Uncertainties and practical work

The aim of physics in studying natural phenomena is to develop explanations based on empirical evidence. Hence there is a central concern about the quality of evidence and of the explanations that are based on it. This involves an appreciation of the causes of uncertainties that can arise in practical work and how they should be dealt with, both in planning an experiment to minimise these uncertainties and in forming a valid conclusion.

There is no practical examination in this qualification and a set of practical skills has been identified as appropriate for indirect assessment. Practical skills should be developed by carrying out practical work throughout the course, for example by carrying out the Core Practicals listed in the Specification. The assessment of these skills will be through examination questions, in particular for Unit 3: Practical Skills in Physics I and Unit 6: Practical Skills in Physics II.

It is clearly important that the words used within the practical context have a precise and scientific meaning as distinct from their everyday usage. The terms used for this assessment will be those described in the publication by the Association for Science Education (ASE) entitled The Language of Measurement (ISBN 9780863574245). In adopting this terminology, it should be noted that **certain terms will have a meaning different to that in the previous specification**. In accordance with common practice, this qualification will adopt the Uncertainty Approach to measurement. Using this approach assumes that the measurement activity produces an interval of reasonable values together with a statement of the confidence that the true value lies within this interval.

The following Glossary is a selection of terms from the list in The Language of Measurement published by ASE (ISBN 9780863574245).

Glossary

Term	Meaning and notes
Validity	A measurement is valid if it measures what it is supposed to be measuring – this depends both on the method and the instruments.
True value	The value that would have been obtained in an ideal measurement – with the exception of a fundamental constant the true value is considered unknowable.
Accuracy	A measurement result is considered accurate if it is judged to be close to the true value. It is a quality denoting the closeness of agreement between measurement and true value – it cannot be quantified and is influenced by random and systematic errors.
Precision	A quality denoting the closeness of agreement (consistency) between values obtained by repeated measurement – this is influenced only by random effects and can be expressed numerically by measures such as standard deviation. A measurement is precise if the values 'cluster' closely together.
Repeatability	The precision obtained when measurement results are obtained by a single operator using a single method over a short timescale. A measurement is repeatable when similar results are obtained by students from the same group using the same method. Students can use the precision of their measurement results to judge this.
Reproducibility	The precision obtained when measurement results are obtained by different operators using different pieces of apparatus. A measurement is reproducible when similar results are obtained by students from different groups using different methods or apparatus. This is a harder test of the quality of data.
Uncertainty	The interval within which the true value can be considered to lie with a given level of confidence or probability – any measurement will have some uncertainty about the result, this will come from variation in the data obtained and be subject to systematic or random effects. This can be estimated by considering the instruments and the method and will usually be expressed as a range such as 20 °C ± 2 °C. The confidence will be qualitative and based on the goodness of fit of the line of best fit and the size of the percentage uncertainty.
Error	The difference between the measurement result and the true value if a true value is thought to exist. This is not a mistake in the measurement. The error can be due to both systematic and random effects and an error of unknown size is a source of uncertainty.
Resolution	The smallest measuring interval and the source of uncertainty in a single reading.
Significant figures (SF)	The number of SF used in recording the measurements depends on the resolution of the measuring instruments and should usually be the same as given in the instrument with the fewest SF in its reading.

Uncertainties in practice

What are uncertainties and why are they important?

When you repeat a measurement you often get different results. There is an **uncertainty** in the measurement that you have taken. It is important to be able to determine the uncertainty in measurements so that its effect can be taken into consideration when drawing conclusions about experimental results. The uncertainty might be the **resolution of the instrument** or, if the readings were repeated, the uncertainty might be **half the range** of the repeats. If the uncertainty is predictable, i.e. it is **systematic**, then the uncertainty should be subtracted from each reading, for example if there is a zero error on an instrument.

Recording data

Results should be recorded to the resolution of the measuring instrument which means there will be a consistent number of decimal places for the readings for any one variable. If raw data goes over a power of ten we would not penalise a mixture of significant figures. When data is processed, these values should be recorded to a consistent number of significant figures which is usually 3 as this is what we can usually confidently plot on a graph.

Calculating uncertainties

There are several techniques that will produce an **estimate** of the uncertainty in the value of the mean. Since we are expecting students to produce an estimate of the uncertainty any suitable value that indicates half the range is acceptable.

