Paper Reference 8FM0/01
Pearson Edexcel
Level 3 GCE
Further Mathematics
Advanced Subsidiary
Paper 1: Core Pure Mathematics

Monday 14 May 2018 – Afternoon

Time: 1 hour 40 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION

Mathematical Formulae and Statistical Tables

Calculator

ITEMS INCLUDED WITH QUESTION PAPERS

Diagram Book

Answer Book

V58302A

Pearson

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **9** questions in this question paper. The total mark for this paper is **80**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1. $$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

(a) Find $\mathbf{M}^{-1}$ giving each element in exact form.

(2 marks)

(b) Solve the simultaneous equations

$$\begin{aligned} 2x + y - 3z &= -4 \\ 4x - 2y + z &= 9 \\ 3x + 5y - 2z &= 5 \end{aligned}$$

(2 marks)

(continued on the next page)

Turn over

1. continued.

(c) Interpret the answer to part (b) geometrically.

(1 mark)

(Total for Question 1 is 5 marks)

2. The cubic equation

$$z^3 - 3z^2 + z + 5 = 0$$

has roots α, β and γ

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(Total for Question 2 is 5 marks)

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3. (a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 1 - i| \leq 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leq \arg(z - 2) \leq \frac{3\pi}{4}\right\}$$

(5 marks)

The complex number w satisfies

$$|w - 1 - i| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

(b) Find, in simplest form, the exact value of $|w|^2$

(4 marks)

(Total for Question 3 is 9 marks)

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4. Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface.

The water pipes are modelled as straight line segments.

One water pipe, W, is buried beneath a particular road.

With respect to a fixed origin O, the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and W passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$

The units are in metres.

(continued on the next page)

4. continued.

(a) Use the model to calculate the acute angle between W and the road surface.
(5 marks)

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to W from C

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect C to W
(6 marks)

(Total for Question 4 is 11 marks)

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5. $\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$

(a) Describe fully the single geometrical transformation U represented by the matrix A

(3 marks)

The transformation V, represented by the 2×2 matrix B, is a reflection in the line $y = -x$

(b) Write down the matrix B

(1 mark)

(continued on the next page)

Turn over

5. continued.

Given that $\mathbf{U}$ followed by $\mathbf{V}$ is the transformation $\mathbf{T}$, which is represented by the matrix $\mathbf{C}$,

(c) find the matrix $\mathbf{C}$

(2 marks)

(d) Show that there is a real number k for which the point $(1, k)$ is invariant under $\mathbf{T}$

(4 marks)

(Total for Question 5 is 10 marks)

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6. (a) Use the standard results for

$$\sum_{r=1}^{n} r^2 \text{ and } \sum_{r=1}^{n} r$$

to show that

$$\sum_{r=1}^{n} (3r-2)^2 = \frac{1}{2}n\left[6n^2 - 3n - 1\right]$$

for all positive integers n

(5 marks)

(continued on the next page)

6. continued.

(b) Hence find any values of n for which

$$\sum_{r=5}^{n}(3r-2)^2+103\sum_{r=1}^{28}r\cos\left(\frac{r\pi}{2}\right)=3n^3$$

(5 marks)

(Total for Question 6 is 10 marks)

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7. $f(z) = z^3 + z^2 + pz + q$

where p and q are real constants.

The equation $f(z) = 0$ has roots z_1, z_2 and z_3

When plotted on an Argand diagram, the points representing z_1, z_2 and z_3 form the vertices of a triangle of area 35

Given that $z_1 = 3$, find the values of p and q

(Total for Question 7 is 7 marks)

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8. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^n = \begin{pmatrix} 4n+1 & -8n \\ 2n & 1-4n \end{pmatrix}$$

(6 marks)

(ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6 marks)

(Total for Question 8 is 12 marks)

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9. Look at Figure 1 and Figure 2 for Question 9 in the Diagram Book.

A mathematics student is modelling the profile of a glass bottle of water. Figure 1 shows a sketch of a central vertical cross–section ABCDEFGHA of the bottle with the measurements taken by the student.

The horizontal cross–section between CF and DE is a circle of diameter 8 cm and the horizontal cross–section between BG and AH is a circle of diameter 2 cm

(continued on the next page)

9. continued.

The student thinks that the curve GF could be modelled as a curve with equation

$$y = ax^2 + b \qquad 1 \leq x \leq 4$$

where a and b are constants and O is the fixed origin, as shown in Figure 2

(a) Find the value of a and the value of b according to the model.
(2 marks)

(continued on the next page)

9. continued.

(b) Use the model to find the volume of water that the bottle can contain.

(7 marks)

(c) State a limitation of the model.

(1 mark)

(continued on the next page)

9. continued.

The label on the bottle states that the bottle holds approximately $750\ cm^3$ of water.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.
(1 mark)

(Total for Question 9 is 11 marks)

TOTAL FOR PAPER IS 80 MARKS

END OF PAPER
