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Origami photography: Pearson Education Ltd/Naki Kouyioumtzis

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Introduction

The Pearson Edexcel Level 3 Advanced GCE in Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet ‘Mathematical Formulae and Statistical Tables’ will be provided for use with these assessments and can be downloaded from our website, qualifications.pearson.com.
General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked unless the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

2. Abbreviations
   These are some of the traditional marking abbreviations that may appear in the mark schemes.
   - **bod** benefit of doubt
   - **ft** follow through
   - √ this symbol is used for correct ft
   - **cao** correct answer only
   - **cso** correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** ignore subsequent working
   - **awrt** answers which round to
   - **SC:** special case
   - **o.e.** or equivalent (and appropriate)
   - **d...** dependent or **dep**
   - **indep** independent
   - **dp** decimal places
   - **sf** significant figures
   - * The answer is printed on the paper or ag- answer given
• The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don’t logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.
The second mark is dependent on gaining the first mark.

All M marks are follow through.

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don’t logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

Ignore wrong working or incorrect statements following a correct answer.

Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.
1. The curve $C$ has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(b) Verify that $C$ has a stationary point when $x = 2$

(c) Determine the nature of this stationary point, giving a reason for your answer.
Question 1 continued

(Total for Question 1 is 7 marks)
The shape $ABCDOA$, as shown in Figure 1, consists of a sector $COD$ of a circle centre $O$ joined to a sector $AOB$ of a different circle, also centre $O$.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and $AOD$ is a straight line of length 12 cm,

(a) find the length of $OD$,

(b) find the area of the shaded sector $AOB$.

(Total for Question 2 is 5 marks)
3. A circle $C$ has equation

$$x^2 + y^2 - 4x + 10y = k$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$. (2)

(b) State the range of possible values for $k$. (2)

(Total for Question 3 is 4 marks)
4. Given that $a$ is a positive constant and

$$\int_{a}^{2a} \frac{t + 1}{t} \, dt = \ln 7$$

show that $a = \ln k$, where $k$ is a constant to be found.  

(Total for Question 4 is 4 marks)
5. A curve $C$ has parametric equations

\[ x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0 \]

Show that the Cartesian equation of the curve $C$ can be written in the form

\[ y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1 \]

where $a$ and $b$ are integers to be found.

(Total for Question 5 is 3 marks)
6. A company plans to extract oil from an oil field.

The daily volume of oil $V$, measured in barrels that the company will extract from this oil field depends upon the time, $t$ years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.

![Graphs of two possible models](image)

(a) (i) Use model $A$ to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model $A$.  

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model $B$.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.
7. Figure 2 shows a sketch of a triangle $ABC$.

Given $\overrightarrow{AB} = 2i + 3j + k$ and $\overrightarrow{BC} = i - 9j + 3k$,

show that $\angle BAC = 105.9^\circ$ to one decimal place. (5)
Question 7 continued

(Total for Question 7 is 5 marks)
8. \( f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5 \)

(a) Show that \( f(x) = 0 \) has a root \( \alpha \) in the interval \([3.5, 4]\)  

A student takes 4 as the first approximation to \( \alpha \).

Given \( f(4) = 3.099 \) and \( f'(4) = 16.67 \) to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for \( \alpha \),

giving your answer to 3 significant figures.

(c) Show that \( \alpha \) is the only root of \( f(x) = 0 \)
Question 8 continued

(b) apply the Newton-Raphson procedure once to obtain a second approximation for

Given \( f(4) = 3.099 \) and \( f'(4) = 16.67 \) to 4 significant figures,

(a) Show that \( f(x) = 0 \) has a root \( x = \alpha \) in the interval \([3.5, 4]\).

\( x \) is the only root of \( f(x) = 0 \).

\( 2 - 30, x > 2.5 \)

(Total for Question 8 is 6 marks)
9. (a) Prove that

\[ \tan \theta + \cot \theta = 2 \csc 2\theta, \quad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z} \]  

(b) Hence explain why the equation

\[ \tan \theta + \cot \theta = 1 \]

does not have any real solutions.

(Total for Question 9 is 5 marks)
10. Given that \( \theta \) is measured in radians, prove, from first principles, that the derivative of \( \sin \theta \) is \( \cos \theta \)

You may assume the formula for \( \sin(A \pm B) \) and that as \( h \to 0 \), \( \frac{\sin h}{h} \to 1 \) and \( \frac{\cos h - 1}{h} \to 0 \)

(Total for Question 10 is 5 marks)
11. An archer shoots an arrow.

The height, $H$ metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0$$

where $d$ is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model. (3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula. (1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where $A$, $B$ and $C$ are constants to be found. (3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height. (2)
Question 11 continued

(Total for Question 11 is 9 marks)
12. In a controlled experiment, the number of microbes, \( N \), present in a culture \( T \) days after the start of the experiment were counted.

\( N \) and \( T \) are expected to satisfy a relationship of the form

\[
N = aT^b,
\]

where \( a \) and \( b \) are constants

(a) Show that this relationship can be expressed in the form

\[
\log_{10} N = m \log_{10} T + c
\]

giving \( m \) and \( c \) in terms of the constants \( a \) and/or \( b \).

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(d) With reference to the model, interpret the value of the constant \( a \).
Question 12 continued

---

With reference to the model, interpret the value of the constant.

Explain why the information provided could not reliably be used to estimate the day giving...

In a controlled experiment, the number of microbes, present in a culture after 3 days, are expected to satisfy a relationship of the form

\[
N = aT^b + cN_0.
\]

Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

Show that this relationship can be expressed in the form

\[
\log_{10} N = \log_{10} a + m \log_{10} T + \log_{10} N_0.
\]

Figure 3 shows the line of best fit for values of \( \log_{10} N \) plotted against values of \( \log_{10} T \). The relationship is given by

\[
\log_{10} N = a + b \log_{10} T + \log_{10} N_0.
\]

\( a \) and \( b \) are constants.

---

**Note:** The remaining text is not visible in the provided image.
13. The curve $C$ has parametric equations
\[ x = 2 \cos t, \quad y = \sqrt{3} \cos 2t, \quad 0 \leq t \leq \pi \]

(a) Find an expression for $\frac{dy}{dx}$ in terms of $t$. 

(b) Show that an equation for $l$ is
\[ 2x - 2\sqrt{3}y - 1 = 0 \]

You must show clearly how you obtained your answers.
Question 13 continued
Question 13 continued
Question 13 continued

(Total for Question 13 is 13 marks)
14.

Figure 4 shows a sketch of part of the curve $C$ with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region $S$, shown shaded in Figure 4, is bounded by the curve $C$, the line with equation $x = 1$, the $x$-axis and the line with equation $x = 3$.

The table below shows corresponding values of $x$ and $y$ with the values of $y$ given to 4 decimal places as appropriate.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3</td>
<td>2.3041</td>
<td>1.9242</td>
<td>1.9089</td>
<td>2.2958</td>
</tr>
</tbody>
</table>

(a) Use the trapezium rule, with all the values of $y$ in the table, to obtain an estimate for the area of $S$, giving your answer to 3 decimal places.

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of $S$.

(c) Show that the exact area of $S$ can be written in the form $\frac{a}{b} + \ln c$, where $a$, $b$ and $c$ are integers to be found.

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)
Question 14 continued
Question 14 continued

(Total for Question 14 is 10 marks)
Figure 5 shows a sketch of the curve with equation \( y = f(x) \), where

\[
f(x) = \frac{4\sin 2x}{e^{\sqrt{2x} - 1}}, \quad 0 \leq x \leq \pi
\]

The curve has a maximum turning point at \( P \) and a minimum turning point at \( Q \) as shown in Figure 5.

(a) Show that the \( x \) coordinates of point \( P \) and point \( Q \) are solutions of the equation

\[
\tan 2x = \sqrt{2}
\]

(b) Using your answer to part (a), find the \( x \)-coordinate of the minimum turning point on the curve with equation

(i) \( y = f(2x) \).

(ii) \( y = 3 - 2f(x) \).
Question 15 continued
Question 15 continued

(Total for Question 15 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS
### Paper 1: Pure Mathematics 1 Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>(i) ( \frac{dy}{dx} = 12x^3 - 24x^2 )</td>
<td>M1 1.1b</td>
<td>A1 1.1b</td>
</tr>
<tr>
<td></td>
<td>(ii) ( \frac{d^2y}{dx^2} = 36x^2 - 48x )</td>
<td>A1ft 1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Substitutes ( x = 2 ) into their ( \frac{dy}{dx} = 12 \times 2^3 - 24 \times 2^2 )</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shows ( \frac{dy}{dx} = 0 ) and states &quot;hence there is a stationary point&quot;</td>
<td>A1 2.1</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Substitutes ( x = 2 ) into their ( \frac{d^2y}{dx^2} = 36 \times 2^2 - 48 \times 2 )</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{d^2y}{dx^2} = 48 &gt; 0 ) and states &quot;hence the stationary point is a minimum&quot;</td>
<td>A1ft 2.2a</td>
<td></td>
</tr>
</tbody>
</table>

**(7 marks)**

### Notes:

- **(a)(i)**
  - **M1:** Differentiates to a cubic form
  - **A1:** \( \frac{dy}{dx} = 12x^3 - 24x^2 \)
- **(a)(ii)**
  - **A1ft:** Achieves a correct \( \frac{d^2y}{dx^2} \) for their \( \frac{dy}{dx} = 36x^2 - 48x \)
- **(b)**
  - **M1:** Substitutes \( x = 2 \) into their \( \frac{dy}{dx} \)
  - **A1:** Shows \( \frac{dy}{dx} = 0 \) and states "hence there is a stationary point" All aspects of the proof must be correct
- **(c)**
  - **M1:** Substitutes \( x = 2 \) into their \( \frac{d^2y}{dx^2} \)
  - Alternatively calculates the gradient of \( C \) either side of \( x = 2 \)
  - **A1ft:** For a correct calculation, a valid reason and a correct conclusion.
    Follow through on an incorrect \( \frac{d^2y}{dx^2} \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td>Uses $s = r\theta \Rightarrow 3 = r \times 0.4$</td>
<td>M1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow OD = 7.5 \text{ cm}$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Uses angle $AOB = (\pi - 0.4)$ or uses radius is $(12 - 7.5')$ cm</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Uses area of sector $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$= 27.8 \text{ cm}^2$</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(5 marks)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

**(a)**

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: $OD = 7.5 \text{ cm}$ (An answer of 7.5 cm implies the use of a correct formula and scores both marks)

**Notes:**

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of $AOB = \text{awrt } 2.74$ or uses radius is $(12 - \text{their} '7.5')$

M1: Follow through on their radius $(12 - \text{their } OD)$ and their angle

A1ft: Allow awrt 27.8 cm$^2$. (Answer 27.75862562). Follow through on their $(12 - \text{their } '7.5')$

Note: Do not follow through on a radius that is negative.
### Question 3(a)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attempts ((x - 2)^2 + (y + 5)^2 = \ldots)</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td>Centre ((2, -5))</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(2 marks)

### Notes:

(a)  
**M1:** Attempts to complete the square so allow \((x - 2)^2 + (y + 5)^2 = \ldots\)  
**A1:** States the centre is at \((2, -5)\). Also allow written separately \(x = 2, y = -5\)  
\((2, -5)\) implies both marks

(b)  
**M1:** Deduces that the right hand side of their \((x \pm \ldots)^2 + (y \pm \ldots)^2 = \ldots\) is \(> 0\) or \(\geq 0\)  
**A1ft:** \(k > -29\) Also allow \(k \geq -29\) Follow through on their rhs of \((x \pm \ldots)^2 + (y \pm \ldots)^2 = \ldots\)

### Question 4

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writes (\int \frac{t+1}{t} , dt = \int 1 + \frac{1}{t} , dt) and attempts to integrate</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td>(= t + \ln t + c)</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td>((2a + \ln 2a) - (a + \ln a) = \ln 7)</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(a = \ln \frac{7}{2}) with (k = \frac{7}{2})</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(4 marks)

### Notes:

**M1:** Attempts to divide each term by \(t\) or alternatively multiply each term by \(t^{-1}\)  
**M1:** Integrates each term and knows \(\int \frac{1}{t} \, dt = \ln t\). The + \(c\) is not required for this mark  
**M1:** Substitutes in both limits, subtracts and sets equal to \(\ln 7\)  
**A1:** Proceeds to \(a = \ln \frac{7}{2}\) and states \(k = \frac{7}{2}\) or exact equivalent such as 3.5
### Question 5

