A Level Mathematics

Sample Assessment Materials

Pearson Edexcel Level 3 Advanced GCE in Mathematics (9MA0)

First teaching from September 2017

First certification from 2018

Issue 1

Pearson

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Contents

Introduction

The Pearson Edexcel Level 3 Advanced GCE in Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet '*Mathematical Formulae and Statistical Tables*' will be provided for use with these assessments and can be downloaded from our website, qualifications.pearson.com.

General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
2. Abbreviations

   These are some of the traditional marking abbreviations that may appear in the mark schemes.

   - **bod** benefit of doubt
   - **ft** follow through
   - $\sqrt{}$ this symbol is used for correct ft
   - **cao** correct answer only
   - **cso** correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** ignore subsequent working
   - **awrt** answers which round to
   - **SC:** special case
   - **o.e.** or equivalent (and appropriate)
   - **d…** or **dep** dependent
   - **indep** independent
   - **dp** decimal places
   - **sf** significant figures
   - ***** The answer is printed on the paper or ag- answer given

- or d… The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference **9MA0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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S 5 4 2 5 9 A 0 1 3 2

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

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Question 1 continued

(Total for Question 1 is 7 marks)

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2.

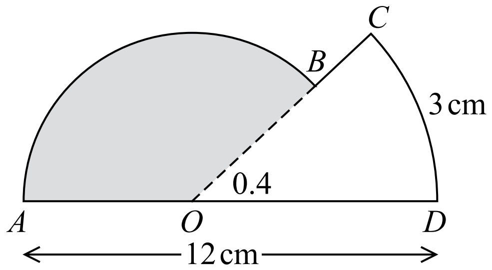

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

(Total for Question 2 is 5 marks)

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3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 3 is 4 marks)

4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 4 is 4 marks)

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5. A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1$$

where a and b are integers to be found.

(3)

(Total for Question 5 is 3 marks)

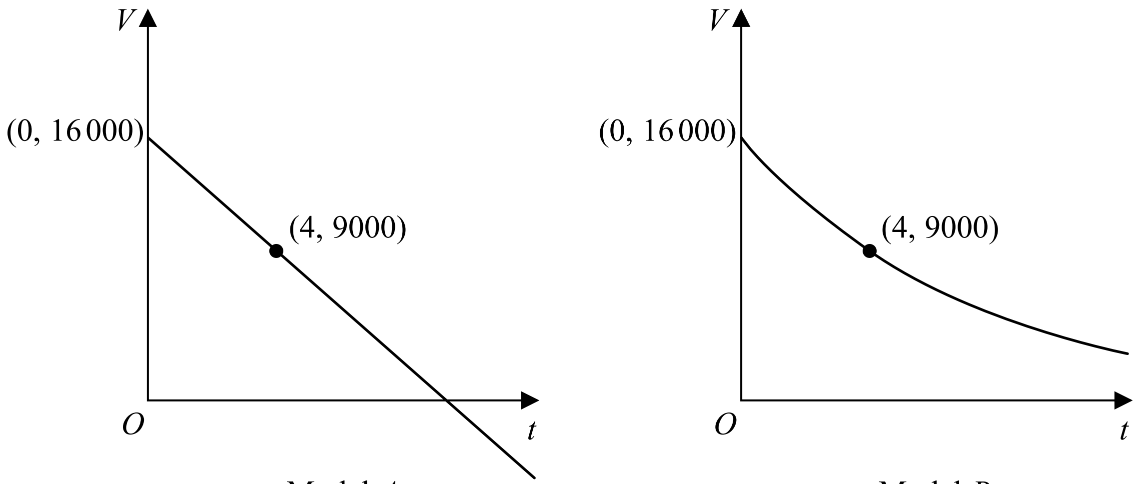
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Model A Model B

(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

Question 6 continued

(Total for Question 6 is 7 marks)

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7.

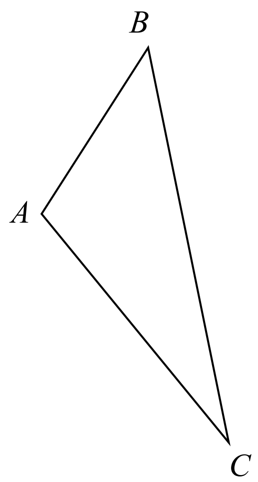

Figure 2

Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

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Question 7 continued

(Total for Question 7 is 5 marks)

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

Question 8 continued

(Total for Question 8 is 6 marks)

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

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10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total for Question 10 is 5 marks)

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.
(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.
(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.
(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.
(2)

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Question 11 continued

(Total for Question 11 is 9 marks)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

