Pearson Edexcel
Level 3 Advanced Subsidiary
GCE in Further Mathematics (8FM0)

Pearson Edexcel
Level 3 Advanced
GCE in Further Mathematics (9FM0)

June 2019 – Decision Mathematics Exemplar

Student answers with examiner comments

First teaching from September 2017

Contents

About this booklet

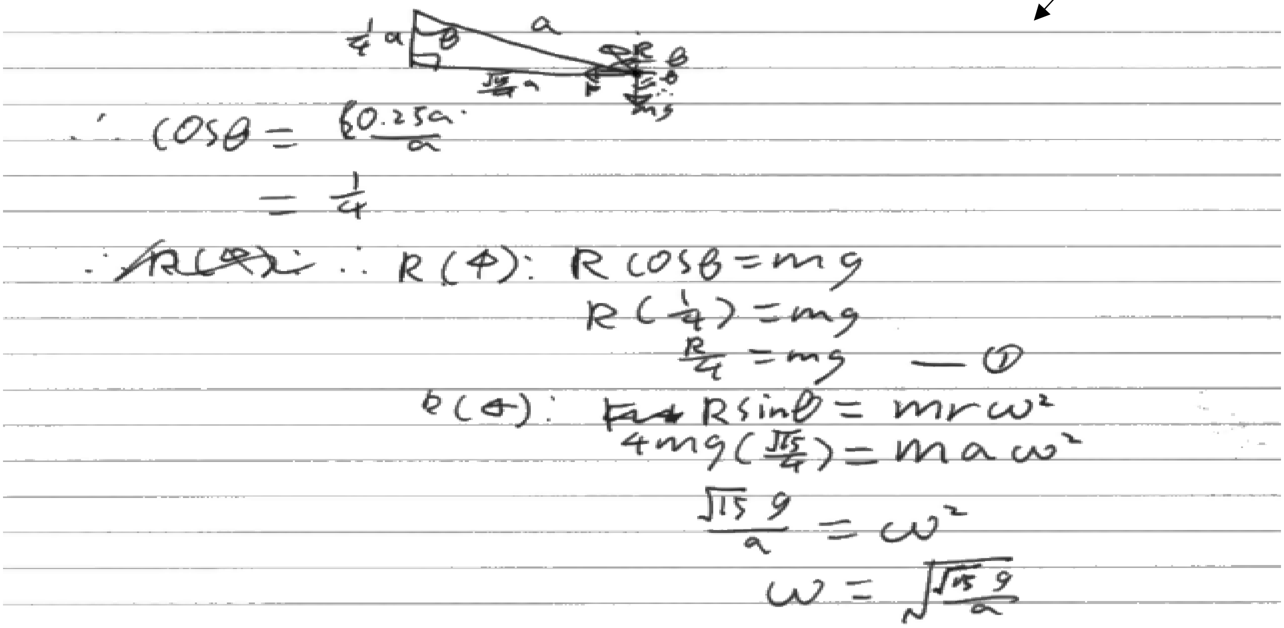
This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Advance Level GCE in Further Mathematics specification (8FM0 & 9FM0). The booklet looks at questions from the AS and A Level Further Mathematics – Decision Mathematics June 2019 Examination Papers. It shows student responses to questions, and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to all questions from the June 2018 Examination Papers. Following each question, you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student response

Student Response B



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Examiner Comments

The solution starts with a correct statement of a relevant trig. ratio. This is followed by correct equations for vertical resolution and for the circular motion. The candidate does go on to use the equations to solve for ω, but they assume that $r = a$ so they do not score the third M mark.

M1M1A1B1M0A0

Examiner commentary on the student response

Marks awarded for the question or question parts

AS Further Mathematics – Decision 1 (8FM0 27)

Exemplar Question 1

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1. (*a*) Draw the graph K_5

(1)

(*b*) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.

(ii) Draw two semi-Eulerian subgraphs of K_5, each having five vertices but with a different number of edges.

(3)

(*c*) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree.

(2)

(Total for Question 1 is 6 marks)

Mean Score 3.1 out of 6

Examiner Comments

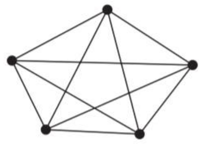
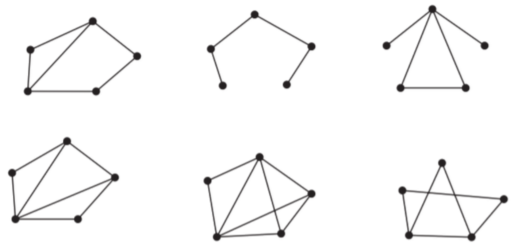
Part (a), as a 'new specification' question, was met with varying degrees of success on the part of candidates. There were some perfect or near perfect responses but also many responses which lacked understanding and/or knowledge. While most were able to draw K_5 in part (a), others were clearly confused by the terminology. Some drew a pentagon, others drew K_6. Some candidates missed out a single arc and many other subgraphs of K_5 appeared here.

In part (b)(i), most candidates were familiar with the term 'semi-Eulerian', however often definitions fell short of the rigour required. It was common to see comments which did not imply 'exactly' two nodes of odd order. Examiners often commented that the phrase 'at least two odd nodes' was seen on a number of occasions. Many candidates also provided non credit-worthy descriptions involving the ability to 'traverse each arc'.

Part (b)(ii) was often well answered, even also when part (a) had not been. Most candidates drew semi-Eulerian subgraphs of K_5 but were sometimes let down by both graphs having the same number of arcs. Others incorrectly drew graphs with multiple arcs between pairs of nodes and some drew graphs with fewer than 5 nodes. Examiners noted that a few Eulerian graphs were provided here as well as some non-semi-Eulerian graphs with multiple nodes of order 1.

Part (c) discriminated well. It was common to see no response here and extremely common to see incorrect attempts. Nonetheless, the most able candidates were able to provide coherent and concise arguments which focussed on the key issues – most usually, the number of arcs required for a tree with the stated orders compared to the number of arcs on a tree with five nodes. Examiners noted that some candidates gave arguments based on the direct connection between the vertex of order 4 and each of the other vertices together with the resulting need for all other nodes to have order 1 for a tree. Other valid arguments were rarer but also appeared from time to time. For some candidates, there was a lot of confusion between arcs and nodes and many stated that there must be 6 nodes on a graph with the stated orders. Other candidates incorrectly deduced that 'because the graph is Semi-Eulerian' it could not be a tree, and some candidates seemed to think that there was only one odd node here. There were a substantial number of candidates who were able to pick up one mark out of two here for making some progress towards a correct explanation.

Mark Scheme

Question	Scheme	Marks	AOs
1(a)		B1	1.2
		(1)	
(b)(i)	A semi-Eulerian graph contains exactly two nodes of odd order (and any number of nodes of even order)	B1	2.5
(b)(ii)	e.g. (two semi-Eulerian subgraphs of K_5 with a different number of edges) 	B1 B1	1.1b 1.1b
		(3)	
(c)	e.g. The graph with five vertices has $\frac{1+2+2+3+4}{2}=6$ arcs but a tree on five nodes would contain only 4 arcs	B1 B1dep	2.2a 2.4
		(2)	
		(6 marks)	

Notes

(a)

B1: CAO (give bod for position of nodes)

(b)(i)

B1: CAO (accept 'there are exactly two odd nodes' but must contain exact oe (e.g. 'only two odd nodes' or 'all but 2 nodes have an even order' but not 'the graph has two odd nodes'))

(b)(ii)

B1: One correct semi-Eulerian subgraph of K_5 with five nodes

B1: Two correct semi-Eulerian subgraphs of K_5 with five nodes – note that the graphs must have a different number of edges

(c)

B1: Deducing that the graph has 6 arcs **or** a tree on five nodes has 4 arcs **or** the node of order 4 must be connected to the other 4 nodes **or** an argument based on the sum of the orders of both the graph and the tree (but must relate the orders to the number of arcs and not the number of nodes) **or** the node with order 4 and one of the nodes of orders 2 or 3 would create a cycle **or** a tree must have two nodes of order 1

B1dep: Complete argument – graph has 6 arcs and the tree would only have 4 arcs **or** the sum of the orders is 12 compared to 8 for the tree **or** the node of order 4 must be connected to the other 4 nodes therefore all the other vertices would have to have order 1 **or** the graph has 6 arcs and therefore with 5 vertices there would have to be cycles **or** the node of order 4 is connected to the other 4 nodes and so together with the node of order 3 (or 2) a cycle would be formed **or** a tree must have at least two nodes of order 1 as otherwise a cycle would be formed

Note: no marks in **(c)** for attempts based only on examples of graphs drawn with the vertex orders as stated

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)</td><td>(i)
<table>
<tr><th>A</th><th>B</th><th>N</th><th>H</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>1</td><td>3</td><td>4</td><td>0.5</td><td>0.25</td><td>0</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>82</td><td>1</td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td></td><td>202.25</td><td>3</td><td>50.5625</td></tr>
</table>
(ii) Final output = 50.5625</td><td>M1
A1
A1
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>$\int_1^3 x^4 \mathrm{d}x = 48.4$
$\left(\frac{50.5625 - 48.4}{48.4}\right) \times 100$
4.47%</td><td>B1
M1
A1ft</td><td>1.1b
1.1a
3.2b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(7 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
<tr><td colspan="4">(a)(i)
M1: At least three rows of cells completed (so at least two values of D and E given) with either a correct first row or 82 found for D – condone repeated values in all columns or a single value in each row
A1: CAO – the values in the second, third and fourth rows correct (so up to the 92.125 in column D and the 2 in column E) – accept exact equivalent fractions
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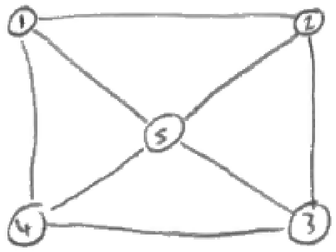
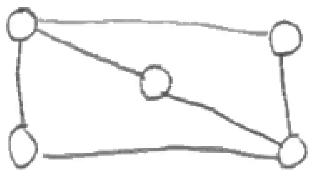
A1: CAO (output = 50.5625) (or equivalent e.g. $50\frac{9}{16}$) – allow if stated only in column F
(b)
B1: CAO (48.4)
M1: Correct method (including multiplying by 100) using candidate’s final output from (a)(ii) and their value for I
A1ft: Follow through their final output from (a)(ii) (for reference: 4.4679752…) must be using 48.4 - dependent on M mark in (a) and percentage error being < 10% (answer must be given to 3 significant figures)</td></tr>
</table>

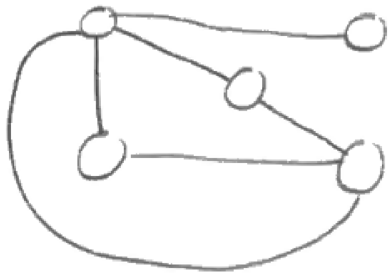
Student Response A

1. a)



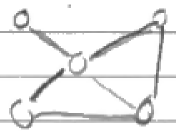
b) II)





b) I) has exactly two odd order nodes

c) It will always contain a cycle



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Examiner Comments

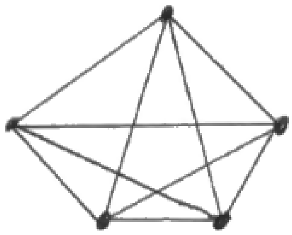
Part (a) is incorrect (this is not the complete graph on five nodes)

In part (b)(i) the candidate has correctly stated that a semi-Eulerian graph contains exactly two nodes with odd order (1 mark). In part (b)(ii) the candidate has correctly drawn two semi-Eulerian subgraphs of K_5 but to score both marks the two graphs had to have a different number of edges (so only 1 mark was awarded). The candidate's answer to part (c) did not contain any pertinent information to award any marks.

Student Response B

1.

a)



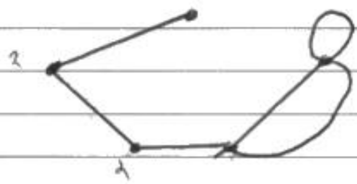
b) i) A connected graph in which there are an even number of ~~odd~~ nodes with odd degree

ii)

Question 1 continued

ii)

c) ~~A tree is a path with no cycles, for~~
A tree is a simply connected ~~graph~~ path with no cycle,
~~for this graph~~ this graph can't be a tree
as it would need a loop to ~~work~~ have the correct order.



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Examiner Comments

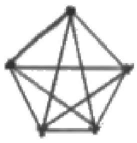
Part (a) is correct (1 mark)

In part (b)(i) the candidate has not stated that a semi-Eulerian graph contains exactly two nodes of odd order but in part (ii) they have drawn two semi-Eulerian subgraphs of K_5 with a different number of edges (2 marks).

Part (c) scored no marks as no mathematical reasoning was given for why the graph would have cycles/loops (and therefore not be a tree).

Student Response C

1. a)



b) ii:

b) i: Complete graph with exactly 2 odd-order nodes.

c) If one vertex has order 4, it must connect to every other vertex, and so any other edges would form a cycle, which makes it not a tree. As the other nodes are not all order 1, a cycle must ~~have~~ be formed.

5/6

Examiner Comments

The only error in this response was in part (b)(i) – a semi-Eulerian graph is not a 'complete' graph and so the mark in part (b)(i) could not be awarded.

Exemplar Question 2

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2. The following algorithm produces a numerical approximation for the integral

$$I = \int_A^B x^4\,\mathrm{d}x$$

Step 1	Start
Step 2	Input the values of A, B and N
Step 3	Let H = (B – A) / N
Step 4	Let C = H / 2
Step 5	Let D = 0
Step 6	Let D = D + A^4 + B^4
Step 7	Let E = A
Step 8	Let E = E + H
Step 9	If E = B go to Step 12
Step 10	Let D = D + 2 × E^4
Step 11	Go to Step 8
Step 12	Let F = C × D
Step 13	Output F
Step 14	Stop

For the case when A = 1, B = 3 and N = 4,

(*a*) (i) complete the table in the answer book to show the results obtained at each step of the algorithm.

(ii) State the final output.

(4)

(*b*) Calculate, to 3 significant figures, the percentage error between the exact value of *I* and the value obtained from using the approximation to *I* in this case.

(3)

(Total for Question 2 is 7 marks)

Mean Score 4.6 out of 7

Examiner Comments

Examiners commented on the fact that many fully complete and correct responses were seen to this question. However, they further commented that a number of applications of the algorithm in part (a)(i) went wildly awry. This was often precipitated by the first application of stage 10. Most candidates who ended up with approximations to *I* running into the tens and hundreds of thousands did not seem perturbed, perhaps indicating a lack of engagement with the problem. There were many approaches employed when completing the table and some candidates wasted time filling in every cell for *A*, *B*, *N*, *H* and *C* for the first several rows; this did not lose marks but certainly lost time. Others spread their values out across the table with one entry per row which was acceptable. Unfortunately, some candidates omitted key elements of the first and second row completely – quite often, 0 was missing from line one and sometimes 1 from line two. Most candidates worked with decimals with a significant minority using the fraction equivalents. Most candidates kept the output exact following their table, but some used a rounded value in part (b) which usually lost the final accuracy mark. More worryingly, some miscopied or lost digits transferring their answer from part (a) to part (b). It seems evident that candidates are less well prepared for questions such as these involving the application of relatively straightforward (but unseen) algorithms.

Part (b) asked for the evaluation of a definite integral and the calculation of the percentage error for their approximation from part (a). The vast majority were able to determine the exact value of *I* although there was a significant number who did not use their calculator and either spent longer than necessary calculating the value or made errors in the integration. Candidates should be advised that in such cases calculator use is perfectly acceptable and, indeed, is advisable.

It was surprising the number of candidates who were unable to calculate the percentage error correctly. Despite being GCSE level work, candidates seemed stumped and were creative with their own version of the percentage error formula. Often, the denominator was the approximation rather than the exact answer. Sometimes, candidates simply found the ratio of the exact and approximate value. Others were not alarmed by huge percentage errors obtained by incorrect answers from part (a). The question requested that answers were given to 3 significant figures and a small number of candidates fell at this final hurdle giving answers to just 2 significant figures.

Mark Scheme

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(i)

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<tr><td></td><td></td><td></td><td></td><td></td><td>202.25</td><td>3</td><td>50.5625</td></tr>
</table>

(ii) Final output = 50.5625

</td><td>M1
A1
A1
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>

$\int_1^3 x^4 \mathrm{d}x = 48.4$

$\left(\frac{50.5625 - 48.4}{48.4}\right) \times 100$

4.47%

</td><td>B1
M1
A1ft</td><td>1.1b
1.1a
3.2b</td></tr>
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(a)(i)

M1: At least three rows of cells completed (so at least two values of D and E given) with either a correct first row or 82 found for D – condone repeated values in all columns or a single value in each row

A1: CAO – the values in the second, third and fourth rows correct (so up to the 92.125 in column D and the 2 in column E) – accept exact equivalent fractions

A1: CAO – all values correct in columns A to E – accept exact equivalent fractions

(ii)

A1: CAO (output = 50.5625) (or equivalent e.g. $50\frac{9}{16}$) – allow if stated only in column F

(b)

B1: CAO (48.4)

M1: Correct method (including multiplying by 100) using candidate's final output from (a)(ii) and their value for *I*

A1ft: Follow through their final output from (a)(ii) (for reference: 4.4679752…) must be using 48.4 - dependent on M mark in (a) and percentage error being < 10% (answer must be given to 3 significant figures)

</td></tr>
</table>

Student Response A

You may not need to use all the rows in this table.

It may not be necessary to complete all boxes in each row.

A	B	N	H	C	D	E	F
1	3	4					
			$\frac{1}{2}$				
				$\frac{1}{4}$			
					0		
					82		
						1	
						1.5	
					425.25		
						2.0	
					6836		
						2.5	
					6877.0625		
						3.0	
							1719.265625

Question 2 continued

(a)(ii)

F = ~~1719.267~~

1719.265625 – output – Final output

(b) Actual value of F =

$= \left[\frac{1}{5}x^5\right]_1^3$

$= \left[\frac{1}{5}(3)^5\right] - \left[\frac{1}{5}(1)^5\right]$

$= 48.4$

$= \frac{48.4 - 1719.265625}{48.4} \times 100$

$= 3452.2\ \%$

3/7

Examiner Comments

In part (a) the candidate has earnt the method mark for at least three rows completed and a correct first row. The third value in the *D* column (425.25) is incorrect and so no further marks could be awarded in this part (so 1 mark).

In part (b) the candidate has correctly stated the value of the definite integral and has shown the correct method for finding the percentage error, however, the final follow through mark could not be awarded as their answer of 3452.2% was out of the acceptable range (of less than 10%) – so a total for two marks were awarded in part (b).

Student Response B

You may not need to use all the rows in this table.

It may not be necessary to complete all boxes in each row.

1	A	B	N	H	C	D	E	F
2	1	3	4					
3				$\frac{1}{2}$				
4					$\frac{1}{4}$			
5						0		
6						82		
7							1	
8							$\frac{3}{2}$	
9								
10						92.125		
8							2	
9								
10						124.215		
8							$\frac{5}{2}$	
9								
10						202.34		
8							3	
9								
12								50.585.
13								50.585

Final output: 50.585 .

Question 2 continued

b) $\int_1^3 x^4 \, dx \Rightarrow \left[\frac{1}{5}x^5\right]_1^3$

$\frac{1}{5}(3^5) - \frac{1}{5}(1^5) = 48.4.$

$\frac{50.585 - 48.4}{48.4} \times 100 = 4.514\ldots\ldots\%$

$= 4.51\%$ error.

5/7

Examiner Comments

In part (a) started off well with the first two marks being awarded for the values in the first to fourth rows being correct. The final two accuracy marks could not be awarded in (a) as the candidate has incorrectly stated 124.215 (and not 124.125) in column *D*.

All three marks were awarded in part (b) as even though their answer of 4.51% was not strictly correct it did follow through from their answer to part (a) and was within the required bounds to award the final mark in part (b) on the follow through.

Student Response C

You may not need to use all the rows in this table.

It may not be necessary to complete all boxes in each row.

A	B	N	H	C	D	E	F
1	3	4					
			0.5				
				0.25			
					0		
					82		
						1	
						1.5	
					92.125		
						2	
					124.125		
						2.5	
					202.25		
						3	
							50.5625

Question 2 continued

ii) 50.5625.

b) $\int_1^3 x^4 \, dx = \left[\frac{x^5}{5}\right]_1^3$

$= \frac{(3)^5}{5} - \frac{(1)^5}{5}$

$= 48.6 - 0.2$

$= 48.4$

% error $= \frac{50.5625 - 48.4}{48.4} \times 100$

$= 4.5\%$

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Examiner Comments

The only mark that could not be awarded was the final mark in part (b) as the final answer was not given to the required accuracy of 3 significant figures.

Exemplar Question 3

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3.

Activity	Immediately preceding activities
A	-
B	-
C	A
D	A
E	A
F	B, C
G	B, C
H	D
I	D, E, F, G
J	D, E, F, G
K	G

(*a*) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain the minimum number of dummies.

(5)

Every activity shown in the precedence table has the same duration.

(*b*) Explain why activity B cannot be critical.

(1)

(*c*) State which other activities are not critical.

(1)

(Total for Question 3 is 7 marks)

Mean Score 3.9 out of 7

Examiner Comments

Candidates were on more familiar territory here and most candidates were able to earn a good number of marks in part (a). Most candidates were completing 'activity on arc' networks, although examiners noted several 'activity on node' networks - probably more than was the case with the legacy specification. Furthermore, examiners noted that several networks had more than one source node, no finish node and networks with fewer than two dummy activities.

Usually, candidates were able to pick up the first two marks and errors usually arose either with the first two precedence dummies or with the omission of activity *H* or activity *K*. Whilst most candidates now seem to be aware of the importance of arrows on dummies, there are still some candidates who make the costly mistake of not having arrows on their dummies. This makes it impossible to determine the preceding activities for *H, I, J* and *K* and ultimately lost three marks.

Usually, candidates were able to place the uniqueness dummy for activities *I* and *J* although this was omitted on several occasions. Some lost the final accuracy mark for lack of arrows on activities (excluding dummies): Some candidates placed arrows only on dummies whereas others made slips and missed out one or two arrows along the away. Some candidates peppered their networks with extra dummies, often at the end of *B* or *H* or *K*. These candidates seem reluctant to extend their activities in order to meet the required event and instead place a dummy to 'fill the gap'. Of course, while not strictly incorrect it is inefficient.

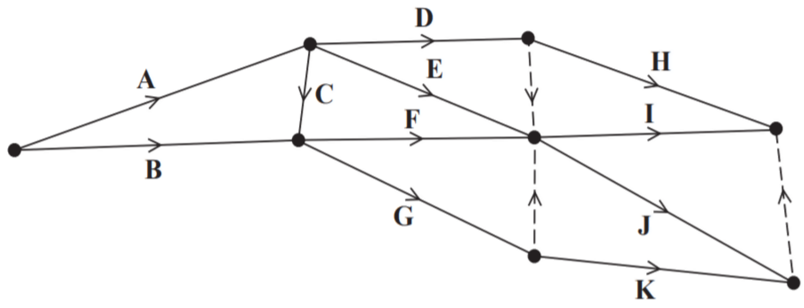
Whilst most responses were clearly set out and often resembled the version given in the printed mark scheme or a correct equivalent, there were several candidates who had multiple arcs crossing over each-other. This is condoned but can make it difficult to see exactly where activities start and finish. It may be advisable for candidates to sketch out a rough diagram in order to see the best placement of activities before completing their final diagram. A word of caution however, sometimes when candidates do exactly this, they fail to copy their initial diagram accurately and miss off arrows and sometimes, more disastrously, activities.

In part (b), many candidates could clearly identify the reasons why activity *B* is not critical. However, articulating these reasons sometimes proved to be more of a challenge. Common responses which were insufficient for the mark included statements such as: "because *F* and *G* depend on *B* and *C*" which did not draw attention to the fact that *C* also depends on *A*; or "*B* is not on the shortest path" which did not give enough detail. Similarly, "*B* does not have a zero float" which was also too vague and was perhaps purely a learned definition for a critical activity. The most successful responses highlighted the dependency of *C* on *A* and the dependency of other activities (*F* and *G*) on both *C* and *B*. Other successful responses involved discussion of event times and the duration of *A* + the duration of *C* compared to the duration of activity *B*.

In Part (c) was challenging for many candidates and it was relatively rare to see a correct answer here perhaps highlighting a lack of understanding of critical paths. Usually, candidates listed several activities. Some candidates were almost correct but incorrectly believed that *K* was not critical – perhaps failing to spot the path through *K* from *A*.

Overall this question was successful in providing both access for less able candidates and differentiation amongst the more able ones.

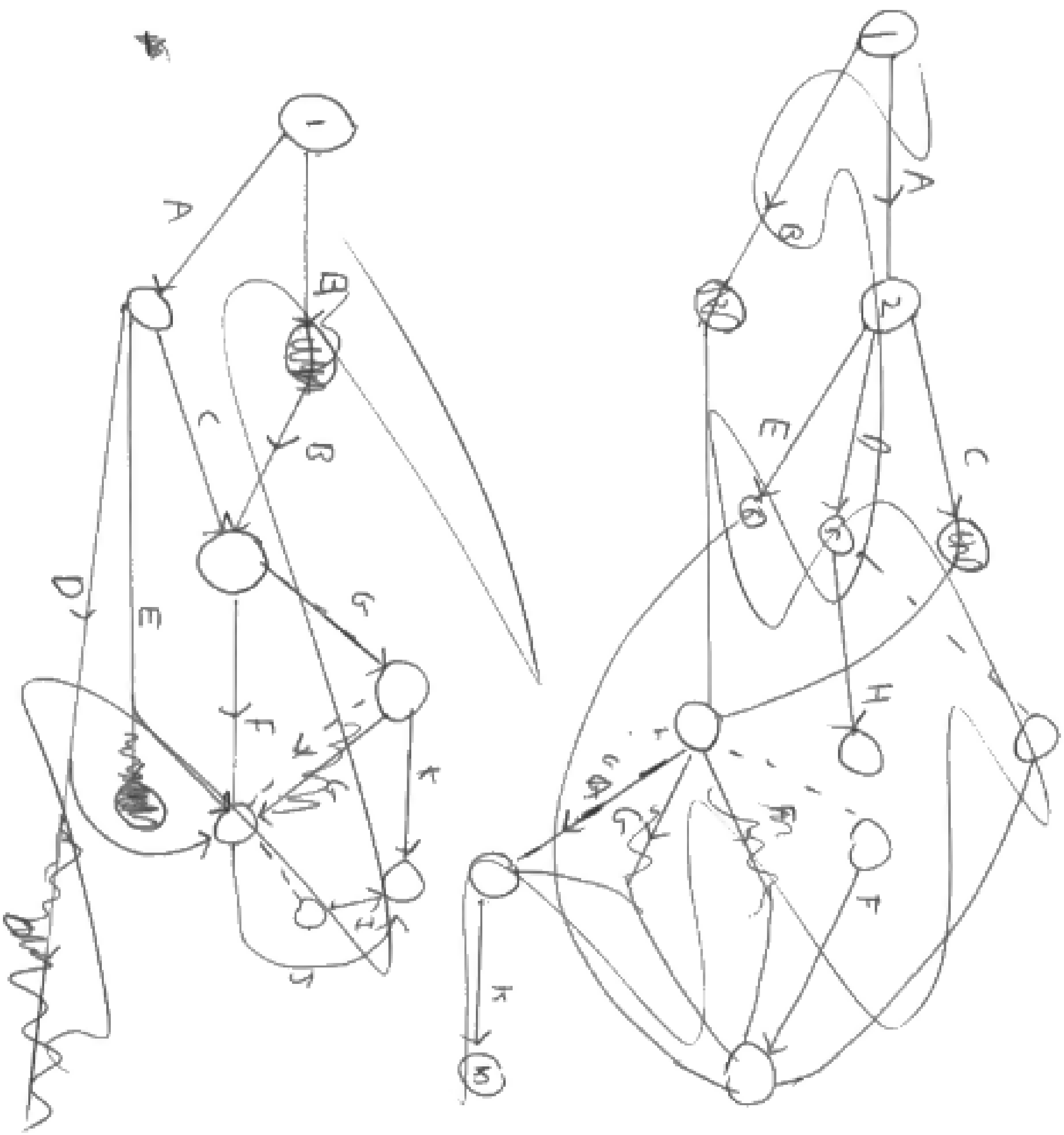
Mark Scheme

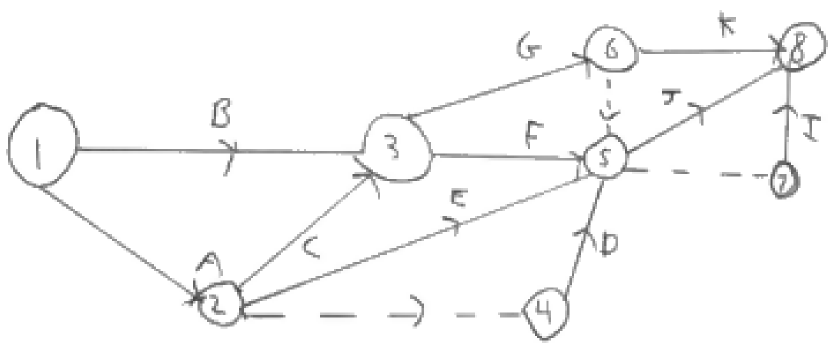
Question	Scheme	Marks	AOs
3(a)		M1 A1 A1 A1 A1	1.1b 1.1b 1.1b 1.1b 1.1b
		(5)	
(b)	Activity F (and/or G) requires activity B and the two activities A and C to be completed before F (and/or G) can begin. The time to complete A and C is double that of B and so B can be delayed waiting for A and C to be completed and so B is therefore not critical.	B1	2.4
		(1)	
(c)	Activities D, E and H	B1	2.2a
		(1)	
			(7 marks)

Notes
In (a) condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'G dealt with correctly' requires the correct precedences for this activity, i.e. B and C labelled correctly and leading into the same node and G starting from that node but do not consider the end node for G. **Activity on node is M0** If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the corrct place for where a dummy should be) **(a)** **M1:** At least eight activities (labelled on arc), one start, and at least two dummies placed **A1:** Activities A – G dealt with correctly (bod if no arrow on activity C) **A1:** First two required dummies + arrows dealt with correctly **A1:** Activities H – K dealt with correctly (A0 if no arrows on preceding dummies (oe)) **A1:** CSO – Final required dummy + all arrows present and correctly placed with one finish and no additional dummies. Note that the arrow for the final dummy could be reversed. Note that there are several correct viable positions for the final dummy **Note that additional (but unecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **(b)** **B1:** CAO - some mention of the time required to complete A + C compared with B (for the next activity to begin (either F and/or G)) oe e.g. paths through B have a maximum length of 3 (non-dummy) activities and there is at least one path of length 4 which does not include B so B cannot be critical **OR** the late time for B must be the same as the late time for A + C which is twice the duration of B and therefore B is not critical. Give bod to responses that imply that B and C meet at the same event, but C is also dependent on A (the key point for awarding this mark is that activities A and C imply that B is not critical) **(c)** **B1:** All three correct with no extras (ignore any mention of activity B)

Student Response A

3. a)





Question 3 continued

b) Nothing relies on just B

c) F, E, C, D, J, I

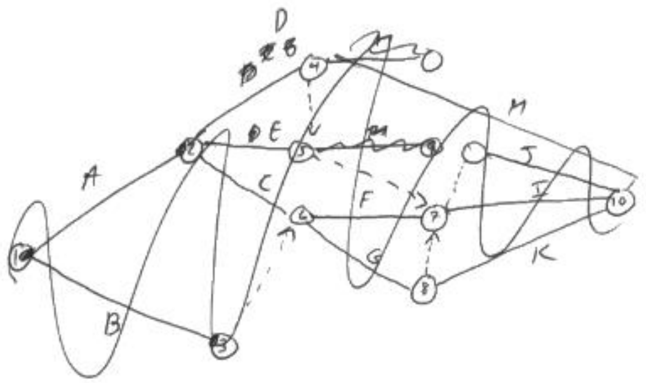
2/7

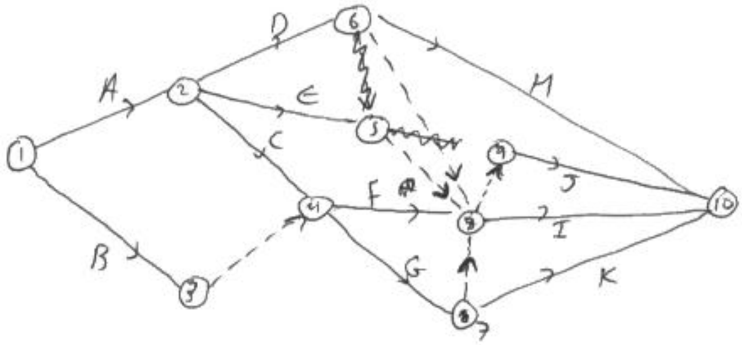
Examiner Comments

After making multiple attempts at drawing the activity network the response at the bottom scored 2 of the 5 marks available as this network had at least eight activities, one start, two dummied placed and activities *A* to *G* having been dealt with correctly. No other marks could be awarded as there is no dummy at the end of *D* and activity *H* is missing.

Parts (b) and (c) scored no marks.

Student Response B





Question 3 continued

b) The two activities ~~events~~ dependent on B already have 2 activities before them so if ~~they~~ all activities have the same duration there will be a float of one activity length ~~at event 3.~~ for B

c) M

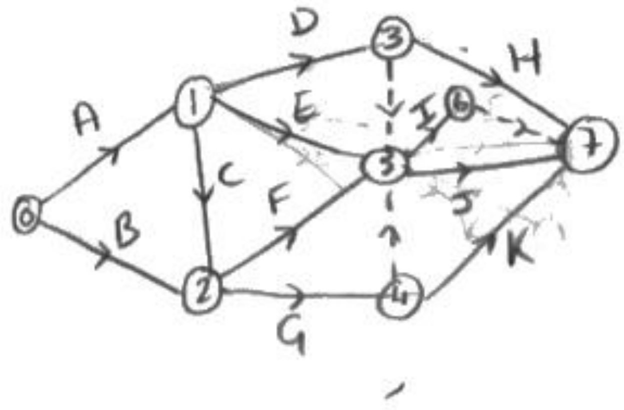
4/7

Examiner Comments

This candidate has produced an activity network which scored 4 out of the 5 marks available. While there is nothing incorrect in their diagram, they have used 5 dummy activities in their network when only three were required (there is no need for the dummy at the end of *B* or between the ends of *E* and *F*) and so the final accuracy mark could not be awarded.

Part (b) was not detailed enough (the candidate did not mention activities A and C in their response) and (c) was incorrect.

Student Response C



Question 3 continued

b) Take activity duration as x.
~~As~~ Activities ~~is~~ F and G depend on B and C.
B does not depend on another activity so the duration is x
C depends on A so the duration to finish C will be $2x$.
this means B has twice the amount of available time ~~that~~
than it needs

c) D, E, H, K

A ✓
B ✗
C ✓
D ~~W~~ ✗
E ~~W~~ ✗
F ✓
G ✓
H ✗
I ✓
J ✓
K ✗

6/7

Examiner Comments

Parts (a) and (b) are fully correct. The only error is in part (c) in which activity *K* has been correctly stated as being critical too.

Exemplar Question 4

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4.

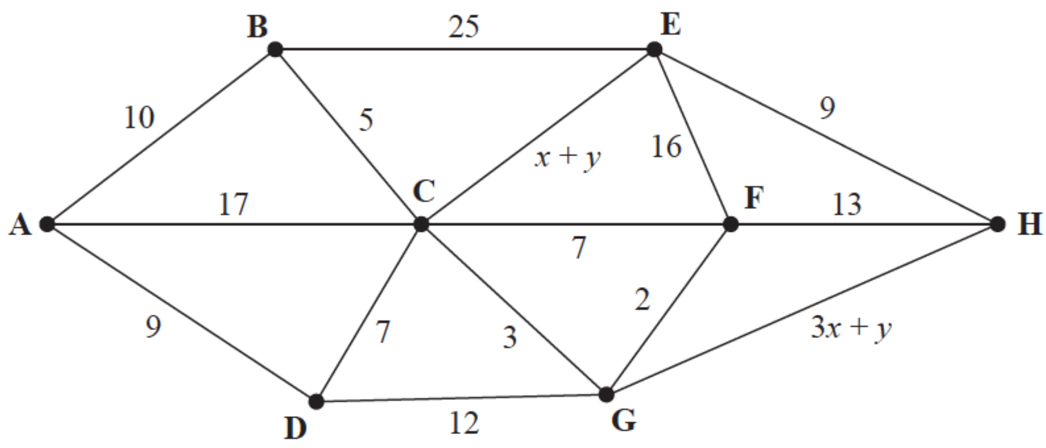


Figure 1

[The total weight of the network is 135 + $4x$ + $2y$]

The weights on the arcs in Figure 1 represent distances. The weights on the arcs **CE** and **GH** are given in terms of x and y, where x and y are positive constants and $7 < x + y < 20$

There are three paths from **A** to **H** that have the same minimum length.

(*a*) Use Dijkstra's algorithm to find x and y.

(7)

An inspection route starting at **A** and finishing at **H** is found. The route traverses each arc at least once and is of minimum length.

(*b*) State the arcs that are traversed twice.

(1)

(*c*) State the number of times that vertex **C** appears in the inspection route.

(1)

(*d*) Determine the length of the inspection route.

