

# AS Further Mathematics



## Sample Assessment Materials

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**Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)**

*First teaching from September 2017*

*First certification from 2018*

Issue 1

## Edexcel, BTEC and LCCI qualifications

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# Introduction

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The Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet '*Mathematical Formulae and Statistical Tables*' will be provided for use with these assessments and can be downloaded from our website, [qualifications.pearson.com](http://qualifications.pearson.com).



# General marking guidance

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- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked unless the candidate has replaced it with an alternative response.

## Specific guidance for mathematics

### 1. These mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

### 2. Abbreviations

These are some of the traditional marking abbreviations that may appear in the mark schemes.

- |                        |                                                                                                    |         |                                                        |
|------------------------|----------------------------------------------------------------------------------------------------|---------|--------------------------------------------------------|
| • bod                  | benefit of doubt                                                                                   | • SC:   | special case                                           |
| • ft                   | follow through                                                                                     | • o.e.  | or equivalent (and appropriate)                        |
| • $\sqrt{\phantom{x}}$ | this symbol is used for correct ft                                                                 | • d...  | dependent or dep                                       |
| • cao                  | correct answer only                                                                                | • indep | independent                                            |
| • cso                  | correct solution only.<br>There must be no errors in this part of the question to obtain this mark | • dp    | decimal places                                         |
| • isw                  | ignore subsequent working                                                                          | • sf    | significant figures                                    |
| • awrt                 | answers which round to                                                                             | • *     | The answer is printed on the paper or ag- answer given |

-  or d... The second mark is dependent on gaining the first mark

3. All M marks are follow through.

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is  $>1$  or  $<0$ , should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.



Write your name here

Surname

Other names

**Pearson Edexcel**  
**Level 3 GCE**

Centre Number

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Candidate Number

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# Further Mathematics

**Advanced Subsidiary**

**Paper 1: Core Pure Mathematics**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/01**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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S 5 4 4 5 6 A 0 1 1 9



Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.  $f(z) = z^3 + pz^2 + qz - 15$

where  $p$  and  $q$  are real constants.

Given that the equation  $f(z) = 0$  has roots

$$\alpha, \frac{5}{\alpha} \text{ and } \left( \alpha + \frac{5}{\alpha} - 1 \right)$$

(a) solve completely the equation  $f(z) = 0$

(5)

(b) Hence find the value of  $p$ .

(2)

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2. The plane  $\Pi$  passes through the point  $A$  and is perpendicular to the vector  $\mathbf{n}$

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where  $O$  is the origin,

- (a) find a Cartesian equation of  $\Pi$ .

(2)

With respect to the fixed origin  $O$ , the line  $l$  is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line  $l$  intersects the plane  $\Pi$  at the point  $X$ .

- (b) Show that the acute angle between the plane  $\Pi$  and the line  $l$  is  $21.2^\circ$  correct to one decimal place.

(4)

- (c) Find the coordinates of the point  $X$ .

(4)

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[illegible]

3. Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%
- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

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4. The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

Without solving the equation, find the cubic equation whose roots are  $(\alpha - 1)$ ,  $(\beta - 1)$  and  $(\gamma - 1)$ , giving your answer in the form  $w^3 + pw^2 + qw + r = 0$ , where  $p$ ,  $q$  and  $r$  are integers to be found.

(5)



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5.

$$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

- (a) Show that  $\mathbf{M}$  is non-singular.

(2)

The hexagon  $R$  is transformed to the hexagon  $S$  by the transformation represented by the matrix  $\mathbf{M}$ .

Given that the area of hexagon  $R$  is 5 square units,

- (b) find the area of hexagon  $S$ .

(1)

The matrix  $\mathbf{M}$  represents an enlargement, with centre  $(0, 0)$  and scale factor  $k$ , where  $k > 0$ , followed by a rotation anti-clockwise through an angle  $\theta$  about  $(0, 0)$ .

- (c) Find the value of  $k$ .

(2)

- (d) Find the value of  $\theta$ .

(2)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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[illegible]

7.

Diagrams not drawn to scale

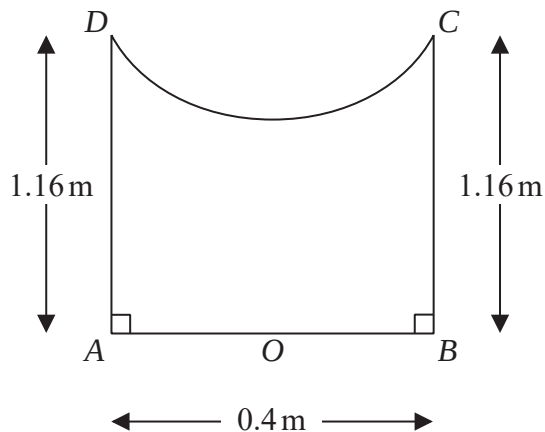


Figure 1

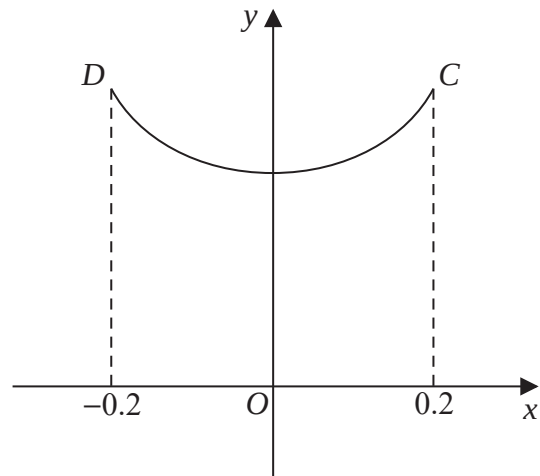


Figure 2

Figure 1 shows the central cross-section  $AOBCD$  of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve  $CD$ , shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \quad -0.2 \leq x \leq 0.2$$

where  $k$  is a constant and where  $O$  is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter,  $AB$ , of the base of the bird bath measured 0.40 m, as shown in Figure 1.

- Suggest the maximum depth of the bird bath. (1)
- Find the value of  $k$ . (2)
- Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in  $\text{m}^3$ , correct to 3 significant figures. (7)
- State a limitation of the model. (1)

It was later discovered that the volume of concrete used to make the bird bath was  $0.127 \text{ m}^3$  correct to 3 significant figures.

- Using this information and the answer to part (c), evaluate the model, explaining your reasoning. (1)

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[illegible]

8. (a) Shade on an Argand diagram the set of points

$$\left\{ z \in \mathbb{C} : |z - 4i| \leq 3 \right\} \cap \left\{ z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4i) \leq \frac{\pi}{4} \right\}$$

(6)

The complex number  $w$  satisfies

$$|w - 4i| = 3$$

- (b) Find the maximum value of  $\arg w$  in the interval  $(-\pi, \pi]$ .  
Give your answer in radians correct to 2 decimal places.

(2)



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9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish  $F$  swims from a point  $A$  to a point  $B$ .

The octopus is modelled as a fixed particle at the origin  $O$ .

Fish  $F$  is modelled as a particle moving in a straight line from  $A$  to  $B$ .

Relative to  $O$ , the coordinates of  $A$  are  $(-3, 1, -7)$  and the coordinates of  $B$  are  $(9, 4, 11)$ , where the unit of distance is metres.

- (a) Use the model to determine whether or not the octopus is able to catch fish  $F$ . (7)
- (b) Criticise the model in relation to fish  $F$ . (1)
- (c) Criticise the model in relation to the octopus. (1)

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Paper 1: Core Pure Mathematics Mark Scheme

| Question                                                                                                                                                                      | Scheme                                                                                                                                                                                                                       | Marks | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>1(a)</b>                                                                                                                                                                   | $\alpha\left(\frac{5}{\alpha}\right)\left(\alpha + \frac{5}{\alpha} - 1\right) = 15$                                                                                                                                         | M1    | 1.1b |
|                                                                                                                                                                               |                                                                                                                                                                                                                              | A1    | 1.1b |
|                                                                                                                                                                               | $\Rightarrow 5\alpha + \frac{25}{\alpha} - 5 = 15 \Rightarrow \alpha^2 - 4\alpha + 5 = 0$<br>$\Rightarrow \alpha = \frac{- -4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$ or $(\alpha - 2)^2 - 4 + 5 = 0 \Rightarrow \alpha = \dots$ | M1    | 3.1a |
|                                                                                                                                                                               | $\Rightarrow \alpha = 2 \pm i$                                                                                                                                                                                               | A1    | 1.1b |
|                                                                                                                                                                               | Hence the roots of $f(z) = 0$ are $2 + i$ , $2 - i$ and $3$                                                                                                                                                                  | A1    | 2.2a |
|                                                                                                                                                                               |                                                                                                                                                                                                                              | (5)   |      |
| <b>(b)</b>                                                                                                                                                                    | $p = -\left((2 + i) + (2 - i) + 3\right) \Rightarrow p = \dots$                                                                                                                                                              | M1    | 3.1a |
|                                                                                                                                                                               | $\Rightarrow p = -7$ cso                                                                                                                                                                                                     | A1    | 1.1b |
|                                                                                                                                                                               |                                                                                                                                                                                                                              | (2)   |      |
|                                                                                                                                                                               | <b>1(b) alternative</b>                                                                                                                                                                                                      |       |      |
|                                                                                                                                                                               | $f(z) = (z - 3)(z^2 - 4z + 5) \Rightarrow p = \dots$                                                                                                                                                                         | M1    | 3.1a |
|                                                                                                                                                                               | $\Rightarrow p = -7$ cso                                                                                                                                                                                                     | A1    | 1.1b |
|                                                                                                                                                                               |                                                                                                                                                                                                                              | (2)   |      |
| <b>(7 marks)</b>                                                                                                                                                              |                                                                                                                                                                                                                              |       |      |
| Notes:                                                                                                                                                                        |                                                                                                                                                                                                                              |       |      |
| <b>(a)</b>                                                                                                                                                                    |                                                                                                                                                                                                                              |       |      |
| <b>M1:</b> Multiplies the three given roots together and sets the result equal to 15 or -15                                                                                   |                                                                                                                                                                                                                              |       |      |
| <b>A1:</b> Obtains a correct equation in $\alpha$                                                                                                                             |                                                                                                                                                                                                                              |       |      |
| <b>M1:</b> Forms a quadratic equation in $\alpha$ and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha = \dots$ |                                                                                                                                                                                                                              |       |      |
| <b>A1:</b> $\alpha = 2 \pm i$                                                                                                                                                 |                                                                                                                                                                                                                              |       |      |
| <b>A1:</b> Deduces the roots are $2 + i$ , $2 - i$ and $3$                                                                                                                    |                                                                                                                                                                                                                              |       |      |
| <b>(b)</b>                                                                                                                                                                    |                                                                                                                                                                                                                              |       |      |
| <b>M1:</b> Applies the process of finding $-\sum$ (of their three roots found in part (a)) to give $p = \dots$                                                                |                                                                                                                                                                                                                              |       |      |
| <b>A1:</b> $p = -7$ by correct solution only                                                                                                                                  |                                                                                                                                                                                                                              |       |      |
| <b>(b) Alternative</b>                                                                                                                                                        |                                                                                                                                                                                                                              |       |      |
| <b>M1:</b> Applies the process expanding $(z - "3")(z - (\text{their sum})z + \text{their product})$ in order to find $p = \dots$                                             |                                                                                                                                                                                                                              |       |      |
| <b>A1:</b> $p = -7$ by correct solution only                                                                                                                                  |                                                                                                                                                                                                                              |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                                                                                               | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>2(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                  | $\mathbf{r} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$                         | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $3x - y + 2z = 10$                                                                                                                                                                   | A1         | 2.5  |
|                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                      | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                   | $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} = 8$                                                                                   | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $\sqrt{(3)^2 + (-1)^2 + (2)^2} \cdot \sqrt{(-1)^2 + (-5)^2 + (3)^2} \cos \alpha = "-3 + 5 + 6"$                                                                                      | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $\theta = 90^\circ - \arccos\left(\frac{8}{\sqrt{14} \cdot \sqrt{35}}\right)$ or $\sin \theta = \frac{8}{\sqrt{14} \cdot \sqrt{35}}$                                                 | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $\theta = 21.2^\circ$ (1 dp) * cso                                                                                                                                                   | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                      | <b>(4)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                   | $3(7 - \lambda) - (3 - 5\lambda) + 2(-2 + 3\lambda) = 10 \Rightarrow \lambda = \dots$                                                                                                | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $\lambda = -\frac{1}{2}$                                                                                                                                                             | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $\overrightarrow{OX} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$ | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              | $X(7.5, 5.5, -3.5)$                                                                                                                                                                  | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                      | <b>(4)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                      |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                      |            |      |
| <p><b>(a)</b><br/> <b>M1:</b> Attempts to apply the formula <math>\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}</math><br/> <b>A1:</b> Correct Cartesian notation. e.g. <math>3x - y + 2z = 10</math> or <math>-3x + y - 2z = -10</math><br/> <b>Note:</b> Do not allow final answer given as <math>\mathbf{r} \cdot (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 10</math>, o.e.</p>              |                                                                                                                                                                                      |            |      |
| <p><b>(b)</b><br/> <b>B1:</b> <math>\overrightarrow{OA} \cdot \mathbf{n} = 8</math><br/> <b>M1:</b> An attempt to apply the correct dot product formula between <math>\mathbf{n}</math> and <math>\mathbf{d}</math><br/> <b>M1:</b> Depends on previous M mark. Applies the dot product formula to find the angle between <math>l</math> and <math>l</math><br/> <b>A1*:</b> <math>21.2^\circ</math> cso</p> |                                                                                                                                                                                      |            |      |

Question 2 notes continued:

**(c)**

**M1:** Substitutes  $l$  into  $II$  and solves the resulting equation to give  $\lambda = \dots$

**A1:**  $\lambda = -\frac{1}{2}$  o.e.

**M1:** Depends on previous M mark. Substitutes their  $\lambda$  into  $l$  and finds at least one of the coordinates

**A1ft:** (7.5, 5.5, -3.5) but follow through on their value of  $\lambda$

| Question | Scheme                                                                                                                                                                                                                                                  | Marks | AOs  |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>3</b> | $x = \text{value of savings account, } y = \text{value of property bond account,}$<br>$z = \text{value of share dealing account}$<br>$x + y + z = 5000$<br>$x + 400 = y$<br>$0.015x + 0.035y - 0.025z = 79 \text{ or } 1.015x + 1.035y + 0.975z = 5079$ | M1    | 3.1b |
|          |                                                                                                                                                                                                                                                         | A1    | 1.1b |
|          | $\text{Let } \mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.015 & 1.035 & 0.975 \end{pmatrix}$                                                   |       |      |
|          | $\text{e.g. } \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$                                                             | M1    | 3.1a |
|          |                                                                                                                                                                                                                                                         | A1    | 1.1b |
|          | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}^{-1} \begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$             | M1    | 1.1b |
|          | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1800 \\ 2200 \\ 1000 \end{pmatrix}$                                                                                                                                                        | A1    | 1.1b |
|          | Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account                                                                                                                                  | A1ft  | 3.2a |

**(7 marks)**

Notes:

**M1:** Attempts to set up 3 equations with 3 unknowns

**A1:** At least 2 equations are correct with the appropriate variables defined

**M1:** Sets up a matrix equation of the form, e.g.  $\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$ , where “...” are numerical values

**A1:** Correct matrix equation (or equivalent)

**M1:** Depends on previous M mark. Applies  $(\text{their } \mathbf{A})^{-1} \begin{pmatrix} 5000 \\ \text{their "-400"} \\ \text{their "79"} \end{pmatrix}$  and obtains at least one value of  $x, y$  or  $z$

**A1:** Correct answer

**A1ft:** Correct follow through answer in context



| Question    | Scheme                                                                                                                                      | Marks | AOs  |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 4           | $\{w = x - 1 \Rightarrow\} x = w + 1$                                                                                                       | B1    | 3.1a |
|             | $(w + 1)^3 + 3(w + 1)^2 - 8(w + 1) + 6 = 0$                                                                                                 | M1    | 3.1a |
|             | $w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w + 1) - 8w - 8 + 6 = 0$                                                                                    |       |      |
|             | $w^3 + 6w^2 + w + 2 = 0$                                                                                                                    | M1    | 1.1b |
|             |                                                                                                                                             | A1    | 1.1b |
|             |                                                                                                                                             | A1    | 1.1b |
|             |                                                                                                                                             | (5)   |      |
|             | Alternative                                                                                                                                 |       |      |
|             | $\alpha + \beta + \gamma = -3, \alpha\beta + \beta\gamma + \alpha\gamma = -8, \alpha\beta\gamma = -6$                                       | B1    | 3.1a |
|             | sumroots = $\alpha - 1 + \beta - 1 + \gamma - 1$                                                                                            | M1    | 3.1a |
|             | $= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$                                                                                               |       |      |
|             | pairsum = $(\alpha - 1)(\beta - 1) + (\alpha - 1)(\gamma - 1) + (\beta - 1)(\gamma - 1)$                                                    |       |      |
|             | $= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha + \beta + \gamma) + 3$                                                               |       |      |
|             | $= -8 - 2(-3) + 3 = 1$                                                                                                                      |       |      |
|             | product = $(\alpha - 1)(\beta - 1)(\gamma - 1)$                                                                                             |       |      |
|             | $= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$                                          |       |      |
|             | $= -6 - (-8) - 3 - 1 = -2$                                                                                                                  |       |      |
|             | $w^3 + 6w^2 + w + 2 = 0$                                                                                                                    | M1    | 1.1b |
|             |                                                                                                                                             | A1    | 1.1b |
|             |                                                                                                                                             | A1    | 1.1b |
|             |                                                                                                                                             | (5)   |      |
|             | (5 marks)                                                                                                                                   |       |      |
| Notes:      |                                                                                                                                             |       |      |
| B1:         | Selects the method of making a connection between $x$ and $w$ by writing $x = w + 1$                                                        |       |      |
| M1:         | Applies the process of substituting their $x = w + 1$ into $x^3 + 3x^2 - 8x + 6 = 0$                                                        |       |      |
| M1:         | Depends on previous M mark. Manipulating their equation into the form<br>$w^3 + pw^2 + qw + r = 0$                                          |       |      |
| A1:         | At least two of $p, q, r$ are correct                                                                                                       |       |      |
| A1:         | Correct final equation                                                                                                                      |       |      |
| Alternative |                                                                                                                                             |       |      |
| B1:         | Selects the method of giving three correct equations each containing $\alpha, \beta$ and $\gamma$                                           |       |      |
| M1:         | Applies the process of finding sum roots, pair sum and product                                                                              |       |      |
| M1:         | Depends on previous M mark. Applies<br>$w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their } \alpha\beta\gamma = 0$ |       |      |
| A1:         | At least two of $p, q, r$ are correct                                                                                                       |       |      |
| A1:         | Correct final equation                                                                                                                      |       |      |

| Question                                                                                                                                                             | Scheme                                                                                        | Marks | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|-------|------|
| <b>5(a)</b>                                                                                                                                                          | $\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$                                           | M1    | 1.1a |
|                                                                                                                                                                      | $\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$  | A1    | 2.4  |
|                                                                                                                                                                      |                                                                                               | (2)   |      |
| <b>(b)</b>                                                                                                                                                           | $\text{Area}(S) = 4(5) = 20$                                                                  | B1ft  | 1.2  |
|                                                                                                                                                                      |                                                                                               | (1)   |      |
| <b>(c)</b>                                                                                                                                                           | $k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$                                                   | M1    | 1.1b |
|                                                                                                                                                                      | $= 2$                                                                                         | A1ft  | 1.1b |
|                                                                                                                                                                      |                                                                                               | (2)   |      |
| <b>(d)</b>                                                                                                                                                           | $\cos \theta = \frac{1}{2}$ or $\sin \theta = \frac{\sqrt{3}}{2}$ or $\tan \theta = \sqrt{3}$ | M1    | 1.1b |
|                                                                                                                                                                      | $\theta = 60^\circ$ or $\frac{\pi}{3}$                                                        | A1    | 1.1b |
|                                                                                                                                                                      |                                                                                               | (2)   |      |
| <b>(7 marks)</b>                                                                                                                                                     |                                                                                               |       |      |
| Notes:                                                                                                                                                               |                                                                                               |       |      |
| <b>(a)</b>                                                                                                                                                           |                                                                                               |       |      |
| <b>M1:</b> An attempt to find $\det(\mathbf{M})$ .                                                                                                                   |                                                                                               |       |      |
| <b>A1:</b> $\det(\mathbf{M}) = 4$ <b>and</b> reference to zero, e.g. $4 \neq 0$ <b>and</b> conclusion.                                                               |                                                                                               |       |      |
| <b>(b)</b>                                                                                                                                                           |                                                                                               |       |      |
| <b>B1ft:</b> 20 or a correct ft based on their answer to part (a).                                                                                                   |                                                                                               |       |      |
| <b>(c)</b>                                                                                                                                                           |                                                                                               |       |      |
| <b>M1:</b> $\sqrt{(\text{their } \det \mathbf{M})}$                                                                                                                  |                                                                                               |       |      |
| <b>A1ft:</b> 2                                                                                                                                                       |                                                                                               |       |      |
| <b>(d)</b>                                                                                                                                                           |                                                                                               |       |      |
| <b>M1:</b> <b>Either</b> $\cos \theta = \frac{1}{(\text{their } k)}$ <b>or</b> $\sin \theta = \frac{\sqrt{3}}{(\text{their } k)}$ <b>or</b> $\tan \theta = \sqrt{3}$ |                                                                                               |       |      |
| <b>A1:</b> $\theta = 60^\circ$ or $\frac{\pi}{3}$ . Also accept any value satisfying $360n + 60^\circ$ , $n \in \mathbb{Z}$ , o.e.                                   |                                                                                               |       |      |

| Question          | Scheme                                                                                                                                                                                                                              | Marks      | AOs  |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>6(a)</b>       | $n=1, \sum_{r=1}^1 r^2 = 1 \text{ and } \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$                                                                                                                                         | B1         | 2.2a |
|                   | Assume general statement is true for $n = k$<br>So assume $\sum_{r=1}^k r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true                                                                                                                      | M1         | 2.4  |
|                   | $\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$                                                                                                                                                                          | M1         | 2.1  |
|                   | $= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$                                                                                                                                                                                                 | A1         | 1.1b |
|                   | $= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$                                                                                                                                                           | A1         | 1.1b |
|                   | Then the general result is <u>true for <math>n = k + 1</math></u><br>As the general result has been shown to be <u>true for <math>n = 1</math></u> , then the general result is <u>true for all <math>n \in \mathbb{Z}^+</math></u> | A1         | 2.4  |
|                   |                                                                                                                                                                                                                                     | <b>(6)</b> |      |
| <b>(b)</b>        | $\sum_{r=1}^n r(r+6)(r-6) = \sum_{r=1}^n (r^3 - 36r)$                                                                                                                                                                               |            |      |
|                   | $= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$                                                                                                                                                                                      | M1         | 2.1  |
|                   |                                                                                                                                                                                                                                     | A1         | 1.1b |
|                   | $= \frac{1}{4}n(n+1)[n(n+1) - 72]$                                                                                                                                                                                                  | M1         | 1.1b |
|                   | $= \frac{1}{4}n(n+1)(n-8)(n+9) \quad * \quad \text{cso}$                                                                                                                                                                            | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                     | <b>(4)</b> |      |
| <b>(c)</b>        | $\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$                                                                                                                                                                            | M1         | 1.1b |
|                   | $\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$                                                                                                                                                                                        | M1         | 1.1b |
|                   | $3n^2 - 65n - 250 = 0$                                                                                                                                                                                                              | A1         | 1.1b |
|                   | $(3n+10)(n-25) = 0$                                                                                                                                                                                                                 | M1         | 1.1b |
|                   | (As $n$ must be a positive integer,) $n = 25$                                                                                                                                                                                       | A1         | 2.3  |
|                   |                                                                                                                                                                                                                                     | <b>(5)</b> |      |
| <b>(15 marks)</b> |                                                                                                                                                                                                                                     |            |      |

|                   |                                                                                                                                                                                    |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 6 notes: |                                                                                                                                                                                    |
| <b>(a)</b>        |                                                                                                                                                                                    |
| <b>B1:</b>        | Checks $n = 1$ works for both sides of the general statement                                                                                                                       |
| <b>M1:</b>        | Assumes (general result) true for $n = k$                                                                                                                                          |
| <b>M1:</b>        | Attempts to add $(k + 1)^{\text{th}}$ term to the sum of $k$ terms                                                                                                                 |
| <b>A1:</b>        | Correct algebraic work leading to <b>either</b> $\frac{1}{6}(k + 1)(2k^2 + 7k + 6)$<br><b>or</b> $\frac{1}{6}(k + 2)(2k^2 + 5k + 3)$ <b>or</b> $\frac{1}{6}(2k + 3)(k^2 + 3k + 2)$ |
| <b>A1:</b>        | Correct algebraic work leading to $\frac{1}{6}(k + 1)(\{k + 1\} + 1)(2\{k + 1\} + 1)$                                                                                              |
| <b>A1:</b>        | cso leading to a correct induction statement conveying all three underlined points                                                                                                 |
| <b>(b)</b>        |                                                                                                                                                                                    |
| <b>M1:</b>        | Substitutes at least one of the standard formulae into their expanded expression                                                                                                   |
| <b>A1:</b>        | Correct expression                                                                                                                                                                 |
| <b>M1:</b>        | Depends on previous M mark. Attempt to factorise at least $n(n + 1)$ having used                                                                                                   |
| <b>A1*:</b>       | Obtains $\frac{1}{4}n(n + 1)(n - 8)(n + 9)$ by cso                                                                                                                                 |
| <b>(c)</b>        |                                                                                                                                                                                    |
| <b>M1:</b>        | Sets their part (a) answer equal to $\frac{17}{6}n(n + 1)(2n + 1)$                                                                                                                 |
| <b>M1:</b>        | Cancels out $n(n + 1)$ from both sides of their equation                                                                                                                           |
| <b>A1:</b>        | $3n^2 - 65n - 250 = 0$                                                                                                                                                             |
| <b>M1:</b>        | A valid method for solving a 3 term quadratic equation                                                                                                                             |
| <b>A1:</b>        | Only one solution of $n = 25$                                                                                                                                                      |

| Question   | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                          | Marks | AOs  |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|-------|------|
| 7(a)       | Depth = 0.16 (m)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                          | B1    | 2.2b |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          | (1)   |      |
| (b)        | $y = 1 + kx^2 \Rightarrow 1.16 = 1 + k(0.2)^2 \Rightarrow k = \dots$                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                          | M1    | 3.3  |
|            | $\Rightarrow k = 4$ cao {So $y = 1 + 4x^2$ }                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                          | A1    | 1.1b |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          | (2)   |      |
| (c)        | $\frac{\pi}{4} \int (y - 1) dy$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | $\frac{\pi}{4} \int y dy$                                                                | B1ft  | 1.1a |
|            | $= \left\{ \frac{\pi}{4} \right\} \int_1^{1.16} (y - 1) dy$                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $= \left\{ \frac{\pi}{4} \right\} \int_0^{0.16} y dy$                                    | M1    | 3.3  |
|            | $= \left\{ \frac{\pi}{4} \right\} \left[ \frac{y^2}{2} - y \right]_1^{1.16}$                                                                                                                                                                                                                                                                                                                                                                                                                                             | $= \left\{ \frac{\pi}{4} \right\} \left[ \frac{y^2}{2} \right]_0^{0.16}$                 | M1    | 1.1b |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          | A1    | 1.1b |
|            | $= \frac{\pi}{4} \left( \left( \frac{1.16^2}{2} - 1.16 \right) - \left( \frac{1}{2} - 1 \right) \right) \{ = 0.0032\pi \}$                                                                                                                                                                                                                                                                                                                                                                                               | $= \frac{\pi}{4} \left( \left( \frac{0.16^2}{2} \right) - (0) \right) \{ = 0.0032\pi \}$ |       |      |
|            | $V_{\text{cylinder}} = \pi(0.2)^2(1.16) \{ = 0.0464\pi \}$                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                          | B1    | 1.1b |
|            | Volume = $0.0464\pi - 0.0032\pi \{ = 0.0432\pi \}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                          | M1    | 3.4  |
|            | $= 0.1357168026\dots = 0.136(\text{m}^3) \text{ (3sf)}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                          | A1    | 1.1b |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | (7)                                                                                      |       |      |
| (d)        | Any one of e.g.<br>the measurements may not be accurate<br>the inside surface of the bowl may not be smooth<br>there may be wastage of concrete when making the bird bath                                                                                                                                                                                                                                                                                                                                                |                                                                                          | B1    | 3.5b |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          | (1)   |      |
| (e)        | Some comment consistent with their values. We do need a reason<br>e.g. $\left[ \left( \frac{0.136 - 0.127}{0.127} \right) \times 100 = 7.0866\dots \right]$<br>so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model<br><br><b>or</b><br>We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used |                                                                                          | B1ft  | 3.5a |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          | (1)   |      |
| (12 marks) |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                          |       |      |

|                   |                                                                                                                                                                                                      |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 7 notes: |                                                                                                                                                                                                      |
| (a)               |                                                                                                                                                                                                      |
| <b>B1:</b>        | Infers that the maximum depth of the bird bath could be 0.16 (m)                                                                                                                                     |
| (b)               |                                                                                                                                                                                                      |
| <b>M1:</b>        | Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$<br>and rearranges to give $k = \dots$                                                                                         |
| <b>A1:</b>        | $k = 4$ cao                                                                                                                                                                                          |
| (c)               |                                                                                                                                                                                                      |
| <b>B1ft:</b>      | Uses the model to obtain either $\frac{\pi}{(\text{their } k)} \int (y-1) dy$ <b>or</b> $\frac{\pi}{(\text{their } k)} \int y dy$                                                                    |
| <b>M1:</b>        | Chooses limits that are appropriate to their model                                                                                                                                                   |
| <b>M1:</b>        | Integrates $y$ (with respect to $y$ ) to give $\pm \lambda y^2$ , where $\lambda \neq 0$ is a constant                                                                                               |
| <b>A1:</b>        | Uses their model correctly to give either $y-1 \rightarrow \frac{y^2}{2} - y$ or $y \rightarrow \frac{y^2}{2}$                                                                                       |
| <b>B1:</b>        | $V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or $0.0464\pi$ <b>or</b> $\frac{29}{625}\pi$ , o.e.                                                                                                         |
| <b>M1:</b>        | Depends on <b>both</b> previous M marks<br>Uses the model to find $V_{\text{their cylinder}}$ – their integrated volume                                                                              |
| <b>A1:</b>        | 0.136 cao                                                                                                                                                                                            |
| (d)               |                                                                                                                                                                                                      |
| <b>B1:</b>        | States an acceptable limitation of the model                                                                                                                                                         |
| (e)               |                                                                                                                                                                                                      |
| <b>B1ft:</b>      | Compares the actual volume with their answer to (c). Makes an assessment of the model.<br>E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Scheme                                                              | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|------------|------|
| <b>8(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | A1         | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$ | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $= 2.42 \text{ (2dp) } \text{cao}$                                  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                     | <b>(2)</b> |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                     |            |      |
| <p><b>(a)</b></p> <p><b>M1:</b> Circle</p> <p><b>A1:</b> Centre (0, 4) and above the real axis</p> <p><b>M1:</b> Half-line</p> <p><b>A1:</b> (−3, 4) positioned correctly and the half-line intersects the top of the circle on the y-axis</p> <p><b>M1:</b> Depends on <b>both</b> previous M marks Shades in a region inside the circle and below the half-line</p> <p><b>A1:</b> cso</p> <p><b>Note:</b> Final A1 mark is dependent on all previous marks being scored in part (a)</p> |                                                                     |            |      |
| <p><b>(b)</b></p> <p><b>M1:</b> Uses trigonometry to give an expression for an angle in the range <math>\left(\frac{\pi}{2}, \pi\right)</math> or <math>(90^\circ, 180^\circ)</math></p> <p><b>A1:</b> 2.42 cao</p>                                                                                                                                                                                                                                                                       |                                                                     |            |      |

| Question    | Scheme                                                                                                                                                                                                                                                                                                          | Marks      | AOs  |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>9(a)</b> | $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$                                                                                      | M1         | 3.1a |
|             | $\{\overrightarrow{OF} = \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}\}$                                                                                                                                                                      | M1         | 1.1b |
|             | $\{\overrightarrow{OF} \cdot \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix} -3 + 12\lambda \\ 1 + 3\lambda \\ -7 + 18\lambda \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$<br>$\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$ | dM1        | 1.1b |
|             | $\Rightarrow \lambda = \frac{1}{3}$                                                                                                                                                                                                                                                                             | A1         | 1.1b |
|             | $\{\overrightarrow{OF} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$<br>and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$                                                                          | dM1        | 3.1a |
|             | $= \sqrt{6}$ or 2.449...                                                                                                                                                                                                                                                                                        | A1         | 1.1b |
|             | $> 2$ , so the octopus is not able to catch the fish $F$                                                                                                                                                                                                                                                        | A1ft       | 3.2a |
|             |                                                                                                                                                                                                                                                                                                                 | <b>(7)</b> |      |



| Question | Scheme                                                                                                                                                                                                                                                                                            | Marks |      |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
|          | <b>9(a) Alternative 1</b>                                                                                                                                                                                                                                                                         |       |      |
|          | $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$                                                                        | M1    | 3.1a |
|          | $\left\{ \overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right.$                    | M1    | 1.1b |
|          | $\cos \theta = \frac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{ \overrightarrow{OA}   \overrightarrow{AB} } = \frac{\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} \cdot \sqrt{(12)^2 + (3)^2 + (18)^2}}$ | dM1   | 1.1b |
|          | $\left\{ \cos \theta = \frac{-36 + 3 - 126}{\sqrt{59} \cdot \sqrt{477}} = \frac{-159}{\sqrt{59} \cdot \sqrt{477}} \right\}$                                                                                                                                                                       |       |      |
|          | $\theta = 161.4038029... \text{ or } 18.59619709... \text{ or } \sin \theta = 0.3188964021...$                                                                                                                                                                                                    | A1    | 1.1b |
|          | minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2} \sin(18.59619709...)$                                                                                                                                                                                                                          | dM1   | 3.1a |
|          | $= \sqrt{6} \text{ or } 2.449...$                                                                                                                                                                                                                                                                 | A1    | 1.1b |
|          | $> 2$ , so the octopus is not able to catch the fish $F$                                                                                                                                                                                                                                          | A1ft  | 3.2a |
|          |                                                                                                                                                                                                                                                                                                   | (7)   |      |
|          | <b>9(a) Alternative 2</b>                                                                                                                                                                                                                                                                         |       |      |
|          | $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$                                                                        | M1    | 3.1a |
|          | $\left\{ \overrightarrow{OF} = \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right.$                                                                                                                                            | M1    | 1.1b |
|          | $ \overrightarrow{OF} ^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$                                                                                                                                                                                                            | dM1   | 1.1b |
|          | $= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$                                                                                                                                                                                                     |       |      |
|          | $= 477\lambda^2 - 318\lambda + 59$                                                                                                                                                                                                                                                                | A1    | 1.1b |
|          | $= 53(3\lambda - 1)^2 + 6$                                                                                                                                                                                                                                                                        | dM1   | 3.1a |
|          | minimum distance $= \sqrt{6} \text{ or } 2.449...$                                                                                                                                                                                                                                                | A1    | 1.1b |
|          | $> 2$ , so the octopus is not able to catch the fish $F$                                                                                                                                                                                                                                          | A1ft  | 3.2a |
|          |                                                                                                                                                                                                                                                                                                   | (7)   |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Scheme                                                                                                                                                                                                          | Marks | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>9(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | e.g.<br>Fish $F$ may not swim in an exact straight line from $A$ to $B$<br>Fish $F$ may hit an obstacle whilst swimming from $A$ to $B$<br>Fish $F$ may deviate his path to avoid being caught by the octopus   | B1    | 3.5b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                 | (1)   |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | e.g.<br>Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed<br>Octopus may during the fish $F$ 's motion move away from its fixed location at $O$ | B1    | 3.5b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                 | (1)   |      |
| <b>(9 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                 |       |      |
| Question 9 notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                 |       |      |
| <p><b>(a)</b></p> <p><b>M1:</b> Attempts to find <math>\overrightarrow{OB} - \overrightarrow{OA}</math> or <math>\overrightarrow{OA} - \overrightarrow{OB}</math> or the direction vector <math>\mathbf{d}</math></p> <p><b>M1:</b> Applies <math>\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})</math> or equivalent</p> <p><b>M1:</b> Depends on previous M mark. Writes down<br/>(their <math>\overrightarrow{OF}</math> which is in terms of <math>\lambda</math>) • (their <math>\overrightarrow{AB}</math>) = 0. Can be implied</p> <p><b>A1:</b> Lambda is correct. e.g. <math>\lambda = \frac{1}{3}</math> for <math>\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}</math> or <math>\lambda = 1</math> for <math>\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}</math></p> <p><b>M1:</b> Depends on previous M mark. Complete method for finding <math> \overrightarrow{OF} </math></p> <p><b>A1:</b> <math>\sqrt{6}</math> or awrt 2.4</p> <p><b>A1ft :</b> Correct follow through conclusion, which is in context with the question</p> |                                                                                                                                                                                                                 |       |      |
| <p><b>Alternative 1</b></p> <p><b>(a)</b></p> <p><b>M1:</b> Attempts to find <math>\overrightarrow{OB} - \overrightarrow{OA}</math> or <math>\overrightarrow{OA} - \overrightarrow{OB}</math> or the direction vector <math>\mathbf{d}</math></p> <p><b>M1:</b> Realisation that the dot product is required between <math>\overrightarrow{OA}</math> and their <math>\overrightarrow{AB}</math>. (o.e.)</p> <p><b>M1:</b> Depends on previous M mark. Applies dot product formula between <math>\overrightarrow{OA}</math> and their <math>\overrightarrow{AB}</math> (o.e.)</p> <p><b>A1:</b> <math>\theta = \text{awrt } 161.4</math> or <math>\text{awrt } 18.6</math> or <math>\sin \theta = \text{awrt } 0.319</math></p> <p><b>M1:</b> Depends on previous M mark. (their <math>OA</math>)sin(their <math>\theta</math>)</p> <p><b>A1:</b> <math>\sqrt{6}</math> or awrt 2.4</p> <p><b>A1ft :</b> Correct follow through conclusion, which is in context with the question</p>                                                                                                                                                                             |                                                                                                                                                                                                                 |       |      |

Question 9 notes continued:

**Alternative 2**

**(a)**

**M1:** Attempts to find  $\overrightarrow{OB} - \overrightarrow{OA}$  **or**  $\overrightarrow{OA} - \overrightarrow{OB}$  **or** the direction vector **d**

**M1:** Applies  $\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$  or equivalent

**M1:** Depends on previous M mark. Applies Pythagoras by finding  $|\overrightarrow{OF}|^2$ , o.e.

**A1:**  $|\overrightarrow{OF}|^2 = 477\lambda^2 - 318\lambda + 59$

**M1:** Depends on previous M mark. Method of completing the square or differentiating their  $|\overrightarrow{OF}|^2$  w.r.t.  $\lambda$

**A1:**  $\sqrt{6}$  or awrt 2.4

**A1ft :** Correct follow through conclusion, which is in context with the question

**(b)**

**B1:** An acceptable criticism for fish F, which is in context with the question

**(c)**

**B1:** An acceptable criticism for the octopus, which is in context with the question



Write your name here

Surname

Other names

**Pearson Edexcel**  
**Level 3 GCE**

Centre Number

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2A: Further Pure Mathematics 1 and Further Pure Mathematics 2**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2A**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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## SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution  $t = \tan\left(\frac{x}{2}\right)$  to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \quad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

- (b) Use the substitution  $t = \tan\left(\frac{x}{2}\right)$  and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \quad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

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2. The value,  $V$  hundred pounds, of a particular stock  $t$  hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \quad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula  $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$  to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)



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3. Use algebra to find the set of values of  $x$  for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

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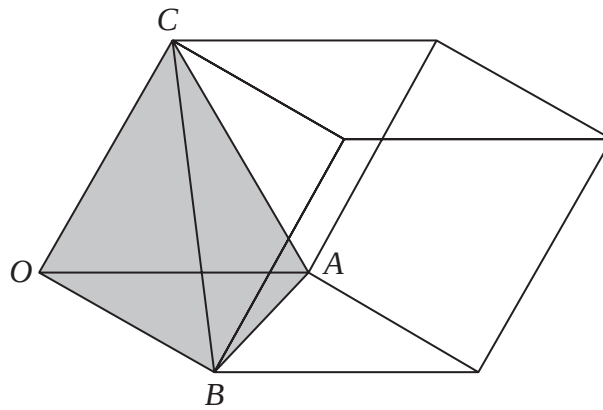
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[illegible]

4.



### Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are  $O(0, 0, 0)$ ,  $A(2, 0, 0)$ ,  $B(0, 3, 1)$  and  $C(1, 1, 2)$ , where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

- (a) Find the surface area of the glued face of the tetrahedron. (4)
- (b) Find the volume of glass contained in this parallelepiped. (5)
- (c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture. (1)

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**Question 4 continued**

**(Total for Question 4 is 10 marks)**

5.

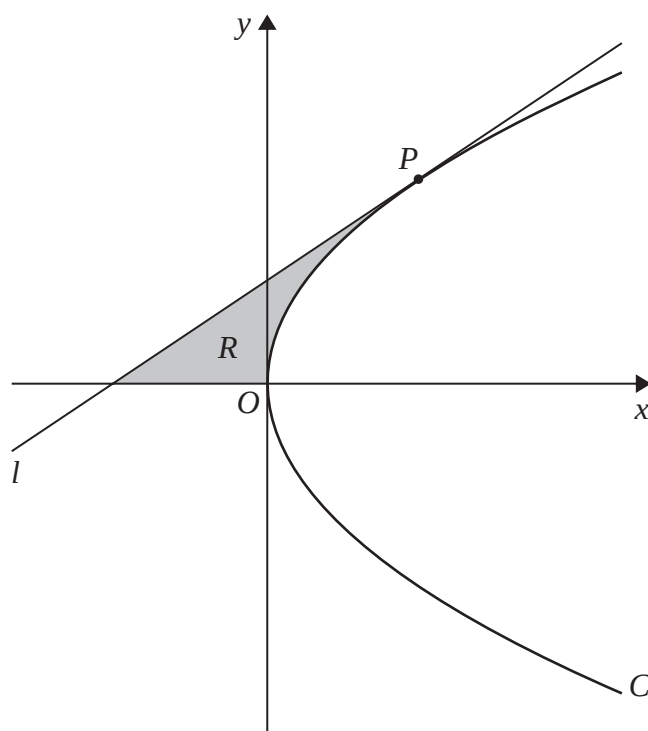
Diagram not  
drawn to scale

Figure 2

[ You may quote without proof that for the general parabola  $y^2 = 4ax$ ,  $\frac{dy}{dx} = \frac{2a}{y}$  ]

The parabola  $C$  has equation  $y^2 = 16x$ .

(a) Deduce that the point  $P(4p^2, 8p)$  is a general point on  $C$ .

(1)

The line  $l$  is the tangent to  $C$  at the point  $P$ .

(b) Show that an equation for  $l$  is

$$py = x + 4p^2$$

(3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the line  $l$ , the  $x$ -axis and the parabola  $C$ .

The line  $l$  intersects the directrix of  $C$  at the point  $B$ , where the  $y$  coordinate of  $B$  is  $\frac{10}{3}$

Given that  $p > 0$

(c) show that the area of  $R$  is 36

(8)

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[illegible]

**Question 5 continued**

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**(Total for Question 5 is 12 marks)**

**TOTAL FOR SECTION A IS 40 MARKS**



**SECTION B**

Answer ALL questions. Write your answers in the spaces provided.

6. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix  $\mathbf{A}$ .

(2)

(b) Hence show that  $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$ .

(3)

**Question 6 continued**

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**(Total for Question 6 is 5 marks)**

7. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

- (ii) Use the Euclidean algorithm to find integers  $a$  and  $b$  such that

$$27a + 31b = 1$$

(4)

**Question 7 continued**

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**(Total for Question 7 is 6 marks)**

(2)

**Question 8 continued**

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**(Total for Question 8 is 8 marks)**

9. The operation  $*$  is defined on the set  $S = \{0, 2, 3, 4, 5, 6\}$  by  $x*y = x + y = xy \pmod{7}$

| $*$ | 0 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|
| 0   |   |   |   |   |   |   |
| 2   |   | 0 |   |   |   |   |
| 3   |   |   |   |   |   | 5 |
| 4   |   |   |   |   |   |   |
| 5   |   |   |   |   |   |   |
| 6   |   |   |   |   |   |   |

- (a) (i) Complete the Cayley table shown above  
 (ii) Show that  $S$  is a group under the operation  $*$   
 (You may assume the associative law is satisfied.)

