{6}$	Or exact equivalent	A1
	Both of $x < -2 + \sqrt{6}$ or $x > 2 + \sqrt{6}$	No others seen. Exact equivalents allowed Allow "or" or "and" but not $\cap$ if set notation used	A1

ALT	$(x^2-2)^2=16x^2$	Square both sides **and** attempt to solve quadratic in x^2 may be implied by correct value(s) (allow decimals 19.79... -0.202..)	M1
	$x^2=10\pm\sqrt{96}$	$x^2=10\pm4\sqrt{6}$ oe	A1
	$x=2\pm\sqrt{6}$ and $x=-2\pm\sqrt{6}$ $\left(x=2+\sqrt{6} \text{ and } x=-2+\sqrt{6} \text{ sufficient}\right)$	Valid attempt required to find exact form for x e.g. $(a+\sqrt{b})^2=10\pm\sqrt{96}$	M1A1
	$x>$ largest root or $x<$ 2nd largest root	As main scheme	dM1
	As main scheme	As main scheme	A1,A1
			Total 7

Question Number	Scheme	Notes	Marks
5.	$y\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}-3y^2=0$		
(a)	$y\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}$	M1: Use of Product Rule on $y\dfrac{d^2y}{dx^2}$, 2 terms added with at least one term correct. A1: Fully correct derivative of $y\dfrac{d^2y}{dx^2}$	M1,A1
	$+3x\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}$	Correct derivative of $3x\dfrac{dy}{dx}$	B1
	$-6y\dfrac{dy}{dx}$	oe.	B1
	At $x=0$, $2\dfrac{d^2y}{dx^2}+3(0)(1)-3(4)=0\Rightarrow\dfrac{d^2y}{dx^2}=...$ **and** $2\dfrac{d^3y}{dx^3}+(1)(6)+3(1)-6(2)(1)=0\Rightarrow\dfrac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\dfrac{dy}{dx}=1$ (must use these values) leading to numerical values for $\dfrac{d^2y}{dx^2}$ **and** $\dfrac{d^3y}{dx^3}$	M1
	$\dfrac{d^3y}{dx^3}=\dfrac{3}{2}$**	**Given answer** cso	A1cso(6)
ALT 1	**Divide by *y* before differentiating:**		
(a)	$\dfrac{d^2y}{dx^2}+\dfrac{3x}{y}.\dfrac{dy}{dx}-3y=0$		
	$\left(\dfrac{3y-3x\dfrac{dy}{dx}}{y^2}\right)\dfrac{dy}{dx}+\dfrac{3x}{y}\times\dfrac{d^2y}{dx^2}$ oe	M1 Use of Product Rule on $\dfrac{3x}{y}\times\dfrac{dy}{dx}$, 2 terms added with at least one term correct A1 Correct derivative	M1A1
	$\dfrac{d^3y}{dx^3}$	oe	B1
	$-3\dfrac{dy}{dx}$	oe	B1
	$\dfrac{d^3y}{dx^3}+\left(\dfrac{3\times2-0}{2^2}\right)\times1+0\times\dfrac{d^2y}{dx^2}-3\times1\rightarrow\dfrac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\dfrac{dy}{dx}=1$ (must use these values) leading to numerical value for $\dfrac{d^3y}{dx^3}$ (value for $\dfrac{d^2y}{dx^2}$ not needed)	M1
	$\dfrac{d^3y}{dx^3}=\dfrac{3}{2}$**	**Given answer** cso	A1cso