(1)

(Total for Question 4 is 10 marks)

Mean Score 5.8 out of 10

Examiner Comments

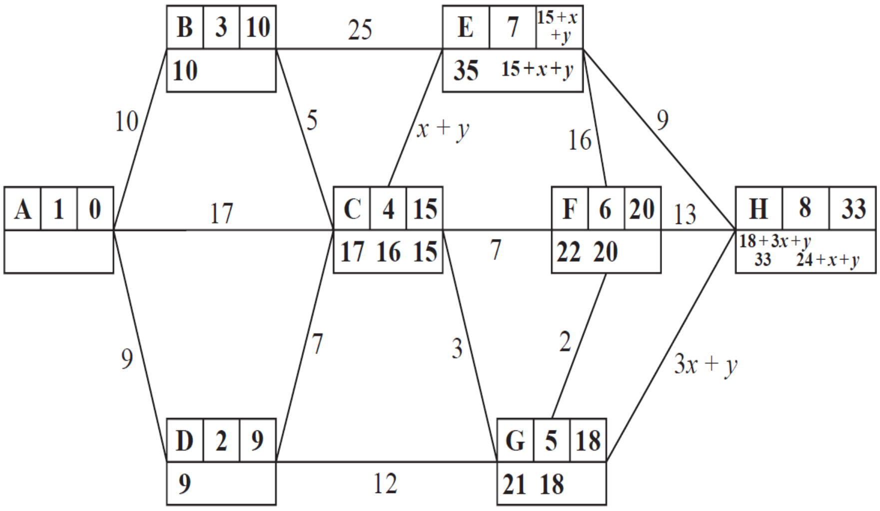
Most candidates were able to earn at least some of the marks in part (a) for applying Dijkstra’s algorithm for the network at least up to vertex E. There were the usual issues with order of working values with issues occasionally cropping up at C, F and G. Also, order of labelling was sometimes problematic with repeated labels occurring, from time to time at B and D or F and G. Up to vertex E was, however, comfortable territory for most and correspondingly well completed. The introduction of the algebraic weights for CE and GH was, of course, less familiar and some candidates were unsure how to proceed. A good proportion however, were unfazed and correctly stated working values at E and H. Sometimes only one working value was given at H (and the other two often shown later in working) probably because candidates had been told that the three paths to H have equal length. Sometimes 36 made an appearance at E and this was penalised. Of course, earlier errors in the network sometimes led to incorrect expressions at E and/or H.

Some candidates believed that they had completed part (a) with the diagram and did not proceed to determine the values of x and y. Those that did however, were often successful provided they had managed to identify the expressions for the three paths. Occasionally, an extra value of 44 appeared at H which sometimes led to incorrect calculations. Other times, candidates did not identify the three different lengths of the paths and so ended up with a single equation to solve in two variables.

It was surprising to examiners that part (b) seemed to be problematic for so many. The majority of candidates were able to identify the four vertices with odd degree, but many had not read the question carefully and did not see that the route would start at A and end at H. Thus requiring, only, the repeat of arcs from B to D. Often, candidates proceeded down the usual route inspection method of pairing odd vertices and choosing the pairing of smallest weight – usually AD and BH. Candidates should perhaps be advised to consider the amount of work required in relation to the number of marks available and this may have been a flag to re-read the question. It was indeed rare to see the correct answer stated here.

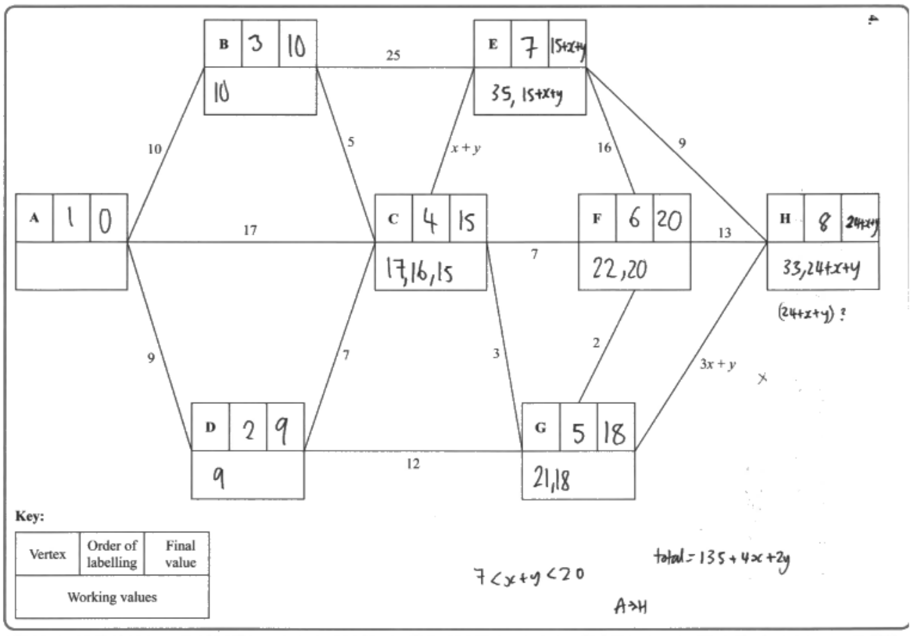
Part (c) also presented a challenge for many candidates and it was uncommon to see the correct answer of ‘4 times’ stated here. An answer of 8 appeared frequently for candidates who appear to be counting the number of times an arc incident to C would be travelled along rather than the number of times C itself would be visited. An answer of 3 was also quite common.
Due to a lack of success with part (b), the correct answer was relatively rarely seen in part (d). The value of 91 was often given which related to AD and BH (BC, CG, GF, FH) being repeated.

Mark Scheme

Question	Scheme	Marks	AOs
4(a)	 Attempt to form a pair of simultaneous equations using their three working values from H e.g. $3x + y = 15$ or $x + y = 9$ $x = 3, y = 6$	M1 A1 A1ft A1 M1dep A1 A1	1.1b 1.1b 1.1b 1.1b 3.1a 2.1 2.2a
		(7)	
(b)	Arcs BC and CD need to be traversed twice	B1	1.1b
		(1)	
(c)	Vertex C would appear 4 times	B1	2.2a
		(1)	
(d)	135 + 4(3) + 2(6) + 12 = 171	B1ft	1.1b
		(1)	
			(10 marks)

Notes
(a) **M1:** For a larger number replaced by a smaller one in the working value boxes at C, F or G **A1:** For all values correct (and in correct order) at A, D, B and C (condone order of labelling starting at A with 0) **A1ft:** For all values correct (and in correct order) at G and F following through from A, D, B and C **A1:** For all **working values** correct at E and H (order of working values must be correct at E but condone any order of working values at H) **however, at H if only one working value is seen e.g. 18 + 3*x* + *y* then both 33 and 24 + *x* + *y* must be seen (or clearly implied) in later working for this mark to be awarded (e.g. 3*x* + *y* = 15 and *x* + *y* = 9 would imply this). Similarly, if only two working values seen (e.g. 18 + 3*x* + *y* and 33) at H then the third (24 + *x* + *y*) must be implied by later working. Any incorrect working values seen at H though will score A0** **M1dep:** Forming two equations from the candidate's three working values at H (so two of their $18+3x+y=24+x+y$, $18+3x+y=33$ and $24+x+y=33$) – allow all three working values stated anywhere in their solution – dependent on previous M mark. Must be a complete method – so for those finding x from $18+3x+y=24+x+y$ they must also either state or use one of the other two equations (so candidates must be interacting with all three paths from A to H) **A1:** Two correct equations formed (dependent on correct working values either seen at H or in their subsequent working) – can be unsimplified but must come from correct working **A1:** CAO for x and y (x = 3 and y = 6) – must come from correct working If all three correct working values at H are seen (either at H or subsequent working) together with both correct answers (with no other working) then award M1A1A1. **(b)** **B1:** CAO (arcs BC and CD) **(c)** **B1:** CAO (4 times) **(d)** **B1ft:** Follow through only for 135 + 12 + 4x + 2y (for their x and y values provided $7 < x+y < 20$ and x and y are positive constants)

Student Response A



Question 4 continued

largest is 33 or 24 + x + y ?

24 + x + y =

b) ~~ABCDEFGH~~

BC + DC

c) 8

d)

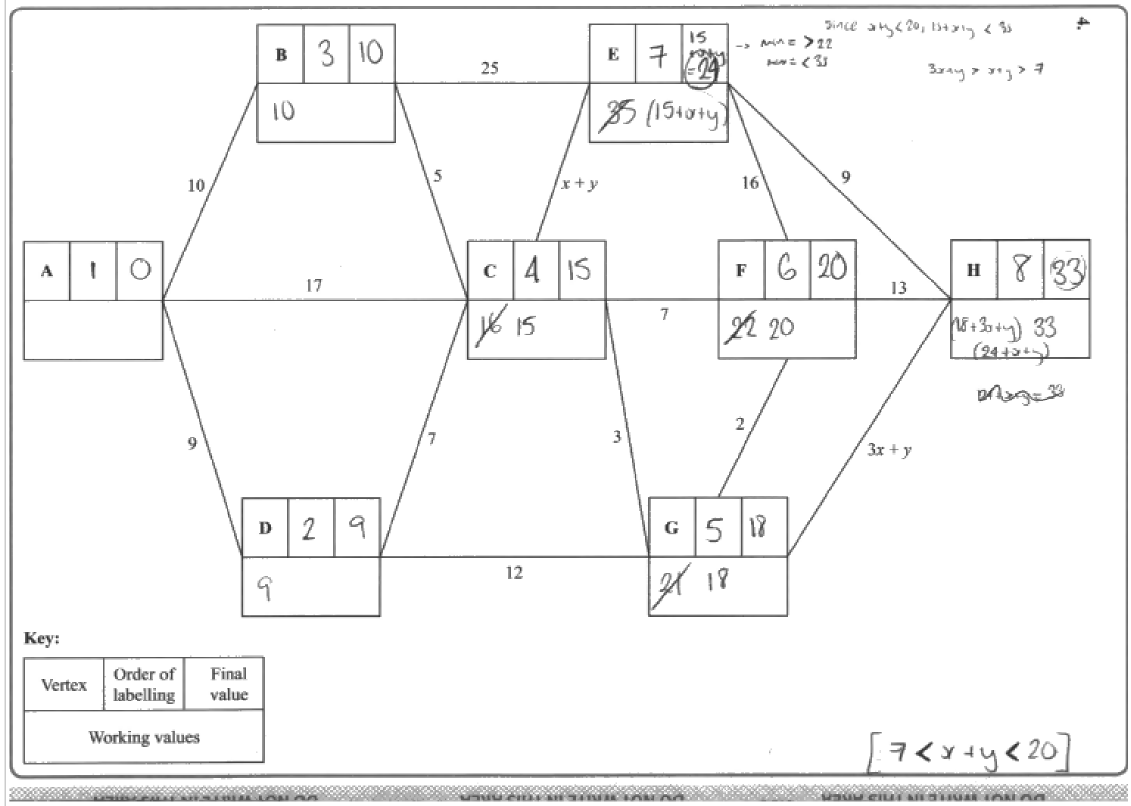
4/10

Examiner Comments

The first three marks were awarded in part (a) as Dijkstra's algorithm has been applied correctly to nodes A, D, B, C, G and F. The third accuracy mark could not be awarded as the working value of $18 + 3x + y$ is not seen or implied at H. The candidate does not form two equations from the candidate's three working values at H and so no further marks could be awarded in this part.

Part (b) is correct (1 mark) but part (c) was incorrect and there was no attempt at part (d).

Student Response B



Question 4 continued

(a) $18 + 3x + y = \underbrace{24 + x + y}$ $= 33$

$x + y = 9$

$x = 9 - y$

(sub)

$18 + 3(9 - y) + y = 33$

$18 + 27 - 3y + y = 33$

$45 - 2y = 33$

$2y = 12$

$y = 6$

$\therefore x = 3$

$x = 3,\ y = 6$

(b).

Vertex	Odd/Even (valency)
A	O
B	O
C	E
D	O
E	E
F	E
G	E
H	O

AB DH	AD BH	AH BD
10 + 27	9 + 23	33 ....
= 37	= 32	

arcs AD and BH are repeated

(c). 3 times

(d). Total weight + 32 = 133 + 4(3) + 2(6) + 32 = 191 units

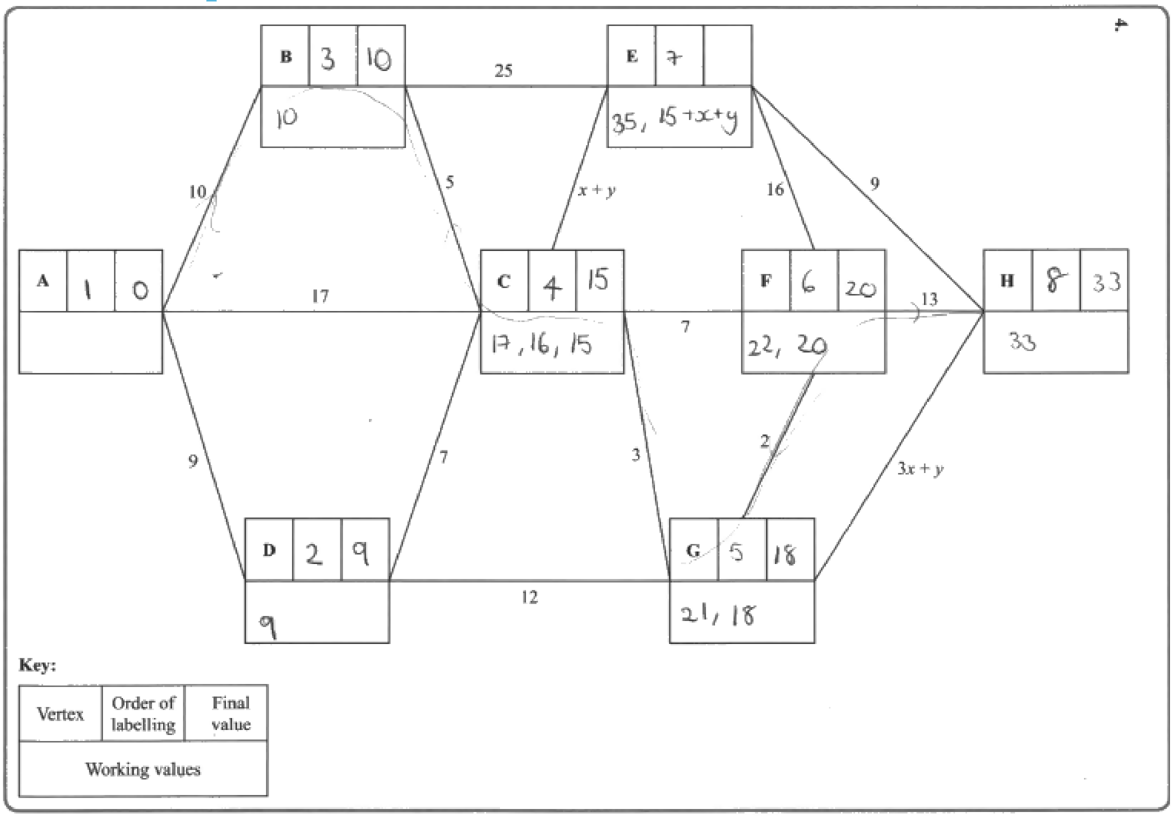
6/10

Examiner Comments

The only error in part (a) is the lack of a working value of 17 at C.

Parts (b), (c) and (d) are incorrect.

Student Response C



Question 4 continued

a. $15 + x + y + 9 = 33 \Rightarrow x + y = 9 \Rightarrow y = 9 - x$

$18 + 3x + y = 33 \Rightarrow 3x + y = 15$

$3x + (9 - x) = 15$

$2x + 9 = 15$

$2x = 6$

$$\boxed{\begin{array}{l} x = 3 \\ y = 6 \end{array}}$$

~~b. odd nodes:~~

~~A, B, D, H~~

~~$\Rightarrow$ AB and DH $= 10 + 24 = 34$~~

~~$\Rightarrow$ AD and BH $= 9 + 25 = 34$~~

~~$\Rightarrow$~~

~~Arcs joining A and B~~

b. Arcs that are traversed twice: BC and CD

c. 4 times

d. $135 + 4x + 2y + 5 + 7 = 147 + 4 \times 3 + 2 \times 6 = 171$

10/10

Examiner Comments

A completely correct response which applies Dijkstra's algorithm and shows the required working as evidence that the correct calculations have been performed to find the values of *x* and *y* in part (a).

Parts (b), (c) and (d) are also correct.

Exemplar Question 5

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5. Ben is a wedding planner. He needs to order flowers for the weddings that are taking place next month. The three types of flower he needs to order are roses, hydrangeas and peonies.

Based on his experience, Ben forms the following constraints on the number of each type of flower he will need to order.

- At least three-fifths of all the flowers must be roses.
- For every 2 hydrangeas there must be at most 3 peonies.
- The total number of flowers must be exactly 1000

The cost of each rose is £1, the cost of each hydrangea is £5 and the cost of each peony is £4

Ben wants to minimise the cost of the flowers.

Let x represent the number of roses, let y represent the number of hydrangeas and let z represent the number of peonies that he will order.

(*a*) Formulate this as a linear programming problem in x and y only, stating the objective function and listing the constraints as simplified inequalities with integer coefficients.

(7)

Ben decides to order the minimum number of roses that satisfy his constraints.

(*b*) (i) Calculate the number of each type of flower that he will order to minimise the cost of the flowers.

(ii) Calculate the corresponding total cost of this order.

(3)

(Total for Question 5 is 10 marks)

Mean Score 3.8 out of 10

Examiner Comments

It was noted by examiners that this final question was attempted by most candidates and thus indicated that time pressure was unlikely to have been an issue for many. Some responses were incomplete but most of the time it seems that candidates completed as much as they were able to do.

This question gave rise to a mixed bag of responses. Despite involving three variables, the work required, at least initially in part (a), was standard and the question gave rise to the usual errors that have been observed over many previous sessions.

Almost all candidates worked with the provided variables x, y and z. Some candidates initially began formulating the problem in terms of r, p and h although usually they switched to x, y and z and rewrote their work with the required variables. Those that didn't convert their variables usually petered out before getting very far with the question.

Most candidates stated the objective function correctly and many remembered to 'minimise' although this was certainly not universally the case. Many candidates were able to state the equality constraint for the total number of flowers although some candidates believed there were 100 flowers rather than the given 1000. As is usually the case, the inequality constraints were generally more problematic. Most candidates had success with the roses constraint and many immediately deduced that $x \geq 600.$ Sometimes though, this inequality was in the wrong direction and les able candidates gave other incorrect interpretations including $3x \geq 5(y+z).$ By far the most challenging constraint was the hydrangea/peony constraint which was often incorrect. It was common to see the incorrect inequality $2y \geq 3z$ but also $3y \leq 2z$ or $2y \leq 3z.$ Sometimes all the constraints were stated as equations with no inequalities and examiners also observed strict inequalities being used from time to time.

Often candidates did not seem to realise that they had been asked to eliminate z and many simply worked with the constraints in all three variables. Of those that did attempt to eliminate z however, some candidates set the hydrangea/peony constraint to be an equality and used this to eliminate z rather than using the correct $x + y + z = 1000$ constraint. Some candidates who did make headway eliminating z from the constraints, neglected to eliminate z from the objective function.

Often candidates were able to pick up marks in part (b) despite incomplete responses to part (a). Most recognised the need for the total number of flowers to equal 1000 and correspondingly stated values to fit their constraints. Occasionally though, the minimum value of x was not used as had been stipulated and quite often candidates did not state their solution in context despite being asked for the number of each type of flower. Indeed, some candidates seemed to become muddled with the variables and flowers and stated solutions that were not compatible with their constraints - seemingly mixing up hydrangeas and peonies. On several occasions, examiners saw responses where candidates had only stated the required values for x and y and omitted the value for z. Occasionally the total cost was not stated. Overall, the question performed well providing a wide range of marks and the opportunity for differentiation amongst the candidates.

Mark Scheme

Question	Scheme	Marks	AOs
5(a)	Minimise $(P=)\,x+5y+4z$	B1	3.3
	Subject to $x \geq \frac{3}{5}(x+y+z)\,(\Rightarrow 2x \geq 3y+3z)$	B1	3.3
	$3y \geq 2z$	B1	3.3
	$x+y+z=1000$	B1	3.3
	$z=1000-x-y$ substituted into objective and constraints gives	M1	3.1a
	Minimise $(P=)\,y-3x(+\ 4000)$ subject to $x \geq 600$ and $2x+5y \geq 2000$	A1 A1	1.1b 1.1b
		(7)	
(b)	**(i)** Using least value of x to find y and z 600 roses, 160 hydrangeas and 240 peonies **(ii)** £2360	M1 A1 A1	3.4 3.2a 1.1b
		(3)	
			(10 marks)

Notes

(a)

B1: CAO (for objective) – must contain 'minimise' or 'min' only (so not 'minimum') either when stated in terms of x, y and z or x and y only

B1: $x \geq \frac{3}{5}(x+y+z)$ oe – need not be simplified for this mark, accept $x \geq \frac{3}{5}(1000)$

B1: $3y \geq 2z$ or any equivalent form (need not be simplified nor integer coefficients for this mark)

B1: $x+y+z=1000$ (could be implied by earlier/later working)

M1: Eliminating z from either the objective **or** both constraints using the constraint $x+y+z=1000$

A1: Correct objective in terms of x and y only – condone lack of 'minimise'

A1: Both constraints correct ($x \geq 600$ and $2x+5y \geq 2000$ - must be integer coefficients for this mark)

(b)(i)

M1: Using their least value of x to find both y and z (with both y and z being positive integers) – note that all values must satisfy the constraint $x+y+z=1000$ (and must all be integers)

A1: All three types of flowers correct (in context – so not just in terms of x, y and z) – must come from correct constraints in (a)

(ii)

A1: CAO for cost (condone lack of units but not 2360p) – must come from correct constraints in (a)

SC for (b) – for those candidates with the constraint $2y \geq 3z$ in (a) leading to 600 roses, 240 hydrangeas and 160 peonies (so not just in terms of x, y and z) together with (£)2440 award **SC M1A1A0** in (b)

Student Response A

Question 5 continued

x = number of roses
y = number of hydrangeas
z = number of peonies

Objective function:

$x + y + z = 1000$

$z = 1000 - x - y$

$C = x + 5y + 4z$

$C = x + 5y + 4(1000 - x - y)$

$C = x + 5y + 4000 - 4x - 4y$

$C = -3x + y + 4000$

$x > \frac{3}{5}(x+y+z)$

$x > \frac{3}{5}(1000)$

$x > 600$

$\frac{3}{5}z \leq \frac{2}{5}y$

$3z \leq 2y$

$3(1000 - x - y) \leq 2y$

$3000 - 3x - 3y \leq 2y$

$3000 - 3x \leq 5y$

$x > 600$

$3000 - 3x \leq 5y$

b) i)

3/10

Examiner Comments

In part (a) the candidate earnt three marks for correctly stating the constraint $x + y + z = 1000$ and for re-writing the objective function correctly in terms of x and y only.

The candidate did not attempt part (b).

Student Response B

5.

a) $x+y+z=1000$ Objective: $C=x+5y+4z$ is as small as possible

$x \geq 600$

$2y \leq 3z$

$C=x+5y+4(1000-x-y)$

$=x+5y+4000-4x-4y$

$z=1000-x-y$

$=y-3x+4000$

$2y \leq 3(1000-x-y)$

$2y \leq 3000-3x-3y$

$5y \leq 3000-3x$ $x \geq 600$

b) i) Minimum number of roses $=600$

$5y \leq 3000-3(600)$ $C=y-3(600)+4000$

$5y \leq 1200$ $=y+2200$

$y \leq 240$

~~y+z~~ $y+z=400$

To minimise cost $y=240$ ∴ $z=160$

ii) $1(600)+5(240)+4(160)=£2440$

5/10

Examiner Comments

In part (a) the candidate scored B0 (no mention of ‘minimise’ with the objective function), B1 (correct condition that $x \geq 600$), B0 ($2y \leq 3z$ is incorrect and should have been $3y \geq 2z$), B1 ($x + y + z = 1000$) and then M1 A1 (for correctly re-writing the objective in terms of x and y only).

In part (b) the candidate scored the method mark for using their least value of x to find both a y and z value but no accuracy marks could be awarded due to the answers being incorrect and not given in context.

Student Response C

5.

a) $x = 600$

$5y \geqslant 2z$

$x + y + z = 1000$

$3y \geqslant 2x$

$z = 1000 - 600 - y = 400 - y$

$x \geqslant 0 \quad y \geqslant 0 \quad z \geqslant 0$

$3y \geqslant 2z$

objective function: minimise: $x + 5y + 4z = C$ ← minimum cost

constraints:

$x \geqslant 600$

$3y \geqslant 2z \qquad 3y \geqslant 2(400 - y) = 3y \geqslant 800 - 2y$

$5y \geqslant 800 \qquad y \geqslant 160$

$x + y + z = 1000 \qquad x \geqslant 600$

$x + y + (400 - y) = 1000 \qquad y \geqslant 160$

$x = 600$

$z = 1000 - x - y$

b) i) roses : 600 $\qquad z = 400 - 160$

hydrangeas : 160

peonies : 240

ii) $600 + 160(5) + 240(4) = £2360$

7/10

Examiner Comments

In part (a) the candidate correctly stated all the constraints and corresponding objective of the LP problem in terms of *x*, *y* and *z* (so scoring the first four marks) but then could not use the constraint that $x + y + z = 1000$ to eliminate *z* and so the final three marks in this part could not be awarded.

Part (b) was fully correct (including giving the number of each type of flower in context) and so scored the three marks in this part.

AS Further Mathematics – Decision 2 (8FM0 28)

Exemplar Question 1

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1. Three workers, A, B and C, are each to be assigned to one of four tasks, P, Q, R and S.

Each worker must be assigned to at most one task, and each task must be done by at most one worker.

The amount, in pounds, that each worker will earn while assigned to each task is shown in the table below.

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

The Hungarian algorithm is to be used to find the maximum total amount that can be earned by the three workers.

(*a*) Explain how the table should be modified.

(2)

(*b*) (i) Reducing rows first, use the Hungarian algorithm to obtain an allocation which maximisesthe total earnings.

(ii) Explain how any initial row and column reductions were made and also how you determined if the table was optimal at each stage.

(7)

(Total for Question 1 is 9 marks)

Mean Score 5.7 out of 9

Examiner Comments

This question on applying the Hungarian algorithm to obtain an allocation which maximised the total earnings proved to be accessible to nearly all candidates with the majority scoring the marks for the application of the algorithm. When errors were seen these were usually down to numerical slips rather than errors in method as nearly all candidates correctly applied row and column reduction followed by correct augmentations. A small number of candidates failed to deduce the optimal allocation from the location of the zeros in their final table. However, many candidates failed to read the question carefully and did not adequately explain how the table should be modified in part (a) (by subtracting each entry from a constant and adding a dummy row with equal values) or in part (b) many failed to explain how any initial row/column reductions were made and also how to determine if the table was optimal at each stage.

Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Subtract each entry from a constant (e.g. 42) to covert from maximisation problem to minimisation	B1	1.1a
	Add an additional dummy row with equal values (e.g. 42, 0, etc.) to create a square array	B1	3.5c
		(2)	
(b)	e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ X \end{array} & \begin{pmatrix} 10 & 2 & 5 & 0 \\ 13 & 10 & 7 & 1 \\ 5 & 9 & 3 & 2 \\ 42 & 42 & 42 & 42 \end{pmatrix} \end{array}$	B1	1.1b
	No reduction for row A, reduce row B by 1, reduce row C by 2 and row X by 42 (or equivalent). No reduction of columns	B1	2.4
	Reducing rows and columns gives $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ X \end{array} & \begin{pmatrix} 10 & 2 & 5 & 0 \\ 12 & 9 & 6 & 0 \\ 3 & 7 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$	M1	1.1b
	Two lines required to cover the zeros hence solution is not optimal (augment by 1) $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ X \end{array} & \begin{pmatrix} 9 & 1 & 4 & 0 \\ 11 & 8 & 5 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$	M1	1.1b
	Three lines required to cover the zeros hence solution is not optimal (augment by 1) e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ X \end{array} & \begin{pmatrix} 8 & 0 & 3 & 0 \\ 10 & 7 & 4 & 0 \\ 2 & 6 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \end{array}$ or $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ X \end{array} & \begin{pmatrix} 8 & 0 & 4 & 0 \\ 10 & 7 & 5 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \end{array}$	M1	1.1b
	Four lines required to cover the zeros hence solution is optimal	B1	2.4
	A – Q, B – S, C – R, (X – P)	A1	2.2a
		(7)	
		(9 marks)	

Notes
(a) **B1**: Valid statement regarding converting a maximisation problem to a minimisation problem – must imply subtracting each entry from a constant (although the value of this constant need not be stated) **B1**: Explain the need to add a valid dummy row (to create a square array) e.g. allow mention of adding an additional worker or the need to have a square array **(b)** **B1:** Mark awarded when both steps complete (a valid subtraction and addition of a correct extra row) **B1:** Correct statements regarding row and column reduction – if explicit values not stated then it must be clear that reduction is done by subtracting the least value in each row/column from each element of that row/column **M1:** Simplifying the initial matrix by reducing rows and then columns – allow one error **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 2 lines needed to 3 lines needed **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table) **B1:** Correct statement(s) regarding the minimum number of lines to cover zeros at each stage or a general statement that covers all augmentations (e.g. if we have an *n* by *n* array then if the minimum number of lines required to cover the zeros is less than *n* then the solution is not optimal but if the minimum number of lines to cover the zeros is *n* then it is optimal – as an absolute minimum allow mention that until there are 4 lines covering the zeros then the solution is not optimal (oe) – however, in all cases, there must have been two augmentations taking place (2 to 3 lines and then 3 to 4)) **A1:** CSO for the application of the Hungarian algorithm - so correct application of the algorithm from a correct initial matrix (so in (b) candidates must have scored at least B1B0M1M1M1B0) – together with the deduction of the correct allocation **Note that if array is not square or if the additional row is given entries of 'infinity' then only the second B mark in (b) can be awarded**

Student Response A

1. (a)

~~[illegible] A Dummy row must be inserted~~
The table must be turned into minimising be subtracting each value from the greatest value in the table.

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

You may not need to use all of these tables
You may not need to use all the rows and columns

	P	Q	R	S	
A	10	2	5	0	
B	13	10	7	1	
C	5	9	3	2	
Dummy					

b) i.
Subtract each value from 42.

	P	Q	R	S	
A	10	2	5	0	
B	12	9	6	0	
C	3	7	1	0	

b) ii.
Row Reduction.
$R_1 - 0$
$R_2 - 1$
$R_3 - 2$

	P	Q	R	S	
A	7	0	4	0	
B	9	7	5	0	
C	0	5	0	0	
D	0	0	0	0	

Column reduction
$C_1 - 3$
$C_2 - 2$
$C_3 - 1$
$C_4 - 0$
- Then insert Dummy another with zero.

Question 1 continued

	P	Q	R	S	
A	7	0	4	0	
B	9	7	5	0	
C	0	5	0	0	
D	0	0	0	0	

b) ii.
Cover all zeros using minimum num of lines. num of lines = num of workers, ∴ optimal.

	P	Q	R	S	
A					
B					
C					

b) i. A — Q
B — S
C — R
3. A should do Q
B should do S
C should do R.

1/9

Examiner Comments

In part (a) the candidate correctly stated that each entry had to be subtracted from a constant (so in essence changing the problem from a maximisation to a minimisation one) and so scored the first mark. However, the candidate failed to also state that an additional dummy row with equal values needed to be added to create a square array.

In the remainder of the question the candidate attempted to apply the Hungarian algorithm on a non-square array and so therefore no further marks could be awarded.

Student Response B

1. (a)

Add a dummy row with 0 entries, then subtract ~~the largest~~ all the entries ~~by the~~ from the largest entry

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

You may not need to use all of these tables
You may not need to use all the rows and columns

	P	Q	R	S	
A	32	40	37	42	
B	29	32	35	41	
C	37	33	39	40	
D	0	0	0	0	

Add dummy row with 0 entries

	P	Q	R	S	
A	10	2	5	0	
B	13	10	7	1	
C	5	9	3	2	
D	42	42	42	42	

42 - each element
↳ largest

	P	Q	R	S	
A	10	2	5	0	
B	12	9	8	0	
C	3	7	1	0	
D	0	0	0	0	

Reduce row
can be covered by 2 lines, so solution isn't optimal
Augment by 1
Column doesn't need to be reduced since each column has a 0

Question 1 continued

	P	Q	R	S	
A	9	7	4	0	
B	11	8	7	0	
C	2	6	0	0	
D	0	0	0	1	

Can be covered by 3 lines, so solution isn't optimal

Augment by 1

	P	Q	R	S	
A	8	0	4	0	
B	10	7	6	0	
C	2	6	0	1	
D	0	0	0	2	

Can be covered by 4 lines, so solution is optimal

A → Q

B → S

C → R

(40 + 41 + 39) = 120

	P	Q	R	S	
A					
B					
C					

6/9

Examiner Comments

Part (a) was fully correct (2 marks).

In part (b) the candidate failed to include a correct statement regarding row and column reductions and also failed to explicitly mention that an optimal solution is reached when there is a minimum of four lines covering the zeros. Also, in the initial row reduction of row B the value in the R column has been increased by 1 (to 8) rather than being reduced by 1 (to 6). Therefore in (b) the candidate scored B1B0M1M1M1B0A0 (4 marks).

Student Response C

1. (a)

Take ~~away~~ away each value from the largest value in the table

I.e replace each box with 42-x with x being its current value

Then add a dummy row with values twice the largest value in the table

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

You may not need to use all of these tables
You may not need to use all the rows and columns

	P	Q	R	S	
A	10	2	5	0	
B	13	10	7	1	
C	5	9	3	2	
D	26	26	26	26	

Subtract each value from 42

	P	Q	R	S	
A	10	2	5	0	
B	12	9	6	0	
C	3	7	1	0	
D	0	0	0	0	

Take away the smallest value in each row from each row

	P	Q	R	S	
A	10	2	5	0	
B	12	9	6	0	
C	3	7	1	0	
D	0	0	0	0	

Take away the smallest value in each column

Then attempt to cross out all zeroes in minimal lines

2 ≠ 4 so find the smallest value subtract from all non-crossed and add to intersections

2

Question 1 continued

	P	Q	R	S	
A	9	1	4	0	
B	11	8	5	0	
C	3	7	0	0	
0	0	0	0	1	

r=1

Attempt to cross all zeroes until no. of lines = n (in n×n matrix)

	P	Q	R	S	
A	8	0 x	4	0	
B	10	7	5	0 x	
C	2	6	0 x	0	
	0 x	0	1	2	

r=1

4 lines used, now assign tasks for each worker

~~Dummy does P~~

A does Q

C does R

B does S

	P	Q	R	S	
A					
B					
C					

8/9

Examiner Comments

Part (a) was fully correct.

In part (b) all but the final accuracy mark was awarded (as when the first augmentation of 1 takes places the candidate has forgot to reduce the entries in cells CP and CQ by 1).

Exemplar Question 2

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2. (*a*) Find the general solution of the recurrence relation

$$u_{n+1} = 3u_n + 2^n \qquad n \geqslant 1$$

(4)

(*b*) Find the particular solution of this recurrence relation for which $u_1 = u_2$

(2)

(Total for Question 2 is 6 marks)

Mean Score 1.4 out 6

Examiner Comments

As stated in the introduction, the responses to this question on the relatively new topic of recurrence relations were mixed. While some candidates scored full marks, a significant number made very little progress after correctly stating the complementary function in part (a). Examiners noted that many candidates seemed to need to re-write the recurrent relation in terms of u_n instead of working with the given form in terms of u_{n+1} (which seemed to display a lack of understanding of how these types of equations work) and many incorrectly re-wrote $u_{n+1} = 3u_n + 2^n$ as $u_n = 3u_{n-1} + 2^n$ so scoring nothing but possibly the first mark in part (a). Those who correctly stated the complementary function as $A(3)^n$ and then used $u_n = k\left(2^n\right)$ as a trial solution in the given recurrence relation usually went on to derive the correct general solution but some incorrectly stated this as $u_{n+1} = A(3)^n - 2^n$ instead of in terms of u_n.

In part (b) those who had a general solution of the correct form could usually score the method mark for finding a value for their constant based on the condition that $u_1 = u_2$. However, a correct particular solution was rarely seen in this part.

Mark Scheme

Question	Scheme	Marks	AOs
	$u_{n+1} = 3u_n + 2^n \; n \geq 1$		
2(a)	(aux equation $m - 3 = 0 \Rightarrow$) complementary function is $A(3)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = k(2^n)$ so $2k(2^n) = 3k(2^n) + 2^n$	M1	1.1b
	$k = -1$	A1	1.1b
	General solution is $u_n = A(3)^n - 2^n$	A1	1.1b
		(4)	
(b)	$u_1 = u_2 \Rightarrow 3A - 2 = 9A - 4 \Rightarrow A = ...$	M1	3.1a
	$u_n = 3^{n-1} - 2^n$	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: CAO for complementary function

M1: substituting correct trial solution into the recurrence relation – allow substitution of $u_n = k(2^n)$ into $u_n = 3u_{n-1} + 2^{n-1}$ but not $u_n = 3u_{n-1} + 2^n$

A1: CAO $k = -1$

A1: CAO for the general solution – must include $u_n = ...$

(b)

M1: using the condition $u_1 = u_2$ to calculate a value for their A $\left(= \frac{1}{3}\right)$ - this mark is dependent on the general solution being of the form $\pm\lambda(3)^n \pm \mu(2)^n$

A1: CAO for the particular solution (oe) – must include $u_n = ...$ - however, if neither (general nor particular) solution is given in terms of u_n then award this mark if correct expression in terms of n seen (or if both solutions are given in terms of say u_{n+1})

Student Response A

2.

a) $u_{n+1} = 3u_n + 2^n$ $\qquad kp^n \Rightarrow \lambda p^n$

$u_n = c(3)^n$ $\qquad$ P.S. $= \lambda p^n$

$\lambda p^{n-1} = 3\lambda p^{n-1} + 2^{n-1}$

$0 = 2\lambda p^{n-1} + 2^{n-1}$

$0 = \cancel{2(\lambda p}$

$-2^{n-1} = 2\lambda p^{n-1}$

$-1^{n-1} = \lambda p^{n-1}$

$\frac{-1^{n-1}}{p^{n-1}} = \lambda$

$\therefore \lambda = -\frac{1}{p}$ $\qquad u_n = c(3)^n - \frac{1}{p}$

b) $u_1 = u_2$

$3c - \frac{1}{p} = 9c - \frac{1}{p}$

$3c = 9c$

$0 = 6c$

$\therefore c = 0$ $\qquad u_n = 0 \times 3^n - \frac{1}{p}$

$u_n = -\frac{1}{p}$

1/6

Examiner Comments

Unfortunately, this type of response was all too common with only the first mark for a correct complementary function in part (a) being awarded. The candidate did not have a correct form for the trial solution and so therefore no further marks could be awarded.

