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AS
and A Level
Physics

STUDENT MATHS SUPPORT

PEARSON

Contents

Introduction

Congratulations on choosing A level physics! You will probably have heard that it is challenging or that there's 'lots of maths' in it. You might even know that the syllabus has changed this year and there is now 'even more maths' in it. Many physics students are very competent at and interested in maths but, if that's not you, don't worry: it's true that there are several number-related skills to master to be good at physics (indeed, that's one of the reasons why physics graduates are in demand for all the best jobs) but you won't need most of the topics you studied for your maths GCSE.

Physics is about real-life, practical, useful things. We've discovered that many things in nature follow the same kinds of patterns: for example, you might have learned about electricity using an analogy about water flow or about radioactivity by rolling dice.

We use maths in physics because it's good for describing patterns clearly and because it lets us put a number on things: rather than just saying something **increases** we'd like to be able to say by **how much**.

So we tend to look at equations a bit differently than your maths teachers. We see our equations as pictures: pictures of the real life situation that the maths is describing. The other thing that makes our equations different is that, unlike the maths department, we almost never mean them to be exact. Nature is complex so most situations don't quite **perfectly** fit the patterns we have discovered and in any case we can never measure things perfectly. So all our equations are approximate to some extent, or only work in certain situations.

How to use maths in physics

The better you are able to visualise the patterns and understand the limits of equations in physics, the easier you will find it. This booklet aims to give you some pointers to, and some practice, in those bits of maths that come up a lot in A level physics. It also shows you the calculation skills you need to practise so that you can find answers quickly and check they are reasonable. They might seem daunting at first but don't be put off: like anything else that is worth doing, you will only get good at them through practice.

Maths Skills for Physicists

Standard Form, Units, Prefixes and Significant Figures

Mathematical skills: C0.1, C0.2, C1.1

You will probably be familiar with these ideas from GCSE maths and science but you will meet more challenging examples at A level such as:

- conversion of complex units such as those for density, for example from $g\,cm^{-3}$ to $kg\,m^{-3}$
- using more unit prefixes e.g. nano-, giga- and micro-
- learning more about measurements and significant figures (sf).

Tip: two voltmeter readings of, say, 0.935 V and 1.025 V are correctly recorded despite different sf as the meter resolution is ±0.005 V.

Worked example
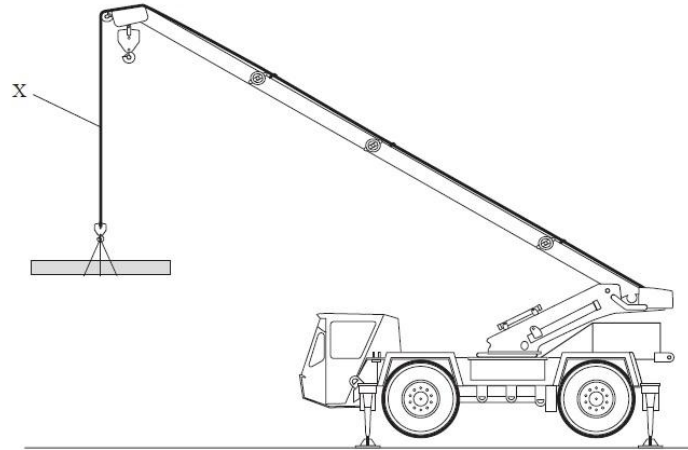
An ultrasound pulse travels through a metal bar and reflects off a defect inside the bar. The time for the pulse echo to be detected is 12 µs. The speed of ultrasound in the bar is $1.1\,km\,s^{-1}$. Calculate the distance between the defect and the bar's surface in mm.

Answer

$d = vt$

$= 1.1 \times 10^3 \times (½ \times 12 \times 10^{-6})$

$= 6.6 \times 10^{-3}\,m$

$= 6.6\,mm$

Practice question 1

The diagram shows a crane lifting a concrete beam.

Weight of beam = 13 kN diameter of steel cable = 1.1 cm

Show that the stress in the cable at point X is about 0.1 GPa.

As you will be using standard form all the time you really need to be an expert in using your calculator. On your calculator, do you know how to do the following? (Key sequences in the text refer to the Casio FX-82ES.)