Example: A student measures the diameter of a metal canister using a ruler graduated in mm and records these results:

Diameter / mm			
Reading 1	Reading 2	Reading 3	Mean
66	65	61	64

The uncertainty in the mean value (64 mm) can be calculated as follows:

a. Using the half range

The range of readings is 61 mm – 66 mm so half the range is used to determine the uncertainty.

Uncertainty in the mean diameter= (66 mm – 61 mm)/2 = 2.5 mm
Therefore, the diameter of the metal canister is 64 mm ± 2.5 mm.

Since a ruler graduated in mm could easily be read to ± 0.5 mm, it is acceptable to quote the uncertainty as ± 2.5 mm for this experiment.

b. Using the reading furthest from the mean

In this case, the measurement of 61 mm is further from the average value than 66 mm therefore we can use this value to calculate the uncertainty in the mean.

   Uncertainty in the mean diameter= 64 mm – 61 mm = 3 mm.
Therefore, the diameter of the metal canister is 64 mm ± 3 mm.

c. Using the resolution of the instrument

This is used if a single reading is taken or if repeated readings have the same value. This is because there is an uncertainty in the measurement because the instrument used to take the measurement has its own limitations. If the three readings obtained above were all 64 mm then the value of the diameter being measured lies somewhere between 63.5 mm and 64.5 mm since a metre rule could easily be read to half a millimetre. In this case, the uncertainty in the diameter is 0.5 mm.

Therefore, the diameter of the metal canister is 64 mm ± 0.5 mm.

This also applies to digital instruments. An ammeter records currents to 0.1 A. A current of 0.36 A would be displayed as 0.4 A, and a current of 0.44 A would also be displayed as 0.4 A. The resolution of the instrument is 0.1 A but the uncertainty in a reading is 0.05 A.

Calculating percentage uncertainties

The percentage uncertainty in a measurement can be calculated using:

Percentage uncertainty = (Uncertainty of measurement/Measurement) × 100%

In the above example the percentage uncertainty in the diameter of the metal canister is:

Percentage uncertainty = (3/64) × 100% = 4.7%

Often the radius would be used in a calculation, for example in a calculation of volume. In this case, the percentage uncertainty for the radius of the canister is the same as its diameter, i.e. 4.7%, and not half of the percentage uncertainty. This is one reason why the percentage uncertainty in a measurement is useful.

Additionally, the value is less than 5%, which shows that the measurement is probably repeatable. Note that a percentage uncertainty would normally be quoted to 1 or 2 sf.

Compounding uncertainties

Calculations often use more than one measurement. Each measurement will have its own uncertainty, so it is necessary to combine the uncertainties for each measurement to calculate the overall uncertainty in the calculation provided all the measured quantities are independent of one another.

There are three methods of compounding uncertainties depending on whether the measurements in a calculation are raised to a power, multiplied/divided, or added/subtracted.

a. Raising a measurement to a power

If a measurement is raised to a power, for example squared or cubed, then the percentage uncertainty is multiplied by that power to give the total percentage uncertainty.

Example: A builder wants to calculate the area of a square tile. He uses a rule to measure the two adjacent sides of a square tile and obtains the following results:

Length of one side = 84 mm ± 0.5 mm

Length of perpendicular side = 84 mm ± 0.5 mm

The percentage uncertainty in the length of each side of this square tile is given by:

Percentage uncertainty = (0.5/84) × 100% = 0.59 % = 0.6 %

The area of the tile A is given by A = 84 × 84 = 7100 mm^2

Note that this is to 2 sf since the measurements are to 2 sf.

The percentage uncertainty in the area of the square tile is calculated by multiplying the percentage uncertainty in the length by 2.

Percentage uncertainty in A = 2 × 0.6% = 1.2%

Therefore the uncertainty in A = 7100 × 1.2% = 85 mm^2

So A = 7100 mm^2 ± 1.2% or A = 7100 mm^2 ± 85 mm^2

b. Multiplying or dividing measurements

The total percentage uncertainty is calculated by adding together the percentage uncertainties for each measurement.

Example: A metallurgist is determining the purity of a sample of an alloy that is in the shape of a cube by determining the density of the material.

The following readings are taken:

Length of each side of the cube= 24.0 mm ± 0.5 mm

Mass of cube = 48.23 g ± 0.05 g

She calculates (i) the density of the material and (ii) the percentage uncertainty in the density of the material.