Student Response B

a) $u_{n+1} = 3u_n + 2^n$ $\qquad n \geq 1$

$u_{n+1} = k(3)^n + 2^n$

$u_n = \lambda(2)^n$

$u_{n+1} = \lambda(2)^{n+1}$

$\lambda(2)^{n+1} = 3\lambda(2)^n + 2^n$

$\lambda(2 \times 2^n) = 3\lambda(2^n) + 2^n$

$2\lambda = 3\lambda + 1$

$-\lambda = 1$

$\lambda = -1$

$u_{n+1} = k(3^n) - 2^n$

b) $u_1 = u_2$

$u_2 = k(3^1) - 2^1$ $\qquad u_1 = k(3^0) - 2^0$

$u_2 = 3k - 2$ $\qquad u_1 = k - 1$

$3k - 2 = k - 1$

$2k = 1 \Rightarrow k = 0.5$

$u_{n+1} = 0.5(3^n) - 2^n$

4/6

Examiner Comments

An almost correct response to part (a) with the only error being that the answer should have been of the form u_n = ... and not u_{n+1} = … (so scoring the first three marks only in part (a)).

In part (b) the candidate has the correct method for finding the particular solution but due to their error in (a) the value they find for their k is incorrect (so scoring only the 1 mark in part (b)).

Student Response C

2.

a) $u_{n+1} = 3u_n + 2^n$

$C.F = c(3)^n$ $\quad$ $P.S = \cancel{\lambda 2^n}\ k(2)^n$

$k(2)^{n+1} = 3k(2)^n + 2^n$

$2k(2)^n = 3k(2)^n + 2^n$

$-2^n = k(2)^n$

$-1 = k$

$P.S = -(2)^n$

$u_{n+1} = c(3)$

a) $u_{n+1} = 3u_n + 2^n$

$u_n = 3u_{n-1} + 2^{n-1}$

$u_n = 3u_{n-1} + \frac{1}{2}(2)^n$

$C.F = c(3)^n$ $\quad$ $P.S = kp^n = k(2)^n$

$k(2)^n = 3k(2)^{n-1} + \frac{1}{2}(2)^n$

$k(2)^n = \cancel{3k}\ \frac{3}{2}k(2)^n + \frac{1}{2}(2)^n$

$-\frac{1}{2}(2)^n = \frac{1}{2}k(2)^n$

$-\frac{1}{2} = \frac{1}{2}k$

$-1 = k$

$P.S = -(2)^n$

$u_{n+1} = c(3)^n - 2^n.$

b) $c(3)^2 - 2^2 = c(3)^1 - 2^1$

$9c - 4 = 3c - 2$

$6c = 2$

$c = \frac{1}{3}$

$u_{n+1} = \frac{1}{3}(3)^n - 2^n$

5/6

Examiner Comments

Like the previous response in that the general solution was not given in the form $u_n = \ldots$ but instead was written as $u_{n+1} = \ldots$ (so scoring three out of four marks available in part (a)).

However, this notational error was not penalised twice, and this candidate went on to score both marks in (b) for a correct expression for the particular solution.

Exemplar Question 3

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3.

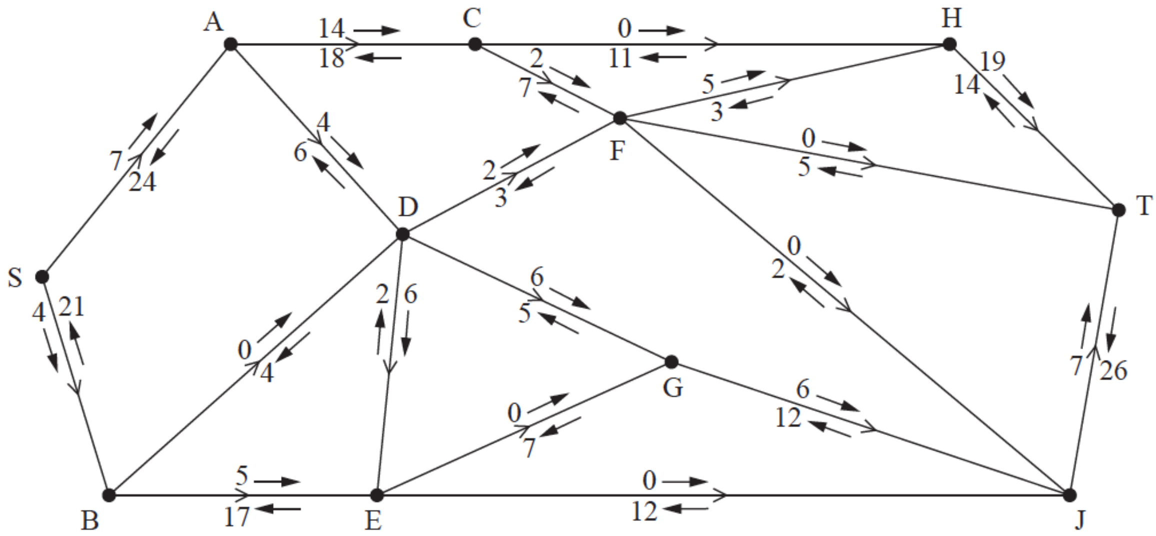


Figure 1

Alexa is monitoring a system of pipes through which fluid can flow from the source, S, to the sink, T. Currently, fluid is flowing through the system from S to T.

Alexa initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

(*a*) State the value of the initial flow.

(1)

(*b*) Explain why arcs DF and DG can never both be full to capacity.

(1)

(*c*) Obtain the capacity of the cut that passes through the arcs AC, AD, BD, DE, EG and EJ.

(1)

(*d*) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(*e*) Use your answers to part (*d*)to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book.

(1)

(*f*) Prove that the answer to part (*e*)is optimal.

(3)

(Total for Question 3 is 10 marks)

Mean Score 5.2 out of 10

Examiner Comments

In part (a) nearly all candidates correctly stated the initial flow as 45.

In part (b) many candidates struggled to explain why arcs DF and DG can never **both** by full to capacity. Very few explained that the total capacity of these two arcs is 16 but the maximum flow of the two arcs that flow into D (AD and BD) is only 14. Many incorrectly based their argument on the flow out of D without realising that flow could leave D via arc DE.

In part (c) the most common error in calculating the value of the given cut was to include a value for arc DE even though this arc was directed from the sink set of nodes to the source set of nodes for this cut.

Most candidates could correctly apply the labelling procedure in part (d) to find at least two correct flow-augmenting routes (and corresponding flows) but not all managed to increase the flow by the correct amount of 8 (and some surprisingly managed to increase the flow to 10).

In part (e) candidates are reminded of the following two points when showing a flow on a diagram.

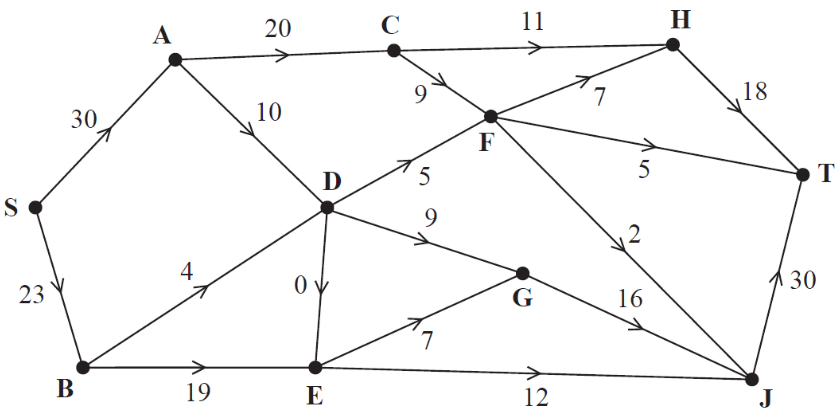
- There should only be one number on each arc and
- all arcs should be assigned a value (even if this value is zero).

Both points were condoned by examiners this series, but this may not be so in the future.

Part (f) discriminated well with very few **proving** that the flow in part (e) was optimal. To assist for future series please find below a way in which candidates can set out such a proof:

- It is best to state (and not just draw) a cut (that passes through saturated arcs that are directed from the source set to the sink set of nodes together with any arcs with zero flow that are directed from the sink set to source set) - so for this network as a list of arcs this was CH, CF, AD, BD, DE, EG and EJ or as set of nodes $\{S, A, B, C, E\}, \{D, F, G, H, J, T\}$.
- State the capacity of this cut and hence what this implies about the minimum cut e.g. the value of this cut is 53 which implies that the minimum cut ≤ 53
- State the value of the flow through the network after augmentation and what this implied about the maximum flow e.g. the current flow through the network is 53 which implies that the maximum flow is ≥ 53
- Conclude the proof by referring to the maximum flow-minimum cut theorem e.g. the min. cut is ≤ 53 and the max. flow is ≥ 53 but by the maximum flow-minimum cut theorem the max. flow is equal to min. cut therefore the maximum flow is 53 and therefore the flow in part (e) is indeed optimal.

Mark Scheme

Question	Scheme	Marks	AOs
3(a)	45	B1	1.1b
		(1)	
(b)	e.g. the total capacity of arcs DF and DG is 5 + 11 = 16. The capacity of the two arcs leading into D are 10 (from AD) and 4 (from BD) giving a total capacity into D of 14. As 14 < 16 arcs DF and DG cannot both be full to capacity	B1	2.4
		(1)	
(c)	Value of cut = 32 + 10 + 4 + 7 + 12 = 65	B1	1.1b
		(1)	
(d)	e.g. SACFHT – 2; SADGJT – 4; SBEDFHT – 2 e.g. SACFHT – 2; SADGJT – 2; SADFHT – 2; SBEDGJT – 2 e.g. SACFHT – 2; SADGJT –4; SBEDGJT – 2	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(e)	e.g.  Alternative: DG 11, GJ 18, JT 32 DF 3, FH 5, HT 16	B1	1.1b
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through CH, CF, AD, BD, DE, EG and EJ Value of flow = 53 Therefore by the max-flow min-cut theorem it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(10 marks)

Notes
(a) **B1:** CAO (45) **(b)** **B1:** CAO (as a minimum accept mention that the max flow into D is 14 and max flow out of D is 16 together with comparison of these two values – (node) D must be mentioned) **(c)** **B1:** CAO (65) **(d)** **M1:** One correct flow augmenting route found from S to T + flow value **or** two correct routes **A1:** Two correct routes + correct flow values **A1:** CSO – increasing the flow by 8 only **(e)** **B1:** CAO – condone more than one value on an arc only if one of these values is circled – mark those that have been circled only **(f)** **M1:** Construct the start of an argument based on the max-flow min-cut theorem (that is an attempt to find a genuine cut together with the value of either their cut or flow (but not re-iterating the cut given in (c) – AC, AD, BD, DE, EG, EJ)) **A1:** Use appropriate process of finding a minimum cut – must see correct cut + value correct (accept '53' and the cut either stated or drawn on either diagram) **A1:** Correct deduction that the flow is maximal by stating 'maximum flow (equal to) minimum cut' – dependent on previous A mark and the correct flow in (e)

Student Response A

3.

a. 45

b) Because the maximum flow which can come into D is 10. ~~And then~~ An if DF and DG where saturated it would at least need a flow of 21. However, a flow of 10 can only enter D therefore 21 can not leave from D.

c) 32 + 10 + 4 + 7 + 12 = 65

d) • Additional flow of 2 via SACFHT
• 2 additional flow via SADFHT
• 2 additional flow via SBEDGJT

f) Minimum cut = Maximum flow theorm
minimum cut = 51
Maximum flow = 51

∴ Minimum cut = maximum flow, ∴ optimal.

Question 3 continued

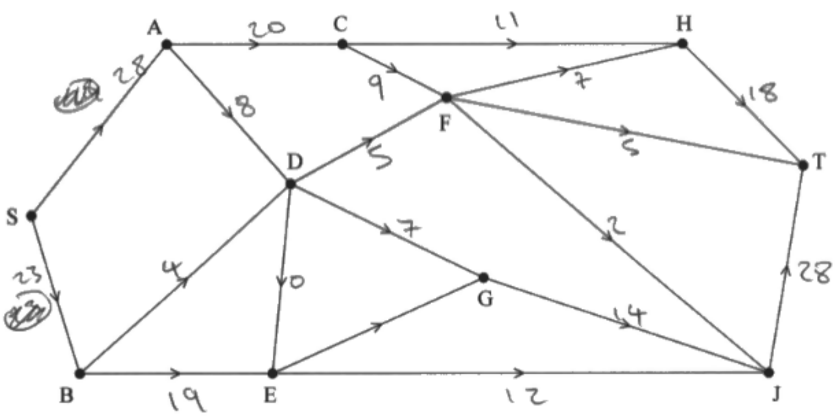


Diagram 1

4/10

Examiner Comments

Part (a) was correct (1 mark)

In part (b) the candidate has stated the total capacity of DF and DG as 21 which is incorrect (it should have been 16).

Part (c) is correct (1 mark) and in part (d) the candidate has two correct routes and corresponding flows so scored the method and first accuracy marks in this part. However, the candidate has only increased the flow by 6 and not to the maximum possible value of 8.

Part (e) is incorrect and there is no attempt at a cut in part (f) so no marks could be awarded here either.

Student Response B

3.

a. 45

b. Only a maximum of 14 can flow into D and DF + DG = 16 which is >14 so they can never both be at full capacity.

c. 47

d. 2 additional flow via SACFHT
2 additional flow via SADFHT
2 additional flow via SADGJT
2 additional flow via SBEDGJT

Question 3 continued

e.

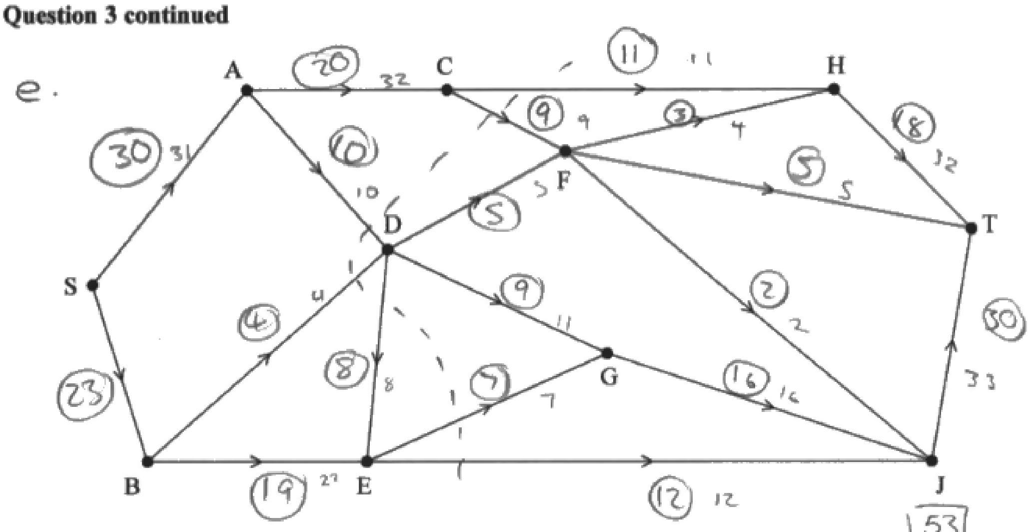


Diagram 1

f. Maxflow = 53
Mincut = 53 CH, CF, AD, BD, ED, EG, EJ.

Max flow = min cut theorem
53 = 53

7/10

Examiner Comments

Parts (a) and (b) were correct (so 2 marks in total)

Part (c) is incorrect – the value of the cut was 65 not 47

Part (d) was fully correct with the flow correctly increased by 8 (3 marks)

Part (e) was incorrect (the flow along DE should have been 0 not 8) which therefore meant that in

Part (f) only the first two marks could be awarded for proving that the flow was maximal (as the final mark was dependent on a correct flow in (e)).

Student Response C

a. ~~init~~ initial flow = 45

b. capacity for 14 into D but combined capacity of DF and DG is 16 so they cannot both be saturated at ~~one~~ any one time.

c. capacity of cut = 65

D. Add^n 2 via SBEDGJT
Add^n 4 via SADGJT
Add^n 2 via SACFHT
8 additional flow found.

Question 3 continued

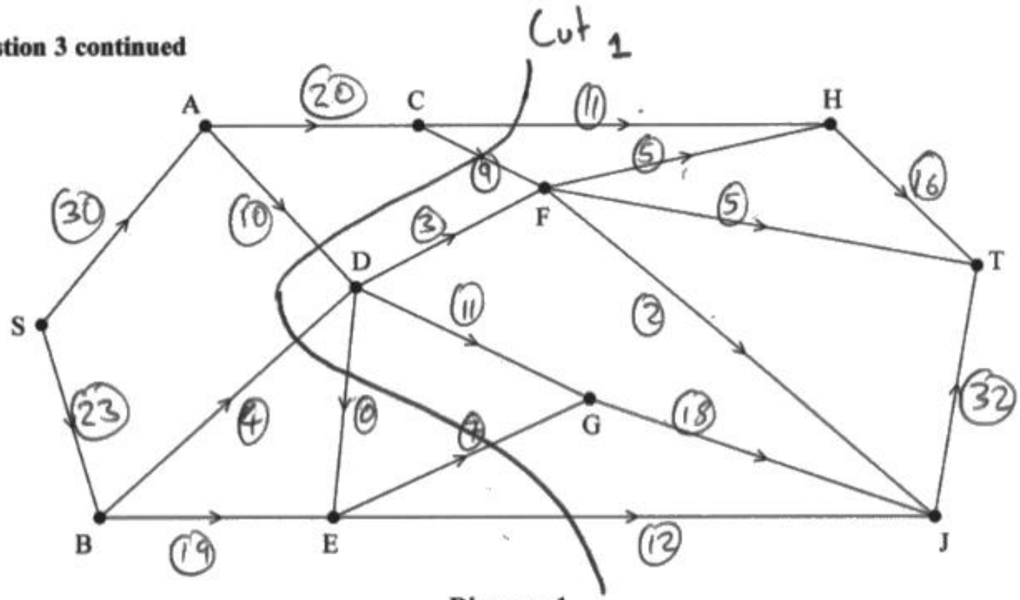


Diagram 1

f. Part e is optimal ~~dut~~ due to minimum cut = maximum flow Theorum

max flow = 53

10/10

Examiner Comments

A fully correct response showing the level of detail required in part (f) to prove that a flow is maximal by using the maximum flow-minimum cut theorem (that is showing a correct flow through the network by placing a single value on every arc (including those with zero flow), showing or stating a cut with capacity equal to the flow through the network and the stating of the theorem).

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Exemplar Question 4

4. The table below gives the pay-off matrix for a zero-sum game between two players, Aljaz and Brendan. The values in the table show the pay-offs for Aljaz.

Brendan

		Option X	Option Y	Option Z
Aljaz	Option P	–6	–1	2
	Option Q	5	4	–7
	Option R	5	6	3

(*a*) (i) Show that this game is stable.

(ii) State the value of this game to Brendan.

(3)

Option R is removed from Aljaz's choices and the reduced game, with option R removed, is no longer stable.

(*b*) Find the best strategy for Aljaz in this reduced game, defining any variable you use.

(7)

(*c*) Explain why Brendan should never play option Y

(1)

Let Brendan play option X with probability q

(*d*) (i) Explain why q satisfies the equation $6q - 2(1 - q) = 1.6$

(ii) Hence find the best strategy for Brendan in this reduced game.

(4)

(Total for Question 4 is 15 marks)

Mean Score 8.0 out of 15

Examiner Comments

Part (a)(i) was done very well, with most candidates finding correct row minimum and column maximum values, with very few errors. A small number of candidates failed to either correctly identify the row maximin and column minimax, or state that the game was stable with correct justification.

In part (ii) most correctly stated the value of the game to Brendan (*B*) but unsurprisingly gave the answer as 3 (which was the value to Aljaz (*A*)).

In (b) most candidates set up three correct probability expressions (though some had errors when simplifying these expressions) and then most subsequently went on to draw a graph with 3 lines; a few candidates attempted to just solve three pairs of simultaneous equations, scoring no marks.

It was noted that some graphs:

- were poorly drawn without rulers,
- went beyond the axes at $p < 0$ and $p > 1$,
- had uneven or missing scales on the vertical axes,
- were so cramped that it was difficult to identify the correct optimum point.

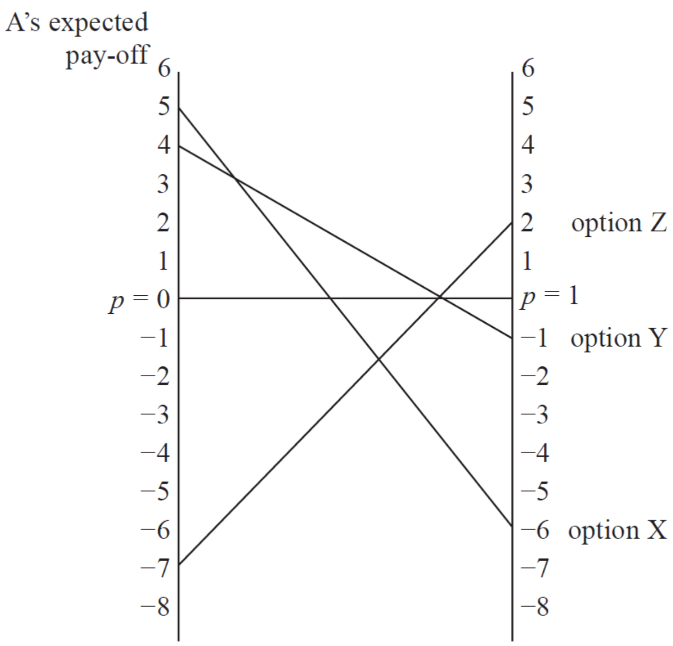
Most candidates attempted to solve the pair of equations for which they considered to be their optimal point from their graph. Those that solved the correct pair usually went on to list the correct options for A (that is, that *A* should play option P with probability 0.6 and option *Q* with probability 0.4) although a number did not state their answer in context or did not define *p* initially as the probability of A playing option P (and hence 1 – *p* as the probability of A playing option *Q*).

Very few candidates could come up with a correct explanation for why B should never play option Y; many candidates gave a response that danced around the correct answer, but examiners were looking for an answer that strictly indicated that the graph indicated that for all value of *p* Brendan could gain more by playing either option *X* or *Z*.

The responses to part (d)(i) were very mixed with many candidates failing to **explain** why *q* satisfied the given equation – many simply stated the given equation or derived it without explaining that if A plays option P then B can expect to gain $-\left(-6q+2\left(1-q\right)\right)$ (as the values in the table are the pay-offs for player A) and the value of the game to player B is $-\left(5-11\left(0.6\right)\right)=1.6$ which implies that $-\left(-6q+2\left(1-q\right)\right)=1.6$ which leads to the given equation. Many candidates simply stated the value of 1.6 without any indication of where it had come from and therefore did not 'show it'.

Part (d)(ii) was done much better but some candidates are clearly not conversant with certain mathematical command words and many derived the value of *q* from scratch and did not use the given equation in (d)(i). While many correctly found the value of *q* as 0.45 many did not give the best strategy for B in context or refer to the fact that B should 'play' option *X* with prob. 0.45 (and option *Z* with prob. 0.55).

Mark Scheme

Question	Scheme	Marks	AOs
4(a)	**(i)** Row minima: –6, –7, 3 max is 3 Column maxima: 5, 6, 3 min is 3 Row(maximin) = Col(minimax) therefore game is stable **(ii)** value of the game to B is –3	M1 A1 B1	1.1b 2.4 2.2a
		(3)	
(b)	Let A play option P with probability p and option Q with probability $1 - p$	B1	3.3
	If B plays option X, A's gains are $-6p + 5(1 - p) = 5 - 11p$ If B plays option Y, A's gains are $-p + 4(1 - p) = 4 - 5p$ If B plays option Z, A's gains are $2p + (-7)(1 - p) = -7 + 9p$  $5 - 11p = -7 + 9p \Rightarrow p = 3/5$ A should play option P with probability 0.6 and option Q with probability 0.4	M1 A1 M1dep A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b 1.1b 3.2a
		(7)	
(c)	As indicated by the graph Brendan can, for all values of p, gain more by playing either options X or Z e.g. for $0 \le p \le \frac{3}{5}$ Brendan would be better off playing Z and for $\frac{3}{5} < p \le 1$ Brendan would be better off playing X than playing Y	B1	3.2a
		(1)	

Question	Scheme	Marks	AOs
(d)	**(i)** If A plays option P then B can expect to gain $-\left(-6q+2\left(1-q\right)\right)$ as the values in the table are the pay-offs for A	B1	2.2a
	The value of the game to B is $-\left(5-11\left(0.6\right)\right)$ = 1.6 (or equivalent calculation e.g. $-\left(-7+9\left(0.6\right)\right)$)	B1	2.1
	(ii) $6q-2(1-q)=1.6 \Rightarrow q=0.45$	B1	1.1b
	B should play option X with probability 0.45 and option Z with probability 0.55	B1	3.2a
		(4)	
			(15 marks)

Notes

(a)
M1: finding row minimums and column maximums – condone one error
A1: row maximin (3) = col minimax (3) so stable (dependent on correct row minimums and col maximums) - as a minimum accept '3 = 3 so stable'
B1: CAO (–3)
(b)
B1: defining variable p (must mention 'probability' – as a minimum accept 'P with probability p and Q with probability $1 – p$')
M1: setting up three expressions in terms of p (need not be simplified)
A1: all three expressions correctly simplified
M1dep: axes correct, at least one line correctly drawn from their expressions – dependent on previous M mark in **(b)**
A1: correct graph – if no scaling on vertical axis assume 1 line = 1 unit (A0 if graph extends for $p< 0$ and/or $p> 1$)
A1: using the graph to obtain the correct probability expressions leading to the correct value of p
A1ft: interpret their value of p in the context of the question – must refer to 'play' and the associated probabilities (need not say 'probability' again) – this mark is dependent on both previous M marks in this part
(c)
B1: correct explanation in context (of playing only X and Z <u>for all</u> p) with specific reference to the modelling of the problem by the graph
(d)(i):
B1: correctly deducing the lhs of the given equation (with clear reasoning for the change in sign) – only allow stating $6q – 2(1 – q)$ (without seeing the change of sign) if the game is restated for player B
B1: correctly deriving the rhs of the given equation (must indicate that this is the value of the game to B although as a minimum accept V(B)) – stating ± 1.6 without any working is B0 - n**ote that for either mark in (d)(i) candidates must <u>explain</u> where the two parts of the given equation came from**
Note that candidates may explain the formulation of the given equation by considering $-6q+2\left(1-q\right)=-1.6$ (which is what player B can expect to lose if player A plays option P which is equal to the value of the game to player A)
(d)(ii):
B1: CAO for the value of q (must come from solving $6q – 2(1 – q) = 1.6$)
B1: CAO in context and must refer to 'play'

Student Response A

4.

			Brendan		
		q			
		Option X	Option Y	Option Z	worst Aljaz
r	Option P	−6	−1	2	−6
Aljaz	Option Q	5	4	−7	−7
	Option R	5	6	3	3
	Worst Brenda	5	6	3	

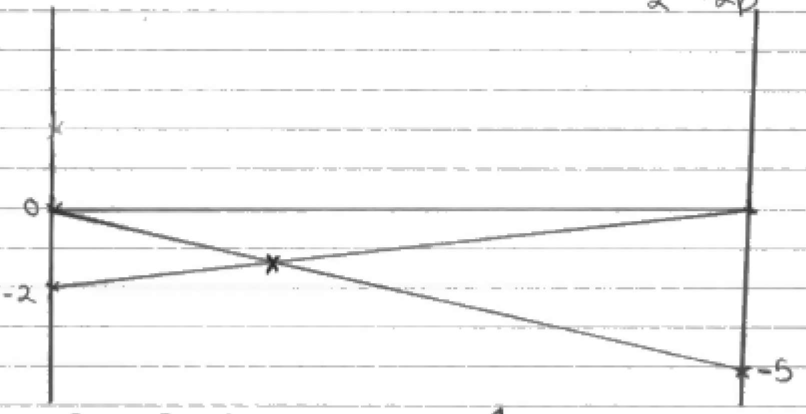
a) i) This game is stable as

row maximin = column maximin
3 = 3 .

ii) The value of this game to Brendan is −3

		Brendan	
	OPT X	OPT Y	OPT Z
Aljaz p OPT P	−6	−1	2
1−p OPT Q	5	4	−7

b) Option P → probability p $-6p - 1p + 2p = -5p$
Option Q → probability $1-p$. $5(1-p) + 4(1-p) - 7(1-p)$
$2 - 2p$



$-5p = 2 - 2p$ $p = 1/2$
$-2 = -3p$

Question 4 continued

d) i) ~~$-6q + 5q + 5q = 4q$~~

~~$6q - 2(1 - q) = 6q - 2 + 2q$~~

4/15

Examiner Comments

Part (a) was fully correct (3 marks)

In part (b) the candidate has defined the variable *p* (so scoring the first mark in this part) but has failed to set up all three expressions in terms of *p* and so therefore no further marks could be awarded in this part.

Part (c) was not attempted and while part (d) was attempted and then crossed out the crossed-out work was considered (as it had not been replaced) but was not worthy of any credit.

Student Response B

4.

Brendan

Aljaz		Option X	Option Y	Option Z	
	Option P	−6	−1	2	-1
	Option Q	5	4	−7	-7
	Option R	5	6	3	3
		5	6	3	

a) ~~Rows~~ 3 = 3, Row maximin = column minimax

∴ the game is stable

ii) ~~...~~ -1

b)

	X	Y	Z
P	-6	-1	2
Q	5	4	-7

let p = probability Aljaz plays P

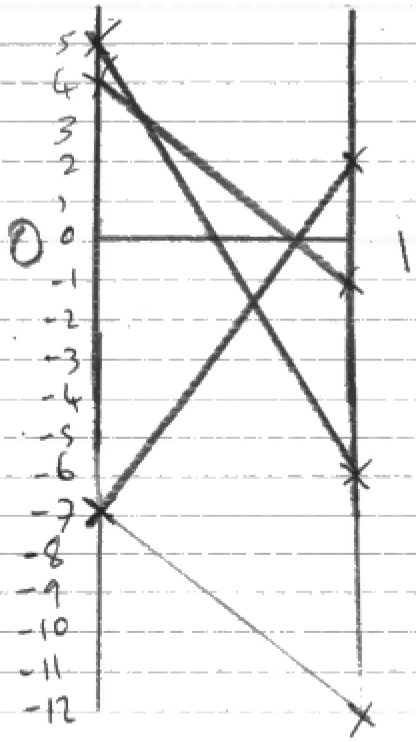
let 1-p = probability Aljaz plays Q

$-6p + 5(1-p) = -11p + 5$

$-1p + 4(1-p) = -5p + 4$

$2p - 7(1-p) = \cancel{...}9p - 7$

Question 4 continued



~~Aljaz plays P all the time~~

$-11p+5 = 9p-7$

~~b)~~ $-20p = -12$

$p = \frac{3}{5}$ ~~Aljaz~~

$\therefore$ Aljax plays P $\frac{3}{5}$ of the time and Q $\frac{2}{5}$ of the time, randomly

c) Because there is no way he could win ~~it~~ over time as he will lose 4 ~~and~~ if ~~only ever gain~~ 1 if he Aljax plays Q and only gain 1 if Aljaz plays P.

Question 4 continued

d)
$$\begin{array}{c|cc} & P & Q \\ \hline X & 6 & -5 \\ Y & 1 & -4 \\ Z & 2 & 7 \end{array}$$

take the negative transpose of the matrix,

let probability he plays $X = q$. as stated

8/15

Examiner Comments

In part (a) the candidate has attempted to find the row minimums and column maximums but has made an error in the first row and so only the first mark was awarded in this part.

Part (b) was fully correct (and so scored all 7 marks).

In part (c) the candidate failed to give a correct explanation in context for why *B* should never play *Y* (very few candidates made use of the graph from part (b) in this part).

No marks were awarded in part (d) as this response failed to adequately explain how the given equation was derived (and the candidate did not go and use this given equation to work out the strategy for player *B*).

Student Response C

4.

		Brendan		
		Option X	Option Y	Option Z
Aljaz	Option P	−6	−1	2
	Option Q	5	4	−7
	Option R	5	6	3

$$\text{a) i. } \begin{pmatrix} -6 & -1 & 2 \\ 5 & 4 & -7 \\ 5 & 6 & 3 \end{pmatrix} \begin{array}{c} \text{max min} \\ -6 \\ -7 \\ \boxed{3} \end{array}$$

$$\begin{array}{cccc} \text{mini max} & 5 & 6 & \boxed{3} \end{array}$$

maximin = 3
minimax = 3

Because maximin = minimax this game is stable.

(ii) −3

b) Let p be probability Aljaz plays option P
∴ $(1-p)$ is probability Aljaz plays option Q

$$P_{\text{option } X} = -6p + 5(1-p)$$
$$= -6p + 5 - 5p$$
$$= 5 - 11p \quad 5 \quad -6$$

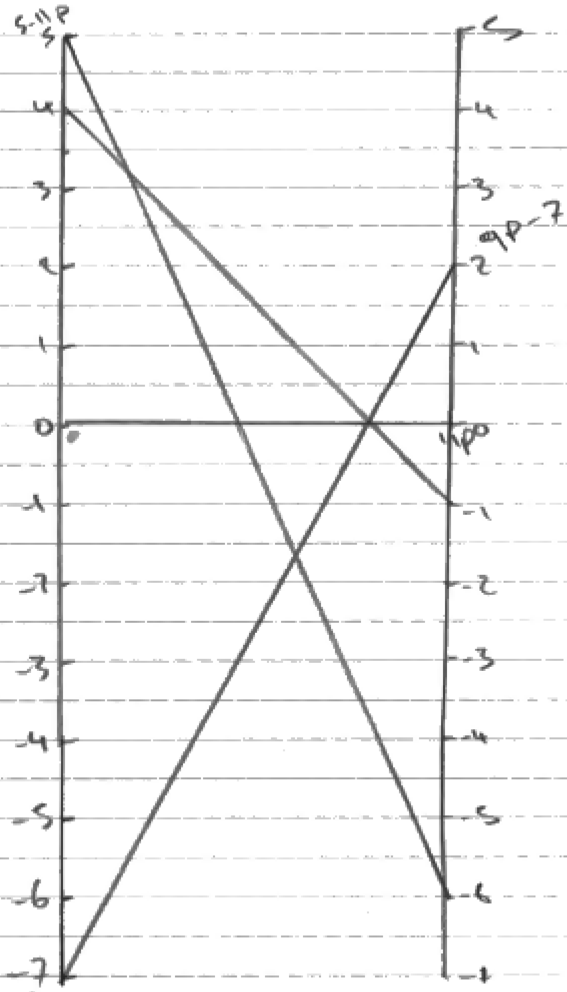
$$P_{\text{option } Y} = -p + 4(1-p)$$
$$= -p + 4 - 4p$$
$$= 4 - 5p \quad 4 \quad -1$$

$$P_{\text{option } Z} = 2p + -7(1-p)$$
$$= 2p - 7 + 7p$$
$$= 9p - 7 \quad -7 \quad 2$$

$$9p - 7 = 5 - 11p$$
$$20p = 12$$
$$p = \frac{12}{20} = \frac{3}{5}$$
$$1 - p = \frac{2}{5}$$

Aljaz should play option P ~~with probability~~ 3/5 of the games and option Q 2/5 of the games but he should play randomly.

Question 4 continued



c) If Brenden plays Option Y He is always going to lose.

d) i. value of game to Brenden if he plays x is $-[5-11(3/5)] = -[-1.6] = 1.6$

Probability if Alyaz plays P is:

$6(q) + -2(1-q)$, which equals the value of the game to him. $6q - 2(1-q) = 1.6$

Question 4 continued

ii. $6q - 2 + 2q = 1.6$

$8q = {}^{18}/_{5}$

$q = {}^{9}/_{20}$

$1 - q = {}^{19}/_{20}$

Brenden should play option X ~~as the~~ ${}^{9}/_{20}$ of the time and Option Z ${}^{19}/_{20}$ of the time, but should play randomly.

12/15

Examiner Comments

Parts (a) and (b) were fully correct (10 marks)

In part (c) the candidate failed to give a correct explanation in context for why B should never play *Y* (many candidates wrote answers such as this which gave no mathematical reason for why *Y* should not be chosen).

In part (d)(i) the candidate did not give sufficient written explanation for where the left-hand side of the given equation came from (again it was all too common for candidates to simply quote this expression without any real justification), however, this candidate has adequately explained why the right-hand side of the equation is 1.6 (by referring to the value of the game to player B).

In (d)(ii) the candidate has correctly found the value of q as $\frac{9}{20}$ but then gone on to incorrect state the value of $1 - q$ as $\frac{19}{20}$ rather than $\frac{11}{20}$ so therefore the final mark (for stating the strategy for player *B*) could not be awarded.

A Level Further Mathematics – Decision 1 (9FM0 3D)

Exemplar Question 1

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1.

2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(*a*) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(2)

The list of numbers is now to be sorted into descending order.

(*b*) Perform a quick sort on the original list to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

For a list of n numbers, the quick sort algorithm has, on average, order $n \log n$.

Given that it takes 2.32 seconds to run the algorithm when $n = 450$

(*c*) calculate approximately how long it will take, to the nearest tenth of a second, to run the algorithm when $n = 11\ 250$. You should make your method and working clear.

(2)

(Total for Question 1 is 8 marks)

Mean Score 6.4 out of 8

Examiner Comments

Examiners reported that a significant number of candidates struggled in applying the first-fit bin packing algorithm in part (a). This was mainly down to not applying the algorithm correctly. First fit is just that; candidates must decide if the current item under consideration will fit in the first bin rather than the most recent bin used. In this part several candidates placed the 1.2 in the third bin (and not the first bin) and others did not place the 0.2 in the third bin.

Many correct solutions were seen in part (b), but several candidates did not choose their pivots consistently, switching between middle-left and middle-right pivots during the quick sort algorithm. Several candidates either lost an item or changed an item during the sort, and in a small number of cases only one pivot was chosen per iteration. As stated in previous examiners' reports candidates must make it clear that the sort is complete by either explicitly stating that the sort is complete or by choosing each item as a pivot or by rewriting the final list. A common error included the 1.5 not being used as a pivot for the fifth pass; candidates should be reminded that items should remain in the order from the previous pass as they move into sub-lists. There were only a few instances where candidates selected the first or last items as the pivot. Pivots were usually chosen consistently although the spacing and notation on some solutions made these difficult for examiners to follow. Some candidates over complicated the process by insisting on using a different 'symbol' to indicate the pivots for each pass. Those candidates who sorted into ascending order usually remembered to reverse their list at the end to gain full credit although several candidates left their list in ascending order.