- Reset it back to 'normal' mode (SHIFT 9 3 = AC).
- Make sure × and ÷ happen in the right order, perhaps using brackets to make sure. (What is the value of $3.2 \times 10^7 \div 2.5 \times 10^4 + 3.0 \times 10^3$?)
- Enter standard form accurately (using EXP or $x10^x$) including $x10^{-x}$. (Which minus button do you use?)
- Convert answers so you can use prefixes like k, M, μ (the S<->D, ENG buttons).
- Change final answer to given sf using the SCI mode or similar.
- Change to and from radian mode.
- Use a single button to do *one-over* ($\frac{1}{x}$ or x^{-1}).

Tip: standard form has a single digit before the decimal point. However, enter numbers with unit prefixes as they occur i.e. enter 650 nm as 6 5 0 EXP − 9 rather than converting to standard form first. Let the calculator do the work

Considering the units of quantities will help both in certain exam questions such as the example below but also in problem solving where the units of the answer can show you how to get started with the problem (see later).

> **Tip:** think of the **number of** electrons, photons etc. like a unit – that way electrons and anything else that is **per electron** will cancel. Never write them as if they are a unit however because they are not part of SI and therefore technically not correct as units.

Worked example

The current I in a length of aluminium of cross-sectional area A is given by the formula

$$I = nevA$$

where e is the charge on an electron.

Show that the units on the left-hand side of the equation are consistent with those on the right-hand side.

Answer

Write out the units for each quantity. In this example we're thinking of the ones as being electrons (but not writing it this way).

$$nevA = \frac{\cancel{1}}{\cancel{m^3}} \quad \frac{C}{\cancel{1}} \quad \frac{\cancel{m}}{s} \quad \cancel{m^2}$$

$$= C\,s^{-1}$$

$$= A$$

When converting unfamiliar units it might help you to think of a conversion factor having two units: for example, there are 1000 m per km so it's 1000 m km^{-1}. Then when you multiply km by km^{-1} (or if you divide m by m) they cancel.

Worked example

The specific heat capacity of aluminium is 900 J kg^{-1} K^{-1}.

Convert this to kJ mol^{-1} K^{-1} (M_r aluminium is 27 g mol^{-1}).

Answer

M_r is 27 g mol^{-1} = 27 $\cancel{g}$ mol^{-1} ÷ 1000 $\cancel{g}$ kg^{-1} = 0.027 kg mol^{-1}

For kJ the conversion is 1000 J kJ^{-1}

Specific heat capacity = 900 J kg^{-1} K^{-1}

$\qquad\qquad$ = (900 $\cancel{J}$ $\cancel{kg}^{-1}$ K^{-1} ÷ 1000 $\cancel{J}$ kJ^{-1}) × 0.027 $\cancel{kg}$ mol^{-1}

$\qquad\qquad$ = 0.0243 kJ mol^{-1} K^{-1}

Practice question 2

A submarine has a volume of 7100 m³. Calculate the upthrust force.

Density of sea water = 1.03 g cm⁻³

It is useful to be able to do a sum roughly in your head to make sure the answer is sensible and that you didn't make a calculator error. You will find this hard at first, but keep practising: your physics teacher is probably very good at it following many years of practice! Using standard form, do all the numbers roughly first, then work out the powers separately: finally change the answer to standard form. For example:

$$\frac{1.60 \times 10^{-19}}{9.11 \times 10^{-31}} \approx \frac{2}{10} \times 10^{(-19 - -31)} = 0.2 \times 10^{12} = 2 \times 10^{11}$$

Further examples are given in the section on estimation below.

Transformation of Formulae

Mathematical skills: C2.2–4

In maths this is called 'changing the subject of a formula'. You might have used the 'magic triangle' approach before, but for A level you will meet more complex formulae and you will need to learn the 'proper' maths technique now. Practice makes perfect!