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attempts to substitute $t = \frac{x + 1}{2}$ into $y \Rightarrow y = 4\left(\frac{x + 1}{2}\right) - 7 + \frac{6}{(x + 1)}$</td>
</tr>
<tr>
<td>Attempts to write as a single fraction $y = \frac{(2x - 5)(x + 1) + 6}{(x + 1)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 2.1</td>
<td></td>
</tr>
<tr>
<td>M1 2.1</td>
<td></td>
</tr>
<tr>
<td>A1 1.1b</td>
<td></td>
</tr>
</tbody>
</table>

### Notes:

- **M1:** Score for an attempt at substituting $t = \frac{x + 1}{2}$ or equivalent into $y = 4t - 7 + \frac{3}{t}$

- **M1:** Award this for an attempt at a single fraction with a correct common denominator. Their $4\left(\frac{x + 1}{2}\right) - 7$ term may be simplified first

- **A1:** Correct answer only $y = \frac{2x^2 - 3x + 1}{x + 1}$ $a = -3, b = 1$ (3 marks)
(a)(i) 10750 barrels

(ii) Gives a valid limitation, for example
- The model shows that the daily volume of oil extracted would become negative as $t$ increases, which is impossible
- States when $t = 10, V = -1500$ which is impossible
- States that the model will only work for $0 \leq t \leq \frac{64}{7}$

(b)(i) Suggests a suitable exponential model, for example $V = Ae^{kt}$

\[
\Rightarrow k = \frac{1}{4} \ln \left( \frac{9}{16} \right) \quad \text{awrt} -0.144
\]

\[
V = 16000e^{\frac{1}{4} \ln \left( \frac{9}{16} \right)} \quad \text{or} \quad V = 16000e^{-0.144t}
\]

(ii) Uses their exponential model with $t = 3 \Rightarrow V = \text{awrt} 10400$ barrels

(7 marks)

Notes:
(a)(i) B1: 10750 barrels
(a)(ii) B1: See scheme

(b)(i) M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for $b$.
dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{kt}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^t$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{kt} + b$ where $b$ is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.
A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0, 16000)$ and $(4, 9000)$. Possible equations for the model could be for example

\[
V = 16000e^{-0.144t} \quad V = 16000 \times (0.866)^t \quad V = 15800e^{-0.146t} + 200
\]

(b)(ii) B1ft: Follow through on their exponential model
Question 7

Attempts
\[ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 2i + 3j + k + i - 9j + 3k = 3i - 6j + 4k \]

Attempts to find any one length using 3-d Pythagoras

Finds all of \( |AB| = \sqrt{14}, \quad |AC| = \sqrt{61}, \quad |BC| = \sqrt{91} \)

\[ \cos BAC = \frac{14 + 61 - 91}{2 \sqrt{14} \sqrt{61}} \]

\[ \text{angle } BAC = 105.9^\circ \]

Notes:

M1: Attempts to find \( \overrightarrow{AC} \) by using \( \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \)

M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their \( |AC| \)

M1: Attempts to find \( BAC \) using \( \cos BAC = \frac{|AB|^2 + |AC|^2 - |BC|^2}{2|AB||AC|} \)

Allow this to be scored for other methods such as \( \cos BAC = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|AB||AC|} \)

A1*: This is a show that and all aspects must be correct. Angle \( BAC = 105.9^\circ \)
## Question 8 (a)

\[ f(3.5) = -4.8, \ f(4) = (+)3.1 \]

Change of sign and function continuous in interval 
\[ [3.5, 4] \Rightarrow \text{Root} \]

<table>
<thead>
<tr>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1b</td>
<td></td>
</tr>
</tbody>
</table>

\[ M1 \]

\[ (2) \]

## Question 8 (b)

Attempts 
\[ x_i = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_i = 4 - \frac{3.099}{16.67} \]

\[ x_i = 3.81 \]

\[ y = \ln(2x - 5) \]

\[ (2) \]

<table>
<thead>
<tr>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>1.1b</td>
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</tr>
</tbody>
</table>

\[ M1 \]

\[ (2) \]

## Question 8 (c)

Attempts to sketch both \( y = \ln(2x - 5) \) and \( y = 30 - 2x^2 \)

States that \( y = \ln(2x - 5) \) meets \( y = 30 - 2x^2 \) in just one place, therefore \( y = \ln(2x - 5) = 30 - 2x \) has just one root \( \Rightarrow f(x) = 0 \) has just one root

\[ (2) \]

\[ (6 \text{ marks}) \]

## Notes:

### (a)

- **M1**: Attempts \( f(x) \) at both \( x = 3.5 \) and \( x = 4 \) with at least one correct to 1 significant figure
- **A1**: \( f(3.5) \) and \( f(4) \) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or \( f(3.5) \times f(4) < 0 \) or similar with \( f(x) \) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

### (b)

- **M1**: Attempts 
\[ x_i = x_0 - \frac{f(x_0)}{f'(x_0)} \] evidenced by 
\[ x_i = 4 - \frac{3.099}{16.67} \]

- **A1**: Correct answer only \( x_i = 3.81 \)

### (c)

- **M1**: For a valid attempt at showing that there is only one root. This can be achieved by
  - Sketching graphs of \( y = \ln(2x - 5) \) and \( y = 30 - 2x^2 \) on the same axes
  - Showing that \( f(x) = \ln(2x - 5) + 2x^2 - 30 \) has no turning points
  - Sketching a graph of \( f(x) = \ln(2x - 5) + 2x^2 - 30 \)
- **A1**: Scored for correct conclusion
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
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</tr>
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<tbody>
<tr>
<td>9(a)</td>
<td>(\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta})</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>(\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta})</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(\equiv \frac{1}{\sin 2\theta})</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>(\equiv 2\csc 2\theta) *</td>
<td>A1*</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>States (\tan \theta + \cot \theta = 1 \Rightarrow \sin 2\theta = 2) AND no real solutions as (-1 \leq \sin 2\theta \leq 1)</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td></td>
<td></td>
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<tr>
<td>(5 marks)</td>
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</tbody>
</table>

**Notes:**

(a)

**M1:** Writes \(\tan \theta = \frac{\sin \theta}{\cos \theta}\) and \(\cot \theta = \frac{\cos \theta}{\sin \theta}\)  
**A1:** Achieves a correct intermediate answer of \(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}\)  
**M1:** Uses the double angle formula \(\sin 2\theta = 2\sin \theta \cos \theta\)  
**A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

\[\tan \theta + \cot \theta = \tan \theta + \frac{1}{\tan \theta} = \tan^2 \theta + 1 = \sec^2 \theta = \frac{1}{\cos^2 \theta \times \frac{\sin \theta}{\cos \theta}} = \frac{1}{\cos \theta \times \sin \theta}\]  
then as the main scheme.

(b)

**B1:** Scored for sight of \(\sin 2\theta = 2\) and a reason as to why this equation has no real solutions. Possible reasons could be \(-1 \leq \sin 2\theta \leq 1\)......and therefore \(\sin 2\theta \neq 2\)  
or \(\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2\) which has no answers as \(-1 \leq \sin 2\theta \leq 1\)
Question Scheme Marks AOs

10 Use of \( \frac{\sin(\theta + h) - \sin \theta}{(\theta + h) - \theta} \) B1 2.1

Uses the compound angle identity for \( \sin(A + B) \) with \( A = \theta, B = h \)
\( \Rightarrow \sin(\theta + h) = \sin \theta \cos h + \cos \theta \sin h \) M1 1.1b

Achieves \( \frac{\sin(\theta + h) - \sin \theta}{h} = \frac{\sin \theta \cos h + \cos \theta \sin h - \sin \theta}{h} \) A1 1.1b

\( = \frac{\sin h}{h} \cos \theta + \left( \frac{\cos h - 1}{h} \right) \sin \theta \) M1 2.1

Uses \( h \to 0, \frac{\sin h}{h} \to 1 \) and \( \frac{\cos h - 1}{h} \to 0 \)

Hence the limit \( \lim_{h \to 0} \frac{\sin(\theta + h) - \sin \theta}{(\theta + h) - \theta} = \cos \theta \) and the gradient of the chord \( \Rightarrow \frac{dy}{d\theta} = \cos \theta \) A1* 2.5

(5 marks)

Notes:

B1: States or implies that the gradient of the chord is \( \frac{\sin(\theta + h) - \sin \theta}{h} \) or similar such as
\( \frac{\sin(\theta + \delta \theta) - \sin \theta}{\delta \theta} \) for a small \( h \) or \( \delta \theta \)

M1: Uses the compound angle identity for \( \sin(A + B) \) with \( A = \theta, B = h \) or \( \delta \theta \)

A1: Obtains \( \frac{\sin \theta \cos h + \cos \theta \sin h - \sin \theta}{h} \) or equivalent

M1: Writes their expression in terms of \( \frac{\sin h}{h} \) and \( \frac{\cos h - 1}{h} \)

A1*: Uses correct language to explain that \( \frac{dy}{d\theta} = \cos \theta \)

For this method they should use all of the given statements \( h \to 0, \frac{\sin h}{h} \to 1, \frac{\cos h - 1}{h} \to 0 \) meaning that the limit \( \lim_{h \to 0} \frac{\sin(\theta + h) - \sin \theta}{(\theta + h) - \theta} = \cos \theta \)

and therefore the gradient of the chord \( \Rightarrow \frac{dy}{d\theta} = \cos \theta \)
<table>
<thead>
<tr>
<th>Question</th>
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</thead>
<tbody>
<tr>
<td>10alt</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Use of $\frac{\sin(\theta + h) - \sin \theta}{(\theta + h) - \theta}$</td>
<td>B1</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Sets $\frac{\sin(\theta + h) - \sin \theta}{(\theta + h) - \theta} = \frac{\sin \left( \frac{\theta + h + h}{2} \right) - \sin \left( \frac{\theta + h - h}{2} \right)}{h}$</td>
<td>M1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>and uses the compound angle identity for $\sin(A + B)$ and $\sin(A - B)$ with $A = \theta + \frac{h}{2}$, $B = \frac{h}{2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Achieves $\frac{\sin(h) - \sin \theta}{h} = $</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>$\left[ \sin \left( \frac{\theta + h}{2} \right) \cos \left( \frac{h}{2} \right) + \cos \left( \frac{\theta + h}{2} \right) \sin \left( \frac{h}{2} \right) \right] \left[ \sin \left( \frac{\theta + h}{2} \right) \cos \left( \frac{h}{2} \right) - \cos \left( \frac{\theta + h}{2} \right) \sin \left( \frac{h}{2} \right) \right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{\sin \left( \frac{h}{2} \right)}{h} \times \cos \left( \frac{\theta + h}{2} \right)$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td>Uses $h \to 0$, $\frac{h}{2} \to 0$ hence $\frac{\sin \left( \frac{h}{2} \right)}{h} \to 1$ and $\cos \left( \frac{\theta + h}{2} \right) \to \cos \theta$</td>
<td>A1*</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Therefore the $\lim_{h \to 0} \frac{\sin(h) - \sin \theta}{(\theta + h) - \theta} = \cos \theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \frac{dy}{d\theta} = \cos \theta$</td>
<td></td>
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</tbody>
</table>

(5 marks)

**Additional notes:**

A1*: Uses correct language to explain that $\frac{dy}{d\theta} = \cos \theta$. For this method they should use the (adapted) given statement $h \to 0$, $\frac{h}{2} \to 0$ hence $\frac{\sin \left( \frac{h}{2} \right)}{h} \to 1$ with $\cos \left( \frac{\theta + h}{2} \right) \to \cos \theta$

meaning that the $\lim_{h \to 0} \frac{\sin(h) - \sin \theta}{(\theta + h) - \theta} = \cos \theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \frac{dy}{d\theta} = \cos \theta$
<table>
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</thead>
<tbody>
<tr>
<td><strong>11(a)</strong></td>
<td>Sets $H = 0 \Rightarrow 1.8 + 0.4d - 0.002d^2 = 0$</td>
<td>M1 3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solves using an appropriate method, for example $d = \frac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(1.8)}}{2 \times -0.002}$</td>
<td>dM1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Distance = awrt 204(m) only</td>
<td>A1 2.2a</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>States the initial height of the arrow above the ground.</td>
<td>B1 3.4</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$1.8 + 0.4d - 0.002d^2 = -0.002(d^2 - 200d) + 1.8$</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= -0.002(d - 100)^2 - 10000 + 1.8$</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 21.8 - 0.002(d - 100)^2$</td>
<td>A1 1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>(i) 22.1 metres</td>
<td>B1ft 3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) 100 metres</td>
<td>B1ft 3.4</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)  
M1: Sets $H = 0 \Rightarrow 1.8 + 0.4d - 0.002d^2 = 0$  
M1: Solves using formula, which if stated must be correct, by completing square (look for $(d - 100)^2 = 10900 \Rightarrow d = ..$) or even allow answers coming from a graphical calculator  
A1: Awrt 204 m only