ALT 2	**Re-arrange and divide by *y* before differentiating:**		
	$\frac{d^2y}{dx^2}=\frac{1}{y}\left(3y^2-3\frac{dy}{dx}x\right)$		
	$\frac{d^3y}{dx^3},=+\frac{1}{y}\left(6y\frac{dy}{dx}-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}\right),$ $-\frac{1}{y^2}\frac{dy}{dx}\left(3y^2-3x\frac{dy}{dx}\right)$	B1 $\frac{d^3y}{dx^3}$, M1 Differentiate using product rule. 2 terms added with at least one term correct A1: $+\frac{1}{y}\left(6y\frac{dy}{dx}-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}\right)$ B1 $-\frac{1}{y^2}\frac{dy}{dx}\left(3y^2-3x\frac{dy}{dx}\right)$	B1, M1A1, B1
	$\frac{d^3y}{dx^3}=\frac{1}{2}(6\times2\times1-3\times1-3\times0\times6)$ $-\frac{1}{4}\times1(3\times4-3\times0\times1)\Rightarrow\frac{d^3y}{dx^3}=...$	Sub $x=0, y=2$ and $\frac{dy}{dx}=1$ (must use these values) leading to numerical value for $\frac{d^3y}{dx^3}$ (value for $\frac{d^2y}{dx^2}$ not needed)	M1
	$\frac{d^3y}{dx^3}=\frac{3}{2}$**	**Given answer** cso	A1cso
(b)	$(y=)\ 2+x$	Use the given values to form the first 2 terms of the series	B1
	$(y=)\ 2+x+\frac{6}{2!}x^2+\frac{\frac{3}{2}}{3!}x^3\ (+...)$	Find a numerical value for $\frac{d^2y}{dx^2}$ (may be seen in (a)) and use with the **given** value of $\frac{d^3y}{dx^3}$ to form the x^2 and x^3 terms of the series expansion	M1
	$y=\ 2+x+3x^2+\frac{1}{4}x^3\ (+...)$	Follow through their value of $\frac{d^2y}{dx^2}$ used correctly. Must start *y* = ... Allow f(*x*) **only** if this has been defined anywhere in the question to be equal to *y*	A1ft
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
6.(a)	$6\frac{d^2y}{dx^2}+5\frac{dy}{dx}-6y=x-6x^2$		
	$6m^2+5m-6=0 \Rightarrow (3m-2)(2m+3)=0$ $m=\frac{2}{3},\frac{-3}{2}$	M1 Forms and solves auxiliary equation A1 Correct roots	M1A1
	Complementary Function $Ae^{\frac{2}{3}x}+Be^{-\frac{3}{2}x}$	CF of the form shown formed using their 2 real roots Can be awarded if seen in gen solution	B1ft NB A1 on e-PEN
	Particular Integral $(y=)\ Cx^2+Dx+E$	May include higher powers	B1
	$\frac{dy}{dx}=2Cx+D, \frac{d^2y}{dx^2}=2C$	Differentiates their PI twice All powers of x to decrease by 1	M1
	$6(2C)+5(2Cx+D)-6(Cx^2+Dx+E)\equiv -6x^2+x$		
	$-6C=-6$ $10C-6D=1$ $12C+5D-6E=0$	Substitutes their derivatives into the equation and equates at least one pair of coefficients	M1
	$C=1$ $10-6D=1 \Rightarrow D=\frac{3}{2}$ $12+5\left(\frac{3}{2}\right)-6E=0 \Rightarrow E=\frac{13}{4}$	Attempt to solve 3 equations. Must reach a numerical value for all 3 coefficients	M1
	General Solution $y=Ae^{\frac{2}{3}x}+Be^{-\frac{3}{2}x}+x^2+\frac{3}{2}x+\frac{13}{4}$	Must start $y=$ ... cao	A1
			(8)
(b)	$\frac{dy}{dx}=\frac{2}{3}Ae^{\frac{2}{3}x}-\frac{3}{2}Be^{-\frac{3}{2}x}+2x+\frac{3}{2}$	Differentiates their GS – min 4 terms in their GS	M1
	$y=0,\ \frac{dy}{dx}=\frac{3}{2}, x=0 \qquad 0=A+B+\frac{13}{4}$ $\frac{3}{2}=\frac{2}{3}A-\frac{3}{2}B+\frac{3}{2}$	Forms 2 simultaneous equations using given boundary values	M1
	$4A+4B=-13, 4A-9B=0$,	Attempt to solve Must reach $A=$ ... or $B=$ ...	M1
	$A=-\frac{9}{4}, B=-1$	Both correct	A1
	$y=x^2+\frac{3}{2}x+\frac{13}{4}-\frac{9}{4}e^{\frac{2}{3}x}-e^{-\frac{3}{2}x}$	Must start $y=$ ...	A1 (5)
			Total 13