> **Tip:** when a new formula is introduced, practise transposing it to make each variable the subject

One situation you might find confusing is if you have a fraction on the bottom of a fraction. For example:

$$\frac{\frac{14.6}{7.3}}{2}$$

In this situation just move the bottom of the bottom to the top – in other words:

$$\frac{2 \times 14.6}{7.3}$$

Practice question

Calculate the angular velocity of a satellite that makes 8 complete orbits in 3 days.

Estimation

Mathematical skills: C0.4, 1.4

The new exams might ask you to estimate the value of a familiar quantity such as the volume of a house. This kind of question can be daunting at first but you will find it becomes easier with practice and it can be fun to make up questions of your own.

To become good at estimating you need to be able to make **assumptions** to simplify the problem, to have a rough idea of the sizes of some everyday things (at least the nearest factor of 10) and to be able to combine them using physics ideas.

For example, to estimate the weight of the air in an average house we might use

$$\text{mass} = \text{density} \times \text{volume}$$

A useful number to know is the density of air: about 1 kg m^{-3} (it's actually 1.2 but that doesn't matter). What about the volume? If we assume the house is a cube we know it's bigger than 1 m on a side but probably not as much as 10 m on a side. Assuming it's a 5 m cube is near enough; so the volume is 125 m^3.

So the answer to the question is probably about 1300 N (did you remember we wanted weight?).

Practice question

Estimate:

- the number of photocopies made in your school in a year
- the sea level rise caused if the entire world's population went swimming
- the rate at which rubber wears off the tyres of a family car (in mm^3 s^{-1}).

Proportion, Percentage Change and Uncertainty

Mathematical skills: C0.3, 1.5, 2.1

Proportional reasoning is a skill we use as physicists all the time: when this thing doubles, this other thing doubles (or halves, or quadruples…). It saves time in recalculating if we can see the answer using a pattern. This skill appears often in A level questions but rarely at GCSE so it is worth practising.

Questions can be set using numbers: in these questions you can just recalculate with the new value, but this is much slower than using proportion.

Worked example

The current through a bulb connected to a 6 V cell is 2 A. The bulb is replaced with one of half the original resistance. Calculate the new current.

The long way (by calculation)

Resistance of original bulb, using Ohm's law, $R = \dfrac{V}{I} = \dfrac{6}{2} = 3\,\Omega$.

Therefore new bulb has resistance 1.5 Ω.

New current, using Ohm's law, $I = \dfrac{V}{R} = \dfrac{6}{1.5} = 4$ A.

Using proportion

Ohm's law tells us $I \propto \dfrac{1}{R}$ (if V constant) so if R halves then I doubles.

So the answer is 4 A.

Practice question 1

A football has a diameter of 22.5 cm. It contains air at a temperature of 20°C and a pressure of 1.65×10^5 Pa. When the football is left in direct sunlight, the temperature of the air in the football increases to 40°C.

Show that the new pressure exerted by the air in the football is about 2×10^5 Pa. Assume that the volume of the football remains constant.

Questions can also be set with letters (algebraic symbols) rather than values: in this case you can only use proportion.

Practice question 2

Two protons, separated by a distance x, experience a repulsive force F.

If the separation is reduced to $\dfrac{x}{3}$ the force between the protons will be:

       A $\dfrac{F}{9}$ **B** $\dfrac{F}{3}$ **C** $3F$ **D** $9F$

Another common question type asks you to check whether some experimental results are in direct or inverse proportion (or sometimes inverse square or exponential relationships). These can be tested by checking that the data values multiply or divide to give a constant.

If two things are proportional, $y \propto x$, then $y = constant \times x$ and so $\dfrac{y}{x}$ is constant.

y/cm	1.2	2.9	3.4	4.6
x/V	0.245	0.579	0.684	0.922
$\dfrac{y}{x}$	4.90	5.01	4.97	4.99

Calculate $\dfrac{y}{x}$ for each pair of values (as shown above).

This gives the same answer (to within the uncertainty in your experiment) so we conclude that $y \propto x$.

Practice question 3

A wire carries a constant current. A Hall probe is used to investigate how the magnetic flux density produced by the wire varies with distance from the wire.

r/cm	V/V
1.0	0.725
1.5	0.483
2.0	0.363
2.5	0.29
3.0	0.242
3.5	0.21

The potential difference V was recorded for a range of distances r.