(6)

- (b) Show that the element 4 has order 3

(2)

- (c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

**Question 9 continued**

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**(Total for Question 9 is 11 marks)**





**Question 10 continued**

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**Question 10 continued**

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**(Total for Question 10 is 10 marks)**

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**

## Paper 2 Option A

### Further Pure Mathematics 1 Mark Scheme (Section A)

| Question                                                                                                                                                                                                                                                                                                                                                                        | Scheme                                                                               | Marks      | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                                                     | $\sec x - \tan x = \frac{1}{\frac{1-t^2}{1+t^2}} - \frac{2t}{1-t^2}$                 | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                 | $= \frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2} = \frac{1-2t+t^2}{1-t^2}$                  | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                 | $= \frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t} *$                                   | A1*        | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                      | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                      | $\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$          | M1         | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                                                 | $= \frac{1+t^2-2t}{1+t^2+2t}$                                                        | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                 | $= \frac{(1-t)^2}{(1+t)^2} = \left(\frac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2 *$ | A1*        | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                      | <b>(3)</b> |      |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                |                                                                                      |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                          |                                                                                      |            |      |
| <b>(a)</b><br><b>M1:</b> Uses $\sec x = \frac{1}{\cos x}$ and the $t$ -substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of $t$<br><b>M1:</b> Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$<br><b>A1*:</b> Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer |                                                                                      |            |      |
| <b>(b)</b><br><b>M1:</b> Uses the $t$ -substitution for $\sin x$ in both numerator and denominator<br><b>M1:</b> Multiplies through by $1+t^2$ in numerator and denominator<br><b>A1*:</b> Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result                                                                         |                                                                                      |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                            | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>2</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | £300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$ ;<br>half an hour after purchase $\Rightarrow t_2 = 1.5$ , so step $h$ required is 0.25 | B1         | 3.3  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $t_0 = 1, V_0 = 3, \left(\frac{dV}{dt}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$                                                                              | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_1 \approx V_0 + h \left(\frac{dV}{dt}\right)_0 = 3 + 0.25 \times 2 = \dots$                                                                                    | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $= 3.5$                                                                                                                                                           | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\left(\frac{dV}{dt}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$                                                | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_2 \approx V_1 + h \left(\frac{dV}{dt}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\dots$ , so £396<br>(nearest £)                                       | A1         | 3.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                   | <b>(6)</b> |      |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                   |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                   |            |      |
| <p><b>B1:</b> Identifies the correct initial conditions and requirement for <math>h</math></p> <p><b>M1:</b> Uses the model to evaluate <math>\frac{dV}{dt}</math> at <math>t_0</math>, using their <math>t_0</math> and <math>V_0</math></p> <p><b>M1:</b> Applies the approximation formula with their values</p> <p><b>A1ft:</b> 3.5 or exact equivalent. Follow through their step value</p> <p><b>M1:</b> Attempt to find <math>\left(\frac{dV}{dt}\right)_1</math> with their 3.5</p> <p><b>A1:</b> Applies the approximation and interprets the result to give £396</p> |                                                                                                                                                                   |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                   | Marks      | AOs         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|------------|-------------|
| <b>3</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{1}{x} < \frac{x}{x+2}$                                                                            |            |             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$                                              | M1         | 2.1         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$         | M1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | At least two correct critical values from $-2, -1, 0, 2$                                                 | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | All four correct critical values $-2, -1, 0, 2$                                                          | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$ | M1<br>A1   | 2.2a<br>2.5 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                          | <b>(6)</b> |             |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                          |            |             |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                          |            |             |
| <p><b>M1:</b> Gathers terms on one side and puts over common denominator, or multiply by <math>x^2(x+2)^2</math> and then gather terms on one side</p> <p><b>M1:</b> Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors</p> <p><b>A1:</b> At least 2 correct critical values found</p> <p><b>A1:</b> Exactly 4 correct critical values</p> <p><b>M1:</b> Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means</p> <p><b>A1:</b> Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept <math>\mathbb{R} - ([-2, -1] \cup [0, 2])</math> or <math>\{x \in \mathbb{R} : x &lt; -2 \text{ or } -1 &lt; x &lt; 0 \text{ or } x &gt; 2\}</math></p> |                                                                                                          |            |             |

| Question    | Scheme                                                                                                                                                                                 | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b> | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2} \mathbf{AB} \times \mathbf{AC} $                                          | M1         | 3.1a |
|             | $\frac{1}{2} \mathbf{AB} \times \mathbf{AC}  = \frac{1}{2} (-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 2\mathbf{k}) $                                 | M1         | 1.1b |
|             | $= \frac{1}{2} 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} $                                                                                                                                | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35}(\text{m}^2)$                                                                                                                                                   | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
|             | <b>Alternative</b>                                                                                                                                                                     |            |      |
|             | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{ \mathbf{AB} ^2 \mathbf{AC} ^2 - (\mathbf{AB} \cdot \mathbf{AC})^2}$ | M1         | 3.1a |
|             | $ \mathbf{AB} ^2 = 4 + 9 + 1 = 14$ , $ \mathbf{AC} ^2 = 1 + 1 + 4 = 6$<br>and $\mathbf{AB} \cdot \mathbf{AC} = 2 + 3 + 2 = 7$                                                          | M1         | 1.1b |
|             | So area of glue is $= \frac{1}{2}\sqrt{(14)(6) - (7)^2}$                                                                                                                               | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35} (\text{m}^2)$                                                                                                                                                  | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
| <b>(b)</b>  | Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC} \cdot (\mathbf{OA} \times \mathbf{OB}))$                                                                | M1         | 3.1a |
|             | $= \frac{1}{6}(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} \times (3\mathbf{j} + \mathbf{k}))$                                                                           | M1         | 1.1b |
|             | $= \frac{10}{6} = \frac{5}{3}$                                                                                                                                                         | A1         | 1.1b |
|             | Volume of parallelepiped is $6 \times$ volume of tetrahedron ( $= 10$ ),<br>so volume of glass is difference between these, viz. $10 - \frac{5}{3} = \dots$                            | M1         | 3.1a |
|             | Volume of glass $= \frac{25}{3}(\text{m}^3)$                                                                                                                                           | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(5)</b> |      |



| Question                                                                                                                                | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                                                                                                                                         | <b>4(b) Alternative</b>                                                                                                                                                                                                                                               |            |      |
|                                                                                                                                         | $-\mathbf{j} + 3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                                         | M1         | 3.1a |
|                                                                                                                                         | Area $AOB = \frac{1}{2} \times  \mathbf{OA}  \times  \mathbf{OB}  = \frac{1}{2} \times 2 \times \sqrt{10} = \sqrt{10}$                                                                                                                                                | A1         | 1.1b |
|                                                                                                                                         | $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k}) \Rightarrow p = \frac{1}{2}$<br>and so height of tetrahedron is<br>$h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10}$ | M1         | 3.1a |
|                                                                                                                                         | Volume of glass is $V = 5 \times \text{Volume of tetrahedron}$<br>$= 5 \times \frac{1}{3} \sqrt{10} \times \frac{1}{2} \sqrt{10}$                                                                                                                                     | M1         | 1.1b |
|                                                                                                                                         | $= \frac{25}{3} (\text{m}^3)$                                                                                                                                                                                                                                         | A1         | 1.1b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(5)</b> |      |
| <b>(c)</b>                                                                                                                              | The glued surfaces may distort the shapes / reduce the volume of concrete<br>Measurements in m may not be accurate<br>The surface of the concrete tetrahedron may not be smooth<br>Pockets of air may form when the concrete is being poured                          | B1         | 3.2b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Question 4 notes:                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Accept use of column vectors throughout                                                                                                 |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a)</b>                                                                                                                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Shows an understanding of what is required via an attempt at finding the area of triangle $ABC$                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Any correct method for the triangle area is fine                                                                             |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds <b>AB</b> and <b>AC</b> or any other appropriate pair of vectors to use in the vector product and attempts to use them |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent             |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a) Alternative</b>                                                                                                                  |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides                               |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> May use different sides to those shown                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Correct full method to find the area of the triangle using their two sides                                                   |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> $\frac{1}{2}\sqrt{35}$ or exact equivalent                                                                                   |                                                                                                                                                                                                                                                                       |            |      |

|                             |                                                                                                                                                                                                                                                                                                                  |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 4 notes continued: |                                                                                                                                                                                                                                                                                                                  |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Attempts volume of concrete by finding volume of tetrahedron with appropriate method                                                                                                                                                                                                                             |
| <b>M1:</b>                  | Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted                                                                                                                                                                                                |
| <b>A1:</b>                  | Correct value for the volume of concrete                                                                                                                                                                                                                                                                         |
| <b>M1:</b>                  | Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped                                                                                                                                    |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(b)</b>                  | <b>Alternative</b>                                                                                                                                                                                                                                                                                               |
| <b>M1:</b>                  | Notes (or works out using scalar products) that $-\mathbf{j} + 3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                           |
| <b>A1:</b>                  | Finds (using that $\mathbf{OA}$ and $\mathbf{OB}$ are perpendicular), area of $AOB = \sqrt{10}$                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Solves $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k})$ to get the height of the tetrahedron<br>$\left[ (\mu = \lambda =) p = \frac{1}{2}, \text{ so } h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10} \right]$ |
| <b>M1:</b>                  | Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)                                                                                                                                                                                          |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(c)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>B1:</b>                  | Any acceptable reason in context                                                                                                                                                                                                                                                                                 |

| Question    | Scheme                                                                                                                     | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>5(a)</b> | $y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$<br>$\Rightarrow P(4p^2, 8p)$ is a general point on $C$                 | B1         | 2.2a |
|             |                                                                                                                            | <b>(1)</b> |      |
| <b>(b)</b>  | $y^2 = 16x$ gives $a = 4$ , or $2y \frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$                                    | M1         | 2.2a |
|             | $l: y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$                                                                          | M1         | 1.1b |
|             | leading to $py = x + 4p^2$ *                                                                                               | A1*        | 2.1  |
|             |                                                                                                                            | <b>(3)</b> |      |
| <b>(c)</b>  | $B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                            | M1         | 3.1a |
|             | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                 | M1         | 1.1b |
|             | $p = \frac{3}{2}$ and $l$ cuts $x$ -axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$     | M1         | 2.1  |
|             | $x = -9$                                                                                                                   | A1         | 1.1b |
|             | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}} dx$ | M1         | 2.1  |
|             | $\int 4x^{\frac{1}{2}} dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} (+c)$ or $\frac{8}{3}x^{\frac{3}{2}} (+c)$   | M1         | 1.1b |
|             |                                                                                                                            | A1         | 1.1b |
|             | $\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *                     | A1*        | 1.1b |
|             |                                                                                                                            | <b>(8)</b> |      |

| Question          | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                   | <b>5(c) Alternative 1</b>                                                                                                                                                                                                                                             |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                                | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                            | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ into } l \text{ gives } \frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                                 | M1         | 2.1  |
|                   | $x = \frac{3}{2}y - 9$                                                                                                                                                                                                                                                | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left( \frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right) \right) dy$                                                                                                                     | M1         | 2.1  |
|                   | $\int \left( \frac{1}{16}y^2 - \frac{3}{2}y + 9 \right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y (+c)$                                                                                                                                                              | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                       | A1         | 1.1b |
|                   | $\text{Area}(R) = \left( \frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12) \right) - (0)$<br>$= 36 - 108 + 108 = 36 *$                                                                                                                                                   | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                       | <b>(8)</b> |      |
|                   | <b>5(c) Alternative 2</b>                                                                                                                                                                                                                                             |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                                | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                            | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ and } l \text{ cuts px-axis when } \frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                    | M1         | 2.1  |
|                   | $x = -9$                                                                                                                                                                                                                                                              | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \text{ and } x = 0 \text{ in } l: y = \frac{2}{3}x + 6 \text{ gives } y = 6$<br>$\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left( \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dx$ | M1         | 2.1  |
|                   | $\int \left( \frac{2}{3}x + 6 - 4x^{\frac{1}{2}} \right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} (+c)$                                                                                                                                                  | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                       | A1         | 1.1b |
|                   | $\text{Area}(R) = 27 + \left( \left( \frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}(9^{\frac{3}{2}}) \right) - (0) \right)$<br>$= 27 + (27 + 54 - 72) = 27 + 9 = 36 *$                                                                                                         | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                       | <b>(8)</b> |      |
| <b>(12 marks)</b> |                                                                                                                                                                                                                                                                       |            |      |

|                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 5 notes:        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| (a)                      | <p><b>B1:</b> Substitutes <math>y_p = 8p</math> into <math>y^2</math> to obtain <math>64p^2</math> and substitutes <math>x_p = 4p^2</math> into <math>16x</math> to obtain <math>64p^2</math> and concludes that <math>P</math> lies on <math>C</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| (b)                      | <p><b>M1:</b> Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it</p> <p><b>M1:</b> Applies <math>y - 8p = m(x - 4p^2)</math>, with their tangent gradient <math>m</math>, which is in terms of <math>p</math>. Accept use of <math>8p = m(4p^2) + c</math> with a clear attempt to find <math>c</math></p> <p><b>A1*:</b> Obtains <math>py = x + 4p^2</math> by <b>cs0</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| (c)                      | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math></p> <p><b>M1:</b> Substitutes their <math>p</math> (which must be positive) and <math>y = 0</math> into <math>l</math> and solves to give <math>x = \dots</math></p> <p><b>A1:</b> Finds that <math>l</math> cuts the <math>x</math>-axis at <math>x = -9</math></p> <p><b>M1:</b> Fully correct method for finding the area of <math>R</math><br/> i.e. <math>\frac{1}{2}(\text{their } x_p - "-9")(\text{their } y_p) - \int_0^{\text{their } x_p} 4x^2 dx</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda x^{\frac{1}{2}}</math> to give <math>\pm \mu x^{\frac{3}{2}}</math>, where <math>\lambda, \mu \neq 0</math></p> <p><b>A1:</b> Integrates <math>4x^{\frac{1}{2}}</math> to give <math>\frac{8}{3}x^{\frac{3}{2}}</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p>                                                             |
| (c) <b>Alternative 1</b> | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math><br/> Substitutes their <math>p</math> (which must be positive) into <math>l</math> and rearranges to give <math>x = \dots</math></p> <p><b>M1:</b> Finds <math>l</math> as <math>x = \frac{3}{2}y - 9</math></p> <p><b>A1:</b> Fully correct method for finding the area of <math>R</math></p> <p><b>M1:</b> i.e. <math>\int_0^{\text{their } y_p} \left( \frac{1}{16}y^2 - \text{their} \left( \frac{3}{2}y - 9 \right) \right) dy</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda y^2 \pm \mu y \pm \nu</math> to give <math>\pm \alpha y^3 \pm \beta y^2 \pm \nu y</math>, where <math>\lambda, \mu, \nu, \alpha, \beta \neq 0</math></p> <p><b>A1:</b> Integrates <math>\frac{1}{16}y^2 - \left( \frac{3}{2}y - 9 \right)</math> to give <math>\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p> |

Question 5 notes continued:

**(c) Alternative 2**

**M1:** Substitutes their  $x = "-a"$  and  $y = \frac{10}{3}$  into  $l$

**M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  $p = \dots$

**M1:** Substitutes their  $p$  (which must be positive) and  $y = 0$  into  $l$  and solves to give  $x = \dots$

**A1:** Finds that  $l$  cuts the  $x$ -axis at  $x = -9$

**M1:** Fully correct method for finding the area of  $R$

i.e.  $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_p} \left( \text{their } \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dy$

**M1:** Integrates  $\pm \lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$  to give  $\pm \alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$ , where  $\lambda, \mu, \nu, \alpha, \beta \neq 0$

**A1:** Integrates  $\left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right)$  to give  $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$ , simplified or un-simplified

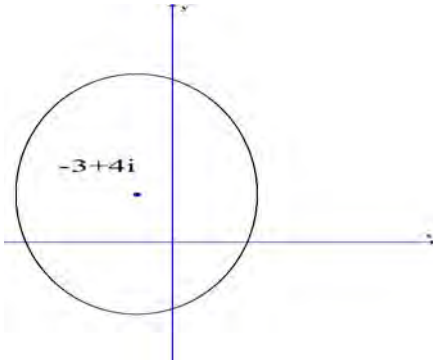
**A1\*:** Fully correct proof leading to a correct answer of 36

Further Pure Mathematics 2 Mark Scheme (Section B)

| Question                                                                                                                                                                      | Scheme                                                                                                             | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>6(a)</b>                                                                                                                                                                   | Consider $\det \begin{pmatrix} 3-\lambda & 1 \\ 6 & 4-\lambda \end{pmatrix} = (3-\lambda)(4-\lambda) - 6$          | M1         | 1.1b |
|                                                                                                                                                                               | So $\lambda^2 - 7\lambda + 6 = 0$ is characteristic equation                                                       | A1         | 1.1b |
|                                                                                                                                                                               |                                                                                                                    | <b>(2)</b> |      |
|                                                                                                                                                                               | So $\mathbf{A}^2 = 7\mathbf{A} - 6\mathbf{I}$                                                                      | B1ft       | 1.1b |
| <b>(b)</b>                                                                                                                                                                    | Multiplies both sides of their equation by $\mathbf{A}$ so $\mathbf{A}^3 = 7\mathbf{A}^2 - 6\mathbf{A}$            | M1         | 3.1a |
|                                                                                                                                                                               | Uses $\mathbf{A}^3 = 7(7\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$ So $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}^*$ | A1*cso     | 1.1b |
|                                                                                                                                                                               |                                                                                                                    | <b>(3)</b> |      |
| <b>(5 marks)</b>                                                                                                                                                              |                                                                                                                    |            |      |
| Notes:                                                                                                                                                                        |                                                                                                                    |            |      |
| <b>(a)</b>                                                                                                                                                                    |                                                                                                                    |            |      |
| <b>M1:</b> Complete method to find characteristic equation                                                                                                                    |                                                                                                                    |            |      |
| <b>A1:</b> Obtains a correct three term quadratic equation – may use variable other than $\lambda$                                                                            |                                                                                                                    |            |      |
| <b>(b)</b>                                                                                                                                                                    |                                                                                                                    |            |      |
| <b>B1ft:</b> Uses Cayley Hamilton Theorem to produce equation replacing $\lambda$ with $\mathbf{A}$ and constant term with constant multiple of identity matrix, $\mathbf{I}$ |                                                                                                                    |            |      |
| <b>M1:</b> Multiplies equation by $\mathbf{A}$                                                                                                                                |                                                                                                                    |            |      |
| <b>A1*:</b> Replaces $\mathbf{A}^2$ by linear expression in $\mathbf{A}$ and achieves printed answer with no errors                                                           |                                                                                                                    |            |      |

| Question                                                                                                                                                                                                                                                                                                      | Scheme                                                                                                                                                                    | Marks      | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>7(i)</b>                                                                                                                                                                                                                                                                                                   | Adding digits $8 + 1 + 8 + 4 = 21$ which is divisible by 3 ( or continues to add digits giving $2+1=3$ which is divisible by 3 ) so concludes that 8184 is divisible by 3 | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                               | 8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6                                                                              | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                           | <b>(2)</b> |      |
| <b>(ii)</b>                                                                                                                                                                                                                                                                                                   | Starts Euclidean algorithm $31=27 \times 1 + 4$ and $27 = 4 \times 6 + 3$                                                                                                 | M1         | 1.2  |
|                                                                                                                                                                                                                                                                                                               | $4 = 3 \times 1 + 1$ ( so hcf = 1 )                                                                                                                                       | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                               | So $1 = 4 - 3 \times 1 = 4 - (27 - 4 \times 6) \times 1 = 4 \times 7 - 27 \times 1$                                                                                       | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                               | $(31 - 27 \times 1) \times 7 - 27 \times 1 = 31 \times 7 - 27 \times 8$<br>$a = -8$ and $b = 7$                                                                           | A1cso      | 1.1b |
|                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                           | <b>(4)</b> |      |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                              |                                                                                                                                                                           |            |      |
| Notes:                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                           |            |      |
| <b>(i)</b><br><b>M1:</b> Explains divisibility by 3 rule in context of this number by adding digits<br><b>A1:</b> Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6                                                                             |                                                                                                                                                                           |            |      |
| <b>(ii)</b><br><b>M1:</b> Uses Euclidean algorithm showing two stages<br><b>A1:</b> Completes the algorithm. Does not need to state that hcf = 1<br><b>M1:</b> Starts reversal process, doing two stages and simplifying<br><b>A1cso:</b> Correct completion, giving clear answer following complete solution |                                                                                                                                                                           |            |      |



| Question                                                                                                                                                                                                                                                                                                                                                                                      | Scheme                                                                            | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|------------|------|
| <b>8(a)</b>                                                                                                                                                                                                                                                                                                                                                                                   | $(x - 9)^2 + (y + 12)^2 = 4[x^2 + y^2]$                                           | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                               | $3x^2 + 3y^2 + 18x - 24y - 225 = 0$ which is the equation of a circle             | A1*        | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                               | As $x^2 + y^2 + 6x - 8y - 75 = 0$ so $(x + 3)^2 + (y - 4)^2 = 10^2$               | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                               | Giving centre at $(-3, 4)$ and radius = 10                                        | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | <b>(4)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                    |  | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                    | Values range from <b>their</b> $-3 - 10$ to $-3 + 10$                             | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                               | So $-13 \leq \text{Re}(w) \leq 7$                                                 | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                   | <b>(2)</b> |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                                                                                                                                              |                                                                                   |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                   |            |      |
| <p><b>(a)</b><br/> <b>M1:</b> Obtains an equation in terms of <math>x</math> and <math>y</math> using the given information<br/> <b>A1:</b> Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle<br/> <b>M1:</b> Completes the square for their equation to find centre and radius<br/> <b>A1ft:</b> Both correct</p> |                                                                                   |            |      |
| <p><b>(b)</b><br/> <b>M1:</b> Draws a circle with centre and radius as given from <b>their</b> equation<br/> <b>A1:</b> Correct circle drawn, as above, with centre at <math>-3 + 4i</math> and passing through all four quadrants</p>                                                                                                                                                        |                                                                                   |            |      |
| <p><b>(c)</b><br/> <b>M1:</b> Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using “their <math>-3 - 10</math>” to “their <math>-3 + 10</math>”<br/> <b>A1ft:</b> Correctly obtains the correct answer for their centre and radius</p>                                                                                |                                                                                   |            |      |

| Question   | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Marks  | AOs          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------|--------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----------|--------------|
| 9(a)(i)    | <table><tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>2</td><td>2</td><td>0</td><td></td><td></td><td>4</td><td></td></tr><tr><td>3</td><td>3</td><td></td><td></td><td></td><td></td><td>5</td></tr><tr><td>4</td><td>4</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>5</td><td>5</td><td>4</td><td></td><td></td><td></td><td></td></tr><tr><td>6</td><td>6</td><td></td><td>5</td><td></td><td></td><td></td></tr></table>                     | *      | 0            | 2 | 3 | 4 | 5 | 6 | 0 | 0 | 2 | 3 | 4 | 5 | 6 | 2 | 2 | 0 |   |   | 4 |   | 3 | 3 |   |   |   |   | 5 | 4 | 4 |   |   |   |   |   | 5 | 5 | 4 |   |   |   |   | 6 | 6 |   | 5 |   |   |   | M1       | 1.1b         |
|            | *                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 0      | 2            | 3 | 4 | 5 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 0          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 2      | 3            | 4 | 5 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 2          | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 0      |              |   | 4 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 3          | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |        |              |   |   | 5 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 4          | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |        |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 5          | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 4      |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 6          | 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |        | 5            |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            | <table><tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>2</td><td>2</td><td>0</td><td>6</td><td>5</td><td>4</td><td>3</td></tr><tr><td>3</td><td>3</td><td>6</td><td>4</td><td>2</td><td>0</td><td>5</td></tr><tr><td>4</td><td>4</td><td>5</td><td>2</td><td>6</td><td>3</td><td>0</td></tr><tr><td>5</td><td>5</td><td>4</td><td>0</td><td>3</td><td>6</td><td>2</td></tr><tr><td>6</td><td>6</td><td>3</td><td>5</td><td>0</td><td>2</td><td>4</td></tr></table> | *      | 0            | 2 | 3 | 4 | 5 | 6 | 0 | 0 | 2 | 3 | 4 | 5 | 6 | 2 | 2 | 0 | 6 | 5 | 4 | 3 | 3 | 3 | 6 | 4 | 2 | 0 | 5 | 4 | 4 | 5 | 2 | 6 | 3 | 0 | 5 | 5 | 4 | 0 | 3 | 6 | 2 | 6 | 6 | 3 | 5 | 0 | 2 | 4 | M1<br>A1 | 1.1b<br>1.1b |
| *          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 2      | 3            | 4 | 5 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 0          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 2      | 3            | 4 | 5 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 2          | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 0      | 6            | 5 | 4 | 3 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 3          | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 6      | 4            | 2 | 0 | 5 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 4          | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 5      | 2            | 6 | 3 | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 5          | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 4      | 0            | 3 | 6 | 2 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| 6          | 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 3      | 5            | 0 | 2 | 4 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| (ii)       | Identity is zero and there is closure as shown above                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | M1     | 2.1          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            | 3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse, 0 is identity so is self-inverse                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | M1     | 2.5          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            | Associative law may be assumed so S forms a group                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | A1     | 1.1b         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | (6)    |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| (b)        | $4*4*4 = 4* (4 * 4) = 4 * 6$ or $4*4*4 = (4* 4) * 4 = 6* 4$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | M1     | 2.1          |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            | $= 0$ (the identity) so 4 has order 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | A1     | 2.2a         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | (2)    |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| (c)        | 3 and 5 each have order 6 so either generates the group                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1     | 3.1a         |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            | <b>Either</b> $3^1 = 3, 3^2 = 4, 3^3 = 2, 3^4 = 6, 3^5 = 5, 3^6 = 0$<br><b>Or</b> $5^1 = 5, 5^2 = 6, 5^3 = 2, 5^4 = 4, 5^5 = 3, 5^6 = 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                 | A1, A1 | 1.1b<br>1.1b |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | (3)    |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |
| (11 marks) |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |        |              |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |          |              |

|                   |                                                                                                                  |
|-------------------|------------------------------------------------------------------------------------------------------------------|
| Question 9 notes: |                                                                                                                  |
| <b>(a)(i)</b>     |                                                                                                                  |
| <b>M1:</b>        | Begins completing the table – obtaining correct first row and first column and using symmetry                    |
| <b>M1:</b>        | Mostly correct – three rows or three columns correct (so demonstrates understanding of using *)                  |
| <b>A1:</b>        | Completely correct                                                                                               |
| <b>(a)(ii)</b>    |                                                                                                                  |
| <b>M1:</b>        | States closure and identifies the identity as zero                                                               |
| <b>M1:</b>        | Finds inverses for each element                                                                                  |
| <b>A1:</b>        | States that associative law is satisfied and so all axioms satisfied and $S$ is a group                          |
| <b>(b)</b>        |                                                                                                                  |
| <b>M1:</b>        | Clearly begins process to find $4*4*4$ reaching $6*4$ or $4*6$ with clear explanation                            |
| <b>A1:</b>        | Gives answer as zero, states identity and deduces that order is 3                                                |
| <b>(c)</b>        |                                                                                                                  |
| <b>M1:</b>        | Finds either 3 or 5 or both                                                                                      |
| <b>A1:</b>        | Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself) |
| <b>A1:</b>        | Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)                           |

| Question                                                                                                                                                                                                                                                                                                                                | Scheme                                                                                                                                                                                                          | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>10(a)</b>                                                                                                                                                                                                                                                                                                                            | $P_{n-1}$ is the population at the end of year $n - 1$ and this is increased by 10% by the end of year $n$ , so is multiplied by $110\% = 1.1$ to give $1.1 \times P_{n-1}$ as new population by natural causes | B1         | 3.3  |
|                                                                                                                                                                                                                                                                                                                                         | $Q$ is subtracted from $1.1 \times P_{n-1}$ as $Q$ is the number of deer removed from the estate                                                                                                                | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                         | So $P_n = 1.1P_{n-1} - Q$ , $P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$                                                                                                                | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                 | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                              | Let $n = 0$ , then $P_0 = (5000 - 10Q)(1.1)^0 + 10Q = 5000$ so result is true when $n = 0$                                                                                                                      | B1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                         | Assume result is true for $n = k$ , $P_k = (1.1)^k (5000 - 10Q) + 10Q$ , then as $P_{k+1} = 1.1P_k - Q$ , so $P_{k+1} = \dots$                                                                                  | M1         | 2.4  |
|                                                                                                                                                                                                                                                                                                                                         | $P_{k+1} = 1.1 \times 1.1^k (5000 - 10Q) + 1.1 \times 10Q - Q$                                                                                                                                                  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                         | So $P_{k+1} = (5000 - 10Q)(1.1)^{k+1} + 10Q$ ,                                                                                                                                                                  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                         | Implies result holds for $n = k + 1$ and so by induction $P_n = (5000 - 10Q)(1.1)^n + 10Q$ , is true for all integer $n$                                                                                        | B1         | 2.2a |
|                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                 | <b>(5)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                              | For $Q < 500$ the population of deer will grow, for $Q > 500$ the population of deer will fall                                                                                                                  | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                         | For $Q = 500$ the population of deer remains steady at 5000,                                                                                                                                                    | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                 | <b>(2)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                 |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                 |            |      |
| <b>(a)</b><br><b>B1:</b> Need to see 10% increase linked to multiplication by scale factor 1.1<br><b>B1:</b> Needs to explain that subtraction of $Q$ indicates the removal of $Q$ deer from population<br><b>B1:</b> Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q$ , $P_0 = 5000$ being the initial number of deer |                                                                                                                                                                                                                 |            |      |
| <b>(b)</b><br><b>B1:</b> Begins proof by induction by considering $n = 0$<br><b>M1:</b> Assumes result is true for $n = k$ and uses iterative formula to consider $n = k + 1$<br><b>A1:</b> Correct algebraic statement<br><b>A1:</b> Correct statement for $k + 1$ in required form<br><b>B1:</b> Completes the inductive argument     |                                                                                                                                                                                                                 |            |      |
| <b>(c)</b><br><b>B1:</b> Consideration of both possible ranges of values for $Q$ as listed in the scheme<br><b>B1:</b> Gives the condition for the steady state                                                                                                                                                                         |                                                                                                                                                                                                                 |            |      |

Write your name here

Surname

Other names

**Pearson Edexcel**  
**Level 3 GCE**

Centre Number

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2B: Further Pure Mathematics 1 and Further Statistics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2B**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

S58539A

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## SECTION A

**Answer ALL questions. Write your answers in the spaces provided.**

1. (a) Use the substitution  $t = \tan\left(\frac{x}{2}\right)$  to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \quad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

- (b) Use the substitution  $t = \tan\left(\frac{x}{2}\right)$  and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \quad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

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2. The value,  $V$  hundred pounds, of a particular stock  $t$  hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \quad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula  $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$  to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)



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[illegible]

3. Use algebra to find the set of values of  $x$  for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

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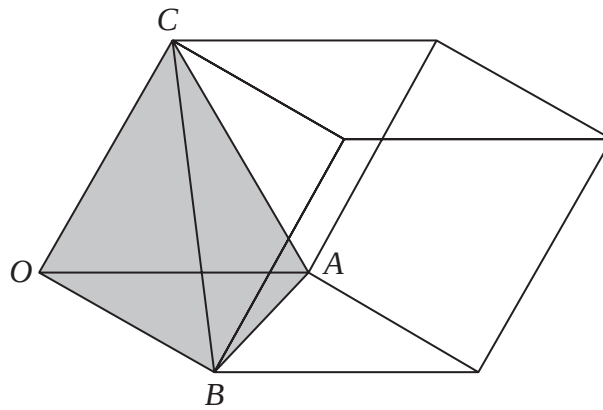
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(Total for Question 3 is 6 marks)

4.



### Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are  $O(0, 0, 0)$ ,  $A(2, 0, 0)$ ,  $B(0, 3, 1)$  and  $C(1, 1, 2)$ , where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

- (a) Find the surface area of the glued face of the tetrahedron. (4)
- (b) Find the volume of glass contained in this parallelepiped. (5)
- (c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture. (1)

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5.

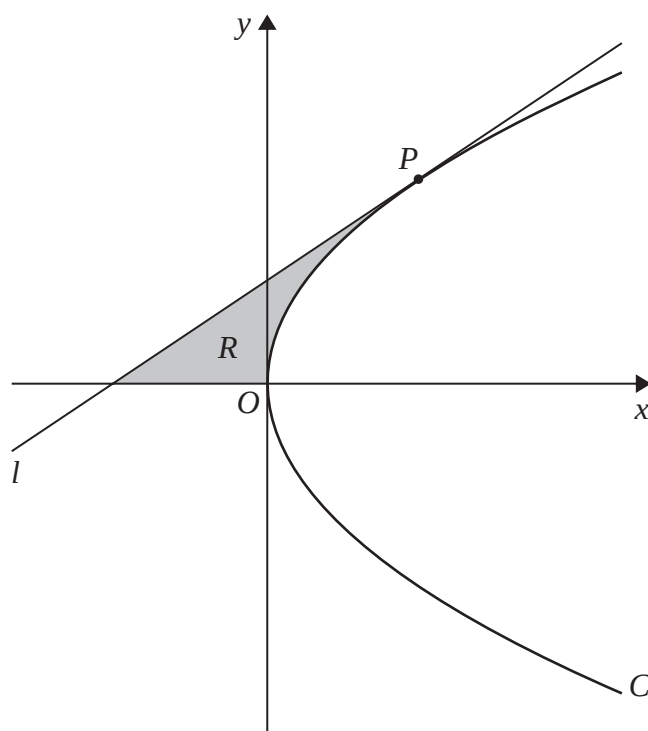
Diagram not  
drawn to scale

Figure 2

[ You may quote without proof that for the general parabola  $y^2 = 4ax$ ,  $\frac{dy}{dx} = \frac{2a}{y}$  ]

The parabola  $C$  has equation  $y^2 = 16x$ .

(a) Deduce that the point  $P(4p^2, 8p)$  is a general point on  $C$ .

(1)

The line  $l$  is the tangent to  $C$  at the point  $P$ .

(b) Show that an equation for  $l$  is

$$py = x + 4p^2$$

(3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the line  $l$ , the  $x$ -axis and the parabola  $C$ .

The line  $l$  intersects the directrix of  $C$  at the point  $B$ , where the  $y$  coordinate of  $B$  is  $\frac{10}{3}$

Given that  $p > 0$

(c) show that the area of  $R$  is 36

(8)

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[illegible]

**Question 5 continued**

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**Question 5 continued**

(Total for Question 5 is 12 marks)

**TOTAL FOR SECTION A IS 40 MARKS**

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|        |        | Language |         |          |
|--------|--------|----------|---------|----------|
|        |        | French   | Spanish | Mandarin |
| Gender | Male   | 23       | 22      | 20       |
|        | Female | 38       | 32      | 15       |

(a) State the null hypothesis for this test. (1)

- (b) Show that the expected frequency for females choosing Spanish is 30.6

- (c) Calculate the test statistic for this test, stating the expected frequencies you have used. (3)

- (d) State whether or not the null hypothesis is rejected. Justify your answer. (2)

- (e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance. (1)

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**Question 6 continued**

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7. The discrete random variable  $X$  has probability distribution given by

|            |     |     |     |     |     |
|------------|-----|-----|-----|-----|-----|
| $x$        | -1  | 0   | 1   | 2   | 3   |
| $P(X = x)$ | $c$ | $a$ | $a$ | $b$ | $c$ |

The random variable  $Y = 2 - 5X$

Given that  $E(Y) = -4$  and  $P(Y \geq -3) = 0.45$

- (a) find the probability distribution of  $X$ .

(7)

Given also that  $E(Y^2) = 75$

- (b) find the exact value of  $\text{Var}(X)$

(2)

- (c) Find  $P(Y > X)$

(2)

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**Question 7 continued**

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[illegible]

8. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

- (a) In a 1 hour period, find the probability that each company hires exactly 2 cars. (2)

- (b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3 (2)

- (c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9 (2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

- (d) (i) find the mean of the number of cars with a defect,  
(ii) find the variance of the number of cars with a defect. (2)

- (e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.  
(ii) Give a reason to support the use of a Poisson approximation. (2)

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**Question 8 continued**

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[illegible]

9. The discrete random variable  $X$  follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i)  $P(X = 1)$

(ii)  $P(X \leq 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

|                                     |    |    |   |   |   |   |   |
|-------------------------------------|----|----|---|---|---|---|---|
| <b>Number of mortgages approved</b> | 0  | 1  | 2 | 3 | 4 | 5 | 6 |
| <b>Frequency</b>                    | 10 | 16 | 7 | 4 | 2 | 0 | 1 |

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

|                                     |      |     |      |      |      |           |
|-------------------------------------|------|-----|------|------|------|-----------|
| <b>Number of mortgages approved</b> | 0    | 1   | 2    | 3    | 4    | 5 or more |
| <b>Expected frequency</b>           | 9.86 | $r$ | 9.67 | 4.51 | 1.58 | $s$       |

(c) Find the value of  $r$  and the value of  $s$  giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

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**Question 9 continued**

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**TOTAL FOR SECTION B IS 40 MARKS**



## Paper 2 Option B

### Further Pure Mathematics 1 Mark Scheme (Section A)

| Question                                                                                                                                      | Scheme                                                                               | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                   | $\sec x - \tan x = \frac{1}{\frac{1-t^2}{1+t^2}} - \frac{2t}{1-t^2}$                 | M1         | 2.1  |
|                                                                                                                                               | $= \frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2} = \frac{1-2t+t^2}{1-t^2}$                  | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t} *$                                   | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                    | $\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$          | M1         | 1.1a |
|                                                                                                                                               | $= \frac{1+t^2-2t}{1+t^2+2t}$                                                        | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1+t)^2} = \left(\frac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2 *$ | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(6 marks)</b>                                                                                                                              |                                                                                      |            |      |
| Notes:                                                                                                                                        |                                                                                      |            |      |
| <b>(a)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses $\sec x = \frac{1}{\cos x}$ and the $t$ -substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of $t$ |                                                                                      |            |      |
| <b>M1:</b> Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer                       |                                                                                      |            |      |
| <b>(b)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses the $t$ -substitution for $\sin x$ in both numerator and denominator                                                          |                                                                                      |            |      |
| <b>M1:</b> Multiplies through by $1+t^2$ in numerator and denominator                                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result                      |                                                                                      |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                            | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | £300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$ ;<br>half an hour after purchase $\Rightarrow t_2 = 1.5$ , so step $h$ required is 0.25 | B1    | 3.3  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $t_0 = 1, V_0 = 3, \left(\frac{dV}{dt}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$                                                                              | M1    | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_1 \approx V_0 + h \left(\frac{dV}{dt}\right)_0 = 3 + 0.25 \times 2 = \dots$                                                                                    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $= 3.5$                                                                                                                                                           | A1ft  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\left(\frac{dV}{dt}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$                                                | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_2 \approx V_1 + h \left(\frac{dV}{dt}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\dots$ , so £396<br>(nearest £)                                       | A1    | 3.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                   | (6)   |      |
| (6 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                   |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                   |       |      |
| <p><b>B1:</b> Identifies the correct initial conditions and requirement for <math>h</math></p> <p><b>M1:</b> Uses the model to evaluate <math>\frac{dV}{dt}</math> at <math>t_0</math>, using their <math>t_0</math> and <math>V_0</math></p> <p><b>M1:</b> Applies the approximation formula with their values</p> <p><b>A1ft:</b> 3.5 or exact equivalent. Follow through their step value</p> <p><b>M1:</b> Attempt to find <math>\left(\frac{dV}{dt}\right)_1</math> with their 3.5</p> <p><b>A1:</b> Applies the approximation and interprets the result to give £396</p> |                                                                                                                                                                   |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                   | Marks      | AOs         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|------------|-------------|
| <b>3</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{1}{x} < \frac{x}{x+2}$                                                                            |            |             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$                                              | M1         | 2.1         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$         | M1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | At least two correct critical values from $-2, -1, 0, 2$                                                 | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | All four correct critical values $-2, -1, 0, 2$                                                          | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$ | M1<br>A1   | 2.2a<br>2.5 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                          | <b>(6)</b> |             |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                          |            |             |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                          |            |             |
| <p><b>M1:</b> Gathers terms on one side and puts over common denominator, or multiply by <math>x^2(x+2)^2</math> and then gather terms on one side</p> <p><b>M1:</b> Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors</p> <p><b>A1:</b> At least 2 correct critical values found</p> <p><b>A1:</b> Exactly 4 correct critical values</p> <p><b>M1:</b> Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means</p> <p><b>A1:</b> Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept <math>\mathbb{R} - ([-2, -1] \cup [0, 2])</math> or <math>\{x \in \mathbb{R} : x &lt; -2 \text{ or } -1 &lt; x &lt; 0 \text{ or } x &gt; 2\}</math></p> |                                                                                                          |            |             |

| Question    | Scheme                                                                                                                                                                                 | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b> | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2} \mathbf{AB} \times \mathbf{AC} $                                          | M1         | 3.1a |
|             | $\frac{1}{2} \mathbf{AB} \times \mathbf{AC}  = \frac{1}{2} (-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 2\mathbf{k}) $                                 | M1         | 1.1b |
|             | $= \frac{1}{2} 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} $                                                                                                                                | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35}(\text{m}^2)$                                                                                                                                                   | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
|             | <b>Alternative</b>                                                                                                                                                                     |            |      |
|             | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{ \mathbf{AB} ^2 \mathbf{AC} ^2 - (\mathbf{AB} \cdot \mathbf{AC})^2}$ | M1         | 3.1a |
|             | $ \mathbf{AB} ^2 = 4 + 9 + 1 = 14$ , $ \mathbf{AC} ^2 = 1 + 1 + 4 = 6$<br>and $\mathbf{AB} \cdot \mathbf{AC} = 2 + 3 + 2 = 7$                                                          | M1         | 1.1b |
|             | So area of glue is $= \frac{1}{2}\sqrt{(14)(6) - (7)^2}$                                                                                                                               | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35} (\text{m}^2)$                                                                                                                                                  | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
| <b>(b)</b>  | Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC} \cdot (\mathbf{OA} \times \mathbf{OB}))$                                                                | M1         | 3.1a |
|             | $= \frac{1}{6}(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} \times (3\mathbf{j} + \mathbf{k}))$                                                                           | M1         | 1.1b |
|             | $= \frac{10}{6} = \frac{5}{3}$                                                                                                                                                         | A1         | 1.1b |
|             | Volume of parallelepiped is $6 \times$ volume of tetrahedron ( $= 10$ ),<br>so volume of glass is difference between these, viz. $10 - \frac{5}{3} = \dots$                            | M1         | 3.1a |
|             | Volume of glass $= \frac{25}{3}(\text{m}^3)$                                                                                                                                           | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(5)</b> |      |

| Question                                                                                                                                | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                                                                                                                                         | <b>4(b) Alternative</b>                                                                                                                                                                                                                                               |            |      |
|                                                                                                                                         | $-\mathbf{j} + 3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                                         | M1         | 3.1a |
|                                                                                                                                         | Area $AOB = \frac{1}{2} \times  \mathbf{OA}  \times  \mathbf{OB}  = \frac{1}{2} \times 2 \times \sqrt{10} = \sqrt{10}$                                                                                                                                                | A1         | 1.1b |
|                                                                                                                                         | $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k}) \Rightarrow p = \frac{1}{2}$<br>and so height of tetrahedron is<br>$h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10}$ | M1         | 3.1a |
|                                                                                                                                         | Volume of glass is $V = 5 \times \text{Volume of tetrahedron}$<br>$= 5 \times \frac{1}{3} \sqrt{10} \times \frac{1}{2} \sqrt{10}$                                                                                                                                     | M1         | 1.1b |
|                                                                                                                                         | $= \frac{25}{3} (\text{m}^3)$                                                                                                                                                                                                                                         | A1         | 1.1b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(5)</b> |      |
| <b>(c)</b>                                                                                                                              | The glued surfaces may distort the shapes / reduce the volume of concrete<br>Measurements in m may not be accurate<br>The surface of the concrete tetrahedron may not be smooth<br>Pockets of air may form when the concrete is being poured                          | B1         | 3.2b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Question 4 notes:                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Accept use of column vectors throughout                                                                                                 |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a)</b>                                                                                                                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Shows an understanding of what is required via an attempt at finding the area of triangle $ABC$                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Any correct method for the triangle area is fine                                                                             |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds <b>AB</b> and <b>AC</b> or any other appropriate pair of vectors to use in the vector product and attempts to use them |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent             |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a) Alternative</b>                                                                                                                  |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides                               |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> May use different sides to those shown                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Correct full method to find the area of the triangle using their two sides                                                   |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> $\frac{1}{2}\sqrt{35}$ or exact equivalent                                                                                   |                                                                                                                                                                                                                                                                       |            |      |

|                             |                                                                                                                                                                                                                                                                                                                  |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 4 notes continued: |                                                                                                                                                                                                                                                                                                                  |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Attempts volume of concrete by finding volume of tetrahedron with appropriate method                                                                                                                                                                                                                             |
| <b>M1:</b>                  | Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted                                                                                                                                                                                                |
| <b>A1:</b>                  | Correct value for the volume of concrete                                                                                                                                                                                                                                                                         |
| <b>M1:</b>                  | Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped                                                                                                                                    |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(b)</b>                  | <b>Alternative</b>                                                                                                                                                                                                                                                                                               |
| <b>M1:</b>                  | Notes (or works out using scalar products) that $-\mathbf{j} + 3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                           |
| <b>A1:</b>                  | Finds (using that $\mathbf{OA}$ and $\mathbf{OB}$ are perpendicular), area of $AOB = \sqrt{10}$                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Solves $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k})$ to get the height of the tetrahedron<br>$\left[ (\mu = \lambda =) p = \frac{1}{2}, \text{ so } h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10} \right]$ |
| <b>M1:</b>                  | Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)                                                                                                                                                                                          |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(c)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>B1:</b>                  | Any acceptable reason in context                                                                                                                                                                                                                                                                                 |



| Question    | Scheme                                                                                                                     | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>5(a)</b> | $y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$<br>$\Rightarrow P(4p^2, 8p)$ is a general point on $C$                 | B1         | 2.2a |
|             |                                                                                                                            | <b>(1)</b> |      |
| <b>(b)</b>  | $y^2 = 16x$ gives $a = 4$ , or $2y \frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$                                    | M1         | 2.2a |
|             | $l: y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$                                                                          | M1         | 1.1b |
|             | leading to $py = x + 4p^2$ *                                                                                               | A1*        | 2.1  |
|             |                                                                                                                            | <b>(3)</b> |      |
| <b>(c)</b>  | $B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                            | M1         | 3.1a |
|             | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                 | M1         | 1.1b |
|             | $p = \frac{3}{2}$ and $l$ cuts $x$ -axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$     | M1         | 2.1  |
|             | $x = -9$                                                                                                                   | A1         | 1.1b |
|             | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}} dx$ | M1         | 2.1  |
|             | $\int 4x^{\frac{1}{2}} dx = \frac{4x^{\frac{3}{2}}}{(\frac{3}{2})} (+c)$ or $\frac{8}{3}x^{\frac{3}{2}} (+c)$              | M1         | 1.1b |
|             |                                                                                                                            | A1         | 1.1b |
|             | $\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *                     | A1*        | 1.1b |
|             |                                                                                                                            | <b>(8)</b> |      |

| Question | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|          | <b>5(c) Alternative 1</b>                                                                                                                                                                                                                                             |            |      |
|          | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                                | M1         | 3.1a |
|          | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                            | M1         | 1.1b |
|          | $p = \frac{3}{2} \text{ into } l \text{ gives } \frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                                 | M1         | 2.1  |
|          | $x = \frac{3}{2}y - 9$                                                                                                                                                                                                                                                | A1         | 1.1b |
|          | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left( \frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right) \right) dy$                                                                                                                     | M1         | 2.1  |
|          | $\int \left( \frac{1}{16}y^2 - \frac{3}{2}y + 9 \right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y (+c)$                                                                                                                                                              | M1         | 1.1b |
|          |                                                                                                                                                                                                                                                                       | A1         | 1.1b |
|          | $\text{Area}(R) = \left( \frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12) \right) - (0)$<br>$= 36 - 108 + 108 = 36 *$                                                                                                                                                   | A1*        | 1.1b |
|          |                                                                                                                                                                                                                                                                       | <b>(8)</b> |      |
|          | <b>5(c) Alternative 2</b>                                                                                                                                                                                                                                             |            |      |
|          | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                                | M1         | 3.1a |
|          | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                            | M1         | 1.1b |
|          | $p = \frac{3}{2} \text{ and } l \text{ cuts px-axis when } \frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                    | M1         | 2.1  |
|          | $x = -9$                                                                                                                                                                                                                                                              | A1         | 1.1b |
|          | $p = \frac{3}{2} \Rightarrow P(9, 12) \text{ and } x = 0 \text{ in } l: y = \frac{2}{3}x + 6 \text{ gives } y = 6$<br>$\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left( \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dx$ | M1         | 2.1  |
|          | $\int \left( \frac{2}{3}x + 6 - 4x^{\frac{1}{2}} \right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} (+c)$                                                                                                                                                  | M1         | 1.1b |
|          |                                                                                                                                                                                                                                                                       | A1         | 1.1b |
|          | $\text{Area}(R) = 27 + \left( \left( \frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}(9^{\frac{3}{2}}) \right) - (0) \right)$<br>$= 27 + (27 + 54 - 72) = 27 + 9 = 36 *$                                                                                                         | A1*        | 1.1b |
|          |                                                                                                                                                                                                                                                                       | <b>(8)</b> |      |
|          | <b>(12 marks)</b>                                                                                                                                                                                                                                                     |            |      |

|                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 5 notes:        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| (a)                      | <p><b>B1:</b> Substitutes <math>y_p = 8p</math> into <math>y^2</math> to obtain <math>64p^2</math> and substitutes <math>x_p = 4p^2</math> into <math>16x</math> to obtain <math>64p^2</math> and concludes that <math>P</math> lies on <math>C</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| (b)                      | <p><b>M1:</b> Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it</p> <p><b>M1:</b> Applies <math>y - 8p = m(x - 4p^2)</math>, with their tangent gradient <math>m</math>, which is in terms of <math>p</math>. Accept use of <math>8p = m(4p^2) + c</math> with a clear attempt to find <math>c</math></p> <p><b>A1*:</b> Obtains <math>py = x + 4p^2</math> by <b>cs0</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| (c)                      | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math></p> <p><b>M1:</b> Substitutes their <math>p</math> (which must be positive) and <math>y = 0</math> into <math>l</math> and solves to give <math>x = \dots</math></p> <p><b>A1:</b> Finds that <math>l</math> cuts the <math>x</math>-axis at <math>x = -9</math></p> <p><b>M1:</b> Fully correct method for finding the area of <math>R</math><br/> i.e. <math>\frac{1}{2}(\text{their } x_p - "-9")(\text{their } y_p) - \int_0^{\text{their } x_p} 4x^2 dx</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda x^{\frac{1}{2}}</math> to give <math>\pm \mu x^{\frac{3}{2}}</math>, where <math>\lambda, \mu \neq 0</math></p> <p><b>A1:</b> Integrates <math>4x^{\frac{1}{2}}</math> to give <math>\frac{8}{3}x^{\frac{3}{2}}</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p>                                                             |
| (c) <b>Alternative 1</b> | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math><br/> Substitutes their <math>p</math> (which must be positive) into <math>l</math> and rearranges to give <math>x = \dots</math></p> <p><b>M1:</b> Finds <math>l</math> as <math>x = \frac{3}{2}y - 9</math></p> <p><b>A1:</b> Fully correct method for finding the area of <math>R</math></p> <p><b>M1:</b> i.e. <math>\int_0^{\text{their } y_p} \left( \frac{1}{16}y^2 - \text{their} \left( \frac{3}{2}y - 9 \right) \right) dy</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda y^2 \pm \mu y \pm \nu</math> to give <math>\pm \alpha y^3 \pm \beta y^2 \pm \nu y</math>, where <math>\lambda, \mu, \nu, \alpha, \beta \neq 0</math></p> <p><b>A1:</b> Integrates <math>\frac{1}{16}y^2 - \left( \frac{3}{2}y - 9 \right)</math> to give <math>\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p> |