In part (c) many candidates scored both marks for showing correct working (for example, $\dfrac{2.32(11250\log 11250)}{450\log 450}$ or starting with an equation of the form $t = k(n\log n)$ with $t = 2.32$ and $n = 450$ to find k) followed by a final answer of 88.6. A sizeable minority did not score the accuracy mark for giving an answer of 88.56 or 90, as the question required an answer to the nearest tenth of a second. Those candidates with a correct answer, but no supporting working, scored one mark only, as the question explicitly stated that both method and working needed to be shown.

Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Bin 1: $\underline{2.1 1.7 1.2}$ Bin 2: $\underline{3.0 1.9}$ Bin 3: $\underline{3.2}$ 1.4 0.2 Bin 4: 3.3 1.5	M1 A1	1.1b 1.1b
		(2)	
(b)	e.g. middle right 2.1 1.7 3.0 1.9 3.2 $\boxed{1.2}$ 3.3 1.4 1.5 0.2 Pivot: 1.2 2.1 1.7 3.0 1.9 $\boxed{3.2}$ 3.3 1.4 1.5 $\underline{1.2}$ 0.2 Pivot(s): 3.2 (0.2) 3.3 $\underline{3.2}$ 2.1 1.7 3.0 $\boxed{1.9}$ 1.4 1.5 $\underline{1.2}$ 0.2 Pivot(s): (3.3) 1.9 3.3 $\underline{3.2}$ 2.1 $\boxed{3.0}$$\underline{1.9}$ 1.7 $\boxed{1.4}$ 1.5 $\underline{1.2}$ 0.2 Pivots: 3.0 1.4 3.3 $\underline{3.2 3.0}$ 2.1 $\underline{1.9}$ 1.7 $\boxed{1.5}$$\underline{1.4 1.2}$ 0.2 Pivot(s): (2.1) 1.5 3.3 $\underline{3.2 3.0}$ 2.1 $\underline{1.9}$ 1.7 $\underline{1.5 1.4 1.2}$ 0.2	M1 A1 A1ft A1	1.1b 1.1b 1.1b 1.1b
		(4)	
(c)	$\frac{2.32(11\,250\log 11\,250)}{450\log 450}$ =88.6 seconds	M1 A1	1.1a 1.1b
		(2)	
		(8 marks)	

Notes for Question 1

PLEASE NOTE NO MISREADS IN PARTS (a) and (b) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING ORDER IN (b)

(a) M1: First six items placed correctly and at least eight values placed in bins - condone cumulative totals for M1 only (the underlined values)
A1: CSO – all correct (so no additional/repeated values)

(b) M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1A1 only** – Bubble sort is not a MR and scores M0

A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot value of 3.2 for the second pass or (if middle left) a pivot value of 1.9

A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fifth pass with the 1.5 used as a pivot or if middle left a fourth pass with the 1.7 used as a pivot

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(b)**
- If the list is not reversed in **(b)** then remove the last two A marks earned in **(b)**. If the candidate says that the list needs reversing in **(b)** but does not actually show the reversed list in **(b)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle left

2.1	1.7	3.0	1.9	$\boxed{3.2}$	1.2	3.3	1.4	1.5	0.2
3.3	$\underline{3.2}$	2.1	1.7	3.0	$\boxed{1.9}$	1.2	1.4	1.5	0.2
3.3	$\underline{3.2}$	$\boxed{2.1}$	3.0	$\underline{1.9}$	1.7	1.2	$\boxed{1.4}$	1.5	0.2
3.3	$\underline{3.2}$	3.0	$\underline{2.1}$	$\underline{1.9}$	$\boxed{1.7}$	1.5	$\underline{1.4}$	$\boxed{1.2}$	0.2
3.3	$\underline{3.2}$	3.0	$\underline{2.1}$	$\underline{1.9}$	$\underline{1.7}$	1.5	$\underline{1.4}$	$\underline{1.2}$	0.2

Middle right ascending (which requires a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	3.2	$\boxed{1.2}$	3.3	1.4	1.5	0.2
0.2	$\underline{1.2}$	2.1	1.7	3.0	1.9	$\boxed{3.2}$	3.3	1.4	1.5
0.2	$\underline{1.2}$	2.1	1.7	3.0	$\boxed{1.9}$	1.4	1.5	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	1.7	$\boxed{1.4}$	1.5	$\underline{1.9}$	2.1	$\boxed{3.0}$	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	1.7	$\boxed{1.5}$	$\underline{1.9}$	2.1	$\underline{3.0}$	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	$\underline{1.5}$	1.7	$\underline{1.9}$	2.1	$\underline{3.0}$	$\underline{3.2}$	3.3

Middle left ascending (which required a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	$\boxed{3.2}$	1.2	3.3	1.4	1.5	0.2
2.1	1.7	3.0	$\boxed{1.9}$	1.2	1.4	1.5	0.2	$\underline{3.2}$	3.3
1.7	1.2	$\boxed{1.4}$	1.5	0.2	$\underline{1.9}$	$\boxed{2.1}$	3.0	$\underline{3.2}$	3.3
$\boxed{1.2}$	0.2	$\underline{1.4}$	$\boxed{1.7}$	1.5	$\underline{1.9}$	$\underline{2.1}$	3.0	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	1.5	$\underline{1.7}$	$\underline{1.9}$	$\underline{2.1}$	3.0	$\underline{3.2}$	3.3

(c) M1: Complete correct method – allow reciprocal – allow slips in values only e.g. 1250 for 11250

A1: CAO – the exact value of 88.6 must be stated at some point (as question specifically asked for the answer to the nearest tenth of a second) –isw if 90 follows 88.6 seen. 90 with no working scores no marks. An answer of 88.6 with no working scores M1A0 – condone lack of units (but if present must be correct)

Student Response A

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

a) ~~2, 2.1~~ 1.7

Bin 1 : 2.1, 1.7
Bin 2 : 3.0, 1.9
Bin 3 : 3.2, 1.2
Bin 4 : 3.3, 1.4
Bin 5 : 1.5, 0.2.

b) 2.1 1.7 3 1.9 3.2 1.2 3.3 1.4 1.5 0.2

2.1 1.7 3 1.9 3.2 3.3 1.4 1.5 1.2 0.2

3.3 3.2 2.1 1.7 3 1.9 1.4 1.5 1.2 0.2

3.3 3.2 2.1 3 1.9 1.7 1.4 1.5 1.2 0.2

3.3 3.2 2.1 3 1.9 1.7 1.4 1.5 1.2 0.2

3.3 3.2 3 2.1 1.9 1.7 1.4 1.5 1.2 0.2

Question 1 continued

2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

c) $t = K n \log n$

$2.32 = (450 \log 450) \times K$

$K = \dfrac{2.32}{450 \log 450}$

~~$t =$~~

$t = \dfrac{2.32}{450 \log 450} \times 11250 \log 11250$

$t = 88.557\ldots$

$t = 88.6 \text{ s}$

4/8

Examiner Comments

Part (a) - this was a common incorrect answer which scored no marks; the 1.2 is in the wrong bin (and so therefore this candidate failed to apply the first-fit algorithm at its first real test).

In part (b) the first pivot was chosen correctly, and list partitioned correctly. This response therefore scores the method mark. The first pass is correct, and the candidate has correctly selected the second pivot which scores the first accuracy mark. However, the candidate has only chosen one pivot per iteration and therefore cannot access the last two accuracy marks. (2 (out of 4) marks)

Part (c) is fully correct (2 marks)

Student Response B

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

a) Bin 1 = 2.1, 1.7, 1.2
Bin 2 = 3.0, 1.9
Bin 3 = 3.2, 1.4, 0.2
Bin 4 = 3.3, 1.5

b)

	PIVOTS
2.1 1.7 3.0 1.9 $\boxed{3.2}$ 1.2 3.3 1.4 1.5 0.2	3.2
2.1 1.7 3.0 $\boxed{1.9}$ 1.2 1.4 1.5 0.2 3.2 $\boxed{3.3}$	1.9, 3.3
1.7 1.2 $\boxed{1.4}$ 1.5 0.2 1.9 $\boxed{2.1}$ 3.0 3.2 3.3	1.4, 2.1
$\boxed{1.2}$ 0.2 1.4 $\boxed{1.7}$ 1.5 1.9 2.1 3.0 3.2 3.3	1.2, 1.7
0.2 1.2 1.4 ~~1.5~~ 1.5 1.7 1.9 2.1 3.0 3.2 3.3	SORTED LIST

c) 2.32 seconds → n = 450, order nlogn

450 log 450 = 1193.945631

when n = 11250

11250 log 11250 = 45575.46588

11250 ÷ 450 = 25

2.32 x 25 = 58 seconds

4/8

Examiner Comments

Part (a) is fully correct (2 marks)

Part (b) – his response highlights a commonly seen issue. This candidate has sorted into ascending order. This amounts to a misread of the question which is penalised by withholding two accuracy marks. In this case two marks is the maximum available for a list sorted into ascending order unless the final answer is reversed and in which case recovery is allowed and full marks could have been awarded. (2 marks)

In part (c) the candidate just multiplies the time by 25 and so scores no marks in this part.

Student Response C

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

a) ~~Bin 1: 2.1, 1.7~~ Bin 1: 2.1, 1.7, 1.2
~~Bin 2: 3.0, 1.9~~ Bin 2: 3.0, 1.9
~~Bin 3: 3.2, 1.2~~ Bin 3: 3.2, 1.4, 0.2
~~Bin 4: 3.3, 1.4~~ Bin 4: 3.3, 1.5
~~Bin 5: 1.5~~

b) 2.1, 1.7, 3.0, 1.9, 3.2, 1.2, 3.3, 1.4, 1.5, 0.2
2.1, 1.7, 3.0, 1.9, 3.2, 3.3, 1.4, 1.5, $\boxed{1.2}$, 0.2
3.3, $\boxed{3.2}$ 2.1, 1.7, 3.0, 1.9, 1.4, 1.5, $\boxed{1.2}$, $\boxed{0.2}$
$\boxed{3.3}$, $\boxed{3.2}$, 2.1, 3.0, $\boxed{1.9}$, 1.7, 1.4, 1.5, $\boxed{1.2}$, $\boxed{0.2}$
$\boxed{3.3}$ $\boxed{3.2}$ $\boxed{3.0}$ 2.1, $\boxed{1.9}$ 1.7, 1.5, $\boxed{1.4}$ $\boxed{1.2}$, $\boxed{0.2}$
$\boxed{3.3}$ $\boxed{3.2}$ $\boxed{3.0}$ $\boxed{2.1}$ $\boxed{1.9}$ 1.7 $\boxed{1.5}$ $\boxed{1.4}$ $\boxed{1.2}$ $\boxed{0.2}$
∴ Sorted list = 3.3, 3.2, 3.0, 2.1, 1.9, 1.7, 1.5, 1.4, 1.2, 0.2.

c) ~~11250 log~~

$$\frac{11250 \times \log(11250)}{450 \times \log(450)} \times 2.32 = 38.17 \text{ seconds}$$
$$= 38.2 \text{ seconds (1dp).}$$

7/8

Examiner Comments

Part (a) is fully correct, the crossed-out working was ignored as it has been replaced. (2 marks)

Part (b) is full correct too, it was common to see single-value lists considered as pivots, while this is unnecessary it is not incorrect and therefore was not penalised. (4 marks)

The method in part (c) was correct and a corresponding fully correct expression seen. However, the answer given omits the multiplication by 2.32. This therefore gained the method mark but not the corresponding accuracy mark. (1 mark)

Exemplar Question 2

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2.

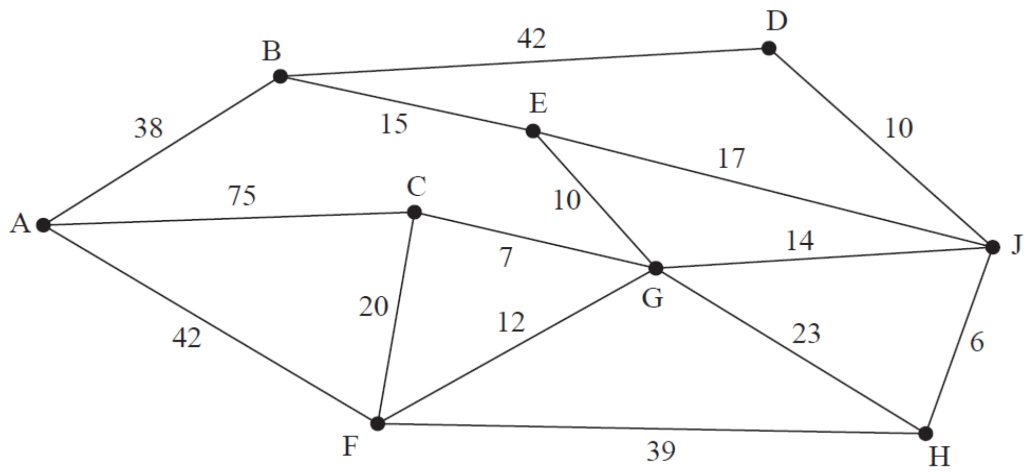


Figure 1

[*The total weight of the network is* 370]

Figure 1 represents a network of corridors in a building. The number on each arc represents the length, in metres, of the corresponding corridor.

(*a*) Use Dijkstra's algorithm to find the shortest path from A to D, stating the path and its length.

(6)

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.

(*b*) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

(*c*) Find the length of Naasir's route.

(1)

On a different day, all the corridors that start or finish at B are closed for redecorating. Naasir needs to check all the remaining corridors and may now start at any vertex and finish at any vertex. A route is required that excludes all those corridors that start or finish at B.

(*d*) (i) Determine the possible starting and finishing points so that the length of Naasir's route is minimised. You must give reasons for your answer.

(ii) Find the length of Naasir's new route.

(3)

(Total for Question 2 is 14 marks)

Mean Score 10.2 out of 14

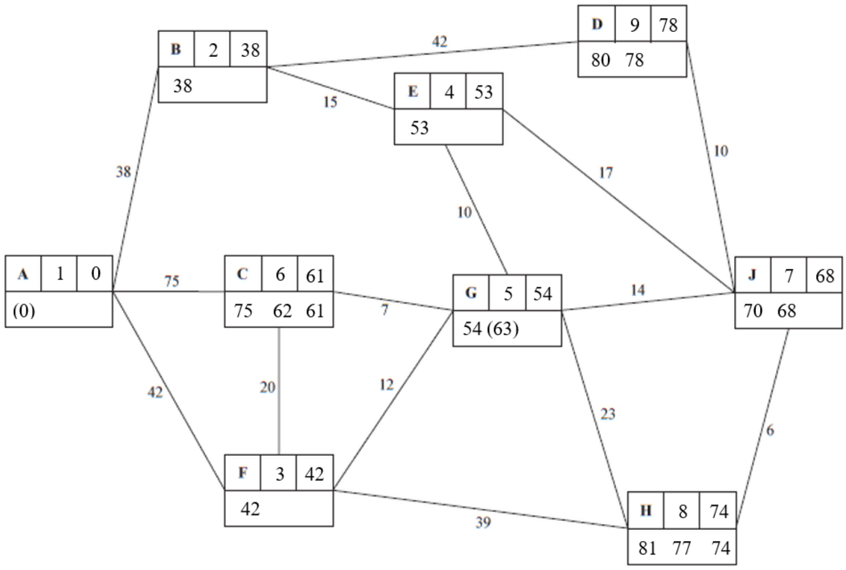
Examiner Comments

This question was generally well attempted. Most candidates were clearly wellprepared for Dijkstra's algorithm with most errors in part (a) arising from slips rather than lack of understanding. Values at nodes A, B, F, E and G were generally correct with errors most commonly occurring at C and/or H. Most candidates were able to correctly state the path from A to D and the corresponding length. There were the standard errors in order of working values and/or extra or missing working values and examiners saw several cases where there was no replacement of working values whatsoever. As is often the case, handwriting presented something of a challenge when deciphering working values. Candidates should be reminded of the importance of working values in judging the application of the method by examiners and so candidates should ensure their presentation is clear and it cannot be stressed enough that working values should **NOT** be crossed out.

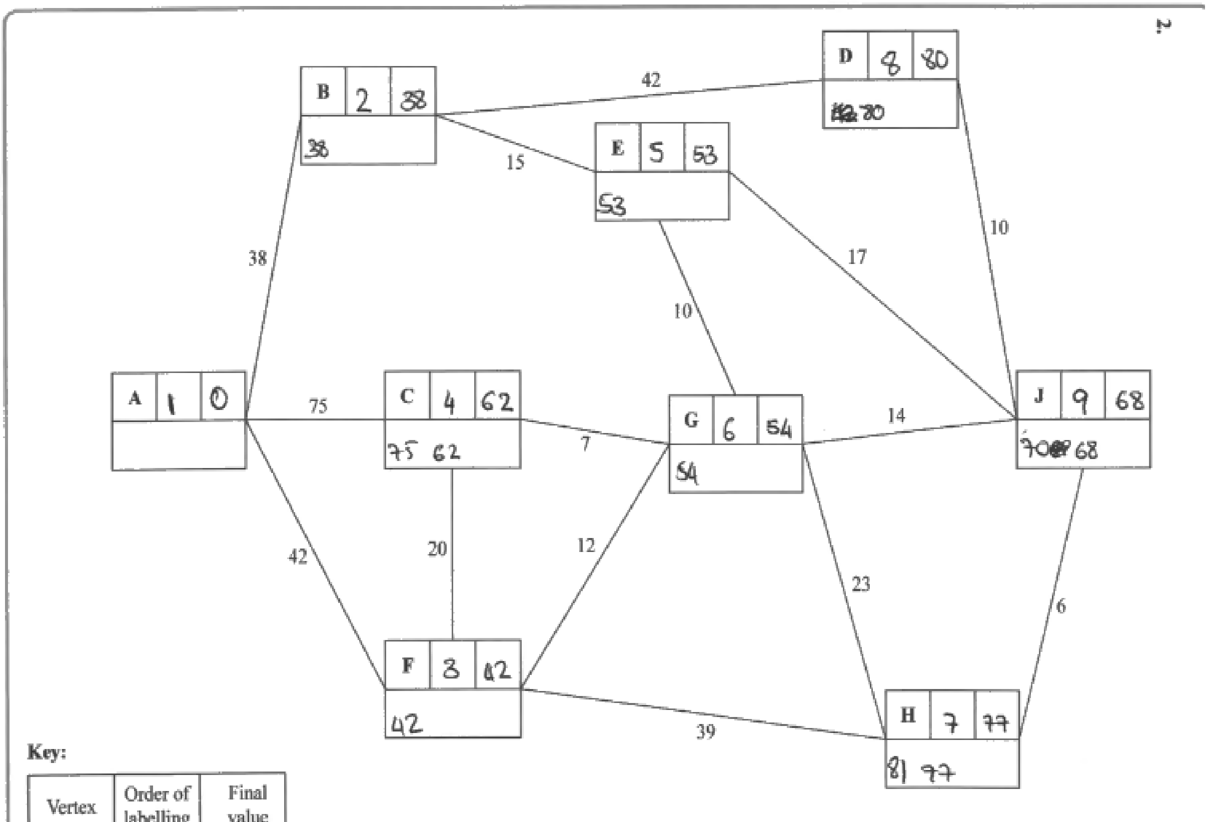
In part (b), most candidates were able to identify the correct four odd nodes and most paired them correctly. There were thankfully few candidates who made the error of considering less (or more) than the three pairings. There were however, perhaps surprisingly, frequent errors in the pairing totals. A common error arose in the pairing AH + CE. However, errors in the totals often did not affect the choice of repeated arcs which were usually stated correctly. Candidates should however note the requirement for repeated *arcs* rather than repeated *pairings* as there were several candidates who lost a mark for stating simply AE + CH. Some candidates were clearly on 'autopilot' and stated the length of the route here rather than (or as well as) in part (c).

Part (d) was the most testing part of the question for a lot of candidates, with some making no attempt. Of those that did many were limited to the mark for identifying which nodes were now odd, and some candidates got stuck after "finding" there were three or five odd nodes. Some candidates who had identified the four odd nodes seemed unsure how to proceed, giving unjustified start and finish points and ignoring the arc that needed to be repeated. Other candidates were meticulous in their working listing every possible pairing with their length, then picking the shortest CG and identifying D and H as the start/finish points. Some candidates lost the accuracy mark by selecting a shortest pairing before homing in on CG. A small but worrying number of candidates persist in using the "avoid the longest of the repeats" misconception. In the final part of (d), of those who got here successfully, not all removed the three arcs from B not currently available before adding on the repeated arc CG.

Mark Scheme

Qu	Scheme	Marks	AOs
2(a)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Path from A to D is AFGJD	A1	2.2a
	Length of path from A to D is 78 metres	A1ft	2.2a
		(6)	
(b)		M1	3.1b
	A(FG)C + E(J)H = 61 + 23 = 84 A(B)E + C(GJ)H = 53 + 27 = 80* A(FGJ)H + C(G)E = 74 + 17 = 91	 A1ft A1	 1.1b 1.1b
	Repeat arcs: AB, BE, CG, GJ and JH	A1	2.2a
		(4)	
(c)	Length of the route is 370 + 80 = 450 metres	B1ft	2.2a
		(1)	
(d)(i)	If node B is removed this makes D, C, G and H odd	M1	3.1b
	The shortest path between any two odd nodes is CG (so repeat CG) so the route should start at D and finish at H (or vice-versa)	A1	2.2a
(ii)	Length of new route is 370 – 38 – 42 – 15 + 7 = 282 metres	B1	2.2a
		(3)	
			(14 marks)

Student Response A



Shortest path from A to D: A B D

Length of shortest path from A to D: 80 metres

Question 2 continued

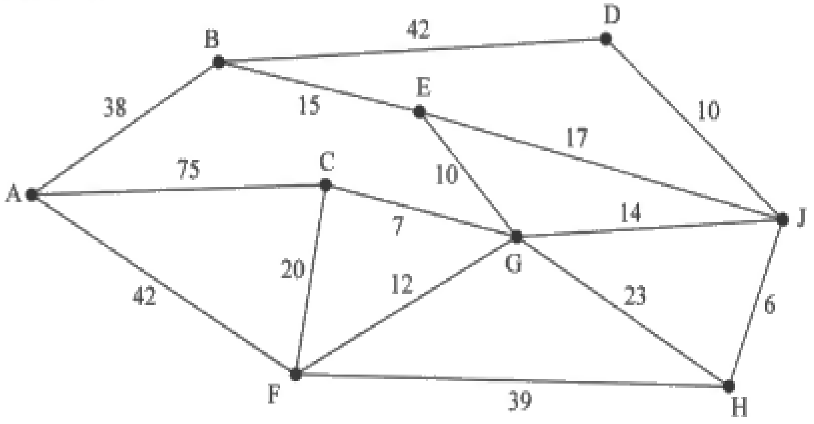


Figure 1

(b) A ~~B~~ C E ~~G~~ H

AC EH
62 33 = 95

AE CH
53 30 = 83

AH ECE
81 17 = 98

AE & CH must be traversed twice

(c) 370 + 83 = 453 metres

(d)
(i) Possible starting & finishing points include C G H D as they are all odd nodes and therefore Naasir will be able to travel around all arcs once

6/14

Examiner Comments

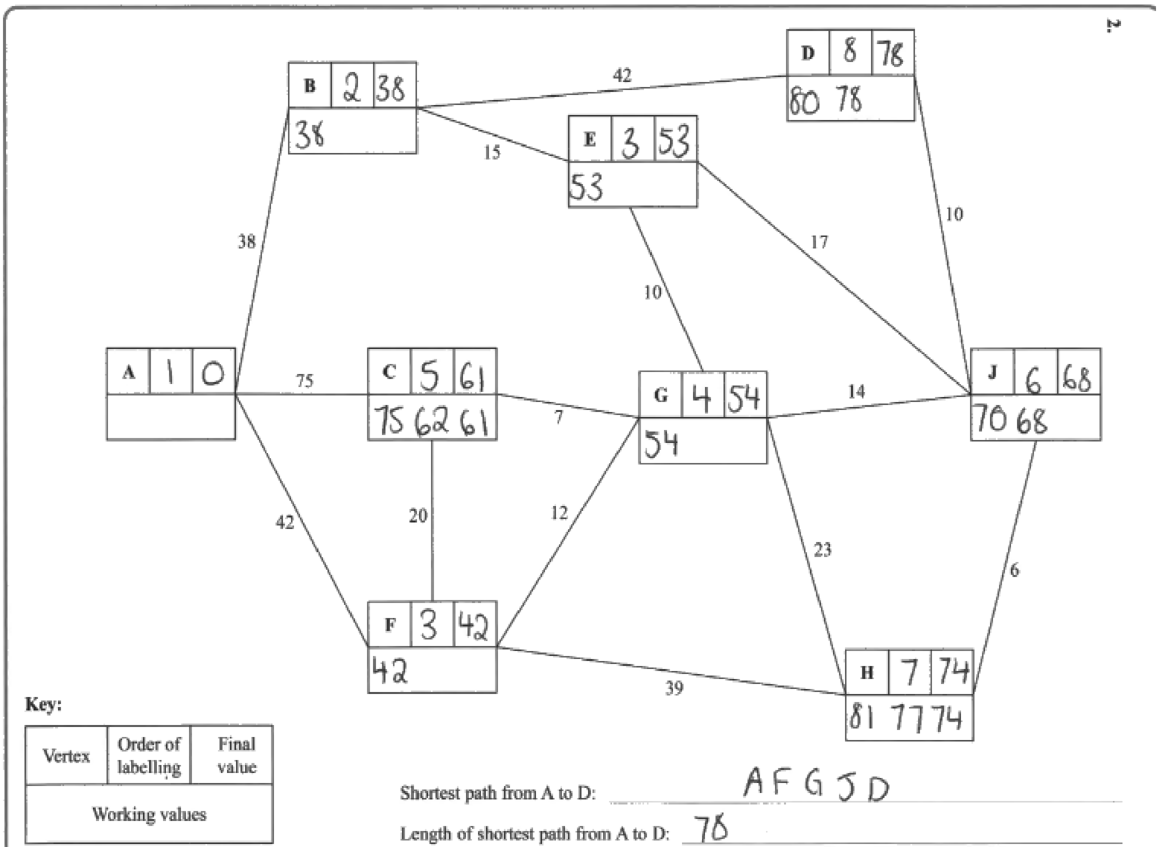
In part (a) a smaller value replaces a larger value at the working values at C, H and J (any two of these would have sufficed) so the method mark was awarded. The values at A, B, F and E are correct and the order of labelling is increasing and so the first accuracy mark was also awarded. The values at C are incorrect (and C has been labelled out of order, but this was not penalised here as other errors have occurred) and so the second accuracy mark could not be awarded. Finally, while values at H and D do follow from their other nodes, the candidate has incorrect order of labelling which is penalised here. The path is incorrect but the length of the shortest path from A to D is correct on the follow through from their final value at D giving a total of 3 marks in (a).

In part (b) the candidate has the correct three pairings of the required four odd nodes and so the method mark was awarded. However, no further marks could be awarded in this part as none of the totals were correct and the candidate did not state the correct arcs that needed to be repeated (so 1 mark).

In part (c) the mark was awarded on the follow through for 370 + their shortest repeat from part (b) (1 marks).

In part (d) the correct four odd nodes were given but there was no indication of choosing the correct shortest path between any pair of these odd nodes and finally the correct length was not given (1 mark).

Student Response B



Shortest path from A to D: AFGJD

Length of shortest path from A to D: 78

Question 2 continued

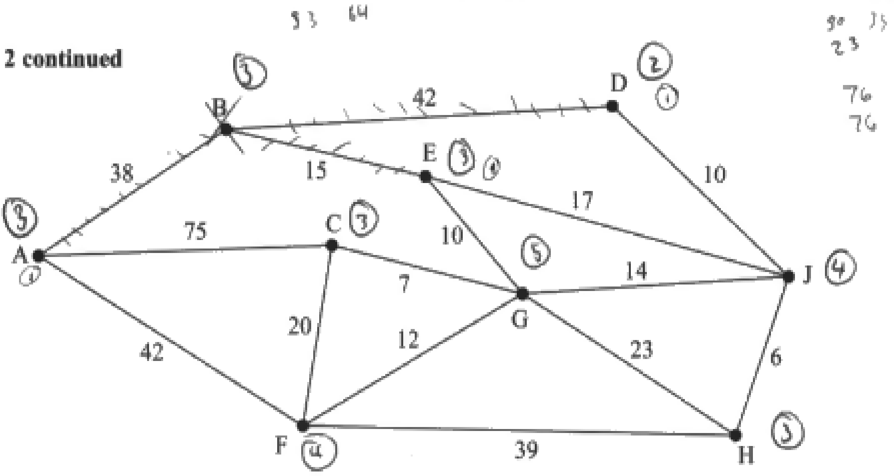


Figure 1

b. Odd: A, B, C, E, G, H

B and G remain odd.

AC + EH: AFGC + EJH = 61 + 23 = 84
AE + CH: ABE + CGJH = 53 + 20 = 73 → chosen
AH + CE: ABEJH + CGE = 76 + 17 = 93

Arcs AB, BE, CG, GJ and JH will need to be traversed twice.

c. length = 370 + 73 = 443 metres

~~d without B, there will be 5 odd nodes. To minimise the route we should start and end at the same vertex.~~
~~He must either start or end at D because it is only linked to~~
~~Start at D, finish at H~~
~~" " H, " " D~~

Question 2 continued

CD GH CA DG C DGH
CG DH

Start	Finish	(possibilities
D	H	CG traversed twice +7
H	D	
C	G	DJH traversed twice +16
G	C	
D	C	GJH +20
C	D	
G	H	~~EJD~~ ~~+20~~ CGJD +31
H	G	
D	G	~~E~~ ~~+13~~ CGJH +27
G	D	
C	H	GJD +24
H	C	

~~minimum route = 370+~~

Start and finish at D and H

ii. length = 370 + 7 = 377 metres

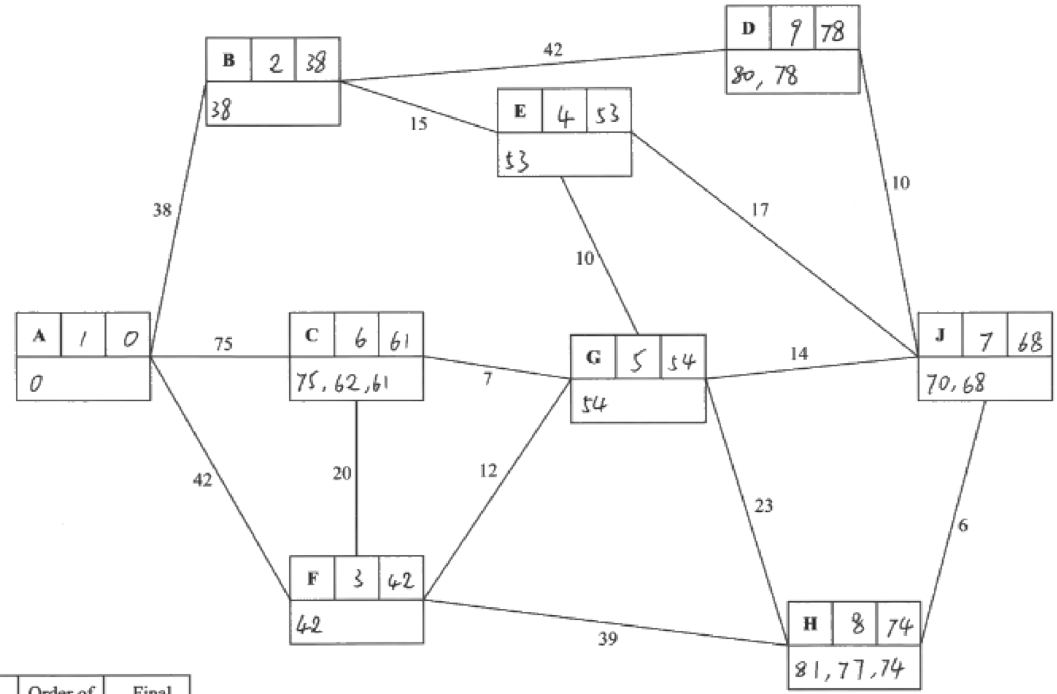
10/14

Examiner Comments

In part (a) the only error was the fact that the candidate used the same order of labelling at both nodes E and F (so scored 5 out of the 6 possible marks).

In part (b) the candidate had the three correct pairings of the four odd nodes but only one correct total (the 84 for AC + EH). However, they did correctly state the arcs that need to be repeated so scoring 3 out of the 4 possible marks in this part. In part (c) the candidate considered the correct odd nodes (D, C, G and H) but does not explicitly state CG has the shortest path and the length of the new route should have been given as 282 (and not 377) and so only 1 mark was awarded in part (c).

Student Response C



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to D: D: 78 − 10 = 68 → J: 68 − 14 = 54 → G: 54 − 12 = 42 → F: 42 − 42 = 0 ∴ AFGJD

Length of shortest path from A to D: 78 m

Question 2 continued

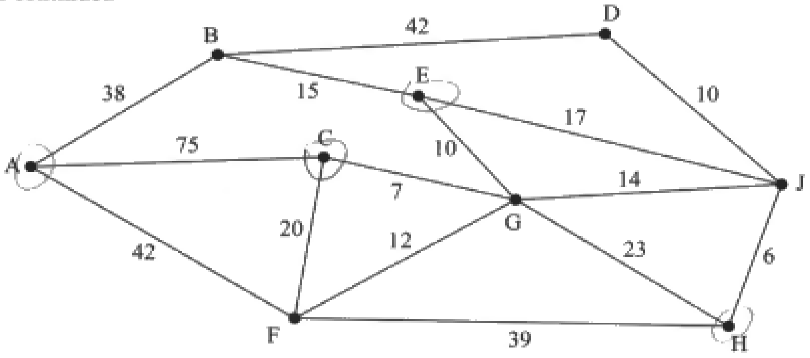


Figure 1

b) A, C, E, H are odd nodes.

AC + EH = 61 + 23 = 84

AE + HC = AB+BE + CG+~~GH~~ GJ + JH

= 38 + 15 + 7+14+6 = 80 ✓ the smallest

AH + CG = AF + FG + GJ + HJ + 7 = 91

∴ AB, BE, CG, GJ, JH will be traversed twice to minimise the length.

c) Length: 370 + 80 = 450 m

d i) Now AB, BD, BE are removed.

∴ D, C, G, H are odd.

DC + GH = 31 + 20 = 51

DG + CH = 24 + 27 = 51

DH + GC = 16 + 7 = 23 ✓

∴ DH + GC is the smallest

GC is the smallest

∴ GC is added.

We can start at D finish at H.

ii) 370 - 38 - 42 - 15 + 7 = 282 m

13/14

Examiner Comments

Parts (a), (b) and (c) are fully correct so scored 11 marks.

In part (d) the candidate has correctly considered the required four odd nodes (D, C, G and H) and while they have selected the shortest path (CG) this has come from first selecting the shortest total pairing first which is an incorrect method (and will not give the optimum route in all cases) – this lost the accuracy mark in this part but the total of 282 was correct and therefore the final independent accuracy mark could be awarded (giving a total of 2 marks for this part).

Exemplar Question 3

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3.

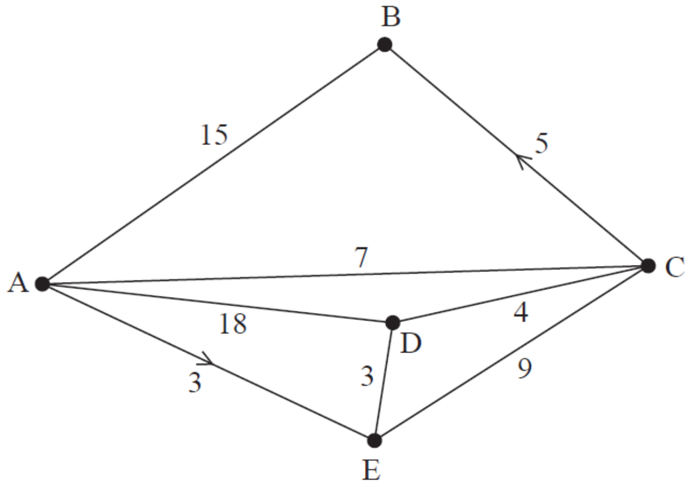

Figure 2

The network in Figure 2 shows the direct roads linking five villages, A, B, C, D and E.
The number on each arc represents the length, in miles, of the corresponding road.
The roads from A to E and from C to B are one-way, as indicated by the arrows.

(*a*) Complete the initial distance and route tables for the network provided in the answer book.

(2)

(*b*) Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each of the three iterations.

(5)

After five iterations of Floyd's algorithm the final distance table and partially completed final route table are shown below.

Distance table

	A	**B**	**C**	**D**	**E**
A	–	12	7	6	3
B	15	–	22	21	18
C	7	5	–	4	7
D	11	9	4	–	3
E	14	12	7	3	–

Route table

	A	**B**	**C**	**D**	**E**
A	A				
B	A	B			
C	A	B	C		
D	C	C	C	D	
E	D	D	D	D	E

(*c*) (i) Explain how the partially completed final route table can be used to find the shortest route from E to A.

(ii) State this route.

(3)

Mabintou decides to use the distance table to try to find the shortest cycle that passes through each vertex. Starting at D, she applies the nearest neighbour algorithm to the final distance table.

(*d*) (i) State the cycle obtained using the nearest neighbour algorithm.

(ii) State the length of this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(iv) Prove that Mabintou's cycle is not optimal.

(4)

(Total for Question 3 is 14 marks)

Mean Score 8.6 out of 14

Examiner Comments

Many candidates seemed to find the first part of this question on Floyd's algorithm relatively straight forward, even though it is a newly examined topic, with many scoring a significant number of marks. The route table in part (a) was almost unanimously done correctly, and most of the distance tables were too. A small number had one or two incorrect values in the distance table, and a very small number of candidates confused the rows and columns in the distance table.

It was clear that most candidates were using the method displayed in the new Pearson textbook to complete the iterations of the distance and route tables, though of course those using alternative approaches were given equal credit for their responses. A significant number of candidates made one or two errors causing them to drop the final accuracy mark in part (b), but a mark of 5 or 6 out of total of 7 (for parts (a) and (b)) was very common.

Only a very small proportion of candidates lost method marks for changing entries in rows/columns that they should have been leaving unchanged for that iteration.
It was part (c) and (d) that caused most problems for candidates. In part (c) several candidates were able to score full marks for a near full (but not totally complete) response, due to the varying nature of the different methods. Many lost marks though as they were not clear about how to use the tables, many did not mention rows and columns specifically, which lost the first two marks in this part. Also, the second (dependent) B mark was especially rarely scored as many candidates did not fully consider the route from D to A.