It is suggested that V and r are related by the equation

$$V = \frac{k}{r}$$

where k is a constant.

Determine by calculation whether this suggestion is valid.

At A level you will also learn about how to combine experimental uncertainties. Imagine you have made measurements of a and b. If a were actually 1% bigger and b 2% bigger than measured, then the product increases by about 3%. We can show this by writing:

$1.01a \times 1.02b = (1 + 0.01)(1 + 0.02)ab = (1 + 0.01 + 0.02 + …)ab ≈ 1.03ab$

where the three dots (…) show we have ignored some very small values.

This is useful in questions where you are asked what happens when one quantity changes by a small amount.

Worked example

A 1 m length of wire of cross-section 1.00×10^{-2} mm² has resistance 2.00 Ω.

a Calculate the resistivity of the material.

b What is the effect on the resistance of a 2% increase in diameter?

Answer

a $\rho = \dfrac{RA}{l} = \dfrac{2 \times 1 \times 10^{-2} \times 10^{-6}}{1} = 2 \times 10^{-8}$ Ω m

b

Expert teacher solution

$R \propto \dfrac{1}{d^2}$ so increasing d by 2% makes R decrease by 4%.

So the answer is 96% of 2 Ω which is 1.92 Ω to 3 sf.

Transitioning student's solution

$R = \dfrac{\rho I}{A} = \dfrac{2 \times 10^{-8} \times 1}{A}$

$A = \pi(\dfrac{d}{2})^2$ so $d = 2\sqrt{\dfrac{A}{\pi}} = 0.113$ mm

So new $d = 0.115$ mm, $A = 1.04 \times 10^{-2}$ mm²

So $R = \dfrac{2 \times 10^{-8} \times 1}{1.04 \times 10^{-2}} = 1.92$ Ω (3 sf)

Vectors

Mathematical skills: C0.6, C4.1–5

Physics makes use of the idea of a vector to show direction. Lots of quantities in physics have an associated direction (quantities like time or mass that don't have direction are called **scalars**.) You will be used to this from studying forces at GCSE.

You will also be familiar with writing column vectors from maths but, for A level physics, the idea of writing a vector in this column notation is not so important. Instead, it is more important to be able to add two vectors (at right angles) and carry out the reverse process (to split a single vector into two **components** at right angles). You will remember that to do this you need to use Pythagoras' theorem and a bit of trigonometry. Although this is the same as the ideas you've met in maths, students often say that it doesn't look the same when you first meet it in physics.

Practice question 1

Which set of quantities is all scalar?

A acceleration, displacement, velocity

B energy, mass, power

C extension, force, gravitational potential energy

D weight, kinetic energy, work

Practice question 2
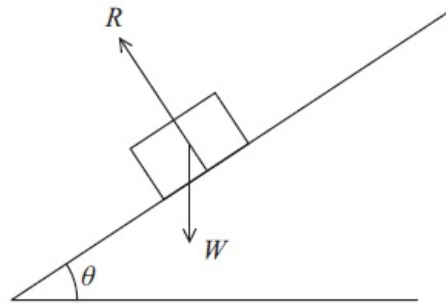
A model boat is crossing a stream. The stream is travelling east at a speed of 1.5 m s^{-1}.

The boat is heading north at a speed of 0.5 m s^{-1}.

The magnitude of the resultant velocity is:

A $(1.5 + 0.5)\text{ m s}^{-1}$

B $(1.5^2 + 0.5^2)\text{ m s}^{-1}$

C $\sqrt{1.5 + 0.5}\text{ m s}^{-1}$

D $\sqrt{(1.5^2 + 0.5)^2}\text{ m s}^{-1}$

Practice question 3

The diagram shows an object on an inclined surface.

The component of the weight *W* parallel to the surface is

A 0

B 1

C *W* cos θ

D *W* sin θ

Practice question 4

Films made to be watched in three dimensions (3D) are produced by projecting two slightly different images on to the screen, one to be seen by each eye.

In one technique the images are polarised. The viewers wear special glasses where the lenses are replaced by two separate plane polarising filters.