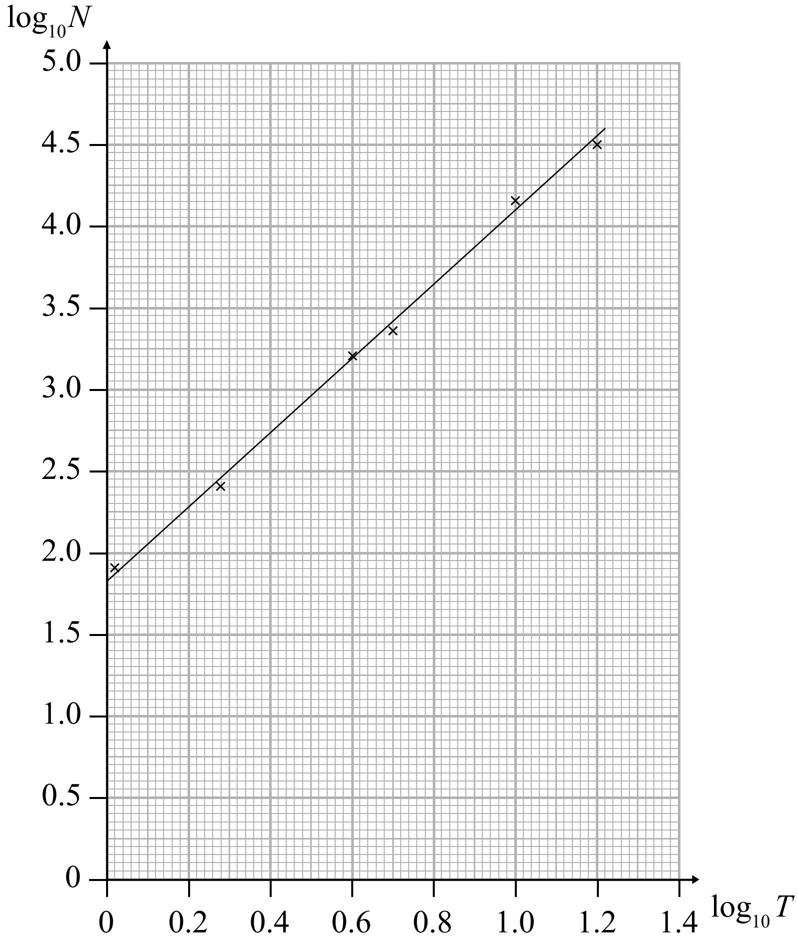

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

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Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

Question 13 continued

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Question 13 continued

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Question 13 continued

(Total for Question 13 is 13 marks)

14.

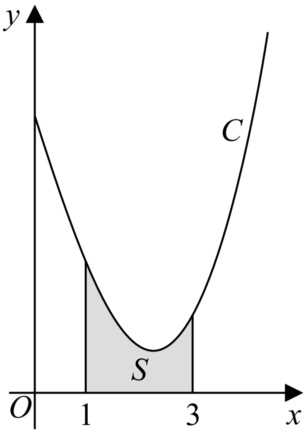

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

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Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15.

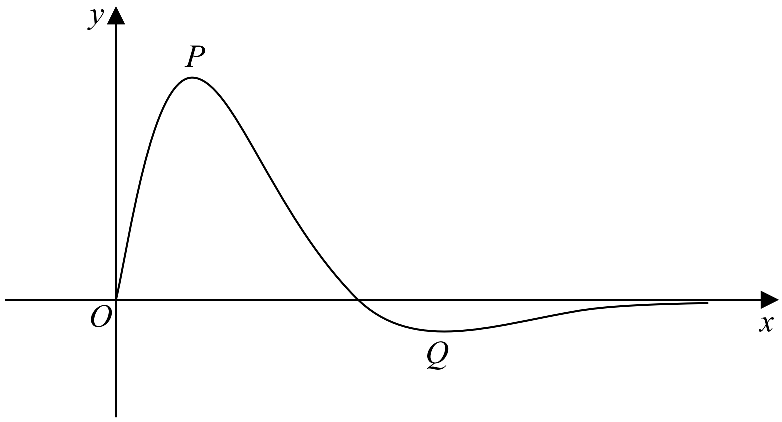


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

Question 15 continued

Question 15 continued

(Total for Question 15 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS

Paper 1: Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	(i) $\frac{dy}{dx}=12x^3-24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=36x^2-48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x=2$ into their $\frac{dy}{dx}=12\times2^3-24\times2^2$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x=2$ into their $\frac{d^2y}{dx^2}=36\times2^2-48\times2$	M1	1.1b
	$\frac{d^2y}{dx^2}=48>0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\frac{dy}{dx}=12x^3-24x^2$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}=36x^2-48x$

(b)

M1: Substitutes $x=2$ into their $\frac{dy}{dx}$

A1: Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x=2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x=2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5\text{ cm}$	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
		(5 marks)	

Notes:

(a)

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: OD = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of AOB = awrt 2.74 or uses radius is (12 – their '7.5')

M1: Follow through on their radius (12 – their OD) and their angle

A1ft: Allow awrt 27.8 cm^2. (Answer 27.75862562). Follow through on their (12 – their '7.5')
Note: Do not follow through on a radius that is negative.

Question	Scheme	Marks	AOs
3(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre (2, −5)	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

A1: States the centre is at (2, −5). Also allow written separately $x=2, y=-5$

(2, −5) implies both marks

(b)

M1: Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is >0 or $\geqslant 0$

A1ft: $k>-29$ Also allow $k\geqslant -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

Question	Scheme	Marks	AOs
4	Writes $\int \frac{t+1}{t}\,\mathrm{d}t=\int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$=t+\ln t\ (+c)$	M1	1.1b
	$(2a+\ln 2a)-(a+\ln a)=\ln 7$	M1	1.1b
	$a=\ln\frac{7}{2}$ with $k=\frac{7}{2}$	A1	1.1b
			(4 marks)

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int \frac{1}{t}\,\mathrm{d}t=\ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a=\ln\frac{7}{2}$ and states $k=\frac{7}{2}$ or exact equivalent such as 3.5

Question	Scheme	Marks	AOs
5	Attempts to substitute $=\dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right)-7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1} \qquad a=-3, b=1$	A1	1.1b
			(3 marks)

Notes:

M1: Score for an attempt at substituting $t = \dfrac{x+1}{2}$ or equivalent into $y = 4t - 7 + \dfrac{3}{t}$

M1: Award this for an attempt at a single fraction with a correct common denominator.

Their $4\left(\dfrac{x+1}{2}\right)-7$ term may be simplified first

A1: Correct answer only $y = \dfrac{2x^2-3x+1}{x+1} \qquad a=-3, b=1$

Question	Scheme	Marks	AOs
6 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t = 3 \Rightarrow V =$ awrt $10\,400$ barrels	B1ft	3.4
		(5)	
			(7 marks)

Notes:

(a)(i)

B1: 10750 barrels

(a)(ii)

B1: See scheme

(b)(i)

M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^{t}$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.

dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^{t}$ candidates need to proceed to $9000 = 16000r^{4}$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.