Meanwhile many candidates who lost these first two B marks still earnt the third for a correct route EDCA. Candidates should be aware of the level of detail required for a question that requires them to explain a method.

In part (d), many candidates failed to appreciate that the standard method of applying the nearest neighbour method on a table with undirected arcs would not apply in the same way with directed arcs, so incorrectly stated the nearest neighbour route. In nearly all cases this therefore led to the rest of part (d) being incorrect so unfortunately no marks in this part was not an uncommon score. Parts (d)(iii) and (iv) proved particularly challenging with very few candidates realising that (iii) was requesting the actual route used and that (iv) required candidates to find a shorter cycle. In general, proof that a solution to any Decision Mathematics problem is non-optimal only requires a counter-example i.e. a better solution.

Mark Scheme

Qu	Scheme	Marks	AOs
3(a)	Distance table and Route table (below)	B1 B1	1.1b 1.1b

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	∞	∞	∞
C	7	5	-	4	9
D	18	∞	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

Qu	Scheme	Marks	AOs
		(2)	
(b)	1st iteration:	M1 A1	1.1b 1.1b

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	22	33	18
C	7	5	-	4	9
D	18	33	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

Qu	Scheme	Marks	AOs
	2nd iteration:	A1ft	1.1b

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	22	33	18
C	7	5	-	4	9
D	18	33	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

Qu	Scheme	Marks	AOs
	3rd iteration (**SEE NOTES FOR VALID ALTERNATIVE**):	M1 A1	1.1b 1.1b

Distance table

	A	B	C	D	E
A	-	12	7	11	3
B	15	-	22	26	18
C	7	5	-	4	9
D	11	9	4	-	3
E	16	14	9	3	-

Route table

	A	B	C	D	E
A	A	C	C	C	E
B	A	B	A	C	A
C	A	B	C	D	E
D	C	C	C	D	E
E	C	C	C	D	E

Qu	Scheme	Marks	AOs
		(5)	
(c)(i)	Start at E(5th row) and read across to the A (1st column), there is a D there so the route from E to A is via D	B1	2.4
	Now consider both E to D and D to A – for E reading across to the D (4th column), there is a D indicating that the shortest path from E to D is ED. For D reading across to the A (1st column), there is a C indicating that the shortest path from D to A is via C	dB1	2.4
(ii)	EDCA	B1	2.2a
		(3)	

(d)(i)	NNA: D – E – C – B – A – D	B1	1.1b
(ii)	3 + 7 + 5 + 15 + 6 = 36 miles	B1	1.1b
(iii)	D – E – D – C – B – A – E – D	B1	3.2a
(iv)	e.g. the cycle A – E – D – C – B – A has a length of 30 miles < 36 miles so Mabintou's route is not optimal	B1	2.4
		(4)	
			(14 marks)

Notes for Question 3

IN THE DISTANCE AND ROUTE TABLE FOR PARTS (a) and (b) IGNORE WHATEVER IS WRITTEN IN THE LEAD DIAGONAL (TOP LEFT TO BOTTOM RIGHT)

(a) B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a 'large' number in these cells)

B1: Correct route table

(b)M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete

A1: CAO (condone dashes, etc. in cells EA and EB)

A1ft: No change from candidate's first iteration to second iteration for either table or ft from candidate's first iteration

M1: No change in the third row and third column of both tables with at least two values in the distance table correctly reduced from their second iteration and two values in the route table correctly changed

A1: CAO for third iteration (**note that the entry in row B column D for the route table could be an A**)

(c) B1: Row E column A is D so the route is E to A via D (or implies that the order of the nodes in the route is EDA) **or** D implied from general argument **or** Row E column A is D therefore the route begins ED (in all cases must clearly imply **row** E and **column** A)

dB1: Row D column A is C therefore the route goes via C (before A) **or** complete general argument that allows the route from D to A to be found **or** allow those who say that row D column A is C so the route is EDC and then row C column A is A

B1: CAO (EDCA)

(d)(i)B1: CAO (D – E – C – B – A – D)

(ii) B1: CAO (36 – no units required)

(iii) B1: CAO (D – E – D – C – B – A – E – D) **or** mentions that the cycle would visit E twice and D three times (or visit D before the end of the cycle – if D visited once stated and it is not clear that this isn't the start or finish then B0) **or** mention of E to C via D and A to D via E

(iv) B1: A correct cycle stated (e.g. a cyclic permutation of A – E – D – C – B – A) with corresponding correct length – **dependent on second B mark in this part** (so must have had 36 in **(d)(ii)**)

Student Response A

3. (a)

Initial distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	∞	∞	∞
C	7	5	–	4	9
D	18	∞	4	–	3
E	∞	∞	9	3	–

Initial route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b) 1st iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	22	33	18
C	7	5	–	4	9
D	18	33	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

2nd iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	22	33	18
C	7	5	–	4	9
D	18	33	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

3rd iteration

Distance table

	A	B	C	D	E
A	–	12	7	16	3
B	15	–	22	26	18
C	7	5	–	4	9
D	11	9	4	–	3
E	16	14	9	3	–

Route table

	A	B	C	D	E
A	A	C	C	C	E
B	A	B	A	C	A
C	A	B	C	D	E
D	C	C	C	D	E
E	C	C	C	D	E

Turn to page 11 for spare copies of these tables if you need to correct your work.

Question 3 continued

c)i) The shortest route from E to A goes via D

E to D is direct → D to C is direct

D to A goes via C → C to A is direct

ii) EDCA

d)i)

	A	B	C	D	E
	5	6	4	1	2
A	–	12	7	6	3
B	15	–	22	21	18
C	7	5	–	4	7
D	11	9	4	–	3
E	14	12	7	3	–

Add DE(3), Add AE(3), Add AC(7), Add BC(22) Add BD(9)

DBCAED

ii) $3+3+7+22+9 = 44$ miles

iii) DCBACAED

iv) Consider the cycle DECBAED - visits every vertex

Length $= 3+9+5+15+3+3$

$= 38$ miles

7/14

Examiner Comments

In part (a) the candidate correctly stated the initial distance and route tables (2 marks). In part (b), the first two iterations of Floyd's algorithm have been completed correctly and so the first three marks in this part were awarded. While the candidate has the correct method for the third iteration (in which the key feature is that there are no changes in the third row/columns of both tables) there is a 16 in row A column D instead of 11 and so this loses the final accuracy mark in this part (so scoring 4 marks).

Part (c) required candidates to explain how the route table could be used to find the required route and without references to either rows or columns, there was no real indication of how the table has been used and so the first two marks in this part could not be awarded. However, the correct route of EDCA was stated and so the final mark in this part was awarded (1 mark). In part (d)(i) the candidate did not use the final distance table correctly and ended up with the common incorrect nearest neighbour route of DBCAED which gave a total of 44 (and not the correct 36). The route given in (iii) was incorrect and the mark in (iv) was dependent on a correct nearest neighbour route seen in (ii).

Student Response B

3. (a)

Initial distance table

	A	B	C	D	E
A	x	15	7	18	3
B	15	x	x	x	x
C	7	5	x	4	9
D	18	x	4	x	3
E	x	x	9	3	x

Initial route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b) 1st iteration

Distance table

	A	B	C	D	E
A	x	12	7	6	3
B	15	x	x	x	x
C	7	5	x	4	9
D	11	x	4	x	3
E	14	x	9	3	x

Route table

	A	B	C	D	E
A	A	C	C	E	E
B	A	B	C	D	E
C	A	B	C	D	E
D	C	B	C	D	E
E	D	B	C	D	E

2nd iteration

Distance table

	A	B	C	D	E
A	x	12	7	6	3
B	15	x	22	21	18
C	7	5	x	4	9
D	11	9	4	x	3
E	14	12	9	3	x

Route table

	A	B	C	D	E
A	A	C	C	E	E
B	A	B	A	A	A
C	A	B	C	D	E
D	C	C	C	D	E
E	D	C	C	D	E

3rd iteration

Distance table

	A	B	C	D	E
A	x	12	7	6	3
B	15	x	22	21	18
C	7	5	x	4	7
D	11	9	4	x	3
E	14	12	7	3	x

Route table

	A	B	C	D	E
A	A	C	C	E	E
B	A	B	A	A	A
C	A	B	C	D	D
D	C	C	C	D	E
E	D	C	D	D	E

Turn to page 11 for spare copies of these tables if you need to correct your work.

c To go from E to A, view the E row and A column, the corresponding letter is D. Look therefore at the D row and A column, the letter is C. In the C row and A column the letter is A. This means the route must be EDCA for the shortest distance.

~~AD~~

~~$A_3 E_3 D_4 C_5 B_{15} A$~~

~~AEDCBA~~ ~~Weight = 30~~

~~$D_3 E_9 C_5 B_{15} A_{18} D$~~

~~DECBAD~~ ~~Weight~~

d i $D_3 E_7 C_5 B_{15} A_6 D$

ii DECBAD Weight = 36

iii DEDCBAED

iv If Mabintou uses this algorithm, she'll go to D three times and E twice, travelling 36 miles. If Mabintou starts at A then she'll be able to visit them all once (except D) at a distance of 30 miles

8/14

Examiner Comments

Both marks were awarded in part (a) for a correct distance and route table (examiners condoned x for ∞). In the first iteration several values in the first row and column have been changed and so the first method mark (and dependent on accuracy marks) could not be awarded.

The same issue is evident in the third iteration when a number of values in the third row and third column have been changed – therefore no marks could be awarded in part (b).

In part (c) the candidate has given a clear indication of how the final route table is used to get from E to A via the other nodes in the network and correctly stated the route so all three marks could be awarded in (c). Part (d) was almost fully correct apart from in part (iv) the candidate did state the correct length of a cycle that was shorter than the one found in (ii), however, without stating the actual cycle the final mark in part (d) could not be awarded.

Student Response C

3. (a)

Initial distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	∞	∞	∞
C	7	5	–	4	9
D	18	∞	4	–	3
E	∞	∞	9	3	–

Initial route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b) 1st iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	[22]	[33]	[18]
C	7	5	–	4	9
D	18	[35]	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	[A]	[A]	[A]
C	A	B	C	D	E
D	A	[A]	C	D	E
E	A	B	C	D	E

2nd iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	22	33	18
C	7	5	–	4	9
D	18	33	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

3rd iteration

Distance table

	A	B	C	D	E
A	–	[12]	7	[11]	3
B	15	–	22	[26]	18
C	7	5	–	4	9
D	[11]	[9]	4	–	3
E	[16]	[17]	9	3	–

Route table

	A	B	C	D	E
A	A	C	C	C	E
B	A	B	A	C	A
C	A	B	C	D	E
D	C	C	C	D	E
E	C	C	C	D	E

Turn to page 11 for spare copies of these tables if you need to correct your work.

Question 3 continued

(c)(i) ~~It is symmetrical~~. It shows how to get TO A FROM E. You do not need TO E FROM A.

(ii) E D C A

(d) (i)

~~B E A E B D~~

D E C B A D

(ii) ~~$3+3+7+22+9=44$ miles~~

$3+7+5+15+6 = 36$ miles

(iii) E to C ~~ee~~ is visited via D.

and A to D is visited via C.

(iv) MST weight $= 3+3+4+5 = 15$

$15 \times 2 = 30$

$30 < 36$

$\therefore$ it is not optimal.

9/14

Examiner Comments

Part (a) is fully correct.

In part (b) the first and second iterations of Floyd’s algorithm have been completed correctly and so the first three marks in (b) were awarded. In the third iteration the candidate has shown sufficient understanding of the algorithm to be awarded the corresponding method mark (that is no change in the third row or column in either table) but there is an error in the entry for row E, column B of the Distance table (the 17 should be a 14) and so the final accuracy mark in part (b) could not be awarded (therefore giving a total of 4 marks in part (b)).

In part (c) the candidate has not explained sufficiently how to use the Route table to find the shortest path from E to A but has correctly stated the route as EDCA (so 1 mark was awarded in this part). In part (d) the nearest neighbour route and corresponding length have been stated correctly and so the first two marks were awarded. In (iii) the candidate should have said that A to D is visited via E and not C, and in part (iv) no cycle is given to justify the answer of 30 and so the final two marks in this part could not be awarded.

Exemplar Question 4

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4.

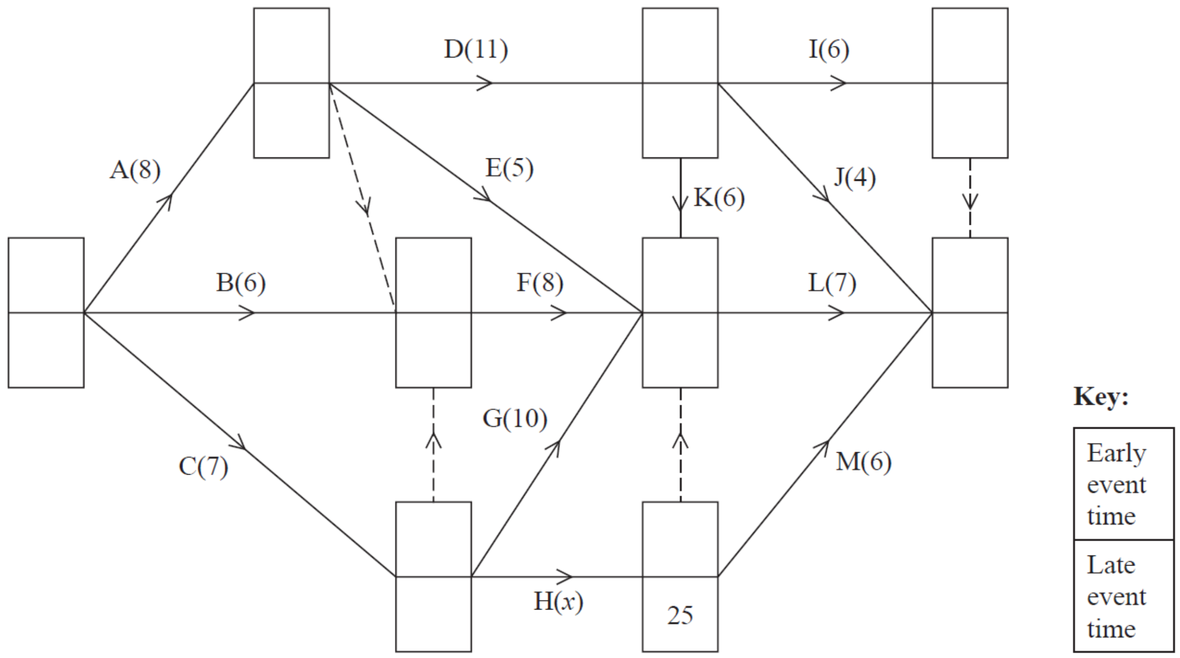


Figure 3

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and one late event time has been completed for you.

The total float of activity H is 7 days.

(*a*) Explain, with detailed reasoning, why $x = 11$

(2)

(*b*) Determine the missing early event times and late event times, and hence complete Diagram 1 in your answer book.

(3)

Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.

(*c*) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

(1)

(*d*) Schedule the activities using Grid 1 in the answer book.

(3)

(Total for Question 4 is 9 marks)

Mean Score 5.2 out of 9

Examiner Comments

In part (a) very few candidates explained, with detailed reasoning, why $x = 11$. For the first mark candidates had to give a correct reason why the total float on activity H was given by $25 – 7 – x$ (e.g. mention that the early event time at the end of activity C is 7) and for the second mark (which was dependent on the first) a reason why the expression $25 – 7 – x$ leads to the given answer. Far too many candidates gave no reasoning at all and simply wrote down the equation $25 – 7 – x = 7$ and then stated that $x = 11$.

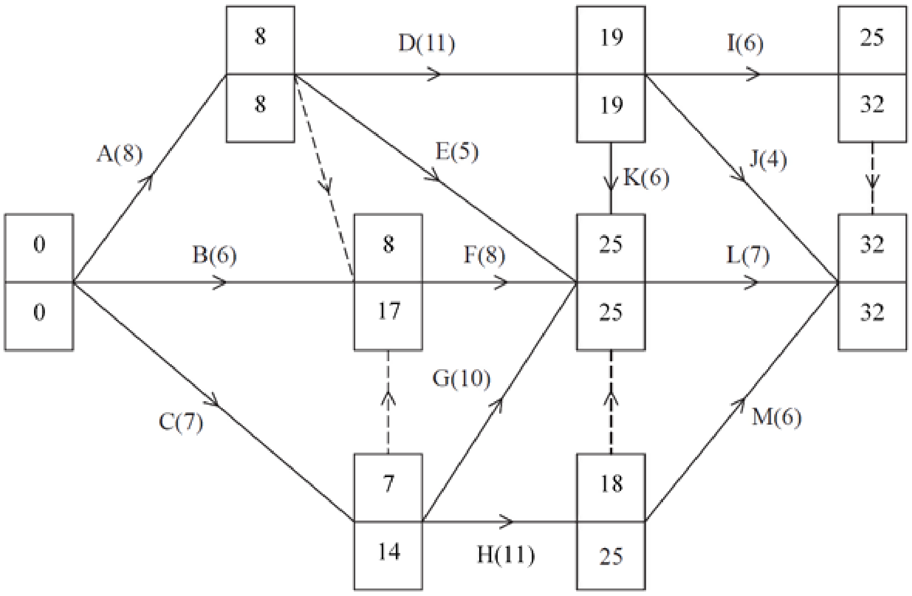
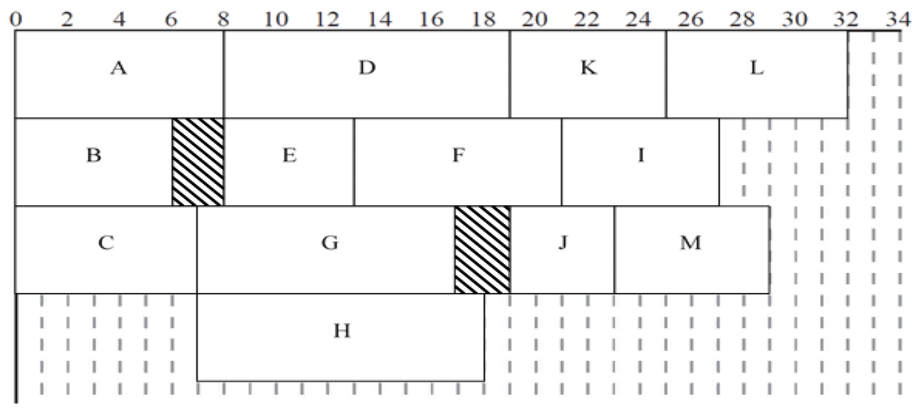
Part (b) was answered extremely well with many candidates correctly complete the diagram with the early event times and late event times. When errors did occur, they mostly occurred at the ends of activities B, F and/or H.

Examiners noted that part (c) was generally answered correctly with nearly all showing a correct calculation.

Most candidates did attempt to produce a schedule in part (d). However, a significant proportion of candidates tried to construct a schedule with only three workers (possibly due to their answer for part (c)), therefore scoring only one mark and completely disregarding the significance of the Immediately Preceding Activities (IPA). Of those candidates who did have four workers in their schedule, a pleasing number were correct, though errors were sometimes seen in either the duration, time interval or IPA for one or more activities.

It was pleasing to note that cascade charts were rarely seen.

Mark Scheme

Qu	Scheme	Marks	AOs
4(a)	The early event time at the end of activity C is 7 (as no other activity leads into this event). Therefore the float on activity H is 25 – 7 – x The float on activity H is given as 7 and so therefore 25 – 7 – x = 7 implies that the value of x is equal to 25 – 7 – 7 = 11	B1 dB1	3.1a 2.4
		(2)	
(b)		M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(c)	$\frac{95}{32} = 2.968... = 3$ workers	B1	2.2a
		(1)	
(d)	e.g. 	M1 A1 A1	2.1 1.1b 1.1b
		(3)	
			(9 marks)

Notes for Question 4

(a) B1: correct reasoning for why the float on activity H is given by 25 – 7 – x, must mention that the early event time at the end of activity C is 7 or the early event time at the start ofH is 7 **and** that the total float for H is therefore 25 – 7 – x (or 25 – x – 7 but not just 18 – x) (no reason for why the early event time at the end of C is 7 is required)

dB1: correct explanation for why x = 11 (dependent on previous B mark) – as a minimum must equate 25 – 7 – x to 7 (allow 18 – x = 7 as they must have shown where the 18 comes from to get the first B mark) and hence x = 11

SCB1B0 – for those who write or imply 25 – 7 – x = 7(but not just 18 – x = 7) and state x = 11 without any mention of the early event time at the end of C or the total float of activity H. However, 25 – 7 – 7 = 11 only is no marks in this part

(b) M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0s at the source node or the 32 in the bottom box at the sink node for M only. Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

A1:CAO -Top boxes (including zero at the source node)

A1: CAO - Bottom boxes (including zero at the sink node)

(c) B1: Correct calculation seen then 3 – an answer of 3 with no working scores B0

(d) M1: Not a cascade chart. 4 'workers' used at most and at least 10 different activities placed

A1:4 workers. All 13 activities present (just once – so if an activity appears for two different workers and is happening at the same time this is A0). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

A1: 4 workers. All 13 activities present (just once). No errors

Activity	Duration	Time interval	IPA
A	8	0 -8	-
B	6	0 –17	-
C	7	0 – 14	-
D	11	8 – 19	A
E	5	8 – 25	A
F	8	8 – 25	A, B, C
G	10	7 – 25	C
H	11	7 – 25	C
I	6	19 – 32	D
J	4	19 – 32	D
K	6	19 – 25	D
L	7	25 – 32	E, F, G, H, K
M	6	18 – 32	H

Student Response A

4.

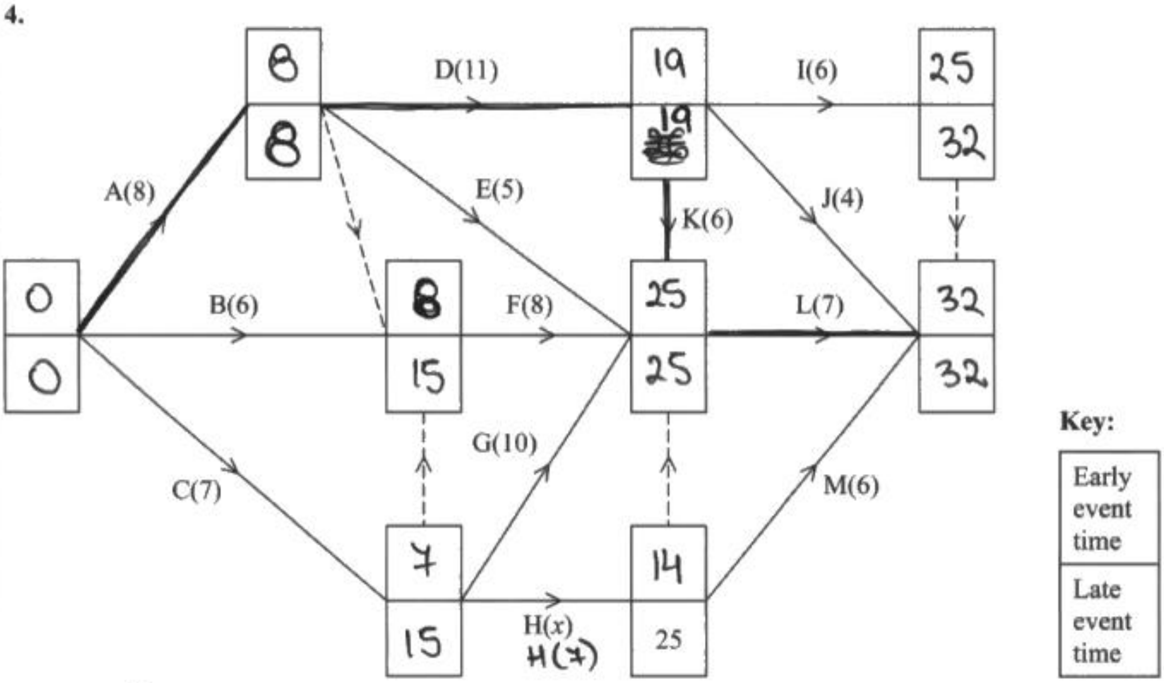


Diagram 1

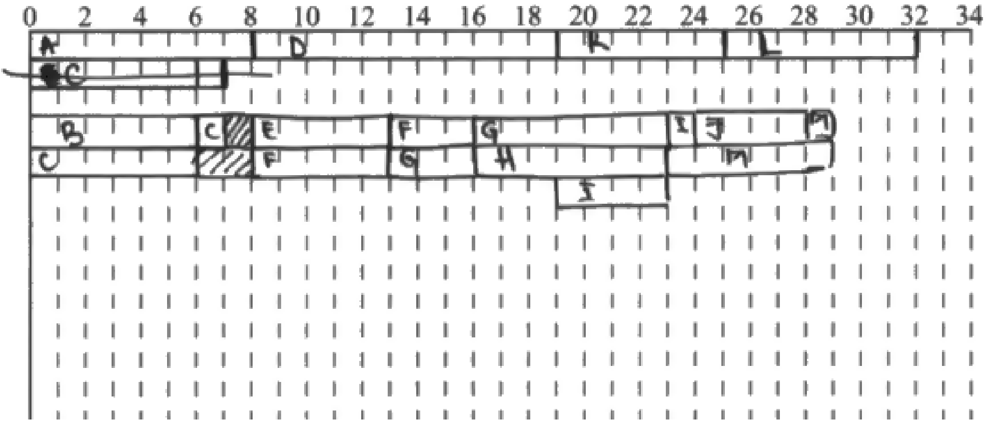
a) C(7) 7 H(x) 25

float $25 - x - 7 = 11 \Leftrightarrow x = 7$

b).

c) $\frac{\sum \text{duration of activities}}{\text{total time}} = \frac{91}{32} = 2.84 \sim 3$ workers minimum

schedule.



2/9

Examiner Comments

In part (a) no marks were awarded as the candidate did not give any detailed reasoning for why $x = 11$ – in fact they wrongly stated that $25 - x - 7 = 11$ which lead to the incorrect answer of $x = 7$.

In part (b) the method mark was awarded as all the boxes were completed, with the top boxes generally increasing left to right (in the direction of the arrows), and bottom boxes generally decreasing right to left (in the opposite direction to the arrows). However, no accuracy marks could be awarded due to the errors at the event boxes at the beginning and end of activity H.

No marks could be awarded in (c) due to an incorrect value in the candidate's calculation (91 not 95)

In part (d) the method mark was awarded for a scheduling diagram that had no more than four workers with at least 10 activities placed. However, no accuracy marks could be awarded as activity M is being completed by two different workers at the same time (between times 28 to 29)

Student Response B

4.

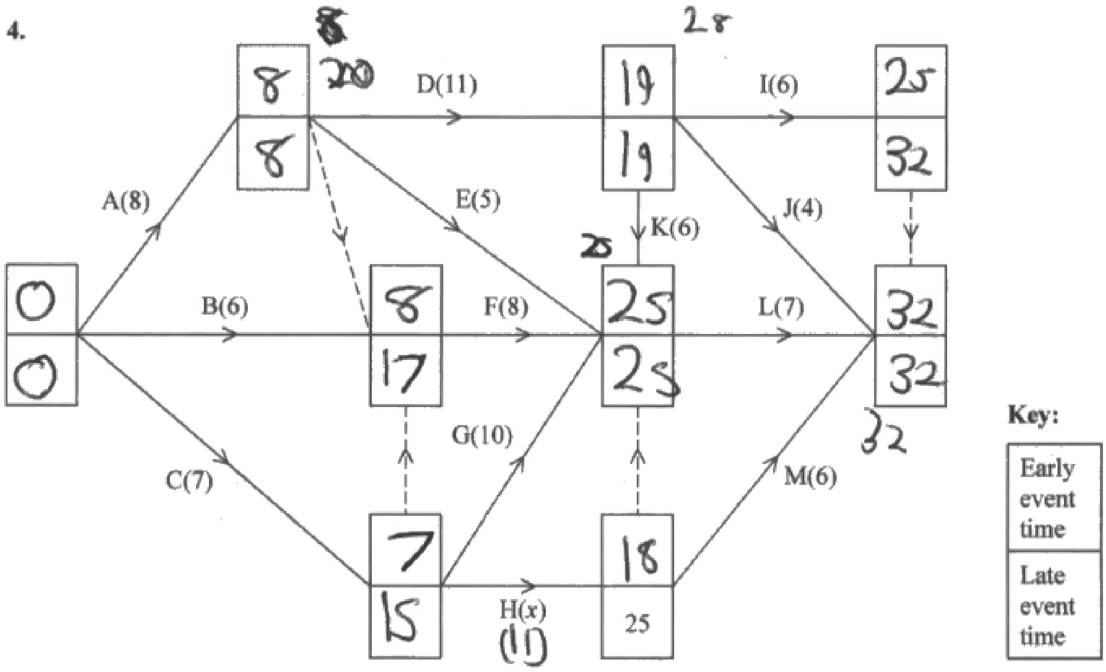


Diagram 1

a)

Float = 25 − x − 7 = 7

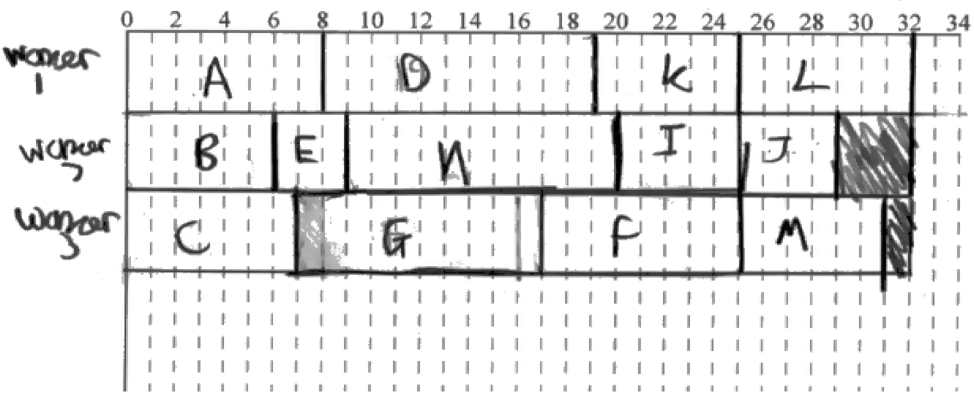
∴ x = 25 − 7 − 7 = 11

c)

Total work time = 95

Total time 32

95 ÷ 32 = 2.968 so LB = 3 workers



5/9

Examiner Comments

In part (a) although no detailed reasoning was given for why $x = 11$ this response earned the special case (SC) of one mark for a correct calculation leading to the correct given answer.

In part (b) the candidate earned the first two marks with the final mark being withheld for the error in the early event time at the start of activity H (15 should have been 14).

In part (c) the mark was awarded for the correct lower bound with corresponding calculation seen. Finally, in part (d) the candidate scored the method mark for correctly producing a scheduling diagram with no more than 4 workers with at least 10 activities placed. However, no accuracy marks could be awarded as the minimum number of workers required to schedule all the activities was in fact 4 (and not 3).

Student Response C

4.

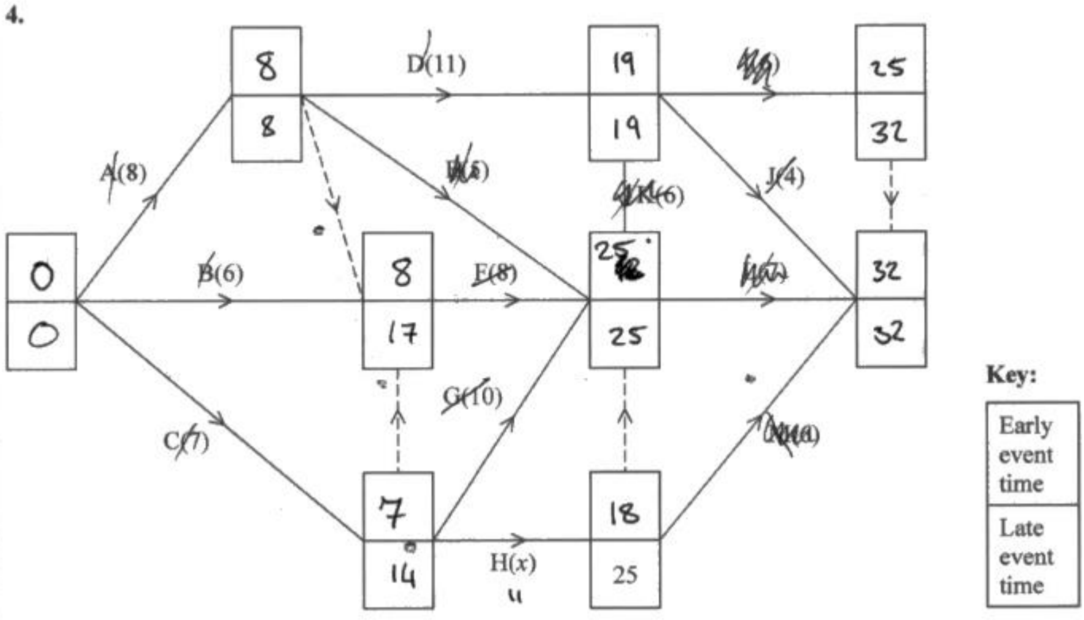


Diagram 1

lowest late

a) as the early time for H is 7
3 float equals 7 so $7 = 25 - x - 7$
$x = 11$

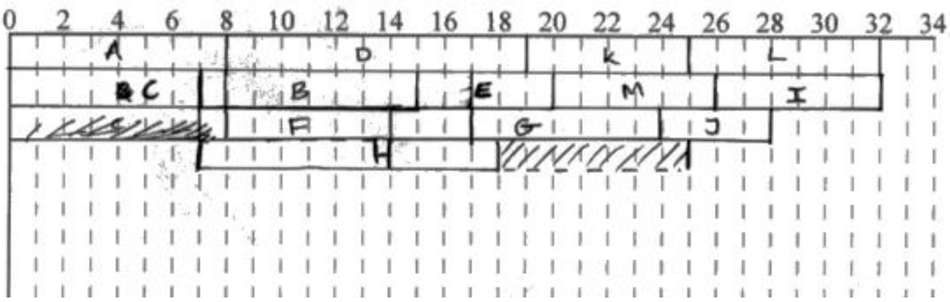
b)

c) shortest time = 32 days

89 days of jobs

$89 \div 32 = 2.78$

So 3 workers is the lower bound for 32 days.



6/9

Examiner Comments

Part (a) scored both marks – the candidate has given enough detail in explaining why x = 11

Part (b) is fully correct (three marks), but unfortunately their calculation in (c) is incorrect (89 has been used instead of 95).

In part (d) the candidate was awarded the method mark for a scheduling diagram with no more than 4 workers with at least 10 activities placed. However, no accuracy marks could be awarded as the duration of activity B is incorrect, F is taking place in an incorrect time interval and is also happening before B has been completed (for the first accuracy mark examiners allowed up to two errors in the scheduling).

Exemplar Question 5

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5.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	C
F	B, C, D
G	A
H	B, C, D
I	B, C, D, G
J	B, C, D, G
K	E, H

(*a*) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

Given that all the activities shown in the precedence table have the same duration,

(*b*) state the critical path for the network.

(1)

(Total for Question 5 is 6 marks)

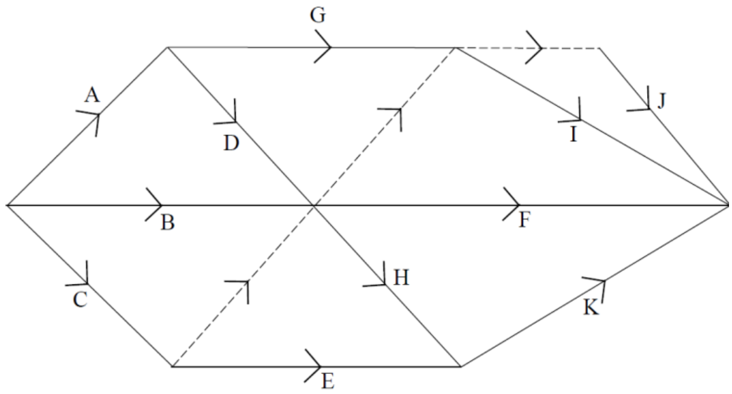
Mean Score 4.0 out of 6

Examiner Comments

Candidates generally showed a good understanding of the process of constructing an activity network from a precedence table in part (a), using arcs drawn with arrows and labelled for activities. Some scripts lacked a sink node at the end and a small number did not have a single source node. Some of the diagrams and labels were challenging to read, especially when they were very small and/or drawn with lines that crossed over. Some candidates were unsure about the placement of their dummies, putting them in 'anywhere' so that precedence was guaranteed. A very small number of candidates put activity on node, and some failed to check that they had all activities present, with activity K being the activity that was missing most often. Usually candidates were able to pick up the first two marks and errors usually arose either with the first two precedence dummies. Whilst most candidates now seem to be aware of the importance of arrows on dummies, there are still some candidates who make the costly mistake of not having arrows on their dummies. This makes it impossible to determine the preceding activities for F, H, I, J and K and ultimately lost three marks.

In part (b) only the most able candidates realised that if all the activities have the same duration then ADHK was the critical path.

Mark Scheme

Qu	Scheme	Marks	AOs
5(a)	e.g. 	M1 A1 A1 A1 A1	1.1b 1.1b 1.1b 1.1b 1.1b
		(5)	
(b)	Critical path: ADHK	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. B, C and D labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

(a) M1: At least nine activities (labelled on arc), one start, at least two dummies placed

A1: Activities A, B, C, D, E, G dealt with correctly

A1: Activities F, H and first two dummies + arrows dealt with correctly (the first two dummies are those that are required at the event at the end of activity B)

A1: Activities I, J and K dealt with correctly (note that I and J can start directly after the end of G)

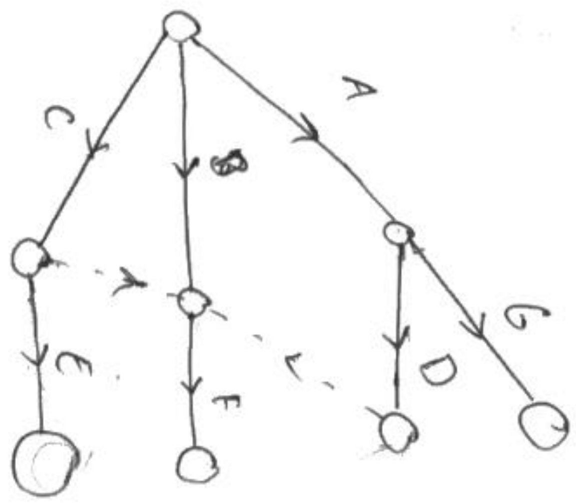
A1:CSO – Final dummy + arrow, all arrows present for every activity with one finish and no additional dummies. Note that this is not a unique solution e.g. I, J could be interchanged, or the dummy could come after I or J, F and K could lead into the dummy etc. so please check these carefully. **Please check all arcs carefully for arrows – if there are no arrows on dummies then M1A1max**

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

(b)B1: CAO (ADHK only)

Student Response A

5.