Question Number	Scheme	Notes	Marks
7. (a)	$r = 2 + \sqrt{3}\cos\theta$		
Way 1	$y = r\sin\theta = 2\sin\theta + \sqrt{3}\cos\theta\sin\theta$	Multiplies r by $\sin\theta$	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = 2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\sin^2\theta$	M1 Differentiates using product rule A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\left(1-\cos^2\theta\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = \ldots$	M1
	$\cos\theta = \frac{-2 \pm \sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2 + \sqrt{3}\cos\theta$	A1cso
			(6)
Way 2	$y = r\sin\theta = (2 + \sqrt{3}\cos\theta)\sin\theta$	Leaves y as a product	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = (2 + \sqrt{3}\cos\theta)\cos\theta - \sqrt{3}\sin\theta\sin\theta$	M1 Differentiates using product rule A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\cos^2\theta - \sqrt{3}\left(1-\cos^2\theta\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = \ldots$	M1
	$\cos\theta = \frac{-2 \pm \sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$	Must show substitution of correct, exact $\cos\theta$ in $r = 2 + \sqrt{3}\cos\theta$	A1cso
			(6)
Way 3	$y = r\sin\theta = 2\sin\theta + \frac{\sqrt{3}}{2}\sin 2\theta$	Uses a double angle formula	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = 2\cos\theta + \sqrt{3}\cos 2\theta$	M1 Differentiates A1 Correct derivative	M1A1
	$2\cos\theta + \sqrt{3}\left(2\cos^2\theta - 1\right) = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use a double angle identity to form a 3TQ in $\cos\theta$. $\cos 2\theta = \left(2\cos^2\theta - 1\right)$ Attempt to solve their 3TQ. Reach $\cos\theta = \ldots$	M1
	$\cos\theta = \frac{-2 \pm \sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1

	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso(6)
Way 4	$y = r\sin\theta$		
	$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -\sqrt{3}\sin\theta$	Correct derivative	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = \frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta$	Differentiate using product rule	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right) = -\sqrt{3}\sin^2\theta + \left(2+\sqrt{3}\cos\theta\right)\cos\theta$	Correct derivative as a function of θ	A1
	$-\sqrt{3}\left(1-\cos^2\theta\right) + 2\cos\theta + \sqrt{3}\cos^2\theta = 0$ $2\sqrt{3}\cos^2\theta + 2\cos\theta - \sqrt{3} = 0$	Use $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\cos\theta$ and attempt to solve. Reach $\cos\theta = ...$	M1
	$\cos\theta = \frac{-2\pm\sqrt{28}}{4\sqrt{3}}$ or $\frac{\sqrt{21}-\sqrt{3}}{6}$ oe	Accept $\pm$ or $+$ Any exact equivalent – need not be simplified.	A1
	$OP = r = 2 + \frac{-2+\sqrt{28}}{4} = \frac{1}{2}\left(3+\sqrt{7}\right)$ **	Must show substitution of correct, exact $\cos\theta$ in $r = 2+\sqrt{3}\cos\theta$	A1cso
			(6)
Special Case	$y = r\cos\theta$	**NOT** $x = r\cos\theta$	
	$r\cos\theta = 2\cos\theta + \sqrt{3}\cos^2\theta$		B0
	$\left(\frac{\mathrm{d}y}{\mathrm{d}\theta} =\right) -2\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$	Differentiates Cannot obtain correct derivative	M1 A0
	No further marks available		