A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0.144t}$ $\quad V = 16000 \times (0.866)^{t}$ $\quad V = 15800e^{-0.146t} + 200$

(b)(ii)

B1ft: Follow through on their exponential model

Question	Scheme	Marks	AOs
7	Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k}=3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert=\sqrt{14},\quad \lvert AC\rvert=\sqrt{61},\quad \lvert BC\rvert=\sqrt{91}$	A1ft	1.1b
	$\cos BAC=\dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC=105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

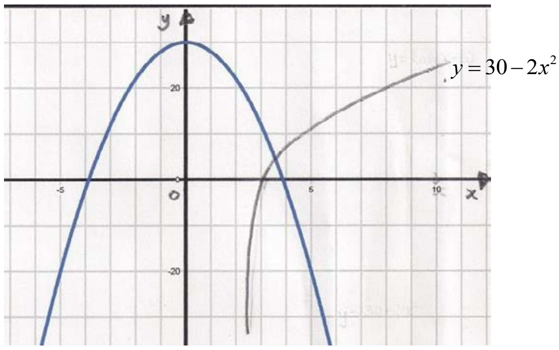
M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC=\dfrac{\lvert AB\rvert^2+\lvert AC\rvert^2-\lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC=\dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC=$ 105.9 °

Question	Scheme	Marks	AOs
8 (a)	$f(3.5) = -4.8,\ f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow \quad x_1 = 4 - \frac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x-5)$	**(2)**	
(c)	 Attempts to sketch both $y = \ln(2x-5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x-5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x-5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(x) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: f(3.5) and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $f(3.5) \times f(4) < 0$ or similar with f(x) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ evidenced by $x_1 = 4 - \frac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x-5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $f(x) = \ln(2x-5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $f(x) = \ln(2x-5) + 2x^2 - 30$

A1: Scored for correct conclusion

Question	Scheme	Marks	AOs
9(a)	$\tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\text{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta + \cot\theta = 1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \leqslant \sin 2\theta \leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta + \cot\theta \equiv \tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan\theta} \equiv \dfrac{\sec^2\theta}{\tan\theta} \equiv \dfrac{1}{\cos^2\theta \times \dfrac{\sin\theta}{\cos\theta}} \equiv \dfrac{1}{\cos\theta \times \sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions.
Possible reasons could be $-1 \leqslant \sin 2\theta \leqslant 1$........and therefore $\sin 2\theta \neq 2$
or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \leqslant \sin 2\theta \leqslant 1$

Question	Scheme	Marks	AOs
10	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1,$ $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

Question	Scheme	Marks	AOs
10alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for $\sin(A+B)$ and $\sin(A-B)$ with $A=\theta+\frac{h}{2},\quad B=\frac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ Therefore the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$

Question	Scheme	Marks	AOs
11(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times-0.002}$	dM1	1.1b
	Distance $=$ awrt $204(\text{m})$ only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$	M1	1.1b
	$=-0.002\left((d-100)^2-10000\right)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

Notes:

(a)

M1: Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$

M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator

A1: Awrt 204 m only

(b)

B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

(c)

M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms

M1: For completing the square for their $\left(d^2-200d\right)$ term

A1: $=21.8-0.002(d-100)^2$ or exact equivalent

(d)

B1ft: For their '21.8+0.3' =22.1m

B1ft: For their 100m

Question	Scheme	Marks	AOs
12 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either *a* or *b* $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both *a* and *b* $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their *a* and *b*	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that '*a*' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Question 12 continued
Notes:
(a) **M1:** Takes logs of both sides and shows the addition law **M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$
(b) **M1:** Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$ **M1:** Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$ **M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work. There is an alternative to this using a graphical approach. **M1**: Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$ **M1**: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48" Accept $\log_{10} N \approx 2.9$ **M1:** Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work
(c) **M1** For using N = 1000000 and stating that $\log_{10} N = 6$ **A1:** Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula. **M1:** Use N = 1000000 in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$. **A1:** The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d) **B1:** Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T=1

Question	Scheme	Marks	AOs
13(a)	Attempts $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$	M1	1.1b
	$\dfrac{dy}{dx}=\dfrac{\sqrt{3}\sin 2t}{\sin t}\quad\left(=2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t=\dfrac{2\pi}{3}$ in $\dfrac{dy}{dx}=\dfrac{\sqrt{3}\sin 2t}{\sin t}=\left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal = $-\dfrac{1}{dy/dx}=\left(\dfrac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P=\left(-1,-\dfrac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y+\dfrac{\sqrt{3}}{2}=\dfrac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x-2\sqrt{3}y-1=0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$	M1	3.1a
	Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t-4\cos t-5=0$	A1	1.1b
	Finds $\cos t=\dfrac{5}{6},\cancel{-\dfrac{1}{2}}$	M1	2.4
	Substitutes their $\cos t=\dfrac{5}{6}$ into $x=2\cos t,\ y=\sqrt{3}\cos 2t$,	M1	1.1b
	$Q=\left(\dfrac{5}{3},\dfrac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
		(13 marks)	

Question 13 continued
Notes:
(a) **M1:** Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t\cos t}{\sin t}$ **A1:** Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$
(b) **M1:** For substituting $t=\frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t **M1:** Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l. **B1:** States or uses (in their tangent or normal) that $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$ **M1:** Uses their numerical value of $-1\Big/\frac{dy}{dx}$ with their $\left(-1,-\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P **A1*:** This is a proof and all aspects need to be correct. Correct answer only $2x-2\sqrt{3}y-1=0$
(c) **M1:** For substituting $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$ to produce an equation in t. Alternatively candidates could use $\cos 2t=2\cos^2 t-1$ to set up an equation of the form $y=Ax^2+B$. **M1:** Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic equation in $\cos t$ In the alternative method it is for combining their $y=Ax^2+B$ with $2x-2\sqrt{3}y-1=0$ to get an equation in just one variable **A1:** For the correct quadratic equation $12\cos^2 t-4\cos t-5=0$ Alternatively the equations in x and y are $3x^2-2x-5=0$ $12\sqrt{3}y^2+4y-7\sqrt{3}=0$ **M1:** Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P. **M1:** Substitutes their $\cos t=\frac{5}{6}$ or their $t=\arccos\left(\frac{5}{6}\right)$ in $x=2\cos t$ and $y=\sqrt{3}\cos 2t$ If a value of x or y has been found it is for finding the other coordinate. **A1:** $Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$. Allow $x=\frac{5}{3}, y=\frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Question	Scheme	Marks	AOs
14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\left\{3+2.2958+2\left(2.3041+1.9242+1.9089\right)\right\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$=\frac{x^3}{3}\ln x-\int \frac{x^2}{3}\, dx$	A1	1.1b
	$\int -2x+5\, dx = -x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5\, dx = \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S = \frac{28}{27} + \ln 27 \quad (a = 28, b = 27, c = 27)$	A1	1.1b
		(6)	
			(10 marks)