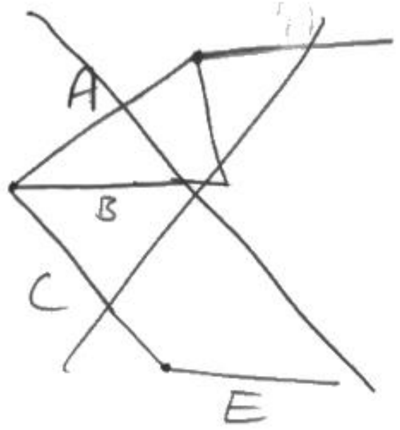
0/6

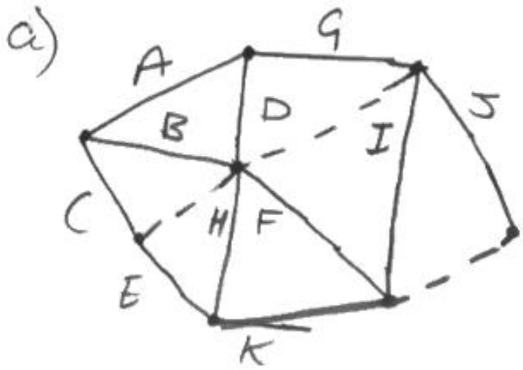
Examiner Comments

No marks could be awarded in part (a) as the candidate has only placed seven activities – for the method mark (and hence any later accuracy marks) a minimum of nine activities had to be seen on the activity network. Part (b) was not attempted by the candidate.

Student Response B

5.





Question 5 continued

b) ~~AG~~ ADHK

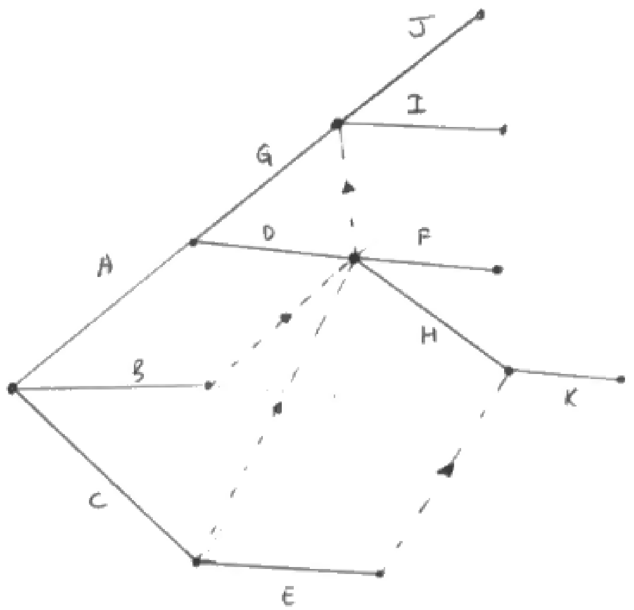
3/6

Examiner Comments

The candidate scored the first two marks in part (a) for an activity network with at least nine activities placed and at least two dummies seen together with activities A, B, C, D, E and G having been dealt with correctly. However, as the dummy activities did not contain any arrows no further marks could be awarded in this part. Part (b) was correct giving three marks in total.

Student Response C

5.



Question 5 continued

b) Critical path : A D H K

5/6

Examiner Comments

In part (a) the candidate scored four out of a possible five marks. This type of response is all too common in which candidates do not bring together the final activities into a single end/sink node. It is also worth noting that an additional dummy separating I and J would have been required to do this. Finally, this is a good example to remind future candidates to only use dummy activities where strictly necessary; the dummies at the end of both activities B and E are not required and would have been penalised with the final mark in this part if it had been earned. Part (b) was fully correct.

Exemplar Question 6

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6. A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to
$$3x + y + 2z \leq 30$$
$$x - y + z \geq 8$$
$$4y + 2z \geq 15$$
$$x, y, z \geq 0$$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	−0.25	26.25
a_1	1	0	1.5	0	−1	−0.25	1	0.25	11.75
y	0	1	0.5	0	0	−0.25	0	0.25	3.75
P	$-(2 + M)$	0	$2 - 1.5M$	0	M	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for Question 6 is 12 marks)

Mean Score 7.6 out of 12

Examiner Comments

In part (a) many candidates gave a correct response, but these varied from statements such as “the problem contains $\geq$ constraints”, to “simplex can only deal with $\leq$ constraints” through to a thorough explanation that “for this problem the origin is not in the feasible region as $x-y+z\geq 8$ and therefore the problem had to be modified to accommodate this”. Incorrect responses included statements such as “the problem has three variables” or “it needs artificial variables”.

In part (b) most candidates made a good attempt at setting up the initial tableau, generally converting the three constraints into equations of the correct form. Most used the notation provided in the question, although a small number used *r*, *s* and *t* instead of s_1, s_2 and s_3, although only some candidates changed the headings in their tableau. A small number made sign errors with the second and third constraints, adding surplus variables and subtracting artificial variables. A significant number of candidates made mistakes with their objective function, ranging from the basic form with errors such as $P-2x-2y+z-M\left(a_1+a_2\right)=0$ or $P=2x+2y-z+M\left(a_1+a_2\right)$ seen or an expression with the equals sign omitted. Other errors followed when attempting to substitute for a_1+a_2 or when simplifying the equation. There were some common errors seen in the three constraints when they were written in the tableau, such as 0 4 2 in the third row being entered as 4 2 0, the b.v. column being left empty or an *x*,*y* or *z* appearing in a cell. Some candidates left one or more cells blank. There were a significant number of errors made in the objective row, many of which came from simplification errors noted above. Common errors included 0 instead of $-23M$ in the value column and $-M$ instead of $+M$ in the s_2 and s_3 columns.

In part (c) most candidates stated the correct values for the three non-zero variables although a small number made errors here. Some failed to state the value of the five zero variables and a small number of candidates either omitted one of these or added an extra one, sometimes duplicating one of the non-zero variables.

In part (d) the majority of candidates failed to give a correct reason or explanation of why the solution was not feasible, with many stating that there were negative values in the objective row or making a general comment such as “not all the artificial variables are zero” or “the sum of the artificial variables is not zero”. Other incorrect statements included “a_1 has a value”. Correct answers included “a_1 is not zero” and “a_1 = 11.75” or even a full statement to explain that $x-y+z\geq 8$ is not satisfied with *x* = 0, *y* = 3.75 and *z* = 0 and therefore it is not a feasible solution.

In part (e) some candidates were unsure of the choice of pivot column with a significant number incorrectly choosing the *x* column. Some of those who chose the correct *z* column failed to state a reason for their choice. Of those who chose the *z* column and gave the correct reason, some then went on to simply stated that 0.5 had the smallest ratio value without showing the three calculations and others, who did show all their calculations correctly, stated that 7.5 was the next pivot.

Mark Scheme

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>Simplex can only be applied when the non-negativity constraints are $\leq$</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$3x+y+2z\leq 30 \Rightarrow 3x+y+2z+s_1=30$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$x-y+z\geq 8 \Rightarrow x-y+z-s_2+a_1=8$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>$4y+2z\geq 15 \Rightarrow 4y+2z-s_3+a_2=15$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$P=2x+2y-z \Rightarrow P=2x+2y-z-M\left(a_1+a_2\right)$ together with
$a_1+a_2=23-x-3y-3z+s_2+s_3$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$P-\left(2+M\right)x-\left(2+3M\right)y-\left(-1+3M\right)z+Ms_2+Ms_3=-23M$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td><table>
<tr><th>b.v</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>a_1</th><th>a_2</th><th>Value</th></tr>
<tr><td>s_1</td><td>3</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
<tr><td>a_1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>8</td></tr>
<tr><td>a_2</td><td>0</td><td>4</td><td>2</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>15</td></tr>
<tr><td>P</td><td>$-\left(2+M\right)$</td><td>$-\left(2+3M\right)$</td><td>$-\left(3M-1\right)$</td><td>0</td><td>M</td><td>M</td><td>0</td><td>0</td><td>$-23M$</td></tr>
</table></td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
<tr><td>(c)</td><td>$s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>The solution after the 1st iteration is not feasible because $a_1=11.75$ is an artificial variable which must be zero in a feasible solution</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)</td><td>The most negative value in the objective row is $2-1.5M$ so the pivot is a value from the z-column</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>The 0.5 in the y row is the pivot because $\frac{3.75}{0.5}$ is less than both $\frac{26.25}{1.5}$ and $\frac{11.75}{1.5}$</td><td>dB1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4" align="right">(12 marks)</td></tr>
</table>

Notes for Question 6
(a) **B1:** CAO – e.g. not all of the constraints are $\le$, the origin is not a (basic feasible) solution of the LP **(b)** **B1:** CAO $3x+y+2z+s_1=30$ (may be seen in the simplex tableau – allow any s_i (or s) for s_1) **B1:**CAO $x-y+z-s_2+a_1=8$ (may be seen in the simplex tableau – allow any consistent s_i for s_2 (or t say) but not the same s_i as in the previous mark and allow any a_i for a_1) **B1:**CAO $4y+2z-s_3+a_2=15$ (may be seen in the simplex tableau – same conditions as above) **M1:** setting up the new objective which must be $P=2x+2y-z-M(a_1+a_2)$ and substituting for their a_1 and a_2 (if no working then the **correct** objective line in the tableau implies this mark) **A1:** CAO $P-(2+M)x-(2+3M)y-(-1+3M)z+Ms_2+Ms_3=-23M$ (any equivalent form – need not be factorised and does not need to be re-arranged into this form - if no working then the **correct** objective line in the tableau implies this mark) **M1:**setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark) **A1:** CAO (any equivalent correct form) **(c)** **B1:** CAO $s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$ (ignore expression for P if given) **(d)** **B1:** correct reasoning of why the solution is not feasible e.g. a_1 is not zero but B0 for just stating that the artificial variable is non-zero (so must see either a_1 or 11.75 being stated as non-zero) **(e)** **B1:** correct reasoning of why the pivot comes from a value from the z-column so must say that the most negative value (in the objective row) is $2 - 1.5M$ (or this expression clearly implied) **dB1:** correct justification of why the 0.5 in the third row is the next pivot (dependent on previous B mark) – so must compare or state that $\frac{3.75}{0.5}$ or 7.5 is less than both $\frac{26.25}{1.5}$ or 17.5 and $\frac{11.75}{1.5}$ or 7.8(3333….) – just stating that the 0.5 in the third row is the next pivot without reasoning is no marks in this part

Student Response A

6.

a) Simplex cannot be used as there are greater than inequalities

b) ~~$P-2x-2y+z=0$~~

$3x+y+2z+S_1=30$

$x-y+z+S_2-a_1=8$

$4y+2z+S_3-a_2=15$

~~$I=-(a_1+a_2)$~~

~~$I=-(x-y+z+S_2-8+4y+2z+S_3-15)$~~

~~$I=-x-3y-3z+23-S_2-S_3$~~

~~$I+x+3y+3z+S_2+S_3=23$~~

$P=2x+2y-z-M(a_1+a_2)$

$P=2x+2y-z-M(x+3y+3z+S_2+S_3-23)$

~~$P+(M-2)x$~~

$P=2x+2y-z-Mx-3My-3Mz-MS_2-MS_3+23M$

$P=x(2-M)+y(2-3M)+z(-1-3M)-MS_2-MS_3+23M$

$P+(M-2)x+y(3M-2)+z(1+3M)+MS_2+MS_3=23M$

c) $x=0,\ y=3.75,\ z=0,\ P=7.5-11.25M$.

d) there is an M in the value of profit, so it is negative profit.

e) most negative in P row $=2-1.5M$.

θ values

17.5

7.83

7.5 ←

$\therefore$ pivot $=0.5$ in z collumn.

(b)

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	1	2	1	0	0	0	0	30
a_1	1	−1	1	0	1	0	−1	0	8
a_2	~~4~~0	4	2	0	0	1	0	−1	15
P	M−2	3M−2	3M+1	0	M	M	0	0	23M

5/12

Examiner Comments

The answer in part (a) is a correct reason for why Simplex cannot be used to solve this LP problem. In part (b) the first constraint has been rewritten correctly with a slack variable, but the next two equations are incorrect (the candidate’s signs on their artificial and surplus variables are incorrect). The candidate’s method for finding the new objective is sound but due to the previous sign errors it is incorrect. Finally, in part (b) the candidate attempts to set up an initial tableau but as it does not contain two correct rows it did not score any marks (giving a total of 2 marks in part (b)). In part (c) the candidate failed to state the values for all the variables (and only gave the values for x, y and z). In (d) the candidate has failed to comment on the fact that one of the artificial variables is not zero, but they do go on to earn both marks in part (e) for correctly finding and justifying what would be the pivot for the next iteration.

Student Response B

6.

a) ~~there are positives on the original profit row meaning it would be impossible to find an optimal solution~~

a) there are positives and negatives on profit row making it impossible to find optimal solution

$P - 2x - 2y + z = 0$

$3x + y + 2z + s_1 = 30$ $\qquad$ $x - y + z - s_2 + a_1 = 8$

$4y + 2z - s_3 + a_2 = 15$

(b)

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	1	2	1	0	0	0	0	30
a_1	1	−1	1	0	−1	0	1	0	8
a_2	0	4	2	0	0	−1	0	1	15
P	−2 − M	−2 − M	1 −2M	0	0	0	0	0	−15.5M

c) $x = 0$ $y = 3.75$ $z = 0$
$s_1 = 26.25$ $s_2 = 0$ $s_3 = 0$
$a_1 = 11.75$ $a_2 = 0$

d) There is still a negative value on the bottom row so is not Feasible

~~Most~~ Most negative entry = $2 - 1.5M$ so look at column z.

Ratios are $\frac{26.25}{1.5} = 17.5$

$\frac{11.75}{1.5} = 7.83$ ← gives lowest ratio value

$\frac{3.75}{0.5} = 7.5$

so the pivot would be row a_1 column z.

6/12

Examiner Comments

The reason given in part (a) is incorrect for why Simplex cannot be used. In part (b) the candidate has correctly rewritten all three inequalities as equation (and so scores the first three marks), however, the candidate has shown no working in deriving the new objective function and as the objective row in the tableau is incorrect the next two marks could not be awarded. Finally, in part (b) the final method mark could be awarded for two correct rows in the tableau but not the corresponding accuracy mark due to the earlier mentioned error in the objective row (so a total of 4 marks in part (b)).

Part (c) was fully correct (1 mark) but the common (incorrect) answer in (d) of negative values in the bottom row scored no credit. Finally, in part (e) the candidate scored the first mark for giving the correct reason that the pivot came from a value in the *z*-column and even though they had the correct calculations for the pivot they incorrectly assumed that 7.83 was less than 7.5 and therefore selected the wrong pivot value.

Student Response C

(a) the simplex algorithm cannot be used because it only deals with $\leq$ constraints whereas this problem contains $\geq$ constraints where artificial variables must be introduced ~~to~~ to minimise.

(b)

$$P = 2x + 2y - z - M(a_1 + a_2 + \dots)$$

$$3x + y + 2z + s_1 = 30$$

$$x - y + z - s_2 + a_1 = 8$$

$$4y + 2z - s_3 + a_2 = 15$$

for $x, y, z \geq 0$

$$a_1 = 8 - x + y - z + s_2$$

$$a_2 = 15 - 4y - 2z + s_3$$

$$\therefore a_1 + a_2 = 23 - x - 3y - 3z + s_2 + s_3$$

$$\therefore P = 2x + 2y - z - M(23 - x - 3y - 3z + s_2 + s_3)$$

$$P = 2x + 2y - z - 23M + Mx + 3My + 3Mz - Ms_2 - Ms_3$$

$$P = x(2 + M) + y(2 + 3M) + z(-1 + 3M) - 23M - Ms_2 - Ms_3$$

$$P - x(2+M) - y(2+3M) - z(3M-1) + Ms_2 + Ms_3 = -23M$$

(b)

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	4	2	1	0	0	0	0	30
a_1	1	−1	1	0	−1	0	1	0	8
a_2	0	4	2	0	0	−1	0	1	15
P	$-(2+M)$	$-(2+3M)$	$-(3M-1)$	0	M	M	0	0	$-23M$

(c) $s_1 = 26.25$
$a_1 = 11.75$
$y = 3.75$
$x = 0$
$z = 0$
$s_2 = 0$
$s_3 = 0$
$a_2 = 0$
$P = 7.5 - 11.75M$

(d) Not feasible because the value of P includes an M, meaning the artificial variables are yet to be reduced to zero.

(e) pivot; most −ve p-val =

compare $-(2+M)$, $2 - 1.5M$
$= -(-2+1.5M)$
↗ smaller

∴ z-column θ values

$= \frac{3.75}{0.5} = 7.5$ ← least +ve

$= \frac{11.75}{1.5} = \frac{47}{6} = 7\frac{5}{6}$

$\frac{26.25}{1.5} = 17.5$

∴ pivot = 0.5 in z-column.

10/12

Examiner Comments

Part (a) is correct (1 mark) and the only error in part (b) is the first row of the tableau where the candidate has incorrectly stated the value in the y column as y and not 1 (6 marks). Part (c) is correct (1 mark) but the reason given in part (d) for why after the 1st iteration the solution was not feasible was incorrect; examiners had to see evidence that the candidate knew that a_1 was not zero and not just a general comment regarding artificial variables. Finally, part (e) was fully correct (2 marks).

Exemplar Question 7

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7. A shop sells two types of watch, analogue watches and digital watches.

The shop manager knows that, each month, she should order at least 60 watches in total. In addition, at most 80% of the watches she orders must be digital.

Let x be the number of analogue watches ordered and let y be the number of digital watches ordered.

(*a*) Write down inequalities, in terms of x and y, to model these constraints.

(2)

Two further constraints are

$$y + 3x \geq 140$$

$$4y + x \geq 80$$

(*b*) Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

The cost to the shop of ordering an analogue watch is five times the cost of ordering a digital watch. The shop manager wishes to minimise the total cost.

(*c*) Determine the number of each type of watch the shop manager should order. You must make your method clear.

(3)

Given that the minimum total cost of ordering the watches is £4455

(*d*) determine the cost of ordering one analogue watch and the cost of ordering one digital watch. You must make your method clear.

(3)

(Total for Question 7 is 12 marks)

Mean Score 6.4 out of 12

Examiner Comments

In part (a) the first constraint (based on the manager ordering at least 60 watches) was usually stated correctly. The constraint which required “at most 80% of the watches to be digital” was either dealt with very well by candidates or not attempted at all with only a minority giving either the inequality or the variables transposed.

Most candidates were able to draw the required lines correctly in (b) although some were unable to draw lines sufficiently accurately (some drew lines without a ruler) or sufficiently long enough. As stated in previous reports on the legacy specification the following general principle should always be adopted by candidates.

- Lines should always be drawn which cover the entire graph paper supplied in the answer book and therefore,
- lines with negative gradient should always be drawn from axis to axis.

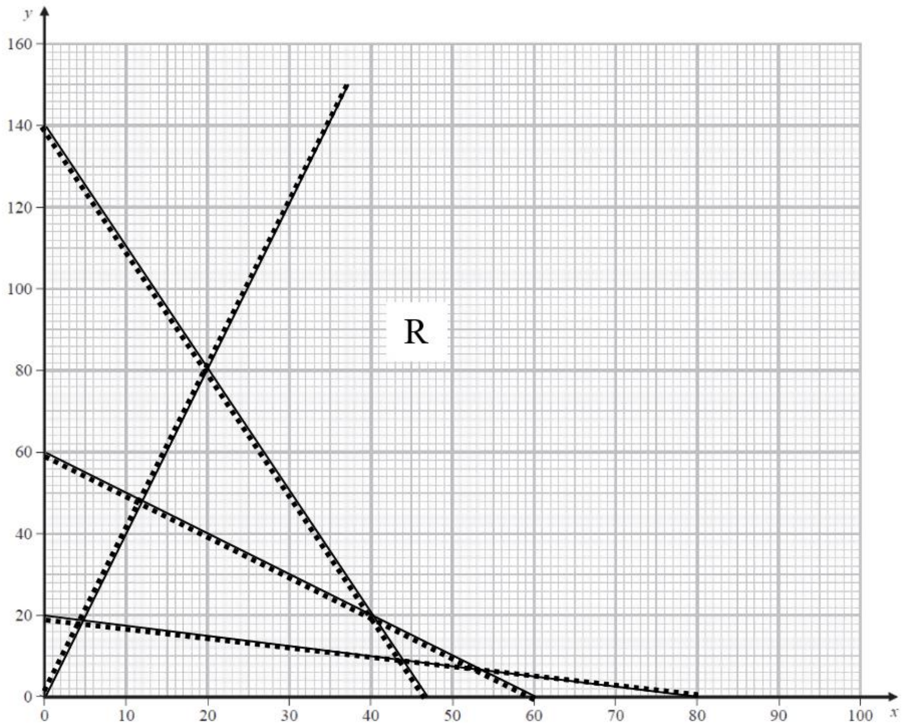
The rationale behind this is that until all the lines are drawn (and shaded accordingly) it is unclear which lines (or parts of lines) will define the boundary of the feasible region. If candidates only draw the line segments that they believe define the boundary of the feasible region then examiners are unaware of the order in which the lines were drawn and therefore it is unclear to examiners why some parts of the lines have been omitted. In general, the lines $x + y = 60$ and $y = 4x$ were drawn correctly. Furthermore, a significant number of candidates were unable to select (or even label) the correct feasible region.

In part (c) of the two possible methods that could be adopted the objective line method was predominantly the one seen and it could be argued the more appropriate for this problem.

Reciprocal gradient objective lines were shown by a significant number, which of course usually meant choosing the wrong vertex. Where point testing was used there were a surprisingly large number of calculation errors. When vertices were read off the graph, they were rarely checked algebraically to confirm if they were correct. If candidates are to use point testing it would be advisable (in most cases) to find the intersection points using simultaneous equations rather than reading off the graph. Some candidates did not show a method at all and hence gained no marks in this part. A small number of candidates stated the correct values for x and y but failed to give these in terms of the number of watches the manager should order and so failed to gain the final accuracy mark in part (c).

It was often the case that the final part of this question was left blank. Those candidates who did attempt this part usually scored at most two marks as they had failed to find the correct optimum point in part (c). Many candidates in part (d) attempted to combine the two relationships, which involved the total cost of the watches and the relative costs of the two types of watches, into one step. While many candidates completed this successfully, a significant number made errors confusing *analogue = 5(digital)* with *digital = 5(analogue)*. Several candidates in part (d) stated an incorrect equation that related their objective function with the total cost of the watches. This meant that the equation *analogue + 5(digital) = 4455* was often seen by examiners. Finally, several candidates made no use of the total cost of the watches in any part of their solution.

Mark Scheme

Qu	Scheme	Marks	AOs
7(a)	$x+y \geq 60$ $y \leq \frac{4}{5}(x+y)$	B1 B1	3.3 3.3
		(2)	
(b)		B1 B1 B1 B1	1.1b 1.1b 1.1b 2.2a
		(4)	
(c)	objective line drawn or point-testing	M1 A1	3.1a 1.1b
	(20, 80) so 20 analogue watches and 80 digital watches	A1	3.2a
		(3)	
(d)	$20a+80d=4\,455$	B1ft	3.1b
	$a=5d$	B1	2.1
	Leading to $a=123.75$ and $d=24.75$ so an analogue watch costs £123.75 and a digital watch costs £24.75	dB1	2.2a
		(3)	
			(12 marks)

Notes for Question 7

(a) B1: CAO – allow any equivalent form of $x+y \geq 60$ - do not condone strict inequality

B1: CAO – allow any equivalent form of $y \leq \frac{4}{5}(x+y)$ (but not $y \leq 80\%(x+y)$ only) and need not be simplified - do not condone strict inequality – isw if correct answer is incorrectly simplified

In **(b)**, lines must be long enough to define the correct feasible region and would pass if extended through one small square of the points stated:
$x+y=60$ must pass within one small square of its intersection with the axes – (0, 60) and (60, 0)
$y+3x=140$ must pass within one small square of its intersection with the axes – (0, 140) and $\left(\frac{140}{3}, 0\right)$ (so at 46.666…, 0)
$4y+x=80$ must pass within one small square of its intersection with the axes – (0, 20) and (80, 0)
$y=4x$ must pass within one small square of (0, 0) and (25, 100)
In (b) condone for full marks lines which are drawn as dashed rather than solid

(b) B1: 2 lines drawn correctly
B1: 3 lines drawn correctly
B1: 4 lines drawn correctly
B1: Region, *R*, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part

(c)M1: Drawing the correct objective line (with gradient – 5) or its reciprocal (with gradient $-\frac{1}{5}$) . Line must be correct to within one small square if extended from axis to axis. If lines shorter than (5, 0) to (0, 25) or (0, 5) to (25, 0) then M0. Or point testing at least two exact coordinates of their *R* using their objective function which must be of the form $k(5x+y)$ or $k(x+5y)$ for some positive real value *k*
A1: Correct objective line – condone lack of labelling of the objective line. Or point testing at least two of the correct exact coordinates which are (20, 80), (40, 20), (80, 0) and $\left(\frac{160}{3}, \frac{20}{3}\right)$ using a correct objective function of the form $k(5x+y)$
A1: Correct number of watches – **must be in context** (and not just in terms of *x* and *y*) – dependent on a correct feasible region in **(b)** (so must have scored the first three marks in **(b)** but may not have labelled the FR as *R*)

Condone use of *x*for *a*and *y*for *d* in part (d)
(d) B1ft: A 'correct' equation (e.g. $20a+80d=4455$) involving their optimal point from **(c)** (accept any values even if non-integer) and 4455 –- note that for those who have done point testing in **(c)** the calculation 4455 / (their value for *P*) where *P* = $5x+y$ or $x+5y$ using their optimal point implies this mark
B1: CAO on the relationship between the costs of the two types of watches (*a* = 5*d*) – this mark may be implied e.g. 20(5*d*) + 80*d* = 4455 would score the first two marks in this part – note that for those who have done point testing in **(c)**the calculation 4455 / (their value for *P*) where *P* = $5x+y$ using their optimal point implies this marke.g.just seeing$\frac{4455}{180}$is the first two marks in this part

dB1: CAO (dependent on first two B marks) – this mark is dependent on having the correct optimal point (20, 80) and is dependent on a correct feasible region in **(b)** (so must have scored at least the first three marks in **(b)**) – allow for $a = 123.75$ and $d = 24.75$ (so does not need to be in context or units) – the correct answers with no working scores no marks in this part (however, note that $\frac{4455}{180}$ is the minimum amount of working that is acceptable)

Student Response A

7.

7a) $x + y \geq 60$

$y \leq 0.8(x+y)$

$y \leq 0.8x + 0.8y$

$0.2y \leq 0.8x$

$y \leq 4x$

Answer : $x + y \geq 60$

$y \leq 4x$

$x, y \geq 0$

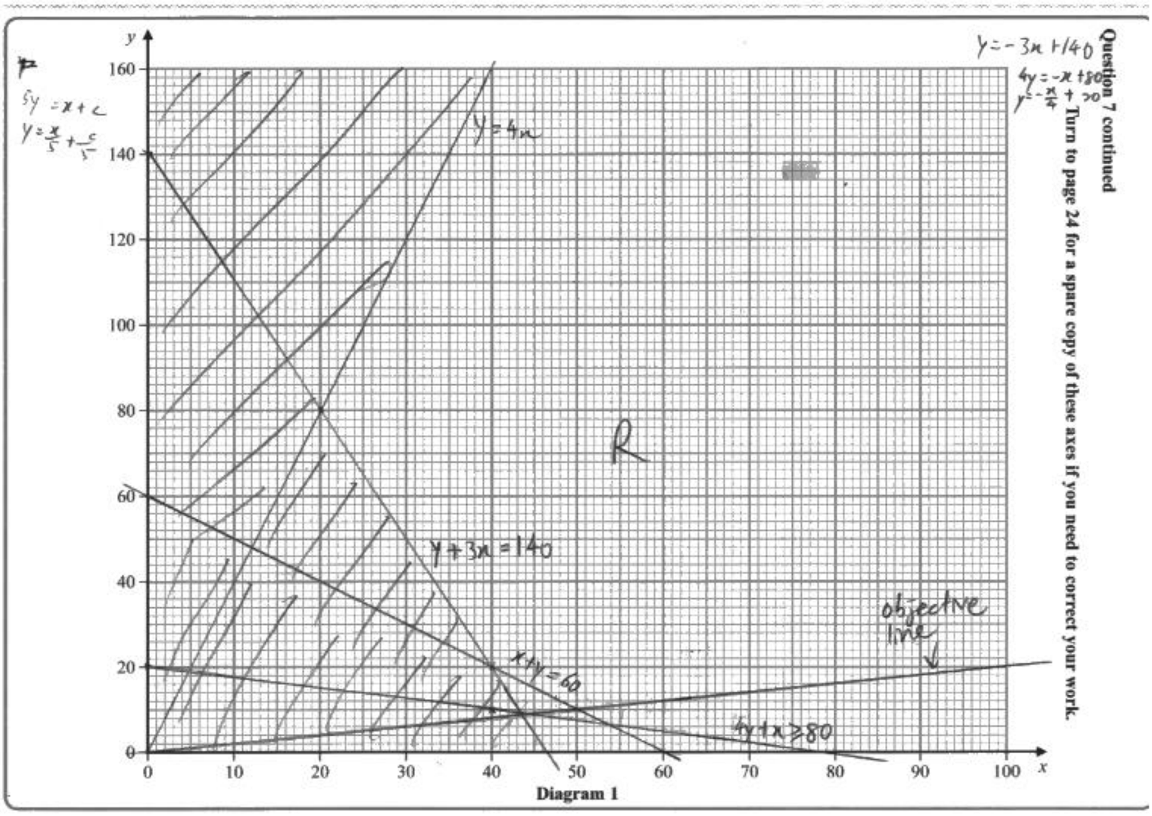
c) $x = 5y$

Minimise : $P = x - 5y$

$4y + x = 80$ $\quad x = 53.3,\ \frac{160}{3}$ (analogue ~~values~~ (53))

$y + x = 60$ $\quad y = 6.67,\ \frac{20}{3}$ (digital $\cdot$ (7))

d)



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Examiner Comments

Parts (a) and (b) were fully correct. However, in part (c) the candidate's objective line has a positive gradient and therefore no marks could be awarded in this part (only an objective line with a gradient of either -5 or -0.2 could be accepted for the method mark in this part).

The candidate did not attempt part (d).

Student Response B

7. a) $0.8(x+y) \geq \cancel{x}y$

$x+y \geq 60$

$80x+80y \geq 100\cancel{x}y$

$80\cancel{y}x \geq 20\cancel{x}y$

$\cancel{y \geq \frac{1}{4}x}$ $4x \geq y$

$x+y \geq 60$

c) min $C = 5x+y$

$y = C-5x$

first one objective line hits is

$y=140-3x$ and $y=4x$

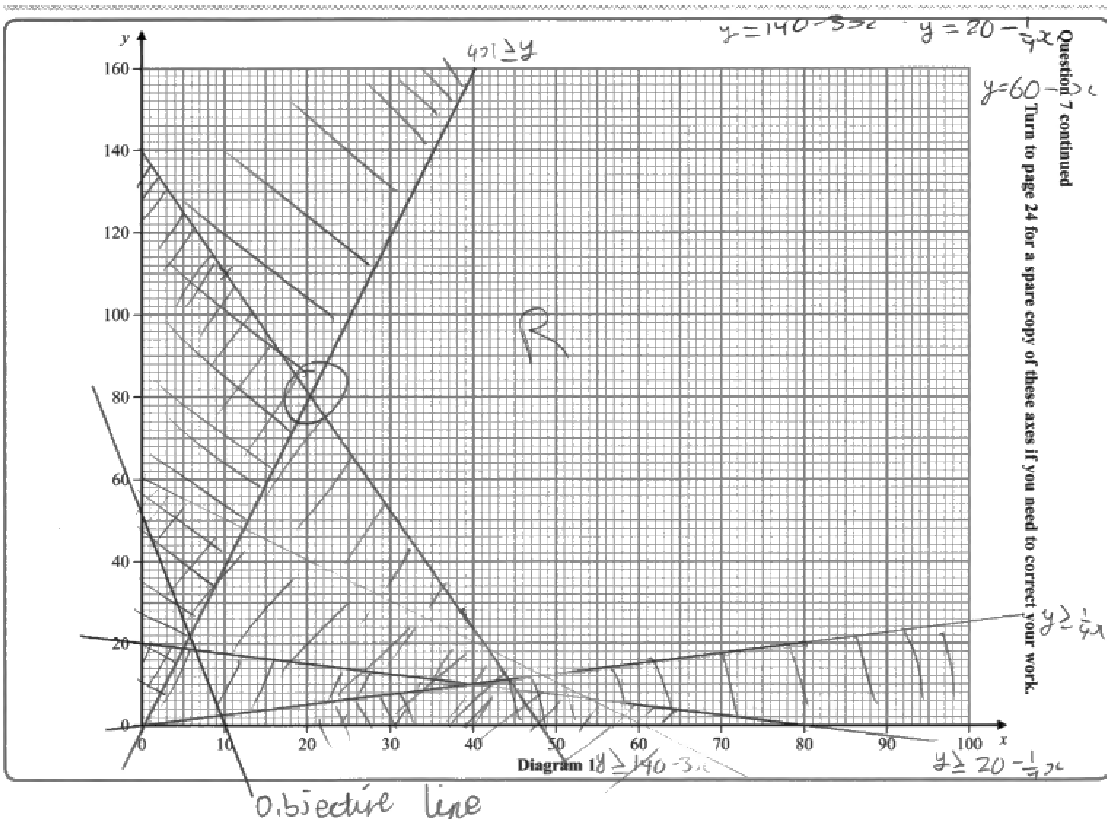
$4x=140-3x$

$7x=140$

$x=20$

$y=4\times20$

$y=80$



Question 7 continued

7d

~~4455 = 5x~~

$5 \times 20 + 80 = 180$

$\frac{4455}{180} = £24.75$

cost of ordering digital is £24.75
cost of ordering one analogue is £123.75

8/12

Examiner Comments

Part (a) was fully correct. In part (b) only 3 lines have been drawn correctly (it is quite clear that a correct line has been rejected and replaced) and so only the first two marks could be awarded in this part. In part (c) the candidate has drawn a correct objective line so therefore earning the first 2 marks in this part and while the number of watches has been correctly stated it has not been given in context (however, even if the candidate had given their answer in context this final mark in part (c) would still not have been awarded as the feasible region from part (b) is incorrect and therefore the correct answer in (c) has come from previously incorrect working). Finally, in part (d) the correct working and answer is seen for the cost of each type of watch but once again the final mark cannot be awarded due to previous incorrect feasible region.

Student Response C

7.

7a) ~~xyz≥0~~

$x, y \geq 0$

$x + y \geq 60$

$xy \leq 0.8(x+y)$

b) $y \leq 0.8(x+y) \Rightarrow 10y \leq 8x + 8y$

$2y \leq 8x$

$y \leq 4x$

c) $C = 5x + y$

2 digital

10 analogue

objective line method

$y + 3x = 140$

$y - 4x = 0$

$x = 20, y = 80$ //

d) $£4455 = \lambda(5x + y)$

$4455 = 180\lambda$

$\lambda = \frac{99}{4}$

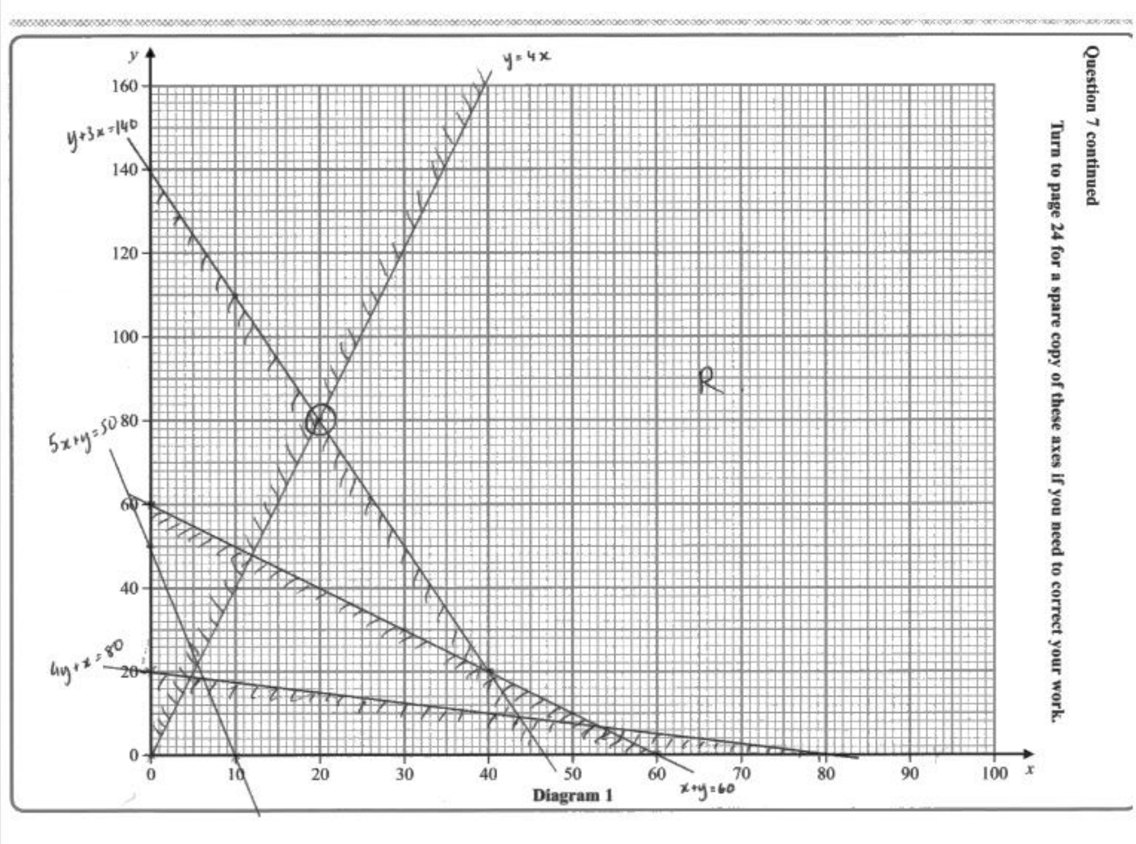
$= 24.75$

~~$C = \lambda(5x+y)$~~

~~$C = \frac{99}{4}(5x+y)$~~

£123.75 for analogue.

