Calculators may NOT be used in this examination.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided — there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets — use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
1. Find

\[ \int \left( 2x^5 - \frac{1}{4x^3} - 5 \right) \, dx \]

giving each term in its simplest form. (4)
Question 1 continued

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(Total 4 marks)
2. Given

\[ y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \quad x > 0 \]

find the value of \( \frac{dy}{dx} \) when \( x = 8 \), writing your answer in the form \( a\sqrt{2} \), where \( a \) is a rational number.

(5)
Question 2 continued
3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

\[ a_1 = 1 \]
\[ a_{n+1} = \frac{k(a_n + 1)}{a_n}, \quad n \geq 1 \]

where $k$ is a positive constant.

(a) Write down expressions for $a_2$ and $a_3$ in terms of $k$, giving your answers in their simplest form.

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for $k$. 
Question 3 continued

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(Total 6 marks)
4. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by $d$ each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of $d$. (2)

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1. (5)
Question 4 continued
5. \[ f(x) = x^2 - 8x + 19 \]

(a) Express \( f(x) \) in the form \((x + a)^2 + b\), where \( a \) and \( b \) are constants. 

The curve \( C \) with equation \( y = f(x) \) crosses the \( y \)-axis at the point \( P \) and has a minimum point at the point \( Q \).

(b) Sketch the graph of \( C \) showing the coordinates of point \( P \) and the coordinates of point \( Q \).

(c) Find the distance \( PQ \), writing your answer as a simplified surd.
Question 5 continued
6. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)
Question 6 continued
7. The curve $C$ has equation $y = f(x), x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on $C$,

(a) find the equation of the tangent to $C$ at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants. (4)

(b) Find $f(x)$, giving each term in its simplest form. (5)
Question 7 continued

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(Total 9 marks)
8.

The straight line $l_1$, shown in Figure 1, has equation $5y = 4x + 10$.
The point $P$ with $x$ coordinate 5 lies on $l_1$.
The straight line $l_2$ is perpendicular to $l_1$ and passes through $P$.

(a) Find an equation for $l_2$, writing your answer in the form $ax + by + c = 0$ where $a$, $b$, and $c$ are integers.

(b) Calculate the area of triangle $SPT$. 

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Figure 1

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Question 8 continued
9. (a) On separate axes sketch the graphs of

(i) \( y = -3x + c \), where \( c \) is a positive constant,

(ii) \( y = \frac{1}{x} + 5 \)

On each sketch show the coordinates of any point at which the graph crosses the \( y \)-axis and the equation of any horizontal asymptote.

(b) show that \((5 - c)^2 > 12\)

(c) Hence find the range of possible values for \( c \).
Question 9 continued

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10.

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant $k$,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant $c$.

(3) Points $A$ and $B$ are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at $A$ is equal to the gradient of the curve at $B$.

Given that point $A$ has $x$ coordinate 3

(c) find the $x$ coordinate of point $B$.

(5)
Question 10 continued
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(Total 11 marks)

END