Question 14 continued
Notes:
(a) **B1:** States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule **M1:** For the correct form of the bracket in the trapezium rule. Must be y values rather than x values $\left\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times\left(\text{sum of other } y \text{ values}\right)\right\}$ **A1:** 4.393 **(b)** **B1:** See scheme **(c)** **M1:** Uses integration by parts the right way around. Look for $\int x^2 \ln x \, \mathrm{d}x = Ax^3 \ln x - \int Bx^2 \, \mathrm{d}x$ **A1:** $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, \mathrm{d}x$ **B1:** Integrates the $-2x+5$ term correctly $= -x^2 + 5x$ **M1:** All integration completed and limits used **M1:** Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$ **A1:** Correct answer only $\frac{28}{27} + \ln 27$

Question	Scheme	Marks	AOs
15(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$\mathrm{f}'(x)=\dfrac{\mathrm{e}^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}\mathrm{e}^{\sqrt{2}x-1}}{\left(\mathrm{e}^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $\mathrm{f}'(x)=0$ and divides/ factorises out the $\mathrm{e}^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x=1.02$	A1	1.1b
	(ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x=0.478$	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to differentiate by using the quotient rule with $u=4\sin 2x$ and $v=\mathrm{e}^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u=4\sin 2x$ and $v=\mathrm{e}^{1-\sqrt{2}x}$

A1: For achieving a correct $\mathrm{f}'(x)$. For the product rule

$\mathrm{f}'(x)=\mathrm{e}^{1-\sqrt{2}x}\times 8\cos 2x+4\sin 2x\times-\sqrt{2}\mathrm{e}^{1-\sqrt{2}x}$

M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

A1*: Proceeds to $\tan 2x=\sqrt{2}$. This is a given answer.

(b) (i)

M1: Solves $\tan 4x=\sqrt{2}$ attempts to find the 2nd solution. Look for $x=\dfrac{\pi+\arctan\sqrt{2}}{4}$

Alternatively finds the 2nd solution of $\tan 2x=\sqrt{2}$ and attempts to divide by 2

A1: Allow awrt $x=1.02$. The correct answer, with no incorrect working scores both marks

(b)(ii)

M1: Solves $\tan 2x=\sqrt{2}$ attempts to find the 1st solution. Look for $x=\dfrac{\arctan\sqrt{2}}{2}$

A1: Allow awrt $x=0.478$. The correct answer, with no incorrect working scores both marks

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced
Paper 2: Pure Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 2 hours

Paper Reference
9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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S 5 4 2 6 0 A 0 1 2 6

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for Question 2 is 3 marks)

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3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n, A and B are constants to be found.

(4)

(Total for Question 3 is 4 marks)

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, \ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

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5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for Question 5 is 4 marks)

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6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \dfrac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

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Question 7 continued

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(Total for Question 7 is 5 marks)

8.

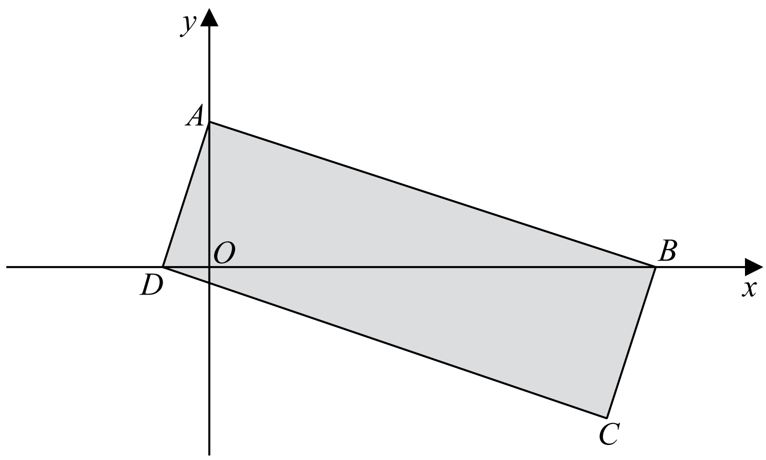

Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle $ABCD$.

(3)

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Question 8 continued

(Total for Question 8 is 7 marks)

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) \mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 9 is 5 marks)

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 10 is 4 marks)

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11.

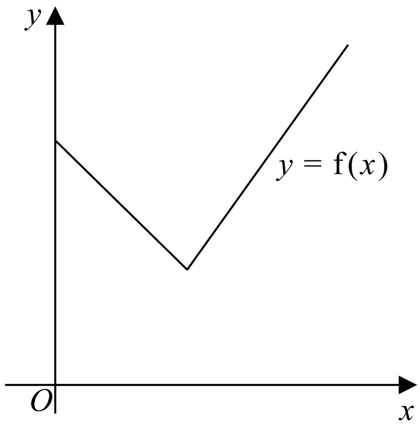

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

Question 11 continued

(Total for Question 11 is 6 marks)

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

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Question 12 continued

(Total for Question 12 is 8 marks)

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

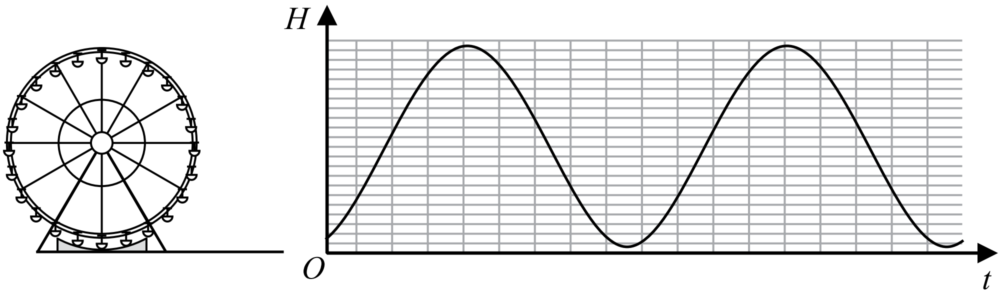

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

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Question 13 continued

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

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Question 14 continued

(Total for Question 14 is 9 marks)

15.

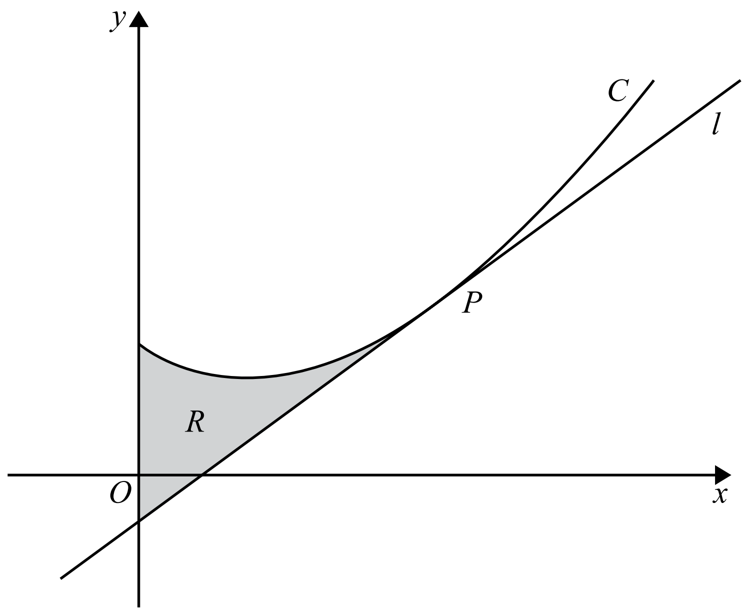

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \; x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

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Question 15 continued

(Total for Question 15 is 10 marks)

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11 - 2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

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Question 16 continued

Question 16 continued

(Total for Question 16 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
			(3 marks)

Notes:

M1: Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$
Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=..\Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

Notes:

(a)

B1: Accept a response of the type 'They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\dfrac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ±5

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{\mathrm{d}y}{\mathrm{d}x} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Question	Scheme	Marks	AOs
4 (a)	$\text{gf}(x) = 3\ln e^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x}$ = their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$ or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{d}{dt}\left(e^{-0.05t}\right) = \pm C\,e^{-0.05t}$	M1	2.1
	$\frac{dm}{dt} = -0.05\times25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times0.5}$

A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks

(b)

M1: Applies the rule $\frac{d}{dt}\left(e^{kx}\right) = k\,e^{kx}$ in this context by stating or using $\frac{d}{dt}\left(e^{-0.05t}\right) = \pm C\,e^{-0.05t}$

A1: $\frac{dm}{dt} = -0.05\times25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$

Question	Scheme	Marks	AOs
6(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2+1 \geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used	B1	2.4
		(1)	
			(5 marks)

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\frac{2}{5}$

B1: States that y coordinate of $A = 2$

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}$ '5.8'$\times$'2'

A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$

Question	Scheme		Marks	AOs
9	$\int \left(3x^{0.5}+A\right)\mathrm{d}x = 2x^{1.5}+Ax\left(+c\right)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow \left(2\times 8+4A\right)-\left(2\times 1+A\right)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
				(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax\left(+c\right)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_\infty = \frac{8}{7}\times S_6 \Rightarrow \frac{a}{1-r}=\frac{8}{7}\times\frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1=\frac{8}{7}\times(1-r^6)$	M1	2.1
	$\Rightarrow r^6=\frac{1}{8}\Rightarrow r=..$	M1	1.1b
	$\Rightarrow r=\pm\frac{1}{\sqrt{2}}$ (so $k=2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty=\frac{8}{7}\times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r=\pm\frac{1}{\sqrt{2}}$ giving $k=2$

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$	A1	2.5
		(2)	
			(6 marks)

Notes:

(a)

B1: $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1-\sin^2 x \Rightarrow 3\sin^2 x+\sin x+8=9\left(1-\sin^2 x\right)$	M1	3.1a
	$\Rightarrow 12\sin^2 x+\sin x-1=0$	A1	1.1b
	$\Rightarrow (4\sin x-1)(3\sin x+1)=0$	M1	1.1b
	$\Rightarrow \sin x=\frac{1}{4},-\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x=14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta-30°=-19.47°$	M1	3.1a
	$\Rightarrow \theta=5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Substitutes $\cos^2 x=1-\sin^2 x$ into $3\sin^2 x+\sin x+8=9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x+\sin x-1=0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x=\frac{1}{4},-\frac{1}{3}$

M1: Obtains two correct values for their $\sin x=k$

A1: All four of $x=14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta-30°=$ their $'-19.47°'$

A1ft: $\theta=5.26°$ but allow a follow through on their $'-19.47°'$

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\frac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos\left(\theta+16.70°\right)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos\left(80t+16.70\right)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\frac{540-"16.70"}{80}=\left(6.54\right)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha=\pm\frac{3}{10}$

A1: $\alpha=16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11+$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t+"16.70"=540$. Follow through on their 16.70

M1: Solves their $80t+"16.70"=540$ correctly to find t

A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant	M1	2.1
	Radius $= 4.30\,\text{cm}$	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60\,\text{cm}$	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3\,\text{cm}$ and $h = 8.6\,\text{cm}$ the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
		9 marks	

Notes:

(a)

B1: Uses the correct volume formula with V=500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question	Scheme	Marks	AOs
15	$\dfrac{dy}{dx}=\dfrac{15}{2}x^{\frac{1}{2}}-9$	M1 A1	3.1a 1.1b
	Substitutes $x=4 \Rightarrow \dfrac{dy}{dx}=6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y-15=6(x-4)$	M1	2.1
	Equation of l is $y=6x-9$	A1	1.1b
	Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,dx$	M1	3.1a
	$=\left[2x^{\frac{5}{2}}-\dfrac{15}{2}x^2+20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}}-\dfrac{15}{2}x^2+20x\right]_0^4=2\times 4^{\frac{5}{2}}-\dfrac{15}{2}\times 4^2+20\times 4-0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
		(10 marks)	

Question 15 continued
Notes:

M1: Differentiates $5x^{\frac{3}{2}}-9x+11$ to a form $Ax^{\frac{1}{2}}+B$

A1: $\dfrac{dy}{dx}=\dfrac{15}{2}x^{\frac{1}{2}}-9$ but may not be simplified

M1: Substitutes $x=4$ in their $\dfrac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,dx$ following through on their $y=6x-9$

Look for a form $Ax^{\frac{5}{2}}+Bx^2+Cx$

A1: $=\left[2x^{\frac{5}{2}}-\dfrac{15}{2}x^2+20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $=\displaystyle\int_0^4\left(5x^{\frac{3}{2}}-9x+11\right)=\left[Ax^{\frac{5}{2}}+Bx^2+Cx\right]_0^4$

A1: $=\left[2x^{\frac{5}{2}}-\dfrac{9}{2}x^2+11x\right]_0^4=36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R=$ their $36-\dfrac{1}{2}\times15\times\left(4-\text{their }\dfrac{3}{2}\right)+\dfrac{1}{2}\times\text{their }9\times\text{their }\dfrac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP\Rightarrow A\ or\ B$	M1	1.1b
	$\dfrac{1}{P(11-2P)}=\dfrac{1/11}{P}+\dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int\frac{22}{P(11-2P)}\mathrm{d}P=\int 1\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int\frac{2}{P}+\frac{4}{(11-2P)}\mathrm{d}P=t+c$	M1	1.1b
	$2\ln P-2\ln(11-2P)=t+c$	A1	1.1b
	Substitutes $t=0,P=1\Rightarrow t=0,P=1\Rightarrow\ c=(-2\ln 9)$	M1	3.1a
	Substitutes $P=2\Rightarrow t=2\ln 2+2\ln 9-2\ln 7$	M1	3.1a
	Time $=1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P-2\ln(11-2P)=t-2\ln 9$ $\Rightarrow\ln\left(\dfrac{9P}{11-2P}\right)=\dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow\left(\dfrac{9P}{11-2P}\right)=e^{\frac{1}{2}t}$ $\Rightarrow 9P=(11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P=f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P=f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P=\dfrac{11}{2+9e^{-\frac{1}{2}t}}\Rightarrow A=11,B=2,C=9$	A1	1.1b
		(3)	
		(12 marks)	

Question 16 continued

Notes:

(a)

B1: Sets $\dfrac{1}{P(11-2P)}=\dfrac{A}{P}+\dfrac{B}{(11-2P)}$

M1: Substitutes $P=0$ or $P=\dfrac{11}{2}$ into $1=A(11-2P)+BP\Rightarrow A\ or\ B$

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\dfrac{1}{P(11-2P)}=\dfrac{{}^{1}\!/_{11}}{P}+\dfrac{{}^{2}\!/_{11}}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)}=\dfrac{1}{11P}+\dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\displaystyle\int\frac{22}{P(11-2P)}\mathrm{d}P=\int 1\,\mathrm{d}t$ or equivalent

M1: Uses part (a) and $\displaystyle\int\frac{A}{P}+\frac{B}{(11-2P)}\mathrm{d}P=A\ln P\pm C\ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg $2\ln P-2\ln\left(11-2P\right)=t+c$

M1: Substitutes t = 0 and P =1 to find c

M1: Substitutes $P=2$ to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P-2\ln\left(11-2P\right)=t+c$ to $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right)=\dfrac{1}{2}t+d\Rightarrow\dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example $\dfrac{P}{11-2P}=\mathrm{e}^{\frac{1}{2}t+d}\Rightarrow\dfrac{11-2P}{P}=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)}\Rightarrow\dfrac{11}{P}-2=\mathrm{e}^{-\left(\frac{1}{2}t+d\right)}\Rightarrow\dfrac{11}{P}=2+\mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P=\dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}}\Rightarrow A=11,B=2,C=9$ oe

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced

Paper 3: Statistics and Mechanics

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference

9MA0/03

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over

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S 5 4 2 6 1 A 0 1 2 5

Pearson

SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leqslant y < 5$	$5 \leqslant y < 8$	$8 \leqslant y < 11$	$11 \leqslant y < 12$	$12 \leqslant y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leqslant y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leqslant y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow.
Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

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Question 1 continued

(Total for Question 1 is 13 marks)

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

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Question 3 continued

(Total for Question 3 is 12 marks)

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

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Question 4 continued

(Total for Question 4 is 10 marks)

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

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Question 5 continued

(Total for Question 5 is 9 marks)

TOTAL FOR SECTION A IS 50 MARKS

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

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Question 6 continued

(Total for Question 6 is 6 marks)

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

Question 7 continued

(Total for Question 7 is 8 marks)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j}) \text{ m s}^{-1}$.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$.
(2)

(b) Find $\mathbf{r}$ in terms of t.
(2)

(c) Find the value of t when the boat is north-east of O.
(3)

(d) Find the value of t when the boat is moving in a north-east direction.
(3)

Question 8 continued

(Total for Question 8 is 10 marks)

9.

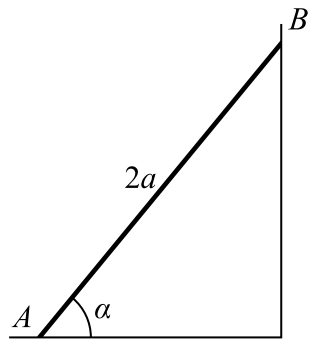

Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 13 marks)

10.

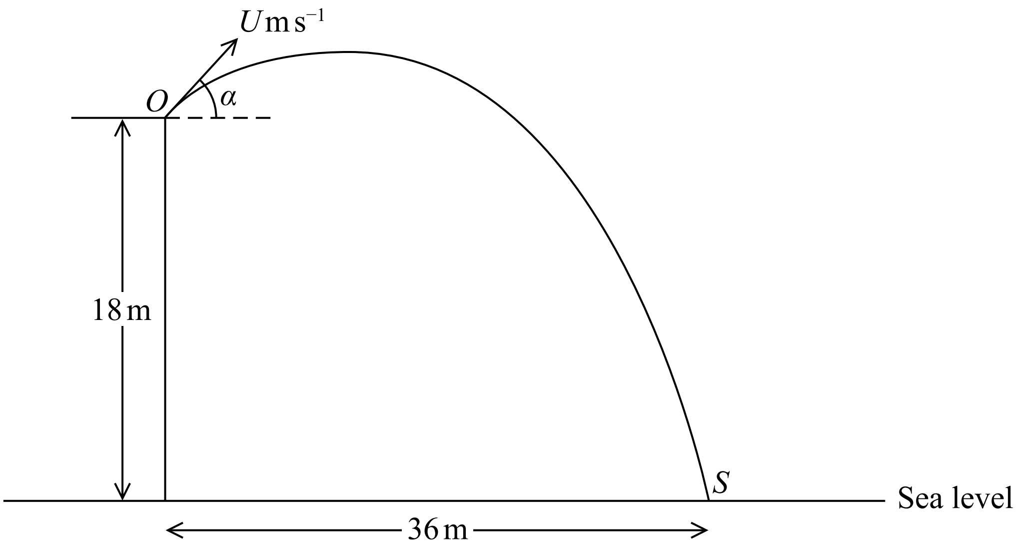

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \dfrac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U, **(6)**

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures. **(5)**

(c) Suggest two improvements that could be made to the model. **(2)**

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 13 marks)

TOTAL FOR SECTION B IS 50 MARKS
TOTAL FOR PAPER IS 100 MARKS

Paper 3: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	**Area** = $8\times1.5=12$ cm^2 **Frequency** = 8 so 1 cm^2 = $\frac{2}{3}$ **hour (o.e.)**	M1	3.1a
	Frequency of 12 corresponds to area of 18 so height = $18\div2.5$ = **7.2 (cm)**	A1	1.1b
	Width = 5×0.5 = **2.5 (cm)**	B1cao	1.1b
		(3)	
(b)	$[\bar{y}=]\ \frac{205.5}{31}$ = awrt 6.63	B1cao	1.1b
	$[\sigma_y=]\ \sqrt{\frac{1785.25}{31}-\bar{y}^2} = \sqrt{13.644641}$ = **awrt 3.69** allow $[s=]\ \sqrt{\frac{1785.25-31\bar{y}^2}{30}}$ = **awrt 3.75**	M1 A1	1.1a 1.1b
		(3)	
(c)	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		(2)	
(d)	$\bar{x}+\sigma\approx10.3$ so number of days is e.g. $\frac{(11-"10.3")}{3}\times8\ (+5)$	M1	1.1b
	= 6.86 so **7 days**	A1	1.1b
		(2)	
(e)	[H = no. of hours] P(H > 10.3) or P(Z > 1) = [0.15865…]	M1	3.4
	Predict $31\times0.15865...$ = **<u>4.9 or 5 days</u>**	A1	1.1b
		(2)	
(f)	(5 or) 4.9 days < (7 or) 6.9 days so model may **not** be suitable	B1	3.5a
		(1)	
			(13 marks)

Question 1 continued	
Notes:	
(a)	
M1:	for clear attempt to relate the area to frequency. Can also award if their height $\times$ their width = 18
A1:	for height = 7.2 (cm)
(b)	
M1:	for a correct expression for σ or s, can ft their value for mean
A1:	awrt 3.69 (allow s = 3.75)
(c)	
M1:	for a suitable comparison of standard deviations to comment on reliability.
A1:	for stating Hurn is south of Heathrow and a correct conclusion
(d)	
M1:	for a correct expression – ft their $\bar{x} + \sigma \approx 10.3$
A1:	for 7 days but accept 6 (rounding down) following a correct expression
(e)	
M1:	for a correct probability attempted
A1:	for a correct prediction
(f)	
B1:	for a suitable comparison and a compatible conclusion

Question	Scheme	Marks	AOs
2(a)	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
(b)	e.g. Linear association between w and t	B1	1.2
		(1)	
(c)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
(d)	Higher $\bar{t}$ suggests overseas and not Perth…lower wind speed so perhaps not close to the sea so suggest **Beijing**	B1	2.4
		(1)	
			(6 marks)

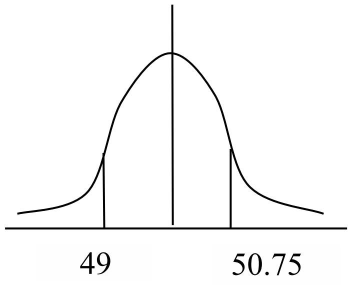
Notes:

(a)
B1: for a correct statement (unreliable) with a suitable reason

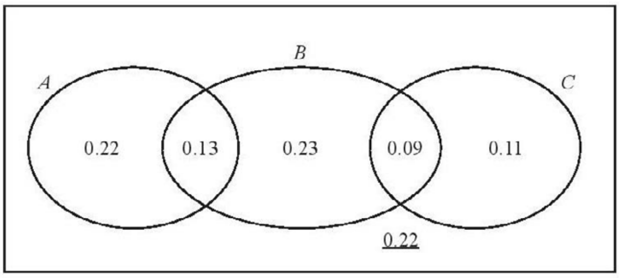
(b)
B1: for a correct statement

(c)
B1: for both hypotheses in terms of ρ
M1: for selecting a suitable 5% critical value compatible with their H_1
A1: for a correct conclusion stated

(d)
B1: for suggesting Beijing with some supporting reason based on t or w
Allow Jacksonville with a reason based just on higher $\bar{t}$

Question	Scheme	Marks	AOs
Q3(a)			
	P $(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \dfrac{50.98-\mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \mu = 50$	A1cao	1.1b
	$\text{P}(49 < L < 50.75)$	M1	3.4
	$= 0.9104\ldots$ awrt **<u>0.910</u>**	A1ft	1.1b
		(5)	
(b)	S = number of strips that cannot be used so $S \sim \text{B}(10, 0.090)$	M1	3.3
	$= \text{P}(S \leqslant 3) = 0.991166\ldots$ awrt 0.991	A1	1.1b
		(2)	
(c)	$\text{H}_0 : \mu = 50.1$ $\text{H}_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim \text{N}\left(50.1, \dfrac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$\text{P}(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ <u>or</u> $z = 1.936\ldots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the **<u>mean length</u>** of strips is **<u>greater than 50.1</u>**	A1	2.2b
		(5)	
			(12 marks)

Question 3 continued
Notes:
(a) **1st M1:** for standardizing with μ and 0.5 and setting equal to a z value ($\|z\| > 1$) **2nd M1:** for attempting the correct probability for strips that can be used **2nd A1ft:** awrt 0.910 (allow ft of their μ)
(b) **M1:** for identifying a suitable binomial distribution **A1:** awrt 0.991 (from calculator)
(c) **B1:** hypotheses stated correctly **M1:** for selecting a correct model (stated or implied) **1st A1:** for use of the correct model to find p = awrt 0.0264 (allow z = awrt 1.94) **2nd A1:** for a correct calculation, comparison and correct statement **3rd A1:** for a correct conclusion in context mentioning "mean length" and 50.1

Question	Scheme	Marks	AOs
4(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e.	M1	1.1b
	= 0.44	A1	1.1b
		(2)	
			(10 marks)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[S = no. of seeds out of 24 that germinate, $S \sim \text{B}(24, 0.55)$]		
	T = no. of trays with at least 15 germinating. $T \sim \text{B}(10, p)$	M1	3.3
	$p = \text{P}(S \geqslant 15) = 0.299126...$	A1	1.1b
	So $\text{P}(T \geqslant 5) = 0.1487…$ awrt $\underline{\mathbf{0.149}}$	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim \text{N}(132, 59.4)$	B1	3.4
	$\text{P}(X \geqslant 149.5) = \text{P}\left(Z \geqslant \frac{149.5-132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158…$ awrt $\underline{\mathbf{0.0116}}$	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			(9 marks)

Notes:

(a)
B1: cao

(b)
M1: for selection of an appropriate model for T
1st A1: for a correct value of the parameter p (accept 0.3 or better)
2nd A1: for awrt 0.149

(c)
B1: both correct conditions

(d)
B1: for correct normal distribution
M1: for correct use of continuity correction
A1: cso

(e)
B1: correct statement

Question	Scheme	Marks	AOs
6	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\,\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1**ft**	1.1b
			(6 marks)

Notes:

1st M1: for integrating **a** w.r.t. time (powers of *t* increasing by 1)
1st A1: for a correct **v** expression without **C**
2nd A1: for a correct **v** expression including **C**
2nd M1: for putting $t = 4$ into their **v** expression
3rd M1: for finding magnitude of their **v**
3rd A1: **ft** for 100 m s^{-1}, follow through on an incorrect **v**

Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 **ft**	2.2a
		(2)	
			(8 marks)

Notes:

(a)

B1: for $R = mg\cos\alpha$

1st M1: for resolving parallel to the plane

1st A1: for a correct equation

2nd M1: for use of $F = \mu R$

3rd M1: for eliminating F and R to give a value for μ

2nd A1: for $\mu = \frac{1}{4}$

(b)

M1: comparing size of limiting friction with weight component down the plane

A1ft: for an appropriate conclusion from their values

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 **ft**	1.1b
		(3)	
			(10 marks)

Notes:

(a)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for **r** in terms of t

(c)

M1: for equating the **i** and **j** components of their **r**

A1ft: for a correct equation following their **r**

A1: for $t = 1.5$

(d)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t

M1: for equating the **i** and **j** components of their **v**

A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha\ +\ 7W2a\cos\alpha\ = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha\ =\ \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{\text{MAX}} = 3W + F$ or $P_{\text{MIN}} = 3W - F$	M1	3.4
	$P_{\text{MAX}} = 5W$ or $P_{\text{MIN}} = W$	A1	1.1b
	$W \leq\ P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 9 continued
Notes:
(a) **1st M1:** for producing an equation in S, W and α only **1st A1:** for an equation that is correct, or which has one error or omission **2nd A1:** for a fully correct equation **2nd M1:** for use of $\tan\alpha = \frac{5}{2}$ to obtain S in terms of W only **3rd A1*:** for given answer $S = 3W$ correctly obtained
(b) **B1:** for $R = 8W$ **1st M1:** for use of $F = \frac{1}{4}R$ **2nd M1:** for either $P = (3W + \text{their } F)$ or $P = (3W - \text{their } F)$ **1st A1:** for a correct max or min value for a correct range for P **2nd A1:** for a correct range for P
(c) **1st M1:** for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder **2nd M1:** for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder **3rd M1:** for concluding that this increases the limiting friction at A

Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s $^{-1}$ (2sf)	A1 **ft**	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 10 continued
Notes:
(a) **1st M1:** for use of $s = ut$ horizontally **1st A1:** for a correct equation **2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1:** for a correct equation **3rd M1:** for correct strategy (need both equations) **2nd A1:** for $U = 15$
(b) **B1:** for $U\cos\alpha$ used as horizontal velocity component **1st M1:** for attempt to find vertical component **1st A1:** for 15 **2nd M1:** for correct strategy (need both components) **2nd A1ft:** for 19 m s^{-1} (2sf) following through on incorrect component(s)
(c) **B1, B1:** for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion

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