£24.75 for digital



11/12

Examiner Comments

Parts (a) and (b) are fully correct. In part (c) the candidate has drawn a correct objective line and found the correct vertex, but the final accuracy mark could not be awarded as the answer was not given in context. Part (d) was fully correct.

A Level Further Mathematics – Decision 2 (9FM0 4D)

Exemplar Question 1

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1. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of four demand points, P, Q, R and S. It also shows the stock held at each
supply point and the stock required at each demand point. A minimum cost solution is required.

	P	Q	R	S	Supply
A	15	14	17	11	23
B	10	9	16	12	42
C	11	13	8	10	18
D	15	13	16	17	19
Demand	25	45	12	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	P	Q	R	S
A	23			
B	2	40		
C		5	12	1
D				19

Table 2

(*a*) Taking DQ as the entering cell, use the stepping-stone method to find an improved solution. Make your method clear.

(2)

(*b*) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by stating the

- shadow costs
- improvement indices
- route
- entering cell and exiting cell.

(4)

(*c*) Determine whether the solution obtained from this second iteration is optimal, giving a reason for your answer.

(3)

(*d*) State the cost of the solution found in (*b*).

(1)

(Total for Question 1 is 10 marks)

Mean Score 6.5 out of 10

Examiner Comments

Part (a) was well answered. Most students were able to find a valid route. Some students lost the accuracy mark when they left ‘0’ in CQ in their improved solution.

Most students found the correct shadow costs and route in (b) for the two method marks; however, a reasonable minority made errors in their improvement indices, thus losing an accuracy mark here. Some students found too many or two few improvement indices which was costly. It should be noted that the setting out of the shadow costs and improvement indices as shown in the published mark scheme (in which both sets of values are shown on the same table) is probably the clearest form of presentation; students who show numerous calculations at the side of the table can make it somewhat difficult for the examiner to find, or even to follow, their corresponding method/working. Furthermore, a compact method of presentation makes it more straightforward for students to ensure that they have the correct number of both shadow costs and improvement indices. A number of students failed to state the entry/exit cells.

Part (c) was generally well understood by students; again, many made slips in their calculations but understood that a negative improvement index meant a non-optimized solution. A surprising number of students did not seem to understand that they would need to recalculate shadow costs and improvement indices in this part and referred instead to shadow costs and improvement indices from the previous iteration. Others calculated more improvement indices than were strictly needed in order to ascertain whether an optimal solution had been reached – as soon as a negative improvement index is reached it is perfectly acceptable to stop calculating more indices.

The correct value in (d) was reached by a majority of students who had scored well in the previous three parts.

The presentation of students’ work was sometimes difficult to follow and, at times, cost marks when it was unclear which values were costs and which values were improvement indices. Students would be well advised to set their work out carefully and to clearly identify the different elements of their solution.

Mark Scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1 (a)</td><td>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>A</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td></td><td>$5-\theta$</td><td></td><td>$1+\theta$</td></tr>
<tr><td>D</td><td></td><td>θ</td><td></td><td>$19-\theta$</td></tr>
</table>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>A</td><td>23</td><td></td><td></td><td></td></tr>
<tr><td>B</td><td>2</td><td>40</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>12</td><td>6</td></tr>
<tr><td>D</td><td></td><td>5</td><td></td><td>14</td></tr>
</table>
</td><td>1M1
1A1</td><td>2.1
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td></td><td></td><td>15</td><td>14</td><td>16</td><td>18</td></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>0</td><td>1</td><td>-7</td></tr>
<tr><td>-5</td><td>B</td><td>X</td><td>X</td><td>5</td><td>-1</td></tr>
<tr><td>-8</td><td>C</td><td>4</td><td>7</td><td>X</td><td>X</td></tr>
<tr><td>-1</td><td>D</td><td>1</td><td>X</td><td>1</td><td>X</td></tr>
</table>
</td><td>1M1
1A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>A</td><td>$23-\theta$</td><td></td><td></td><td>θ</td></tr>
<tr><td>B</td><td>$2+\theta$</td><td>$40-\theta$</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td></td><td></td></tr>
<tr><td>D</td><td></td><td>$5+\theta$</td><td></td><td>$14-\theta$</td></tr>
</table>
<table>
<tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>A</td><td>9</td><td></td><td></td><td>14</td></tr>
<tr><td>B</td><td>16</td><td>26</td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>12</td><td>6</td></tr>
<tr><td>D</td><td></td><td>19</td><td></td><td></td></tr>
</table>
Entering cell is AS and exiting cell is DS</td><td>2M1
2A1</td><td>1.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td></td><td></td><td>15</td><td>14</td><td>9</td><td>11</td></tr>
<tr><td></td><td></td><td>P</td><td>Q</td><td>R</td><td>S</td></tr>
<tr><td>0</td><td>A</td><td>X</td><td>0</td><td>8</td><td>X</td></tr>
<tr><td>-5</td><td>B</td><td>X</td><td>X</td><td>12</td><td>6</td></tr>
<tr><td>-1</td><td>C</td><td>-3</td><td>0</td><td>X</td><td>X</td></tr>
<tr><td>-1</td><td>D</td><td>1</td><td>X</td><td>8</td><td>7</td></tr>
</table>
A negative II so solution is not optimal</td><td>1M1
1A1
2A1</td><td>2.1
1.1b
2.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>(£)1086</td><td>1B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Notes:
(a) 1M1: A valid route, only one empty square used, θ's balance. **Note:** If not entering in DQ, allow this mark for a valid route and entry in CP or DP only. **1A1:** CAO **(b) 1M1**: Finding (exactly) 8 shadow costs and (exactly) 9 improvement indices for their improved solution **1A1:** Shadow costs [Alt: A(15), B(10), C(7), D(14), P(0), Q(-1), R(1), S(3)] and II CAO. Please check top table for Shadow Costs. **2M1:** A valid route, their most negative II chosen, only one empty square used, θ's balance **2A1:** CAO – including the deduction of all entering and exiting cells **(c) 1M1:** finding all 8 shadow costs **and** all 9 negative improvement indices **or** sufficient number of shadow costs for at least 1 negative II found (May just see SC: A, C, P and S and II: PC). This mark is dependent on the previous M mark in (b) which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation or not of the optimality of the current solution **1A1:** CAO negative II from correct working **2A1:** CSO for (a), (b), (c) including the correct reasoning that the solution is not optimal because there is a negative II. Do not allow for 'some IIs are not positive' o.e. [Alt shadow costs: A(15), B(10), C(14), D(14), P(0), Q(-1), R(-6), S(-4)]. **(d) 1B1:** CAO ignore lack of or incorrect units.

Student Response A

1.

	P	Q	R	S	Supply
A	15	14	17	11	23
B	10	9	16	12	42
C	11	13	8	10	18
D	15	13	16	17	19
Demand	25	45	12	20	

Table 1

a)

	P	Q	R	S
A	–			
B	–	–		
C		$5-\theta$	–	$1+\theta$
D		θ		$19-\theta$

$\theta = 5$

Exiting cell = CQ

	P	Q	R	S
A	23			
B	2	40		
C			12	6
D		5		14

Improved solution

b) shadow costs

	P (15)	Q (14)	R (16)	S (18)	
A	15	14	17	11	0
B	10	9	16	12	-5
C	11	13	8	10	-8
D	15	13	16	17	-1

	P (15)	Q (14)	R (16)	S (18)	
A	0	0	1	-7	0
B	0	0	5	-1	-5
C	4	7	0	0	-8
D	1	0	1	0	-1

Improvement Indices

= cost − shadow costs

Improve most ~~positive~~ negative

Entering cell = ~~CQ~~ SA

	P	Q	R	S
A	[illegible] $-\theta$			θ
B	$2+\theta$	$40-\theta$		
C		θ	–	–
D		$5+\theta$		$14-\theta$

$\theta = 14$

Exiting cell = SD

	P	Q	R	S
A	9			14
B	16	26		
C			12	6
D		19		

Improved solution

c)

		15	14	9	11
		P	Q	R	S
0	A	15	0	8	11
-5	B	10	9	12	6
-1	C	-3	0	8	10
-1	D	1	13	8	7

Shadow costs & improvement indices

costs in route underlined

There is still a negative improvement index ∴ solution isn't optimal

	P	Q	R	S
A				
B				
C				
D				

d) £1086

10/10

Examiner Comments

A, not uncommon, perfect solution.
In part (a), a correct stepping stone route is found and the correct theta-value is used to find the correct improved solution.

In part (b), the correct 8 shadow costs and the 9 correct improvement indices are found. Note that the zeros in the table which are not improvement indices are distinguished from the improvement indices because they are underlined. The cell corresponding to the most negative improvement index is selected as the entering cell followed by a correct stepping stone route and a correct improved solution. Both the entering and exiting cells are stated.

In part (c), all 8 shadow costs and 9 improvement indices are found correctly and a correct deduction given (the solution is not optimal due to the existence of a negative improvement index).

In part (d), a correct value for the cost of the solution is stated.

Student Response B

1.

	P	Q	R	S	Supply
A	15	14	17	11	23
B	10	9	16	12	42
C	11	13	8	10	18
D	15	13	16	17	19
Demand	25	45	12	20	

Table 1

a)

	P	Q	R	S
A				
B				
C		~~13 5-θ~~	~~8+θ~~	
D		~~16θ~~	~~16-θ~~	

Entering cell = DQ

	P	Q	R	S
A				
B				
C		5-θ		1+θ
D		θ		19-θ

~~θ/n~~

θ = 5

Exiting cell = DS

	P	Q	R	S
A	23			
B	2	40		
C			12	6
D		5		14

b) Sc

Sc	15	14	16	18
	P	Q	R	S
0 A	X	O	6	-2
-5 B	X	X	5	-1
-8 C	4	7	X	X
-1 D	1	X	1	X

$I_{AQ} = 14 - 14 = 0$
$I_{AR} = 17 - 16 + 5 = 6$
$I_{AS} = 11 - 18 + 5 = -2$
$I_{BR} = 16 - 16 + 5 = 5$
$I_{BS} = 12 - 18 + 5 = -1$
$I_{CP} = 11 - 15 + 8 = 4$
$I_{CQ} = 13 - 14 + 8 = 7$
$I_{DP} = 15 - 15 + 1 = 1$
$I_{DR} = 16 - 16 + 1 = 1$

	P	Q	R	S
A	$23-\theta$			θ
B	$2+\theta$	$10-\theta$		
C				
D		$5+\theta$		$14-\theta$

Entering cell = AS
Exiting cell = AP

$\theta = 14$

	P	Q	R	S
A	9			14
B	16	26		
C			12	6
D		22		

	P	Q	R	S
A	9			14
B	16	26		
C			12	6
D		22		

	P	Q	R	S
A				
B				
C				
D				

c) Solution is optimal because ~~demand~~ ~~meets the sup~~ the supply meets the demand.

d) $(15 \times 9) + (14 \times 11) + (16 \times 10) + (26 \times 9) + (12 \times 8) + (6 \times 10) + (22 \times 13) = £1125$

cost = £1125

5/10

Examiner Comments

In part (a), a correct stepping stone route is found and a correct improved solution given.

In part (b), 8 correct shadow costs are found and 9 improvement indices are stated but there are errors in the values for the improvement indices. The most negative improvement index is, however, selected for the entering cell and a valid stepping stone route found. The improved solution is correct.

In part (c), there is no attempt to recalculate shadow costs and improvement indices.

In part (d), the stated cost of the solution is incorrect.

Student Response C

1.

	P	Q	R	S	Supply
A	15	14	17	11	23
B	10	9	16	12	42
C	11	13	8	10	18
D	15	13	16	17	19
Demand	25	45	12	20	102

Table 1

a)

	P	Q	R	S
A	23			
B	2	40		
C		5	12	1
D		θ		19

	P	Q	R	S
A	23			
B	2	40		
C		5	12	1
D		θ		$19-\theta$

	P	Q	R	S
A	23			
B	2	40		
C		5	12	$1+\theta$
D		θ		$19-\theta$

	P	Q	R	S
A	23			
B	2	40		
C		$5-\theta$	12	$1+\theta$
D		θ		$19-\theta$

Biggest value that won't allow negative values - 5

$\therefore$ let $\boxed{\theta = 5}$

	P	Q	R	S
A	23			
B	2	40		
C		0	12	6
D		5		14

$(23 \times 15) + (2 \times 10) + (40 \times 9) + (0 \times 13) + (5 \times 13) + (12 \times 8) + (6 \times \cancel{8}10) + (14 \times 17)$

$= 1184$

b)

	P	Q	R	S
A				
B				
C				
D				

$(23 \times 15) + (2 \times 10) + (40 \times 9) + (5 \times 13) + (12 \times 8) + (1 \times 10) + (19 \times 17)$

$= 1219$

$1184 < 1219$ ∴ improved solution.

		08	38	12	
	A	P	Q	R	S
23	A	23			
2	B	2	40		
0	C		0	12	6
	D		5		14

	P	Q	R	S
A				
B				
C				
D				

	P	Q	R	S
A				
B				
C				
D				

	P	Q	R	S
A				
B				
C				
D				

c) if there are no negatives left then it is an optimal solution, if there are negatives left from the improvement indices then it is not optimal and another iteration can be done.

1/10

Examiner Comments

In part (a), a highly inefficient method for finding and writing down the stepping stone route is used which takes four tables but which is correct. However, the improved solution is incorrect because the improved solution contains an extra zero (in the exiting cell).

In part (b), there are insufficient shadow costs and no improvement indices stated. Consequently, no stepping stone route is attempted.

In part (c), no calculations are completed. Instead a generic statement about conditions for optimality is provided which is insufficient for the marks here.

In part (d), no answer is given.

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Exemplar Question 2

2. Four workers, Ted (T), Harold (H), James (J) and Margaret (M), are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

The profit, in pounds, resulting from allocating each worker to each task, is shown in the table below. The profit is to be maximised.

	1	2	3	4
T	103	97	74	80
H	201	155	145	155
J	111	80	77	92
M	203	188	137	184

(*a*) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit. You must make your method clear and show the table after each stage.

(6)

(*b*) Determine the resulting total profit.

(1)

(Total for Question 2 is 7 marks)

Mean Score 5.8 out of 7

Examiner Comments

A significant proportion of students were able to produce perfect or near perfect solutions. Students who lost marks sometimes did so due to slips in their application of the algorithm. Such slips were not usually too costly due to the follow through marks available.
A small proportion failed to convert their initial matrix to a maximisation problem and thus lost a significant number of marks as they had oversimplified the problem. Some students did not fully undertake row and column reduction, in some cases prematurely augmenting their tables.
Most students however, subtracted from 203 (or more rarely, numbers greater than 203) and then reduced rows and columns as expected. Augmentation was generally carried out very well. The most common way in which candidates lost marks was inaccuracy in calculation or misreading of handwriting. Indeed, students’ handwriting was sometimes challenging to read, especially once lines covering zeros had been added onto the tables.

Mark Scheme

Question	Scheme	Marks	AOs
2(a)	Subtracting each entry from a value ≥ 203 e.g. $\begin{bmatrix} 100 & 106 & 129 & 123 \\ 2 & 48 & 58 & 48 \\ 92 & 123 & 126 & 111 \\ 0 & 15 & 66 & 19 \end{bmatrix}$	1B1	1.1b
	Reduce rows $\begin{bmatrix} 0 & 6 & 29 & 23 \\ 0 & 46 & 56 & 46 \\ 0 & 31 & 34 & 19 \\ 0 & 15 & 66 & 19 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 40 & 27 & 27 \\ 0 & 25 & 5 & 0 \\ 0 & 9 & 37 & 0 \end{bmatrix}$	1M1 1A1ft	2.1 1.1b
	followed by $\begin{bmatrix} 5 & 0 & 0 & 9 \\ 0 & 35 & 22 & 27 \\ 0 & 20 & 0 & 0 \\ 0 & 4 & 32 & 0 \end{bmatrix}$	2M1 2A1ft	2.1 1.1b
	T – 2, H – 1, J – 3, M – 4	3A1	2.2a
		(6)	
(b)	(£)559	1B1	1.1b
		(1)	
		(7 marks)	

Notes:

(a) 1B1: CAO

1M1: simplifying the initial matrix by reducing rows and then columns. Allow no more than a single error in row reduction together with no more than a single error in column reduction. May combine the two stages of converting from maximum to a minimum problem and row reduction which is acceptable.

1A1ft: CAO following on from their earlier conversion to maximising. If 1B1 awarded, the result of row and column reduction must be as in the main scheme. If 1B0 awarded, then row and column reduction must ft correctly from their attempt to convert to maximisation problem.

2M1: develops an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

2A1ft: Improved solution following on from their previous table

3A1: CSO Correct allocation. Must have gained all previous marks in the question.

(b) 1B1: CAO – solution of original problem (ignore units lack of or incorrect units)

SC: Minimising

After row reduction $\begin{bmatrix} 29 & 23 & 0 & 6 \\ 56 & 10 & 0 & 10 \\ 34 & 3 & 0 & 15 \\ 66 & 51 & 0 & 47 \end{bmatrix}$ and then after column reduction $\begin{bmatrix} 0 & 20 & 0 & 0 \\ 27 & 7 & 0 & 4 \\ 5 & 0 & 0 & 9 \\ 37 & 48 & 0 & 41 \end{bmatrix}$

After augmentation $\begin{bmatrix} 0 & 20 & 4 & 0 \\ 23 & 3 & 0 & 0 \\ 5 & 0 & 4 & 9 \\ 33 & 44 & 0 & 37 \end{bmatrix}$ or $\begin{bmatrix} 0 & 24 & 4 & 0 \\ 23 & 7 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 33 & 48 & 0 & 37 \end{bmatrix}$

Scores B0 M1A0 M1A1(ft)A0 B0 (So 3 marks max)

1B0: No Minimisation
1M1: simplifying the initial matrix by reducing rows and then columns – all values ‘correct’
1A0: Must be maximising.
2M1: develops an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed
2A1ft: Improved solution following on from their previous table
3A0: CSO
1B0: Must be maximising

Student Response A

2.

	1	2	3	4
T	103	97	74	80
H	201	155	145	155
J	111	80	77	92
M	203	188	137	184

largest value = 203

	1	2	3	4	
T	100	106	129	123	−100
H	2	48	58	48	−2
J	92	123	126	111	−92
M	0	15	66	19	−0

Max problem, so take all values from largest

	1	2	3	4
T	0	6	29	23
H	0	46	56	46
J	0	31	34	19
M	0	15	66	19
	−0	−6	−29	−19

Rows reduced

	1	2	3	4
T	0	0	0	4
H	0	40	27	27
J	0	25	5	0
M	0	9	37	0

Columns reduced

0s can be covered with 3 lines
e=5, take 5 from uncovered numbers
and add 5 to numbers covered twice

	1	2	3	4
~~T~~	5	0	0	9
~~H~~	0	35	22	27
~~J~~	0	20	0	0
~~M~~	0	4	32	0

4 lines used to cover, so solution reached

T: 2, 3

H: 1

J: 1, 3, 4

M: 1, 4

	1	2	3	4
T				
H				
J				
M				

Assign: Ted to task 2
Harold to task 1
James to task 3
Margaret to task 4

b) Total profit: $97 + 201 + 92 + 184$
$= £574$

6/7

Examiner Comments

In part (a), there is a correct conversion to maximisation problem. Row and column reduction are carried out correctly. Augmentation is carried out correctly to obtain an improved solution moving from ‘3 lines required’ to ‘4 lines required’. The allocation of workers to tasks is stated correctly.

In part (b), there is an error in the calculation of the total profit.

Student Response B

2.

	1	2	3	4
T	103	97	74	80
H	201	155	145	155
J	111	80	77	92
M	203	188	137	184

max.

	1	2	3	4	
T	100	106	129	123	100
H	2	48	58	48	2
J	92	123	126	111	92
M	0	15	66	19	0

invert: subtract each element from 203.

	1	2	3	4
T	0	6	29	23
H	0	46	56	46
J	0	31	34	19
M	0	15	66	19
	0	6	29	19

reduce rows.

	1	2	3	4
T	0	0	0	4
H	0	40	27	27
J	0	25	5	0
M	0	9	37	0

reduce columns.

	1	2	3	4
~~T~~	~~0~~	~~0~~	~~0~~	~~4~~
H	0	40	27	27
J	0	25	5	0
M	0	9	37	0

cross out 0's with fewest lines possible

3 lines ≠ 4 rows/columns
↳ not optimal

	1	2	3	4
T	5	0	0	9
H	0	35	22	29
J	0	0	0	0
M	0	4	52	0

−5 from uncrossed elements
+5 to double crossed elements

4 lines required to cross out 0's
↳ reduced.

Allocation:

T − 3
H − 1
J − 2
M − 4

profit = 74 + 201 + 80 + 184

profit = 539.

b). £539.

	1	2	3	4
T				
H				
J				
M				

5/7

Examiner Comments

In part (a), there is a correct conversion to maximisation problem. Row and column reduction are carried out correctly. Augmentation is carried out correctly to obtain an improved solution moving from ‘3 lines required’ to ‘4 lines required’. The table requiring 4 lines is correct but the candidate misreads their handwriting to write down an incorrect allocation.

In part (b), the total profit stated is incorrect due to the incorrect allocation.

Student Response C

2.

	1	2	3	4
T	103	97	74	80
H	201	155	145	155
J	111	80	77	92
M	203	188	137	184

a)

	1	2	3	4	
T	29	23	0	6	(-74)
H	56	10	0	10	(-145)
J	34	3	0	15	(-77)
M	66	51	0	47	(-137)

3 = smallest uncovered

∴ minus 3

	1	2	3	4
T	26	20	0	3
H	53	7	0	7
J	31	0	0	12
M	63	48	0	44

3 = smallest uncovered

∴ minus 3

	1	2	3	4
T	23	17	0	0
H	50	4	0	4
J	31	0	3	12
M	60	45	0	41

smallest uncovered = 4

∴ minus 4

	1	2	3	4
T	23	17	4	0
H	46	0	0	0
J	31	0	7	12
M	56	41	0	37

smallest uncovered = 23

∴ −23.

	1	2	3	4
T	0	17	4	0
H	23	0	0	0
J	8	0	7	12
M	33	41	0	37

the minimum number of lines all the 0s can be covered with is the same as the number of columns and rows ∴ the solution is optimal

~~T = task~~
~~H = task 2~~
~~J = task 4~~
~~M~~

~~T = task~~
~~H = task 4~~
~~J = task 2~~
~~M = task 3~~

	1	2	3	4
T				
H				
J				
M				

T = task 1
H = task 4
J = task 2
M = task 3

	1	2	3	4
T				
H				
J				
M				

b)

$(23 \times 201) + (4 \times 74) + (12 \times 92) + (41 \times 188)$
$= 8332$

$\boxed{= £8332}$

	1	2	3	4	
T	0	17	$\boxed{4}$	0	17
H	$\boxed{23}$	0	0	0	23
J	8	0	7	$\boxed{12}$	~~12~~
M	33	$\boxed{41}$	0	37	~~41~~
	33	~~41~~	~~7~~	37	

2/7

Examiner Comments

In part (a), there is no conversion to a maximisation problem so this response should be marked as the special case. Row reduction is carried out correctly but no column reduction is seen. Augmentation from ‘3 lines required’ to ‘4 lines required’ is seen to correctly follow through from the previous table.

In part (b), this mark is not available when minimising rather than maximising.

Exemplar Question 3

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3.

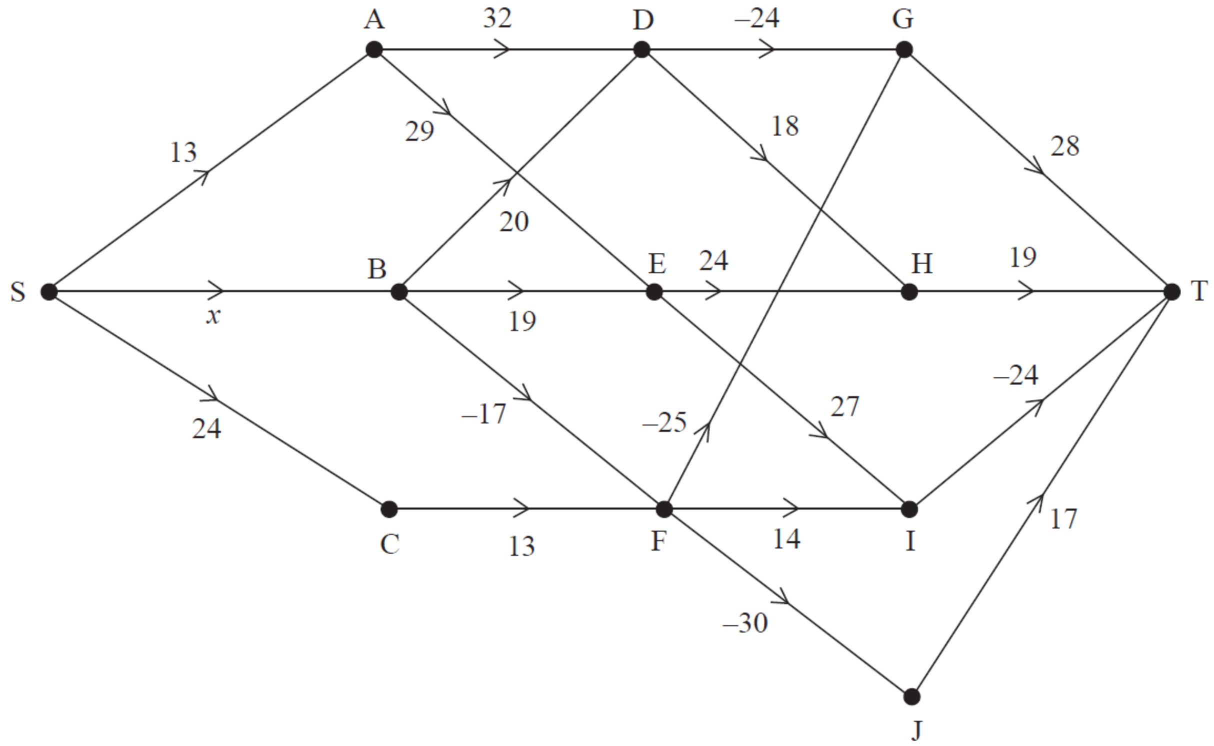

Figure 1

In Figure 1 the weight of arc SB is denoted by x where $x \geq 0$

(*a*) Explain why Dijkstra's algorithm cannot be used on the directed network in Figure 1.

(1)

It is given that the minimum weight route from S to T passes through B.

(*b*) Use dynamic programming to find

(i) the range of possible values of x

(ii) the minimum weight route from S to T.

(12)

(Total for Question 3 is 13 marks)

Mean Score 9.8 out of 13

Examiner Comments

Part (a) was not generally as well answered as the rest of the question, many students failed to correctly indicate that the algorithm can only be applied to weights that are not negative. Others, who did identify the key reason, lost the mark due to the inclusion of additional, erroneous, reasons.

Most students proved to be well prepared for part (b). Those that didn't do well either forgot to carry forward previous optimal values. or had misunderstood the question to be a minimax/maximin problem but these students were in the minority.
One costly error, made by some students, was the inclusion of an insufficient number of rows in stage one of their table. It would be advisable for students to be reminded to perform a straightforward check that their table contains sufficient rows for the number of arcs on the digraph in cases such as these. In fact, most students appeared to find the application of a dynamic programming approach to a minimisation problem based on a digraph relatively straightforward and coped very well with the blank table and lack of scaffolding in the answer book. Occasionally, students mislabelled their states and destinations giving reversed actions and as with the legacy qualification, arithmetic errors were fairly common. A very small number of students did not evaluate their calculations at each stage: calculating only the optimal row. Students should be advised that in decision mathematics they must rigorously apply the algorithm in full. Most students' presentation here, however, was clear and easy to follow although a minority crossed out working and then attempted to squeeze in alternative answers, making it difficult for examiners to follow their working.
Once students had completed the programming, most were able to find a suitable range for x based on the values in their final stage. Some students, however, appeared to be reluctant to carry the value '– 30' into stage 3 and so obtained an incorrect value for SB at the end of the table which in turn led to an incorrect result for the range for x.
A significant number of students misunderstood the final part of the question and quoted the minimum total weight along the route (obtained when setting $x = 0$) rather than giving the route itself.

Mark Scheme

Question	Scheme	Marks	AOs
3(a)	Dijkstra’s algorithm cannot be used on a network with negative weights	1B1	3.5b
		(1)	
(b)(i)			

Stage	State	Action	Destination	Value	Marks	AOs
0	G	GT	T	28*	1B1	3.1a
	H	HT	T	19*		
	I	IT	T	-24*		
	J	JT	T	17*		
1	D	DG	G	$-24+28=4*$	1M1	3.1a
		DH	H	$18+19=37$	1A1	1.1b
	E	EH	H	$24+19=43$	2A1	1.1b
		EI	I	27 – 24 = 3*		
	F	FG	G	$-25+28=3$		
		FI	I	$14-24=-10$		
		FJ	J	$-30+17=-13*$		
2	A	AD	D	32 + 4 = 36	2M1	1.1b
		AE	E	29 + 3 = 32*	3A1ft	1.1b
	B	BD	D	20 + 4 = 24	4A1	1.1b
		BE	E	19 + 3 = 22		
		BF	F	$-17-13=-30*$		
	C	CF	F	13 – 13 = 0*		
3	S	SA	A	13 + 32 = 45	3M1	1.1b
		SB	B	x – 30	5A1ft	1.1b
		SC	C	24 + 0 = 24		

Question	Scheme	Marks	AOs
	$x-30<24$	4dM1	3.1a
	$(0\leq)\ x<54$	6A1	2.2a
(ii)	Route: S – B – F – J – T	1B1	2.2a
		(12)	
			(13 marks)

Notes:

(a) 1B1: CAO – Do not accept ‘Dijkstra’s cannot be used on a directed network’
Condone any reference to negative (weights) or ‘negative edges’. Also condone ‘cannot be used with positive and negative arcs.’ But NOT answers which include incorrect statements.

Throughout (b):
- **Condone lack of destination column and/or reversed stage numbers throughout**
- **Only penalise incorrect result in value – i.e. ignore working values**

• Penalise absence of state or action column with first two A marks earned only
• Penalise empty/errors in stage column with first A mark earned only
• Penalise occurrence of single errors in state/action/destination with the relevant A mark - once only.
• Interchanged state and destination columns penalise with first two A marks

M marks - must bring earlier optimal results into calculations at least once per stage

1B1: Stage 0 correct

1M1: Stage 1 completed with 3 states and at least 7 rows. Bod if something in each cell
1A1: any two states in Stage 1 correct
2A1: CAO all 3 states correct in Stage 1 (must be 7 rows and no extra rows)

2M1: Stage 2 completed with 3 states and at least 6 rows. Bod if something in each cell
3A1ft: CAO any 2 states correct in Stage 2 on the follow through
4A1: CAO all 3 states in Stage 1 (no extra rows)

3M1: Stage 3 completed with 1 state and at least 3 rows. Bod if something in each cell
5A1ft: CAO for Stage 3 following through their optimal values (no extra rows)

4dM1: Dependent on scoring 3rd method mark and at least one of the first two method marks. Award for forming a correct inequality using their SB and the least of their SA and their SC. Allow $\leq$ for this mark. Must follow from stage 3 in their table. So M0 if no *x* appears in stage 3. This M mark could be implied, however by '$x < 54$' if stage 3 is correct.
6A1: CSO correct deduction of range of possible values of *x***.** Strict inequality required for this mark.
1dB1: Correct route. Dependent on 3rd method mark.

<u>SC in part b:</u>
Minimax/Maximin approach
Could score a maximum of B1 M1A0A0 M1A0A0 M1A0 M0A0 (max 4 marks) for the following:
1M1: Stage 1 completed with 3 states and at least 7 rows. Bod if something in each cell
2M1: Stage 2 completed with 3 states and at least 6 rows. Bod if something in each cell
3M1: Stage 3 completed with 1 state and at least 3 rows. Bod if something in each cell
NB must bring earlier optimal results into calculations at least once per stage for each M mark.

Student Response A

(a) there is an unknown value & there are negative constants on arcs (eg -24 on DG)

b) minimise

Stage	State	Action	Destination	Value
4	G	GT	T	28*
	H	HT	T	19*
	I	IT	T	-24*
	J	JT	T	17*
3	D	DG	G	-24+28= 4*
		DH	H	19+18 = 37
	E	EH	H	24+19 = 43
		EI	I	27-24=3*
	F	FG	G	-25+28=3
		FI	I	14-24 = -10
		FJ	J	-30+17= -13*
2	A	AD	D	32+4 = 36
		AE	E	29+3= 32*
	B	BD	D	20+4 = 24
		BE	E	19+3 = 22
		BF	F	-17+-13= -30*
	C	CF	F	15-13=0*
1	S	SA	A	13+32 = 45
		SB	B	x+30 = optimal
		SC	C	24+0 = 24
				as given in the question

Stage	State	Action	Destination	Value

for SB to remain optimal

$x - 30 \leq 24$ (equal to as being the same as would make both optimal)

$x \leq 54$

as the question states $x \geq 0$

$0 \leq x \leq 54$

ii) ∴ minimum weight of SBFJT = -30 where $x = 0$

11/13

Examiner Comments

In part (a), the correct reason is alluded to but the inclusion of an incorrect statement about 'unknown weights' precludes the mark in this part.

In part (b), stages 0, 1, 2 and 3 are fully correct. When determining the range of possible values for x it is necessary to see a strict inequality used for the final accuracy mark in part (i) although the non-strict inequality is condoned for the 4th method mark. The correct route SBFJT is stated.

Student Response B

(a) because there is an unknown value.

Stage	State	Action	Destination	Value
1	G	GT	T	28
	H	HT	T	19
	I	IT	T	-24
	J	JT	T	17
2	D	DG	G	-24 + 28 = 4
		DH	H	18 + 19 = 37
	E	EH	H	24 + 19 = 43
		EI	I	27 - 24 = 3
	F	FG	G	-25 + 28 = 3
		FI	I	14 - 24 = -10
		FJ	J	-30 + 17 = 13
3	A	AD	D	32 + 4 = 36
		AE	E	29 + 3 = 32
	B	BD	D	20 + 4 = 24
		BE	E	19 + 3 = 24
		BF	F	-17 - 10 = -27
	C	CF	F	13 - 10 = 3
4	S	SA	A	13 + 32 = 45
		SB	B	x + 24
		SC	C	24 + 3 = 27

Stage	State	Action	Destination	Value

b) i) $0 \leq x \leq 2$

ii) SBFIT

6/13

Examiner Comments

In part (a), an incorrect reason is given.

In part (b), stage 0 is completed correctly. In stage 1, there is a single error in action FJ where a value of 13 is given in place of -13. Stage 2 has two states correct (A and C) following through from the error in stage 1 but has an error in state B (action BE should have value 22). In stage 3, there is 1 state completed and 3 rows with something in each cell. However, the values do not follow through from earlier optimal values. The value for SB should be '$x - 27$'. When determining the range of values of x, there is no attempt to compare the value for SB (x + 24) with the least of SA and SC (27). We would expect to see either '$x + 24 < 27$' or a simplified '$x < 3$' following from the stated values in the table. The optimal route is incorrect.

Student Response C

(a) it is a flows network so we see weight of max flow on arc and we don't need to find quickest route from source to sink.

Stage	State	Action	Destination	Value
T	J	JT	T	17*
	I	IT	T	-24*
	H	HT	T	19*
	G	GT	T	28*
J3	F	FJ	J	-30*
I3	F	FI	I	14*
	E	EI	I	27*
H	D	DH	H	18*
	E	EH	H	24*
G	D	DG	G	-24*
	F	FG	G	-25*
D	A	AD	D	32
	B	BD	D	20*
E	A	AE	E	29
	B	BE	E	19*
F	B	BF	F	-17*
	C	BCF	F	13
A	S	SA	A	13*
B	S	SB	B	x*

Stage	State	Action	Destination	Value
C	S	SC	C	24 ~~6~~

∴ ~~SA~~ SB F G T → 28 − 24 + 20 + x

~~SB E~~ SB F

~~SB D~~

SB ~~A~~ H

∴ 24 + x

SBF GT

1/13

Examiner Comments

In part (a), an incorrect reason is given.

In part (b), stage 0 is completed correctly. In stage 1, 2 and 3 however, no previous stages' optimal results are brought forward into calculations. This means that none of the first three method marks can be earned. The 4th method mark is dependent on the 3rd method mark and at least one of the first two method marks so cannot be earned here.

The final mark for the route is also dependent on the 3rd method mark and so cannot be earned here.

Exemplar Question 4

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4.

		Player B		
		Option X	Option Y	Option Z
Player A	Option P	3	–2	0
	Option Q	–4	4	–2
	Option R	1	2	–1

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(*a*) Verify that there is no stable solution to this game.

(2)

Player A intends to make a random choice between options P, Q and R, choosing option P with probability p_1, option Q with probability p_2 and option R with probability p_3

Player A wants to find the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm. Player A formulates the following linear programming problem for the game, writing the constraints as inequalities.

Maximise $P = V$

subject to $V \geq 3p_1 - 4p_2 + p_3$
$V \geq -2p_1 + 4p_2 + 2p_3$
$V \geq -2p_2 - p_3$
$p_1 + p_2 + p_3 \leq 1$
$p_1 \geq 0, p_2 \geq 0, p_3 \geq 0, V \geq 0$

(*b*) Correct the errors made by player A in the linear programming formulation of the game, giving reasons for your answer.

(3)

(*c*) Write down an initial Simplex tableau for the corrected linear programming problem.

(3)

The Simplex algorithm is used to solve the corrected linear programming problem.

The optimal values are $p_1 = 0.6$, $p_2 = 0$ and $p_3 = 0.4$

(*d*) Calculate the value of the game to player A.

(2)

(*e*) Determine the optimal strategy for player B, making your reasoning clear.