Question 5 notes continued:

**(c) Alternative 2**

**M1:** Substitutes their  $x = "-a"$  and  $y = \frac{10}{3}$  into  $l$

**M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  $p = \dots$

**M1:** Substitutes their  $p$  (which must be positive) and  $y = 0$  into  $l$  and solves to give  $x = \dots$

**A1:** Finds that  $l$  cuts the  $x$ -axis at  $x = -9$

**M1:** Fully correct method for finding the area of  $R$

i.e.  $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_p} \left( \text{their } \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dy$

**M1:** Integrates  $\pm \lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$  to give  $\pm \alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$ , where  $\lambda, \mu, \nu, \alpha, \beta \neq 0$

**A1:** Integrates  $\left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right)$  to give  $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$ , simplified or un-simplified

**A1\*:** Fully correct proof leading to a correct answer of 36

# Further Statistics 1 Mark Scheme (Section B)

| Question                                                                                                               | Scheme                                                                                                                                                                                                                                                                                                                                       | Marks                | AOs      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|----------|----------|----------|--|--------|---------|----------|--------|------|----------|------|----------|--------|----------|--------|----------|----|-----|
| 6(a)                                                                                                                   | H <sub>0</sub> : There is no association between language and gender                                                                                                                                                                                                                                                                         | B1                   | 1.2      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              | (1)                  |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (b)                                                                                                                    | $\frac{54 \times 85}{150} = 30.6 \quad *$                                                                                                                                                                                                                                                                                                    | B1*cs0               | 1.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              | (1)                  |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (c)                                                                                                                    | <table><tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr><tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43...</td><td>23.4</td><td>15.16...</td></tr><tr><td>Female</td><td>34.56...</td><td>[30.6]</td><td>19.83...</td></tr></table> | Expected frequencies |          | Language |          |  | French | Spanish | Mandarin | Gender | Male | 26.43... | 23.4 | 15.16... | Female | 34.56... | [30.6] | 19.83... | M1 | 2.1 |
|                                                                                                                        | Expected frequencies                                                                                                                                                                                                                                                                                                                         |                      |          | Language |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              |                      | French   | Spanish  | Mandarin |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        | Gender                                                                                                                                                                                                                                                                                                                                       | Male                 | 26.43... | 23.4     | 15.16... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              | Female               | 34.56... | [30.6]   | 19.83... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| $\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \dots + \frac{(15-19.83)^2}{19.83}$                    | M1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Awrt <u>3.6/3.7</u>                                                                                                    | A1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        | (3)                                                                                                                                                                                                                                                                                                                                          |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (d)                                                                                                                    | Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$                                                                                                                                                                                                                                                                 | M1                   | 3.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        | As $\sum \frac{(O-E)^2}{E} < 9.210$ , the null hypothesis is not rejected                                                                                                                                                                                                                                                                    | A1                   | 2.2b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              | (2)                  |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (e)                                                                                                                    | Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$                                                                                                                                                                                                                                                                   | B1                   | 2.4      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              | (1)                  |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (8 marks)                                                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Notes:                                                                                                                 |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (a)                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| B1: For correct hypothesis in context                                                                                  |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (b)                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| B1*: For a correct calculation leading to the given answer and no errors seen                                          |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (c)                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| M1: For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| M1: For applying $\sum \frac{(O-E)^2}{E}$                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| A1: awrt 3.6 or 3.7                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (d)                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| M1: For using degrees of freedom to set up a $\chi^2$ model critical value                                             |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| A1: For correct comparison and conclusion                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| (e)                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| A1ft: For correct conclusion with supporting reason                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                                                                                                                                                                                                                | Marks      | AOs          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|
| <b>7(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $-4 = 2 - 5E(X)$                                                                                                                                                                                                                                                                                                                                      | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $E(X) = 1.2$                                                                                                                                                                                                                                                                                                                                          |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$                                                                                                                                                                                                                                                                               | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a + 2b + 2c = 1.2$ <span style="border: 1px solid black; padding: 0 5px;">1</span>                                                                                                                                                                                                                                                                   |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$<br>i.e. $P(X \leq 1) = 0.45$                                                                                                                                                                                                                                                                   | M1         | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + c = 0.45$ <span style="border: 1px solid black; padding: 0 5px;">2</span>                                                                                                                                                                                                                                                                       |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + b + 2c = 1$ <span style="border: 1px solid black; padding: 0 5px;">3</span>                                                                                                                                                                                                                                                                     | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u> | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | e.g. <span style="border: 1px solid black; padding: 0 5px;">3</span> - <span style="border: 1px solid black; padding: 0 5px;">2</span> $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into <span style="border: 1px solid black; padding: 0 5px;">1</span> $\Rightarrow a = 0.1$ etc                                                                      |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a = 0.1 \quad b = 0.3 \quad c = 0.25$                                                                                                                                                                                                                                                                                                                | A1<br>A1   | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(7)</b> |              |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59                                                                                                                                                                                                                                                                                                            | M1         | 1.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$                                                                                                                                                                                                                                                                            | A1         | 1.2          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$                                                                                                                                                                                                                                                                                             | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(X < \frac{1}{3}) = a + c = 0.35$                                                                                                                                                                                                                                                                                                                   | A1ft       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| <b>(a)</b><br><b>M1:</b> For using given information to find an expression for $E(X)$ i.e. use of $E(Y) = 2 - 5E(X)$<br><b>M1:</b> For use of $\sum xP(X = x) = '1.2'$<br><b>M1:</b> For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$<br><b>M1:</b> For use of $\sum P(X = x) = 1$<br><b>M1:</b> For solving their 3 linear equations (matrix or elimination)<br><b>A1:</b> For any 2 of $a, b$ or $c$ correct<br><b>A1:</b> For all 3 correct values |                                                                                                                                                                                                                                                                                                                                                       |            |              |

|                                        |                                                                                                              |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------|
| Question 7 notes continued:            |                                                                                                              |
| <b>Another method for part (a) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For using given information to find the probability distribution for $Y$ leading to an expression for $E(Y)$ |
| <b>M1:</b>                             | For use of $\sum yP(Y = y) = -4$                                                                             |
| <b>M1:</b>                             | For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$    |
| <b>M1:</b>                             | For use of $\sum P(Y = y) = 1$                                                                               |
| <b>M1:</b>                             | For solving their 3 linear equations (matrix or elimination)                                                 |
| <b>A1:</b>                             | For any 2 of $a$ , $b$ or $c$ correct                                                                        |
| <b>A1:</b>                             | For all 3 correct values                                                                                     |
| <b>(b)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer)                          |
| <b>A1:</b>                             | For use of $\text{Var}(aX) = a^2 \text{Var}(X)$ to reach 2.36 or exact equivalent                            |
| <b>(c)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For rearranging to the form $P(X < k)$                                                                       |
| <b>A1ft:</b>                           | 0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                            |
| <b>Another method for part (c) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For comparing distribution of $X$ with distribution of $Y$ to identify $X = -1$ and $X = 0$                  |
| <b>A1ft:</b>                           | '0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                           |

| Question                                                                                                                                                                                             | Scheme                                                                                                         | Marks | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|-------|------|
| <b>8(a)</b>                                                                                                                                                                                          | $X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$                                                            |       |      |
|                                                                                                                                                                                                      | P(each hire 2 in 1 hour)<br>$= P(X=2) \times P(Y=2) = 0.25104... \times 0.21685...$                            | M1    | 3.3  |
|                                                                                                                                                                                                      | $= 0.05444... \quad \text{awrt } \underline{0.0544}$                                                           | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(b)</b>                                                                                                                                                                                           | $W = X + Y \rightarrow W \sim \text{Po}(3.8)$                                                                  | M1    | 3.4  |
|                                                                                                                                                                                                      | $P(W = 3) = 0.20458... \quad \text{awrt } \underline{0.205}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(c)</b>                                                                                                                                                                                           | $T \sim \text{Po}((2.6+1.2) \times 2)$                                                                         | M1    | 3.3  |
|                                                                                                                                                                                                      | $P(T < 9) = 0.64819... \quad \text{awrt } \underline{0.648}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(d)</b>                                                                                                                                                                                           | <b>(i)</b> Mean = $np = \underline{2.4}$                                                                       | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Variance = $np(1-p) = 2.3904 \quad \text{awrt } \underline{2.39}$                                  | B1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(e)</b>                                                                                                                                                                                           | <b>(i)</b> $[ D \sim \text{Po}(2.4) \quad P(D \leq 4) ]$<br>$= 0.9041... \quad \text{awrt } \underline{0.904}$ | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Since $n$ is large and $p$ is small/mean is approximately equal to variance                        | B1    | 2.4  |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(10 marks)</b>                                                                                                                                                                                    |                                                                                                                |       |      |
| Notes:                                                                                                                                                                                               |                                                                                                                |       |      |
| <b>(a)</b><br><b>M1:</b> For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.0544</b> |                                                                                                                |       |      |
| <b>(b)</b><br><b>M1:</b> For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.205</b>                                                 |                                                                                                                |       |      |
| <b>(c)</b><br><b>M1:</b> For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.648</b>                                   |                                                                                                                |       |      |
| <b>(d)(i)</b><br><b>B1:</b> For <b>2.4</b>                                                                                                                                                           |                                                                                                                |       |      |
| <b>(d)(ii)</b><br><b>B1:</b> For awrt <b>2.39</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(i)</b><br><b>B1:</b> For awrt <b>0.904</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(ii)</b><br><b>B1:</b> For a correct explanation to support use of Poisson approximation in this case                                                                                          |                                                                                                                |       |      |



| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                     | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>9(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | <b>(i)</b> $P(X = 1) = 0.34523\dots$ awrt <b><u>0.345</u></b>                                                                              | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <b>(ii)</b> $P(X \leq 4) = 0.98575\dots$ awrt <b><u>0.986</u></b>                                                                          | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$                        | B1*cs0     | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $r = \mathbf{13.81}$ $s = \mathbf{0.57}$                                                                                                   | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $H_0$ : The Poisson distribution is a suitable model<br>$H_1$ : The Poisson distribution is not a suitable model                           | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [Cells are combined when expected frequencies < 5]<br>So combine the last 3 cells                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$       | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | awrt <b><u>1.1</u></b>                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Degrees of freedom = $4 - 1 - 1 = 2$                                                                                                       | B1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | (Do not reject $H_0$ since $1.10 < \chi^2_{2,(0.05)} = 5.991$ ). The number of mortgages approved each week follows a Poisson distribution | A1         | 3.5a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(6)</b> |      |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                            |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(i)</b><br><b>B1:</b> awrt 0.345                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(ii)</b><br><b>B1:</b> awrt 0.986                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                            |            |      |
| <b>(b)</b><br><b>B1*:</b> For a fully correct calculation leading to given answer with no errors seen                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                            |            |      |
| <b>(c)</b><br><b>M1:</b> For attempt at $r$ or $s$ (may be implied by correct answers)<br><b>A1ft:</b> For both values correct (follow through their answers to part (a))                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                            |            |      |
| <b>(d)</b><br><b>B1:</b> For both hypotheses correct (lambda should not be defined so correct use of the model)<br><b>M1:</b> For understanding the need to combine cells before calculating the test statistic (may be implied)<br><br><b>M1:</b> For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$<br><b>A1:</b> awrt 1.1<br><b>B1:</b> For realising that there are 2 degrees of freedom leading to a critical value of $\chi^2_{2,(0.05)} = 5.991$<br><b>A1:</b> Concluding that a Poisson model is suitable for the number of mortgages approved each week |                                                                                                                                            |            |      |



Write your name here

Surname

Other names

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2C: Further Pure Mathematics 1 and  
Further Mechanics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2C**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

S58540A

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(3)

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2. The value,  $V$  hundred pounds, of a particular stock  $t$  hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \quad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula  $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$  to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

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3. Use algebra to find the set of values of  $x$  for which

$$\frac{1}{x} < \frac{x}{x+2}$$

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4.

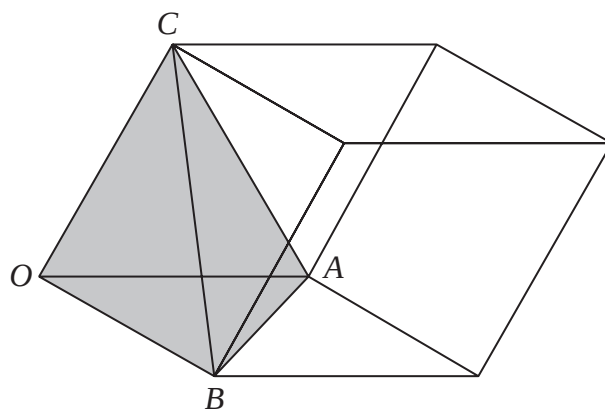
**Figure 1**

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are  $O(0, 0, 0)$ ,  $A(2, 0, 0)$ ,  $B(0, 3, 1)$  and  $C(1, 1, 2)$ , where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

- Find the surface area of the glued face of the tetrahedron. (4)
- Find the volume of glass contained in this parallelepiped. (5)
- Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture. (1)

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5.

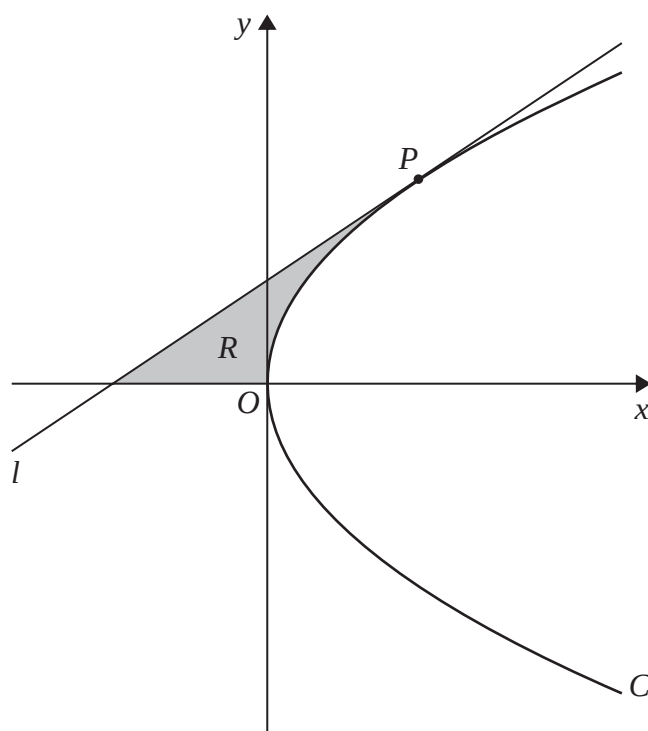
Diagram not  
drawn to scale

Figure 2

[ You may quote without proof that for the general parabola  $y^2 = 4ax$ ,  $\frac{dy}{dx} = \frac{2a}{y}$  ]

The parabola  $C$  has equation  $y^2 = 16x$ .

(a) Deduce that the point  $P(4p^2, 8p)$  is a general point on  $C$ .

(1)

The line  $l$  is the tangent to  $C$  at the point  $P$ .

(b) Show that an equation for  $l$  is

$$py = x + 4p^2$$

(3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the line  $l$ , the  $x$ -axis and the parabola  $C$ .

The line  $l$  intersects the directrix of  $C$  at the point  $B$ , where the  $y$  coordinate of  $B$  is  $\frac{10}{3}$

Given that  $p > 0$

(c) show that the area of  $R$  is 36

(8)

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**Question 5 continued**

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**(Total for Question 5 is 12 marks)**

**TOTAL FOR SECTION A IS 40 MARKS**

## SECTION B

**Answer ALL questions. Write your answers in the spaces provided.**

Unless otherwise indicated, whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ ms}^{-2}$  and give your answer to either 2 significant figures or 3 significant figures.

6. A small ball of mass  $0.1\text{ kg}$  is dropped from a point which is  $2.4\text{ m}$  above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of  $0.6\text{ m}$  above the floor. The ball is modelled as a particle.
- (a) Show that the coefficient of restitution between the ball and the floor is  $0.5$  (6)
- (b) Find the height reached by the ball above the floor after it bounces on the floor for the second time. (3)
- (c) By considering your answer to (b), describe the subsequent motion of the ball. (1)

**Question 6 continued**

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**(Total for Question 6 is 10 marks)**





**Question 7 continued**

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**(Total for Question 7 is 6 marks)**

- A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of  $4 \text{ m s}^{-1}$ . The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

- (3)

$\sin \alpha = \frac{1}{15}$ . Because of the hill, the jogger reduces her speed to  $3 \text{ m s}^{-1}$  and maintains this

constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

- (5)

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**Question 8 continued**

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**(Total for Question 8 is 8 marks)**

9. A particle  $P$  of mass  $3m$  is moving in a straight line on a smooth horizontal table. A particle  $Q$  of mass  $m$  is moving in the opposite direction to  $P$  along the same straight line. The particles collide directly. Immediately before the collision the speed of  $P$  is  $u$  and the speed of  $Q$  is  $2u$ . The velocities of  $P$  and  $Q$  immediately after the collision, measured in the direction of motion of  $P$  before the collision, are  $v$  and  $w$  respectively. The coefficient of restitution between  $P$  and  $Q$  is  $e$ .

(a) Find an expression for  $v$  in terms of  $u$  and  $e$ .

(6)

Given that the direction of motion of  $P$  is changed by the collision,

(b) find the range of possible values of  $e$ .

(2)

(c) Show that  $w = \frac{u}{4}(1 + 9e)$ .

(2)

Following the collision with  $P$ , the particle  $Q$  then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of  $Q$ . The coefficient of restitution between  $Q$  and the wall is  $f$ .

Given that  $e = \frac{5}{9}$ , and that  $P$  and  $Q$  collide again in the subsequent motion,

(d) find the range of possible values of  $f$ .

(6)

**Question 9 continued**

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**Question 9 continued**

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**(Total for Question 9 is 16 marks)**

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**



## Paper 2 Option C

### Further Pure Mathematics 1 Mark Scheme (Section A)

| Question                                                                                                                                      | Scheme                                                                               | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                   | $\sec x - \tan x = \frac{1}{\frac{1-t^2}{1+t^2}} - \frac{2t}{1-t^2}$                 | M1         | 2.1  |
|                                                                                                                                               | $= \frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2} = \frac{1-2t+t^2}{1-t^2}$                  | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t} *$                                   | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                    | $\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$          | M1         | 1.1a |
|                                                                                                                                               | $= \frac{1+t^2-2t}{1+t^2+2t}$                                                        | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1+t)^2} = \left(\frac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2 *$ | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(6 marks)</b>                                                                                                                              |                                                                                      |            |      |
| Notes:                                                                                                                                        |                                                                                      |            |      |
| <b>(a)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses $\sec x = \frac{1}{\cos x}$ and the $t$ -substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of $t$ |                                                                                      |            |      |
| <b>M1:</b> Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer                       |                                                                                      |            |      |
| <b>(b)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses the $t$ -substitution for $\sin x$ in both numerator and denominator                                                          |                                                                                      |            |      |
| <b>M1:</b> Multiplies through by $1+t^2$ in numerator and denominator                                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result                      |                                                                                      |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                            | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>2</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | £300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$ ;<br>half an hour after purchase $\Rightarrow t_2 = 1.5$ , so step $h$ required is 0.25 | B1         | 3.3  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $t_0 = 1, V_0 = 3, \left(\frac{dV}{dt}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$                                                                              | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_1 \approx V_0 + h \left(\frac{dV}{dt}\right)_0 = 3 + 0.25 \times 2 = \dots$                                                                                    | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $= 3.5$                                                                                                                                                           | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\left(\frac{dV}{dt}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$                                                | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_2 \approx V_1 + h \left(\frac{dV}{dt}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\dots$ , so £396<br>(nearest £)                                       | A1         | 3.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                   | <b>(6)</b> |      |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                   |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                   |            |      |
| <p><b>B1:</b> Identifies the correct initial conditions and requirement for <math>h</math></p> <p><b>M1:</b> Uses the model to evaluate <math>\frac{dV}{dt}</math> at <math>t_0</math>, using their <math>t_0</math> and <math>V_0</math></p> <p><b>M1:</b> Applies the approximation formula with their values</p> <p><b>A1ft:</b> 3.5 or exact equivalent. Follow through their step value</p> <p><b>M1:</b> Attempt to find <math>\left(\frac{dV}{dt}\right)_1</math> with their 3.5</p> <p><b>A1:</b> Applies the approximation and interprets the result to give £396</p> |                                                                                                                                                                   |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                   | Marks      | AOs         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|------------|-------------|
| <b>3</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{1}{x} < \frac{x}{x+2}$                                                                            |            |             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$                                              | M1         | 2.1         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$         | M1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | At least two correct critical values from $-2, -1, 0, 2$                                                 | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | All four correct critical values $-2, -1, 0, 2$                                                          | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$ | M1<br>A1   | 2.2a<br>2.5 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                          | <b>(6)</b> |             |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                          |            |             |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                          |            |             |
| <p><b>M1:</b> Gathers terms on one side and puts over common denominator, or multiply by <math>x^2(x+2)^2</math> and then gather terms on one side</p> <p><b>M1:</b> Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors</p> <p><b>A1:</b> At least 2 correct critical values found</p> <p><b>A1:</b> Exactly 4 correct critical values</p> <p><b>M1:</b> Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means</p> <p><b>A1:</b> Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept <math>\mathbb{R} - ([-2, -1] \cup [0, 2])</math> or <math>\{x \in \mathbb{R} : x &lt; -2 \text{ or } -1 &lt; x &lt; 0 \text{ or } x &gt; 2\}</math></p> |                                                                                                          |            |             |

| Question    | Scheme                                                                                                                                                                                 | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b> | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2} \mathbf{AB} \times \mathbf{AC} $                                          | M1         | 3.1a |
|             | $\frac{1}{2} \mathbf{AB} \times \mathbf{AC}  = \frac{1}{2} (-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 2\mathbf{k}) $                                 | M1         | 1.1b |
|             | $= \frac{1}{2} 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} $                                                                                                                                | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35}(\text{m}^2)$                                                                                                                                                   | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
|             | <b>Alternative</b>                                                                                                                                                                     |            |      |
|             | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{ \mathbf{AB} ^2 \mathbf{AC} ^2 - (\mathbf{AB} \cdot \mathbf{AC})^2}$ | M1         | 3.1a |
|             | $ \mathbf{AB} ^2 = 4 + 9 + 1 = 14$ , $ \mathbf{AC} ^2 = 1 + 1 + 4 = 6$<br>and $\mathbf{AB} \cdot \mathbf{AC} = 2 + 3 + 2 = 7$                                                          | M1         | 1.1b |
|             | So area of glue is $= \frac{1}{2}\sqrt{(14)(6) - (7)^2}$                                                                                                                               | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35} (\text{m}^2)$                                                                                                                                                  | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
| <b>(b)</b>  | Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC} \cdot (\mathbf{OA} \times \mathbf{OB}))$                                                                | M1         | 3.1a |
|             | $= \frac{1}{6}(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} \times (3\mathbf{j} + \mathbf{k}))$                                                                           | M1         | 1.1b |
|             | $= \frac{10}{6} = \frac{5}{3}$                                                                                                                                                         | A1         | 1.1b |
|             | Volume of parallelepiped is $6 \times$ volume of tetrahedron ( $= 10$ ),<br>so volume of glass is difference between these, viz. $10 - \frac{5}{3} = \dots$                            | M1         | 3.1a |
|             | Volume of glass $= \frac{25}{3}(\text{m}^3)$                                                                                                                                           | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(5)</b> |      |

| Question                                                                                                                                | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                                                                                                                                         | <b>4(b) Alternative</b>                                                                                                                                                                                                                                               |            |      |
|                                                                                                                                         | $-\mathbf{j} + 3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                                         | M1         | 3.1a |
|                                                                                                                                         | Area $AOB = \frac{1}{2} \times  \mathbf{OA}  \times  \mathbf{OB}  = \frac{1}{2} \times 2 \times \sqrt{10} = \sqrt{10}$                                                                                                                                                | A1         | 1.1b |
|                                                                                                                                         | $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k}) \Rightarrow p = \frac{1}{2}$<br>and so height of tetrahedron is<br>$h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10}$ | M1         | 3.1a |
|                                                                                                                                         | Volume of glass is $V = 5 \times \text{Volume of tetrahedron}$<br>$= 5 \times \frac{1}{3} \sqrt{10} \times \frac{1}{2} \sqrt{10}$                                                                                                                                     | M1         | 1.1b |
|                                                                                                                                         | $= \frac{25}{3} (\text{m}^3)$                                                                                                                                                                                                                                         | A1         | 1.1b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(5)</b> |      |
| <b>(c)</b>                                                                                                                              | The glued surfaces may distort the shapes / reduce the volume of concrete<br>Measurements in m may not be accurate<br>The surface of the concrete tetrahedron may not be smooth<br>Pockets of air may form when the concrete is being poured                          | B1         | 3.2b |
|                                                                                                                                         |                                                                                                                                                                                                                                                                       | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Question 4 notes:                                                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| Accept use of column vectors throughout                                                                                                 |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a)</b>                                                                                                                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Shows an understanding of what is required via an attempt at finding the area of triangle $ABC$                              |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Any correct method for the triangle area is fine                                                                             |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds <b>AB</b> and <b>AC</b> or any other appropriate pair of vectors to use in the vector product and attempts to use them |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent             |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a) Alternative</b>                                                                                                                  |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides                               |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> May use different sides to those shown                                                                                       |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Correct full method to find the area of the triangle using their two sides                                                   |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> $\frac{1}{2}\sqrt{35}$ or exact equivalent                                                                                   |                                                                                                                                                                                                                                                                       |            |      |

|                             |                                                                                                                                                                                                                                                                                                                  |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 4 notes continued: |                                                                                                                                                                                                                                                                                                                  |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Attempts volume of concrete by finding volume of tetrahedron with appropriate method                                                                                                                                                                                                                             |
| <b>M1:</b>                  | Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted                                                                                                                                                                                                |
| <b>A1:</b>                  | Correct value for the volume of concrete                                                                                                                                                                                                                                                                         |
| <b>M1:</b>                  | Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped                                                                                                                                    |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(b) Alternative</b>      |                                                                                                                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Notes (or works out using scalar products) that $-\mathbf{j} + 3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                           |
| <b>A1:</b>                  | Finds (using that $\mathbf{OA}$ and $\mathbf{OB}$ are perpendicular), area of $AOB = \sqrt{10}$                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Solves $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k})$ to get the height of the tetrahedron<br>$\left[ (\mu = \lambda =) p = \frac{1}{2}, \text{ so } h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10} \right]$ |
| <b>M1:</b>                  | Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)                                                                                                                                                                                          |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(c)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>B1:</b>                  | Any acceptable reason in context                                                                                                                                                                                                                                                                                 |

| Question    | Scheme                                                                                                                     | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>5(a)</b> | $y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$<br>$\Rightarrow P(4p^2, 8p)$ is a general point on $C$                 | B1         | 2.2a |
|             |                                                                                                                            | <b>(1)</b> |      |
| <b>(b)</b>  | $y^2 = 16x$ gives $a = 4$ , or $2y \frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$                                    | M1         | 2.2a |
|             | $l: y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$                                                                          | M1         | 1.1b |
|             | leading to $py = x + 4p^2$ *                                                                                               | A1*        | 2.1  |
|             |                                                                                                                            | <b>(3)</b> |      |
| <b>(c)</b>  | $B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                            | M1         | 3.1a |
|             | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                 | M1         | 1.1b |
|             | $p = \frac{3}{2}$ and $l$ cuts $x$ -axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$     | M1         | 2.1  |
|             | $x = -9$                                                                                                                   | A1         | 1.1b |
|             | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}} dx$ | M1         | 2.1  |
|             | $\int 4x^{\frac{1}{2}} dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} (+c)$ or $\frac{8}{3}x^{\frac{3}{2}} (+c)$   | M1         | 1.1b |
|             |                                                                                                                            | A1         | 1.1b |
|             | $\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *                     | A1*        | 1.1b |
|             |                                                                                                                            | <b>(8)</b> |      |

| Question          | Scheme                                                                                                                                                                                                                                                            | Marks      | AOs  |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                   | <b>5(c) Alternative 1</b>                                                                                                                                                                                                                                         |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                            | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                        | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ into } l \text{ gives } \frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                             | M1         | 2.1  |
|                   | $x = \frac{3}{2}y - 9$                                                                                                                                                                                                                                            | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left( \frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right) \right) dy$                                                                                                                 | M1         | 2.1  |
|                   | $\int \left( \frac{1}{16}y^2 - \frac{3}{2}y + 9 \right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y (+c)$                                                                                                                                                          | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | A1         | 1.1b |
|                   | $\text{Area}(R) = \left( \frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12) \right) - (0)$<br>$= 36 - 108 + 108 = 36 *$                                                                                                                                               | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | <b>(8)</b> |      |
|                   | <b>5(c) Alternative 2</b>                                                                                                                                                                                                                                         |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                            | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                        | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ and } l \text{ cuts px-axis when } \frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                | M1         | 2.1  |
|                   | $x = -9$                                                                                                                                                                                                                                                          | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \text{ and } x = 0 \text{ in } l: y = \frac{2}{3}x + 6 \text{ gives } y = 6$<br>$\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left( \left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right) \right) dx$ | M1         | 2.1  |
|                   | $\int \left( \frac{2}{3}x + 6 - 4x^{\frac{1}{2}} \right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} (+c)$                                                                                                                                              | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | A1         | 1.1b |
|                   | $\text{Area}(R) = 27 + \left( \left( \frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}(9^{\frac{3}{2}}) \right) - (0) \right)$<br>$= 27 + (27 + 54 - 72) = 27 + 9 = 36 *$                                                                                                     | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | <b>(8)</b> |      |
| <b>(12 marks)</b> |                                                                                                                                                                                                                                                                   |            |      |



|                          |                                                                                                                                                                                 |
|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 5 notes:        |                                                                                                                                                                                 |
| (a)                      |                                                                                                                                                                                 |
| <b>B1:</b>               | Substitutes $y_p = 8p$ into $y^2$ to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that $P$ lies on $C$                                |
| (b)                      |                                                                                                                                                                                 |
| <b>M1:</b>               | Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it                                                                 |
| <b>M1:</b>               | Applies $y - 8p = m(x - 4p^2)$ , with their tangent gradient $m$ , which is in terms of $p$ .<br>Accept use of $8p = m(4p^2) + c$ with a clear attempt to find $c$              |
| <b>A1*:</b>              | Obtains $py = x + 4p^2$ by <b>cs0</b>                                                                                                                                           |
| (c)                      |                                                                                                                                                                                 |
| <b>M1:</b>               | Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into $l$                                                                                                                    |
| <b>M1:</b>               | Obtains a 3 term quadratic and solves (using the usual rules) to give $p = \dots$                                                                                               |
| <b>M1:</b>               | Substitutes their $p$ (which must be positive) and $y = 0$ into $l$ and solves to give $x = \dots$                                                                              |
| <b>A1:</b>               | Finds that $l$ cuts the $x$ -axis at $x = -9$                                                                                                                                   |
| <b>M1:</b>               | Fully correct method for finding the area of $R$<br>i.e. $\frac{1}{2}(\text{their } x_p - "-9")(\text{their } y_p) - \int_0^{\text{their } x_p} 4x^2 dx$                        |
| <b>M1:</b>               | Integrates $\pm \lambda x^{\frac{1}{2}}$ to give $\pm \mu x^{\frac{3}{2}}$ , where $\lambda, \mu \neq 0$                                                                        |
| <b>A1:</b>               | Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$ , simplified or un-simplified                                                                                |
| <b>A1*:</b>              | Fully correct proof leading to a correct answer of 36                                                                                                                           |
| (c) <b>Alternative 1</b> |                                                                                                                                                                                 |
| <b>M1:</b>               | Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into $l$                                                                                                                    |
| <b>M1:</b>               | Obtains a 3 term quadratic and solves (using the usual rules) to give $p = \dots$<br>Substitutes their $p$ (which must be positive) into $l$ and rearranges to give $x = \dots$ |
| <b>M1:</b>               | Finds $l$ as $x = \frac{3}{2}y - 9$                                                                                                                                             |
| <b>A1:</b>               | Fully correct method for finding the area of $R$                                                                                                                                |
| <b>M1:</b>               | i.e. $\int_0^{\text{their } y_p} \left( \frac{1}{16}y^2 - \text{their} \left( \frac{3}{2}y - 9 \right) \right) dy$                                                              |
| <b>M1:</b>               | Integrates $\pm \lambda y^2 \pm \mu y \pm \nu$ to give $\pm \alpha y^3 \pm \beta y^2 \pm \nu y$ , where $\lambda, \mu, \nu, \alpha, \beta \neq 0$                               |
| <b>A1:</b>               | Integrates $\frac{1}{16}y^2 - \left( \frac{3}{2}y - 9 \right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$ , simplified or un-simplified                                    |
| <b>A1*:</b>              | Fully correct proof leading to a correct answer of 36                                                                                                                           |

Question 5 notes continued:

**(c) Alternative 2**

**M1:** Substitutes their  $x = "-a"$  and  $y = \frac{10}{3}$  into  $l$

**M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  $p = \dots$

**M1:** Substitutes their  $p$  (which must be positive) and  $y = 0$  into  $l$  and solves to give  $x = \dots$

**A1:** Finds that  $l$  cuts the  $x$ -axis at  $x = -9$

**M1:** Fully correct method for finding the area of  $R$

i.e.  $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_p} \left( \text{their } \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dy$

**M1:** Integrates  $\pm \lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$  to give  $\pm \alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$ , where  $\lambda, \mu, \nu, \alpha, \beta \neq 0$

**A1:** Integrates  $\left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right)$  to give  $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$ , simplified or un-simplified

**A1\*:** Fully correct proof leading to a correct answer of 36

# Further Mechanics 1 Mark Scheme (Section B)

| Question                                                                                                                                                                                                                                                                                                                                   | Scheme                                                                                                    | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|------------|------|
| <b>6(a)</b>                                                                                                                                                                                                                                                                                                                                | Using the model and $v^2 = u^2 + 2as$ to find $v$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{4.8g}$                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$ to find $u$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{1.2g}$                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                            | $e = \sqrt{1.2g} / \sqrt{4.8g} = 0.5$ *                                                                   | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                 | Using the model and $e = \text{sep. speed} / \text{app. speed}$ ,<br>$v = 0.5\sqrt{1.2g}$                 | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$                                                                     | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15 \text{ (m)}$                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                 | Ball continues to bounce with the height of each bounce being a quarter of the previous one               | B1         | 2.2b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                          |                                                                                                           |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                     |                                                                                                           |            |      |
| <b>(a)</b><br><b>M1:</b> For a complete method to find $v$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For a complete method to find $u$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For finding both $v$ and $u$ and use of Newton's Law of Restitution<br><b>A1*:</b> For the given answer |                                                                                                           |            |      |
| <b>(b)</b><br><b>M1:</b> For use of Newton's Law of Restitution to find rebound speed<br><b>M1:</b> For a complete method to find $h$<br><b>A1:</b> For 0.15 (m) oe                                                                                                                                                                        |                                                                                                           |            |      |
| <b>(c)</b><br><b>B1:</b> For a clear description including reference to a quarter                                                                                                                                                                                                                                                          |                                                                                                           |            |      |

| Question                                                                                                                                           | Scheme                                                   | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|------------|------|
| <b>7(a)</b>                                                                                                                                        | Energy Loss = KE Loss – PE Gain                          | M1         | 3.3  |
|                                                                                                                                                    | $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 g \times 20$ | A1         | 1.1b |
|                                                                                                                                                    | $= 58.25 = 58 \text{ (J ) or } 58.3 \text{ (J)}$         | A1         | 1.1b |
|                                                                                                                                                    |                                                          | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                         | Using work-energy principle, $20 R = 58.25$              | M1         | 3.3  |
|                                                                                                                                                    | $R = 2.9125 = 2.9 \text{ or } 2.91$                      | A1ft       | 1.1b |
|                                                                                                                                                    |                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                         | Make resistance variable (dependent on speed)            | B1         | 3.5c |
|                                                                                                                                                    |                                                          | <b>(1)</b> |      |
| <b>(6 marks)</b>                                                                                                                                   |                                                          |            |      |
| Notes:                                                                                                                                             |                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For a difference in KE and PE<br><b>A1:</b> For a correct expression<br><b>A1:</b> For either 58 (2sf) or 58.3(3sf)       |                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of work-energy principle<br><b>A1ft:</b> For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a) |                                                          |            |      |
| <b>(c)</b><br><b>B1:</b> For variable resistance oe                                                                                                |                                                          |            |      |

| Question                                                                                                                                                                                                                                                            | Scheme                                          | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|-------|------|
| <b>8(a)</b>                                                                                                                                                                                                                                                         | Force = Resistance (since no acceleration) = 30 | B1    | 3.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 30 $\times$ 4    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 120 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (3)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                          | Resolving parallel to the slope                 | M1    | 3.1b |
|                                                                                                                                                                                                                                                                     | $F - 60g\sin\alpha - 30 = 0$                    | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | $F = 70$                                        | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 70 $\times$ 3    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 210 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (5)   |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                    |                                                 |       |      |
| Notes:                                                                                                                                                                                                                                                              |                                                 |       |      |
| <b>(a)</b><br><b>B1:</b> For force = 30 seen<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 120 (W), follow through on their '30'                                                                                                                            |                                                 |       |      |
| <b>(b)</b><br><b>M1:</b> For resolving parallel to the slope with correct no. of terms and 60g resolved<br><b>A1:</b> For a correct equation<br><b>A1:</b> For $F = 70$<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 210 (W), follow through on their '70' |                                                 |       |      |

| Question    | Scheme                                                                                                                   | Marks      | AOs  |
|-------------|--------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>9(a)</b> | Use of conservation of momentum                                                                                          | M1         | 3.1a |
|             | $3mu - 2mu = 3mv + mw$                                                                                                   | A1         | 1.1b |
|             | Use of NLR                                                                                                               | M1         | 3.1a |
|             | $3ue = -v + w$                                                                                                           | A1         | 1.1b |
|             | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1         | 3.1b |
|             | $v = \frac{u}{4}(1 - 3e)$                                                                                                | A1         | 1.1b |
|             |                                                                                                                          | <b>(6)</b> |      |
| <b>(b)</b>  | $\frac{u}{4}(1 - 3e) < 0$                                                                                                | M1         | 3.1b |
|             | $\frac{1}{3} < e \leq 1$                                                                                                 | A1         | 1.1b |
|             |                                                                                                                          | <b>(2)</b> |      |
| <b>(c)</b>  | Solving for $w$                                                                                                          | M1         | 2.1  |
|             | $w = \frac{u}{4}(1 + 9e)^*$                                                                                              | A1 *       | 1.1b |
|             |                                                                                                                          | <b>(2)</b> |      |
| <b>(d)</b>  | Substitute $e = \frac{5}{9}$                                                                                             | M1         | 1.1b |
|             | $v = -\frac{u}{6}, w = \frac{3u}{2}$                                                                                     | A1         | 1.1b |
|             | Use NLR for impact with wall, $x = fw$                                                                                   | M1         | 1.1b |
|             | Further collision if $x > -v$                                                                                            | M1         | 3.4  |
|             | $f\frac{3u}{2} > \frac{u}{6}$                                                                                            | A1         | 1.1b |
|             | $1 \geq f > \frac{1}{9}$                                                                                                 | A1         | 1.1b |
|             |                                                                                                                          | <b>(6)</b> |      |

**(16 marks)**

Notes:

**(a)**

**M1:** For use of CLM, with correct no. of terms, condone sign errors

**A1:** For a correct equation

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**A1:** For a correct equation

**M1:** For setting up *two* equations and solving their equations for  $v$

**A1:** For a correct expression for  $v$

**(b)**

**M1:** For use of an appropriate inequality

**A1:** For a complete range of values of  $e$

**(c)**

**M1:** For solving their equations for  $w$

**A1:** For the given answer

Question 9 notes continued:

(d)

**M1:** For substituting  $e = \frac{5}{9}$  into their  $v$  and  $w$

**A1:** For correct expressions for  $v$  and  $w$

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**M1:** For use of appropriate inequality

**A1:** For a correct inequality

**A1:** For a correct range





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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2D: Further Pure Mathematics 1 and  
Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2D**

**You must have:**

Decision Mathematics question insert

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.  
Calculators must not have the facility for algebraic manipulation,  
differentiation and integration, or have retrievable mathematical  
formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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(b) Use the substitution  $t = \tan\left(\frac{x}{2}\right)$  and the answer to part (a) to prove that

$$\frac{1 - \sin x}{1 + \sin x} \dots (\sec x - \tan x)^2 \quad x \neq (2n + 1)\frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

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2. The value,  $V$  hundred pounds, of a particular stock  $t$  hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{dV}{dt} = \frac{V^2 - t}{t^2 + tV} \quad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula  $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$  to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

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3. Use algebra to find the set of values of  $x$  for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

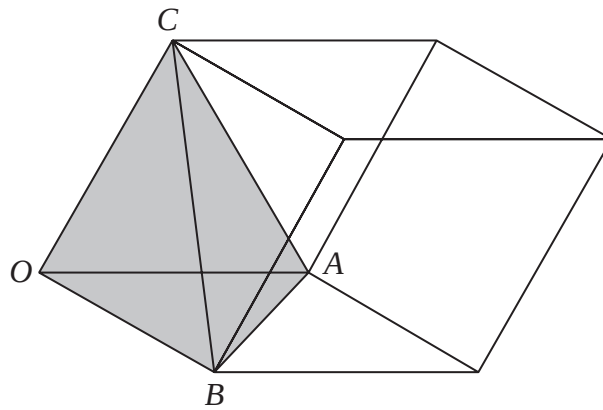
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**Question 3 continued****(Total for Question 3 is 6 marks)**

4.



### Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are  $O(0, 0, 0)$ ,  $A(2, 0, 0)$ ,  $B(0, 3, 1)$  and  $C(1, 1, 2)$ , where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

- (a) Find the surface area of the glued face of the tetrahedron. (4)
- (b) Find the volume of glass contained in this parallelepiped. (5)
- (c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture. (1)



**Question 4 continued****(Total for Question 4 is 10 marks)**

5.

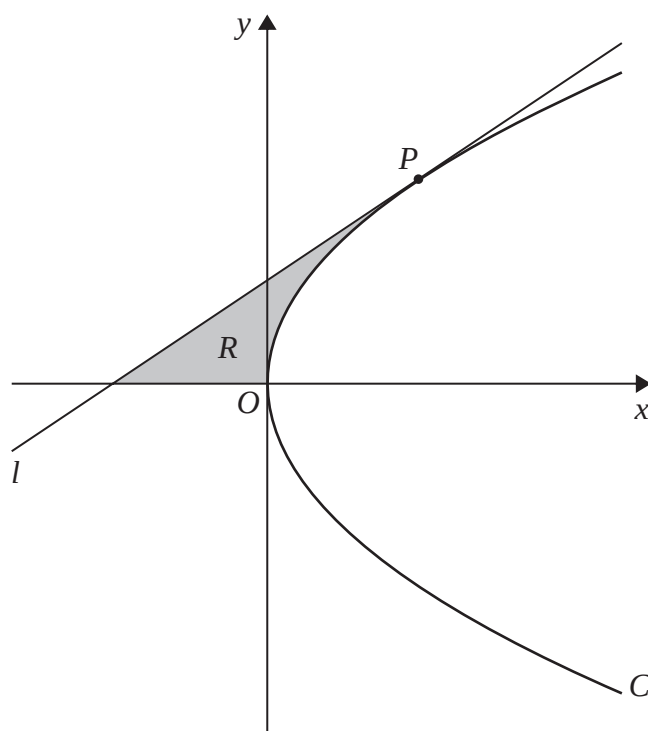
Diagram not  
drawn to scale

Figure 2

[ You may quote without proof that for the general parabola  $y^2 = 4ax$ ,  $\frac{dy}{dx} = \frac{2a}{y}$  ]

The parabola  $C$  has equation  $y^2 = 16x$ .

(a) Deduce that the point  $P(4p^2, 8p)$  is a general point on  $C$ .

(1)

The line  $l$  is the tangent to  $C$  at the point  $P$ .

(b) Show that an equation for  $l$  is

$$py = x + 4p^2$$

(3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the line  $l$ , the  $x$ -axis and the parabola  $C$ .

The line  $l$  intersects the directrix of  $C$  at the point  $B$ , where the  $y$  coordinate of  $B$  is  $\frac{10}{3}$

Given that  $p > 0$

(c) show that the area of  $R$  is 36

(8)

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**Question 5 continued**

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**(Total for Question 5 is 12 marks)**

**TOTAL FOR SECTION A IS 40 MARKS**



**Question 6 continued**

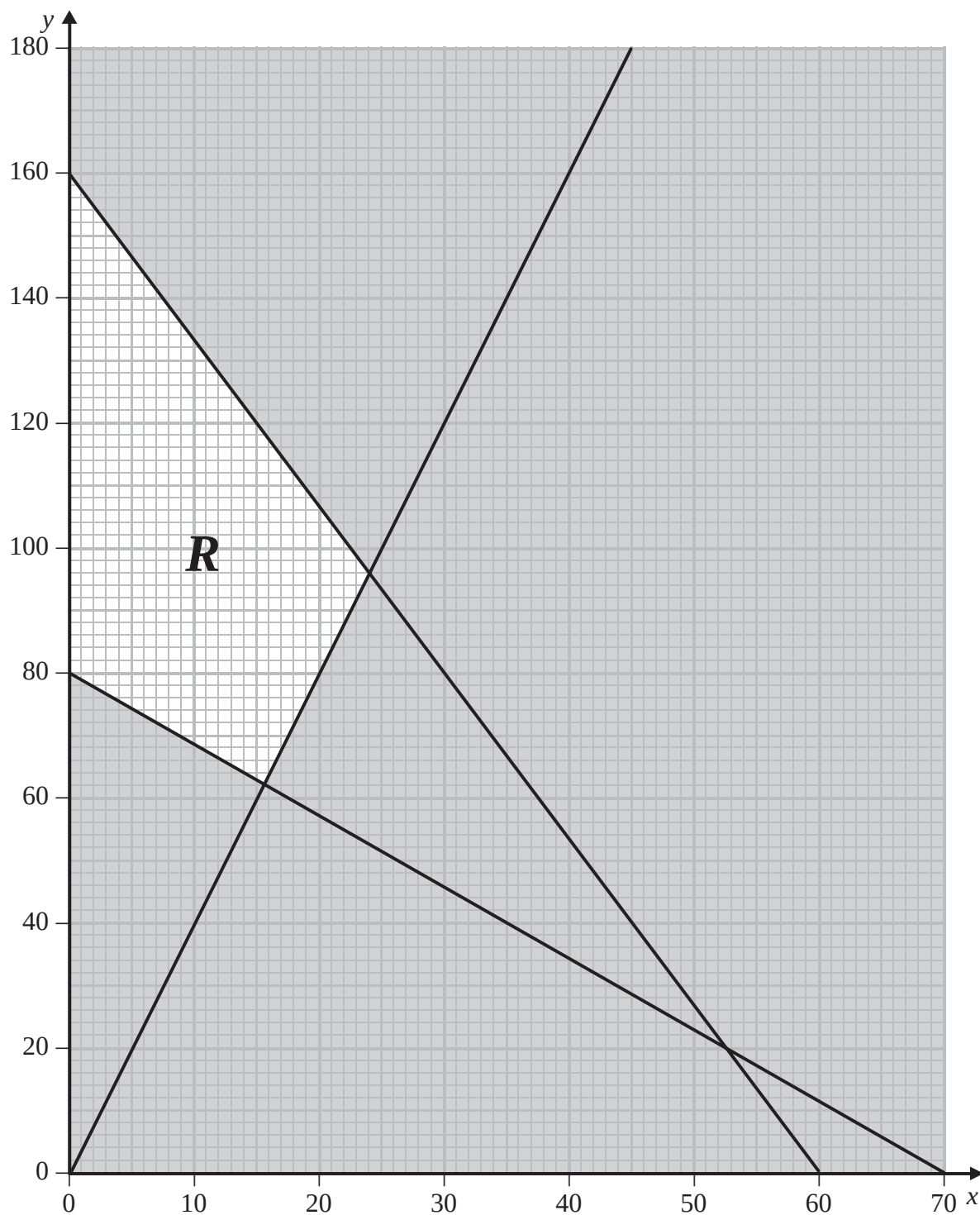
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**(Total for Question 6 is 12 marks)**

7.



**Question 7 continued**

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**(Total for Question 7 is 7 marks)**



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8. (a) and (b)

(Total for Question 8 is 7 marks)

9.

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(Total for Question 9 is 9 marks)

10.

(Total for Question 10 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**



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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2D: Section B Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

**Questions 6 – 10**

Paper Reference

**8FM0/2D**

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Total Marks

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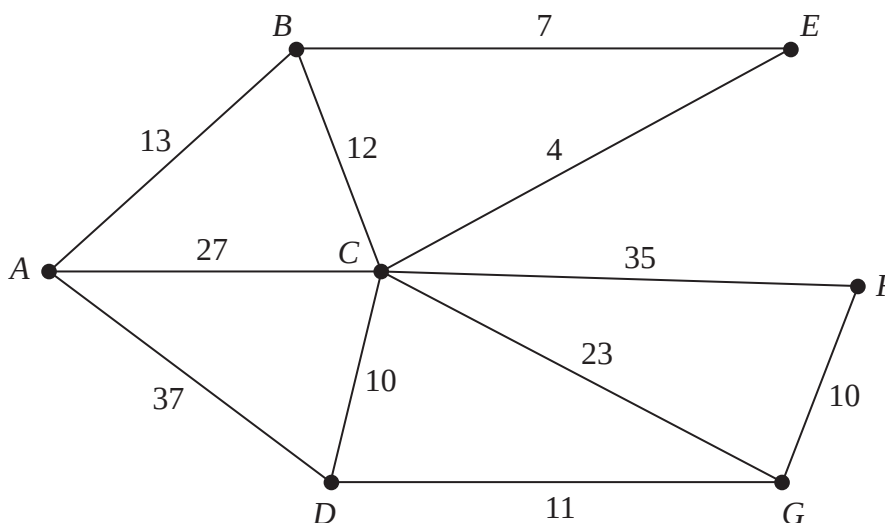


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## SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6.



**Figure 1**

[The total weight of the network is 189]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

- (a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length. (5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

- (b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear. (4)

- (c) State the minimum length of Gabriel's route. (1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

- (d) Calculate the length of BG. You must show your working. (2)

**(Total for Question 6 is 12 marks)**

7.

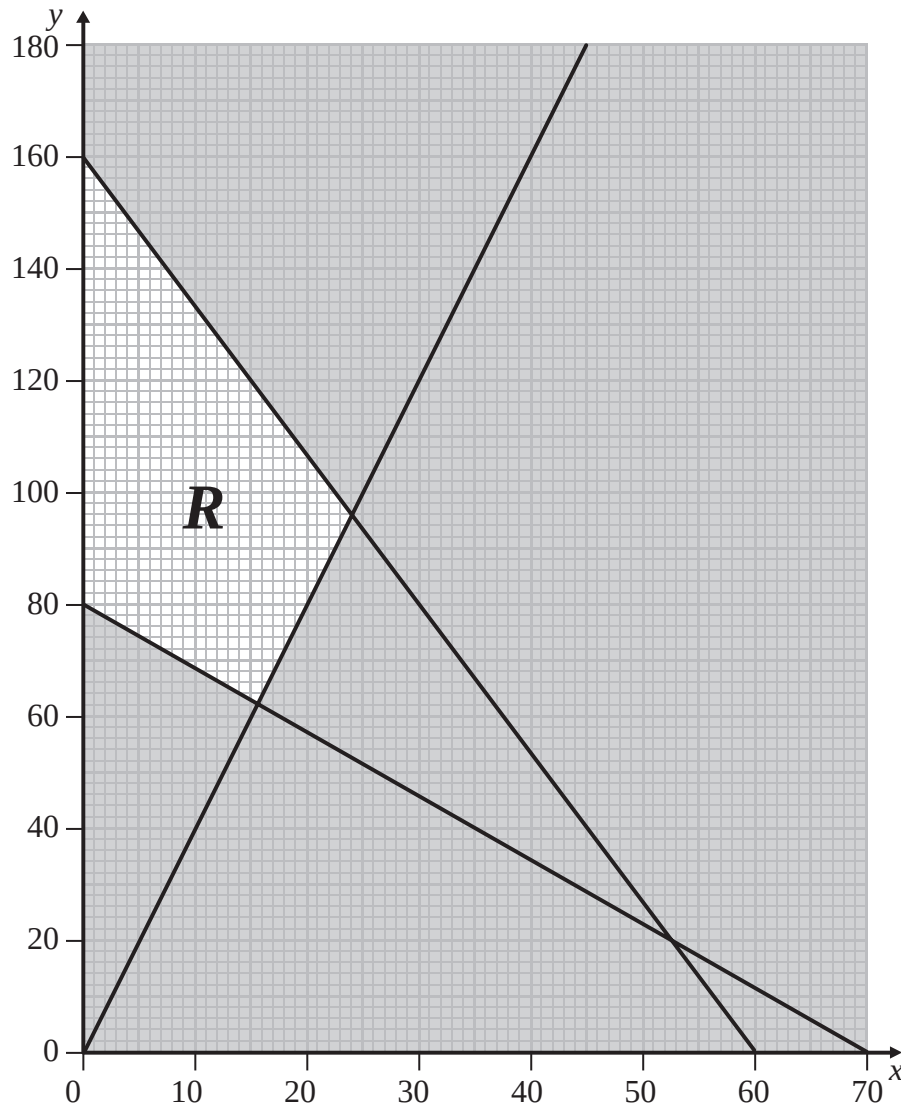


Figure 2

A teacher buys pens and pencils. The number of pens,  $x$ , and the number of pencils,  $y$ , that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 7 is 7 marks)

8.

| Activity | Time taken (days) | Immediately preceding activities |
|----------|-------------------|----------------------------------|
| A        | 5                 | -                                |
| B        | 7                 | -                                |
| C        | 3                 | -                                |
| D        | 4                 | A, B                             |
| E        | 4                 | D                                |
| F        | 2                 | B                                |
| G        | 4                 | B                                |
| H        | 5                 | C, G                             |
| I        | 10                | C, G                             |

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only. (3)
- (b) (i) Show that the project can be completed in 21 days, showing your working. (4)
- (ii) Identify the critical activities.

(Total for Question 8 is 7 marks)



9. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5 (1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are  $2^{2x} - 1$ ,  $2^x$ ,  $x + 1$ ,  $2^{x+1} - 3$  and  $11 - x$ .

- (b) (i) Calculate  $x$ .  
(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer. (6)

- (c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian. (2)

---

(Total for Question 9 is 9 marks)

10. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let  $x$  and  $y$  represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 10 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**

## Paper 2 Option D

### Further Pure Mathematics 1 Mark Scheme (Section A)

| Question                                                                                                                                      | Scheme                                                                               | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                   | $\sec x - \tan x = \frac{1}{\frac{1-t^2}{1+t^2}} - \frac{2t}{1-t^2}$                 | M1         | 2.1  |
|                                                                                                                                               | $= \frac{1+t^2}{1-t^2} - \frac{2t}{1-t^2} = \frac{1-2t+t^2}{1-t^2}$                  | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t} *$                                   | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                    | $\frac{1-\sin x}{1+\sin x} = \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$          | M1         | 1.1a |
|                                                                                                                                               | $= \frac{1+t^2-2t}{1+t^2+2t}$                                                        | M1         | 1.1b |
|                                                                                                                                               | $= \frac{(1-t)^2}{(1+t)^2} = \left(\frac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2 *$ | A1*        | 2.1  |
|                                                                                                                                               |                                                                                      | <b>(3)</b> |      |
| <b>(6 marks)</b>                                                                                                                              |                                                                                      |            |      |
| Notes:                                                                                                                                        |                                                                                      |            |      |
| <b>(a)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses $\sec x = \frac{1}{\cos x}$ and the $t$ -substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of $t$ |                                                                                      |            |      |
| <b>M1:</b> Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer                       |                                                                                      |            |      |
| <b>(b)</b>                                                                                                                                    |                                                                                      |            |      |
| <b>M1:</b> Uses the $t$ -substitution for $\sin x$ in both numerator and denominator                                                          |                                                                                      |            |      |
| <b>M1:</b> Multiplies through by $1+t^2$ in numerator and denominator                                                                         |                                                                                      |            |      |
| <b>A1*:</b> Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result                      |                                                                                      |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                            | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | £300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$ ;<br>half an hour after purchase $\Rightarrow t_2 = 1.5$ , so step $h$ required is 0.25 | B1    | 3.3  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $t_0 = 1, V_0 = 3, \left(\frac{dV}{dt}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$                                                                              | M1    | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_1 \approx V_0 + h \left(\frac{dV}{dt}\right)_0 = 3 + 0.25 \times 2 = \dots$                                                                                    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $= 3.5$                                                                                                                                                           | A1ft  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\left(\frac{dV}{dt}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$                                                | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $V_2 \approx V_1 + h \left(\frac{dV}{dt}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\dots$ , so £396<br>(nearest £)                                       | A1    | 3.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                   | (6)   |      |
| (6 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                   |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                   |       |      |
| <p><b>B1:</b> Identifies the correct initial conditions and requirement for <math>h</math></p> <p><b>M1:</b> Uses the model to evaluate <math>\frac{dV}{dt}</math> at <math>t_0</math>, using their <math>t_0</math> and <math>V_0</math></p> <p><b>M1:</b> Applies the approximation formula with their values</p> <p><b>A1ft:</b> 3.5 or exact equivalent. Follow through their step value</p> <p><b>M1:</b> Attempt to find <math>\left(\frac{dV}{dt}\right)_1</math> with their 3.5</p> <p><b>A1:</b> Applies the approximation and interprets the result to give £396</p> |                                                                                                                                                                   |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                   | Marks      | AOs         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|------------|-------------|
| <b>3</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{1}{x} < \frac{x}{x+2}$                                                                            |            |             |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$                                              | M1         | 2.1         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$         | M1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | At least two correct critical values from $-2, -1, 0, 2$                                                 | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | All four correct critical values $-2, -1, 0, 2$                                                          | A1         | 1.1b        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$ | M1<br>A1   | 2.2a<br>2.5 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                          | <b>(6)</b> |             |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                          |            |             |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                          |            |             |
| <p><b>M1:</b> Gathers terms on one side and puts over common denominator, or multiply by <math>x^2(x+2)^2</math> and then gather terms on one side</p> <p><b>M1:</b> Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors</p> <p><b>A1:</b> At least 2 correct critical values found</p> <p><b>A1:</b> Exactly 4 correct critical values</p> <p><b>M1:</b> Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means</p> <p><b>A1:</b> Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept <math>\mathbb{R} - ([-2, -1] \cup [0, 2])</math> or <math>\{x \in \mathbb{R} : x &lt; -2 \text{ or } -1 &lt; x &lt; 0 \text{ or } x &gt; 2\}</math></p> |                                                                                                          |            |             |

| Question    | Scheme                                                                                                                                                                                 | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b> | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2} \mathbf{AB} \times \mathbf{AC} $                                          | M1         | 3.1a |
|             | $\frac{1}{2} \mathbf{AB} \times \mathbf{AC}  = \frac{1}{2} (-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 2\mathbf{k}) $                                 | M1         | 1.1b |
|             | $= \frac{1}{2} 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} $                                                                                                                                | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35}(\text{m}^2)$                                                                                                                                                   | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
|             | <b>Alternative</b>                                                                                                                                                                     |            |      |
|             | Identifies glued face is triangle $ABC$ and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{ \mathbf{AB} ^2 \mathbf{AC} ^2 - (\mathbf{AB} \cdot \mathbf{AC})^2}$ | M1         | 3.1a |
|             | $ \mathbf{AB} ^2 = 4 + 9 + 1 = 14$ , $ \mathbf{AC} ^2 = 1 + 1 + 4 = 6$<br>and $\mathbf{AB} \cdot \mathbf{AC} = 2 + 3 + 2 = 7$                                                          | M1         | 1.1b |
|             | So area of glue is $= \frac{1}{2}\sqrt{(14)(6) - (7)^2}$                                                                                                                               | M1         | 1.1b |
|             | $= \frac{1}{2}\sqrt{35} (\text{m}^2)$                                                                                                                                                  | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(4)</b> |      |
| <b>(b)</b>  | Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC} \cdot (\mathbf{OA} \times \mathbf{OB}))$                                                                | M1         | 3.1a |
|             | $= \frac{1}{6}(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} \times (3\mathbf{j} + \mathbf{k}))$                                                                           | M1         | 1.1b |
|             | $= \frac{10}{6} = \frac{5}{3}$                                                                                                                                                         | A1         | 1.1b |
|             | Volume of parallelepiped is $6 \times$ volume of tetrahedron ( $= 10$ ),<br>so volume of glass is difference between these, viz. $10 - \frac{5}{3} = \dots$                            | M1         | 3.1a |
|             | Volume of glass $= \frac{25}{3}(\text{m}^3)$                                                                                                                                           | A1         | 1.1b |
|             |                                                                                                                                                                                        | <b>(5)</b> |      |

| Question                                                                                                                                        | Scheme                                                                                                                                                                                                                                                                | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                                                                                                                                                 | <b>4(b) Alternative</b>                                                                                                                                                                                                                                               |            |      |
|                                                                                                                                                 | $-\mathbf{j} + 3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                                         | M1         | 3.1a |
|                                                                                                                                                 | Area $AOB = \frac{1}{2} \times  \mathbf{OA}  \times  \mathbf{OB}  = \frac{1}{2} \times 2 \times \sqrt{10} = \sqrt{10}$                                                                                                                                                | A1         | 1.1b |
|                                                                                                                                                 | $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k}) \Rightarrow p = \frac{1}{2}$<br>and so height of tetrahedron is<br>$h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10}$ | M1         | 3.1a |
|                                                                                                                                                 | Volume of glass is $V = 5 \times \text{Volume of tetrahedron}$<br>$= 5 \times \frac{1}{3} \sqrt{10} \times \frac{1}{2} \sqrt{10}$                                                                                                                                     | M1         | 1.1b |
|                                                                                                                                                 | $= \frac{25}{3} (\text{m}^3)$                                                                                                                                                                                                                                         | A1         | 1.1b |
|                                                                                                                                                 |                                                                                                                                                                                                                                                                       | <b>(5)</b> |      |
| <b>(c)</b>                                                                                                                                      | The glued surfaces may distort the shapes / reduce the volume of concrete<br>Measurements in m may not be accurate<br>The surface of the concrete tetrahedron may not be smooth<br>Pockets of air may form when the concrete is being poured                          | B1         | 3.2b |
|                                                                                                                                                 |                                                                                                                                                                                                                                                                       | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                               |                                                                                                                                                                                                                                                                       |            |      |
| Question 4 notes:                                                                                                                               |                                                                                                                                                                                                                                                                       |            |      |
| Accept use of column vectors throughout                                                                                                         |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a)</b>                                                                                                                                      |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Shows an understanding of what is required via an attempt at finding the area of triangle $ABC$                                      |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Any correct method for the triangle area is fine                                                                                     |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent                     |                                                                                                                                                                                                                                                                       |            |      |
| <b>(a) Alternative</b>                                                                                                                          |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides                                       |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> May use different sides to those shown                                                                                               |                                                                                                                                                                                                                                                                       |            |      |
| <b>M1:</b> Correct full method to find the area of the triangle using their two sides                                                           |                                                                                                                                                                                                                                                                       |            |      |
| <b>A1:</b> $\frac{1}{2}\sqrt{35}$ or exact equivalent                                                                                           |                                                                                                                                                                                                                                                                       |            |      |

|                             |                                                                                                                                                                                                                                                                                                                  |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 4 notes continued: |                                                                                                                                                                                                                                                                                                                  |
| <b>(b)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Attempts volume of concrete by finding volume of tetrahedron with appropriate method                                                                                                                                                                                                                             |
| <b>M1:</b>                  | Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted                                                                                                                                                                                                |
| <b>A1:</b>                  | Correct value for the volume of concrete                                                                                                                                                                                                                                                                         |
| <b>M1:</b>                  | Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped                                                                                                                                    |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(b)</b>                  | <b>Alternative</b>                                                                                                                                                                                                                                                                                               |
| <b>M1:</b>                  | Notes (or works out using scalar products) that $-\mathbf{j} + 3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j} + \mathbf{k}$                                                                                                                           |
| <b>A1:</b>                  | Finds (using that $\mathbf{OA}$ and $\mathbf{OB}$ are perpendicular), area of $AOB = \sqrt{10}$                                                                                                                                                                                                                  |
| <b>M1:</b>                  | Solves $\mathbf{i} + \mathbf{j} + 2\mathbf{k} - p(-\mathbf{j} + 3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j} + \mathbf{k})$ to get the height of the tetrahedron<br>$\left[ (\mu = \lambda =) p = \frac{1}{2}, \text{ so } h = \frac{1}{2}  -\mathbf{j} + 3\mathbf{k}  = \frac{1}{2} \sqrt{10} \right]$ |
| <b>M1:</b>                  | Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)                                                                                                                                                                                          |
| <b>A1:</b>                  | $\frac{25}{3}$ only, ignore reference to units                                                                                                                                                                                                                                                                   |
| <b>(c)</b>                  |                                                                                                                                                                                                                                                                                                                  |
| <b>B1:</b>                  | Any acceptable reason in context                                                                                                                                                                                                                                                                                 |



| Question    | Scheme                                                                                                                     | Marks      | AOs  |
|-------------|----------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>5(a)</b> | $y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$<br>$\Rightarrow P(4p^2, 8p)$ is a general point on $C$                 | B1         | 2.2a |
|             |                                                                                                                            | <b>(1)</b> |      |
| <b>(b)</b>  | $y^2 = 16x$ gives $a = 4$ , or $2y \frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$                                    | M1         | 2.2a |
|             | $l: y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$                                                                          | M1         | 1.1b |
|             | leading to $py = x + 4p^2$ *                                                                                               | A1*        | 2.1  |
|             |                                                                                                                            | <b>(3)</b> |      |
| <b>(c)</b>  | $B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                            | M1         | 3.1a |
|             | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                 | M1         | 1.1b |
|             | $p = \frac{3}{2}$ and $l$ cuts $x$ -axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$     | M1         | 2.1  |
|             | $x = -9$                                                                                                                   | A1         | 1.1b |
|             | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}} dx$ | M1         | 2.1  |
|             | $\int 4x^{\frac{1}{2}} dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} (+c)$ or $\frac{8}{3}x^{\frac{3}{2}} (+c)$   | M1         | 1.1b |
|             |                                                                                                                            | A1         | 1.1b |
|             | $\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *                     | A1*        | 1.1b |
|             |                                                                                                                            | <b>(8)</b> |      |

| Question          | Scheme                                                                                                                                                                                                                                                            | Marks      | AOs  |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
|                   | <b>5(c) Alternative 1</b>                                                                                                                                                                                                                                         |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                            | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                        | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ into } l \text{ gives } \frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                             | M1         | 2.1  |
|                   | $x = \frac{3}{2}y - 9$                                                                                                                                                                                                                                            | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left( \frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right) \right) dy$                                                                                                                 | M1         | 2.1  |
|                   | $\int \left( \frac{1}{16}y^2 - \frac{3}{2}y + 9 \right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y (+c)$                                                                                                                                                          | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | A1         | 1.1b |
|                   | $\text{Area}(R) = \left( \frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12) \right) - (0)$<br>$= 36 - 108 + 108 = 36 *$                                                                                                                                               | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | <b>(8)</b> |      |
|                   | <b>5(c) Alternative 2</b>                                                                                                                                                                                                                                         |            |      |
|                   | $B\left(-4, \frac{10}{3}\right) \text{ into } l \Rightarrow \frac{10p}{3} = -4 + 4p^2$                                                                                                                                                                            | M1         | 3.1a |
|                   | $6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \dots$                                                                                                                                                                                        | M1         | 1.1b |
|                   | $p = \frac{3}{2} \text{ and } l \text{ cuts px-axis when } \frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \dots$                                                                                                                                | M1         | 2.1  |
|                   | $x = -9$                                                                                                                                                                                                                                                          | A1         | 1.1b |
|                   | $p = \frac{3}{2} \Rightarrow P(9, 12) \text{ and } x = 0 \text{ in } l: y = \frac{2}{3}x + 6 \text{ gives } y = 6$<br>$\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left( \left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right) \right) dx$ | M1         | 2.1  |
|                   | $\int \left( \frac{2}{3}x + 6 - 4x^{\frac{1}{2}} \right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} (+c)$                                                                                                                                              | M1         | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | A1         | 1.1b |
|                   | $\text{Area}(R) = 27 + \left( \left( \frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}(9^{\frac{3}{2}}) \right) - (0) \right)$<br>$= 27 + (27 + 54 - 72) = 27 + 9 = 36 *$                                                                                                     | A1*        | 1.1b |
|                   |                                                                                                                                                                                                                                                                   | <b>(8)</b> |      |
| <b>(12 marks)</b> |                                                                                                                                                                                                                                                                   |            |      |

|                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 5 notes:        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| (a)                      | <p><b>B1:</b> Substitutes <math>y_p = 8p</math> into <math>y^2</math> to obtain <math>64p^2</math> and substitutes <math>x_p = 4p^2</math> into <math>16x</math> to obtain <math>64p^2</math> and concludes that <math>P</math> lies on <math>C</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| (b)                      | <p><b>M1:</b> Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it</p> <p><b>M1:</b> Applies <math>y - 8p = m(x - 4p^2)</math>, with their tangent gradient <math>m</math>, which is in terms of <math>p</math>. Accept use of <math>8p = m(4p^2) + c</math> with a clear attempt to find <math>c</math></p> <p><b>A1*:</b> Obtains <math>py = x + 4p^2</math> by <b>cs0</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| (c)                      | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math></p> <p><b>M1:</b> Substitutes their <math>p</math> (which must be positive) and <math>y = 0</math> into <math>l</math> and solves to give <math>x = \dots</math></p> <p><b>A1:</b> Finds that <math>l</math> cuts the <math>x</math>-axis at <math>x = -9</math></p> <p><b>M1:</b> Fully correct method for finding the area of <math>R</math><br/> i.e. <math>\frac{1}{2}(\text{their } x_p - "-9")(\text{their } y_p) - \int_0^{\text{their } x_p} 4x^2 dx</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda x^{\frac{1}{2}}</math> to give <math>\pm \mu x^{\frac{3}{2}}</math>, where <math>\lambda, \mu \neq 0</math></p> <p><b>A1:</b> Integrates <math>4x^{\frac{1}{2}}</math> to give <math>\frac{8}{3}x^{\frac{3}{2}}</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p>                                                             |
| (c) <b>Alternative 1</b> | <p><b>M1:</b> Substitutes their <math>x = "-a"</math> and <math>y = \frac{10}{3}</math> into <math>l</math></p> <p><b>M1:</b> Obtains a 3 term quadratic and solves (using the usual rules) to give <math>p = \dots</math><br/> Substitutes their <math>p</math> (which must be positive) into <math>l</math> and rearranges to give <math>x = \dots</math></p> <p><b>M1:</b> Finds <math>l</math> as <math>x = \frac{3}{2}y - 9</math></p> <p><b>A1:</b> Fully correct method for finding the area of <math>R</math></p> <p><b>M1:</b> i.e. <math>\int_0^{\text{their } y_p} \left( \frac{1}{16}y^2 - \text{their} \left( \frac{3}{2}y - 9 \right) \right) dy</math></p> <p><b>M1:</b> Integrates <math>\pm \lambda y^2 \pm \mu y \pm \nu</math> to give <math>\pm \alpha y^3 \pm \beta y^2 \pm \nu y</math>, where <math>\lambda, \mu, \nu, \alpha, \beta \neq 0</math></p> <p><b>A1:</b> Integrates <math>\frac{1}{16}y^2 - \left( \frac{3}{2}y - 9 \right)</math> to give <math>\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y</math>, simplified or un-simplified</p> <p><b>A1*:</b> Fully correct proof leading to a correct answer of 36</p> |

Question 5 notes continued:

**(c) Alternative 2**

**M1:** Substitutes their  $x = "-a"$  and  $y = \frac{10}{3}$  into  $l$

**M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  $p = \dots$

**M1:** Substitutes their  $p$  (which must be positive) and  $y = 0$  into  $l$  and solves to give  $x = \dots$

**A1:** Finds that  $l$  cuts the  $x$ -axis at  $x = -9$

**M1:** Fully correct method for finding the area of  $R$

i.e.  $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_p} \left( \text{their } \left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right) \right) dy$

**M1:** Integrates  $\pm \lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$  to give  $\pm \alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$ , where  $\lambda, \mu, \nu, \alpha, \beta \neq 0$

**A1:** Integrates  $\left( \frac{2}{3}x + 6 \right) - \left( 4x^{\frac{1}{2}} \right)$  to give  $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$ , simplified or un-simplified

**A1\*:** Fully correct proof leading to a correct answer of 36

Decision Mathematics 1 Mark Scheme (Section **B**)

| Question                                                                                                                                                                                                                                                                                                                                                       | Scheme                              | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|------------|------|
| <b>6(a)</b>                                                                                                                                                                                                                                                                                                                                                    |                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Path: ABECDGF                       | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Length: 55 (metres)                 | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(5)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                     | $AB + DG = 13 + 11 = 24 \leftarrow$ | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BEC)D + B(ECD)G = 34 + 32 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BECD)G + B(EC)D = 45 + 21 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Repeat arcs: AB, DG                 | A1ft       | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(4)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                     | Length = $189 + 24 = 213$ (metres)  | B1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(1)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                     | $189 + x + 34 = 213 + 2x$           | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $x = 10$ so BG is 10 m              | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(2)</b> |      |
| <b>(12 marks)</b>                                                                                                                                                                                                                                                                                                                                              |                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                         |                                     |            |      |
| <b>(a)</b><br><b>M1:</b> For a larger number replaced by a smaller one in the working values boxes at C, D, F or G<br><b>A1:</b> For all values correct (and in correct order) at A, B, C and D<br><b>A1:</b> For all values correct (and in correct order) at E, F & G<br><b>A1:</b> For the correct path<br><b>A1ft:</b> For 55 or ft their final value at F |                                     |            |      |
| <b>(b)</b><br><b>M1:</b> For 3 correct pairings of the four odd nodes (A,B, D & G)<br><b>A1:</b> At least two pairings and totals correct<br><b>A2:</b> All three pairings and totals correct<br><b>A3ft:</b> Selecting their shortest pairing, and stating that these arcs should be repeated                                                                 |                                     |            |      |

Question **6** notes continued:

**(c)**

**B1ft:** For 213 or 189 + their shortest repeat

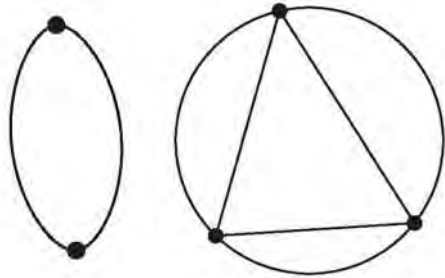
**M1:** For translating the information in the question in to an equation involving  $x$ ,  $2x$  and 34

**A1:** For a correct equation leading to  $BG = 10$  (m)

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                                        | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Objective line drawn or at least two vertices tested                                                                          | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | For solving $y = 4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$ | M1    | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Finding at least two points with integer co-ordinates from $(15 \pm 1, 63 \pm 2)$                                             | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Testing at least two points with integer co-ordinates                                                                         | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15$ and $y = 63$                                                                                                         | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | So the teacher should buy 15 pens and 63 pencils                                                                              | A1ft  | 3.2a |
| (7 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                               |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                               |       |      |
| <p><b>M1:</b> Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C</p> <p><b>M1:</b> Solving simultaneous equations</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of <math>y \geq 4x</math> and <math>8x + 7y \geq 560</math></p> <p><b>M1:</b> Testing at least two integer solutions in <math>y \geq 4x</math> or <math>8x + 7y \geq 560</math> and C</p> <p><b>A1:</b> cao – deducing from tests which integer solution is both valid and optimal</p> <p><b>A1ft:</b> Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils</p> |                                                                                                                               |       |      |

| Question  | Scheme                                                                                                                                                                                                                                                                        | Marks | AOs  |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 8(a)(b)   | <p>The number(s) at the end of activity E indicate this project can be completed in 21 days</p> <p>Critical activities: B, G, I</p>                                                                                                                                           | M1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | A1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | A1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | (3)   |      |
|           |                                                                                                                                                                                                                                                                               | M1    | 2.1  |
| A1        | 1.1b                                                                                                                                                                                                                                                                          |       |      |
| A1ft      | 2.2a                                                                                                                                                                                                                                                                          |       |      |
| A1        | 1.1b                                                                                                                                                                                                                                                                          |       |      |
| (4)       |                                                                                                                                                                                                                                                                               |       |      |
| (7 marks) |                                                                                                                                                                                                                                                                               |       |      |
| Notes:    |                                                                                                                                                                                                                                                                               |       |      |
| M1:       | At least 5 activities and one dummy, one start                                                                                                                                                                                                                                |       |      |
| A1:       | A,B,C,D,F,G and first dummy correct                                                                                                                                                                                                                                           |       |      |
| A1:       | E,H,I correct, second dummy correct and one finish                                                                                                                                                                                                                            |       |      |
| M1:       | All boxes completed, number generally increasing L to R (condone one “rogue”)                                                                                                                                                                                                 |       |      |
| A1:       | All values cao                                                                                                                                                                                                                                                                |       |      |
| A1:       | Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes) |       |      |
| A1:       | Critical activities correct                                                                                                                                                                                                                                                   |       |      |



| Question                                                                         | Scheme                                                                                                                        | Marks | AOs  |
|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>9(a)</b>                                                                      | e.g. a graph cannot contain an odd number of odd nodes<br>e.g. number of arcs = $\frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$ | B1    | 2.4  |
|                                                                                  |                                                                                                                               | (1)   |      |
| <b>(b)(i)</b>                                                                    | $(2^{2x}-1)+(2^x)+(x+1)+(2^{x+1}-3)+(11-x)=2(18)$                                                                             | M1    | 1.1b |
|                                                                                  | $2^{2x}+3(2^x)-28=0 \Rightarrow x=...$                                                                                        | M1    | 1.1b |
|                                                                                  | $(2^x+7)(2^x-4)=0 \Rightarrow x=2$                                                                                            | A1    | 1.1b |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(b)(ii)</b>                                                                   | The order of the nodes are 9, 15, 3, 4, 5                                                                                     | M1    | 2.1  |
|                                                                                  | Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes                                | A1    | 2.4  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(c)</b>                                                                       |                                             | M1    | 2.5  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (2)   |      |
| <b>(9 marks)</b>                                                                 |                                                                                                                               |       |      |
| Notes:                                                                           |                                                                                                                               |       |      |
| <b>(a)</b>                                                                       |                                                                                                                               |       |      |
| <b>B1:</b> Explanation referring to need for an even number of odd nodes oe      |                                                                                                                               |       |      |
| <b>(b)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Forming an equation involving the orders of the 5 odd nodes and 2(18) |                                                                                                                               |       |      |
| <b>M1:</b> Simplifies to a quadratic in $2^x$ and attempts to solve              |                                                                                                                               |       |      |
| <b>A1:</b> 2 cao                                                                 |                                                                                                                               |       |      |
| <b>M1:</b> Construct an argument involving the order of the 5 nodes              |                                                                                                                               |       |      |
| <b>A1:</b> Explanation considering the number of odd nodes                       |                                                                                                                               |       |      |
| <b>A1:</b> Deduction that therefore it is neither Eulerian nor semi-Eulerian     |                                                                                                                               |       |      |
| <b>(c)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Interprets mathematical language to construct a disconnected graph    |                                                                                                                               |       |      |
| <b>A1:</b> Deduce a correct graph                                                |                                                                                                                               |       |      |

| Question                                                                                                                                                                                                                                                                                                                         | Scheme                                                                      | Marks    | AOs        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|----------|------------|
| <b>10</b>                                                                                                                                                                                                                                                                                                                        | Minimise ( $C =$ ) $25x + 35y$                                              | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | Subject to:<br>$(500x + 800y \geq 150\,000 \Rightarrow) 5x + 8y \geq 1500$  | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | $\frac{7}{20}(x + y) \leq x \leq \frac{13}{20}(x + y)$                      | M1<br>M1 | 3.3<br>3.3 |
|                                                                                                                                                                                                                                                                                                                                  | Which simplifies to $7y \leq 13x$ <b>and</b> $13y \geq 7x$<br>$x, y \geq 0$ | A1       | 1.1b       |
| <b>(5 marks)</b>                                                                                                                                                                                                                                                                                                                 |                                                                             |          |            |
| Notes:                                                                                                                                                                                                                                                                                                                           |                                                                             |          |            |
| <b>B1:</b> A correct objective function + minimise<br><b>B1:</b> Translate information in to a correct inequality<br><b>M1:</b> For translating the information given into the LHS inequality<br><b>M1:</b> For translating the information given in to the RHS inequality<br><b>A1:</b> Simplifying to the correct inequalities |                                                                             |          |            |

Write your name here

Surname

Other names

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2E: Further Statistics 1 and Further Mechanics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2E**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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|        |        | Language |         |          |
|--------|--------|----------|---------|----------|
|        |        | French   | Spanish | Mandarin |
| Gender | Male   | 23       | 22      | 20       |
|        | Female | 38       | 32      | 15       |

(a) State the null hypothesis for this test. (1)

- (b) Show that the expected frequency for females choosing Spanish is 30.6

- (c) Calculate the test statistic for this test, stating the expected frequencies you have used. (3)

- (d) State whether or not the null hypothesis is rejected. Justify your answer. (2)

- (e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance. (1)

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**Question 1 continued**

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2. The discrete random variable  $X$  has probability distribution given by

|            |     |     |     |     |     |
|------------|-----|-----|-----|-----|-----|
| $x$        | -1  | 0   | 1   | 2   | 3   |
| $P(X = x)$ | $c$ | $a$ | $a$ | $b$ | $c$ |

The random variable  $Y = 2 - 5X$

Given that  $E(Y) = -4$  and  $P(Y \geq -3) = 0.45$

- (a) find the probability distribution of  $X$ .

(7)

Given also that  $E(Y^2) = 75$

- (b) find the exact value of  $\text{Var}(X)$

(2)

- (c) Find  $P(Y > X)$

(2)



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**Question 2 continued**

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3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

- (a) In a 1 hour period, find the probability that each company hires exactly 2 cars. (2)

- (b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3 (2)

- (c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9 (2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

- (d) (i) find the mean of the number of cars with a defect,  
(ii) find the variance of the number of cars with a defect. (2)

- (e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.  
(ii) Give a reason to support the use of a Poisson approximation. (2)

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**Question 3 continued**

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[illegible]

4. The discrete random variable  $X$  follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i)  $P(X = 1)$

(ii)  $P(X \leq 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

|                                     |    |    |   |   |   |   |   |
|-------------------------------------|----|----|---|---|---|---|---|
| <b>Number of mortgages approved</b> | 0  | 1  | 2 | 3 | 4 | 5 | 6 |
| <b>Frequency</b>                    | 10 | 16 | 7 | 4 | 2 | 0 | 1 |

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

|                                     |      |     |      |      |      |           |
|-------------------------------------|------|-----|------|------|------|-----------|
| <b>Number of mortgages approved</b> | 0    | 1   | 2    | 3    | 4    | 5 or more |
| <b>Expected frequency</b>           | 9.86 | $r$ | 9.67 | 4.51 | 1.58 | $s$       |

(c) Find the value of  $r$  and the value of  $s$  giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)



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**Question 4 continued**

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**Question 4 continued****(Total for Question 4 is 11 marks)****TOTAL FOR SECTION A IS 40 MARKS**

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6. A small stone of mass  $0.5\text{ kg}$  is thrown vertically upwards from a point  $A$  with an initial speed of  $25\text{ ms}^{-1}$ . The stone first comes to instantaneous rest at the point  $B$  which is  $20\text{ m}$  vertically above the point  $A$ . As the stone moves it is subject to air resistance. The stone is modelled as a particle.

- (a) Find the energy lost due to air resistance by the stone, as it moves from  $A$  to  $B$  (3)

The air resistance is modelled as a constant force of magnitude  $R$  newtons.

- (b) Find the value of  $R$ . (2)

- (c) State how the model for air resistance could be refined to make it more realistic. (1)

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7. [In this question use  $g = 10 \text{ m s}^{-2}$ ]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of  $4 \text{ m s}^{-1}$ . The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle  $\alpha$ , where

$\sin \alpha = \frac{1}{15}$ . Because of the hill, the jogger reduces her speed to  $3 \text{ m s}^{-1}$  and maintains this

constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at  $3 \text{ m s}^{-1}$ .