The light from the screen reaching each eye passes through a different filter so each eye sees a different image. The filter for one eye has a plane of polarisation of 45° and the filter for the other eye has a plane of polarisation of 135°.

Explain this choice of angles.

Rate

Mathematical skills: C2.1, C3.5–7

The idea of rate occurs in several places in physics as well as in the other subjects you are probably studying.

> **Examples:** reaction rate in chemistry, photosynthesis rate in biology, GDP growth in economics, attrition rate in a psychological study.

Rate tells us how quickly something is **changing** compared to another, which is related to the gradient of a graph (and to ideas in calculus if you study this in maths). For example, acceleration tells us how much the speed is changing in a given time; or absorption rate tells us how the number of beta particles changes as they pass through different thicknesses of absorbing material.

It is very rare to have a question specifically about rate but many questions expect you to be able to recognise it and understand that we are really talking about the gradient of a graph. This can be tricky when the word **rate** does not appear: for example, acceleration and power both have the idea of rate built in to their definition. This leads to a common calculation mistake we see in exams, confusing power in watts and energy in joules. Perhaps you could use a highlight or icon in your notes to show when a quantity is a rate?

Problem Solving

I understand the lessons: I just can't do the questions.

It's quite hard to define what we mean by problem solving. Sometimes it's hard to see how to connect what you know with the way the examiner has phrased the question. How do you get from what you know to what they want? That's a good enough definition of problem solving as far as it applies to A level physics.

There are two general tips you can use to tackle these questions.

> **Tip**: draw a detailed diagram with all the information on it.

This puts all the information into a visual form and sometimes helps you to see which part of physics the question is about. This might lead you to a plan of attack or at least to a principle you need to use, such as the principle of moments.

> **Tip**: start with what you need and work backwards.

Usually the problem will only require two steps (maybe using two formulae or a principle that isn't given to you in the question, like energy conservation). That means there's probably only 'one degree of separation' between what you know and what's required so you just need to find that missing link.

Worked example

A bulb of resistance 12 Ω is connected to a 6 V battery. Calculate the charge passing through the bulb in 1 minute.

Answer

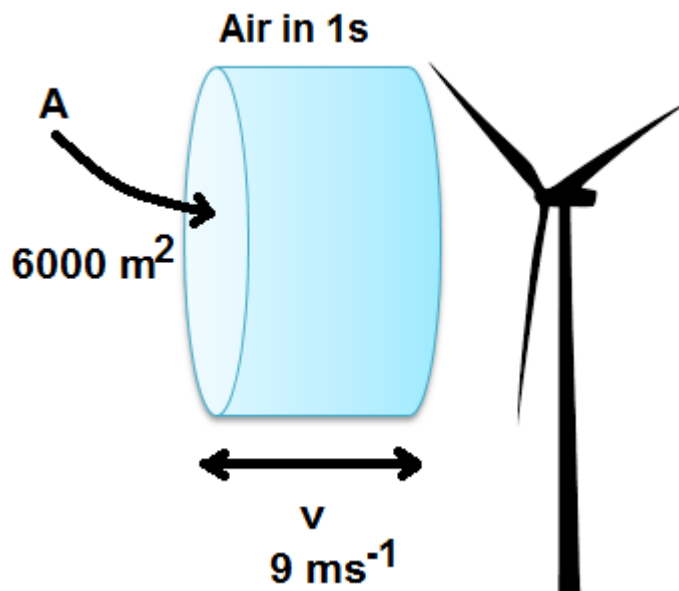The current would tell us how much charge passes in given time ($Q = It$).

We can find current from the information we have using Ohm's law ($V = IR$).

$I = \dfrac{6}{12} = 0.5\,\text{A}$

$Q = 0.5 \times 60 = 30\,\text{C}$

Worked example

Kinetic energy of wind is transferred to electrical energy by a wind turbine as the blades rotate.

The area swept out by one blade, as it turns through 360°, is 6000 m². Wind at a speed of 9 m s^{-1} passes the turbine.

Calculate the maximum power available to the wind turbine.

density of air = 1.2 kg m^{-3}

Answer

We need to find power. $P = \dfrac{E}{t}$

The energy transfer is from the wind's KE to the turbine's.