(b)	$\left(2+\sqrt{3}\cos\theta\right)^2 = 4+4\sqrt{3}\cos\theta+3\cos^2\theta$	Attempt to find r^2 as a 3 term quadratic and use a double angle formula $\cos^2\theta = \pm\frac{1}{2}(\cos 2\theta \pm 1)$	M1
	$= 4+4\sqrt{3}\cos\theta+\frac{3}{2}(\cos 2\theta+1)$	Correct result	A1
	$\int r^2 \mathrm{d}\theta = 4\theta+4\sqrt{3}\sin\theta+3\left(\frac{1}{4}\sin 2\theta+\frac{1}{2}\theta\right)$ oe	dM1 Attempts to integrate their r^2 Depends on first M of (b) $\cos\theta \to \pm\sin\theta$ $\cos 2\theta \to \pm k\sin 2\theta$ $k=1$ or $\frac{1}{2}$ A1 Correct integral	dM1A1
	Check the integration carefully as the sine terms become 0 when limits substituted.		
	$\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta \quad = \frac{1}{2}(8\pi+3\pi-0)$	Substitutes correct limits in $\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta$ or $\left(2\times\frac{1}{2}\right)\int_0^{\pi} r^2 \mathrm{d}\theta$ or $\frac{1}{2}\int_{-\pi}^{\pi} r^2 \mathrm{d}\theta$	ddM1
	$= \frac{11\pi}{2}$	Correct answer must be exact Accept 5.5π No errors in the working	A1cso
			(6)
			Total 12
NB:	$\frac{1}{2}\int_0^{2\pi} r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{2\pi}\left(2+\sqrt{3}\cos\theta\right)^2 \mathrm{d}\theta = \frac{11}{2}\pi$	Integral evaluated on a calculator. Correct answer – send to review. Incorrect answer – 0/6	

Question Number	Scheme	Notes	Marks
8.	$\int 2x^5 e^{-x^2} dx$		
(a)	$t = x^2 \Rightarrow dt = 2x dx$ or $dx = \frac{1}{2} t^{-\frac{1}{2}} dt$ oe	May be implied by subsequent work	M1
	$\int 2x^5 e^{-x^2} dx = \int t^2 e^{-t} dt$	Integral in terms of t only required. dt may be implied Must have attempted to change dx to dt (ie not just used dx = dt)	M1
	$= -t^2 e^{-t} + 2\int te^{-t} dt$	Use of integration by parts Reduce the power of t. Sign errors are allowed. $\int kt^p e^{-t} \to \pm kt^p e^{-t} \pm A\int t^{p-1} e^{-t} dt$	M1
	$= -t^2 e^{-t} - 2te^{-t} + 2\int e^{-t} dt$	Use of integration by parts again in the same direction	dM1
	$= -t^2 e^{-t} - 2te^{-t} - 2e^{-t} (+C)$ oe	Correct integration, constant not needed	A1
	$= -x^4 e^{-x^2} - 2x^2 e^{-x^2} - 2e^{-x^2} (+C)$ oe	Reverse substitution, constant not needed. **This mark cannot be recovered in (b)**	A1
			(6)
ALTs	Attempts without substitution which may merit part marks – send to review.		
	$x\frac{dy}{dx} + 4y = 2x^2 e^{-x^2}$		
(b)	Integrating Factor $e^{\int \frac{4}{x} dx} = x^4$	Use of x^4 seen	B1
	$\frac{d}{dx}(x^4 y) = 2x^5 e^{-x^2}$ or $x^4 y = \int 2x^5 e^{-x^2} dx$	Multiply through by their IF	M1
	$x^4 y = -x^4 e^{-x^2} - 2x^2 e^{-x^2} - 2e^{-x^2} (+C)$	Use their answer for (a), which must be a function of x, to integrate RHS	A1ft
	$y = -e^{-x^2} - \frac{2e^{-x^2}}{x^2} - \frac{2e^{-x^2}}{x^4} + \frac{C}{x^4}$	Complete to $y = ...$ Include the constant and deal with it correctly **Not follow through**	A1
			(4)