(5)



**DO NOT WRITE IN THIS AREA**

[illegible]

8. A particle  $P$  of mass  $3m$  is moving in a straight line on a smooth horizontal table. A particle  $Q$  of mass  $m$  is moving in the opposite direction to  $P$  along the same straight line. The particles collide directly. Immediately before the collision the speed of  $P$  is  $u$  and the speed of  $Q$  is  $2u$ . The velocities of  $P$  and  $Q$  immediately after the collision, measured in the direction of motion of  $P$  before the collision, are  $v$  and  $w$  respectively. The coefficient of restitution between  $P$  and  $Q$  is  $e$ .

(a) Find an expression for  $v$  in terms of  $u$  and  $e$ .

(6)

Given that the direction of motion of  $P$  is changed by the collision,

(b) find the range of possible values of  $e$ .

(2)

(c) Show that  $w = \frac{u}{4}(1 + 9e)$ .

(2)

Following the collision with  $P$ , the particle  $Q$  then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of  $Q$ . The coefficient of restitution between  $Q$  and the wall is  $f$ .

Given that  $e = \frac{5}{9}$ , and that  $P$  and  $Q$  collide again in the subsequent motion,

(d) find the range of possible values of  $f$ .

(6)

**DO NOT WRITE IN THIS AREA**

**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 8 continued**

(Total for Question 8 is 16 marks)

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**



## Paper 2 Option E

### Further Statistics 1 Mark Scheme (Section A)

| Question                                                                                                                      | Scheme                                                                                                                                                                                                                                                                                                                                       | Marks                | AOs      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|----------|----------|----------|--|--------|---------|----------|--------|------|----------|------|----------|--------|----------|--------|----------|----|-----|
| <b>1(a)</b>                                                                                                                   | H <sub>0</sub> : There is no association between language and gender                                                                                                                                                                                                                                                                         | B1                   | 1.2      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    | $\frac{54 \times 85}{150} = 30.6 \quad *$                                                                                                                                                                                                                                                                                                    | B1*cs0               | 1.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    | <table><tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr><tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43...</td><td>23.4</td><td>15.16...</td></tr><tr><td>Female</td><td>34.56...</td><td>[30.6]</td><td>19.83...</td></tr></table> | Expected frequencies |          | Language |          |  | French | Spanish | Mandarin | Gender | Male | 26.43... | 23.4 | 15.16... | Female | 34.56... | [30.6] | 19.83... | M1 | 2.1 |
|                                                                                                                               | Expected frequencies                                                                                                                                                                                                                                                                                                                         |                      |          | Language |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              |                      | French   | Spanish  | Mandarin |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | Gender                                                                                                                                                                                                                                                                                                                                       | Male                 | 26.43... | 23.4     | 15.16... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | Female               | 34.56... | [30.6]   | 19.83... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| $\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \dots + \frac{(15-19.83)^2}{19.83}$                           | M1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Awrt <u><b>3.6/3.7</b></u>                                                                                                    | A1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | <b>(3)</b>                                                                                                                                                                                                                                                                                                                                   |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    | Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$                                                                                                                                                                                                                                                                 | M1                   | 3.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | As $\sum \frac{(O-E)^2}{E} < 9.210$ , the null hypothesis is not rejected                                                                                                                                                                                                                                                                    | A1                   | 2.2b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(2)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    | Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$                                                                                                                                                                                                                                                                   | B1                   | 2.4      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(8 marks)</b>                                                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Notes:                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(a)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1:</b> For correct hypothesis in context                                                                                  |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1*:</b> For a correct calculation leading to the given answer and no errors seen                                          |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For applying $\sum \frac{(O-E)^2}{E}$                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> awrt 3.6 or 3.7                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For using degrees of freedom to set up a $\chi^2$ model critical value                                             |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> For correct comparison and conclusion                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1ft:</b> For correct conclusion with supporting reason                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Scheme                                                                                                                                                                                                                                                                                                                                                | Marks      | AOs          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|
| <b>2(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $-4 = 2 - 5E(X)$                                                                                                                                                                                                                                                                                                                                      | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $E(X) = 1.2$                                                                                                                                                                                                                                                                                                                                          |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$                                                                                                                                                                                                                                                                               | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $a + 2b + 2c = 1.2$ <span style="border: 1px solid black; padding: 0 5px;">1</span>                                                                                                                                                                                                                                                                   |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$<br>i.e. $P(X \leq 1) = 0.45$                                                                                                                                                                                                                                                                   | M1         | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $2a + c = 0.45$ <span style="border: 1px solid black; padding: 0 5px;">2</span>                                                                                                                                                                                                                                                                       |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $2a + b + 2c = 1$ <span style="border: 1px solid black; padding: 0 5px;">3</span>                                                                                                                                                                                                                                                                     | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u> | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | e.g. <span style="border: 1px solid black; padding: 0 5px;">3</span> - <span style="border: 1px solid black; padding: 0 5px;">2</span> $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into <span style="border: 1px solid black; padding: 0 5px;">1</span> $\Rightarrow a = 0.1$ etc                                                                      |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $a = 0.1 \quad b = 0.3 \quad c = 0.25$                                                                                                                                                                                                                                                                                                                | A1<br>A1   | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                       | <b>(7)</b> |              |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59                                                                                                                                                                                                                                                                                                            | M1         | 1.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$                                                                                                                                                                                                                                                                            | A1         | 1.2          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$                                                                                                                                                                                                                                                                                             | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $P(X < \frac{1}{3}) = a + c = 0.35$                                                                                                                                                                                                                                                                                                                   | A1ft       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| <b>(a)</b><br><b>M1:</b> For using given information to find an expression for $E(X)$ i.e. use of $E(Y) = 2 - 5E(X)$<br><b>M1:</b> For use of $\sum xP(X = x) = '1.2'$<br><b>M1:</b> For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$<br><b>M1:</b> For use of $\sum P(X = x) = 1$<br><b>M1:</b> For solving their 3 linear equations (matrix or elimination)<br><b>A1:</b> For any 2 of $a$ , $b$ or $c$ correct<br><b>A1:</b> For all 3 correct values |                                                                                                                                                                                                                                                                                                                                                       |            |              |



|                                        |                                                                                                              |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------|
| Question 2 notes continued:            |                                                                                                              |
| <b>Another method for part (a) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For using given information to find the probability distribution for $Y$ leading to an expression for $E(Y)$ |
| <b>M1:</b>                             | For use of $\sum yP(Y = y) = -4$                                                                             |
| <b>M1:</b>                             | For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$    |
| <b>M1:</b>                             | For use of $\sum P(Y = y) = 1$                                                                               |
| <b>M1:</b>                             | For solving their 3 linear equations (matrix or elimination)                                                 |
| <b>A1:</b>                             | For any 2 of $a$ , $b$ or $c$ correct                                                                        |
| <b>A1:</b>                             | For all 3 correct values                                                                                     |
| <b>(b)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer)                          |
| <b>A1:</b>                             | For use of $\text{Var}(aX) = a^2 \text{Var}(X)$ to reach 2.36 or exact equivalent                            |
| <b>(c)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For rearranging to the form $P(X < k)$                                                                       |
| <b>A1ft:</b>                           | 0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                            |
| <b>Another method for part (c) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For comparing distribution of $X$ with distribution of $Y$ to identify $X = -1$ and $X = 0$                  |
| <b>A1ft:</b>                           | '0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                           |

| Question                                                                                                                                                                                             | Scheme                                                                                                         | Marks | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|-------|------|
| <b>3(a)</b>                                                                                                                                                                                          | $X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$                                                            |       |      |
|                                                                                                                                                                                                      | P(each hire 2 in 1 hour)<br>$= P(X=2) \times P(Y=2) = 0.25104... \times 0.21685...$                            | M1    | 3.3  |
|                                                                                                                                                                                                      | $= 0.05444... \quad \text{awrt } \underline{0.0544}$                                                           | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(b)</b>                                                                                                                                                                                           | $W = X + Y \rightarrow W \sim \text{Po}(3.8)$                                                                  | M1    | 3.4  |
|                                                                                                                                                                                                      | $P(W = 3) = 0.20458... \quad \text{awrt } \underline{0.205}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(c)</b>                                                                                                                                                                                           | $T \sim \text{Po}((2.6+1.2) \times 2)$                                                                         | M1    | 3.3  |
|                                                                                                                                                                                                      | $P(T < 9) = 0.64819... \quad \text{awrt } \underline{0.648}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(d)</b>                                                                                                                                                                                           | <b>(i)</b> Mean = $np = \underline{2.4}$                                                                       | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Variance = $np(1-p) = 2.3904 \quad \text{awrt } \underline{2.39}$                                  | B1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(e)</b>                                                                                                                                                                                           | <b>(i)</b> $[ D \sim \text{Po}(2.4) \quad P(D \leq 4) ]$<br>$= 0.9041... \quad \text{awrt } \underline{0.904}$ | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Since $n$ is large and $p$ is small/mean is approximately equal to variance                        | B1    | 2.4  |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(10 marks)</b>                                                                                                                                                                                    |                                                                                                                |       |      |
| Notes:                                                                                                                                                                                               |                                                                                                                |       |      |
| <b>(a)</b><br><b>M1:</b> For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.0544</b> |                                                                                                                |       |      |
| <b>(b)</b><br><b>M1:</b> For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.205</b>                                                 |                                                                                                                |       |      |
| <b>(c)</b><br><b>M1:</b> For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.648</b>                                   |                                                                                                                |       |      |
| <b>(d)(i)</b><br><b>B1:</b> For <b>2.4</b>                                                                                                                                                           |                                                                                                                |       |      |
| <b>(d)(ii)</b><br><b>B1:</b> For awrt <b>2.39</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(i)</b><br><b>B1:</b> For awrt <b>0.904</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(ii)</b><br><b>B1:</b> For a correct explanation to support use of Poisson approximation in this case                                                                                          |                                                                                                                |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                     | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | <b>(i)</b> $P(X = 1) = 0.34523\dots$ awrt <b><u>0.345</u></b>                                                                              | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <b>(ii)</b> $P(X \leq 4) = 0.98575\dots$ awrt <b><u>0.986</u></b>                                                                          | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$                        | B1*cs0     | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $r = \mathbf{13.81}$ $s = \mathbf{0.57}$                                                                                                   | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $H_0$ : The Poisson distribution is a suitable model<br>$H_1$ : The Poisson distribution is not a suitable model                           | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [Cells are combined when expected frequencies < 5]<br>So combine the last 3 cells                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$       | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | awrt <b><u>1.1</u></b>                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Degrees of freedom = $4 - 1 - 1 = 2$                                                                                                       | B1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | (Do not reject $H_0$ since $1.10 < \chi^2_{2,(0.05)} = 5.991$ ). The number of mortgages approved each week follows a Poisson distribution | A1         | 3.5a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(6)</b> |      |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                            |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(i)</b><br><b>B1:</b> awrt 0.345                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(ii)</b><br><b>B1:</b> awrt 0.986                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                            |            |      |
| <b>(b)</b><br><b>B1*:</b> For a fully correct calculation leading to given answer with no errors seen                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                            |            |      |
| <b>(c)</b><br><b>M1:</b> For attempt at $r$ or $s$ (may be implied by correct answers)<br><b>A1ft:</b> For both values correct (follow through their answers to part (a))                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                            |            |      |
| <b>(d)</b><br><b>B1:</b> For both hypotheses correct (lambda should not be defined so correct use of the model)<br><b>M1:</b> For understanding the need to combine cells before calculating the test statistic (may be implied)<br><br><b>M1:</b> For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$<br><b>A1:</b> awrt 1.1<br><b>B1:</b> For realising that there are 2 degrees of freedom leading to a critical value of $\chi^2_{2,(0.05)} = 5.991$<br><b>A1:</b> Concluding that a Poisson model is suitable for the number of mortgages approved each week |                                                                                                                                            |            |      |

# Further Mechanics 1 Mark Scheme (Section **B**)

| Question                                                                                                                                                                                                                                                                                                                                                                                                  | Scheme                                                                                                    | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|------------|------|
| <b>5(a)</b>                                                                                                                                                                                                                                                                                                                                                                                               | Using the model and $v^2 = u^2 + 2as$ to find $v$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                           | $v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{4.8g}$                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                           | Using the model and $v^2 = u^2 + 2as$ to find $u$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                           | $0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{1.2g}$                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                           | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                           | $e = \sqrt{1.2g} / \sqrt{4.8g} = 0.5$ *                                                                   | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                           | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                | Using the model and $e = \text{sep. speed} / \text{app. speed}$ ,<br>$v = 0.5\sqrt{1.2g}$                 | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                           | Using the model and $v^2 = u^2 + 2as$                                                                     | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                           | $0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15 \text{ (m)}$                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                           | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                | Ball continues to bounce with the height of each bounce being a quarter of the previous one               | B1         | 2.2b |
|                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                           | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                           |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                           |            |      |
| <p><b>(a)</b><br/> <b>M1:</b> For a complete method to find <math>v</math><br/> <b>A1:</b> For a correct value (may be numerical)<br/> <b>M1:</b> For a complete method to find <math>u</math><br/> <b>A1:</b> For a correct value (may be numerical)<br/> <b>M1:</b> For finding both <math>v</math> and <math>u</math> and use of Newton's Law of Restitution<br/> <b>A1*:</b> For the given answer</p> |                                                                                                           |            |      |
| <p><b>(b)</b><br/> <b>M1:</b> For use of Newton's Law of Restitution to find rebound speed<br/> <b>M1:</b> For a complete method to find <math>h</math><br/> <b>A1:</b> For 0.15 (m) oe</p>                                                                                                                                                                                                               |                                                                                                           |            |      |
| <p><b>(c)</b><br/> <b>B1:</b> For a clear description including reference to a quarter</p>                                                                                                                                                                                                                                                                                                                |                                                                                                           |            |      |

| Question                                                                                                                                           | Scheme                                                   | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|------------|------|
| <b>6(a)</b>                                                                                                                                        | Energy Loss = KE Loss – PE Gain                          | M1         | 3.3  |
|                                                                                                                                                    | $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 g \times 20$ | A1         | 1.1b |
|                                                                                                                                                    | $= 58.25 = 58 \text{ (J ) or } 58.3 \text{ (J)}$         | A1         | 1.1b |
|                                                                                                                                                    |                                                          | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                         | Using work-energy principle, $20 R = 58.25$              | M1         | 3.3  |
|                                                                                                                                                    | $R = 2.9125 = 2.9 \text{ or } 2.91$                      | A1ft       | 1.1b |
|                                                                                                                                                    |                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                         | Make resistance variable (dependent on speed)            | B1         | 3.5c |
|                                                                                                                                                    |                                                          | <b>(1)</b> |      |
| <b>(6 marks)</b>                                                                                                                                   |                                                          |            |      |
| Notes:                                                                                                                                             |                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For a difference in KE and PE<br><b>A1:</b> For a correct expression<br><b>A1:</b> For either 58 (2sf) or 58.3(3sf)       |                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of work-energy principle<br><b>A1ft:</b> For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a) |                                                          |            |      |
| <b>(c)</b><br><b>B1:</b> For variable resistance oe                                                                                                |                                                          |            |      |

| Question                                                                                                                                                                                                                                                            | Scheme                                          | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|-------|------|
| <b>7(a)</b>                                                                                                                                                                                                                                                         | Force = Resistance (since no acceleration) = 30 | B1    | 3.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 30 $\times$ 4    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 120 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (3)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                          | Resolving parallel to the slope                 | M1    | 3.1b |
|                                                                                                                                                                                                                                                                     | $F - 60g\sin\alpha - 30 = 0$                    | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | $F = 70$                                        | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 70 $\times$ 3    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 210 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (5)   |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                    |                                                 |       |      |
| Notes:                                                                                                                                                                                                                                                              |                                                 |       |      |
| <b>(a)</b><br><b>B1:</b> For force = 30 seen<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 120 (W), follow through on their '30'                                                                                                                            |                                                 |       |      |
| <b>(b)</b><br><b>M1:</b> For resolving parallel to the slope with correct no. of terms and 60g resolved<br><b>A1:</b> For a correct equation<br><b>A1:</b> For $F = 70$<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 210 (W), follow through on their '70' |                                                 |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                  | Scheme                                                                                                                   | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>8(a)</b>                                                                                                                                                                                                                                                                                                                                                                               | Use of conservation of momentum                                                                                          | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                           | $3mu - 2mu = 3mv + mw$                                                                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Use of NLR                                                                                                               | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                           | $3ue = -v + w$                                                                                                           | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $v = \frac{u}{4}(1 - 3e)$                                                                                                | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                | $\frac{u}{4}(1 - 3e) < 0$                                                                                                | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $\frac{1}{3} < e \leq 1$                                                                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                | Solving for $w$                                                                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                           | $w = \frac{u}{4}(1 + 9e)^*$                                                                                              | A1 *       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                | Substitute $e = \frac{5}{9}$                                                                                             | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $v = -\frac{u}{6}, w = \frac{3u}{2}$                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Use NLR for impact with wall, $x = fw$                                                                                   | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Further collision if $x > -v$                                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                           | $f\frac{3u}{2} > \frac{u}{6}$                                                                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $1 \geq f > \frac{1}{9}$                                                                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(6)</b> |      |
| <b>(16 marks)</b>                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                          |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For use of CLM, with correct no. of terms, condone sign errors<br><b>A1:</b> For a correct equation<br><b>M1:</b> For use of Newton's Law of Restitution, with $e$ on the correct side<br><b>A1:</b> For a correct equation<br><b>M1:</b> For setting up <i>two</i> equations and solving their equations for $v$<br><b>A1:</b> For a correct expression for $v$ |                                                                                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of an appropriate inequality<br><b>A1:</b> For a complete range of values of $e$                                                                                                                                                                                                                                                                         |                                                                                                                          |            |      |
| <b>(c)</b><br><b>M1:</b> For solving their equations for $w$<br><b>A1:</b> For the given answer                                                                                                                                                                                                                                                                                           |                                                                                                                          |            |      |

Question 8 notes continued:

(d)

**M1:** For substituting  $e = \frac{5}{9}$  into their  $v$  and  $w$

**A1:** For correct expressions for  $v$  and  $w$

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**M1:** For use of appropriate inequality

**A1:** For a correct inequality

**A1:** For a correct range



Write your name here

Surname

Other names

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Centre Number

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2F: Further Statistics 1 and Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2F**

**You must have:**

Decision Mathematics 1 question insert for Section B  
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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|        |        | Language |         |          |
|--------|--------|----------|---------|----------|
|        |        | French   | Spanish | Mandarin |
| Gender | Male   | 23       | 22      | 20       |
|        | Female | 38       | 32      | 15       |

(a) State the null hypothesis for this test. (1)

- (b) Show that the expected frequency for females choosing Spanish is 30.6

- (c) Calculate the test statistic for this test, stating the expected frequencies you have used. (3)

- (d) State whether or not the null hypothesis is rejected. Justify your answer. (2)

- (e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance. (1)

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**Question 1 continued**

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2. The discrete random variable  $X$  has probability distribution given by

|            |     |     |     |     |     |
|------------|-----|-----|-----|-----|-----|
| $x$        | -1  | 0   | 1   | 2   | 3   |
| $P(X = x)$ | $c$ | $a$ | $a$ | $b$ | $c$ |

The random variable  $Y = 2 - 5X$

Given that  $E(Y) = -4$  and  $P(Y \geq -3) = 0.45$

- (a) find the probability distribution of  $X$ .

(7)

Given also that  $E(Y^2) = 75$

- (b) find the exact value of  $\text{Var}(X)$

(2)

- (c) Find  $P(Y > X)$

(2)

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**Question 2 continued**

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**Question 2 continued****(Total for Question 2 is 11 marks)**

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

- (a) In a 1 hour period, find the probability that each company hires exactly 2 cars. (2)

- (b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3 (2)

- (c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9 (2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

- (d) (i) find the mean of the number of cars with a defect,  
(ii) find the variance of the number of cars with a defect. (2)

- (e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.  
(ii) Give a reason to support the use of a Poisson approximation. (2)

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**Question 3 continued**

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4. The discrete random variable  $X$  follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i)  $P(X = 1)$

(ii)  $P(X \leq 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

|                                     |    |    |   |   |   |   |   |
|-------------------------------------|----|----|---|---|---|---|---|
| <b>Number of mortgages approved</b> | 0  | 1  | 2 | 3 | 4 | 5 | 6 |
| <b>Frequency</b>                    | 10 | 16 | 7 | 4 | 2 | 0 | 1 |

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

|                                     |      |     |      |      |      |           |
|-------------------------------------|------|-----|------|------|------|-----------|
| <b>Number of mortgages approved</b> | 0    | 1   | 2    | 3    | 4    | 5 or more |
| <b>Expected frequency</b>           | 9.86 | $r$ | 9.67 | 4.51 | 1.58 | $s$       |

(c) Find the value of  $r$  and the value of  $s$  giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

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**Question 4 continued**

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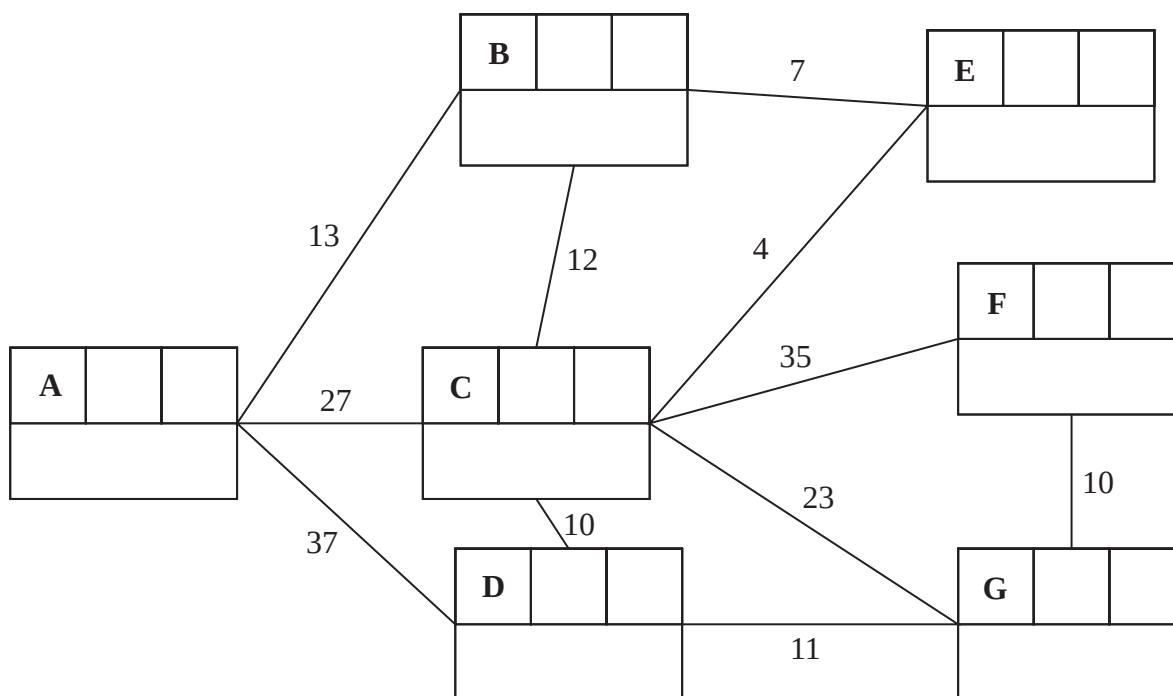


**Question 4 continued****(Total for Question 4 is 11 marks)****TOTAL FOR SECTION A IS 40 MARKS**

## SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics 1 question insert.

5.



**Key:**

| Vertex         | Order of labelling | Final value |
|----------------|--------------------|-------------|
| Working values |                    |             |

Shortest path: \_\_\_\_\_

Length of shortest path: \_\_\_\_\_

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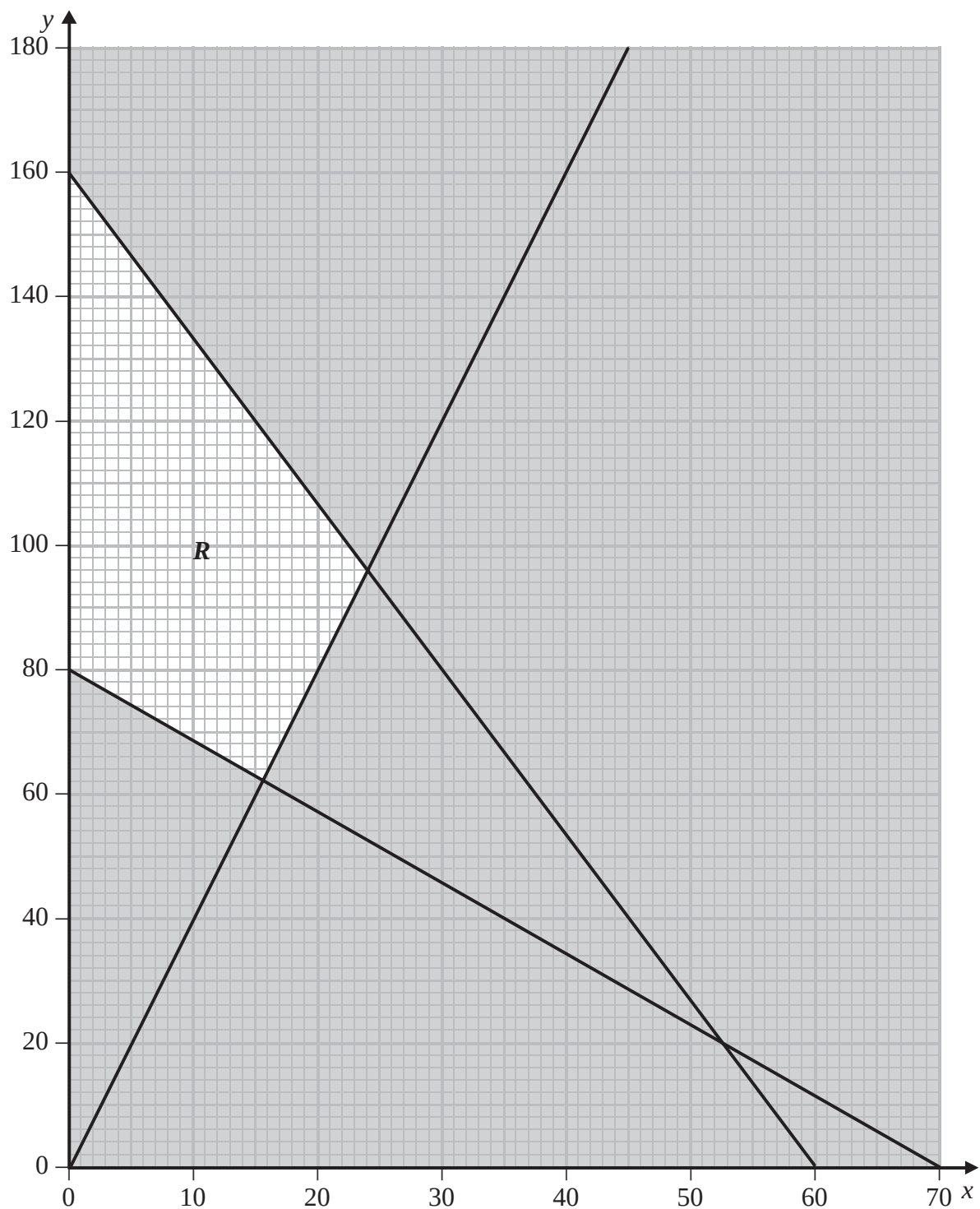
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6.



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7. (a) and (b)

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(Total for Question 7 is 7 marks)

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(Total for Question 9 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**



**Pearson Edexcel Level 3 GCE**

# **Further Mathematics**

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2F: Section B Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

Paper Reference

**8FM0/2F**

**Decision Mathematics 1 question insert for Section B**

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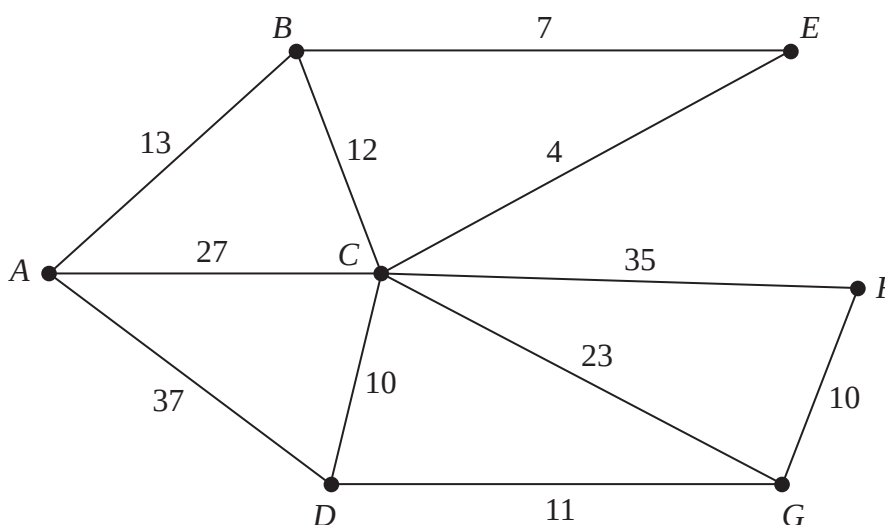


  
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## SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.



**Figure 1**

*[The total weight of the network is 189]*

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

- (a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length. (5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

- (b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear. (4)

- (c) State the minimum length of Gabriel's route. (1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

- (d) Calculate the length of BG. You must show your working. (2)

**(Total for Question 5 is 12 marks)**

6.

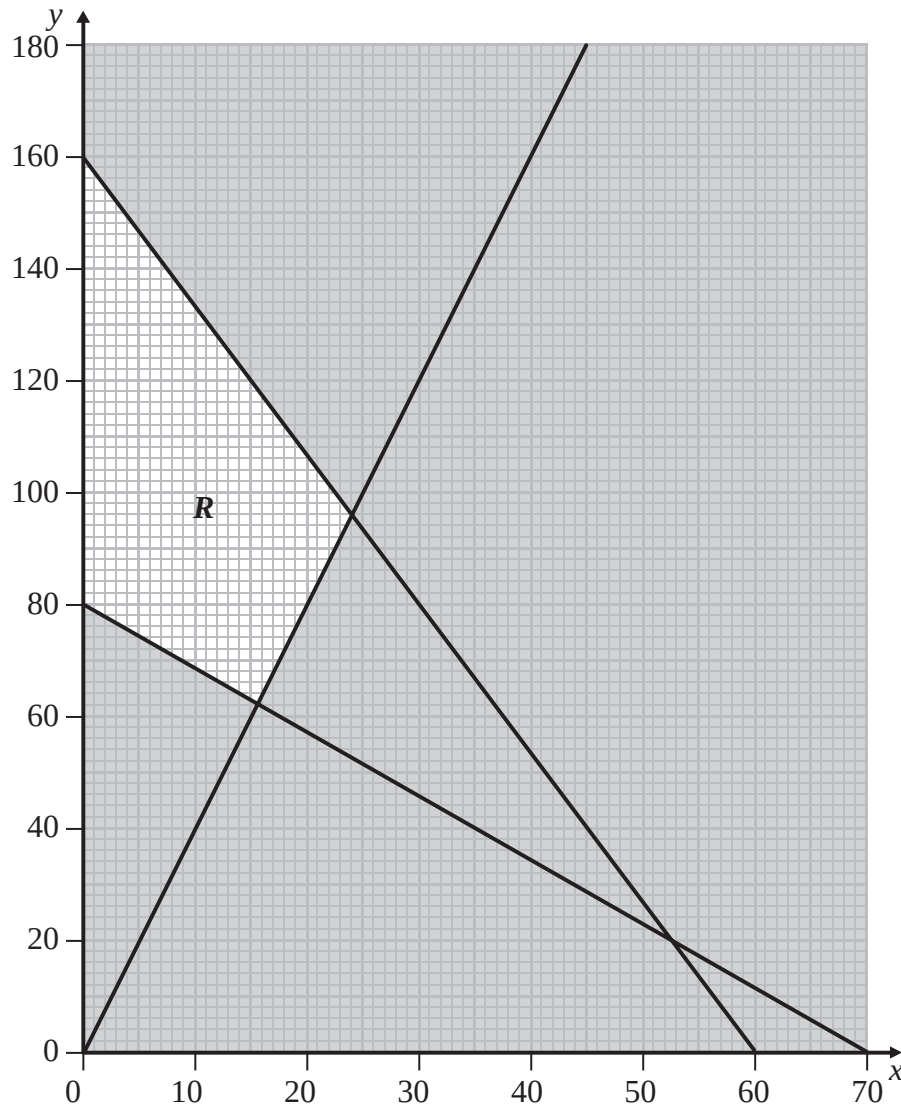


Figure 2

A teacher buys pens and pencils. The number of pens,  $x$ , and the number of pencils,  $y$ , that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

| Activity | Time taken (days) | Immediately preceding activities |
|----------|-------------------|----------------------------------|
| A        | 5                 | -                                |
| B        | 7                 | -                                |
| C        | 3                 | -                                |
| D        | 4                 | A, B                             |
| E        | 4                 | D                                |
| F        | 2                 | B                                |
| G        | 4                 | B                                |
| H        | 5                 | C, G                             |
| I        | 10                | C, G                             |

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only. (3)
- (b) (i) Show that the project can be completed in 21 days, showing your working. (4)
- (ii) Identify the critical activities.

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5 (1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are  $2^{2x} - 1$ ,  $2^x$ ,  $x + 1$ ,  $2^{x+1} - 3$  and  $11 - x$ .

- (b) (i) Calculate  $x$ .  
(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer. (6)

- (c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let  $x$  and  $y$  represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**

## Paper 2 Option F

### Further Statistics 1 Mark Scheme (Section A)

| Question                                                                                                                      | Scheme                                                                                                                                                                                                                                                                                                                                       | Marks                | AOs      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|----------|----------|----------|--|--------|---------|----------|--------|------|----------|------|----------|--------|----------|--------|----------|----|-----|
| <b>1(a)</b>                                                                                                                   | H <sub>0</sub> : There is no association between language and gender                                                                                                                                                                                                                                                                         | B1                   | 1.2      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    | $\frac{54 \times 85}{150} = 30.6 \quad *$                                                                                                                                                                                                                                                                                                    | B1*cs0               | 1.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    | <table><tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr><tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43...</td><td>23.4</td><td>15.16...</td></tr><tr><td>Female</td><td>34.56...</td><td>[30.6]</td><td>19.83...</td></tr></table> | Expected frequencies |          | Language |          |  | French | Spanish | Mandarin | Gender | Male | 26.43... | 23.4 | 15.16... | Female | 34.56... | [30.6] | 19.83... | M1 | 2.1 |
|                                                                                                                               | Expected frequencies                                                                                                                                                                                                                                                                                                                         |                      |          | Language |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              |                      | French   | Spanish  | Mandarin |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | Gender                                                                                                                                                                                                                                                                                                                                       | Male                 | 26.43... | 23.4     | 15.16... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | Female               | 34.56... | [30.6]   | 19.83... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(23 - 26.43)^2}{26.43} + \dots + \frac{(15 - 19.83)^2}{19.83}$                     | M1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Awrt <u><b>3.6/3.7</b></u>                                                                                                    | A1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | <b>(3)</b>                                                                                                                                                                                                                                                                                                                                   |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    | Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$                                                                                                                                                                                                                                                                 | M1                   | 3.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | As $\sum \frac{(O - E)^2}{E} < 9.210$ , the null hypothesis is not rejected                                                                                                                                                                                                                                                                  | A1                   | 2.2b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(2)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    | Still not rejected since $\sum \frac{(O - E)^2}{E} < \chi^2_{2,0.1} = 4.605$                                                                                                                                                                                                                                                                 | B1                   | 2.4      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(8 marks)</b>                                                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Notes:                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(a)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1:</b> For correct hypothesis in context                                                                                  |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1*:</b> For a correct calculation leading to the given answer and no errors seen                                          |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For applying $\sum \frac{(O - E)^2}{E}$                                                                            |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> awrt 3.6 or 3.7                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For using degrees of freedom to set up a $\chi^2$ model critical value                                             |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> For correct comparison and conclusion                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1ft:</b> For correct conclusion with supporting reason                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                                                                                                                                                                                                                | Marks      | AOs          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|
| <b>2(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $-4 = 2 - 5E(X)$                                                                                                                                                                                                                                                                                                                                      | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $E(X) = 1.2$                                                                                                                                                                                                                                                                                                                                          |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$                                                                                                                                                                                                                                                                               | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a + 2b + 2c = 1.2$ <span style="border: 1px solid black; padding: 0 5px;">1</span>                                                                                                                                                                                                                                                                   |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$<br>i.e. $P(X \leq 1) = 0.45$                                                                                                                                                                                                                                                                   | M1         | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + c = 0.45$ <span style="border: 1px solid black; padding: 0 5px;">2</span>                                                                                                                                                                                                                                                                       |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + b + 2c = 1$ <span style="border: 1px solid black; padding: 0 5px;">3</span>                                                                                                                                                                                                                                                                     | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u> | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | e.g. <span style="border: 1px solid black; padding: 0 5px;">3</span> - <span style="border: 1px solid black; padding: 0 5px;">2</span> $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into <span style="border: 1px solid black; padding: 0 5px;">1</span> $\Rightarrow a = 0.1$ etc                                                                      |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a = 0.1 \quad b = 0.3 \quad c = 0.25$                                                                                                                                                                                                                                                                                                                | A1<br>A1   | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(7)</b> |              |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59                                                                                                                                                                                                                                                                                                            | M1         | 1.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$                                                                                                                                                                                                                                                                            | A1         | 1.2          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$                                                                                                                                                                                                                                                                                             | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(X < \frac{1}{3}) = a + c = 0.35$                                                                                                                                                                                                                                                                                                                   | A1ft       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                       | <b>(2)</b> |              |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                       |            |              |
| <b>(a)</b><br><b>M1:</b> For using given information to find an expression for $E(X)$ i.e. use of $E(Y) = 2 - 5E(X)$<br><b>M1:</b> For use of $\sum xP(X = x) = '1.2'$<br><b>M1:</b> For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$<br><b>M1:</b> For use of $\sum P(X = x) = 1$<br><b>M1:</b> For solving their 3 linear equations (matrix or elimination)<br><b>A1:</b> For any 2 of $a, b$ or $c$ correct<br><b>A1:</b> For all 3 correct values |                                                                                                                                                                                                                                                                                                                                                       |            |              |



|                                        |                                                                                                              |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------|
| Question 2 notes continued:            |                                                                                                              |
| <b>Another method for part (a) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For using given information to find the probability distribution for $Y$ leading to an expression for $E(Y)$ |
| <b>M1:</b>                             | For use of $\sum yP(Y = y) = -4$                                                                             |
| <b>M1:</b>                             | For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$    |
| <b>M1:</b>                             | For use of $\sum P(Y = y) = 1$                                                                               |
| <b>M1:</b>                             | For solving their 3 linear equations (matrix or elimination)                                                 |
| <b>A1:</b>                             | For any 2 of $a$ , $b$ or $c$ correct                                                                        |
| <b>A1:</b>                             | For all 3 correct values                                                                                     |
| <b>(b)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer)                          |
| <b>A1:</b>                             | For use of $\text{Var}(aX) = a^2 \text{Var}(X)$ to reach 2.36 or exact equivalent                            |
| <b>(c)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For rearranging to the form $P(X < k)$                                                                       |
| <b>A1ft:</b>                           | 0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                            |
| <b>Another method for part (c) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For comparing distribution of $X$ with distribution of $Y$ to identify $X = -1$ and $X = 0$                  |
| <b>A1ft:</b>                           | '0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                           |

| Question                                                                                                                                                                                                    | Scheme                                                                                           | Marks | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|-------|------|
| <b>3(a)</b>                                                                                                                                                                                                 | $X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$                                              |       |      |
|                                                                                                                                                                                                             | P(each hire 2 in 1 hour)<br>= $P(X=2) \times P(Y=2) = 0.25104... \times 0.21685...$              | M1    | 3.3  |
|                                                                                                                                                                                                             | = 0.05444... awrt <b><u>0.0544</u></b>                                                           | A1    | 1.1b |
|                                                                                                                                                                                                             |                                                                                                  | (2)   |      |
| <b>(b)</b>                                                                                                                                                                                                  | $W = X + Y \rightarrow W \sim \text{Po}(3.8)$                                                    | M1    | 3.4  |
|                                                                                                                                                                                                             | $P(W = 3) = 0.20458....$ awrt <b><u>0.205</u></b>                                                | A1    | 1.1b |
|                                                                                                                                                                                                             |                                                                                                  | (2)   |      |
| <b>(c)</b>                                                                                                                                                                                                  | $T \sim \text{Po}((2.6+1.2) \times 2)$                                                           | M1    | 3.3  |
|                                                                                                                                                                                                             | $P(T < 9) = 0.64819...$ awrt <b><u>0.648</u></b>                                                 | A1    | 1.1b |
|                                                                                                                                                                                                             |                                                                                                  | (2)   |      |
| <b>(d)</b>                                                                                                                                                                                                  | <b>(i)</b> Mean = $np = \underline{2.4}$                                                         | B1    | 1.1b |
|                                                                                                                                                                                                             | <b>(ii)</b> Variance = $np(1 - p) = 2.3904$ awrt <b><u>2.39</u></b>                              | B1    | 1.1b |
|                                                                                                                                                                                                             |                                                                                                  | (2)   |      |
| <b>(e)</b>                                                                                                                                                                                                  | <b>(i)</b> [ $D \sim \text{Po}(2.4) \quad P(D \leq 4)$ ]<br>= 0.9041... awrt <b><u>0.904</u></b> | B1    | 1.1b |
|                                                                                                                                                                                                             | <b>(ii)</b> Since $n$ is large and $p$ is small/mean is approximately equal to variance          | B1    | 2.4  |
|                                                                                                                                                                                                             |                                                                                                  | (2)   |      |
| <b>(10 marks)</b>                                                                                                                                                                                           |                                                                                                  |       |      |
| Notes:                                                                                                                                                                                                      |                                                                                                  |       |      |
| <b>(a)</b><br><b>M1:</b> For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)<br><b>A1:</b> awrt <b><u>0.0544</u></b> |                                                                                                  |       |      |
| <b>(b)</b><br><b>M1:</b> For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)<br><b>A1:</b> awrt <b><u>0.205</u></b>                                                 |                                                                                                  |       |      |
| <b>(c)</b><br><b>M1:</b> For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)<br><b>A1:</b> awrt <b><u>0.648</u></b>                                   |                                                                                                  |       |      |
| <b>(d)(i)</b><br><b>B1:</b> For 2.4                                                                                                                                                                         |                                                                                                  |       |      |
| <b>(d)(ii)</b><br><b>B1:</b> For awrt 2.39                                                                                                                                                                  |                                                                                                  |       |      |
| <b>(e)(i)</b><br><b>B1:</b> For awrt <b><u>0.904</u></b>                                                                                                                                                    |                                                                                                  |       |      |
| <b>(e)(ii)</b><br><b>B1:</b> For a correct explanation to support use of Poisson approximation in this case                                                                                                 |                                                                                                  |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                     | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | <b>(i)</b> $P(X = 1) = 0.34523\dots$ awrt <b><u>0.345</u></b>                                                                              | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <b>(ii)</b> $P(X \leq 4) = 0.98575\dots$ awrt <b><u>0.986</u></b>                                                                          | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$                        | B1*cs0     | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $r = \mathbf{13.81}$ $s = \mathbf{0.57}$                                                                                                   | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $H_0$ : The Poisson distribution is a suitable model<br>$H_1$ : The Poisson distribution is not a suitable model                           | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [Cells are combined when expected frequencies < 5]<br>So combine the last 3 cells                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$       | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | awrt <b><u>1.1</u></b>                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Degrees of freedom = $4 - 1 - 1 = 2$                                                                                                       | B1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | (Do not reject $H_0$ since $1.10 < \chi^2_{2,(0.05)} = 5.991$ ). The number of mortgages approved each week follows a Poisson distribution | A1         | 3.5a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(6)</b> |      |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                            |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(i)</b><br><b>B1:</b> awrt 0.345                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(ii)</b><br><b>B1:</b> awrt 0.986                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                            |            |      |
| <b>(b)</b><br><b>B1*:</b> For a fully correct calculation leading to given answer with no errors seen                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                            |            |      |
| <b>(c)</b><br><b>M1:</b> For attempt at $r$ or $s$ (may be implied by correct answers)<br><b>A1ft:</b> For both values correct (follow through their answers to part (a))                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                            |            |      |
| <b>(d)</b><br><b>B1:</b> For both hypotheses correct (lambda should not be defined so correct use of the model)<br><b>M1:</b> For understanding the need to combine cells before calculating the test statistic (may be implied)<br><br><b>M1:</b> For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$<br><b>A1:</b> awrt 1.1<br><b>B1:</b> For realising that there are 2 degrees of freedom leading to a critical value of $\chi^2_{2,(0.05)} = 5.991$<br><b>A1:</b> Concluding that a Poisson model is suitable for the number of mortgages approved each week |                                                                                                                                            |            |      |