So we need the KE available from the wind in 1 s. $KE = \frac{1}{2}mv^2$

We don't know m but we know how much wind passes through the turbine in 1 s from our diagram: it is $A\rho x$. x is the distance the wind goes in 1 s so that's v.

So the power is the wind's KE available in 1 s which is $\frac{1}{2}(A\rho v)v^2$.

So the answer is 2.62 MW.

Practice question 1

The heating element of an electric shower has a power of 6.0 kW.

Water enters the shower at a temperature of 7.5°C.

Calculate the water flow rate required to give an output temperature of 37.5°C.

specific heat capacity of water = 4200 J kg^{-1} K^{-1}

Practice question 2

A CCD sensor of area 1 mm^2 is illuminated with light of wavelength 700 nm and intensity 2.84 mW m^{-2}. The quantum efficiency of the CCD is 70%. Find the rate at which electrons are released in the sensor.

Common Mathematical Models in Physics

As you learn more physics you will find that the same mathematical patterns keep appearing in different parts of the course. Many students find it very satisfying to see how all the apparently very different bits of physics you've studied are actually connected and, in some cases, are just two sides of the same coin. Four of these common patterns are explained in this section.

Simple Harmonic Motion

Mathematical skills: C0.6, C4.5–7

You will probably study simple harmonic motion in Year 13 but you will meet ideas about circular motion and waves beforehand and they are closely connected.

Lots of things start to oscillate if you nudge them. In simple harmonic motion (SHM) a graph of the object's motion is a sine or cosine curve; this curve is the connection to waves and to circular motion. (To see the latter connection, visit http://www.acoustics.salford.ac.uk/feschools/waves/flash/sinewavedisc.swf.)

Questions using the ideas of period, frequency, angular velocity and so on appear in several parts of the syllabus.

Worked example

A toy for cats consists of a plastic mouse of mass **m** attached to a spring. When the mouse is on a low-friction horizontal surface, with the spring attached to a rigid support as shown, it performs simple harmonic motion when given a small displacement **x** from its equilibrium position and released.
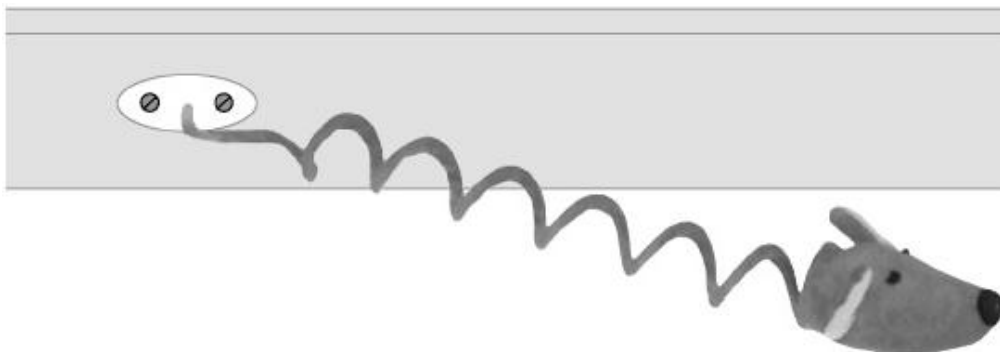
The mouse has a mass $m = 0.15$ kg and the spring extends by 20 cm when the mouse is supported vertically by the spring.