Question Number	Scheme	Notes	Marks
ALT:	Use the same substitution as in (a) Following work uses the work shown in (a) rather than just the final answer. No marks until a first order exact equation in y and t reached and an attempt is made to solve this.		
	$t = x^2 \Rightarrow \mathrm{d}t = 2x\mathrm{d}x$ or $\mathrm{d}x = \frac{1}{2}t^{-\frac{1}{2}}\mathrm{d}t,$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$ $\quad x\frac{\mathrm{d}y}{\mathrm{d}x} = 2t\frac{\mathrm{d}y}{\mathrm{d}t}$ $\quad$ Equation becomes $2t\frac{\mathrm{d}t}{\mathrm{d}x} + 4y = 2t\mathrm{e}^{-t}$		
	Integrating Factor $\mathrm{e}^{\int \frac{2}{t}\mathrm{d}x} = t^2$	Use of t^2 seen	B1
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(t^2 y\right) = t^2\mathrm{e}^{-t}$ or $t^2 y = \int t^2 \mathrm{e}^{-t}\mathrm{d}t$	Multiply through by IF	M1
	$t^2 y = -t^2\mathrm{e}^{-t} - 2t\mathrm{e}^{-t} - 2\mathrm{e}^{-t}(+C)$ oe	Use their work in (a) to integrate RHS	A1ft
	$y = -\mathrm{e}^{-x^2} - \frac{2\mathrm{e}^{-x^2}}{x^2} - \frac{2\mathrm{e}^{-x^2}}{x^4} + \frac{C}{x^4}$	Reverse the substitution Complete to $y = ...$ Include the constant and deal with it correctly **Not follow through**	A1
			(4)
(c)	$0 = -\mathrm{e}^{-1} - 2\mathrm{e}^{-1} - 2\mathrm{e}^{-1} + C$	Attempt to substitute $x = 1, y = 0$ into their y provided it includes a constant	M1
	$\Rightarrow C = 5\mathrm{e}^{-1}$ oe	NB: **Not ft** so must have been obtained using a correct expression for y	A1
	$y = -\mathrm{e}^{-x^2} - \frac{2\mathrm{e}^{-x^2}}{x^2} - \frac{2\mathrm{e}^{-x^2}}{x^4} + \frac{5\mathrm{e}^{-1}}{x^4}$	Must start $y = ...$ Follow through their C and expression for y	A1ft
			(3)
			Total 13
	Some common alternative forms for the answers: **NB:** *This list is not exhaustive.*		
(a)	1) $-x^4\mathrm{e}^{-x^2} - 2x^2\mathrm{e}^{-x^2} - 2\mathrm{e}^{-x^2}(+C)$ 2) $\mathrm{e}^{-x^2}\left(-x^4 - 2x^2 - 2\right)$ $(+C)$ 3) $-\mathrm{e}^{-x^2}\left(x^4 + 2x^2 + 2\right)$ $(+C)$ 4) $\frac{-\left(x^4 + 2x^2 + 2\right)}{\mathrm{e}^{x^2}}$ $(+C)$		

Question Number	Scheme	Notes	Marks
(b)	1) $y=-e^{-x^2}-\frac{2e^{-x^2}}{x^2}-\frac{2e^{-x^2}}{x^4}+\frac{C}{x^4}$ 2) $y=e^{-x^2}\left(-1-\frac{2}{x^2}-\frac{2}{x^4}\right)+\frac{C}{x^4}$ 3) $y=-e^{-x^2}\left(1+\frac{2}{x^2}+\frac{2}{x^4}\right)+\frac{C}{x^4}$ 4) $y=\frac{-\left(x^4+2x^2+2\right)}{x^4e^{x^2}}+\frac{C}{x^4}$		
(c)	1) $y=-e^{-x^2}-\frac{2e^{-x^2}}{x^2}-\frac{2e^{-x^2}}{x^4}+\frac{5e^{-1}}{x^4}$ 2) $y=e^{-x^2}\left(-1-\frac{2}{x^2}-\frac{2}{x^4}\right)+\frac{5e^{-1}}{x^4}$ 3) $y=-e^{-x^2}\left(1+\frac{2}{x^2}+\frac{2}{x^4}\right)+\frac{5e^{-1}}{x^4}$ 4) $y=\frac{-\left(x^4+2x^2+2\right)}{x^4e^{x^2}}+\frac{5e^{-1}}{x^4}$		

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom