Principal Examiner Feedback

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Pearson Edexcel GCE Mathematics

Core Mathematics C4 (6666/01)

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Core Mathematics 4 (6666) – Principal Examiner's report

General Introduction

This paper offered good discrimination between candidates of all abilities and there were sufficient opportunities for a typical E grade candidate to gain some marks across all the questions. There were some testing questions involving parametric equations, differential equations, vectors and integration that allowed the paper to discriminate well between the higher grades.

In summary, Q1(a), Q2(a), Q3(i), Q4, Q5, Q7(a), Q7(d) and Q8(a) were a good source of marks for the average candidate, mainly testing standard ideas and techniques whereas Q1(b), Q2(b), Q3(ii), Q3(iii), Q6, Q7(b), Q7(c) and Q8(b) were discriminating at the higher grades. Q7(e) proved to be the most challenging question on the paper.

Question 1

Part (a) was accessible with most candidates scoring full marks and part (b) offered good discrimination for the more able candidates.

In part (a), most candidates manipulated $\sqrt{4-9x}$ to give $2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$, with the 2 outside the brackets sometimes written incorrectly as either $4, 1$ or $\frac{1}{2}$. Many candidates used a correct method for expanding a binomial expansion of the form $(1+kx)^n$, with some applying incorrect $k=\frac{9}{4}$ or $n=-\frac{1}{2}$. Common mistakes in this part included sign errors, bracketing errors, simplification errors and forgetting to multiply the correct $\left(1-\frac{9}{8}x-\frac{81}{128}x^2+...\right)$ by 2.

In part (b), most candidates solved the equation $\sqrt{4-9x}=\sqrt{310}$ to give $x=-34.$ Many candidates substituted $x=-34$ (or sometimes $x=34$) into their part (a) answer to give an estimate of –1384.563 for $\sqrt{310}$, even though the question clearly stated that the binomial expansion is valid for $|x|<\frac{4}{9}$. Only a minority of candidates (including some who had previously used $x=-34$), simplified $\sqrt{310}$ to give $10\sqrt{3.1}$ and deduced that they needed to substitute $x=0.1$ into their binomial expansion. Many of these candidates arrived at the correct answer 17.623, although a few gave their final answer as 1.762 by forgetting to multiply their correct $1.76234...$ by 10.

Question 2

This was a well-answered question with many candidates scoring full marks in part (a) and part (b) providing most of the discrimination.

In part (a), most candidates differentiated correctly, factorised out $\frac{dy}{dx}$ and rearranged their equation to arrive at the correct gradient function. Some candidates did not apply the product rule correctly when differentiating xy, while a few candidates left the constant term '$+1$' as part of their differentiated equation or did not differentiate the term y^2.

In part (b), most candidates set the numerator of their $\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ equal to 0. Some candidates gave up at this point while others set the denominator of their $\frac{dy}{dx}$ equal to 0 and attempted to solve their $2x + y - 4 = 0$ and their $5 - x - 2y = 0$ simultaneously. Many candidates substituted their $y = 4 - 2x$ into the curve C, but some struggled to obtain a correct $x^2 - 2x - 1 = 0$ (or equivalent), mainly due to algebraic, sign or bracketing errors. Those who progressed this far usually produced a correct method for solving their 3-term quadratic. Only a minority of candidates found the correct exact values $x = 1 \pm \sqrt{2}$, and some of them wasted their time by needlessly finding the corresponding values for y. A few candidates applied the alternative method of substituting their $x = \frac{4-y}{2}$ into the curve C, finding both values of y and using their $x = \frac{4-y}{2}$ to find both values of x.

Question 3

Part (i) was generally well answered, with most candidates scoring full marks. Part (ii) and part (iii) were discriminating with many candidates unable to score in part (ii).

In part (i), most candidates used the given $\frac{13 - 4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$ to write down the correct identity $13 - 4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$. The incorrect identity $13 - 4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ was used by a few candidates. Most candidates used either a method of substituting values into their identity or a method of comparing coefficients and were generally successful in finding the correct values for A, B and C. Most candidates used their constants in a correct method of integrating the resulting partial fractions, with many finding a correct $-\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3) + c$ which some simplified to give

$\ln\left(\dfrac{x+3}{2x+1}\right)-\dfrac{3}{2x+1}+c$. Common errors included integrating $\dfrac{-2}{(2x+1)}$ to give $-2\ln(2x+1)$ or integrating $\dfrac{6}{(2x+1)^2}$ to give $-\dfrac{6}{2x+1}$ or $-3\ln(2x+1)^2$.

In part (ii), a minority of candidates expanded $(\mathrm{e}^x+1)^3$ to give $\mathrm{e}^{3x}+3\mathrm{e}^{2x}+3\mathrm{e}^x+1$, and many of these integrated to give a correct $\dfrac{1}{3}\mathrm{e}^{3x}+\dfrac{3}{2}\mathrm{e}^{2x}+3\mathrm{e}^x+x+c$, with a few giving the incorrect $3\mathrm{e}^{3x}+6\mathrm{e}^{2x}+3\mathrm{e}^x+x+c$. Some candidates applied the substitution $u=\mathrm{e}^x$ to achieve the correct $\displaystyle\int\left(u^2+3u+3+\frac{1}{u}\right)\mathrm{d}u$, with many of them integrating to achieve a correct answer in terms of x. It was rare to see a correct solution for those candidates who used the substitution $u=\mathrm{e}^x+1$. Most of them achieved the correct $\displaystyle\int\frac{u^3}{(u-1)}\mathrm{d}u$, but only a few applied a method of long division to give $\displaystyle\int\left(u^2+u+1+\frac{1}{u-1}\right)\mathrm{d}u$. Some candidates did not attempt part (ii), while many gave incorrect answers such as $\dfrac{1}{4\mathrm{e}^x}(\mathrm{e}^x+1)^4$ or $\dfrac{1}{4}(\mathrm{e}^x+1)^4$.

In part (iii), many candidates differentiated $u^3=x$ to give $\dfrac{\mathrm{d}x}{\mathrm{d}u}=3u^2$ and applied the substitution correctly to give $\displaystyle\int\frac{3u^2}{4u^3+5u}\mathrm{d}u$. Some candidates made no further progress while many simplified the integral to give the correct $\displaystyle\int\frac{3u}{4u^2+5}\mathrm{d}u$ with some simplifying this further to give the incorrect $\displaystyle\int\left(\frac{3}{4u}+\frac{3u}{5}\right)\mathrm{d}u$. Those candidates who found a correct $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{3}x^{-\frac{2}{3}}$ were less successful in obtaining a correct integral in terms of u. Some candidates recognised that $\displaystyle\int\frac{3u}{4u^2+5}\mathrm{d}u$ could be expressed in the form $\displaystyle\int\frac{\mathrm{f}'(u)}{\mathrm{f}(u)}\mathrm{d}u$ and integrated to give $\lambda\ln(4u^2+5)+c$. While many of these candidates obtained the correct $\lambda=\dfrac{3}{8}$, common incorrect values of λ included $\dfrac{8}{3}$, $\dfrac{1}{8}$ or 3. Many candidates gave their final answer in terms of u, and only a minority applied $u=x^{\frac{1}{3}}$ to arrive at a final answer in terms of x.

Question 4

This question discriminated well between the average and more able candidates with part (b) more accessible than part (a). There were a few candidates, however, who made no creditable progress in this question.

In part (a), some candidates struggled to make any creditable progress. Some candidates wrote down $r = \frac{h}{\sqrt{3}}$ (or equivalent) with no supported working, while others used the given result to write down $\frac{1}{9}\pi h^3 = \frac{1}{3}\pi r^2 h \Rightarrow r = \frac{h}{\sqrt{3}}$. These ploys, which usually led to candidates arriving at the given answer, received no marks. Many candidates used trigonometry to write down a correct equation such as $\frac{r}{h} = \tan 30$, $\frac{h}{r} = \tan 60$, $\frac{r}{h} = \frac{50\tan 30°}{50}$ or $\frac{r}{\sin 30} = \frac{h}{\sin 60}$, and a few candidates applied Pythagoras' Theorem to give the correct equation $h^2 + r^2 = (2r)^2$. Many of these candidates substituted either $r = \frac{h}{\sqrt{3}}$, $r = \frac{\sqrt{3}}{3}h$ or $r^2 = \frac{1}{3}h^2$ into $V = \frac{1}{3}\pi r^2 h$ and arrived at the given result $V = \frac{1}{9}\pi h^3$.

In part (b), most candidates differentiated to give $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{3}\pi h^2$ and many used the chain rule to write down a correct equation for $\frac{\mathrm{d}h}{\mathrm{d}t}$. Many candidates divided 200 by their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and substituted $h = 15$ to arrive at an exact value for $\frac{\mathrm{d}h}{\mathrm{d}t}$. Common errors included: applying $200 \times \left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)$; applying $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \div 200$ and leaving their final answer as either $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{200}{75\pi}$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{8\pi}{3}$.

Question 5

This question discriminated well between candidates of all abilities with many candidates struggling to access the final mark in part (a) and the final mark in part (b).

In part (a), most candidates applied a full method of setting $y = 2$ to find t and substituting their t into $x = 1 + t - 5\sin t$ to find the exact value of k. Only a minority of candidates found and applied $t = -\frac{\pi}{2}$ to give the correct answer $k = 6 - \frac{\pi}{2}$ or $k = \frac{12 - \pi}{2}$. Many candidates used the incorrect value $t = \frac{\pi}{2}$ which led to the incorrect answer $k = \frac{\pi}{2} - 4$. Only a few of these candidates realised that $k = \frac{\pi}{2} - 4$ was negative and so some of them opted to use $t = \frac{3\pi}{2}$ (which was outside the given $-\pi \le t \le \pi$) to give the incorrect $k = 6 + \frac{3\pi}{2}$. Some candidates found both $k = 6 - \frac{\pi}{2}$ and $k = \frac{\pi}{2} - 4$ with many of them identifying the correct $k = 6 - \frac{\pi}{2}$ as their final answer. Other common mistakes included: working in degrees; deducing $k = -\frac{\pi}{2}$ after correctly finding $t = -\frac{\pi}{2}$; deducing $k = 4 - \frac{\pi}{2}$ after finding $x = \frac{\pi}{2} - 4$; and not finding k as an exact value.

In part (b), most candidates applied the complete process of using a correct method of parametric differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, using their value of t to find the gradient of the curve at A and applying a correct straight-line method for finding the equation of the tangent. Occasionally, sign errors were seen in the differentiation of both x and y and a few candidates wrote down an incorrect $\frac{\mathrm{d}x}{\mathrm{d}t} = -5\cos t$. Only a minority of candidates used $t = -\frac{\pi}{2}$ to find a correct $y = -4x + 26 - 2\pi$. Many candidates arrived at the incorrect tangent equation $y = 4x + 18 - 2\pi$ by using their $t = \frac{\pi}{2}$ from part (a). It was rare to see these candidates enquire why they were applying a tangent gradient of 4 when it was clear from the diagram that the gradient of the tangent to C at the point A is negative. Other common mistakes in part (b) included: bracketing and sign errors when finding the equation of their tangent; and working in degrees.

Question 6

This question discriminated well between candidates of all abilities.

There were a few candidates who made no creditable progress in this question. Some of these candidates substituted $x = -\frac{\pi}{8}$ and $y = 2$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x}$ and attempted to find the equation of the tangent to the curve at the point $\left(-\frac{\pi}{8},\, 2\right)$.

Many candidates separated the variables correctly and integrated $\frac{1}{y^2}$ correctly. Some candidates did not give a correct method for integrating $\frac{1}{3\cos^2 2x}$. Common errors included simplifying $\frac{1}{3\cos^2 2x}$ to give $3\sec^2 2x$, $\frac{1}{3}\cos^2\left(\frac{1}{2}x\right)$ or $\frac{1}{3}\cos^2 2x$. and applying $\int \frac{1}{3\cos^2 2x}\mathrm{d}x$ to give $\int \frac{2}{3(1+\cos 4x)}\,\mathrm{d}x = \frac{1}{6}\ln(1+\cos 4x)$.

Many candidates applied the boundary conditions $y = 2$ and $x = -\frac{\pi}{8}$ to their integrated equation which contained a constant of integration.

Only a minority of candidates rearranged a correct $-\frac{1}{y} = -\frac{1}{3} + \frac{1}{6}\tan 2x$ to give a correct answer in the form $y = \mathrm{f}(x)$. Many candidates, applied the misconception $\frac{1}{A} = B + C \;\Rightarrow\; A = \frac{1}{B} + \frac{1}{C}$ to give the incorrect answer $y = 3 - 6\cot 2x$.

Question 7

This question discriminated well between candidates of all abilities with many candidates scoring full marks in part (a), part (b) and part (d). Part (c) and part (e) provided good discrimination for the more able candidates.

In part (a), many candidates found a correct $B(1, 1, 4)$, while a few made an arithmetic error when adding $\overrightarrow{OA}$ to $\overrightarrow{AB}$ to give the incorrect $B(1, 1, 0)$. Common errors included: applying $\overrightarrow{OP} - \overrightarrow{AB}$ to give $(5, 7, 6)$ and finding the difference between $\overrightarrow{OA}$ and $\overrightarrow{AB}$.

In part (b), most candidates applied the scalar product formula between $\overrightarrow{AB}$ and $\overrightarrow{AP}$ (or between $\overrightarrow{BA}$ and $\overrightarrow{PA}$) with many arriving at a correct $\cos(P\hat{A}B) = \frac{4}{21}\sqrt{21}$ or $\cos(P\hat{A}B) = \frac{4}{\sqrt{21}}$. The most common mistake was to apply the scalar product formula between $\overrightarrow{AB}$ and $\overrightarrow{PA}$ (or between $\overrightarrow{BA}$ and $\overrightarrow{AP}$) which resulted in giving $\cos(P\hat{A}B) = -\frac{4}{21}\sqrt{21},$ although the negative sign was dropped by some of these candidates. A minority of candidates applied the scalar product formula between non-relevant vectors such as $\overrightarrow{OP}$ and $\overrightarrow{OB}$ or $\overrightarrow{PA}$ and $\overrightarrow{OB}$. A few candidates arrived at the correct answer by applying a cosine rule method or by applying a vector product method.

Part (c) proved challenging with many candidates unable to find the exact value for $\sin(P\hat{A}B)$. Most of these candidates found an approximation for the angle PAB and applied the formula '$\frac{1}{2}ab\sin C$' to give $\frac{1}{2}\sqrt{216}\sqrt{56}\sin(29.205...) = 26.8$ (3 sf A minority of candidates used $\sqrt{1-\cos^2(P\hat{A}B)}$ or used a right-angled triangle method to find the exact value of $\sin(P\hat{A}B)$. Those candidates who wrote the area expression $\frac{1}{2}\sqrt{216}\sqrt{56}\left(\sin\left(\cos^{-1}\left(\frac{4}{21}\sqrt{21}\right)\right)\right)$ only gained the method mark for applying the correct area formula. Only a few of these candidates received full credit by simplifying this expression to give the correct $12\sqrt{5}$. Other common errors in this part included applying $\frac{1}{2}\sqrt{216}\sqrt{56}\sin\left(\frac{4}{21}\sqrt{21}\right)$ or $\frac{1}{2}\sqrt{216}\sqrt{56}\left(1-\left(\frac{4}{21}\sqrt{21}\right)^2\right)$.

In part (d), most candidates wrote down a correct equation for the line l_2, while some candidates lost the final mark for writing an expression rather than an equation for l_2.

In part (e), some candidates applied the complete process of expressing $\overrightarrow{BQ}$ in terms of μ, forming and solving the equation $\overrightarrow{BQ} \bullet \overrightarrow{AP} = 0$ to find μ and substituting their

μ into the line l_2 to find the coordinates of Q. Most candidates who applied this process usually found the correct coordinates of Q, while only a few made manipulation errors in solving their equation $\overrightarrow{BQ} \bullet \overrightarrow{AP} = 0$ to find μ or in evaluating the coordinates of Q from their correct μ. Some candidates applied $\overrightarrow{BQ} \bullet \overrightarrow{AP} = 0$ to give the equation $12x - 6y + 6z = 30$. These candidates did not receive any credit until they formed an equation in one unknown by applying, for example, $x = 9 + 4\mu$, $y = 1 - 6\mu$ and $z = 8 + 2\mu$ to their equation in x, y and z. Many candidates, however, made no progress with part (e), with many unable to start and some attempting to solve the incorrect equation $\overrightarrow{OQ} \bullet \overrightarrow{AP} = 0$. A few candidates used a method of similar triangles with some of them using ratios to correctly deduce $\left|\overrightarrow{PQ}\right| = \frac{5}{4}\left|\overrightarrow{AB}\right|$, where $\overrightarrow{OQ} = \begin{pmatrix} 9 + 4\mu \\ 1 - 6\mu \\ 8 + 2\mu \end{pmatrix}$. Only a few of these candidates applied $\mu = -\frac{5}{4}$ to find the correct coordinates of Q.

Question 8

This was a well-answered question with part (b) providing most of the discrimination.

In part (a), most candidates applied the method of integration by parts in the correct direction and many produced a fully correct solution. Some candidates integrated $\cos 4x$ incorrectly to give either $-\frac{1}{4}\sin 4x$, $\pm\sin 4x$ or $\pm 4\sin 4x$. After correctly obtaining $\frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\mathrm{d}x$, a few candidates integrated to give the incorrect $\frac{1}{4}x\sin 4x + \frac{1}{8}\cos 4x + c$.

In part (b), most candidates applied the volume formula $\pi\int y^2\,\mathrm{d}x$ to give the correct $\pi\int_0^{\frac{\pi}{4}} x\sin^2 2x\,\mathrm{d}x$, although a few used incorrect formulae such as $2\pi\int y^2\,\mathrm{d}x$, $\int y^2\,\mathrm{d}x$ or even $\int y\,\mathrm{d}x$. Many candidates realised that a strategy of using a double-angle formula was required with some applying incorrect formulae such as $\cos 4x = 2\sin^2 2x - 1$ and $\cos 2x = 1 - 2\sin^2 2x$. Some candidates applied the formula $\cos 4x = 1 - 2\sin^2 2x$ to give the correct $\pi\int_0^{\frac{\pi}{4}} x\left(\frac{1-\cos 4x}{2}\right)\mathrm{d}x$. Most of these candidates integrated by using their answer to part (a) to give an expression of the form $\pm Ax^2 \pm Bx\sin 4x \pm C\cos 4x;\ A, B, C \neq 0$. Other candidates arrived at this expression by using an integration by parts method with $u = x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1-\cos 4x}{2}$. Most candidates applied the limits of $\frac{\pi}{4}$ and 0 correctly, with some making the mistake of assuming the limit of 0 would give 0 when substituted into their integrated result. There were many bracketing, manipulation and sign errors seen in this part with only a minority of candidates using a correct method to achieve a correct exact answer $\frac{1}{64}\pi^3 + \frac{1}{16}\pi$. A few candidates used the elaborate method of integrating by parts twice, firstly with $u = \sin^2 2x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = x$ followed by $u = x^2$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \sin 4x$. It was rare to see these candidates obtaining full marks.

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