(b)  
B1: States it is the initial height of the arrow above the ground. Do not allow "it is the height of the archer"

(c)  
M1: Score for taking out a common factor of $-0.002$ from at least the $d^2$ and $d$ terms  
M1: For completing the square for their $(d^2 - 200d)$ term  
A1: $= 21.8 - 0.002(d - 100)^2$ or exact equivalent

(d)  
B1ft: For their '21.8+0.3'=22.1m  
B1ft: For their 100m
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>12 (a)</strong></td>
<td>[ N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b ]</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>[ \Rightarrow \log_{10} N = \log_{10} a + b \log_{10} T ] so [ m = b \text{ and } c = \log_{10} a ]</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Uses the graph to find either ( a ) or ( b ) ( a = 10^{\text{intercept or } b} = \text{gradient} )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Uses the graph to find both ( a ) and ( b ) ( a = 10^{\text{intercept and } b} = \text{gradient} )</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Uses ( T = 3 ) in ( N = aT^b ) with their ( a ) and ( b )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Number of microbes ( \approx 800 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>[ N = 1000000 \Rightarrow \log_{10} N = 6 ]</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>We cannot ‘extrapolate’ the graph and assume that the model still holds</td>
<td>A1</td>
<td>3.5b</td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>States that ‘( a )’ is the number of microbes 1 day after the start of the experiment</td>
<td>B1</td>
<td>3.2a</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td><strong>(9 marks)</strong></td>
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<td></td>
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<tr>
<td>Notes:</td>
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**Question 12 continued**

### (a)

**M1:** Takes logs of both sides and shows the addition law

**M1:** Uses the power law, writes \( \log_{10} N = \log_{10} a + b \log_{10} T \) and states \( m = b \) and \( c = \log_{10} a \)

### (b)

**M1:** Uses the graph to find either \( a \) or \( b \) \( a = 10^{\text{intercept}} \) or \( b = \text{gradient} \). This would be implied by the sight of \( b = 2.3 \) or \( a = 10^{1.8} \approx 63 \)

**M1:** Uses the graph to find both \( a \) and \( b \) \( a = 10^{\text{intercept}} \) and \( b = \text{gradient} \). This would be implied by the sight of \( b = 2.3 \) and \( a = 10^{1.8} \approx 63 \)

**M1:** Uses \( T = 3 \Rightarrow N = a T^b \) with their \( a \) and \( b \). This is implied by an attempt at \( 63 \times 3^{2.3} \)

**A1:** Accept a number of microbes that are approximately 800. Allow \( 800 \pm 150 \) following correct work.

There is an alternative to this using a graphical approach.

**M1:** Finds the value of \( \log_{10} T \) from \( T = 3 \). Accept as \( T = 3 \Rightarrow \log_{10} T \approx 0.48 \)

**M1:** Then using the line of best fit finds the value of \( \log_{10} N \) from their "0.48"

Accept \( \log_{10} N \approx 2.9 \)

**M1:** Finds the value of \( N \) from their value of \( \log_{10} N \). \( \log_{10} N \approx 2.9 \Rightarrow N = 10^{2.9} \)

**A1:** Accept a number of microbes that are approximately 800. Allow \( 800 \pm 150 \) following correct work

### (c)

**M1** For using \( N = 1000000 \) and stating that \( \log_{10} N = 6 \)

**A1:** Statement to the effect that "we only have information for values of \( \log N \) between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate"

There is an alternative approach that uses the formula.

**M1:** Use \( N = 1000000 \) in their \( N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \frac{\log_{10} \left( \frac{1000000}{63} \right)}{2.3} \approx 1.83 \).

**A1:** The reason would be similar to the main scheme as we only have \( \log_{10} T \) values from 0 to 1.2. We cannot ‘extrapolate’ the graph and assume that the model still holds

### (d)

**B1:** Allow a numerical explanation \( T = 1 \Rightarrow N = a T^b \Rightarrow N = a \) giving \( a \) is the value of \( N \) at \( T = 1 \)
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<tbody>
<tr>
<td><strong>13(a)</strong></td>
<td>Attempts $\frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t} = (2\sqrt{3} \cos t)$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t}$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t} = (-\sqrt{3})$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Uses gradient of normal $= -\frac{1}{\frac{dy}{dx}} = \left(\frac{1}{\sqrt{3}}\right)$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$</td>
<td>A1*</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Substitutes $x = 2\cos t$ and $y = \sqrt{3} \cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Finds $\cos t = \frac{5}{6}, \frac{\sqrt{3}}{2}$</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t, y = \sqrt{3} \cos 2t,$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$Q = \left(\frac{5}{3}, \frac{7}{18} \sqrt{3}\right)$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6)</td>
<td></td>
</tr>
<tr>
<td><strong>(13 marks)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Question 13 continued

### Notes:

#### (a)

**M1:** Attempts \( \frac{dy}{dx} = \frac{dy}{dt} \) and achieves a form \( k \frac{\sin 2t}{\sin t} \) and achieves a form \( k \frac{\sin t \cos t}{\sin t} \). Alternatively candidates may apply the double angle identity for \( \cos 2t \) and achieve a form \( k \frac{\sin t \cos t}{\sin t} \).

**A1:** Scored for a correct answer, either \( \frac{\sqrt{3} \sin 2t}{\sin t} \) or \( 2\sqrt{3} \cos t \)

#### (b)

**M1:** For substituting \( t = \frac{2\pi}{3} \) in their \( \frac{dy}{dx} \) which must be in terms of \( t \).

**M1:** Uses the gradient of the normal is the negative reciprocal of the value of \( \frac{dy}{dx} \). This may be seen in the equation of \( l \).

**B1:** States or uses (in their tangent or normal) that \( P = \left( -1, -\frac{\sqrt{3}}{2} \right) \)

**M1:** Uses their numerical value of \( -1/\frac{dy}{dx} \) with their \( \left( -1, -\frac{\sqrt{3}}{2} \right) \) to form an equation of the normal at \( P \)

**A1\*:** This is a proof and all aspects need to be correct. Correct answer only \( 2x - 2\sqrt{3}y - 1 = 0 \)

#### (c)

**M1:** For substituting \( x = 2\cos t \) and \( y = \sqrt{3} \cos 2t \) into \( 2x - 2\sqrt{3}y - 1 = 0 \) to produce an equation in \( t \). Alternatively candidates could use \( \cos 2t = 2\cos^2 t - 1 \) to set up an equation of the form \( y = Ax^2 + B \).

**M1:** Uses the identity \( \cos 2t = 2\cos^2 t - 1 \) to produce a quadratic equation in \( \cos t \)

In the alternative method it is for combining their \( y = Ax^2 + B \) with \( 2x - 2\sqrt{3}y - 1 = 0 \) to get an equation in just one variable

**A1:** For the correct quadratic equation \( 12\cos^2 t - 4\cos t - 5 = 0 \) alternatively the equations in \( x \) and \( y \) are \( 3x^2 - 2x - 5 = 0 \) \( 12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0 \)

**M1:** Solves the quadratic equation in \( \cos t \) (or \( x \) or \( y \)) and rejects the value corresponding to \( P \).

**M1:** Substitutes their \( \cos t = \frac{5}{6} \) or their \( t = \arccos \left( \frac{5}{6} \right) \) in \( x = 2\cos t \) and \( y = \sqrt{3} \cos 2t \)

If a value of \( x \) or \( y \) has been found it is for finding the other coordinate.

**A1:** \( Q = \left( \frac{5}{3}, \frac{7}{18}, \sqrt{3} \right) \). Allow \( x = \frac{5}{3}, y = \frac{7}{18}, \sqrt{3} \) but do not allow decimal equivalents.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>14(a)</strong></td>
<td>Uses or implies $h = 0.5$</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>For correct form of the trapezium rule =</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{0.5}{2} \left{ 3 + 2.2958 + 2(2.3041 + 1.9242 + 1.9089) \right} = 4.393$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Any valid statement reason, for example</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>• Increase the number of strips</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Decrease the width of the strips</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use more trapezia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>For integration by parts on $\int x^2 \ln x , dx$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} , dx$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$\int -2x + 5 , dx = -x^2 + 5x \quad (+c)$</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>All integration attempted and limits used</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area of $S = \int_1^3 x^2 \ln x - 2x + 5 , dx = \left[ \frac{x^3}{9} \ln x - \frac{x^3}{27} - x^2 + 5x \right]_1^3$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Uses correct ln laws, simplifies and writes in required form</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area of $S = \frac{28}{27} + \ln 27 \quad (a = 28, b = 27, c = 27)$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10 marks)</td>
</tr>
</tbody>
</table>
### Question 14 continued

<table>
<thead>
<tr>
<th>Notes:</th>
</tr>
</thead>
</table>
| (a) B1: States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{\ldots\}$ in the trapezium rule  
M1: For the correct form of the bracket in the trapezium rule. Must be $y$ values rather than $x$ values \{first $y$ value + last $y$ value + $2x$(sum of other $y$ values)\}  
A1: 4.393  
(b) B1: See scheme  
(c) M1: Uses integration by parts the right way around.  
Look for $\int x^2 \ln x \, dx = A x^3 \ln x - \int B x^2 \, dx$  
A1: $\frac{x^3}{3} \ln x - \int \frac{x^2}{3} \, dx$  
B1: Integrates the $-2x + 5$ term correctly $= -x^2 + 5x$  
M1: All integration completed and limits used  
M1: Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$  
A1: Correct answer only $\frac{28}{27} + \ln 27$ |
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>15(a)</strong></td>
<td>Attempts to differentiate using the quotient rule or otherwise</td>
<td>M1 2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( f'(x) = \frac{e^{\sqrt{x-1}} \times 8 \cos 2x - 4 \sin 2x \times \sqrt{2} e^{\sqrt{x-1}}}{\left(e^{\sqrt{x-1}}\right)^2} )</td>
<td>A1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sets ( f'(x) = 0 ) and divides/ factorises out the ( e^{\sqrt{x-1}} ) terms</td>
<td>M1 2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proceeds via ( \frac{\sin 2x}{\cos 2x} = \frac{8}{4\sqrt{2}} ) to ( \Rightarrow \tan 2x = \sqrt{2}^* )</td>
<td>A1* 1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td><strong>(i)</strong> Solves ( \tan 4x = \sqrt{2} ) and attempts to find the 2(^{nd}) solution</td>
<td>M1 3.1a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x = 1.02 )</td>
<td>A1 1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(ii)</strong> Solves ( \tan 2x = \sqrt{2} ) and attempts to find the 1(^{st}) solution</td>
<td>M1 3.1a</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x = 0.478 )</td>
<td>A1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(8 marks)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**M1:** Attempts to differentiate by using the quotient rule with \( u = 4 \sin 2x \) and \( v = e^{\sqrt{x-1}} \) or alternatively uses the product rule with \( u = 4 \sin 2x \) and \( v = e^{\sqrt{x-1}} \)

**A1:** For achieving a correct \( f'(x) \). For the product rule

\[
 f'(x) = e^{1-\sqrt{x}} \times 8 \cos 2x + 4 \sin 2x \times -\sqrt{2} e^{1-\sqrt{x}}
\]

**M1:** This is scored for cancelling/ factorising out the exponential term. Look for an equation in just \( \cos 2x \) and \( \sin 2x \)

**A1:*:** Proceeds to \( \tan 2x = \sqrt{2} \). This is a given answer.

(b)(i)

**M1:** Solves \( \tan 4x = \sqrt{2} \) attempts to find the 2\(^{nd}\) solution. Look for \( x = \frac{\pi + \arctan \sqrt{2}}{4} \)

Alternatively finds the 2\(^{nd}\) solution of \( \tan 2x = \sqrt{2} \) and attempts to divide by 2

**A1:** Allow awrt \( x = 1.02 \). The correct answer, with no incorrect working scores both marks

(b)(ii)

**M1:** Solves \( \tan 2x = \sqrt{2} \) attempts to find the 1\(^{st}\) solution. Look for \( x = \frac{\arctan \sqrt{2}}{2} \)

**A1:** Allow awrt \( x = 0.478 \). The correct answer, with no incorrect working scores both marks
15(a) Attempts to differentiate using the quotient rule or otherwise

\[
\frac{21e^x}{8\cos^2 x - 4\sin^2 x} = \frac{2e^x}{1 - \cos 2x}
\]

M1

(b) (i) Solves \(\tan 2x = \frac{1}{2}\) and attempts to find the 2nd solution

M1

(ii) Solves \(\tan 2x = \frac{1}{2}\) and attempts to find the 1st solution

M1

Notes:

(a) M1: Attempts to differentiate by using the quotient rule with 

\[
\frac{4\sin^2 x}{21e^x - 12e^x}
\]

or alternatively uses the product rule with 

\[
4\sin^2 x
\]

and 

\[
21e^x - 12e^x
\]

A1:

For achieving a correct \(f(x)\). For the product rule

\[
12e^x - 12e^x
\]

M1:

This is scored for cancelling/factorising out the exponential term. Look for an equation in just \(\cos^2 x\) and \(\sin^2 x\). 

A1*:

Proceeds to \(\tan 2x = \frac{1}{2}\). This is a given answer.

(b)(ii) M1:

Solves \(\tan 2x = \frac{1}{2}\) and attempts to find the 1st solution. Look for 

\[
\arctan \left(\frac{\sqrt{2}}{2}\right) = x
\]

Alternatively finds the 2nd solution of \(\tan 2x = \frac{1}{2}\) and attempts to divide by 2

A1:

Allow awrt \(1.02x = 0\). The correct answer, with no incorrect working scores both marks

(b)(ii) M1:

Solves \(\tan 2x = \frac{1}{2}\) and attempts to find the 1st solution. Look for 

\[
\arctan \left(\frac{\sqrt{2}}{2}\right) = x
\]

A1:

Allow awrt \(0.478x = 0\). The correct answer, with no incorrect working scores both marks
Answer ALL questions. Write your answers in the spaces provided.

1. 

\[ f(x) = 2x^3 - 5x^2 + ax + a \]

Given that \( (x + 2) \) is a factor of \( f(x) \), find the value of the constant \( a \). 

(Total for Question 1 is 3 marks)
2. Some A level students were given the following question.

Solve, for $-90^\circ < \theta < 90^\circ$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos \theta = 2 \sin \theta$</td>
<td>$\cos \theta = 2 \sin \theta$</td>
</tr>
<tr>
<td>$\tan \theta = 2$</td>
<td>$\cos^2 \theta = 4 \sin^2 \theta$</td>
</tr>
<tr>
<td>$\theta = 63.4^\circ$</td>
<td>$1 - \sin^2 \theta = 4 \sin^3 \theta$</td>
</tr>
</tbody>
</table>

(a) Identify an error made by student A.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(Total for Question 2 is 3 marks)
3. Given \( y = x(2x + 1)^4 \), show that

\[
\frac{dy}{dx} = (2x + 1)^3 (Ax + B)
\]

where \( n, A \) and \( B \) are constants to be found.

(Total for Question 3 is 4 marks)
Given

\[ f(x) = e^x, \quad x \in \mathbb{R} \]
\[ g(x) = 3 \ln x, \quad x > 0, \quad x \in \mathbb{R} \]

(a) find an expression for \(gf(x)\), simplifying your answer. 

(b) Show that there is only one real value of \(x\) for which \(gf(x) = fg(x)\) 

(Total for Question 4 is 5 marks)
5. The mass, $m$ grams, of a radioactive substance, $t$ years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed, (2)

(b) show that $\frac{dm}{dt} = km$, where $k$ is a constant to be found. (2)

(Total for Question 5 is 4 marks)
6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Always True</th>
<th>Sometimes True</th>
<th>Never True</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>The quadratic equation $ax^2 + bx + c = 0, \ (a \neq 0)$ has 2 real roots.</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
<td>It only has 2 real roots when $b^2 - 4ac &gt; 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac &lt; 0$ it has 0 real roots.</td>
</tr>
<tr>
<td>(i) When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) If $ax &gt; b$ then $x &gt; \frac{b}{a}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii) The difference between consecutive square numbers is odd.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Total for Question 6 is 6 marks)
7. (a) Use the binomial expansion, in ascending powers of $x$, to show that

$$\sqrt{(4 - x)} = 2 - \frac{1}{4}x + kx^2 + ...$$

where $k$ is a rational constant to be found.

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of $x$. 

(Total for Question 7 is 5 marks)
8.

Figure 1 shows a rectangle $ABCD$.

The point $A$ lies on the $y$-axis and the points $B$ and $D$ lie on the $x$-axis as shown in Figure 1.

Given that the straight line through the points $A$ and $B$ has equation $5y + 2x = 10$

(a) show that the straight line through the points $A$ and $D$ has equation $2y - 5x = 4$  \hspace{1cm} (4)

(b) find the area of the rectangle $ABCD$. \hspace{1cm} (3)
Question 8 continued

(Total for Question 8 is 7 marks)
9. Given that $A$ is constant and

$$
\int_1^4 (3\sqrt{x} + A)\,dx = 2A^2
$$

show that there are exactly two possible values for $A$. 

(Total for Question 9 is 5 marks)
10. In a geometric series the common ratio is \( r \) and sum to \( n \) terms is \( S_n \). 

Given  

\[
S_\infty = \frac{8}{7} S_6
\]

show that \( r = \pm \frac{1}{\sqrt{k}} \), where \( k \) is an integer to be found.  

(Total for Question 10 is 4 marks)
11.

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of $f$  \hspace{1cm} (1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$  \hspace{1cm} (3)

Given that the equation $f(x) = k$, where $k$ is a constant, has two distinct roots,

c) state the set of possible values for $k$.  \hspace{1cm} (2)
(Total for Question 11 is 6 marks)
12. (a) Solve, for $-180^\circ \leq x < 180^\circ$, the equation
\[ 3 \sin^2 x + \sin x + 8 = 9 \cos^2 x \]
giving your answers to 2 decimal places. \hfill (6)

(b) Hence find the smallest positive solution of the equation
\[ 3 \sin^2(2\theta - 30^\circ) + \sin(2\theta - 30^\circ) + 8 = 9 \cos^2(2\theta - 30^\circ) \]
giving your answer to 2 decimal places. \hfill (2)
Question 12 continued

(Total for Question 12 is 8 marks)
13. (a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$.

Give the exact value of $R$ and give the value of $\alpha$, in degrees, to 2 decimal places.

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?
Question 13 continued

(Total for Question 13 is 9 marks)
14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius $r$ cm and height $h$ cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, $S$ cm$^2$, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

Given that $r$ can vary,

(b) find the dimensions of a can that has minimum surface area.

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.
(Total for Question 14 is 9 marks)
Figure 4 shows a sketch of the curve $C$ with equation

$$y = \frac{3}{5}x^2 - 9x + 11, \ x \geq 0$$

The point $P$ with coordinates $(4, 15)$ lies on $C$.

The line $l$ is the tangent to $C$ at the point $P$.

The region $R$, shown shaded in Figure 4, is bounded by the curve $C$, the line $l$ and the $y$-axis.

Show that the area of $R$ is 24, making your method clear.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*
Figure 4 shows a sketch of the curve \( y = x^2 + 3x + 11 \). The line \( \text{C} \) at the point \( P \) with coordinates \((4, 15)\) lies on \( \text{R} \), shown shaded in Figure 4, is bounded by the curve \( \text{R} \) is 24, making your method clear. Solutions based entirely on graphical or numerical methods are not acceptable.

Question 15 continued

(Total for Question 15 is 10 marks)
A population of meerkats is being studied.

The population is modelled by the differential equation

\[ \frac{dP}{dt} = \frac{1}{22} P(11 - 2P), \quad t \geq 0, \quad 0 < P < 5.5 \]

where \( P \), in thousands, is the population of meerkats and \( t \) is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(c) show that

\[ P = \frac{A}{B + Ce^{-\frac{t}{2}}} \]

where \( A, B \) and \( C \) are integers to be found.
Question 16 continued
Question 16 continued

(Total for Question 16 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS
### Paper 2: Pure Mathematics 2 Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>Sets ( f(-2) = 0 \Rightarrow 2(-2)^3 - 5(-2)^2 + a(-2) + a = 0 )</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Solves linear equation ( 2a - a = -36 \Rightarrow a = )</td>
<td>dM1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow a = -36 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(3 marks)

**Notes:**

**M1:** Selects a suitable method given that \((x + 2)\) is a factor of \(f(x)\)
Accept either setting \(f(-2) = 0\) or attempted division of \(f(x)\) by \((x + 2)\)

**dM1:** Solves linear equation in \(a\). Minimum requirement is that there are two terms in 'a' which must be collected to get ..\(a = \ldots \Rightarrow a = \ldots\)

**A1:** \(a = -36\)

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td>Identifies an error for student A: They use (\frac{\cos \theta}{\sin \theta} = \tan \theta) (\frac{\sin \theta}{\cos \theta} = \tan \theta) (\cos \theta = \frac{\sin \theta}{\cos \theta})</td>
<td>B1</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>It should be (\frac{\sin \theta}{\cos \theta} = \tan \theta)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>(i) Shows (\cos(-26.6^\circ) \neq 2\sin(-26.6^\circ)), so cannot be a solution</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>(ii) Explains that the incorrect answer was introduced by squaring</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(3 marks)

**Notes:**

(a)

**B1:** Accept a response of the type 'They use \(\frac{\cos \theta}{\sin \theta} = \tan \theta\). This is incorrect as \(\frac{\sin \theta}{\cos \theta} = \tan \theta\)'

It can be implied by a response such as 'They should get \(\tan \theta = \frac{1}{2}\) not \(\tan \theta = 2\)'

Accept also statements such as 'it should be \(\cot \theta = 2\)'

(b)

**B1:** Accept a response where the candidate shows that \(-26.6^\circ\) is not a solution of \(\cos \theta = 2\sin \theta\). This can be shown by, for example, finding both \(\cos(-26.6^\circ)\) and \(2\sin(-26.6^\circ)\) and stating that they are not equal. An acceptable alternative is to state that \(\cos(-26.6^\circ) = +ve\) and \(2\sin(-26.6^\circ) = -ve\) and stating that they therefore cannot be equal.

**B1:** Explains that the incorrect answer was introduced by squaring. Accept an example showing this. For example \(x = \pm 5\) squared gives \(x^2 = 25\) which has answers \(\pm 5\)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Attempts the product and chain rule on $y = x(2x + 1)^4$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = (2x + 1)^4 + 8x(2x + 1)^3$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Takes out a common factor $\frac{dy}{dx} = (2x + 1)^3 { (2x + 1) + 8x }$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = (2x + 1)^3 (10x + 1) \Rightarrow n = 3, A = 10, B = 1$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(4 marks)

**Notes:**

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x + 1)^4 + Bx(2x + 1)^3$

A1: $\frac{dy}{dx} = (2x + 1)^4 + 8x(2x + 1)^3$

M1: Takes out a common factor of $(2x + 1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x + 1)^3 (10x + 1) \Rightarrow n = 3, A = 10, B = 1$
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 (a)</strong></td>
<td>gf((x) = 3\ln e^x)</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[= 3x, \ (x \in \mathbb{R})]</td>
<td>A1 1.1b</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>gf((x) = fg(x) \Rightarrow 3x = x^3)</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\Rightarrow x^3 - 3x = 0 \Rightarrow x =]</td>
<td>M1 1.1b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\Rightarrow x = (\pm)\sqrt[3]{3}] Only as (\ln x) is not defined at (x = 0) and (-\sqrt[3]{3})</td>
<td>M1 2.2a</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a) 
**M1:** For applying the functions in the correct order  
**A1:** The simplest form is required so it must be \(3x\) and not left in the form \(3\ln e^x\)  
An answer of \(3x\) with no working would score both marks

(b) 
**M1:** Allow the candidates to score this mark if they have \(e^{3\ln x}\) = their \(3x\)  
**M1:** For solving their cubic in \(x\) and obtaining at least one solution.  
**A1:** For either stating that \(x = \sqrt[3]{3}\) only as \(\ln x\) (or \(3\ln x\)) is not defined at \(x = 0\) and \(-\sqrt[3]{3}\)  
or stating that \(3x = x^3\) would have three answers, one positive one negative and one zero but \(\ln x\) (or \(3\ln x\)) is not defined for \(x \leq 0\) so therefore there is only one (real) answer.  
Note: Student who mix up \(fg\) and \(gf\) can score full marks in part (b) as they have already been penalised in part (a)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
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<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(a)</td>
<td>Substitutes $t = 0.5$ into $m = 25e^{-0.05t}$ ( \Rightarrow ) $m = 25e^{-0.05\times0.5}$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow m = 24.4g )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(b)</td>
<td>States or uses ( \frac{d}{dt} \left( e^{-0.05t} \right) = \pm C e^{-0.05t} )</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>( \frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**M1:** Substitutes $t = 0.5$ into $m = 25e^{-0.05t}$ \( \Rightarrow \) $m = 25e^{-0.05\times0.5}$

**A1:** $m = 24.4g$ An answer of $m = 24.4g$ with no working would score both marks

(b)

**M1:** Applies the rule \( \frac{d}{dt} \left( e^{kx} \right) = k e^{kx} \) in this context by stating or using \( \frac{d}{dt} \left( e^{-0.05t} \right) = \pm C e^{-0.05t} \)

**A1:** \( \frac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05 \)
<table>
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</tr>
</thead>
<tbody>
<tr>
<td>6(i)</td>
<td>( x^2 - 6x + 10 = (x-3)^2 + 1 )</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Deduces &quot;always true&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>as ((x-3)^2 \geq 0 \Rightarrow (x-3)^2 + 1 \geq 1) and so is always positive</td>
<td>A1</td>
<td>2.2a</td>
</tr>
<tr>
<td>(ii)</td>
<td>For an explanation that it need not (always) be true</td>
<td>M1</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>This could be if (a &lt; 0) then (ax &gt; b \Rightarrow x &lt; \frac{b}{a})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>States 'sometimes' and explains if (a &gt; 0) then (ax &gt; b \Rightarrow x &gt; \frac{b}{a}) if (a &lt; 0) then (ax &gt; b \Rightarrow x &lt; \frac{b}{a})</td>
<td>A1</td>
<td>2.4</td>
</tr>
<tr>
<td>(iii)</td>
<td>Difference = ((n+1)^2 - n^3 = 2n + 1)</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Deduces &quot;Always true&quot; as (2n+1 = \text{(even} +1)) = odd</td>
<td>A1</td>
<td>2.2a</td>
</tr>
</tbody>
</table>

**Notes:**

(i) **M1:** Attempts to complete the square or any other valid reason. Allow for a graph of \(y = x^2 - 6x + 10\) or an attempt to find the minimum by differentiation

(ii) **A1:** States always true with a valid reason for their method

(iii) **M1:** For an explanation that it need not be true (sometimes). This could be if

\[
a < 0 \text{ then } ax > b \Rightarrow x < \frac{b}{a}\text{ or simply } -3x > 6 \Rightarrow x < -2
\]

(iv) **A1:** Correct statement (sometimes true) and explanation

(iii) **M1:** Sets up the proof algebraically.

For example by attempting \((n+1)^2 - n^3 = 2n + 1\) or \(m^2 - n^2 = (m-n)(m+n)\) with \(m = n + 1\)

**A1:** States always true with reason and proof

Accept a proof written in words. For example

If integers are consecutive, one is odd and one is even

When squared odd\(\times\)odd = odd and even\(\times\)even = even

The difference between odd and even is always odd, hence always true

Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.
<table>
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<tr>
<th>Question</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>7(a)</strong></td>
<td>[ \sqrt{(4-x)} = 2 \left(1 - \frac{1}{4}x \right)^{\frac{1}{2}} ]</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>[ \left(1 - \frac{1}{4}x \right)^{\frac{1}{2}} = 1 + \frac{1}{2} \left(\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)!}{2!} \left(\frac{1}{4}x\right)^2 + ... ]</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>[ \sqrt{(4-x)} = 2 \left(1 - \frac{1}{8}x + \frac{1}{128}x^2 + ... \right) ]</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>[ \sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + ... \text{ and } k = -\frac{1}{64} ]</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(b)</td>
<td>The expansion is valid for (</td>
<td>x</td>
<td>&lt; 4), so (x = 1) can be used</td>
</tr>
<tr>
<td></td>
<td>(5 marks)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**M1:** Takes out a factor of 4 and writes \[ \sqrt{(4-x)} = 2 \left(1 \pm ... \right)^{\frac{1}{2}} \]

**M1:** For an attempt at the binomial expansion with \( n = \frac{1}{2} \)

\[ (1 + ax)^{\frac{1}{2}} = 1 + \frac{1}{2} (ax) + \frac{\left(\frac{1}{2}\right)!}{2!} (ax)^2 + ... \]

**A1:** Correct expression inside the bracket \( 1 - \frac{1}{8}x + \frac{1}{128}x^2 + ... \) which may be left unsimplified

**A1:** \[ \sqrt{(4-x)} = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + ... \text{ and } k = -\frac{1}{64} \]

(b)

**B1:** The expansion is valid for \(|x| < 4\), so \(x = 1\) can be used
<table>
<thead>
<tr>
<th>Question</th>
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<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8 (a)</strong></td>
<td>Gradient $AB = -\frac{2}{5}$</td>
<td>B1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$y$ coordinate of $A$ is 2</td>
<td>B1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Uses perpendicular gradients $y = \frac{5}{2}x + c$</td>
<td>M1</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 2y - 5x = 4$ *</td>
<td>A1*</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Uses Pythagoras' theorem to find $AB$ or $AD$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{16}{25}}$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>area $ABCD = 11.6$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

NOTES:

(a) It is important that the student communicates each of these steps clearly

**B1:** States the gradient of $AB$ is $-\frac{2}{5}$

**B1:** States that $y$ coordinate of $A = 2$

**M1:** Uses the form $y = mx + c$ with $m$ = their adapted $-\frac{2}{5}$ and $c$ = their 2

Alternatively uses the form $y - y_i = m(x - x_i)$ with $m$ = their adapted $-\frac{2}{5}$ and $(x_i, y_i) = (0, 2)$

**A1*:** Proceeds to given answer

(b) **M1:** Finds the lengths of $AB$ or $AD$ using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$

Alternatively finds the lengths $BD$ and $AO$ using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2

**M1:** For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2} \times 5.8 \times 2$

**A1:** Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$
### Question 9

\[
\int (3x^{0.5} + A) \, dx = 2x^{1.5} + Ax\left(+c\right)
\]

**Notes:**

- **M1:** Integrates the given function and achieves an answer of the form \( kx^{1.5} + Ax\left(+c\right) \) where \( k \) is a non-zero constant.
- **A1:** Correct answer but may not be simplified.

**Notes:**

- **M1:** Substitutes in limits and subtracts. This can only be scored if \( \int A \, dx = Ax \) and not \( \frac{A^2}{2} \).
- **A1:** Either \( A = -2, \frac{7}{2} \) and states that there are two roots.

\[
\Rightarrow A = -2, \frac{7}{2}
\]

States \( b^2 - 4ac \geq 121 > 0 \) and hence there are two roots.

\( (5 \text{ marks}) \)

### Question 10

\[
\text{Attempts } S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}
\]

\[
\Rightarrow 1 = \frac{8}{7} \times (1-r^6)
\]

\[
\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..
\]

\[
\Rightarrow r = \pm \frac{1}{\sqrt{2}} \text{ (so } k = 2)\]

**Notes:**

- **M1:** Substitutes the correct formulae for \( S_\infty \) and \( S_6 \) into the given equation \( S_\infty = \frac{8}{7} \times S_6 \).
- **M1:** Proceeds to an equation just in \( r \).
- **M1:** Solves using a correct method.
- **A1:** Proceeds to \( r = \pm \frac{1}{\sqrt{2}} \) giving \( k = 2 \).

\( (4 \text{ marks}) \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>11 (a)</strong></td>
<td>$f(x) \geq 5$</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Uses $-2(3-x) + 5 = \frac{1}{2}x + 30$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Attempts to solve by multiplying out bracket, collect terms etc</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2}x = 31$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = \frac{62}{3}$ only</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Makes the connection that there must be two intersections. Implied by either end point $k &gt; 5$ or $k \leq 11$</td>
<td>M1</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>${k : k \in \mathbb{R}, 5 &lt; k \leq 11}$</td>
<td>A1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Notes:**

(a) B1: $f(x) \geq 5$ Also allow $f(x) \in [5, \infty)$

(b) M1: Deduces that the solution to $f(x) = \frac{1}{2}x + 30$ can be found by solving $-2(3-x) + 5 = \frac{1}{2}x + 30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x = \frac{62}{3}$ only. Do not allow 20.6

(c) M1: Deduces that two distinct roots occurs when $y = k$ intersects $y = f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k > 5$ or $k \leq 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5 < k \leq 11\}$
Question | Scheme | Marks | AOs
--- | --- | --- | ---
12(a) | Uses $\cos^2 x = 1 - \sin^2 x$ $\Rightarrow$ $3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$ | M1 | 3.1a

$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$ | A1 | 1.1b

$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$ | M1 | 1.1b

$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$ | A1 | 1.1b

Uses arcsin to obtain two correct values | M1 | 1.1b

All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$ | A1 | 1.1b

(6)

(b) | Attempts $2\theta - 30^\circ = -19.47^\circ$ | M1 | 3.1a

$\Rightarrow \theta = 5.26^\circ$ | A1ft | 1.1b

(2)

(8 marks)

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48^\circ, 165.52^\circ, -19.47^\circ, -160.53^\circ$

(b)

M1: For setting $2\theta - 30^\circ = \text{their}^\circ - 19.47^\circ$

A1ft: $\theta = 5.26^\circ$ but allow a follow through on their $^\circ - 19.47^\circ$
### Question 13(a)

$$R = \sqrt{109}$$

$$\tan \alpha = \frac{3}{10}$$

$$\alpha = 16.70^\circ \text{ so } \sqrt{109} \cos(\theta + 16.70^\circ)$$

(3)

### (b)

(i) e.g. $$H = 11 - 10 \cos(80^\circ) + 3 \sin(80^\circ)$$ or

$$H = 11 - \sqrt{109} \cos(80^\circ + 16.70^\circ)$$

B1 3.3

(ii) $$11 + \sqrt{109}$$ or 21.44 m

B1ft 3.4

(2)

### (c)

Sets $$80t + "16.70" = 540$$

$$t = \frac{540 - "16.70"}{80} = (6.54)$$

$$t = 6 \text{ mins } 32 \text{ seconds}$$

A1 1.1b

(3)

### (d)

Increase the ‘80’ in the formula

For example use $$H = 11 - 10 \cos(90^\circ) + 3 \sin(90^\circ)$$

B1 1.1b

(1)

### Notes:

(a)  

B1:  $$R = \sqrt{109}$$  Do not allow decimal equivalents

M1:  Allow for $$\tan \alpha = \pm \frac{3}{10}$$

A1:  $$\alpha = 16.70^\circ$$

(b)(i)  

B1:  see scheme

(b)(ii)  

B1ft:  their 11 + their $$\sqrt{109}$$  Allow decimals here.

(c)  

M1:  Sets $$80t + "16.70" = 540$$. Follow through on their 16.70

M1:  Solves their $$80t + "16.70" = 540$$ correctly to find $$t$$

A1:  $$t = 6 \text{ mins } 32 \text{ seconds}$$

(d)  

B1:  States that to increase the speed of the wheel the 80’s in the equation would need to be increased.
<table>
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<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>14(a)</strong></td>
<td>Sets $500 = \pi r^2 h$</td>
<td>B1</td>
<td>2.1</td>
</tr>
<tr>
<td>Substitute $h = \frac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$</td>
<td>M1</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Simplifies to reach given answer $S = 2\pi r^2 + \frac{1000}{r}$</td>
<td>A1*</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Differentiates $S$ with both indices correct in $\frac{dS}{dr}$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td>$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant</td>
<td>M1</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Radius = 4.30 cm</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>Substitutes their $r = 4.30$ into $h = \frac{500}{\pi r^2}$ $\Rightarrow$ Height = 8.60 cm</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>States a valid reason such as</td>
<td>B1</td>
<td>3.2a</td>
</tr>
<tr>
<td>- The radius is too big for the size of our hands</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- The radius is too big for us to drink from</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>(1)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**B1:** Uses the correct volume formula with $V = 500$. Accept $500 = \pi r^2 h$

**M1:** Substitutes $h = \frac{500}{\pi r^2}$ or $rh = \frac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get $S$ as a function of $r$

**A1:** $S = 2\pi r^2 + \frac{1000}{r}$ Note that this is a given answer.

(b)

**M1:** Differentiates the given $S$ to reach $\frac{dS}{dr} = Ar \pm Br^{-2}$

**A1:** $\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$ or exact equivalent

**M1:** Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant

**A1:** $R =$ awrt 4.30 cm

**A1:** $H =$ awrt 8.60 cm

(c)

**B1:** Any valid reason. See scheme for alternatives
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>$\frac{dy}{dx} = \frac{15}{2} x^\frac{3}{2} - 9$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Uses $(4, 15)$ and gradient $\Rightarrow y - 15 = 6(x - 4)$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Equation of $l$ is $y = 6x - 9$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Area $R = \int_0^4 \left(5x^\frac{3}{2} - 9x + 11\right) - (6x - 9) , dx$</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>$= \left[2x^\frac{5}{2} - \frac{15}{2}x^2 + 20x + c\right]_0^4$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Uses both limits of 4 and 0</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Area of $R = 24^*$</td>
<td>A1*</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Correct notation with good explanations</td>
<td>A1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

(10 marks)
**Question 15 continued**

**Notes:**

<table>
<thead>
<tr>
<th>M1:</th>
<th>Differentiates $5x^2 - 9x + 11$ to a form $Ax^2 + B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1:</td>
<td>$\frac{dy}{dx} = 15 \frac{1}{2} x^2 - 9$ but may not be simplified</td>
</tr>
</tbody>
</table>

| M1: | Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent |
| M1: | Uses their gradient and the point (4, 15) to find the equation of the tangent |
| A1: | Equation of $l$ is $y = 6x - 9$ |

| M1: | Uses Area $R = \int_0^4 \left(5x^2 - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$ |

Look for a form $Ax^2 + Bx^2 + Cx$

| A1: | $\left[2x^2 - \frac{15}{2} x^2 + 20x\left(+c\right)\right]^4_0$ This must be correct but may not be simplified |

| M1: | Substitutes in both limits and subtracts |
| A1*: | Correct area for $R = 24$ |

A1: Uses correct notation and produces a well explained and accurate solution. Look for
- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of $l$. See scheme.
- Correct explanation in finding the area of $R$. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

| M1: | Area under curve $= \int_0^4 \left(5x^2 - 9x + 11\right) = \left[5x^2 + Bx^2 + Cx\right]^4_0$ |
| A1: | $\left[2x^2 - \frac{9}{2} x^2 + 11x\right]^4_0 = 36$ |

| M1: | This requires a full method with all triangles found using a correct method |

Look for Area $R =$ their $36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$
### Question 16(a)

Sets \( \frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)} \)

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sets ( \frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1.1a</td>
</tr>
</tbody>
</table>

Substitutes either \( P = 0 \) or \( P = \frac{11}{2} \) into \( 1 = A(11-2P) + BP \Rightarrow A \) or \( B \)

\[
\frac{1}{P(11-2P)} = \frac{1}{11} + \frac{2}{11(11-2P)}
\]

| A1     | 1.1b |

(3)

### Question 16(b)

Separates the variables

\[
\int \frac{22}{P(11-2P)} \, dP = \int 1 \, dt
\]

Uses (a) and attempts to integrate

\[
\int \frac{2}{P} + \frac{4}{(11-2P)} \, dP = t + c
\]

<table>
<thead>
<tr>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>3.1a</td>
</tr>
</tbody>
</table>

\[
2 \ln P - 2 \ln (11-2P) = t + c
\]

| A1     | 1.1b |

Substitutes \( t = 0, P = 1 \Rightarrow t = 0, P = 1 \Rightarrow c = (-2 \ln 9) \)

Substitutes \( P = 2 \Rightarrow t = 2 \ln 2 + 2 \ln 9 - 2 \ln 7 \)

Time = 1.89 years

| A1     | 3.2a |

(6)

### Question 16(c)

Uses ln laws

\[
2 \ln P - 2 \ln (11-2P) = t - 2 \ln 9
\]

\[
\Rightarrow \ln \left( \frac{9P}{11-2P} \right) = \frac{1}{2} t
\]

<table>
<thead>
<tr>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Makes 'P' the subject

\[
\Rightarrow \left( \frac{9P}{11-2P} \right) = e^{\frac{1}{2} t}
\]

\[
\Rightarrow 9P = (11-2P)e^{\frac{1}{2} t}
\]

\[
\Rightarrow P = \frac{11}{2 + 9e^{\frac{1}{2} t}} \Rightarrow A = 11, B = 2, C = 9
\]

| A1     | 1.1b |

(3)

(12 marks)
Question 16 continued

Notes:

(a)

B1: \[ \frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)} \]

M1: Substitutes \( P = 0 \) or \( P = \frac{11}{2} \) into \( 1 = A(11-2P) + BP \Rightarrow A \) or \( B \)

Alternatively compares terms to set up and solve two simultaneous equations in \( A \) and \( B \)

A1: \[ \frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)} \quad \text{or equivalent} \quad \frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)} \]

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach \( \int \frac{22}{P(11-2P)} \, dP = \int \frac{1}{11P} \, dt \) or equivalent

M1: Uses part (a) and \( \int \frac{A}{P} + \frac{B}{(11-2P)} \, dP = A \ln P \pm C \ln(11-2P) \)

A1: Integrates both sides to form a correct equation including a 'c' Eg \[ 2 \ln P - 2 \ln (11-2P) = t + c \]

M1: Substitutes \( t = 0 \) and \( P = 1 \) to find \( c \)

M1: Substitutes \( P = 2 \) to find \( t \). This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from \( 2 \ln P - 2 \ln (11-2P) = t + c \) to \( \ln \left( \frac{P}{11-2P} \right) = \frac{1}{2} t + d \)

for their numerical 'c'

M1: Uses a correct method to get \( P \) in terms of \( e^{\frac{1}{2}} \)

This can be achieved from \( \ln \left( \frac{P}{11-2P} \right) = \frac{1}{2} t + d \Rightarrow \frac{P}{11-2P} = e^{\frac{1}{2} t + d} \) followed by cross multiplication and collection of terms in \( P \) (See scheme)

Alternatively uses a correct method to get \( P \) in terms of \( e^{-\frac{1}{2}} \) For example

\[ \frac{P}{11-2P} = e^{\frac{1}{2} t + d} \Rightarrow \frac{11-2P}{P} = e^{\frac{1}{2} t + d} \Rightarrow \frac{11}{P} - 2 = e^{\frac{1}{2} t + d} \Rightarrow \frac{11}{P} = 2 + e^{\frac{1}{2} t + d} \]

followed by division

A1: Achieves the correct answer in the form required. \( P = \frac{11}{2 + 9e^{-\frac{1}{2}}} \Rightarrow A = 11, B = 2, C = 9 \) oe
(a) B1: Sets

\[
(11 - 2) \ (11 - 2) = A \ P \ P \ P \ \Rightarrow \ M1: \ S \ u b s t i t u t e s \ 0 \ P = 0 \ or \ 1 \ 2 \ P = 1 \ I n t o
\]

\[
1 (11 - 2 ) A P B P A o r B \Rightarrow \ A 1: \ \text{Substitutes } t = 0 \ and \ \text{P} = 1 \ t o \ f i n d \ c \ \text{M} 1: \ S \ u b s t i t u t e s \ \text{P} = 2 \ t o \ f i n d \ t . \ \text{This is dependent upon having scored both previous M.'s}
\]

(c) M 1: Uses correct log laws to move from

\[
(2 \ln 2 - \ln 1) \ P \ t \ c \ \Rightarrow \ M 1: \ U s e s \ a \ c o r r e c t \ m e t h o d \ t o \ g e t \ P \ i n \ t e r m s \ o f \ 1 \ 2 e \ t \ (S e e \ s c h e m e)
\]

Alternatively uses a correct method to get

\[
P \ in \ t e r m s \ o f \ 1 \ 2 e \ t \ \Rightarrow \ A 1: \ \text{A c h i e v e s \ t h e \ c o r r e c t \ a n s w e r \ i n \ t h e \ f o r m \ r e q u i r e d}.
\]

\[
11 1 1 22 \ \ctd \ P \ A B \ P P P \ \Rightarrow \ A 1: \ \text{Achieves the correct answer in the form required}.
\]

\[
11 1 1 22 \ \ctd \ P \ A B \ P P P \ \Rightarrow \ A 1: \ \text{Achieves the correct answer in the form required}.
\]

\[
11 1 1 22 \ \ctd \ P \ A B \ P P P \ \Rightarrow \ A 1: \ \text{Achieves the correct answer in the form required}.
\]

\[
11 1 1 22 \ \ctd \ P \ A B \ P P P \ \Rightarrow \ A 1: \ \text{Achieves the correct answer in the form required}.
\]

\[
11 1 1 22 \ \ctd \ P \ A B \ P P P \ \Rightarrow \ A 1: \ \text{Achieves the correct answer in the form required}.
\]
SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. The number of hours of sunshine each day, \( y \), for the month of July at Heathrow are summarised in the table below.

<table>
<thead>
<tr>
<th>Hours</th>
<th>( 0 \leq y &lt; 5 )</th>
<th>( 5 \leq y &lt; 8 )</th>
<th>( 8 \leq y &lt; 11 )</th>
<th>( 11 \leq y &lt; 12 )</th>
<th>( 12 \leq y &lt; 14 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>6</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

A histogram was drawn to represent these data. The \( 8 \leq y < 11 \) group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the \( 0 \leq y < 5 \) group. (3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures. (3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectively. Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas’ belief. (2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean. (2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by \( N(6.6, 3.7^2) \).

(e) Use Helen’s model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean. (2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen’s model. (1)
2. A meteorologist believes that there is a relationship between the daily mean windspeed, \( w \) kn, and the daily mean temperature, \( t \) °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

<table>
<thead>
<tr>
<th>( t )</th>
<th>13.3</th>
<th>16.2</th>
<th>15.7</th>
<th>16.6</th>
<th>16.3</th>
<th>16.4</th>
<th>19.3</th>
<th>17.1</th>
<th>13.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>7</td>
<td>11</td>
<td>8</td>
<td>11</td>
<td>13</td>
<td>8</td>
<td>15</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained \( r = 0.609 \)

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days a location from the large data set gave \( \bar{t} = 27.2 \) and \( \bar{w} = 3.5 \)

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)
Question 2 continued

(Total for Question 2 is 6 marks)
3. A machine cuts strips of metal to length $L$ cm, where $L$ is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips cannot be used.

A second machine cuts strips of metal of length $X$ cm, where $X$ is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm
A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm. Given that 2.5% of the cut lengths exceed 50.98 cm, Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Question 3 continued

(Total for Question 3 is 12 marks)
4. Given that

\[ P(A) = 0.35 \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13 \]

find

(a) \( P(A' \mid B') \) \hspace{2cm} (2)

(b) Explain why the events \( A \) and \( B \) are not independent. \hspace{2cm} (1)

The event \( C \) has \( P(C) = 0.20 \)

The events \( A \) and \( C \) are mutually exclusive and the events \( B \) and \( C \) are statistically independent.

(c) Draw a Venn diagram to illustrate the events \( A, B \) and \( C \), giving the probabilities for each region. \hspace{2cm} (5)

(d) Find \( P( [B \cup C]' \) \hspace{2cm} (2)
Question 4 continued

(Total for Question 4 is 10 marks)
5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company’s claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company’s claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(e) Using your answer to part (d), comment on whether or not the proportion of the company’s pea seeds that germinate is different from the company’s claim of 55%
Question 5 continued

(Total for Question 5 is 9 marks)

TOTAL FOR SECTION A IS 50 MARKS
SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of \( g \) is required, take \( g = 9.8 \, \text{m s}^{-2} \) and give your answer to either 2 significant figures or 3 significant figures.

6. At time \( t \) seconds, where \( t \geq 0 \), a particle \( P \) moves so that its acceleration \( a \, \text{m s}^{-2} \) is given by

\[
a = 5t \mathbf{i} - 15t^2 \mathbf{j}
\]

When \( t = 0 \), the velocity of \( P \) is \( 20 \mathbf{i} \, \text{m s}^{-1} \)

Find the speed of \( P \) when \( t = 4 \)

(6)
Question 6 continued

(Total for Question 6 is 6 marks)
7. A rough plane is inclined to the horizontal at an angle \( \alpha \), where \( \tan \alpha = \frac{3}{4} \).

A particle of mass \( m \) is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is \( \mu \).

The particle moves up the plane with a constant deceleration of \( \frac{4}{5} g \).

(a) Find the value of \( \mu \).  

(b) Determine whether the particle will remain at \( A \), carefully justifying your answer.
(b) Determine whether the particle will remain at rest.

The particle comes to rest at the point P.

(a) Find the value of the frictional force acting on the particle.

The particle moves up the plane with a constant deceleration of $\alpha = \frac{g}{\mu}$. The coefficient of friction between the particle and the plane is $\mu$.

A particle of mass $m$ is placed on the plane and then projected up a line of greatest slope, carefully justifying your answer.

Question 7 continued

(Total for Question 7 is 8 marks)
8. [In this question \( \mathbf{i} \) and \( \mathbf{j} \) are horizontal unit vectors due east and due north respectively]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time \( t = 0 \), the boat is at the fixed point \( O \) and is moving due north with speed 0.6 m s\(^{-1}\).

Relative to \( O \), the position vector of the boat at time \( t \) seconds is \( \mathbf{r} \) metres.

At time \( t = 15 \), the velocity of the boat is \((10.5\mathbf{i} - 0.9\mathbf{j})\) m s\(^{-1}\).

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is \((0.7\mathbf{i} - 0.1\mathbf{j})\) m s\(^{-2}\).

(b) Find \( \mathbf{r} \) in terms of \( t \).

(c) Find the value of \( t \) when the boat is north-east of \( O \).

(d) Find the value of \( t \) when the boat is moving in a north-east direction.
Question 8 continued

The boat is modelled as a particle.

A radio controlled model boat is placed on the surface of a large pond.

\[ \vec{r} \] is the position vector of the boat at time \( t \) seconds is \( \text{metres} \).

At time \( t \) when the boat is moving in a north-east direction.

Relative to \( O \), the position vector of the boat at time \( t \) and is moving due north with speed 0.6 m s\(^{-1}\).

The acceleration of the boat is (0.7 \( i \) - 0.1 \( j \)) m s\(^{-2}\).

The acceleration of the boat is constant.

(b) Find \( \vec{r} \) when the boat is north-east of \( O \).  

(c) Find the value of \( \text{when the boat is north-east of } O \).  

(d) Find the value of \( \text{when the boat is moving in a north-east direction}.  

(Total for Question 8 is 10 marks)
A uniform ladder $AB$, of length $2a$ and weight $W$, has its end $A$ on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end $B$ of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.

To stop the ladder from slipping, the builder’s assistant applies a horizontal force of magnitude $P$ to the ladder at $A$, towards the wall.

The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle $\alpha$ with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at $B$ has magnitude $3W$. 

(b) Find, in terms of $W$, the range of possible values of $P$ for which the ladder remains in equilibrium. 

Often in practice, the builder’s assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.
Question 9 continued
Question 9 continued
Question 9 continued

(Total for Question 9 is 13 marks)
A boy throws a stone with speed $U\text{ms}^{-1}$ from a point $O$ at the top of a vertical cliff. The point $O$ is 18 m above sea level.

The stone is thrown at an angle $\alpha$ above the horizontal, where \(\tan \alpha = \frac{3}{4}\).

The stone hits the sea at the point $S$ which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.

The stone is modelled as a particle moving freely under gravity with $g = 10\text{ms}^{-2}$.

Find

(a) the value of $U$,  
(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(c) Suggest two improvements that could be made to the model.
Question 10 continued
Question 10 continued
Question 10 continued

(Total for Question 10 is 13 marks)

TOTAL FOR SECTION B IS 50 MARKS
TOTAL FOR PAPER IS 100 MARKS
Area = \(28 \times \frac{1.5}{12} \text{ cm}^2\) so frequency = 8 so \(21 \text{ cm}^2\) = \(2.3\) hour (o.e.)

M1

Frequency of 12 corresponds to area of 18 so height = \(\frac{18}{2.5} = 7.2 \text{ (cm)}\)

A1

Width = \(5 \times \frac{0.5}{2.5} = 2.5 \text{ (cm)}\)

B1

cao

(a)(i)

\[y = 205.5 \pm 6.6331\]

\[y = 21785.25 - 31 \pm 13.644641\]

allow \(\pm 3.69\)

M1

A1

1.1a

1.1b

(b)

Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable

M1

2.4

Hurn is South of Heathrow so does not support his belief

A1

2.2b

(d)

\(10.3 \times \sigma \approx\) so number of days is e.g. 
\[\left(11 - 10.3\right) \times 8 \times 5 = 3\]

M1

1

1.1b

= 6.86 so 7 days

A1

1.1b

(e)

\[H = \text{no. of hours}\]

\[P(H > 10.3) = \frac{1}{0.15865...} = 6.9\]

M1

3.4

Predict \(31 \times 0.15865... = 4.9\) or 5 days

A1

1.1b

(f)

(5 or ) 4.9 days < (7 or) 6.9 days so model may not be suitable

B1

3.5a

(1)

(13 marks)
1(a) Area \(= 8 \times 1.5 = 12 \text{ cm}^2\)  
Frequency = 8 so \(1 \text{ cm}^2 = \frac{2}{3} \text{ hour (o.e.)}\)  
M1 3.1a

Frequency of 12 corresponds to area of 18 so  
height = \(18 \div 2.5 = 7.2 \text{ (cm)}\)  
A1 1.1b

Width = \(5 \times 0.5 = 2.5 \text{ (cm)}\)  
B1cao 1.1b

(b) \[
\bar{y} = \frac{205.5}{31} = \text{awrt 6.63}
\]
M1 1.1b

\[
\sigma_y = \sqrt{\frac{1785.25}{31} - \bar{y}^2} = \sqrt{13.644641} = \text{awrt 3.69}
\]
M1 1.1a

allow \[
s = \sqrt{\frac{1785.25 - 31\bar{y}^2}{30}} = \text{awrt 3.75}
\]
M1 A1 1.1b

(c) Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable  
M1 2.4

Hurn is South of Heathrow so does not support his belief  
A1 2.2b

(d) \(\bar{x} + \sigma \approx 10.3\) so number of days is \(\frac{(11 - "10.3")}{3} \times 8 \text{ (} + 5\)\)  
M1 1.1b

= 6.86 so 7 days  
A1 1.1b

(e) \([H = \text{no. of hours}] \quad P(H > 10.3) \text{ or } P(Z > 1) = [0.15865…]\)  
M1 3.4

Predict \(31 \times 0.15865... = 4.9 \text{ or 5 days}\)  
A1 1.1b

(f) (5 or ) 4.9 days < (7 or) 6.9 days so model may not be suitable  
B1 3.5a

(13 marks)
<table>
<thead>
<tr>
<th>Question 1 continued</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notes:</strong></td>
</tr>
<tr>
<td>(a)</td>
</tr>
<tr>
<td><strong>M1:</strong> for clear attempt to relate the area to frequency. Can also award if their height ( \times ) their width = 18</td>
</tr>
<tr>
<td><strong>A1:</strong> for height = 7.2 (cm)</td>
</tr>
<tr>
<td>(b)</td>
</tr>
<tr>
<td><strong>M1:</strong> for a correct expression for ( \sigma ) or ( s ), can fit their value for mean</td>
</tr>
<tr>
<td><strong>A1:</strong> awrt 3.69 (allow ( s = 3.75 ))</td>
</tr>
<tr>
<td>(c)</td>
</tr>
<tr>
<td><strong>M1:</strong> for a suitable comparison of standard deviations to comment on reliability.</td>
</tr>
<tr>
<td><strong>A1:</strong> for stating Hurn is south of Heathrow and a correct conclusion</td>
</tr>
<tr>
<td>(d)</td>
</tr>
<tr>
<td><strong>M1:</strong> for a correct expression – fit their ( \bar{x} + \sigma \approx 10.3 )</td>
</tr>
<tr>
<td><strong>A1:</strong> for 7 days but accept 6 (rounding down) following a correct expression</td>
</tr>
<tr>
<td>(e)</td>
</tr>
<tr>
<td><strong>M1:</strong> for a correct probability attempted</td>
</tr>
<tr>
<td><strong>A1:</strong> for a correct prediction</td>
</tr>
<tr>
<td>(f)</td>
</tr>
<tr>
<td><strong>B1:</strong> for a suitable comparison and a compatible conclusion</td>
</tr>
<tr>
<td>Question</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>2(a)</td>
</tr>
<tr>
<td>(b)</td>
</tr>
<tr>
<td>(c)</td>
</tr>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>(d)</td>
</tr>
</tbody>
</table>

Notes:

(a)  
B1: for a correct statement (unreliable) with a suitable reason

(b)  
B1: for a correct statement

(c)  
B1: for both hypotheses in terms of $\rho$
M1: for selecting a suitable 5% critical value compatible with their $H_1$
A1: for a correct conclusion stated

(d)  
B1: for suggesting Beijing with some supporting reason based on $t$ or $w$
   Allow Jacksonville with a reason based just on higher $\bar{t}$
### Q3(a)

\[
\begin{array}{l}
P(L > 50.98) = 0.025 \\
\therefore \frac{50.98 - \mu}{0.5} = 1.96 \\
\therefore \mu = 50 \\
\end{array}
\]

\[
P(49 < L < 50.75) = 0.9104… \quad \text{awrt 0.910}
\]

(b) \[S = \text{number of strips that cannot be used so } S - \text{B}(10, 0.090)\]

\[
= P(S \leq 3) = 0.991166… \quad \text{awrt 0.991}
\]

(c) \[H_0 : \mu = 50.1 \quad H_1 : \mu > 50.1\]

\[
\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right) \quad \text{and} \quad \bar{X} > 50.4
\]

\[
P(\bar{X} > 50.4) = 0.0264
\]

\[
p = 0.0264 > 0.01 \quad \text{or} \quad z = 1.936… < 2.3263 \quad \text{and not significant}
\]

There is insufficient evidence that the **mean length** of strips is **greater than 50.1**

### Marks and AO

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AO</th>
</tr>
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<tbody>
<tr>
<td>Q3(a)</td>
<td></td>
<td>3.4</td>
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<tr>
<td>(b)</td>
<td></td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>3.3</td>
<td></td>
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<td></td>
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<td>3.4</td>
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<td>3.4</td>
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<td>3.4</td>
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<tr>
<td></td>
<td></td>
<td>2.2b</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>(12 marks)</td>
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</table>
### Question 3 continued

<table>
<thead>
<tr>
<th>Notes:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong></td>
<td></td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; <strong>M1</strong>: for standardizing with $\mu$ and 0.5 and setting equal to a $z$ value ($</td>
<td>z</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>M1</strong>: for attempting the correct probability for strips that can be used</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>A1ft</strong>: awrt 0.910 (allow ft of their $\mu$)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1</strong>: for identifying a suitable binomial distribution</td>
<td></td>
</tr>
<tr>
<td><strong>A1</strong>: awrt 0.991 (from calculator)</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>B1</strong>: hypotheses stated correctly</td>
<td></td>
</tr>
<tr>
<td><strong>M1</strong>: for selecting a correct model (stated or implied)</td>
<td></td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; <strong>A1</strong>: for use of the correct model to find $p = \text{awrt } 0.0264$ (allow $z = \text{awrt } 1.94$)</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>A1</strong>: for a correct calculation, comparison and correct statement</td>
<td></td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; <strong>A1</strong>: for a correct conclusion in context mentioning “mean length” and 50.1</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td><strong>4(a)</strong></td>
<td>[ P(A'</td>
</tr>
<tr>
<td>Marks</td>
<td>AOs</td>
</tr>
<tr>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td>A1</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(2)

| **4(b)** | \[ e.g. \quad P(A) \times P(B) = \frac{7}{30} \times \frac{9}{30} = \frac{63}{900} \neq P(A \cap B) = 0.13 = \frac{52}{400} \] \[ \text{or} \quad P(A' | B') = 0.6 \neq P(A') = 0.65 \] |
| B1 | 2.4 |

(1)

| **4(c)** | <br>![Venn diagram](image) \[ P(B \cap C)' = 0.22 + 0.22 \text{ or } 1-[0.56] \] \[ = 0.44 \] |
| B1 | 2.5 |
| M1 | 3.1a |
| A1 | 1.1b |

(5)

| **4(d)** | \[ P(B \cap C)' = 0.22 + 0.22 \text{ or } 1-[0.56] \] \[ \text{o.e.} \] \[ = 0.44 \] |
| B1 | 2.2b |
| M1 | 3.4 |
| A1 | 1.1b |

(3)

<p>| <strong>Notes:</strong> |
| (a) |
| <strong>M1:</strong> | for a correct ratio of probabilities formula and at least one correct value. |
| <strong>A1:</strong> | a correct answer |
| (b) | for a fully correct explanation: correct probabilities and correct comparisons. |
| (c) |
| <strong>B1:</strong> | for box with B intersecting A and C but C not intersecting A.( Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$) No box is B0 |
| <strong>M1:</strong> | for method for finding $P(B \cap C)$ |
| <strong>A1:</strong> | for 0.09 |
| <strong>M1:</strong> | for 0.13 and their 0.09 in correct places and method for their 0.23 |
| <strong>A1:</strong> | fully correct |
| (d) |
| <strong>M1:</strong> | for a correct expression – fit their probabilities from their Venn diagram. |
| <strong>A1:</strong> | cao |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5 (a)</strong></td>
<td>The seeds would be destroyed in the process so they would have none to sell</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>[S = \text{no. of seeds out of 24 that germinate, } S \sim B(24, 0.55)] [T = \text{no. of trays with at least 15 germinating. } T \sim B(10, p)] [p = P(S \geq 15) = 0.299126...] So (P(T \geq 5) = 0.1487... \text{ awrt } 0.149)</td>
<td>M1</td>
<td>3.3</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>(n) is large and (p) close to 0.5</td>
<td>B1</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>(X \sim N(132, 59.4)) (P(X \geq 149.5) = P\left(Z \geq \frac{149.5 - 132}{\sqrt{59.4}}\right)) = 0.01158... \text{ awrt } 0.0116</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>(e)</strong></td>
<td>e.g. The probability is very small therefore there is evidence that the company’s claim is incorrect.</td>
<td>B1</td>
<td>2.2b</td>
</tr>
</tbody>
</table>

**Notes:**

(a) B1: cao

(b) M1: for selection of an appropriate model for \(T\)

1st A1: for a correct value of the parameter \(p\) (accept 0.3 or better)

2nd A1: for awrt 0.149

(c) B1: both correct conditions

(d) B1: for correct normal distribution

M1: for correct use of continuity correction

A1: cso

(e) B1: correct statement
### Question 6

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrate ( \mathbf{a} ) w.r.t. time</td>
<td>M1</td>
<td>1.1a</td>
</tr>
<tr>
<td>[ v = \frac{5t^2}{2} \mathbf{i} - 10t^\frac{3}{2} \mathbf{j} + \mathbf{C} ] (allow omission of ( \mathbf{C} ))</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>[ v = \frac{5t^2}{2} \mathbf{i} - 10t^\frac{3}{2} \mathbf{j} + 20 \mathbf{i} ]</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>When ( t = 4 ), ( \mathbf{v} = 60 \mathbf{i} - 80 \mathbf{j} )</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td>Attempt to find magnitude: ( \sqrt{(60^2 + 80^2)} )</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td>Speed = 100 m s(^{-1})</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

**Notes:**

1st M1: for integrating \( \mathbf{a} \) w.r.t. time (powers of \( t \) increasing by 1)
1st A1: for a correct \( \mathbf{v} \) expression without \( \mathbf{C} \)
2nd A1: for a correct \( \mathbf{v} \) expression including \( \mathbf{C} \)
2nd M1: for putting \( t = 4 \) into their \( \mathbf{v} \) expression
3rd M1: for finding magnitude of their \( \mathbf{v} \)
3rd A1: ft for 100 m s\(^{-1}\), follow through on an incorrect \( \mathbf{v} \)

(6 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(a)</td>
<td>$R = mg \cos \alpha$</td>
<td>B1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Resolve parallel to the plane</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$- F - mg \sin \alpha = - 0.8mg$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$F = \mu R$</td>
<td>M1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Produce an equation in $\mu$ only and solve for $\mu$</td>
<td>M1</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>$\mu = \frac{1}{4}$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Compare $\mu mg \cos \alpha$ with $mg \sin \alpha$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Deduce an appropriate conclusion</td>
<td>A1 ft</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(8 marks)

Notes:

(a)
B1: for $R = mg \cos \alpha$
1st M1: for resolving parallel to the plane
1st A1: for a correct equation
2nd M1: for use of $F = \mu R$
3rd M1: for eliminating $F$ and $R$ to give a value for $\mu$
2nd A1: for $\mu = \frac{1}{4}$

(b)
M1: comparing size of limiting friction with weight component down the plane
A1 ft: for an appropriate conclusion from their values
<table>
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<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(a)</td>
<td>Use of $v = u + at : (10.5i - 0.9j) = 0.6j + 15a$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$a = (0.7i - 0.1j) \text{ m s}^{-2}$ Given answer</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r} = 0.6j \ t + \frac{1}{2} (0.7i - 0.1j) \ t^2$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Equating the $i$ and $j$ components of $\mathbf{r}$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2} \leftrightarrow 0.7 \ t^2 = 0.6 \ t - \frac{1}{2} \leftrightarrow 0.1 \ t^2$</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$t = 1.5$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>Use of $v = u + at : \ v = 0.6j + (0.7i - 0.1j) \ t$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Equating the $i$ and $j$ components of $\mathbf{v}$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$t = 0.75$</td>
<td>A1 ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(10 marks)

**Notes:**

(a)  
M1: for use of $v = u + at$  
A1: for given answer correctly obtained

(b)  
M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$  
A1: for a correct expression for $\mathbf{r}$ in terms of $t$

(c)  
M1: for equating the $i$ and $j$ components of their $\mathbf{r}$  
A1ft: for a correct equation following their $\mathbf{r}$  
A1: for $t = 1.5$

(d)  
M1: for use of $v = u + at$ for a general $t$  
M1: for equating the $i$ and $j$ components of their $\mathbf{v}$  
A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation
<table>
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<th>Question</th>
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<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9(a)</strong></td>
<td>Take moments about $A$ (or any other complete method to produce an equation in $S$, $W$ and $\alpha$ only)</td>
<td>M1</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>$W\cos\alpha + 7W2\cos\alpha = S2\sin\alpha$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Use of $\tan\alpha = \frac{5}{2}$ to obtain $S$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$S = 3W^*$</td>
<td>A1*</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$R = 8W$</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$F = \frac{1}{4}R$ ($= 2W$)</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{MAX}} = 3W + F$ or $P_{\text{MIN}} = 3W - F$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{MAX}} = 5W$ or $P_{\text{MIN}} = W$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$W \leq P \leq 5W$</td>
<td>A1</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>M($A$) shows that the reaction on the ladder at $B$ is unchanged</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>also $R$ increases (resolving vertically)</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>which increases max $F$ available</td>
<td>M1</td>
<td>2.4</td>
</tr>
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<td><strong>(3)</strong></td>
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</table>

**(13 marks)**
### Question 9 continued

#### Notes:

**(a)**
- **1st M1**: for producing an equation in \( S, W \) and \( \alpha \) only
- **1st A1**: for an equation that is correct, or which has one error or omission
- **2nd A1**: for a fully correct equation
- **2nd M1**: for use of \( \tan \alpha = \frac{5}{2} \) to obtain \( S \) in terms of \( W \) only
- **3rd A1*: for given answer \( S = 3W \) correctly obtained

**(b)**
- **B1**: for \( R = 8W \)
- **1st M1**: for use of \( F = \frac{1}{4} R \)
- **2nd M1**: for either \( P = (3W + \text{their } F) \) or \( P = (3W - \text{their } F) \)
- **1st A1**: for a correct max or min value for a correct range for \( P \)
- **2nd A1**: for a correct range for \( P \)

**(c)**
- **1st M1**: for showing, by taking moments about \( A \), that the reaction at \( B \) is unchanged by the builder’s assistant standing on the bottom of the ladder
- **2nd M1**: for showing, by resolving vertically, that \( R \) increases as a result of the builder’s assistant standing on the bottom of the ladder
- **3rd M1**: for concluding that this increases the limiting friction at \( A \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10(a)</strong></td>
<td>Using the model and horizontal motion: ( s = ut )</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>( 36 = Ut \cos \alpha )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Using the model and vertical motion: ( s = ut + \frac{1}{2}at^2 )</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>(-18 = Ut \sin \alpha - \frac{1}{2}gt^2 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Correct strategy for solving the problem by setting up two equations in ( t ) and ( U ) and solving for ( U )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( U = 15 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td><strong>(6)</strong></td>
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<tr>
<td><strong>(b)</strong></td>
<td>Using the model and horizontal motion: ( U \cos \alpha ) (12)</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Using the model and vertical motion: ( v^2 = (U \sin \alpha)^2 + 2(-10)(-7.2) )</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>( v = 15 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = ( \sqrt{(12^2 + 15^2)} )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{369} = 19 \text{ m s}^{-1} ) (2sf)</td>
<td>A1 ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td><strong>(5)</strong></td>
<td></td>
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<tr>
<td><strong>(c)</strong></td>
<td>Possible improvement (see below in notes)</td>
<td>B1</td>
<td>3.5c</td>
</tr>
<tr>
<td></td>
<td>Possible improvement (see below in notes)</td>
<td>B1</td>
<td>3.5c</td>
</tr>
<tr>
<td></td>
<td><strong>(2)</strong></td>
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<td><strong>(13 marks)</strong></td>
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</table>
### Question 10 continued

#### Notes:

(a)

1<sup>st</sup> **M1**: for use of \( s = ut \) horizontally  
1<sup>st</sup> **A1**: for a correct equation  
2<sup>nd</sup> **M1**: for use of \( s = ut + \frac{1}{2} at^2 \) vertically  
2<sup>nd</sup> **A1**: for a correct equation  
3<sup>rd</sup> **M1**: for correct strategy (need both equations)  
2<sup>nd</sup> **A1**: for \( U = 15 \)

(b)

**B1**: for \( U \cos \alpha \) used as horizontal velocity component  
1<sup>st</sup> **M1**: for attempt to find vertical component  
1<sup>st</sup> **A1**: for 15  
2<sup>nd</sup> **M1**: for correct strategy (need both components)  
2<sup>nd</sup> **A1**<sub>ft</sub>: for 19 m s<sup>-1</sup> (2sf) following through on incorrect component(s)

(c)

**B1, B1**: for any two of  
- e.g. Include air resistance in the model of the motion  
- e.g. Use a more accurate value for \( g \) in the model of the motion  
- e.g. Include wind effects in the model of the motion  
- e.g. Include the dimensions of the stone in the model of the motion
Notes:

(a) 1st M1: for use of \( s = ut \) horizontally

1st A1: for a correct equation

2nd M1: for use of \( s = ut + \frac{1}{2}at^2 \) vertically

2nd A1: for a correct equation

3rd M1: for correct strategy (need both equations)

2nd A1: for \( U = 15 \)

(b) B1: for \( U \cos \alpha \) used as horizontal velocity component

1st M1: for attempt to find vertical component

1st A1: for 15

2nd M1: for correct strategy (need both components)

2nd A1: for 19 \( \text{m s}^{-1} \) (2sf) following through on incorrect component(s)

(c) B1, B1: for any two of

e.g. Include air resistance in the model of the motion

e.g. Use a more accurate value for \( g \) in the model of the motion

e.g. Include wind effects in the model of the motion

e.g. Include the dimensions of the stone in the model of the motion