Decision Mathematics 1 Mark Scheme (Section **B**)

| Question                                                                                                                                                                                                                                                                                                                                                       | Scheme                              | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|------------|------|
| <b>5(a)</b>                                                                                                                                                                                                                                                                                                                                                    |                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Path: ABECDGF                       | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Length: 55 (metres)                 | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(5)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                     | $AB + DG = 13 + 11 = 24 \leftarrow$ | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BEC)D + B(ECD)G = 34 + 32 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BECD)G + B(EC)D = 45 + 21 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Repeat arcs: AB, DG                 | A1ft       | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(4)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                     | Length = $189 + 24 = 213$ (metres)  | B1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(1)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                     | $189 + x + 34 = 213 + 2x$           | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $x = 10$ so BG is 10 m              | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(2)</b> |      |
| <b>(12 marks)</b>                                                                                                                                                                                                                                                                                                                                              |                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                         |                                     |            |      |
| <b>(a)</b><br><b>M1:</b> For a larger number replaced by a smaller one in the working values boxes at C, D, F or G<br><b>A1:</b> For all values correct (and in correct order) at A, B, C and D<br><b>A1:</b> For all values correct (and in correct order) at E, F & G<br><b>A1:</b> For the correct path<br><b>A1ft:</b> For 55 or ft their final value at F |                                     |            |      |
| <b>(b)</b><br><b>M1:</b> For 3 correct pairings of the four odd nodes (A,B, D & G)<br><b>A1:</b> At least two pairings and totals correct<br><b>A2:</b> All three pairings and totals correct<br><b>A3ft:</b> Selecting their shortest pairing, and stating that these arcs should be repeated                                                                 |                                     |            |      |

Question **5** notes continued:

**(c)**

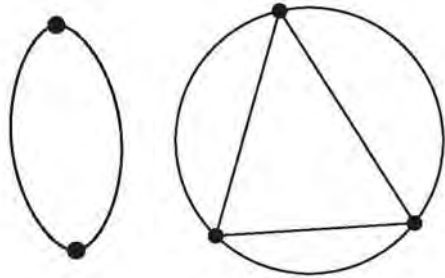
**B1ft:** For 213 or 189 + their shortest repeat

**M1:** For translating the information in the question in to an equation involving  $x$ ,  $2x$  and 34

**A1:** For a correct equation leading to  $BG = 10$  (m)

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                                        | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>6</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Objective line drawn or at least two vertices tested                                                                          | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | For solving $y = 4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$ | M1    | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Finding at least two points with integer co-ordinates from $(15 \pm 1, 63 \pm 2)$                                             | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Testing at least two points with integer co-ordinates                                                                         | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15$ and $y = 63$                                                                                                         | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | So the teacher should buy 15 pens and 63 pencils                                                                              | A1ft  | 3.2a |
| <b>(7 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                               |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                               |       |      |
| <p><b>M1:</b> Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C</p> <p><b>M1:</b> Solving simultaneous equations</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of <math>y \geq 4x</math> and <math>8x + 7y \geq 560</math></p> <p><b>M1:</b> Testing at least two integer solutions in <math>y \geq 4x</math> or <math>8x + 7y \geq 560</math> and C</p> <p><b>A1:</b> cao – deducing from tests which integer solution is both valid and optimal</p> <p><b>A1ft:</b> Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils</p> |                                                                                                                               |       |      |

| Question                                                                                                                                                                                                                                                                                 | Scheme                                                                                                                              | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>7(a)(b)</b>                                                                                                                                                                                                                                                                           | <p>The number(s) at the end of activity E indicate this project can be completed in 21 days</p> <p>Critical activities: B, G, I</p> | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | <b>(3)</b> |      |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1ft       | 2.2a |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | <b>(4)</b> |      |
| <b>(7 marks)</b>                                                                                                                                                                                                                                                                         |                                                                                                                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                   |                                                                                                                                     |            |      |
| <b>M1:</b> At least 5 activities and one dummy, one start                                                                                                                                                                                                                                |                                                                                                                                     |            |      |
| <b>A1:</b> A,B,C,D,F,G and first dummy correct                                                                                                                                                                                                                                           |                                                                                                                                     |            |      |
| <b>A1:</b> E,H,I correct, second dummy correct and one finish                                                                                                                                                                                                                            |                                                                                                                                     |            |      |
| <b>M1:</b> All boxes completed, number generally increasing L to R (condone one “rogue”)                                                                                                                                                                                                 |                                                                                                                                     |            |      |
| <b>A1:</b> All values cao                                                                                                                                                                                                                                                                |                                                                                                                                     |            |      |
| <b>A1:</b> Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes) |                                                                                                                                     |            |      |
| <b>A1:</b> Critical activities correct                                                                                                                                                                                                                                                   |                                                                                                                                     |            |      |

| Question                                                                         | Scheme                                                                                                                        | Marks | AOs  |
|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>8(a)</b>                                                                      | e.g. a graph cannot contain an odd number of odd nodes<br>e.g. number of arcs = $\frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$ | B1    | 2.4  |
|                                                                                  |                                                                                                                               | (1)   |      |
| <b>(b)(i)</b>                                                                    | $(2^{2x}-1)+(2^x)+(x+1)+(2^{x+1}-3)+(11-x)=2(18)$                                                                             | M1    | 1.1b |
|                                                                                  | $2^{2x}+3(2^x)-28=0 \Rightarrow x=...$                                                                                        | M1    | 1.1b |
|                                                                                  | $(2^x+7)(2^x-4)=0 \Rightarrow x=2$                                                                                            | A1    | 1.1b |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(b)(ii)</b>                                                                   | The order of the nodes are 9, 15, 3, 4, 5                                                                                     | M1    | 2.1  |
|                                                                                  | Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes                                | A1    | 2.4  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(c)</b>                                                                       |                                             | M1    | 2.5  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (2)   |      |
| <b>(9 marks)</b>                                                                 |                                                                                                                               |       |      |
| Notes:                                                                           |                                                                                                                               |       |      |
| <b>(a)</b>                                                                       |                                                                                                                               |       |      |
| <b>B1:</b> Explanation referring to need for an even number of odd nodes oe      |                                                                                                                               |       |      |
| <b>(b)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Forming an equation involving the orders of the 5 odd nodes and 2(18) |                                                                                                                               |       |      |
| <b>M1:</b> Simplifies to a quadratic in $2^x$ and attempts to solve              |                                                                                                                               |       |      |
| <b>A1:</b> 2 cao                                                                 |                                                                                                                               |       |      |
| <b>M1:</b> Construct an argument involving the order of the 5 nodes              |                                                                                                                               |       |      |
| <b>A1:</b> Explanation considering the number of odd nodes                       |                                                                                                                               |       |      |
| <b>A1:</b> Deduction that therefore it is neither Eulerian nor semi-Eulerian     |                                                                                                                               |       |      |
| <b>(c)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Interprets mathematical language to construct a disconnected graph    |                                                                                                                               |       |      |
| <b>A1:</b> Deduce a correct graph                                                |                                                                                                                               |       |      |



| Question                                                                                                                                                                                                                                                                                                                         | Scheme                                                                      | Marks    | AOs        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|----------|------------|
| <b>9</b>                                                                                                                                                                                                                                                                                                                         | Minimise ( $C =$ ) $25x + 35y$                                              | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | Subject to:<br>$(500x + 800y \geq 150\,000 \Rightarrow) 5x + 8y \geq 1500$  | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | $\frac{7}{20}(x + y) \leq x \leq \frac{13}{20}(x + y)$                      | M1<br>M1 | 3.3<br>3.3 |
|                                                                                                                                                                                                                                                                                                                                  | Which simplifies to $7y \leq 13x$ <b>and</b> $13y \geq 7x$<br>$x, y \geq 0$ | A1       | 1.1b       |
| <b>(5 marks)</b>                                                                                                                                                                                                                                                                                                                 |                                                                             |          |            |
| Notes:                                                                                                                                                                                                                                                                                                                           |                                                                             |          |            |
| <b>B1:</b> A correct objective function + minimise<br><b>B1:</b> Translate information in to a correct inequality<br><b>M1:</b> For translating the information given into the LHS inequality<br><b>M1:</b> For translating the information given in to the RHS inequality<br><b>A1:</b> Simplifying to the correct inequalities |                                                                             |          |            |



Write your name here

Surname

Other names

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2G: Further Statistics 1 and Further Statistics 2**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2G**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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|        |        | Language |         |          |
|--------|--------|----------|---------|----------|
|        |        | French   | Spanish | Mandarin |
| Gender | Male   | 23       | 22      | 20       |
|        | Female | 38       | 32      | 15       |

(a) State the null hypothesis for this test. (1)

- (b) Show that the expected frequency for females choosing Spanish is 30.6

- (c) Calculate the test statistic for this test, stating the expected frequencies you have used. (3)

- (d) State whether or not the null hypothesis is rejected. Justify your answer. (2)

- (e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance. (1)

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**Question 1 continued**

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2. The discrete random variable  $X$  has probability distribution given by

|            |     |     |     |     |     |
|------------|-----|-----|-----|-----|-----|
| $x$        | -1  | 0   | 1   | 2   | 3   |
| $P(X = x)$ | $c$ | $a$ | $a$ | $b$ | $c$ |

The random variable  $Y = 2 - 5X$

Given that  $E(Y) = -4$  and  $P(Y \geq -3) = 0.45$

- (a) find the probability distribution of  $X$ .

(7)

Given also that  $E(Y^2) = 75$

- (b) find the exact value of  $\text{Var}(X)$

(2)

- (c) Find  $P(Y > X)$

(2)



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**Question 2 continued**

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3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

- (a) In a 1 hour period, find the probability that each company hires exactly 2 cars. (2)

- (b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3 (2)

- (c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9 (2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

- (d) (i) find the mean of the number of cars with a defect,  
(ii) find the variance of the number of cars with a defect. (2)

- (e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.  
(ii) Give a reason to support the use of a Poisson approximation. (2)

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**Question 3 continued**

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4. The discrete random variable  $X$  follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i)  $P(X = 1)$

(ii)  $P(X \leq 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

|                                     |    |    |   |   |   |   |   |
|-------------------------------------|----|----|---|---|---|---|---|
| <b>Number of mortgages approved</b> | 0  | 1  | 2 | 3 | 4 | 5 | 6 |
| <b>Frequency</b>                    | 10 | 16 | 7 | 4 | 2 | 0 | 1 |

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

|                                     |      |     |      |      |      |           |
|-------------------------------------|------|-----|------|------|------|-----------|
| <b>Number of mortgages approved</b> | 0    | 1   | 2    | 3    | 4    | 5 or more |
| <b>Expected frequency</b>           | 9.86 | $r$ | 9.67 | 4.51 | 1.58 | $s$       |

(c) Find the value of  $r$  and the value of  $s$  giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)



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**Question 4 continued**

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**TOTAL FOR SECTION A IS 40 MARKS**

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- 6.** The continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{1}{18}(11 - 2x) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find  $P(X < 3)$

(2)

- (b) State, giving a reason, whether the upper quartile of  $X$  is greater than 3, less than 3 or equal to 3

(1)

Given that  $E(X) = \frac{9}{4}$

- (c) use algebraic integration to find  $\text{Var}(X)$

(3)

The cumulative distribution function of  $X$  is given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}(11x - x^2 + c) & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

- (d) Show that  $c = -10$

(2)

- (e) Find the median of  $X$ , giving your answer to 3 significant figures.

(3)

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**Question 6 continued**

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7. A scientist wants to develop a model to describe the relationship between the average daily temperature,  $x^{\circ}\text{C}$ , and a household's daily energy consumption,  $y\text{kWh}$ , in winter.

A random sample of the average temperature and energy consumption are taken from 10 winter days and are summarised below.

$$\sum x = 12 \quad \sum x^2 = 24.76 \quad \sum y = 251 \quad \sum y^2 = 6341 \quad \sum xy = 284.8$$

$$S_{xx} = 10.36 \quad S_{yy} = 40.9$$

- (a) Find the product moment correlation coefficient between  $y$  and  $x$ . (2)
- (b) Find the equation of the regression line of  $y$  on  $x$  in the form  $y = a + bx$  (3)
- (c) Use your equation to estimate the daily energy consumption when the average daily temperature is  $2^{\circ}\text{C}$  (1)
- (d) Calculate the residual sum of squares (RSS). (2)

The table shows the residual for each value of  $x$ .

|                 |       |       |       |       |      |      |      |      |     |       |
|-----------------|-------|-------|-------|-------|------|------|------|------|-----|-------|
| $x$             | -0.4  | -0.2  | 0.3   | 0.8   | 1.1  | 1.4  | 1.8  | 2.1  | 2.5 | 2.6   |
| <b>Residual</b> | -0.63 | -0.32 | -0.52 | -0.73 | 0.74 | 2.22 | 1.84 | 0.32 | $f$ | -1.88 |

- (e) Find the value of  $f$ . (2)
- (f) By considering the signs of the residuals, explain whether or not the linear regression model is a suitable model for these data. (1)

**DO NOT WRITE IN THIS AREA**

**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**DO NOT WRITE IN THIS AREA**

[illegible]

8. The continuous random variable  $X$  is uniformly distributed over the interval  $[-3, 5]$ .

(a) Sketch the probability density function  $f(x)$  of  $X$ .

(2)

(b) Find the value of  $k$  such that  $P(X < 2[k - X]) = 0.25$

(3)

(c) Use algebraic integration to show that  $E(X^3) = 17$

(3)

**DO NOT WRITE IN THIS AREA**

**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 8 is 8 marks)**

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**



## Paper 2 Option G

### Further Statistics 1 Mark Scheme (Section A)

| Question                                                                                                                      | Scheme                                                                                                                                                                                                                                                                                                                                       | Marks                | AOs      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|-------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|----------|----------|----------|--|--------|---------|----------|--------|------|----------|------|----------|--------|----------|--------|----------|----|-----|
| <b>1(a)</b>                                                                                                                   | $H_0$ : There is no association between language and gender                                                                                                                                                                                                                                                                                  | B1                   | 1.2      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    | $\frac{54 \times 85}{150} = 30.6 \quad *$                                                                                                                                                                                                                                                                                                    | B1*cso               | 1.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    | <table><tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr><tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43...</td><td>23.4</td><td>15.16...</td></tr><tr><td>Female</td><td>34.56...</td><td>[30.6]</td><td>19.83...</td></tr></table> | Expected frequencies |          | Language |          |  | French | Spanish | Mandarin | Gender | Male | 26.43... | 23.4 | 15.16... | Female | 34.56... | [30.6] | 19.83... | M1 | 2.1 |
|                                                                                                                               | Expected frequencies                                                                                                                                                                                                                                                                                                                         |                      |          | Language |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              |                      | French   | Spanish  | Mandarin |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | Gender                                                                                                                                                                                                                                                                                                                                       | Male                 | 26.43... | 23.4     | 15.16... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | Female               | 34.56... | [30.6]   | 19.83... |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| $\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \dots + \frac{(15-19.83)^2}{19.83}$                           | M1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Awrt <u>3.6/3.7</u>                                                                                                           | A1                                                                                                                                                                                                                                                                                                                                           | 1.1b                 |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | <b>(3)</b>                                                                                                                                                                                                                                                                                                                                   |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    | Degrees of freedom $(3-1)(2-1) \rightarrow$ Critical value $\chi^2_{2,0.01} = 9.210$                                                                                                                                                                                                                                                         | M1                   | 3.1b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               | As $\sum \frac{(O-E)^2}{E} < 9.210$ , the null hypothesis is not rejected                                                                                                                                                                                                                                                                    | A1                   | 2.2b     |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(2)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    | Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$                                                                                                                                                                                                                                                                   | B1                   | 2.4      |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
|                                                                                                                               |                                                                                                                                                                                                                                                                                                                                              | <b>(1)</b>           |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(8 marks)</b>                                                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| Notes:                                                                                                                        |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(a)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1:</b> For correct hypothesis in context                                                                                  |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(b)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>B1*:</b> For a correct calculation leading to the given answer and no errors seen                                          |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(c)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For applying $\sum \frac{(O-E)^2}{E}$                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> awrt 3.6 or 3.7                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(d)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>M1:</b> For using degrees of freedom to set up a $\chi^2$ model critical value                                             |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1:</b> For correct comparison and conclusion                                                                              |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>(e)</b>                                                                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |
| <b>A1ft:</b> For correct conclusion with supporting reason                                                                    |                                                                                                                                                                                                                                                                                                                                              |                      |          |          |          |  |        |         |          |        |      |          |      |          |        |          |        |          |    |     |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                                                                                                                                                                                                                         | Marks      | AOs          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------|
| <b>2(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $-4 = 2 - 5E(X)$                                                                                                                                                                                                                                                                                                                               | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $E(X) = 1.2$                                                                                                                                                                                                                                                                                                                                   |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$                                                                                                                                                                                                                                                                        | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a + 2b + 2c = 1.2$ <span style="border: 1px solid black; padding: 0 5px;">1</span>                                                                                                                                                                                                                                                            |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$<br>i.e. $P(X \leq 1) = 0.45$                                                                                                                                                                                                                                                            | M1         | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + c = 0.45$ <span style="border: 1px solid black; padding: 0 5px;">2</span>                                                                                                                                                                                                                                                                |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $2a + b + 2c = 1$ <span style="border: 1px solid black; padding: 0 5px;">3</span>                                                                                                                                                                                                                                                              | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ or | M1         | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | e.g. <span style="border: 1px solid black; padding: 0 5px;">3</span> - <span style="border: 1px solid black; padding: 0 5px;">2</span> $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into <span style="border: 1px solid black; padding: 0 5px;">1</span> $\Rightarrow a = 0.1$ etc                                                               |            |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $a = 0.1 \quad b = 0.3 \quad c = 0.25$                                                                                                                                                                                                                                                                                                         | A1<br>A1   | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                | <b>(7)</b> |              |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\text{Var}(Y) = 75 - (-4)^2$ or 59                                                                                                                                                                                                                                                                                                            | M1         | 1.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$                                                                                                                                                                                                                                                                     | A1         | 1.2          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                | <b>(2)</b> |              |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$                                                                                                                                                                                                                                                                                      | M1         | 3.1a         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $P(X < \frac{1}{3}) = a + c = 0.35$                                                                                                                                                                                                                                                                                                            | A1ft       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                | <b>(2)</b> |              |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                |            |              |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                |            |              |
| <b>(a)</b><br><b>M1:</b> For using given information to find an expression for $E(X)$ i.e. use of $E(Y) = 2 - 5E(X)$<br><b>M1:</b> For use of $\sum xP(X = x) = '1.2'$<br><b>M1:</b> For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$<br><b>M1:</b> For use of $\sum P(X = x) = 1$<br><b>M1:</b> For solving their 3 linear equations (matrix or elimination)<br><b>A1:</b> For any 2 of $a, b$ or $c$ correct<br><b>A1:</b> For all 3 correct values |                                                                                                                                                                                                                                                                                                                                                |            |              |

|                                        |                                                                                                              |
|----------------------------------------|--------------------------------------------------------------------------------------------------------------|
| Question 2 notes continued:            |                                                                                                              |
| <b>Another method for part (a) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For using given information to find the probability distribution for $Y$ leading to an expression for $E(Y)$ |
| <b>M1:</b>                             | For use of $\sum yP(Y = y) = -4$                                                                             |
| <b>M1:</b>                             | For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in $a$ and $c$    |
| <b>M1:</b>                             | For use of $\sum P(Y = y) = 1$                                                                               |
| <b>M1:</b>                             | For solving their 3 linear equations (matrix or elimination)                                                 |
| <b>A1:</b>                             | For any 2 of $a$ , $b$ or $c$ correct                                                                        |
| <b>A1:</b>                             | For all 3 correct values                                                                                     |
| <b>(b)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer)                          |
| <b>A1:</b>                             | For use of $\text{Var}(aX) = a^2 \text{Var}(X)$ to reach 2.36 or exact equivalent                            |
| <b>(c)</b>                             |                                                                                                              |
| <b>M1:</b>                             | For rearranging to the form $P(X < k)$                                                                       |
| <b>A1ft:</b>                           | 0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                            |
| <b>Another method for part (c) is:</b> |                                                                                                              |
| <b>M1:</b>                             | For comparing distribution of $X$ with distribution of $Y$ to identify $X = -1$ and $X = 0$                  |
| <b>A1ft:</b>                           | '0.1' + '025' (provided their $a$ and $c$ and their $a + c$ are all probabilities)                           |

| Question                                                                                                                                                                                             | Scheme                                                                                                         | Marks | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|-------|------|
| <b>3(a)</b>                                                                                                                                                                                          | $X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$                                                            |       |      |
|                                                                                                                                                                                                      | P(each hire 2 in 1 hour)<br>$= P(X=2) \times P(Y=2) = 0.25104... \times 0.21685...$                            | M1    | 3.3  |
|                                                                                                                                                                                                      | $= 0.05444... \quad \text{awrt } \underline{0.0544}$                                                           | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(b)</b>                                                                                                                                                                                           | $W = X + Y \rightarrow W \sim \text{Po}(3.8)$                                                                  | M1    | 3.4  |
|                                                                                                                                                                                                      | $P(W = 3) = 0.20458... \quad \text{awrt } \underline{0.205}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(c)</b>                                                                                                                                                                                           | $T \sim \text{Po}((2.6+1.2) \times 2)$                                                                         | M1    | 3.3  |
|                                                                                                                                                                                                      | $P(T < 9) = 0.64819... \quad \text{awrt } \underline{0.648}$                                                   | A1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(d)</b>                                                                                                                                                                                           | <b>(i)</b> Mean = $np = \underline{2.4}$                                                                       | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Variance = $np(1-p) = 2.3904 \quad \text{awrt } \underline{2.39}$                                  | B1    | 1.1b |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(e)</b>                                                                                                                                                                                           | <b>(i)</b> $[ D \sim \text{Po}(2.4) \quad P(D \leq 4) ]$<br>$= 0.9041... \quad \text{awrt } \underline{0.904}$ | B1    | 1.1b |
|                                                                                                                                                                                                      | <b>(ii)</b> Since $n$ is large and $p$ is small/mean is approximately equal to variance                        | B1    | 2.4  |
|                                                                                                                                                                                                      |                                                                                                                | (2)   |      |
| <b>(10 marks)</b>                                                                                                                                                                                    |                                                                                                                |       |      |
| Notes:                                                                                                                                                                                               |                                                                                                                |       |      |
| <b>(a)</b><br><b>M1:</b> For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.0544</b> |                                                                                                                |       |      |
| <b>(b)</b><br><b>M1:</b> For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.205</b>                                                 |                                                                                                                |       |      |
| <b>(c)</b><br><b>M1:</b> For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)<br><b>A1:</b> awrt <b>0.648</b>                                   |                                                                                                                |       |      |
| <b>(d)(i)</b><br><b>B1:</b> For <b>2.4</b>                                                                                                                                                           |                                                                                                                |       |      |
| <b>(d)(ii)</b><br><b>B1:</b> For awrt <b>2.39</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(i)</b><br><b>B1:</b> For awrt <b>0.904</b>                                                                                                                                                    |                                                                                                                |       |      |
| <b>(e)(ii)</b><br><b>B1:</b> For a correct explanation to support use of Poisson approximation in this case                                                                                          |                                                                                                                |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                                                                     | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | <b>(i)</b> $P(X = 1) = 0.34523\dots$ awrt <b><u>0.345</u></b>                                                                              | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <b>(ii)</b> $P(X \leq 4) = 0.98575\dots$ awrt <b><u>0.986</u></b>                                                                          | B1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$                        | B1*cs0     | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $r = \mathbf{13.81}$ $s = \mathbf{0.57}$                                                                                                   | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $H_0$ : The Poisson distribution is a suitable model<br>$H_1$ : The Poisson distribution is not a suitable model                           | B1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [Cells are combined when expected frequencies < 5]<br>So combine the last 3 cells                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$       | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | awrt <b><u>1.1</u></b>                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Degrees of freedom = $4 - 1 - 1 = 2$                                                                                                       | B1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | (Do not reject $H_0$ since $1.10 < \chi^2_{2,(0.05)} = 5.991$ ). The number of mortgages approved each week follows a Poisson distribution | A1         | 3.5a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                            | <b>(6)</b> |      |
| <b>(11 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                            |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(i)</b><br><b>B1:</b> awrt 0.345                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                            |            |      |
| <b>(a)(ii)</b><br><b>B1:</b> awrt 0.986                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                            |            |      |
| <b>(b)</b><br><b>B1*:</b> For a fully correct calculation leading to given answer with no errors seen                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                            |            |      |
| <b>(c)</b><br><b>M1:</b> For attempt at $r$ or $s$ (may be implied by correct answers)<br><b>A1ft:</b> For both values correct (follow through their answers to part (a))                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                            |            |      |
| <b>(d)</b><br><b>B1:</b> For both hypotheses correct (lambda should not be defined so correct use of the model)<br><b>M1:</b> For understanding the need to combine cells before calculating the test statistic (may be implied)<br><br><b>M1:</b> For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$<br><b>A1:</b> awrt 1.1<br><b>B1:</b> For realising that there are 2 degrees of freedom leading to a critical value of $\chi^2_{2,(0.05)} = 5.991$<br><b>A1:</b> Concluding that a Poisson model is suitable for the number of mortgages approved each week |                                                                                                                                            |            |      |

# Further Statistics 2 Mark Scheme (Section B)

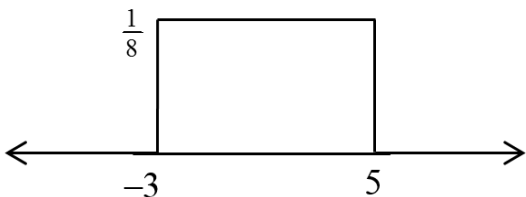
| Question                                                                                                      | Scheme                                                                                                                                               |  |   |   |   |   |   |   |   |   | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|---|---|---|---|-------|------|
| 5(a)                                                                                                          | Competitor                                                                                                                                           |  | A | B | C | D | E | F | G | H | M1    | 1.1b |
|                                                                                                               | Judge 1's ranks                                                                                                                                      |  | 8 | 4 | 7 | 6 | 5 | 1 | 3 | 2 |       |      |
|                                                                                                               | Judge 2's ranks                                                                                                                                      |  | 8 | 5 | 6 | 7 | 3 | 1 | 4 | 2 |       |      |
|                                                                                                               | $d^2$                                                                                                                                                |  | 0 | 1 | 1 | 1 | 4 | 0 | 1 | 0 | M1    | 1.1b |
|                                                                                                               | $\sum d^2 = 8$<br>$r_s = 1 - \frac{6 \times 8}{8(64 - 1)}$                                                                                           |  |   |   |   |   |   |   |   |   | dM1   | 1.1b |
|                                                                                                               | $r_s = 0.90476 \dots$ awrt <b>0.905</b>                                                                                                              |  |   |   |   |   |   |   |   |   | A1    | 1.1b |
|                                                                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   | (4)   |      |
| (b)                                                                                                           | $H_0: \rho_s = 0$ $H_1: \rho_s > 0$                                                                                                                  |  |   |   |   |   |   |   |   |   | B1    | 2.5  |
|                                                                                                               | Critical value $\rho_s = 0.8333$                                                                                                                     |  |   |   |   |   |   |   |   |   | B1    | 1.1b |
|                                                                                                               | $r_s = 0.905$ lies in the critical region/reject $H_0$                                                                                               |  |   |   |   |   |   |   |   |   | M1    | 2.1  |
|                                                                                                               | The two judges are in agreement.                                                                                                                     |  |   |   |   |   |   |   |   |   | A1    | 2.2b |
|                                                                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   | (4)   |      |
| (c)                                                                                                           | E.g. The data is unlikely to be from a bivariate normal distribution (competitor A)/The emphasis here is on the ranks and not the individual scores. |  |   |   |   |   |   |   |   |   | B1    | 2.4  |
|                                                                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   | (1)   |      |
| (d)                                                                                                           | Both show positive correlation, but the judges agree more on the beam (since 0.952 is closer to 1)                                                   |  |   |   |   |   |   |   |   |   | B1    | 2.2b |
|                                                                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   | (1)   |      |
| (10 marks)                                                                                                    |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| Notes:                                                                                                        |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| (a)                                                                                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| M1: For an attempt to rank at least one row (at least four correct)                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| M1: For an attempt at $d^2$ row for their ranks                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| M1: Dependent on 1 <sup>st</sup> M1 for use of $r_s = 1 - \frac{6 \times 8}{8(64 - 1)}$ with their $\sum d^2$ |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| A1: For awrt 0.905                                                                                            |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| (b)                                                                                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| B1: Both hypotheses stated in terms of $\rho_s$                                                               |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| B1: For correct critical value                                                                                |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| M1: For comparing their '0.905' with their '0.8333'                                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| A1: For a correct contextual conclusion with no contradictions seen                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| (c)                                                                                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| B1: For a correct explanation to support the use of Spearman                                                  |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| (d)                                                                                                           |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |
| B1: For a correct comparison of the correlation coefficients                                                  |                                                                                                                                                      |  |   |   |   |   |   |   |   |   |       |      |

| Question                                                                                                                                                                                                                                   | Scheme                                                                                                                         | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>6(a)</b>                                                                                                                                                                                                                                | $P(X < 3) = \int_1^3 \frac{1}{18}(11-2x)dx$ <u>or</u> area of trapezium                                                        | M1         | 1.1a |
|                                                                                                                                                                                                                                            | $= \left[ \frac{1}{18}(11x - x^2) \right]_1^3$                                                                                 |            |      |
|                                                                                                                                                                                                                                            | $= \frac{7}{9}$                                                                                                                | A1         | 1.1b |
|                                                                                                                                                                                                                                            |                                                                                                                                | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                 | Since $P(X < 3) > 0.75$ , the upper quartile is less than 3                                                                    | B1ft       | 2.2a |
|                                                                                                                                                                                                                                            |                                                                                                                                | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                 | $E(X^2) = \int_1^4 \frac{1}{18}x^2(11-2x)dx \left[ = \frac{23}{4} \right]$                                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                            | $\text{Var}(X) = \frac{23}{4} - \left( \frac{9}{4} \right)^2$                                                                  | M1         | 1.1b |
|                                                                                                                                                                                                                                            | $= \frac{11}{16}$                                                                                                              | A1         | 1.1b |
|                                                                                                                                                                                                                                            |                                                                                                                                | <b>(3)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                 | $F(4) = 1 \rightarrow \frac{1}{18}(11(4) - 4^2 + c) = 1$ <u>or</u><br>$F(1) = 0 \rightarrow \frac{1}{18}(11(1) - 1^2 + c) = 0$ | M1         | 2.1  |
|                                                                                                                                                                                                                                            | $c = -10$ *                                                                                                                    | A1*cso     | 1.1b |
|                                                                                                                                                                                                                                            |                                                                                                                                | <b>(2)</b> |      |
| <b>(e)</b>                                                                                                                                                                                                                                 | $F(m) = 0.5$                                                                                                                   | M1         | 1.2  |
|                                                                                                                                                                                                                                            | $\frac{1}{18}(11m - m^2 - 10) = 0.5 \rightarrow m^2 - 11m + 19 = 0$ and attempt to solve                                       | M1         | 1.1b |
|                                                                                                                                                                                                                                            | $m = \frac{11 \pm \sqrt{11^2 - 4(19)}}{2} [= 2.1458 \text{ or } 8.8541\dots]$                                                  |            |      |
|                                                                                                                                                                                                                                            | $m = 2.1458\dots$ <b>2.15</b> (only)                                                                                           | A1         | 2.2a |
|                                                                                                                                                                                                                                            |                                                                                                                                | <b>(3)</b> |      |
| <b>(11 marks)</b>                                                                                                                                                                                                                          |                                                                                                                                |            |      |
| Notes:                                                                                                                                                                                                                                     |                                                                                                                                |            |      |
| <b>(a)</b><br><b>M1:</b> For integrating $f(x)$ with correct limits <b>or</b> for finding area of trapezium<br><b>A1:</b> For $\frac{7}{9}$ (allow awrt 0.778)                                                                             |                                                                                                                                |            |      |
| <b>(b)</b><br><b>B1ft:</b> For comparison of their (a) with 0.75 and concluding that the upper quartile is less than 3                                                                                                                     |                                                                                                                                |            |      |
| <b>(c)</b><br><b>M1:</b> For an attempt to find $E(X^2)$<br><b>M1:</b> For use of $\text{Var}(X) = E(X^2) - \left(\frac{9}{4}\right)^2$<br><b>A1:</b> For $\frac{11}{16}$ (allow awrt 0.688) (M1 marks may be implied by a correct answer) |                                                                                                                                |            |      |

|                             |                                                                          |
|-----------------------------|--------------------------------------------------------------------------|
| Question 6 notes continued: |                                                                          |
| (d)                         |                                                                          |
| <b>M1:</b>                  | For use of $F(4) = 1$ or $F(1) = 0$                                      |
| <b>A1*cs0:</b>              | For a fully correct solution leading to given answer with no errors seen |
| (e)                         |                                                                          |
| <b>M1:</b>                  | For use of $F(m) = 0.5$                                                  |
| <b>M1:</b>                  | For setting up quadratic and attempt to solve                            |
| <b>A1:</b>                  | For 2.15 and rejecting the other solution                                |



| Question                                                                                                                                                        | Scheme                                                                                                 | Marks      | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|------------|------|
| <b>7(a)</b>                                                                                                                                                     | $r = \frac{284.4 - \frac{251(12)}{10}}{\sqrt{10.36 \times 40.9}}$                                      | M1         | 1.1b |
|                                                                                                                                                                 | $r = -0.79671...$ awrt <u><b>-0.797</b></u>                                                            | A1         | 1.1b |
|                                                                                                                                                                 |                                                                                                        | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                                                      | $b = \frac{'-16.4'}{10.36}$                                                                            | M1         | 3.3  |
|                                                                                                                                                                 | $a = \frac{251}{10} - 'b' \frac{12}{10}$                                                               | M1         | 1.1b |
|                                                                                                                                                                 | $y = 27.0 - 1.58x$                                                                                     | A1         | 1.1b |
|                                                                                                                                                                 |                                                                                                        | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                                                      | $y = [27.0 - 1.58(2)] = 23.84$ awrt <u><b>23.8</b></u>                                                 | B1ft       | 3.4  |
|                                                                                                                                                                 |                                                                                                        | <b>(1)</b> |      |
| <b>(d)</b>                                                                                                                                                      | $RSS = 40.9 - \frac{(-16.4)^2}{10.36}$                                                                 | M1         | 1.1b |
|                                                                                                                                                                 | $RSS = 14.938...$ awrt <u><b>14.9</b></u>                                                              | A1         | 1.1b |
|                                                                                                                                                                 |                                                                                                        | <b>(2)</b> |      |
| <b>(e)</b>                                                                                                                                                      | $\sum \text{residuals} = 0 \rightarrow -0.63 + (-0.32) + \dots + f + (-1.88) = 0$                      | M1         | 3.1a |
|                                                                                                                                                                 | $f = \underline{\underline{-1.04}}$                                                                    | A1         | 1.1b |
|                                                                                                                                                                 |                                                                                                        | <b>(2)</b> |      |
| <b>(f)</b>                                                                                                                                                      | The residuals should be randomly scattered above and below zero so linear model may not be appropriate | B1         | 3.5b |
|                                                                                                                                                                 |                                                                                                        | <b>(1)</b> |      |
| <b>(11 marks)</b>                                                                                                                                               |                                                                                                        |            |      |
| Notes:                                                                                                                                                          |                                                                                                        |            |      |
| <b>(a)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>M1:</b> For a complete correct method for finding $r$                                                                                                        |                                                                                                        |            |      |
| <b>A1:</b> For awrt $-0.797$                                                                                                                                    |                                                                                                        |            |      |
| <b>(b)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>M1:</b> For use of a correct model i.e. a correct expression for $b$ (ft their $S_{xy}$ )                                                                    |                                                                                                        |            |      |
| <b>M1:</b> For use of a correct model i.e. a correct (ft) expression for $a$                                                                                    |                                                                                                        |            |      |
| <b>A1:</b> For $y = 27.0 - 1.58x$ [a correct answer here can imply both method marks]                                                                           |                                                                                                        |            |      |
| <b>(c)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>B1:</b> For awrt 23.8 (evaluating their model found in part (b) with $x = 2$ )                                                                               |                                                                                                        |            |      |
| <b>(d)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>M1:</b> For a correct expression for RSS                                                                                                                     |                                                                                                        |            |      |
| <b>A1:</b> For awrt 14.9                                                                                                                                        |                                                                                                        |            |      |
| <b>(e)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>M1:</b> For use of $\sum \text{residuals} = 0$ [Use of regression equation needs correct sign]                                                               |                                                                                                        |            |      |
| <b>A1:</b> For $-1.04$                                                                                                                                          |                                                                                                        |            |      |
| <b>(f)</b>                                                                                                                                                      |                                                                                                        |            |      |
| <b>B1:</b> For identifying that the residuals are not randomly scattered above and below zero and concluding the linear regression model may not be appropriate |                                                                                                        |            |      |

| Question                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                            | Marks          | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|----------------|------|
| <b>8(a)</b>                                                                                                                                                                                                                                                                                                                                                  |  | B1<br>(shape)  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              |                                                                                   | B1<br>(labels) | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              |                                                                                   | <b>(2)</b>     |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                   | $P(X < 2(k - X)) = P(X < \frac{2}{3}k)$                                           | M1             | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                              | $\frac{\frac{2}{3}k - (-3)}{5 - (-3)} = 0.25$                                     | M1             | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              | $k = -\frac{3}{2}$                                                                | A1             | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              |                                                                                   | <b>(3)</b>     |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                   | $E(X^3) = \int_{-3}^5 \frac{1}{5 - (-3)} x^3 dx$                                  | M1             | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                              | $= \left[ \frac{1}{32} x^4 \right]_{-3}^5 = \frac{1}{32} (5^4 - (-3)^4)$          | dM1            | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              | $= 17^*$                                                                          | A1*cs0         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                              |                                                                                   | <b>(3)</b>     |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                                                                                                             |                                                                                   |                |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                       |                                                                                   |                |      |
| <b>(a)</b><br><b>B1:</b> For correct shape<br><b>B1:</b> For correct labels                                                                                                                                                                                                                                                                                  |                                                                                   |                |      |
| <b>(b)</b><br><b>M1:</b> For simplifying to $P(X < \frac{2}{3}k)$<br><b>M1:</b> For equating probability expression to 0.25<br><b>A1:</b> For $-\frac{3}{2}$<br><br><b>Another method for part (b) is:</b><br><b>M1:</b> For understanding $2[k - x] = -1$ and $x = -1$<br><b>M1:</b> For substitution and attempt to solve<br><b>A1:</b> For $-\frac{3}{2}$ |                                                                                   |                |      |
| <b>(c)</b><br><b>B1:</b> For integrating $x^3 f(x)$<br><b>M1:</b> For use of correct limits (dependent on previous M1)<br><b>A1*:</b> For fully correct solution leading to the given answer with no errors seen                                                                                                                                             |                                                                                   |                |      |

Write your name here

Surname

Other names

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**Level 3 GCE**

Centre Number

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2H: Further Mechanics 1 and  
Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2H**

**You must have:**

Decision Mathematics 1 question insert for Section B  
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.  
Calculators must not have the facility for algebraic manipulation,  
differentiation and integration, or have retrievable mathematical  
formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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- (c) By considering your answer to (b), describe the subsequent motion of the ball. (1)

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**Question 1 continued**

**(Total for Question 1 is 10 marks)**

2. A small stone of mass  $0.5\text{ kg}$  is thrown vertically upwards from a point  $A$  with an initial speed of  $25\text{ ms}^{-1}$ . The stone first comes to instantaneous rest at the point  $B$  which is  $20\text{ m}$  vertically above the point  $A$ . As the stone moves it is subject to air resistance. The stone is modelled as a particle.

- (a) Find the energy lost due to air resistance by the stone, as it moves from  $A$  to  $B$ . (3)

The air resistance is modelled as a constant force of magnitude  $R$  newtons.

- (b) Find the value of  $R$ . (2)

- (c) State how the model for air resistance could be refined to make it more realistic. (1)

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3. [In this question use  $g = 10 \text{ m s}^{-2}$ ]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of  $4 \text{ m s}^{-1}$ . The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle  $\alpha$ , where

$\sin \alpha = \frac{1}{15}$ . Because of the hill, the jogger reduces her speed to  $3 \text{ m s}^{-1}$  and maintains this

constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at  $3 \text{ m s}^{-1}$ .

(5)



**Question 3 continued****(Total for Question 3 is 8 marks)**

4. A particle  $P$  of mass  $3m$  is moving in a straight line on a smooth horizontal table. A particle  $Q$  of mass  $m$  is moving in the opposite direction to  $P$  along the same straight line. The particles collide directly. Immediately before the collision the speed of  $P$  is  $u$  and the speed of  $Q$  is  $2u$ . The velocities of  $P$  and  $Q$  immediately after the collision, measured in the direction of motion of  $P$  before the collision, are  $v$  and  $w$  respectively. The coefficient of restitution between  $P$  and  $Q$  is  $e$ .

(a) Find an expression for  $v$  in terms of  $u$  and  $e$ .

(6)

Given that the direction of motion of  $P$  is changed by the collision,

(b) find the range of possible values of  $e$ .

(2)

(c) Show that  $w = \frac{u}{4}(1 + 9e)$ .

(2)

Following the collision with  $P$ , the particle  $Q$  then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of  $Q$ . The coefficient of restitution between  $Q$  and the wall is  $f$ .

Given that  $e = \frac{5}{9}$ , and that  $P$  and  $Q$  collide again in the subsequent motion,

(d) find the range of possible values of  $f$ .

(6)

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This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Question 4 continued**

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**Question 4 continued**

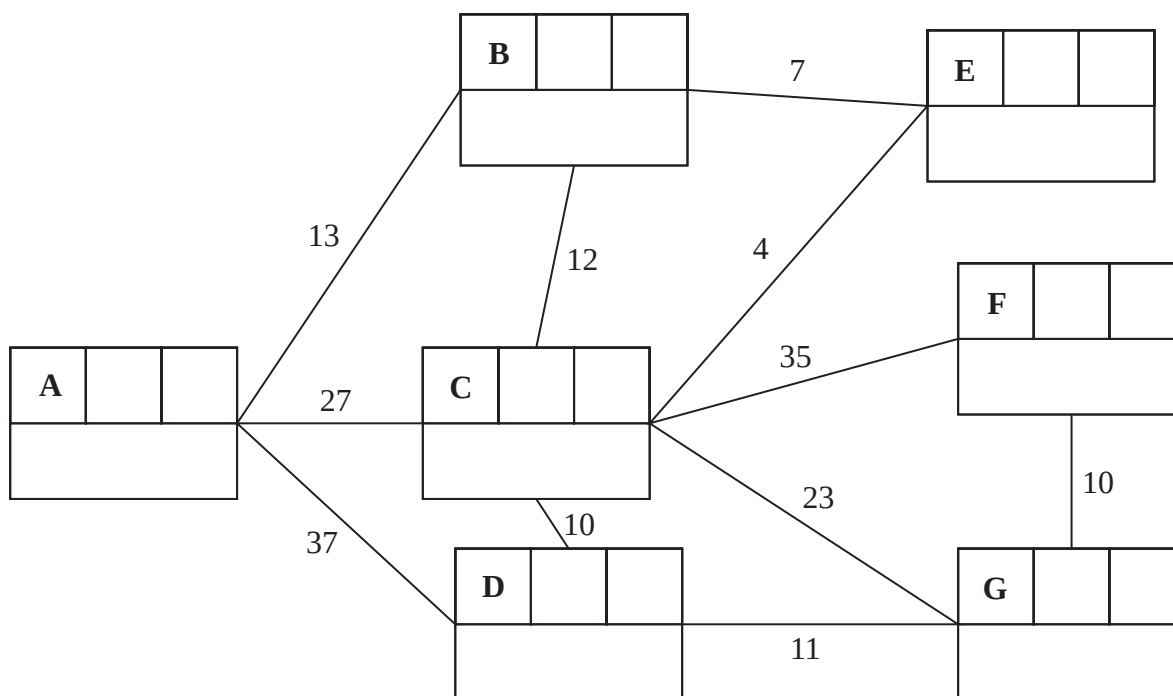
(Total for Question 4 is 16 marks)

**TOTAL FOR SECTION A IS 40 MARKS**

## SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics 1 question insert.

5.