Calculate the frequency of oscillation of the mouse if it is set into oscillation on a low friction horizontal surface.
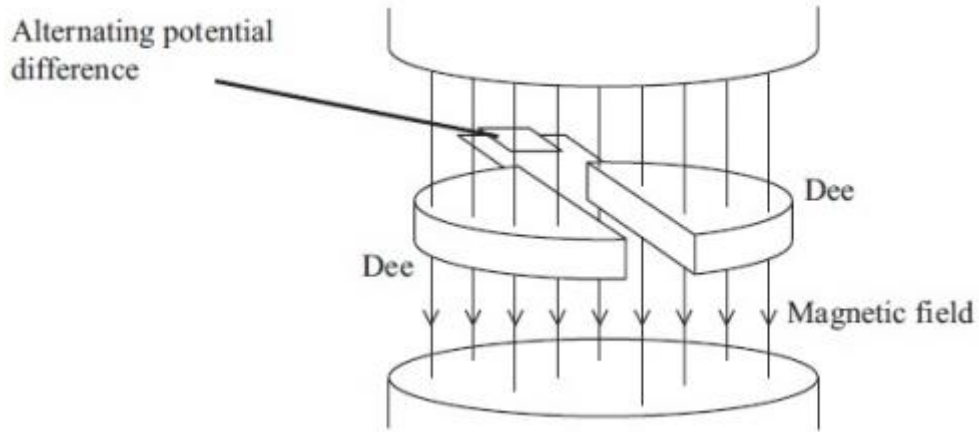
Answer

$$k = \frac{0.15\,\text{kg} \times 9.81\,\text{Nkg}^{-1}}{0.2\,\text{m}} = 7.4\,\text{Nm}^{-1}$$

$$\omega = \sqrt{\frac{7.4\,\text{Nm}^{-1}}{0.15\,\text{kg}}} = 7.0(\text{rad s}^{-1})$$

$$f = \frac{\omega}{2\pi} = \frac{7\,\text{s}^{-1}}{2\pi} = 1.1\,\text{Hz}$$

Practice question 1

A cyclotron can be used to accelerate charged particles.

A beam of low-speed protons is introduced into a cyclotron.

Show that the number of revolutions per second, f, completed by the protons is given by:
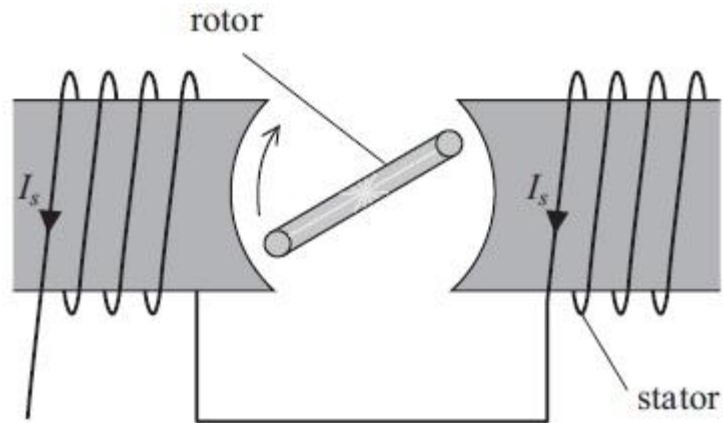
$$f = \frac{eB}{2\pi m}$$

where e is the electronic charge
B is the uniform magnetic flux density within the cyclotron
m is the mass of the proton.

Practice question 2

The diagram represents a simple induction motor. An alternating current I_s is supplied to a stationary coil (stator). This coil is wrapped around an iron core.

A rotating coil (rotor) is shown end on in the diagram.

An induction motor is used to rotate the turntable in a record deck. Long-play records require the turntable to rotate at 33 revolutions per minute.

Calculate the angular velocity of the turntable.

Exponential Decay

Mathematical skills: C0.5, C2.5, C3.10

You will probably remember the idea of half-life from GCSE radioactivity. The remarkable property of that graph (that every time you go one half-life along the *x*-axis the *y*-value halves) is another mathematical pattern. This pattern turns up whenever the amount something changes by depends on how much of it you've got in the first place. For example, the activity of a radioactive source is in proportion to how many radioactive atoms you've got and, loosely speaking, the speed the water goes out of the bath depends on how deep it is.

This pattern is called exponential decay.

If all your followers re-tweet your post twice, and all their followers re-tweet it twice ... soon you are trending, an example of an exponential **growth**. You might meet this if you also study biology e.g. in the growth of populations (like bacteria), or when studying reaction rates in chemistry. The same mathematical pattern is everywhere.

When you learn about the mathematical form of this curve you will need to be able to use logarithms which you might have met in GCSE maths. Using logs is considered a 'Level 2' maths skill so you should expect to see questions that require this in your A level course.