(i) Density of alloy = mass / volume = mass / length^3

$$= (48.23 \times 10^{-3} \text{ kg}) / (24.0 \times 10^{-3} \text{ m})^3 = 3490 \text{ kg m}^{-3}$$

(ii) Percentage uncertainty in the length = 0.5 / 24.0 × 100% = 2.1%

Percentage uncertainty in the mass = 0.05/48.23 × 100% = 0.1%

Percentage uncertainty in density= 3 × 2.1% +0.1% = 6.4% (or 6%)

Therefore, the density of the material= 3490 kg m^{-3} ± 6% or 3490 kg m^{-3} ± 210 kg m^{-3}

Example: A student calculates the volume of a drinks can and the percentage uncertainty for the final value.

The student determines that the radius of the metal can is 33 mm with an uncertainty of 1% so the cross-sectional area A of the canister is:

$A = \pi r^2 = \pi (33)^2 = 3.4 \times 10^3$ mm^2 ± 2%

Notice that the result has been expressed using scientific notation so that we can write down just two significant figures. The calculator answer (3421.1...) gives the impression of far more sf than is justified when the radius is only known to the nearest mm.

The cross-sectional area was calculated by squaring the radius. Since two quantities have in effect been multiplied together, the percentage uncertainty in the value of the cross-sectional area is found by adding the percentage uncertainty of the radius to the percentage uncertainty of the radius – doubling it.

The student measures the length L of the can = 115 mm ± 2 mm

The volume V of the can is

V = 3.4 × 10^3 mm^2 × 115 mm = 3.9 × 10^5 mm^3 = 3.9 × 10^{-4} m^3

The percentage uncertainty in this value is obtained by adding together an appropriate combination of the uncertainties

Percentage uncertainty in L = (2/115) × 100% = 1.7

Therefore, percentage uncertainty in V= 2% + 1.7% = 3.7%

Volume V = 3.91 × 10^{-4} m^3 ± 3.7% = 3.91 × 10^{-4} m^3 ± 1.4 × 10^{-5} m^3

Again, an overall percentage uncertainty of less than 5% suggests that this determination of the volume of a can is repeatable.

c. **Adding or subtracting measurements**

When measurements are added or subtracted in a calculation then the uncertainty for each measurement is added to calculate the total uncertainty.

Example: A student wants to determine the thickness of the walls of a plastic pipe. He measures the internal and external diameters of the pipe using vernier calipers and obtains the following readings:

Internal diameter = 101.4 mm ± 0.1 mm

External diameter = 102.8 mm ± 0.3 mm

The difference between these two measurements is 1.4 mm ± 0.4 mm

Since the difference in the radius is required then both the diameter **and** the uncertainty must be divided by 2 (since the percentage uncertainty remains the same), therefore the thickness of the walls is 0.7 mm ± 0.2 mm.

Using uncertainties in drawing conclusions

Often an experiment will require a comparison to a known value. This is when the uncertainty can be used to assess whether the measured value is accurate or not. This can be achieved in the following ways:

a. **Calculating maximum and minimum values**

The final uncertainty can be used to determine the range in which the measured value may lie. If the known value lies within this range then we can say that the measured value is accurate.

Example: A student used a simple pendulum to obtain a value of $g = 10.1$ m s^{-2}. The experimental percentage uncertainty was calculated as 4%.

Minimum value of $g = 10.1 - (10.1 \times 4\%) = 9.7$ m s^{-2}

Since the accepted value of $g = 9.81$ m s^{-2} lies above the minimum value, then we can conclude that the measured value of g is accurate.

This method should always be used when the percentage uncertainty in the value is known.

b. **Calculating a percentage difference**

If the measured value has been determined from a graph and there is no information about the percentage uncertainty of the measured value, then percentage difference can be used to comment on accuracy. If the percentage difference is less than 5%, then this is an indication that the result is accurate.
In the above example, the percentage difference is calculated as:

Percentage difference = $(10.1 - 9.81)/9.81 \times 100\% = 3\%$

As this is less than 5% we can conclude that the measured value of g is accurate.

c. **Observations from graphs**

There is no expectation that error bars should be added to graphs.
If a straight-line graph through the origin is expected but the line of best fit of the plotted points does not pass through the origin, then this is an indication of a systematic error. If there is a large scatter of points around the line of best fit this is an indication of a large uncertainty possibly due to random errors.