**Key:**

| Vertex         | Order of<br>labelling | Final value |
|----------------|-----------------------|-------------|
| Working values |                       |             |

Shortest path: \_\_\_\_\_

Length of shortest path: \_\_\_\_\_

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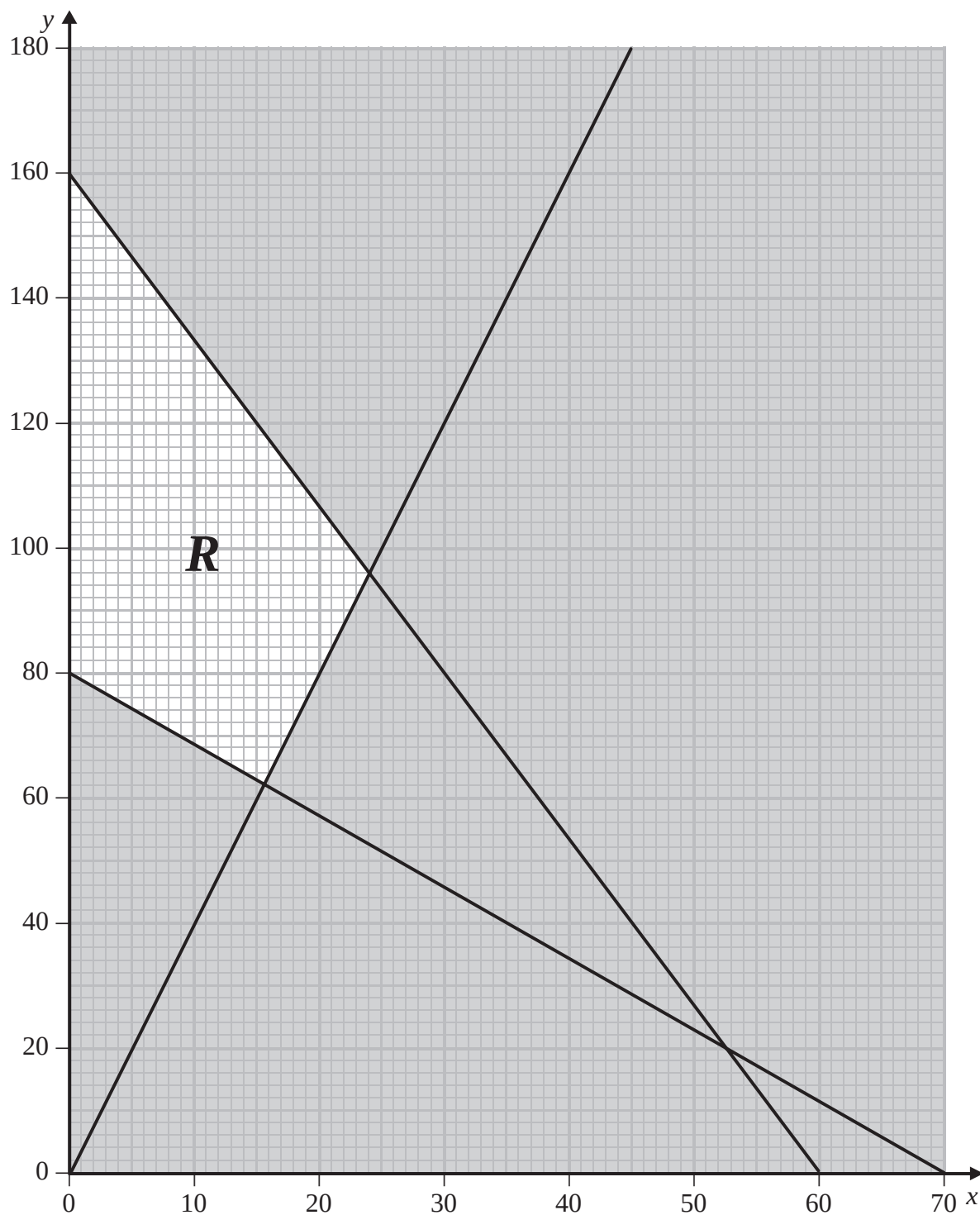
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6.



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7. (a) and (b)

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(Total for Question 7 is 7 marks)

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9.

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(Total for Question 9 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**

**Pearson Edexcel Level 3 GCE**

# **Further Mathematics**

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2H: Section B Decision Mathematics 1**

Sample Assessment Material for first teaching September 2017

Paper Reference

**8FM0/2H**

**Decision Mathematics 1 question insert for Section B**

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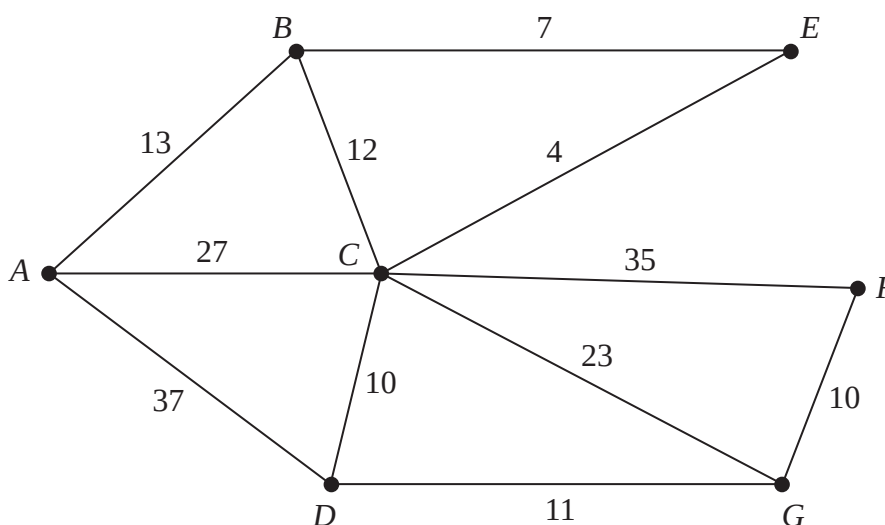


  
**Pearson**

## SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.



**Figure 1**

[The total weight of the network is 189]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

- (a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length. (5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

- (b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear. (4)

- (c) State the minimum length of Gabriel's route. (1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

- (d) Calculate the length of BG. You must show your working. (2)

**(Total for Question 5 is 12 marks)**

6.

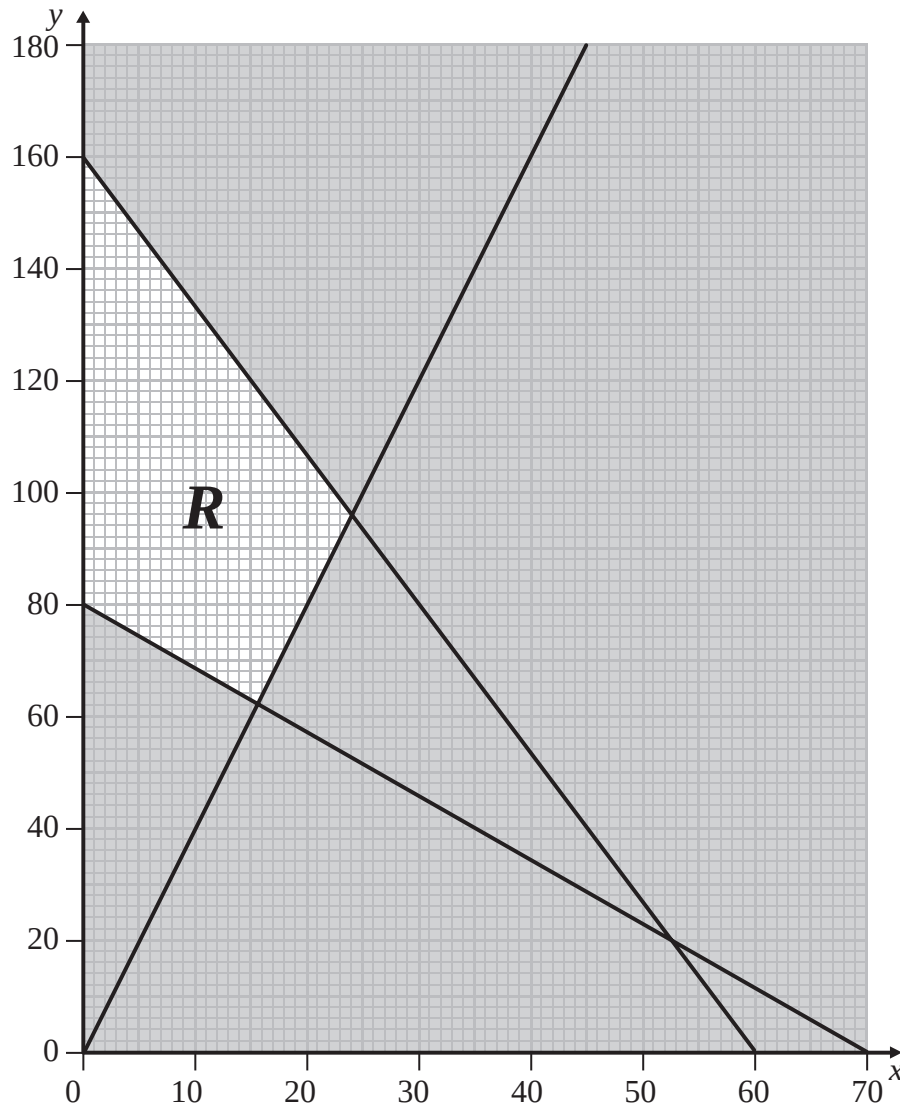


Figure 2

A teacher buys pens and pencils. The number of pens,  $x$ , and the number of pencils,  $y$ , that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

| Activity | Time taken (days) | Immediately preceding activities |
|----------|-------------------|----------------------------------|
| A        | 5                 | -                                |
| B        | 7                 | -                                |
| C        | 3                 | -                                |
| D        | 4                 | A, B                             |
| E        | 4                 | D                                |
| F        | 2                 | B                                |
| G        | 4                 | B                                |
| H        | 5                 | C, G                             |
| I        | 10                | C, G                             |

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only. (3)
- (b) (i) Show that the project can be completed in 21 days, showing your working.  
(ii) Identify the critical activities. (4)

**(Total for Question 7 is 7 marks)**



8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5 (1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are  $2^{2x} - 1$ ,  $2^x$ ,  $x + 1$ ,  $2^{x+1} - 3$  and  $11 - x$ .

- (b) (i) Calculate  $x$ .  
(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer. (6)

- (c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let  $x$  and  $y$  represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

**TOTAL FOR SECTION B IS 40 MARKS**

## Paper 2 Option H

### Further Mechanics 1 Mark Scheme (Section A)

| Question                                                                                                                                                                                                                                                                                                                                   | Scheme                                                                                                    | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                | Using the model and $v^2 = u^2 + 2as$ to find $v$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{4.8g}$                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$ to find $u$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{1.2g}$                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                            | $e = \sqrt{1.2g} / \sqrt{4.8g} = 0.5$ *                                                                   | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                 | Using the model and $e = \text{sep. speed} / \text{app. speed}$ ,<br>$v = 0.5\sqrt{1.2g}$                 | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$                                                                     | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15 \text{ (m)}$                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                 | Ball continues to bounce with the height of each bounce being a quarter of the previous one               | B1         | 2.2b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                          |                                                                                                           |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                     |                                                                                                           |            |      |
| <b>(a)</b><br><b>M1:</b> For a complete method to find $v$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For a complete method to find $u$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For finding both $v$ and $u$ and use of Newton's Law of Restitution<br><b>A1*:</b> For the given answer |                                                                                                           |            |      |
| <b>(b)</b><br><b>M1:</b> For use of Newton's Law of Restitution to find rebound speed<br><b>M1:</b> For a complete method to find $h$<br><b>A1:</b> For 0.15 (m) oe                                                                                                                                                                        |                                                                                                           |            |      |
| <b>(c)</b><br><b>B1:</b> For a clear description including reference to a quarter                                                                                                                                                                                                                                                          |                                                                                                           |            |      |

| Question                                                                                                                                           | Scheme                                                   | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|------------|------|
| <b>2(a)</b>                                                                                                                                        | Energy Loss = KE Loss – PE Gain                          | M1         | 3.3  |
|                                                                                                                                                    | $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 g \times 20$ | A1         | 1.1b |
|                                                                                                                                                    | $= 58.25 = 58 \text{ (J ) or } 58.3 \text{ (J)}$         | A1         | 1.1b |
|                                                                                                                                                    |                                                          | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                         | Using work-energy principle, $20 R = 58.25$              | M1         | 3.3  |
|                                                                                                                                                    | $R = 2.9125 = 2.9 \text{ or } 2.91$                      | A1ft       | 1.1b |
|                                                                                                                                                    |                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                         | Make resistance variable (dependent on speed)            | B1         | 3.5c |
|                                                                                                                                                    |                                                          | <b>(1)</b> |      |
| <b>(6 marks)</b>                                                                                                                                   |                                                          |            |      |
| Notes:                                                                                                                                             |                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For a difference in KE and PE<br><b>A1:</b> For a correct expression<br><b>A1:</b> For either 58 (2sf) or 58.3(3sf)       |                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of work-energy principle<br><b>A1ft:</b> For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a) |                                                          |            |      |
| <b>(c)</b><br><b>B1:</b> For variable resistance oe                                                                                                |                                                          |            |      |

| Question                                                                                                                                                                                                                                                            | Scheme                                          | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|-------|------|
| <b>3(a)</b>                                                                                                                                                                                                                                                         | Force = Resistance (since no acceleration) = 30 | B1    | 3.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 30 $\times$ 4    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 120 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (3)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                          | Resolving parallel to the slope                 | M1    | 3.1b |
|                                                                                                                                                                                                                                                                     | $F - 60g\sin\alpha - 30 = 0$                    | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | $F = 70$                                        | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 70 $\times$ 3    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 210 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (5)   |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                    |                                                 |       |      |
| Notes:                                                                                                                                                                                                                                                              |                                                 |       |      |
| <b>(a)</b><br><b>B1:</b> For force = 30 seen<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 120 (W), follow through on their '30'                                                                                                                            |                                                 |       |      |
| <b>(b)</b><br><b>M1:</b> For resolving parallel to the slope with correct no. of terms and 60g resolved<br><b>A1:</b> For a correct equation<br><b>A1:</b> For $F = 70$<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 210 (W), follow through on their '70' |                                                 |       |      |

| Question    | Scheme                                                                                                                   | Marks      | AOs  |
|-------------|--------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b> | Use of conservation of momentum                                                                                          | M1         | 3.1a |
|             | $3mu - 2mu = 3mv + mw$                                                                                                   | A1         | 1.1b |
|             | Use of NLR                                                                                                               | M1         | 3.1a |
|             | $3ue = -v + w$                                                                                                           | A1         | 1.1b |
|             | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1         | 3.1b |
|             | $v = \frac{u}{4}(1 - 3e)$                                                                                                | A1         | 1.1b |
|             |                                                                                                                          | <b>(6)</b> |      |
| <b>(b)</b>  | $\frac{u}{4}(1 - 3e) < 0$                                                                                                | M1         | 3.1b |
|             | $\frac{1}{3} < e \leq 1$                                                                                                 | A1         | 1.1b |
|             |                                                                                                                          | <b>(2)</b> |      |
| <b>(c)</b>  | Solving for $w$                                                                                                          | M1         | 2.1  |
|             | $w = \frac{u}{4}(1 + 9e)^*$                                                                                              | A1 *       | 1.1b |
|             |                                                                                                                          | <b>(2)</b> |      |
| <b>(d)</b>  | Substitute $e = \frac{5}{9}$                                                                                             | M1         | 1.1b |
|             | $v = -\frac{u}{6}, w = \frac{3u}{2}$                                                                                     | A1         | 1.1b |
|             | Use NLR for impact with wall, $x = fw$                                                                                   | M1         | 1.1b |
|             | Further collision if $x > -v$                                                                                            | M1         | 3.4  |
|             | $f\frac{3u}{2} > \frac{u}{6}$                                                                                            | A1         | 1.1b |
|             | $1 \geq f > \frac{1}{9}$                                                                                                 | A1         | 1.1b |
|             |                                                                                                                          | <b>(6)</b> |      |

**(16 marks)**

Notes:

**(a)**

**M1:** For use of CLM, with correct no. of terms, condone sign errors

**A1:** For a correct equation

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**A1:** For a correct equation

**M1:** For setting up *two* equations and solving their equations for  $v$

**A1:** For a correct expression for  $v$

**(b)**

**M1:** For use of an appropriate inequality

**A1:** For a complete range of values of  $e$

**(c)**

**M1:** For solving their equations for  $w$

**A1:** For the given answer

Question 4 notes continued:

**(d)**

**M1:** For substituting  $e = \frac{5}{9}$  into their  $v$  and  $w$

**A1:** For correct expressions for  $v$  and  $w$

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**M1:** For use of appropriate inequality

**A1:** For a correct inequality

**A1:** For a correct range

Decision Mathematics 1 Mark Scheme (Section **B**)

| Question                                                                                                                                                                                                                                                                                                                                                       | Scheme                              | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|------------|------|
| <b>5(a)</b>                                                                                                                                                                                                                                                                                                                                                    |                                     | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Path: ABECDGF                       | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Length: 55 (metres)                 | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(5)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                     | $AB + DG = 13 + 11 = 24 \leftarrow$ | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BEC)D + B(ECD)G = 34 + 32 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BECD)G + B(EC)D = 45 + 21 = 66$  | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Repeat arcs: AB, DG                 | A1ft       | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(4)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                     | Length = $189 + 24 = 213$ (metres)  | B1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(1)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                     | $189 + x + 34 = 213 + 2x$           | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $x = 10$ so BG is 10 m              | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | <b>(2)</b> |      |
| <b>(12 marks)</b>                                                                                                                                                                                                                                                                                                                                              |                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                         |                                     |            |      |
| <b>(a)</b><br><b>M1:</b> For a larger number replaced by a smaller one in the working values boxes at C, D, F or G<br><b>A1:</b> For all values correct (and in correct order) at A, B, C and D<br><b>A1:</b> For all values correct (and in correct order) at E, F & G<br><b>A1:</b> For the correct path<br><b>A1ft:</b> For 55 or ft their final value at F |                                     |            |      |
| <b>(b)</b><br><b>M1:</b> For 3 correct pairings of the four odd nodes (A,B, D & G)<br><b>A1:</b> At least two pairings and totals correct<br><b>A2:</b> All three pairings and totals correct<br><b>A3ft:</b> Selecting their shortest pairing, and stating that these arcs should be repeated                                                                 |                                     |            |      |



Question **5** notes continued:

**(c)**

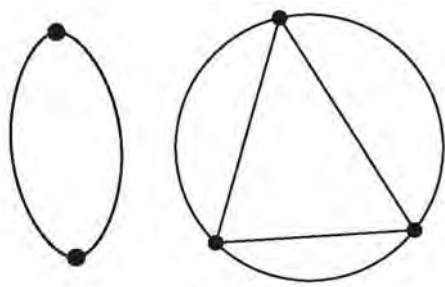
**B1ft:** For 213 or 189 + their shortest repeat

**M1:** For translating the information in the question in to an equation involving  $x$ ,  $2x$  and 34

**A1:** For a correct equation leading to  $BG = 10$  (m)

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                                        | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>6</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Objective line drawn or at least two vertices tested                                                                          | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | For solving $y = 4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$ | M1    | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Finding at least two points with integer co-ordinates from $(15 \pm 1, 63 \pm 2)$                                             | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Testing at least two points with integer co-ordinates                                                                         | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15$ and $y = 63$                                                                                                         | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | So the teacher should buy 15 pens and 63 pencils                                                                              | A1ft  | 3.2a |
| <b>(7 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                               |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                               |       |      |
| <p><b>M1:</b> Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C</p> <p><b>M1:</b> Solving simultaneous equations</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of <math>y \geq 4x</math> and <math>8x + 7y \geq 560</math></p> <p><b>M1:</b> Testing at least two integer solutions in <math>y \geq 4x</math> or <math>8x + 7y \geq 560</math> and C</p> <p><b>A1:</b> cao – deducing from tests which integer solution is both valid and optimal</p> <p><b>A1ft:</b> Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils</p> |                                                                                                                               |       |      |

| Question                                                                                                                                                                                                                                                                                 | Scheme                                                                                                                              | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>7(a)(b)</b>                                                                                                                                                                                                                                                                           | <p>The number(s) at the end of activity E indicate this project can be completed in 21 days</p> <p>Critical activities: B, G, I</p> | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | <b>(3)</b> |      |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1ft       | 2.2a |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                          |                                                                                                                                     | <b>(4)</b> |      |
| <b>(7 marks)</b>                                                                                                                                                                                                                                                                         |                                                                                                                                     |            |      |
| Notes:                                                                                                                                                                                                                                                                                   |                                                                                                                                     |            |      |
| <b>M1:</b> At least 5 activities and one dummy, one start                                                                                                                                                                                                                                |                                                                                                                                     |            |      |
| <b>A1:</b> A,B,C,D,F,G and first dummy correct                                                                                                                                                                                                                                           |                                                                                                                                     |            |      |
| <b>A1:</b> E,H,I correct, second dummy correct and one finish                                                                                                                                                                                                                            |                                                                                                                                     |            |      |
| <b>M1:</b> All boxes completed, number generally increasing L to R (condone one “rogue”)                                                                                                                                                                                                 |                                                                                                                                     |            |      |
| <b>A1:</b> All values cao                                                                                                                                                                                                                                                                |                                                                                                                                     |            |      |
| <b>A1:</b> Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes) |                                                                                                                                     |            |      |
| <b>A1:</b> Critical activities correct                                                                                                                                                                                                                                                   |                                                                                                                                     |            |      |

| Question                                                                         | Scheme                                                                                                                        | Marks | AOs  |
|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>8(a)</b>                                                                      | e.g. a graph cannot contain an odd number of odd nodes<br>e.g. number of arcs = $\frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$ | B1    | 2.4  |
|                                                                                  |                                                                                                                               | (1)   |      |
| <b>(b)(i)</b>                                                                    | $(2^{2x}-1)+(2^x)+(x+1)+(2^{x+1}-3)+(11-x)=2(18)$                                                                             | M1    | 1.1b |
|                                                                                  | $2^{2x}+3(2^x)-28=0 \Rightarrow x=...$                                                                                        | M1    | 1.1b |
|                                                                                  | $(2^x+7)(2^x-4)=0 \Rightarrow x=2$                                                                                            | A1    | 1.1b |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(b)(ii)</b>                                                                   | The order of the nodes are 9, 15, 3, 4, 5                                                                                     | M1    | 2.1  |
|                                                                                  | Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes                                | A1    | 2.4  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(c)</b>                                                                       |                                             | M1    | 2.5  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (2)   |      |
| <b>(9 marks)</b>                                                                 |                                                                                                                               |       |      |
| Notes:                                                                           |                                                                                                                               |       |      |
| <b>(a)</b>                                                                       |                                                                                                                               |       |      |
| <b>B1:</b> Explanation referring to need for an even number of odd nodes oe      |                                                                                                                               |       |      |
| <b>(b)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Forming an equation involving the orders of the 5 odd nodes and 2(18) |                                                                                                                               |       |      |
| <b>M1:</b> Simplifies to a quadratic in $2^x$ and attempts to solve              |                                                                                                                               |       |      |
| <b>A1:</b> 2 cao                                                                 |                                                                                                                               |       |      |
| <b>M1:</b> Construct an argument involving the order of the 5 nodes              |                                                                                                                               |       |      |
| <b>A1:</b> Explanation considering the number of odd nodes                       |                                                                                                                               |       |      |
| <b>A1:</b> Deduction that therefore it is neither Eulerian nor semi-Eulerian     |                                                                                                                               |       |      |
| <b>(c)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Interprets mathematical language to construct a disconnected graph    |                                                                                                                               |       |      |
| <b>A1:</b> Deduce a correct graph                                                |                                                                                                                               |       |      |

| Question                                                                                                                                                                                                                                                                                                                         | Scheme                                                                      | Marks    | AOs        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|----------|------------|
| <b>9</b>                                                                                                                                                                                                                                                                                                                         | Minimise ( $C =$ ) $25x + 35y$                                              | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | Subject to:<br>$(500x + 800y \geq 150\,000 \Rightarrow) 5x + 8y \geq 1500$  | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | $\frac{7}{20}(x + y) \leq x \leq \frac{13}{20}(x + y)$                      | M1<br>M1 | 3.3<br>3.3 |
|                                                                                                                                                                                                                                                                                                                                  | Which simplifies to $7y \leq 13x$ <b>and</b> $13y \geq 7x$<br>$x, y \geq 0$ | A1       | 1.1b       |
| <b>(5 marks)</b>                                                                                                                                                                                                                                                                                                                 |                                                                             |          |            |
| Notes:                                                                                                                                                                                                                                                                                                                           |                                                                             |          |            |
| <b>B1:</b> A correct objective function + minimise<br><b>B1:</b> Translate information in to a correct inequality<br><b>M1:</b> For translating the information given into the LHS inequality<br><b>M1:</b> For translating the information given in to the RHS inequality<br><b>A1:</b> Simplifying to the correct inequalities |                                                                             |          |            |



Write your name here

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2J: Further Mechanics 1 and Further Mechanics 2**

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2J**

**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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- (c) By considering your answer to (b), describe the subsequent motion of the ball. (1)



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[illegible]

2. A small stone of mass  $0.5\text{ kg}$  is thrown vertically upwards from a point  $A$  with an initial speed of  $25\text{ ms}^{-1}$ . The stone first comes to instantaneous rest at the point  $B$  which is  $20\text{ m}$  vertically above the point  $A$ . As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from  $A$  to  $B$ . (3)

The air resistance is modelled as a constant force of magnitude  $R$  newtons.

(b) Find the value of  $R$ . (2)

(c) State how the model for air resistance could be refined to make it more realistic. (1)

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3. [In this question use  $g = 10 \text{ m s}^{-2}$ ]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of  $4 \text{ m s}^{-1}$ . The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle  $\alpha$ , where

$\sin \alpha = \frac{1}{15}$ . Because of the hill, the jogger reduces her speed to  $3 \text{ m s}^{-1}$  and maintains this

constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at  $3 \text{ m s}^{-1}$ .

(5)

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4. A particle  $P$  of mass  $3m$  is moving in a straight line on a smooth horizontal table. A particle  $Q$  of mass  $m$  is moving in the opposite direction to  $P$  along the same straight line. The particles collide directly. Immediately before the collision the speed of  $P$  is  $u$  and the speed of  $Q$  is  $2u$ . The velocities of  $P$  and  $Q$  immediately after the collision, measured in the direction of motion of  $P$  before the collision, are  $v$  and  $w$  respectively. The coefficient of restitution between  $P$  and  $Q$  is  $e$ .

(a) Find an expression for  $v$  in terms of  $u$  and  $e$ .

(6)

Given that the direction of motion of  $P$  is changed by the collision,

(b) find the range of possible values of  $e$ .

(2)

(c) Show that  $w = \frac{u}{4}(1 + 9e)$ .

(2)

Following the collision with  $P$ , the particle  $Q$  then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of  $Q$ . The coefficient of restitution between  $Q$  and the wall is  $f$ .

Given that  $e = \frac{5}{9}$ , and that  $P$  and  $Q$  collide again in the subsequent motion,

(d) find the range of possible values of  $f$ .

(6)

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**Question 4 continued**

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**Question 4 continued****(Total for Question 4 is 16 marks)****TOTAL FOR SECTION A IS 40 MARKS**

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$$v = (t - 2)(3t - 10), \quad t \geq 0$$

(2)

(3)

(3)

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[illegible]

6. A light inextensible string has length  $7a$ . One end of the string is attached to a fixed point  $A$  and the other end of the string is attached to a fixed point  $B$ , with  $A$  vertically above  $B$  and  $AB = 5a$ . A particle of mass  $m$  is attached to a point  $P$  on the string where  $AP = 4a$ . The particle moves in a horizontal circle with constant angular speed  $\omega$ , with both  $AP$  and  $BP$  taut.

(a) Show that

(i) the tension in  $AP$  is  $\frac{4m}{25}(9a\omega^2 + 5g)$

(ii) the tension in  $BP$  is  $\frac{3m}{25}(16a\omega^2 - 5g)$ .

(10)

The string will break if the tension in it reaches a magnitude of  $4mg$ .

The time for the particle to make one revolution is  $S$ .

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}} \quad (5)$$

(c) State how in your calculations you have used the assumption that the string is light.

(1)

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**Question 6 continued**

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**(Total for Question 6 is 16 marks)**

7.

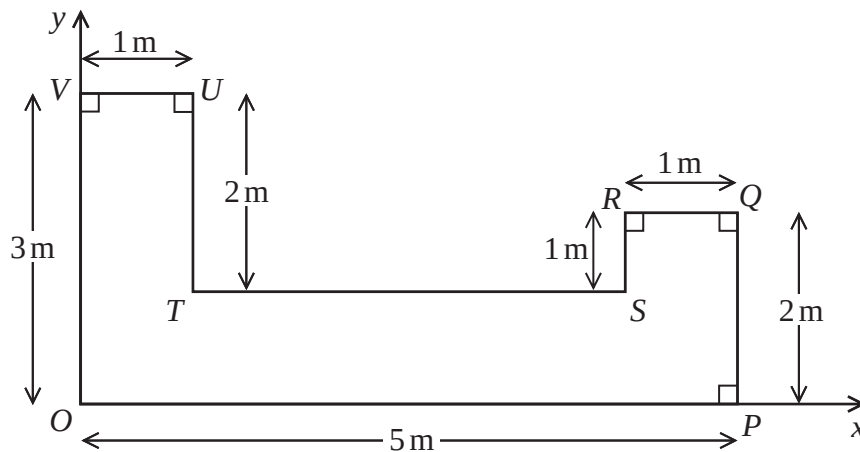


Figure 1

Figure 1 shows the shape and dimensions of a template  $OPQRSTUV$  made from thin uniform metal.

$OP = 5\text{ m}$ ,  $PQ = 2\text{ m}$ ,  $QR = 1\text{ m}$ ,  $RS = 1\text{ m}$ ,  $TU = 2\text{ m}$ ,  $UV = 1\text{ m}$ ,  $VO = 3\text{ m}$ .

Figure 1 also shows a coordinate system with  $O$  as origin and the  $x$ -axis and  $y$ -axis along  $OP$  and  $OV$  respectively. The unit of length on both axes is the metre.

The centre of mass of the template has coordinates  $(\bar{x}, \bar{y})$ .

- (a) (i) Show that  $\bar{y} = 1$   
 (ii) Find the value of  $\bar{x}$ .

(7)

A new design requires the template to have its centre of mass at the point  $(2.5, 1)$ . In order to achieve this, two circular discs, each of radius  $r$  metres, are removed from the template which is shown in Figure 1, to form a new template  $L$ . The centre of the first disc is  $(0.5, 0.5)$  and the centre of the second disc is  $(0.5, a)$  where  $a$  is a constant.

- (b) Find the value of  $r$ .  
 (c) (i) Explain how symmetry can be used to find the value of  $a$ .  
 (ii) Find the value of  $a$ .

(4)

(2)

The template  $L$  is now freely suspended from the point  $U$  and hangs in equilibrium.

- (d) Find the size of the angle between the line  $TU$  and the horizontal.

(3)

**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 16 marks)**

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**

## Paper 2 Option J

### Further Mechanics 1 Mark Scheme (Section A)

| Question                                                                                                                                                                                                                                                                                                                                   | Scheme                                                                                                    | Marks      | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|------------|------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                | Using the model and $v^2 = u^2 + 2as$ to find $v$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{4.8g}$                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$ to find $u$                                                         | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{1.2g}$                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                            | $e = \sqrt{1.2g} / \sqrt{4.8g} = 0.5$ *                                                                   | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                 | Using the model and $e = \text{sep. speed} / \text{app. speed}$ ,<br>$v = 0.5\sqrt{1.2g}$                 | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | Using the model and $v^2 = u^2 + 2as$                                                                     | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                            | $0^2 = 0.25(1.2g) - 2gh \Rightarrow h = 0.15 \text{ (m)}$                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                 | Ball continues to bounce with the height of each bounce being a quarter of the previous one               | B1         | 2.2b |
|                                                                                                                                                                                                                                                                                                                                            |                                                                                                           | <b>(1)</b> |      |
| <b>(10 marks)</b>                                                                                                                                                                                                                                                                                                                          |                                                                                                           |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                     |                                                                                                           |            |      |
| <b>(a)</b><br><b>M1:</b> For a complete method to find $v$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For a complete method to find $u$<br><b>A1:</b> For a correct value (may be numerical)<br><b>M1:</b> For finding both $v$ and $u$ and use of Newton's Law of Restitution<br><b>A1*:</b> For the given answer |                                                                                                           |            |      |
| <b>(b)</b><br><b>M1:</b> For use of Newton's Law of Restitution to find rebound speed<br><b>M1:</b> For a complete method to find $h$<br><b>A1:</b> For 0.15 (m) oe                                                                                                                                                                        |                                                                                                           |            |      |
| <b>(c)</b><br><b>B1:</b> For a clear description including reference to a quarter                                                                                                                                                                                                                                                          |                                                                                                           |            |      |

| Question                                                                                                                                           | Scheme                                                   | Marks      | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|------------|------|
| <b>2(a)</b>                                                                                                                                        | Energy Loss = KE Loss – PE Gain                          | M1         | 3.3  |
|                                                                                                                                                    | $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5 g \times 20$ | A1         | 1.1b |
|                                                                                                                                                    | $= 58.25 = 58 \text{ (J ) or } 58.3 \text{ (J)}$         | A1         | 1.1b |
|                                                                                                                                                    |                                                          | <b>(3)</b> |      |
| <b>(b)</b>                                                                                                                                         | Using work-energy principle, $20 R = 58.25$              | M1         | 3.3  |
|                                                                                                                                                    | $R = 2.9125 = 2.9 \text{ or } 2.91$                      | A1ft       | 1.1b |
|                                                                                                                                                    |                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                         | Make resistance variable (dependent on speed)            | B1         | 3.5c |
|                                                                                                                                                    |                                                          | <b>(1)</b> |      |
| <b>(6 marks)</b>                                                                                                                                   |                                                          |            |      |
| Notes:                                                                                                                                             |                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For a difference in KE and PE<br><b>A1:</b> For a correct expression<br><b>A1:</b> For either 58 (2sf) or 58.3(3sf)       |                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of work-energy principle<br><b>A1ft:</b> For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a) |                                                          |            |      |
| <b>(c)</b><br><b>B1:</b> For variable resistance oe                                                                                                |                                                          |            |      |

| Question                                                                                                                                                                                                                                                            | Scheme                                          | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|-------|------|
| <b>3(a)</b>                                                                                                                                                                                                                                                         | Force = Resistance (since no acceleration) = 30 | B1    | 3.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 30 $\times$ 4    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 120 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (3)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                          | Resolving parallel to the slope                 | M1    | 3.1b |
|                                                                                                                                                                                                                                                                     | $F - 60g\sin\alpha - 30 = 0$                    | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | $F = 70$                                        | A1    | 1.1b |
|                                                                                                                                                                                                                                                                     | Power = Force $\times$ Speed = 70 $\times$ 3    | M1    | 1.1b |
|                                                                                                                                                                                                                                                                     | = 210 W                                         | A1 ft | 1.1b |
|                                                                                                                                                                                                                                                                     |                                                 | (5)   |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                    |                                                 |       |      |
| Notes:                                                                                                                                                                                                                                                              |                                                 |       |      |
| <b>(a)</b><br><b>B1:</b> For force = 30 seen<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 120 (W), follow through on their '30'                                                                                                                            |                                                 |       |      |
| <b>(b)</b><br><b>M1:</b> For resolving parallel to the slope with correct no. of terms and 60g resolved<br><b>A1:</b> For a correct equation<br><b>A1:</b> For $F = 70$<br><b>M1:</b> For use of $P = Fv$<br><b>A1ft:</b> For 210 (W), follow through on their '70' |                                                 |       |      |

| Question                                                                                                                                                                                                                                                                                                                                                                                  | Scheme                                                                                                                   | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|------------|------|
| <b>4(a)</b>                                                                                                                                                                                                                                                                                                                                                                               | Use of conservation of momentum                                                                                          | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                           | $3mu - 2mu = 3mv + mw$                                                                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Use of NLR                                                                                                               | M1         | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                           | $3ue = -v + w$                                                                                                           | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $v = \frac{u}{4}(1 - 3e)$                                                                                                | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(6)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                | $\frac{u}{4}(1 - 3e) < 0$                                                                                                | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $\frac{1}{3} < e \leq 1$                                                                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(2)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                | Solving for $w$                                                                                                          | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                           | $w = \frac{u}{4}(1 + 9e)^*$                                                                                              | A1 *       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                | Substitute $e = \frac{5}{9}$                                                                                             | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $v = -\frac{u}{6}, w = \frac{3u}{2}$                                                                                     | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Use NLR for impact with wall, $x = fw$                                                                                   | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | Further collision if $x > -v$                                                                                            | M1         | 3.4  |
|                                                                                                                                                                                                                                                                                                                                                                                           | $f\frac{3u}{2} > \frac{u}{6}$                                                                                            | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           | $1 \geq f > \frac{1}{9}$                                                                                                 | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                          | <b>(6)</b> |      |
| <b>(16 marks)</b>                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                          |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                          |            |      |
| <b>(a)</b><br><b>M1:</b> For use of CLM, with correct no. of terms, condone sign errors<br><b>A1:</b> For a correct equation<br><b>M1:</b> For use of Newton's Law of Restitution, with $e$ on the correct side<br><b>A1:</b> For a correct equation<br><b>M1:</b> For setting up <i>two</i> equations and solving their equations for $v$<br><b>A1:</b> For a correct expression for $v$ |                                                                                                                          |            |      |
| <b>(b)</b><br><b>M1:</b> For use of an appropriate inequality<br><b>A1:</b> For a complete range of values of $e$                                                                                                                                                                                                                                                                         |                                                                                                                          |            |      |
| <b>(c)</b><br><b>M1:</b> For solving their equations for $w$<br><b>A1:</b> For the given answer                                                                                                                                                                                                                                                                                           |                                                                                                                          |            |      |

Question 4 notes continued:

**(d)**

**M1:** For substituting  $e = \frac{5}{9}$  into their  $v$  and  $w$

**A1:** For correct expressions for  $v$  and  $w$

**M1:** For use of Newton's Law of Restitution, with  $e$  on the correct side

**M1:** For use of appropriate inequality

**A1:** For a correct inequality

**A1:** For a correct range

### Further Mechanics 2 Mark Scheme (Section B)

| Question                                                                                                                      | Scheme                                                                                      | Marks      | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|------------|------|
| <b>5 (a)</b>                                                                                                                  | Multiply out and differentiate wrt $t$                                                      | M1         | 1.1b |
|                                                                                                                               | $v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$                                               | A1         | 1.1b |
|                                                                                                                               |                                                                                             | <b>(2)</b> |      |
| <b>(b)</b>                                                                                                                    | Multiply out and integrate wrt $t$                                                          | M1         | 1.1b |
|                                                                                                                               | $s = \int 3t^2 - 16t + 20 dt = t^3 - 8t^2 + 20t (+C)$                                       | A1         | 1.1b |
|                                                                                                                               | $t = 0, s = 0 \Rightarrow C = 0$                                                            | A1         | 1.1b |
|                                                                                                                               | $t = 2, s = 8 - 32 + 40 = 16$                                                               |            |      |
|                                                                                                                               |                                                                                             | <b>(3)</b> |      |
| <b>(c)</b>                                                                                                                    | $s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$       | M1         | 2.1  |
|                                                                                                                               | Explanation to show that $t^2 - 8t + 20 > 0$ for all $t$ .                                  | M1         | 2.4  |
|                                                                                                                               | So $s = 0$ has no non-zero solutions, so $s$ is never zero again, so never returns to $O^*$ | A1*        | 3.2a |
|                                                                                                                               |                                                                                             | <b>(3)</b> |      |
| <b>(8 marks)</b>                                                                                                              |                                                                                             |            |      |
| Notes:                                                                                                                        |                                                                                             |            |      |
| <b>(a)</b>                                                                                                                    |                                                                                             |            |      |
| <b>M1:</b> For multiplying out and differentiating (powers decreasing by 1)                                                   |                                                                                             |            |      |
| <b>A1:</b> For a correct expression for $a$                                                                                   |                                                                                             |            |      |
| <b>(b)</b>                                                                                                                    |                                                                                             |            |      |
| <b>M1:</b> For multiplying out and integrating (powers increasing by 1)                                                       |                                                                                             |            |      |
| <b>A1:</b> For a correct expression for $s$ with or without $C$                                                               |                                                                                             |            |      |
| <b>A1:</b> For $C = 0$ and correct final answer                                                                               |                                                                                             |            |      |
| <b>(c)</b>                                                                                                                    |                                                                                             |            |      |
| <b>M1:</b> For equating their $s$ to 0 and producing a quadratic                                                              |                                                                                             |            |      |
| <b>M1:</b> For clear explanation that $t^2 - 8t + 20 > 0$ for all $t$ (e.g. completing the square or another complete method) |                                                                                             |            |      |
| <b>A1*:</b> For a correct conclusion in context                                                                               |                                                                                             |            |      |



| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Scheme                                                                                 | Marks       | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|-------------|------|
| <b>6(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\cos \alpha = \frac{4}{5}$ or $\sin \alpha = \frac{3}{5}$                             | B1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $r = 4a \sin \alpha$                                                                   | B1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Resolving vertically                                                                   | M1          | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $T_1 \cos \alpha - T_2 \sin \alpha = mg$                                               | A1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Resolving horizontally                                                                 | M1          | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $T_1 \sin \alpha + T_2 \cos \alpha = m r \omega^2$                                     | A1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $T_1 \sin \alpha + T_2 \cos \alpha = m r \omega^2$                                     | A1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Solving for either tension                                                             | M1          | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$ *                                               | A1*         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $T_2 = \frac{3m}{25}(16a\omega^2 - 5g)$ *                                              | A1*         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                        | <b>(10)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $\frac{4m}{25}(9a\omega^2 + 5g) < 4mg$                                                 | M1          | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\frac{3m}{25}(16a\omega^2 - 5g) > 0$                                                  | M1          | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $\omega > \sqrt{\frac{5g}{16a}}$ or $\omega < \sqrt{\frac{20g}{9a}}$                   | A1          | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $S = \frac{2\pi}{\omega}$                                                              | M1          | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$ *                              | A1*         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                        | <b>(5)</b>  |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | String being light implies that the tension is constant in both portions of the string | B1          | 3.5b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                        | <b>(1)</b>  |      |
| <b>(16 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                        |             |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                        |             |      |
| <b>(a)</b><br><b>B1:</b> For correct trig. ratio seen<br><b>B1:</b> For a correct radius expression seen<br><b>M1:</b> For resolving vertically with correct no. of terms and tensions resolved<br><b>A1:</b> For a correct equation<br><b>M1:</b> For resolving horizontally with correct no. of terms and tensions resolved<br><b>A1A1:</b> For a correct equation<br><b>M1:</b> For solving their two equations to find either tension<br><b>A1*:</b> For the given answer<br><b>A1*:</b> For the given answer |                                                                                        |             |      |

|                             |                                                                 |
|-----------------------------|-----------------------------------------------------------------|
| Question 6 notes continued: |                                                                 |
| <b>(b)</b>                  |                                                                 |
| <b>M1:</b>                  | For use of $T_1 < 4mg$                                          |
| <b>M1:</b>                  | For using $T_2 > 0$                                             |
| <b>A1:</b>                  | For a correct inequality (either) for $\omega$                  |
| <b>M1:</b>                  | For use of $S = \frac{2\pi}{\omega}$ with either critical value |
| <b>A1*:</b>                 | For given answer                                                |
| <b>(c)</b>                  |                                                                 |
| <b>B1:</b>                  | For a clear explanation                                         |

| Question                                                                                                                                                                                                                                                                                                                                       | Scheme                                                                                             | Marks      | AOs  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|------------|------|
| <b>7(a)</b>                                                                                                                                                                                                                                                                                                                                    | Rel. Mass:      2      5      1      8                                                             | B1         | 1.2  |
|                                                                                                                                                                                                                                                                                                                                                | $y:$ 2      0.5      1.5 $\bar{y}$                                                                 | B1         | 1.2  |
|                                                                                                                                                                                                                                                                                                                                                | $x:$ 0.5      2.5      4.5 $\bar{x}$                                                               | B1         | 1.2  |
|                                                                                                                                                                                                                                                                                                                                                | $(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$                                        | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                | $\bar{y} = 1 *$                                                                                    | A1*        | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                | $(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$                                      | M1         | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                | $\bar{x} = 2.25$                                                                                   | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                    | <b>(7)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                     | Use of correct strategy to solve the problem by use of 'moments equation'                          | M1         | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                | $(8 \times 2.25) - (2\pi r^2 \times 0.5) = (8 - 2\pi r^2)2.5$                                      | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                | Solving for $r$                                                                                    | M1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                | $r = \frac{1}{\sqrt{2\pi}} = 0.399$                                                                | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                    | <b>(4)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                     | Since $\bar{y}$ for original plate is 1, holes must be symmetrically placed about the line $y = 1$ | B1         | 2.4  |
|                                                                                                                                                                                                                                                                                                                                                | $a = 1.5$                                                                                          | B1         | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                    | <b>(2)</b> |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                     | Use of tan from an appropriate triangle                                                            | M1         | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                | $\tan \alpha = \frac{2}{1.5} = \frac{4}{3}$                                                        | A1ft       | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                | $\alpha = 53.1^\circ$                                                                              | A1         | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                    | <b>(3)</b> |      |
| <b>(16 marks)</b>                                                                                                                                                                                                                                                                                                                              |                                                                                                    |            |      |
| Notes:                                                                                                                                                                                                                                                                                                                                         |                                                                                                    |            |      |
| <b>(a)</b><br><b>B1:</b> For correct relative masses<br><b>B1:</b> For correct $y$ values<br><b>B1:</b> For correct $x$ values<br><b>M1:</b> For a moments equation, correct no. of terms, condone sign errors<br><b>A1*:</b> For a correct given answer (1)<br><b>M1:</b> For a moments equation, correct no. of terms<br><b>A1:</b> For 2.25 |                                                                                                    |            |      |
| <b>(b)</b><br><b>M1:</b> For a moments equation, correct no. of terms, condone sign errors<br><b>A1ft:</b> For a correct equation, follow through on their $\bar{x}$<br><b>M1:</b> For solving for $r$<br><b>A1:</b> For 0.399 or 0.40                                                                                                         |                                                                                                    |            |      |

|                             |                                                     |
|-----------------------------|-----------------------------------------------------|
| Question 7 notes continued: |                                                     |
| (c)                         |                                                     |
| <b>B1:</b>                  | For consideration of symmetry about $y = 1$         |
| <b>B1:</b>                  | For $a = 1.5$                                       |
| (d)                         |                                                     |
| <b>M1:</b>                  | For use of tan from an appropriate triangle         |
| <b>A1ft:</b>                | For a correct equation, follow through on their $a$ |
| <b>A1:</b>                  | For a correct angle                                 |

# Pearson Edexcel Level 3 GCE

## Further Mathematics

Advanced Subsidiary

Further Mathematics options

Paper 2K: Decision Mathematics 1 and  
Decision Mathematics 2

Sample Assessment Material for first teaching September 2017

**Time: 1 hour 40 minutes**

Paper Reference

**8FM0/2K**

### You must have:

Decision Mathematics Answer Book (enclosed), calculator

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If a pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.
- Do not return this question paper with the answer book.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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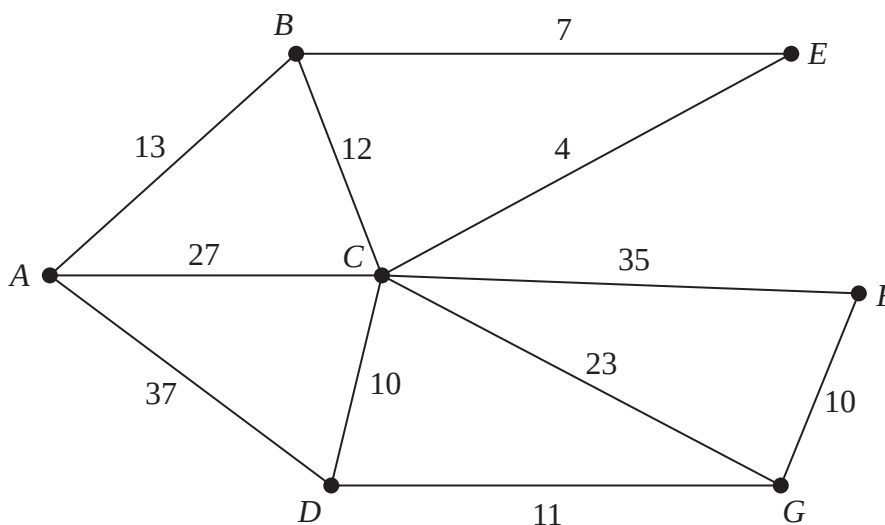
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## SECTION A

Answer ALL questions. Write your answers in the answer book provided.