Worked example

In September 1987, two youngsters in Brazil removed a stainless steel cylinder from a machine in an abandoned clinic. Five days later they sold the cylinder to a scrap dealer who prised open a platinum capsule inside to reveal a glowing blue powder. The powder was found to contain caesium-137 and had an activity of 5.2×10^{13} Bq.

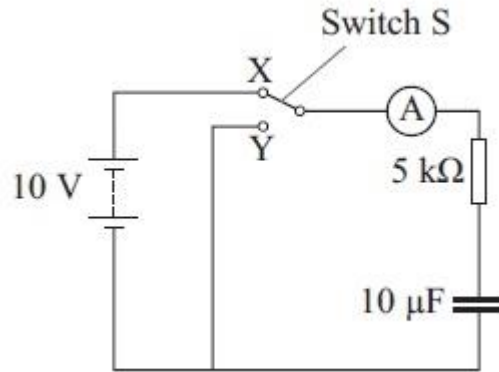Caesium-137 is a β^--emitter with a half-life of 30 years.

Show that the decay constant for the caesium-137 is about 7×10^{-10} s^{-1}.

Answer

$$\lambda = \frac{\log_e 2}{T_{\frac{1}{2}}} = \frac{0.693}{30 \times 365 \times 24 \times 3600 \text{s}} = 7.32 \times 10^{-10} \text{ s}^{-1}$$

Practice question

A student sets up the circuit shown in the diagram.

The student wants to use this circuit to produce a short time delay, equal to the time it takes for the potential difference across the capacitor to fall to 0.07 of its maximum value.

Calculate this time delay.

Inverse Square Laws

Mathematical skills: C3.1-2, C3.12, C4.3, C4.6, C4.7

Inverse square laws describe the pattern $y \propto \dfrac{1}{x^2}$.

This pattern is very common because it occurs whenever something spreads out equally in all directions from a central point. This is because whatever is spreading out (y) gets spread evenly over a sphere which has surface area $4\pi r^2$. (The amount of y passing through unit area is called the **flux** of y.)

You will meet this pattern when studying the intensity of any kind of radiation, in fields and in astronomy.
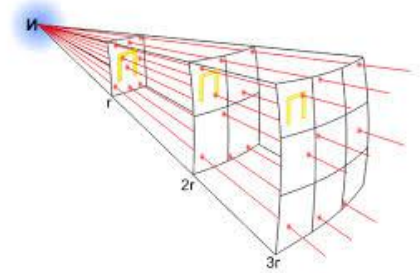
Practice question 1

X and Y are identical stars. When viewed from Earth the flux from star X is 4 times the flux from star Y. Which of the following explanations is possible?

A   X is twice as far away as Y.

B   X is four times as far away as Y.

C   Y is twice as far away as X.

D   Y is four times as far away as X.

Practice question 2

A small satellite has a weight of 1200 N at the Earth's surface. It is launched into a circular orbit with radius equal to twice the radius of the Earth. The weight of the satellite in this orbit is

A   0 N

B   300 N

C   600 N

D   1200 N

Power Laws, Logarithms and Graphs

Mathematical skills: C0.5, C2.5, C3.1-4, C3.10-12

This final pattern is really a summary of several patterns: it is the case when $y \propto x^n$ and we don't know what n is.

This is called a power law relationship – you will find it as one of the trendline options in Excel, along with linear and exponential amongst others.

Often when we meet a new phenomenon in physics we might assume two variables have a power law relationship and try to find out what n is. To do this requires the use of logarithms again, which makes it a favourite of the examiners.

Taking logs of both sides of the equation $y = k\,x^n$ gives:

$$log\ y = log\ k + n\ log\ x$$

So if x and y are indeed related by a power law, the graph of *log y* against *log x* will be a straight line and the gradient will be n. If this graph isn't straight then we know the power law guess was wrong and the relationship is more complicated.

Practice question

A physicist investigates how light intensity varies with distance from a light bulb. She sets up the apparatus as shown.

The relationship between R and d is given by $R = k\,d^p$ where k and p are constants. Explain why a graph of *ln R* against *ln d* should give a straight line.