1.



**Figure 1**

[The total weight of the network is 189]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

- (a) Use Dijkstra's algorithm to find the shortest path from  $A$  to  $F$ . State the path and its length. (5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at  $A$ , needs to be found.

- (b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear. (4)

- (c) State the minimum length of Gabriel's route. (1)

A new pipe,  $BG$ , is added to the network. A route of minimum length that traverses each pipe, including  $BG$ , needs to be found. The route must start and finish at  $A$ .

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of  $BG$ .

- (d) Calculate the length of  $BG$ . You must show your working. (2)

(Total for Question 1 is 12 marks)

2.

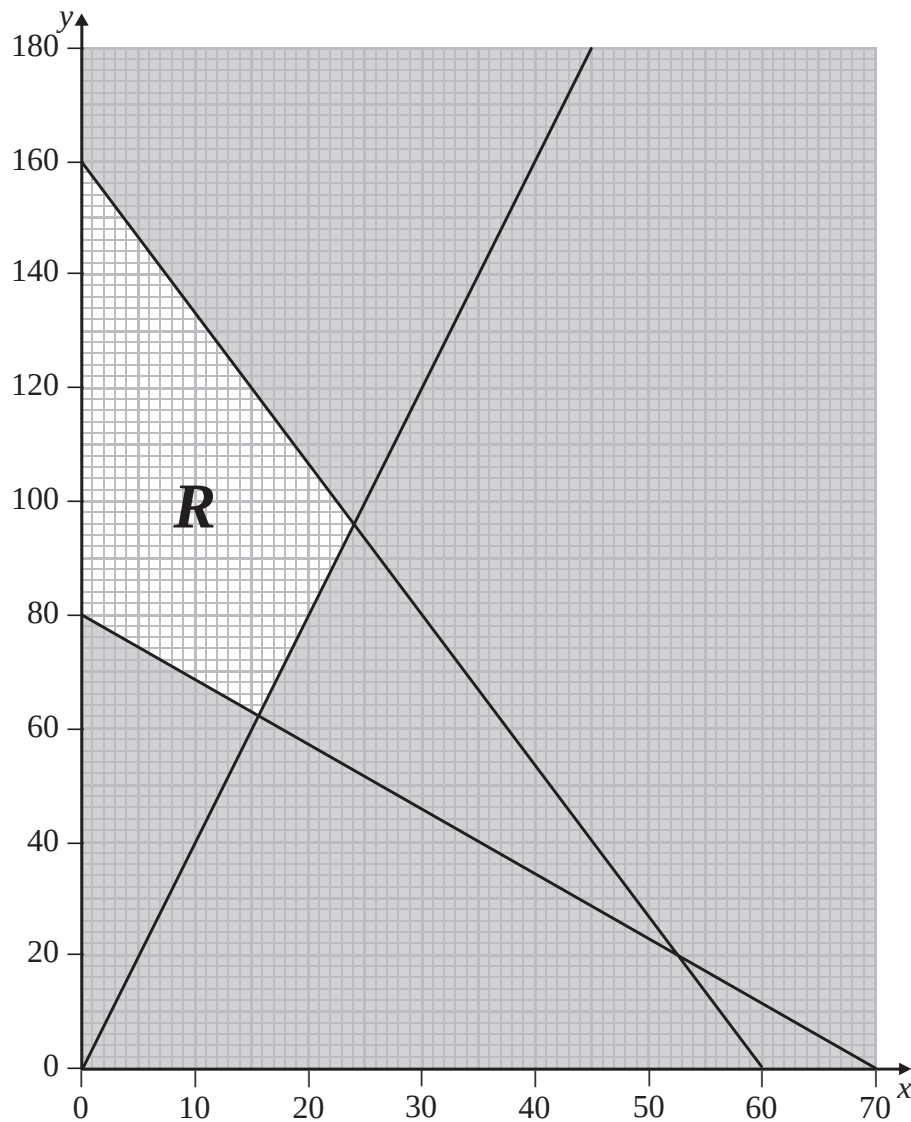


Figure 2

A teacher buys pens and pencils. The number of pens,  $x$ , and the number of pencils,  $y$ , that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 2 is 7 marks)

3.

| Activity | Time taken (days) | Immediately preceding activities |
|----------|-------------------|----------------------------------|
| A        | 5                 | –                                |
| B        | 7                 | –                                |
| C        | 3                 | –                                |
| D        | 4                 | A, B                             |
| E        | 4                 | D                                |
| F        | 2                 | B                                |
| G        | 4                 | B                                |
| H        | 5                 | C, G                             |
| I        | 10                | C, G                             |

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only. (3)
- (b) (i) Show that the project can be completed in 21 days, showing your working.
- (ii) Identify the critical activities. (4)

(Total for Question 3 is 7 marks)



4. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5 (1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are  $2^{2x} - 1$ ,  $2^x$ ,  $x + 1$ ,  $2^{x+1} - 3$  and  $11 - x$ .

- (b) (i) Calculate  $x$ .  
(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer. (6)

- (c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian. (2)

(Total for Question 4 is 9 marks)

5. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let  $x$  and  $y$  represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 5 is 5 marks)

**TOTAL FOR SECTION A IS 40 MARKS**

**SECTION B**

**Answer ALL questions. Write your answers in the answer book provided.**

6. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

|   | P  | Q  | R  | S  | T  |
|---|----|----|----|----|----|
| A | 32 | 32 | 35 | 34 | 33 |
| B | 28 | 35 | 31 | 37 | 40 |
| C | 35 | 29 | 33 | 36 | 35 |
| D | 36 | 30 | 34 | 33 | 35 |
| E | 30 | 31 | 29 | 37 | 36 |
| F | 29 | 28 | 32 | 31 | 34 |

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

**(Total for Question 6 is 9 marks)**

7. In two-dimensional space, lines divide a plane into a number of different regions.

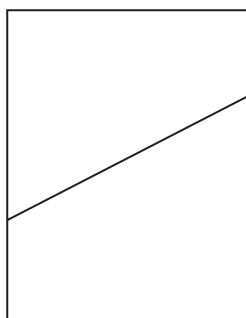


Figure 1

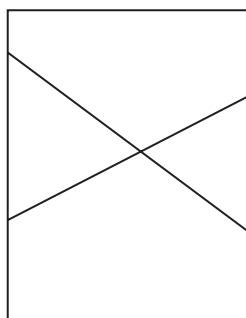


Figure 2

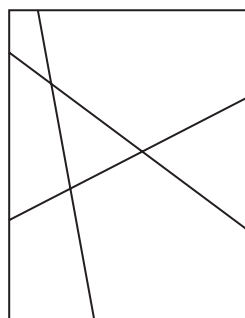


Figure 3

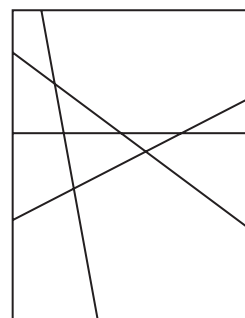


Figure 4

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of  $u_n$ , the recurrence relation for  $u_{n+1}$ , the maximum number of regions when a plane is divided by  $(n + 1)$  lines where  $n \geq 1$

(1)

(c) (i) Solve the recurrence relation for  $u_n$

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 7 is 5 marks)

8.

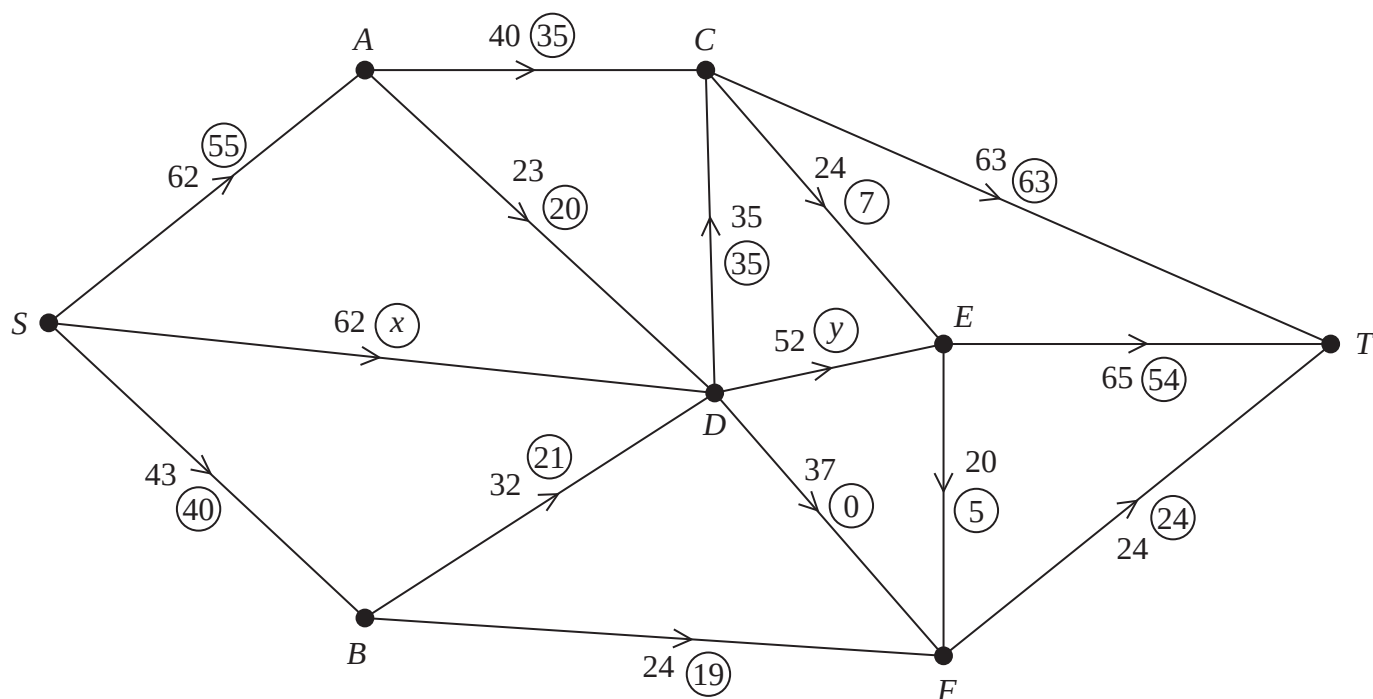


Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11 am on Friday morning, all students leave the hall (S) after assembly and travel to the cybercafé (T). The numbers in circles represent the initial flow of students recorded at 11 am one Friday.

- (a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network. (1)

- (b) Find the value of  $x$  and the value of  $y$ , explaining your reasoning. (3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11 am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

- (c) (i) List all the flow augmenting routes from S to T that increase the flow by at least 5  
(ii) State which route the new students should take, giving a reason for your answer. (3)

- (d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book. (1)

- (e) Prove that the answer to part (d) is optimal. (3)

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

- (f) State, explaining your reasoning,
- (i) which corridor they should widen,
  - (ii) the resulting increase of flow through the network.

(3)

**(Total for Question 8 is 14 marks)**

9. A two person zero-sum game is represented by the following pay-off matrix for player A.

|                  | <i>B</i> plays 1 | <i>B</i> plays 2 | <i>B</i> plays 3 |
|------------------|------------------|------------------|------------------|
| <i>A</i> plays 1 | 4                | 1                | 2                |
| <i>A</i> plays 2 | 2                | 4                | 3                |

- (a) Verify that there is no stable solution. (3)
- (b) (i) Find the best strategy for player A.
- (ii) Find the value of the game to her. (9)

(Total for Question 9 is 12 marks)

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**





Write your name here

Surname

Other names

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# Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**Paper 2K: Decision Mathematics 1 and  
Decision Mathematics 2**

Sample Assessment Material for first teaching September 2017

Paper Reference

**8FM0/2K**

**Answer Book**

Do not return the question paper with the answer book.

Total Marks

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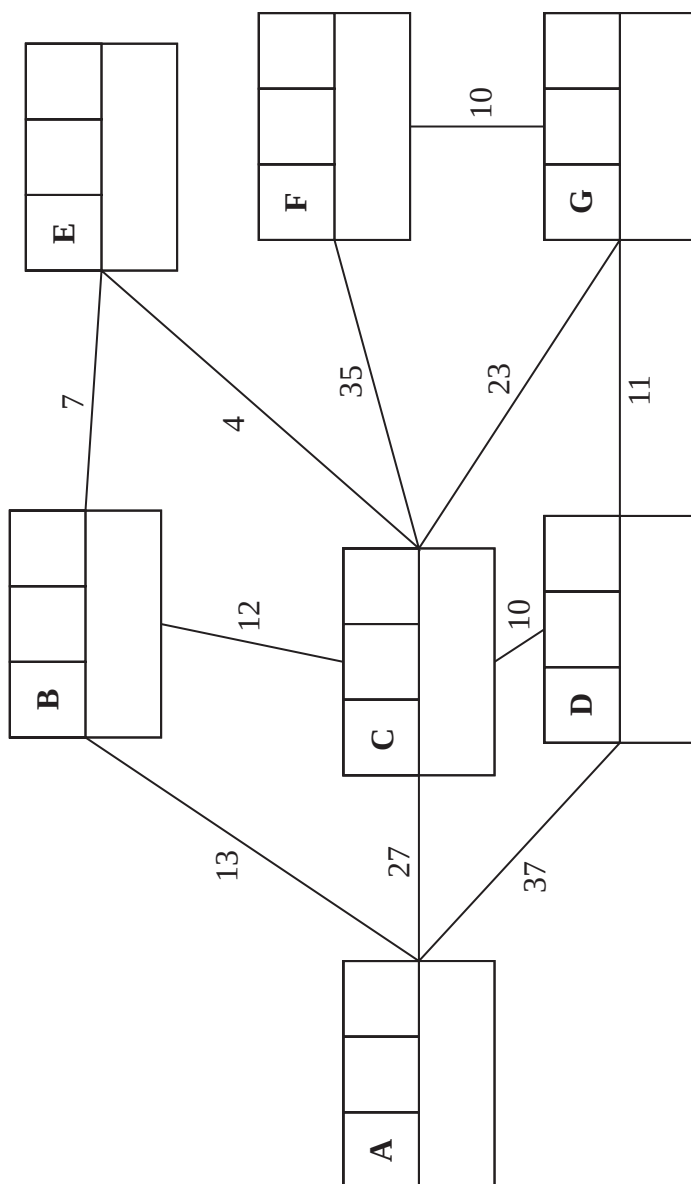
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1.

# SECTION A



Key:

| Vertex        | Order of labelling | Final values |
|---------------|--------------------|--------------|
| Working value |                    |              |

Shortest path: \_\_\_\_\_

Length of shortest path: \_\_\_\_\_

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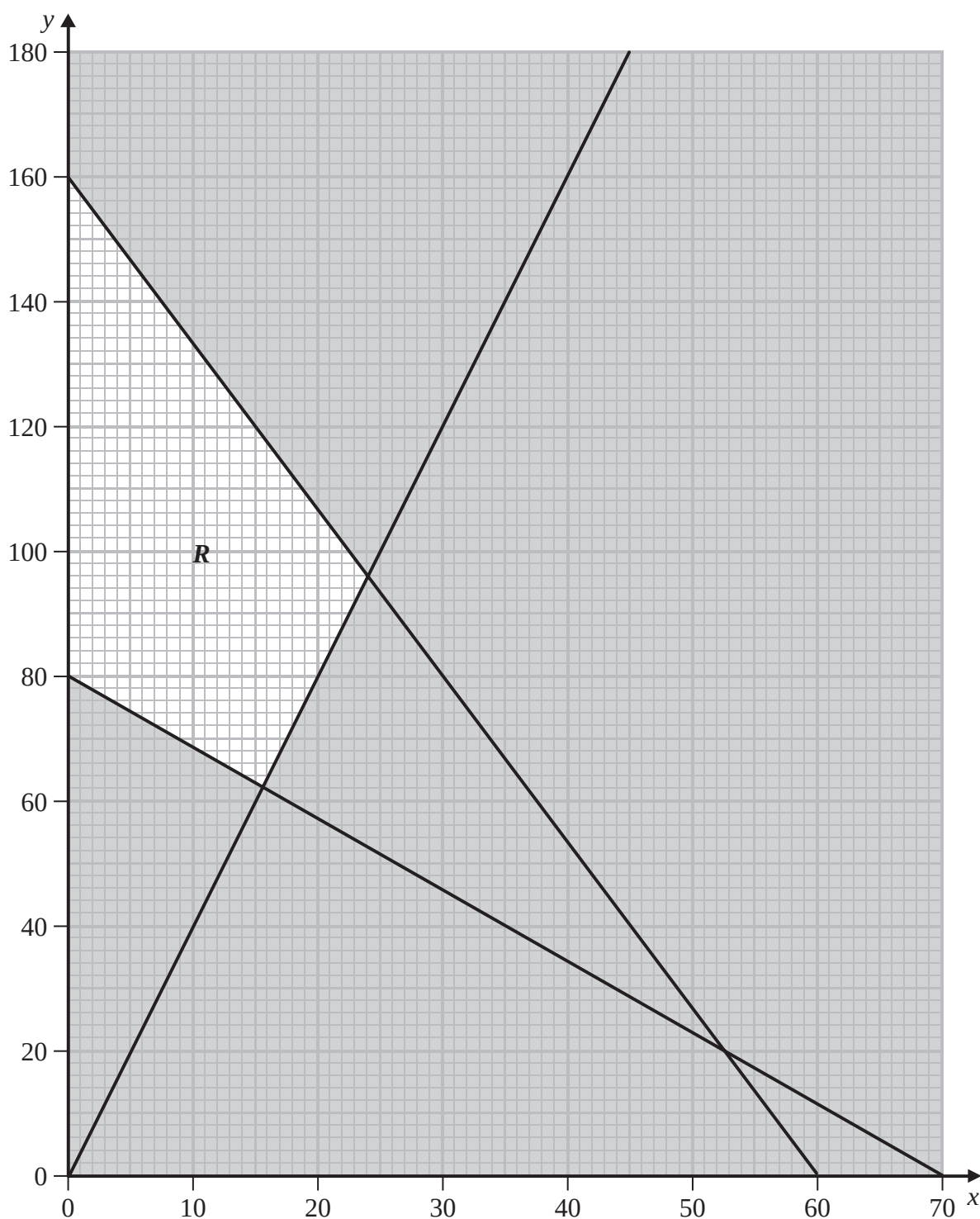
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(Total for Question 1 is 12 marks)

2.



**DO NOT WRITE IN THIS AREA**

3. (a) and (b)

DO NOT WRITE IN THIS AREA

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DO NOT WRITE IN THIS AREA

(Total for Question 3 is 7 marks)

4.

(Total for Question 4 is 9 marks)

5.

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 5 marks)

**TOTAL FOR SECTION A IS 40 MARKS**



## SECTION B

**6.**

|   | P  | Q  | R  | S  | T  |
|---|----|----|----|----|----|
| A | 32 | 32 | 35 | 34 | 33 |
| B | 28 | 35 | 31 | 37 | 40 |
| C | 35 | 29 | 33 | 36 | 35 |
| D | 36 | 30 | 34 | 33 | 35 |
| E | 30 | 31 | 29 | 37 | 36 |
| F | 29 | 28 | 32 | 31 | 34 |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

|   | P | Q | R | S | T |  |
|---|---|---|---|---|---|--|
| A |   |   |   |   |   |  |
| B |   |   |   |   |   |  |
| C |   |   |   |   |   |  |
| D |   |   |   |   |   |  |
| E |   |   |   |   |   |  |
| F |   |   |   |   |   |  |
|   |   |   |   |   |   |  |

(Total for Question 6 is 9 marks)

7. (a)

|                           |   |   |   |    |   |   |   |
|---------------------------|---|---|---|----|---|---|---|
| Number of lines           | 1 | 2 | 3 | 4  | 5 | 6 | 7 |
| Maximum number of regions | 2 | 4 | 7 | 11 |   |   |   |

(Total for Question 7 is 5 marks)

8.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(d)

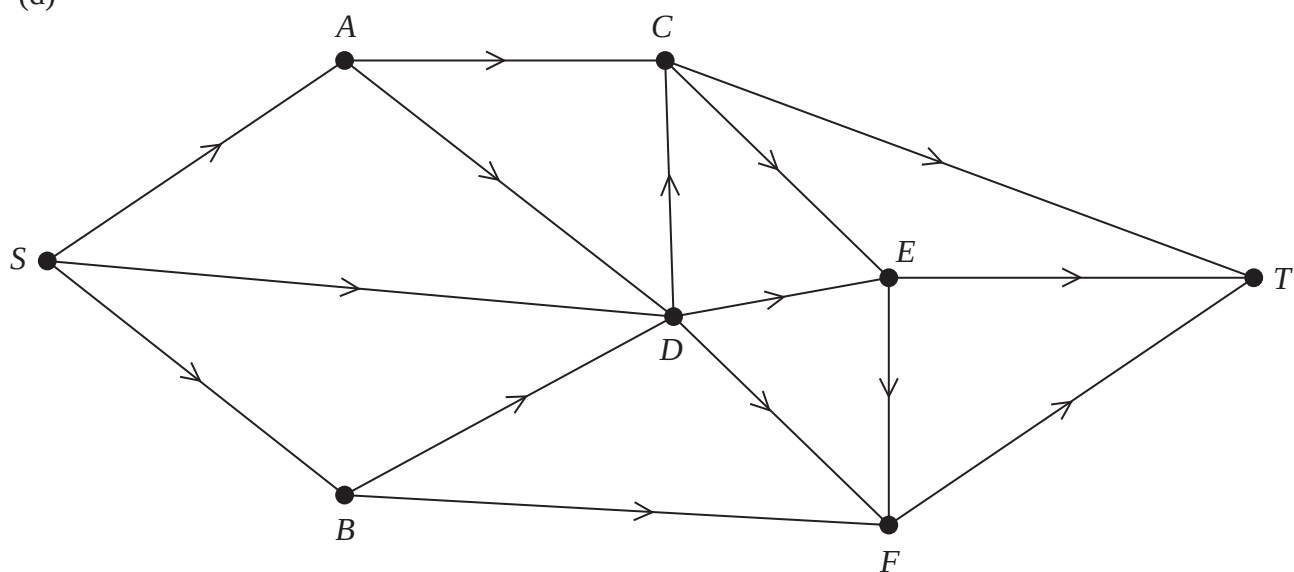


Diagram 1

(Total for Question 8 is 14 marks)

9.

|                  | <i>B</i> plays 1 | <i>B</i> plays 2 | <i>B</i> plays 3 |
|------------------|------------------|------------------|------------------|
| <i>A</i> plays 1 | 4                | 1                | 2                |
| <i>A</i> plays 2 | 2                | 4                | 3                |

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 9 continued**

**(Total for Question 9 is 12 marks)**

**TOTAL FOR SECTION B IS 40 MARKS**  
**TOTAL FOR PAPER IS 80 MARKS**





## Paper 2 Option K

### Decision Mathematics 1 Mark Scheme (Section A)

| Question                                                                                                                                                                                                                                                                                                                                                       | Scheme                              | Marks | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|-------|------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                                    |                                     | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Path: ABECDGF                       | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Length: 55 (metres)                 | A1ft  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | (5)   |      |
|                                                                                                                                                                                                                                                                                                                                                                |                                     |       |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                     | $AB + DG = 13 + 11 = 24 \leftarrow$ | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BEC)D + B(ECD)G = 34 + 32 = 66$  | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $A(BECD)G + B(EC)D = 45 + 21 = 66$  | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                | Repeat arcs: AB, DG                 | A1ft  | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | (4)   |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                     | Length = $189 + 24 = 213$ (metres)  | B1ft  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | (1)   |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                     | $189 + x + 34 = 213 + 2x$           | M1    | 3.1b |
|                                                                                                                                                                                                                                                                                                                                                                | $x = 10$ so BG is 10 m              | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                |                                     | (2)   |      |
| <b>(12 marks)</b>                                                                                                                                                                                                                                                                                                                                              |                                     |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                         |                                     |       |      |
| <b>(a)</b><br><b>M1:</b> For a larger number replaced by a smaller one in the working values boxes at C, D, F or G<br><b>A1:</b> For all values correct (and in correct order) at A, B, C and D<br><b>A1:</b> For all values correct (and in correct order) at E, F & G<br><b>A1:</b> For the correct path<br><b>A1ft:</b> For 55 or ft their final value at F |                                     |       |      |
| <b>(b)</b><br><b>M1:</b> For 3 correct pairings of the four odd nodes (A,B, D & G)<br><b>A1:</b> At least two pairings and totals correct<br><b>A2:</b> All three pairings and totals correct<br><b>A3ft:</b> Selecting their shortest pairing, and stating that these arcs should be repeated                                                                 |                                     |       |      |

Question 1 notes continued:

(c)

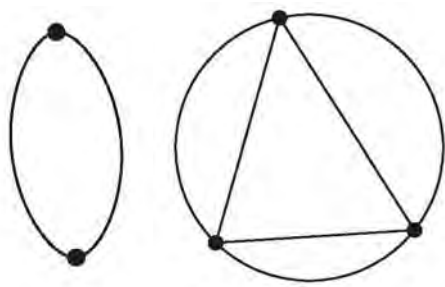
**B1ft:** For 213 or 189 + their shortest repeat

**M1:** For translating the information in the question in to an equation involving  $x$ ,  $2x$  and 34

**A1:** For a correct equation leading to  $BG = 10$  (m)

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Scheme                                                                                                                        | Marks | AOs  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Objective line drawn or at least two vertices tested                                                                          | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | For solving $y = 4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$ | M1    | 1.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Finding at least two points with integer co-ordinates from $(15 \pm 1, 63 \pm 2)$                                             | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Testing at least two points with integer co-ordinates                                                                         | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $x = 15$ and $y = 63$                                                                                                         | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | So the teacher should buy 15 pens and 63 pencils                                                                              | A1ft  | 3.2a |
| (7 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                               |       |      |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                               |       |      |
| <p><b>M1:</b> Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C</p> <p><b>M1:</b> Solving simultaneous equations</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of <math>y \geq 4x</math> and <math>8x + 7y \geq 560</math></p> <p><b>M1:</b> Testing at least two integer solutions in <math>y \geq 4x</math> or <math>8x + 7y \geq 560</math> and C</p> <p><b>A1:</b> cao – deducing from tests which integer solution is both valid and optimal</p> <p><b>A1ft:</b> Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils</p> |                                                                                                                               |       |      |

| Question  | Scheme                                                                                                                                                                                                                                                                        | Marks | AOs  |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 3(a)(b)   | <p>The number(s) at the end of activity E indicate this project can be completed in 21 days</p> <p>Critical activities: B, G, I</p>                                                                                                                                           | M1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | A1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | A1    | 1.1b |
|           |                                                                                                                                                                                                                                                                               | (3)   |      |
|           |                                                                                                                                                                                                                                                                               | M1    | 2.1  |
| A1        | 1.1b                                                                                                                                                                                                                                                                          |       |      |
| A1ft      | 2.2a                                                                                                                                                                                                                                                                          |       |      |
| A1        | 1.1b                                                                                                                                                                                                                                                                          |       |      |
| (4)       |                                                                                                                                                                                                                                                                               |       |      |
| (7 marks) |                                                                                                                                                                                                                                                                               |       |      |
| Notes:    |                                                                                                                                                                                                                                                                               |       |      |
| M1:       | At least 5 activities and one dummy, one start                                                                                                                                                                                                                                |       |      |
| A1:       | A,B,C,D,F,G and first dummy correct                                                                                                                                                                                                                                           |       |      |
| A1:       | E,H,I correct, second dummy correct and one finish                                                                                                                                                                                                                            |       |      |
| M1:       | All boxes completed, number generally increasing L to R (condone one “rogue”)                                                                                                                                                                                                 |       |      |
| A1:       | All values cao                                                                                                                                                                                                                                                                |       |      |
| A1:       | Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes) |       |      |
| A1:       | Critical activities correct                                                                                                                                                                                                                                                   |       |      |

| Question                                                                         | Scheme                                                                                                                        | Marks | AOs  |
|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>4(a)</b>                                                                      | e.g. a graph cannot contain an odd number of odd nodes<br>e.g. number of arcs = $\frac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$ | B1    | 2.4  |
|                                                                                  |                                                                                                                               | (1)   |      |
| <b>(b)(i)</b>                                                                    | $(2^{2x}-1)+(2^x)+(x+1)+(2^{x+1}-3)+(11-x)=2(18)$                                                                             | M1    | 1.1b |
|                                                                                  | $2^{2x}+3(2^x)-28=0 \Rightarrow x=...$                                                                                        | M1    | 1.1b |
|                                                                                  | $(2^x+7)(2^x-4)=0 \Rightarrow x=2$                                                                                            | A1    | 1.1b |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(b)(ii)</b>                                                                   | The order of the nodes are 9, 15, 3, 4, 5                                                                                     | M1    | 2.1  |
|                                                                                  | Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes                                | A1    | 2.4  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (3)   |      |
| <b>(c)</b>                                                                       |                                             | M1    | 2.5  |
|                                                                                  |                                                                                                                               | A1    | 2.2a |
|                                                                                  |                                                                                                                               | (2)   |      |
| <b>(9 marks)</b>                                                                 |                                                                                                                               |       |      |
| Notes:                                                                           |                                                                                                                               |       |      |
| <b>(a)</b>                                                                       |                                                                                                                               |       |      |
| <b>B1:</b> Explanation referring to need for an even number of odd nodes oe      |                                                                                                                               |       |      |
| <b>(b)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Forming an equation involving the orders of the 5 odd nodes and 2(18) |                                                                                                                               |       |      |
| <b>M1:</b> Simplifies to a quadratic in $2^x$ and attempts to solve              |                                                                                                                               |       |      |
| <b>A1:</b> 2 cao                                                                 |                                                                                                                               |       |      |
| <b>M1:</b> Construct an argument involving the order of the 5 nodes              |                                                                                                                               |       |      |
| <b>A1:</b> Explanation considering the number of odd nodes                       |                                                                                                                               |       |      |
| <b>A1:</b> Deduction that therefore it is neither Eulerian nor semi-Eulerian     |                                                                                                                               |       |      |
| <b>(c)</b>                                                                       |                                                                                                                               |       |      |
| <b>M1:</b> Interprets mathematical language to construct a disconnected graph    |                                                                                                                               |       |      |
| <b>A1:</b> Deduce a correct graph                                                |                                                                                                                               |       |      |

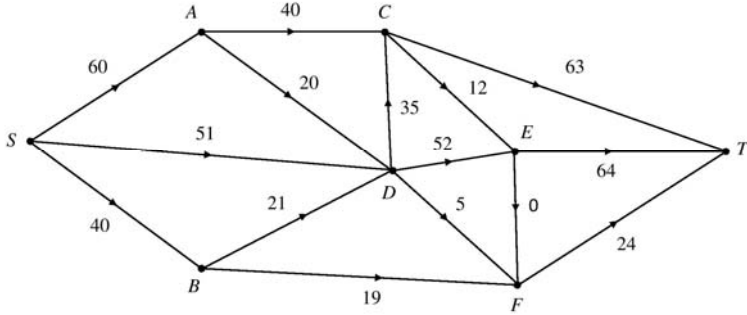
| Question                                                                                                                                                                                                                                                                                                                         | Scheme                                                                      | Marks    | AOs        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|----------|------------|
| 5                                                                                                                                                                                                                                                                                                                                | Minimise ( $C =$ ) $25x + 35y$                                              | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | Subject to:<br>$(500x + 800y \geq 150\,000 \Rightarrow) 5x + 8y \geq 1500$  | B1       | 3.3        |
|                                                                                                                                                                                                                                                                                                                                  | $\frac{7}{20}(x + y) \leq x \leq \frac{13}{20}(x + y)$                      | M1<br>M1 | 3.3<br>3.3 |
|                                                                                                                                                                                                                                                                                                                                  | Which simplifies to $7y \leq 13x$ <b>and</b> $13y \geq 7x$<br>$x, y \geq 0$ | A1       | 1.1b       |
| <b>(5 marks)</b>                                                                                                                                                                                                                                                                                                                 |                                                                             |          |            |
| Notes:                                                                                                                                                                                                                                                                                                                           |                                                                             |          |            |
| <b>B1:</b> A correct objective function + minimise<br><b>B1:</b> Translate information in to a correct inequality<br><b>M1:</b> For translating the information given into the LHS inequality<br><b>M1:</b> For translating the information given in to the RHS inequality<br><b>A1:</b> Simplifying to the correct inequalities |                                                                             |          |            |

# Decision Mathematics 2 Mark Scheme (Section B)

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Marks                    | AOs                          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|------------------------------|
| 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 32 & 32 & 35 & 34 & 33 & 40 \\ B & 28 & 35 & 31 & 37 & 40 & 40 \\ C & 35 & 29 & 33 & 36 & 35 & 40 \\ D & 36 & 30 & 34 & 33 & 35 & 40 \\ E & 30 & 31 & 29 & 37 & 36 & 40 \\ F & 29 & 28 & 32 & 31 & 34 & 40 \end{pmatrix}$                                                                                                                                                                                                                | B1                       | 1.1b                         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Reducing rows and then columns                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                          |                              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 2 & 1 & 8 \\ B & 0 & 7 & 3 & 9 & 12 & 12 \\ C & 6 & 0 & 4 & 7 & 6 & 11 \\ D & 6 & 0 & 4 & 3 & 5 & 10 \\ E & 1 & 2 & 0 & 8 & 7 & 11 \\ F & 1 & 0 & 4 & 3 & 6 & 12 \end{pmatrix}$ then $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 3 & 7 & 11 & 4 \\ C & 6 & 0 & 4 & 5 & 5 & 3 \\ D & 6 & 0 & 4 & 1 & 4 & 2 \\ E & 1 & 2 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 4 & 1 & 5 & 4 \end{pmatrix}$  | M1<br>A1                 | 1.1b<br>1.1b                 |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | e.g. augment by 1 then augment by 1                                                                                                                                                                                                                                                                                                                                                                                                                                                      | M1                       | 1.1b                         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 1 & 1 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 2 & 6 & 10 & 3 \\ C & 6 & 0 & 3 & 4 & 4 & 2 \\ D & 6 & 0 & 3 & 0 & 4 & 1 \\ E & 2 & 3 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 3 & 0 & 4 & 3 \end{pmatrix}$ followed by $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 2 & 2 & 3 & 1 & 0 & 0 \\ B & 0 & 7 & 1 & 6 & 9 & 2 \\ C & 6 & 0 & 2 & 4 & 3 & 1 \\ D & 6 & 0 & 2 & 0 & 3 & 0 \\ E & 3 & 4 & 0 & 7 & 6 & 3 \\ F & 1 & 0 & 2 & 0 & 3 & 2 \end{pmatrix}$ | A1ft<br>M1<br>A1ft<br>A1 | 1.1b<br>1.1b<br>1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | A – T, B – P, C – Q, (D – ), E – R, F – S                                                                                                                                                                                                                                                                                                                                                                                                                                                | A1                       | 2.2a                         |
| (9 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                          |                              |
| Notes:                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                          |                              |
| <p><b>B1:</b> cao – introducing a dummy task and appropriate value</p> <p><b>M1:</b> Simplifying the initial matrix by reducing rows and then columns</p> <p><b>A1:</b> cao</p> <p><b>M1:</b> Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed</p> <p><b>A1ft:</b> ft on their previous table – no errors</p> <p><b>M1:</b> Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)</p> <p><b>A1ft:</b> ft on their previous table – no errors</p> <p><b>A1:</b> cso on final table (so must have scored all previous marks)</p> <p><b>A1:</b> cso – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                          |                              |

| Question                                                                                                                                                                                                      | Scheme                                                                                                         | Marks      | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|------------|------|
| <b>7(a)</b>                                                                                                                                                                                                   | 16, 22, 29                                                                                                     | B1         | 1.1b |
|                                                                                                                                                                                                               |                                                                                                                | <b>(1)</b> |      |
| <b>(b)</b>                                                                                                                                                                                                    | $u_{n+1} = u_n + n + 1$                                                                                        | B1         | 3.3  |
|                                                                                                                                                                                                               |                                                                                                                | <b>(1)</b> |      |
| <b>(c)</b>                                                                                                                                                                                                    | As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$ | M1         | 1.1b |
|                                                                                                                                                                                                               | $u_n = \frac{1}{2}n(n+1) + 1$<br>20 101 (regions)                                                              | A1         | 1.1b |
|                                                                                                                                                                                                               |                                                                                                                | A1ft       | 1.1b |
|                                                                                                                                                                                                               |                                                                                                                | <b>(3)</b> |      |
| <b>(5 marks)</b>                                                                                                                                                                                              |                                                                                                                |            |      |
| Notes:                                                                                                                                                                                                        |                                                                                                                |            |      |
| <b>(a)</b><br><b>B1:</b> cao                                                                                                                                                                                  |                                                                                                                |            |      |
| <b>(b)</b><br><b>B1:</b> Translating problem to mathematical model - correct recurrence relation needed                                                                                                       |                                                                                                                |            |      |
| <b>(c)</b><br><b>M1:</b> An attempt to solve the recurrence relation to determine maximum number of regions<br><b>A1:</b> cao<br><b>A1ft:</b> Substitution of $n = 200$ into their quadratic $u_n$ expression |                                                                                                                |            |      |



| Question          | Scheme                                                                                                                                                                                                                                                                                                                                                                                                | Marks | AOs  |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>8(a)</b>       | Corridors must be one-way                                                                                                                                                                                                                                                                                                                                                                             | B1    | 3.4  |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (1)   |      |
| <b>(b)</b>        | e.g. $55 + x + 40 = 63 + 54 + 24$ <b>or</b> $7 + y = 54 + 5$                                                                                                                                                                                                                                                                                                                                          | M1    | 2.4  |
|                   | $x = 46$                                                                                                                                                                                                                                                                                                                                                                                              | A1    | 1.1b |
|                   | $y = 52$                                                                                                                                                                                                                                                                                                                                                                                              | A1    | 1.1b |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (3)   |      |
| <b>(c)</b>        | (i) SACET (= 5)<br>SDFET (= 5)                                                                                                                                                                                                                                                                                                                                                                        | M1    | 1.1b |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | A1    | 1.1b |
|                   | (ii) Students must choose SACET, as they cannot travel from F to E                                                                                                                                                                                                                                                                                                                                    | A1    | 2.2a |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (3)   |      |
| <b>(d)</b>        |                                                                                                                                                                                                                                                                                                                    | B1    | 1.1b |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (1)   |      |
| <b>(e)</b>        | Use of max-flow min-cut theorem                                                                                                                                                                                                                                                                                                                                                                       | M1    | 2.1  |
|                   | Identification of cut through AC, DC, DE, (EF), FT = 151<br>value of flow = 151                                                                                                                                                                                                                                                                                                                       | A1    | 3.1a |
|                   | Therefore it follows that flow is optimal                                                                                                                                                                                                                                                                                                                                                             | A1    | 2.2a |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (3)   |      |
| <b>(f)</b>        | Consider increasing capacity of arcs in minimum cut                                                                                                                                                                                                                                                                                                                                                   | B1    | 2.1  |
|                   | Explanation based on a valid argument, such as: <ul style="list-style-type: none"> <li>increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152</li> <li>increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)</li> </ul> | B1    | 2.4  |
|                   | Therefore school should choose to widen FT, which could increase the flow through the network by 16                                                                                                                                                                                                                                                                                                   | B1    | 2.2a |
|                   |                                                                                                                                                                                                                                                                                                                                                                                                       | (3)   |      |
| <b>(14 marks)</b> |                                                                                                                                                                                                                                                                                                                                                                                                       |       |      |

|                   |                                                                                          |
|-------------------|------------------------------------------------------------------------------------------|
| Question 8 notes: |                                                                                          |
| <b>(a)</b>        |                                                                                          |
| <b>B1:</b>        | Explanation of assumption to use this model                                              |
| <b>(b)</b>        |                                                                                          |
| <b>M1:</b>        | Either a correct equation, or explanation that flow in = flow out                        |
| <b>A1:</b>        | cao                                                                                      |
| <b>A1:</b>        | cao                                                                                      |
| <b>(c)</b>        |                                                                                          |
| <b>M1:</b>        | One flow augmenting route found from S to T                                              |
| <b>A1:</b>        | Two correct flow augmenting routes 5+                                                    |
| <b>A1:</b>        | Deduce that SACET must be used as students cannot travel from F to E as route is one-way |
| <b>(d)</b>        |                                                                                          |
| <b>B1:</b>        | A consistent flow pattern = 151                                                          |
| <b>(e)</b>        |                                                                                          |
| <b>M1:</b>        | Constructing argument based on max-flow min-cut theorem                                  |
| <b>A1:</b>        | Use appropriate process of finding a minimum cut – cut + value correct                   |
| <b>A1:</b>        | Correct deduction that the flow is maximal                                               |
| <b>(f)</b>        |                                                                                          |
| <b>B1</b>         | Constructing an argument based on arcs in the minimum cut                                |
| <b>B1</b>         | Detailed explanation as to why choosing anything other than FT does not help             |
| <b>B1</b>         | Correct deduction and correct increase in flow of 16                                     |

| Question          | Scheme                                                                                                                                | Marks       | AOs          |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------|-------------|--------------|
| <b>9(a)</b>       | Row minima: 1, 2    max is 2<br>Column maxima: 4, 4, 3    min is 3                                                                    | M1<br>A1    | 1.1b<br>1.1b |
|                   | Row maximin (2) $\neq$ Column minimax (3) so not stable                                                                               | A1          | 2.4          |
|                   |                                                                                                                                       | <b>(3)</b>  |              |
| <b>(b)</b>        | Let A play strategy 1 with probability $p$ and strategy 2 with probability $1-p$ , and using this to get at least one equation in $p$ | M1          | 3.3          |
|                   | Then if B plays strategy 1, A's gains are $4p + 2(1-p) = 2p + 2$                                                                      | A1          | 1.1b         |
|                   | If B plays strategy 2, A's gains are $p + 4(1-p) = 4 - 3p$                                                                            | A1          | 1.1b         |
|                   | If B plays strategy 3, A's gains are $2p + 3(1-p) = 3 - p$                                                                            |             |              |
|                   |                                                                                                                                       |             |              |
|                   | Intersection of $2p + 2$ and $3 - p$ occurs where $p = \frac{1}{3}$                                                                   | dM1<br>A1ft | 1.1b<br>1.1b |
|                   | Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time                     | A1ft        | 3.2a         |
|                   | The value of the game to player A is $2\frac{2}{3}$                                                                                   | A1          | 1.1b         |
|                   |                                                                                                                                       | <b>(9)</b>  |              |
| <b>(12 marks)</b> |                                                                                                                                       |             |              |

|                   |                                                                                                                                                     |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| Question 9 notes: |                                                                                                                                                     |
| <b>(a)</b>        |                                                                                                                                                     |
| <b>M1:</b>        | Finding row minimums and column maximums – condone one error                                                                                        |
| <b>A1:</b>        | Row minima and column maxima correct                                                                                                                |
| <b>A1:</b>        | Explanation involving $2 \neq 3$ and a conclusion                                                                                                   |
| <b>(b)</b>        |                                                                                                                                                     |
| <b>M1:</b>        | Translating situation into model by defining variables and constructing at least one equation                                                       |
| <b>A1:</b>        | One row correct                                                                                                                                     |
| <b>A1:</b>        | All three rows correct                                                                                                                              |
| <b>M1:</b>        | Axes correct, at least one line correctly drawn for their expression                                                                                |
| <b>A1:</b>        | Correct graph                                                                                                                                       |
| <b>M1:</b>        | Using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$ |
| <b>A1ft:</b>      | ft on their optimal intersection                                                                                                                    |
| <b>A1ft:</b>      | Interpret their value of $p$ in the context of the question – must refer to play, player A                                                          |
| <b>A1:</b>        | cao                                                                                                                                                 |



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