(4)

(Total for Question 4 is 14 marks)

Mean Score 4.3 out of 14

Examiner Comments

Part (a) was well answered by almost all students. Occasionally, some marks were lost either due to errors in writing down row minima or column maxima or for incomplete conclusions. Very few inverted the question, finding row maxima and column minima. Sometimes, working was incomplete where there were missing row minimums and column maximums. Students should be reminded that in order to be using the correct “method” they must show the row minimums and column maximums in full.

Part (b) was generally not well answered. The demand of the question was high and required students to provide reasons for the steps required to formulate the linear programming problem. Many students were able to spot the required modification but were unable to articulate the reasons for the changes. A substantial number of students were however, able to go on write down the three constraints correctly. There was, nonetheless, a significant minority who failed to adjust the coefficients of the matrix and/or correct the direction of the inequality signs which was costly in terms of the dependency marks that followed later in the question.

Unfortunately, if students did not correctly amend the constraints, the number of marks available in part (c) was limited. It is clear that setting up a simplex tableau is a far less familiar area of the course and a significant minority did not attempt this part of the question. Of those students that did attempt to populate the tableau, a good number scored full marks. Those that lost marks, did so either due to incorrect constraints from earlier in the question or due to slips and misreading of their own handwriting or simply from a lack of understanding of which rows and columns were required.

Parts d) and e) were somewhat unfamiliar and ultimately more challenging for many students. Whilst some students were able to substitute the values for p_1, p_2 and p_3 into the constraints for V quite a significant proportion either did not attempt this part of the question or simply substituted the values into one constraint only. Sometimes, students who used the transformed constraints forgot to adjust the value to be correct for the original problem. Part (e) was a good discriminator at the top end. It was though, quite frequently not attempted. Of those that did attempt this part of the question, many students did not recognise that $p_2 = 0$ meant that a dominance argument then applied for B which ruled out option X. A number of students instead tried to set up simultaneous equations in three variables and were then unable to make further progress. For those students who did spot the dominance, about half used an approach involving the value of the game to B taken from the value their obtained in part (d) together with an expression in terms of p from the matrix. The other half found two expressions for the value of the game to B from their reduced matrix, equated them and solved to determine the probability. Some students unnecessarily drew graphs to demonstrate the optimality of the value of p they had found. Those that determined the required probability, usually went on to list the correct options for player B although a small number did not state that player B should never play option X. Students should be reminded that their final statement should include a strategy for each of the possible options.

Mark Scheme

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4(a)</td><td colspan="2">Row minima: $-2, -4, -1$ max is -1</td><td>1M1</td><td>1.2</td></tr>
<tr><td></td><td colspan="2">Column maxima: 3, 4, 0 min is 0</td><td></td><td></td></tr>
<tr><td></td><td colspan="2">Row maximin $(-1) \neq$ Column minimax (0) so not stable</td><td>1A1</td><td>2.4</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td colspan="2">As the value of V must be non-negative the coefficients of the three inequalities involving V must be non-negative so add at least 4 to each value $\left[\text{e.g. adding 5 gives} \begin{pmatrix} 8 & 3 & 5 \\ 1 & 9 & 3 \\ 6 & 7 & 4 \end{pmatrix}\right]$</td><td>1B1</td><td>2.3</td></tr>
<tr><td></td><td colspan="2">Furthermore, as V is the minimum that A can expect to win, the constraints should be $V \leq \ldots$</td><td>2B1</td><td>2.3</td></tr>
<tr><td></td><td>e.g. ‘adding 5’
$V \leq 8p_1 + p_2 + 6p_3$
$V \leq 3p_1 + 9p_2 + 7p_3$
$V \leq 5p_1 + 3p_2 + 4p_3$</td><td>e.g. ‘adding 4’
$V \leq 7p_1 + 5p_3$
$V \leq 2p_1 + 8p_2 + 6p_3$
$V \leq 4p_1 + 2p_2 + 3p_3$</td><td>3B1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td colspan="2">‘adding 5’
<table>
<tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr>
<tr><td>r</td><td>1</td><td>-8</td><td>-1</td><td>-6</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>1</td><td>-3</td><td>-9</td><td>-7</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>t</td><td>1</td><td>-5</td><td>-3</td><td>-4</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td>1B1
1M1
1A1</td><td>1.2
3.3
1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td colspan="2">(ii) Substitute p values to obtain $V \leq 4.6$
Value of the game to player A = $4.6 - 5 = -0.4$</td><td>1M1
1A1</td><td>3.4
2.2a</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(e)</td><td colspan="2">Player B’s choices are options Y and Z only</td><td>1B1</td><td>3.4</td></tr>
<tr><td></td><td colspan="2">Either $2q = 0.4$ or $-2q + 1(1 - q) = 0.4$ (where q is the probability Player B plays their option Y and $(1 - q)$ is the corresponding probability for option Z)</td><td>1M1</td><td>3.1a</td></tr>
<tr><td></td><td colspan="2">$q = 0.2$</td><td>1A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2">Player B should never play their option X, they should play their option Y with probability 0.2 and option Z with probability 0.8</td><td>2A1ft</td><td>3.2a</td></tr>
<tr><td></td><td colspan="2"></td><td>(4)</td><td></td></tr>
</table>

(14 marks)
Notes:
(a) 1M1: attempt at row minima and column maxima – condone one error **1A1:** correct reasoning that the game is not stable (accept "$-1 \neq 0$" + statement) – dependent on correct row minima and column maxima **(b) 1B1:** Indicates that <u>coefficients are incorrect</u> because <u>*V* must be non-negative</u>. Must convey both underlined aspects. Condone 'positive' for 'non-negative' **2B1:** Indicates that <u>inequality signs are the wrong way</u> because <u>*V* is the minimum</u> (so expected winnings are $\geq V$). Must convey both aspects but give bod. **3B1:** CAO (all three inequalities). Give this mark for correct equations with slack variables provided 2B1 has been awarded. **(c) 1B1:** All row and column labels correct for Simplex tableau **1M1:** Setting up the Simplex model - any two of my 'r', 's' or 't' rows correct. Or a completely correct answer with either one column or one row missing – condone lack of basic variable column. Should follow from **changed** constraints of the correct form from b). So, constraints must have been of the form $V \leq ap_1 + bp_2 + cp_3$ $(a, b, c \geq 0)$ o.e. **1A1:** CAO on numerical values. **Note:** The B mark is for labelling the simplex tableau correctly; the M and A marks are for values only. **(d) 1M1:** substitutes their *p* values into all three expressions for the upper bound of *V*. Condone use of an equals sign here. But not '$V \geq \ldots$' May see one of: • $V \leq 3(0.6) - 4(0) + 0.4 = 2.2$ $\{=\frac{11}{5}\}$; $V \leq -2(0.6) - 4(0) + 2(0.4) = -0.4$ $\{= -\frac{2}{5},\}$; $V \leq -2(0) - (0.4) = -0.4$ $\{= -\frac{2}{5},\}$ • $V \leq 8(0.6) + (0) + 6(0.4) = 7.2$ $\{=\frac{36}{5}\}$; $V \leq 3(0.6) + 9(0) + 7(0.4) = 4.6$ $\{=\frac{23}{5},\}$; $V \leq 5(0.6) + 3(0) + 4(0.4) = 4.6$ $\{=\frac{23}{5},\}$ • $V \leq 7(0.6) + 5(0.4) = 6.2$ $\{=\frac{31}{5}\}$; $V \leq 2(0.6) + 8(0) + 6(0.4) = 3.6$ $\{=\frac{18}{5},\}$; $V \leq 4(0.6) + 2(0) + 3(0.4) = 3.6$ $\{=\frac{18}{5},\}$ **1A1:** CAO for the value of the game to player A **(e) 1B1:** CAO – uses the model to determine that Player B only plays Y and Z **1M1:** A *correct* equation for B (where value of game to B = – 1 x **their** value of game to A) **1A1:** CAO for *q* (the probability that B plays Y) **2A1ft:** Correct optimal strategy in context (not just in terms of *q*) following through their *q* <u>Alternative for (e)</u> **1B1:** CAO – uses the model to determine that Player B only plays Y and Z **1M1:** Formulates two correct expressions for the expected value of the game to B and finds the intersection: $2q = -2q + 1(1 - q)$ $(= 1- 3q)$ **1A1:** As above

2A1ft: As above

Student Response A

4.

		Player B			
		Option X	Option Y	Option Z	Row maximins
Player A	Option P	3	–2	0	−2
	Option Q	–4	4	–2	−4
	Option R	1	2	–1	−1
	Column Minimax	3	4	0	

a. ∴ There can be no stable solution, since row maximin ≠ column minimax
$(0 \neq -1)$

b. The player has taken no steps to ensure V will be positive, since a value of 5 should be added to every value in the table where V is the value of the transformed game, so V is definitely positive, and the inequalities ~~Also~~ will have to change accordingly.
Also, ~~V~~ the inequalities for V should be $\leq$, since the minimum assured value for V is being maximised.

c. $V - 8p_1 ~~+~~ - p_2 - 6p_3 + r = 0$

$V - 3p_1 - 9p_2 - 7p_3 + s = 0$

$V - 5p_1 - 3p_2 - 4p_3 + t = 0$

$p_1 + p_2 + p_3 + u = 1$

$V, p_1, p_2, p_3, r, s, t, u \geq 0$

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
r	1	−8	−1	−6	1	0	0	0	0
s	1	−3	−9	−7	0	1	0	0	0
t	1	−5	−3	−4	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	−1	0	0	0	0	0	0	0	0

Only use the tableau below if you need to correct your initial tableau.

b.v.									Value
P									

d. ~~When B~~

~~$V \leqslant 3(0.6) - 4(0) + 0.4$~~ ~~$V \leqslant -2(0.6) + 4(0) + 2(0.4)$~~

~~$\leqslant \frac{11}{5}$~~ ~~$\leqslant \frac{18}{5}$~~

~~$V \leqslant -2(0) - 0.4$~~

$V \leqslant 8(0.6) + 0 + 6(0.4)$ $V \leqslant 3(0.6) + 0 + 7(0.4)$

$\leqslant \frac{36}{5}$ $\leqslant \frac{23}{5}$

$V \leqslant 5(0.6) + 3(0) + 4(0.4)$

$\leqslant \frac{23}{5}$

$\therefore V = \frac{23}{5}$ value $= \frac{23}{5} - 5$

$= -\frac{2}{5}$

∴ ~~B should never play~~

Value for B $= -V(A)$

$= \frac{2}{5}$

B will never play X, since it grants A a greater value, so will never be a good strategy to chose ~~Y~~ X,

Let B play Y with probability p, ~~and~~ therefore Z with probability 1-p

~~$V(B) = -2p - (1-p)$~~

~~$\frac{2}{5} = -p - 1$~~

~~$p =$~~

$V(B) = -2p + (1-p)$

$\frac{2}{5} = 1 - 3p$

$3p = \frac{3}{5}$

$p = \frac{1}{5}$

∴ B should play X with probability ~~zero~~ zero, Y with probability $\frac{1}{5}$ and Z with probability $\frac{4}{5}$

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Examiner Comments

In part (a), the row minima and column maxima are stated correctly and correct reasoning that the game is not stable since 'row minima ≠ column maxima' is seen.

In part (b), indication that '5' should be added to coefficients to ensure that *V* is positive. We will condone 'positive' for 'non-negative'. We also see justification for swapping the direction of the inequality signs 'since the minimum value for V is being maximised'. We are looking for three correct modified inequality constraints but since we have awarded the second B mark for the correct justification of the direction of the inequalities, we can give the third B for the three correct constraint equations with slack variables.

In part (c), the row and column labels on the simplex tableau are correct. Furthermore, all numerical entries in the tableau are correct.

In part (d), we see three expressions for the upper bound of V arising from substitution of the *p* values into modified constraints. The correct value of the game to player A is deduced.

In part (e), the student deduces correctly that 'B will never play X' and writes down the value of the game to player B in terms of ***p*.** This is equated to the negative of the value of the game to player A and the correct value of *p* is deduced. Finally, the correct optimal strategy for B is given in context.

Student Response B

4.

		Player B			
		Option X	Option Y	Option Z	Row min
Player A	Option P	3	–2	0	–2
	Option Q	–4	4	–2	–4
	Option R	1	2	–1	–1
	column max	3	4	0	

(a) Row maximin $\neq$ column minimax

$-1 \neq 0$

(b) All rows must be greater than the value of the game

You must add a number n to make all values in table +ve

$V = v + 5$ and must define variables

	X	Y	Z
P	8	3	5
Q	1	9	3
R	6	7	4

$8p_1 + p_2 + 6p_3 \geq V \quad \therefore \quad V - 8p_1 - p_2 - 6p_3 \leq 0$

$3p_1 + 9p_2 + 7p_3 \geq V \quad V - 3p_1 - 9p_2 - 7p_3 \leq 0$

$5p_1 + 3p_2 + 4p_3 \geq V \quad V - 5p_1 - 3p_2 - 4p_3 \leq 0$

$P - V = 0$

(c)

	V	p_1	p_2	p_3	r	s	t	u	value
r	1	–8	–1	–6	1	0	0	0	0
s	1	–3	–9	–7	0	1	0	0	0
t	1	–5	–3	–4	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	–1	0	0	0	0	0	0	0	0

$V - 8p_1 - p_2 - 6p_3 + r = 0$

$V - 3p_1 - 9p_2 - 7p_3 + s = 0$

$V - 5p_1 - 3p_2 - 4p_3 + t = 0$

$p_1 + p_2 + p_3 + u = 1$

where r, s, t, u are slack variables

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
r	1	-8	-1	-6	1	0	0	0	0
s	1	-3	-9	-7	0	1	0	0	0
t	1	-5	-3	-4	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	-1	0	0	0	0	0	0	0	0

Only use the tableau below if you need to correct your initial tableau.

b.v.									Value
P									

(d)

	X	Y	Z
P	3	-2	0
R	1	2	-1

Play P with Play R with

$p_1 = p \quad p_2 = 1-p$

$3(0.6) + (1-0.4)$

$= \frac{12}{5} = 2.4$

(e) using dominance reduce matrix

	~~X~~	Z
P	2	0
R	2	-1

	P	R
Y	2	-2
Z	0	1

Avoid playing X as this will always be a loss

Play Y will probability p and Z with probability 1- p

$2 - 2(1-p) = 2p$

$0 - (1-p) = p-1$

$2p = p-1$

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Examiner Comments

In part (a), row minima and column maxima are stated correctly and correct reasoning that the game is not stable since 'row minima ≠ column maxima' is seen.

In part (b), the constraints are correctly modified but there is no correct justification given.

In part (c), the row and column labels on the simplex tableau are correct. Furthermore, all numerical entries in the tableau are correct.

In part (d), we do not see *p* values substituted into the three expressions for the upper bound of V.

In part (e), the candidate determines that B should not play X but does not make further correct progress to determine the full strategy for B.

Student Response C

4.

		Player B			
		Option X	Option Y	Option Z	
Player A	Option P	3	–2	0	-2
	Option Q	–4	4	–2	-4
	Option R	1	2	–1	-1 ✓
		3	4	0	

(a) for a play-safe ~~strategy~~ method

A plays option R and B plays option Z,

but the value of the game for each players is

$V_{(A)} = -1$, $V_{(B)} = 0$

$-1 \neq 0$ ∴ there is no stable solution.

(b) i) Constraints should be

$V \leq 3P_1 - 4P_2 + P_3$ · the value of the game should be always

$V \leq -2P_1 + 4P_2 + 2P_3$ less or equal to the given situation

$V \leq -2P_2 - P_3$ · normal simplex to be applied, only slack variables can be added. so $\leq$ is required

ii) $V \geq 0 \rightarrow$ X V can be negative value.

(c) $V - 3P_1 + 4P_2 - P_3 + S_1 = 0$

$V + 2P_1 - 4P_2 - 2P_3 + S_2 = 0$

$V + 2P_2 + P_3 + S_3 = 0$

$P_1 + P_2 + P_3 + S_4 = 1$

$P - V = 0$

b.v.	P_1	P_2	P_3	S_1	S_2	S_3	S_4	V	Value
S_1	-3	$+4$	-1	1	0	0	0	1	0
S_2	2	-4	-2	0	1	0	0	1	0
S_3	0	2	1	0	0	1	0	1	0
S_4	1	1	1	0	0	0	1	0	1
P	0	0	0	0	0	0	0	-1	0

Only use the tableau below if you need to correct your initial tableau.

b.v.									Value
P									

(d) $V_{(A)} = 3\times0.6 + 1\times0.4 = \underline{2.2}$ ✓

$V_{(B)} : -2\times0.6 + 2\times0.4 = -0.4$

$V_{(B)} : 0\times0.6 + (-1)\times0.4 = -0.4$

$V_{(A)} = 2.2$

(e) $\underline{V_{(B)} = -2.2}$

let P_1 P_2 P_3, B plays X, Y, Z

Probabilities

~~$P_1 \times 3 + P_2 \times (-2)$~~

For B, option Z gives better results than option Y so delete option Y.

Continued

table for B

			A	
		P	Q	R
B	X	-3	4	-1
	Z	0	2	1

now $-3P_1 = V_{(B)}$ $\qquad P_2 = 0, \quad P_1 + P_3 = 1$

$\therefore P_3 = 1 - P_1$

$4P_1 + 2P_3 = V_{(B)}$

$-P_1 + P_3 = V_{(B)}$

$\Rightarrow \underline{-3P_1 = V_{(B)}}$

$4P_1 + 2(1-P_1) = V_{(B)} \Rightarrow \underline{2 + 2P_1 = V_{(B)}}$

$-P_1 + (1-P_1) = V_{(B)} \Rightarrow \underline{-2P_1 + 1 = V_{(B)}}$

Sol.)

$V_{(B)} = -2.2, \quad (\because V_{(B)} = -V_{(A)}), \quad P_1 \geq 0$

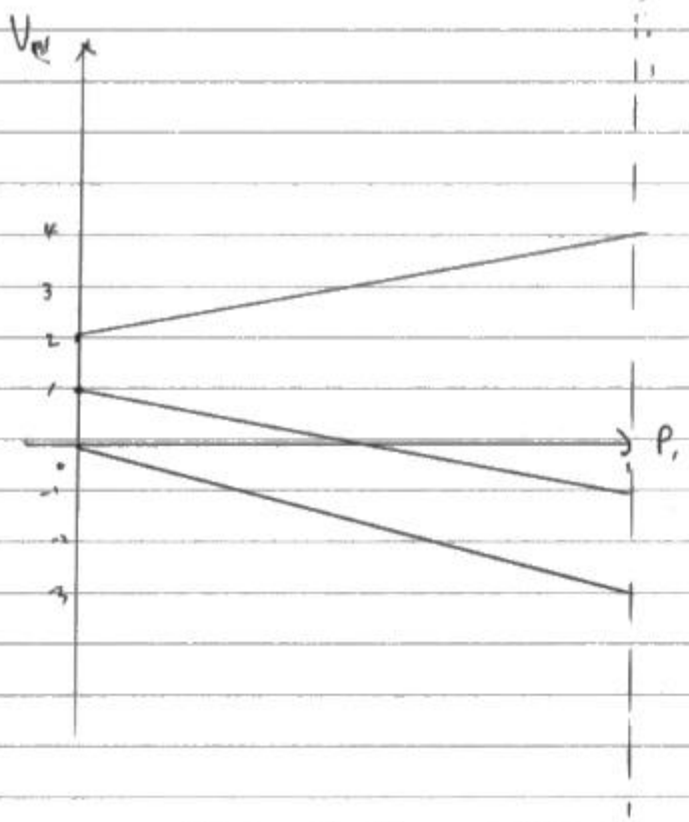
therefore either $-3P_1$, $2+2P_1$ or $-2P_1+1$ is -2.2

$2+2P_1$ can't give -2.2 unless $P_1 < 0$, so

$V_{(B)} = -2P_1 + 1 = -2.2 \quad P_1 < 0$ not possible

$\therefore \cancel{P_1} = \cancel{0}$

$\therefore V_{(B)} = -3P_1 = -2.2 \quad \therefore P_1 = \frac{2.2}{3} = \frac{11}{15}$



~~V_B 2 2+2P, = 2P, + 1~~

→ ~~$4P_1 = -1$~~

∴ B plays X with propability of $\frac{11}{15}$

and plays Z with probability of $\frac{4}{15}$.

and doesn't play Y.

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Examiner Comments

In part (a), row minima and column maxima are stated correctly and correct reasoning that the game is not stable since 'row minima ≠ column maxima' is seen.

In part (b) we see 'the value of the game should be always less or equal to the given situation' which we will accept with benefit of doubt for the justification for the direction of the inequalities in the constraints. There is no attempt to make the coefficients non-negative however.

In part (c), the row and column labels on the simplex tableau are correct but the numerical entries cannot earn further marks in (c) as they do not come from changed constraints of the correct form.

In part (d), we see p values substituted into the original constraints. We will condone the equal sign. The incorrect value of the game to player A is selected.

In part (e), an incorrect dominance argument is applied and no 'correct' equation for B is seen. We can follow through an incorrect value of the game but not an incorrect dominance argument.

Exemplar Question 5

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5. An increasing sequence $\{u_n\}$ for $n \in \mathbb{N}$ is such that the difference between the nth term of $\{u_n\}$ and the mean of the previous two terms of $\{u_n\}$ is always 6

(*a*) Show that, for $n \geq 3$

$$2u_n - u_{n-1} - u_{n-2} = 12$$

(2)

Given that $u_1 = 2$ and $u_2 = 8$

(*b*) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n.

(7)

(*c*) Show that as $n \to \infty$, $u_n \to kn$ where k is a constant to be determined. You must give reasons for your answer.

(2)

(Total for Question 5 is 11 marks)

Mean Score 5.7 out of 11

Examiner Comments

In part (a), a high proportion of students earned both marks. The most common loss of marks being due to sign errors when multiplying out brackets which is quite worrying at this level. Furthermore, a minority of students seemed unsure how to write down the mean of u_{n-1} and u_{n-2}.

Part (b) should have been quite standard and straightforward and for many students this was indeed the case. Worryingly though, some students set the right-hand side of their auxiliary equation equal to 12 and came unstuck almost immediately. However, the majority of students were able to form and solve the correct auxiliary equation although some misconceptions regarding the correct form of complementary function were apparent with some students writing, for example, $u_n = \frac{1}{2}A^n + B^n$.

When determining the particular solution, most students correctly realised they needed to substitute $u_n = \lambda n$ into the recurrence relation and most were able to deduce that $\lambda = 4$. Unfortunately, some students then made the costly error of deducing that the general solution was then $u_n = A + B(-\frac{1}{2})^n + 4$ rather than $u_n = u_n = A + B(-\frac{1}{2})^n + 4n$. Solutions of an incorrect form such as these were then unable to earn later marks. Those students who obtained general solutions of the correct form were almost always able to write down two equations for 'A' and 'B' and solve them simultaneously, often using calculators, to determine the correct solution.

As mentioned above, part (c) was not accessible to students who made critical errors earlier in the question that give rise to general solutions of an incorrect form. Of those students who were eligible for marks here, a majority could state the value of k, but too frequently students only considered the behaviour of the $(-\frac{1}{2})^n$ term, and not the impact of the $-\frac{2}{3}$ term.

Mark Scheme

Question	Scheme	Marks	AOs
5(a)	$u_n - \frac{1}{2}(u_{n-1} + u_{n-2}) = 6 \Rightarrow 2u_n - (u_{n-1} + u_{n-2}) = 12$	1M1	2.1
	$2u_n - u_{n-1} - u_{n-2} = 12$	1A1	2.2a
		(2)	
(b)	aux equation $2m^2 - m - 1 = 0 \Rightarrow m = 1, m = -\frac{1}{2}$	1B1	2.1
	$u_n = A + B\left(-\frac{1}{2}\right)^n$	2B1	1.1b
	particular solution try $u_n = \lambda n$ $\therefore 2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12 \Rightarrow \lambda[= 4]$	1M1	1.1b
	$u_n = A + B\left(-\frac{1}{2}\right)^n + 4n$	1A1	1.1b
	$u_1 = 2 \Rightarrow 2A - B = -4$	2M1	1.1b
	$u_2 = 8 \Rightarrow 4A + B = 0$	3M1	1.1b
	$A = -\frac{2}{3}, B = \frac{8}{3} \Rightarrow u_n = -\frac{2}{3} + \frac{8}{3}\left(-\frac{1}{2}\right)^n + 4n$	2A1	2.2a
		(7)	
(c)	$u_n \to 4n \quad (k = 4)$ As $n \to \infty, \left(-\frac{1}{2}\right)^n \to 0$ and $4n$ is considerably greater than $-\frac{2}{3}$	1M1 1A1ft	2.1 1.1b
		(2)	
		(11 marks)	

Notes:

(a) 1M1: attempt to write given information in terms of a recurrence equation (allow sign errors and errors in notation)

1A1*: CAO (NB equation is provided in the question so must see a correct unsimplified recurrence relation which is simplified with no errors to the required form for this mark).

(b) 1B1: CAO for auxiliary equation and corresponding solutions (this mark can be implied by correct complementary function)

2B1: complementary function CAO. Condone lack of ‘$u_n =$’.

1M1: substitutes $u_n = \lambda n$ into their 2nd-order recurrence relation and solves to obtain $\lambda = \ldots$

Note: $2\lambda n - \lambda(n-1) - \lambda(n-2) = 12$ o.e. $\Rightarrow \lambda = \cdots$ can earn this mark

Note: Do not condone sign errors or errors in coefficients e.g.

$2\lambda n - \lambda(n+1) - \lambda(n-2) = 12$ is M0

$2\lambda n - 2\lambda(n-1) - \lambda(n-2) = 12)$ is M0

1A1: Correct general solution. Condone lack of ‘$u_n =$’.

2M1: Forms one equation in A and B. General Solution must be of the form.

$u_n = A + B\left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$

3dM1: Forms a second equation in A and B. Dependent on the previous method mark.

2A1: Particular solution CAO. Do not condone lack of '$u_n = \ldots$'

(c) M1: Obtains correct limit (ft their particular solution which must be of the correct form i.e. $u_n = A + B\left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$)

A1ft: Provides correct reasoning including comments relating to both of the following:

- $n \to \infty, \left(-\frac{1}{2}\right)^n \to 0$
- '$-\frac{2}{3}$' becomes negligible (compared to $4n$ as $n \to \infty$)

Note: Condone '$-\frac{2}{3}$ becomes insignificant'

Student Response A

5a) difference between nth term and mean of previous two = $u_n - \left(\frac{u_{n-1} + u_{n-2}}{2}\right) = 6$.

$\therefore 2u_n - u_{n-1} - u_{n-2} = 12$ //

b) $2u_n - u_{n-1} - u_{n-2} = 12$.

$2r^2 - r - 1 = 0 \rightarrow$ Auxilliary.

$(2r+1)(r-1) = 0$.

$r = -\frac{1}{2} \quad r = 1$.

$\therefore$ C.F $\rightarrow u_n = A(-\frac{1}{2})^n + B(1)^n$

$= A(-\frac{1}{2})^n + B$ //

where A and B are arbitrary constants.

Let $u_n =$ ~~λn~~ λn.

$\therefore$ ~~$24n - 12(n-1) - (n-2) = 12$.~~

$2\lambda n - \lambda(n-1) - \lambda(n-2) = 12$.

$\cancel{2\lambda n} - \cancel{\lambda n} + \lambda - \cancel{\lambda n} + 2\lambda = 12$.

$3\lambda = 12$

$\lambda = 4$ //.

$\therefore$ G.S $\rightarrow u_n = A\left(-\frac{1}{2}\right)^n + B + 4n$

if $u_1 = 2$ and $u_2 = 8$.

$2 = -\frac{A}{2} + B + 4$.

$-2 = -\frac{A}{2} + B$.

$-4 = -A + 2B$.

$A = \frac{8}{3} \quad B = -\frac{2}{3}$ //

$8 = A\left(\frac{1}{4}\right) + B + 8$.

$0 = \frac{A}{4} + B$.

$-$

$-2 = -\frac{A}{2} + B$.

$2 = \frac{3}{4}A$.

b) ∴ $u_n = \frac{8}{3}\left(-\frac{1}{2}\right)^n - \frac{2}{3} + 4n$.

c) as $n \to \infty$.

$u_n \to \frac{8}{3}\left(-\frac{1}{2}\right)^t - \frac{2}{3} + 4(t)$ where $t \to \infty$.

$u_t \to \frac{8}{3}\left(\frac{1}{4}\right)\left(-\frac{1}{2}\right)^{t-2} - \frac{2}{3} + 4t$ where $t \to \infty$.

$\frac{2}{3}\left(-\frac{1}{2}\right)^{t-2} - \frac{2}{3} + 4t$

$\left(-\frac{1}{2}\right)^{t-2}$ where $t \to \infty$

$\left(-\frac{1}{2}\right)^{t-2} \to 1$ as

$= \frac{2}{3} - \frac{2}{3} + 4t = 4t$

$\to \ = 4n$.

10/11

Examiner Comments

In part (a), we see a correct attempt to write the given information in a recurrence relation which is simplified to the required form.

In part (b), a correct auxiliary equation with correct corresponding solutions is seen. The complementary function is stated correctly and the particular solution is found correctly. The correct general solution is stated. Simultaneous equations are formed in A and B and solved to find the correct expression for u_n.

In part (c), the correct limit is obtained but there is insufficient reasoning for the final mark as we require a comment relating to the negligible size of the $\frac{-2}{3}$ term.

Student Response B

a) $u_n - \left(\frac{u_{n-1} + u_{n-2}}{2}\right) = 6$

$\times$ all by 2

$2u_n - u_{n-1} - u_{n-2} = 12$

as required.

$r = -\frac{1}{2}$

$2r = -1$

$2r + 1$

b) $2r^2 - r - 1$

$(r-1)(2r+1)$

$r = 1, \quad r = -\frac{1}{2}$

$u_n = A(1)^n + B(-\frac{1}{2})^n$

$2u_n - u_{n-1} - u_{n-2} = \lambda n$

$2(\lambda n) - \lambda(\lambda n - 1) - \lambda(\lambda n - 2) = \lambda n$

~~$2\lambda n - \lambda n + 1 - \lambda n + 2 = \lambda n$~~

~~$\lambda n = 3$~~

$2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = \lambda n$

$\lambda n = 3\lambda$

$u_n = A(1)^n + B(-\tfrac{1}{2})^n - 3n$

$2 = A - \tfrac{1}{2}B - 3$

~~$8 = A - \tfrac{1}{4}B$~~ $\quad 5 = A - \tfrac{1}{2}B$

~~$8 = A$~~ $\quad 8 = A - \tfrac{1}{4}B - 6$

$14 = A - \tfrac{1}{4}B$

$A = 23 \quad B = 36$

$u_n = 23(1)^n + 36(-\tfrac{1}{2})^n - 3n$

c) as $n \to \infty$ $\quad 23(1) + 36(-\tfrac{1}{2})^n - 3n$

$\to -\infty$

6/11

Examiner Comments

In part (a), we see a correct attempt to write the given information in a recurrence relation which is simplified to the required form.

In part (b), a correct auxiliary equation with correct corresponding complementary function is seen. Although there is no right-hand side (=0) for the auxiliary equation we will condone this as it is implied by the complementary function. When finding the particular solution, there is an error in the right-hand side of the equation – we see ‘λn’ rather than ’12’. The resulting general solution is, however, of the correct form and we see simultaneous equations in A and B formed and solved. The final particular solution is incorrect.

In part (c) the limit is incorrect and does not follow from their particular solution.

Student Response C

a) $U_n = U_{n-1} + 6$

$2U_n = 2U_{n-1} + 6$

$U_n - U_{n-1} = 6$

$U_{n-1} - U_{n-2} = 6$

$u_n - u_{n-1} + u_n - 2u_{n-2} =$

$(u_n - u_{n-1}) = 6 \quad 6 \quad + 12$

b) CF: $2m^2 - m - 1 = 12$

$2m^2 - m - 13 = 0$

$\frac{1 \pm \sqrt{4\ 105}}{4}$

$A\left(\frac{1+\sqrt{105}}{4}\right)^n + B\left(\frac{1-\sqrt{105}}{4}\right)^n$

PS: $u_n = \lambda n$ $\quad \cancel{2\lambda - \lambda - \lambda = 12}$

$2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12$

$2\lambda n - 2\lambda n + 3\lambda = 12$

$\lambda = 4$

$u_n = A\left(\frac{1+\sqrt{105}}{4}\right)^n + B\left(\frac{1-\sqrt{105}}{4}\right)^n + 4$

$\cancel{8 = A + A\sqrt{105}}$ 1

$4 \cancel{8} = A(1+\sqrt{105}) + B(1-\sqrt{105})$ 4

$(8 \times 16) - 4 = A(1+\sqrt{105})^2 + B(1-\sqrt{105})^2$

$$A = \frac{4 - \cancel{B} \; B(1-\sqrt{105})}{(1+\sqrt{105})}$$

$\cancel{8 \times 16}$ $124 = \cancel{A}(1+\sqrt{105})(4 - B(1-\sqrt{105})$

$+ B(1-\sqrt{105})^2$

2/11

Examiner Comments

In part (a), there is no valid attempt to write the given information in terms of a recurrence equation.

In part (b), the right-hand side of the auxiliary equation is ‘12’ which is not valid and gives rise to an incorrect complementary function. $U_n = \lambda n$ is substituted into the recurrence relation and solved to give $\lambda = 4$ but the general solution is incorrect due to an incorrect complementary function. The third and fourth method marks are dependent on a general solution of the correct form which is not achieved here.

In part (c), the method mark is again dependent on a general solution of the correct form but regardless there is no attempt here to find the limit of u_n

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Exemplar Question 6

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6.

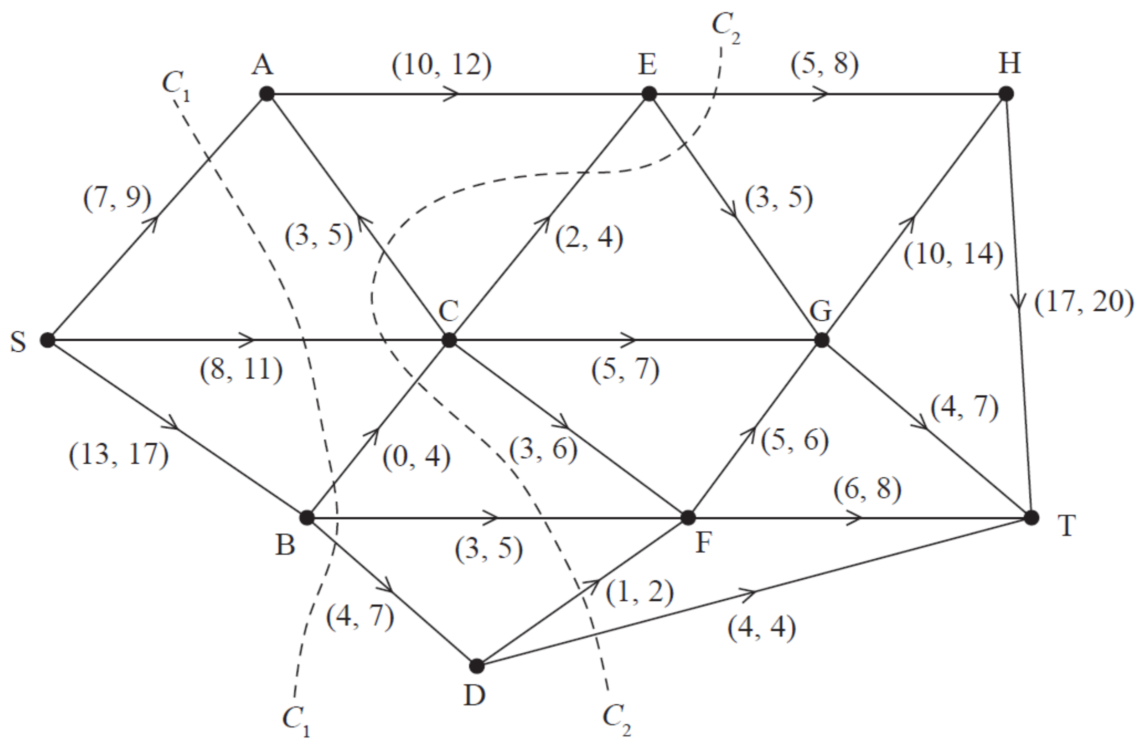


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid flows from a source, S, to a sink, T.

The numbers (l, u) on each arc represent, in litres per second, the lower capacity, l, and the upper capacity, u, of the corresponding pipe.

Two cuts C_1 and C_2 are shown.

(*a*) Find the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(*b*) Explain why the arcs AE and CE cannot be at their upper capacities simultaneously.

(1)

(*c*) Explain why a flow of 31 litres per second through the system is not possible.

(1)

(*d*) Hence determine a minimum feasible flow and a maximum feasible flow through the system. You must draw these feasible flows on the diagrams in the answer book and give reasons to justify your answer. **You should not apply the labelling procedure to find these flows.**

(4)

(Total for Question 6 is 8 marks)

Mean Score 3.0 out of 8

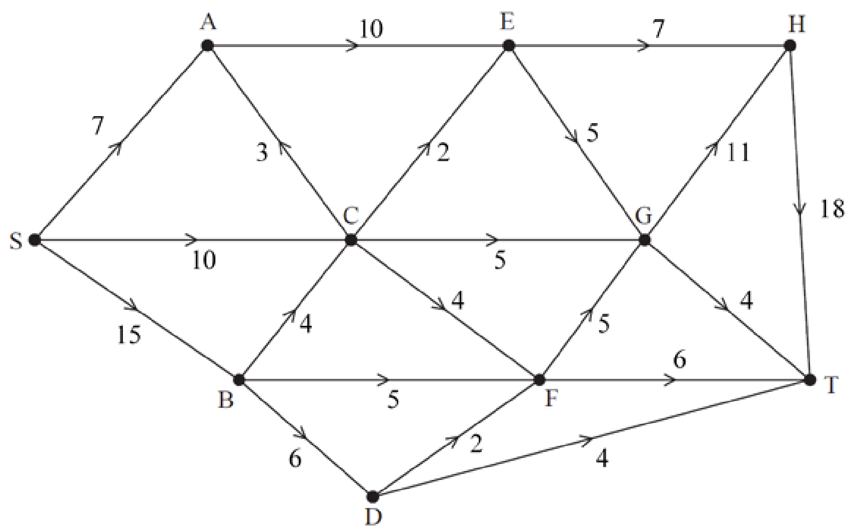
Examiner Comments

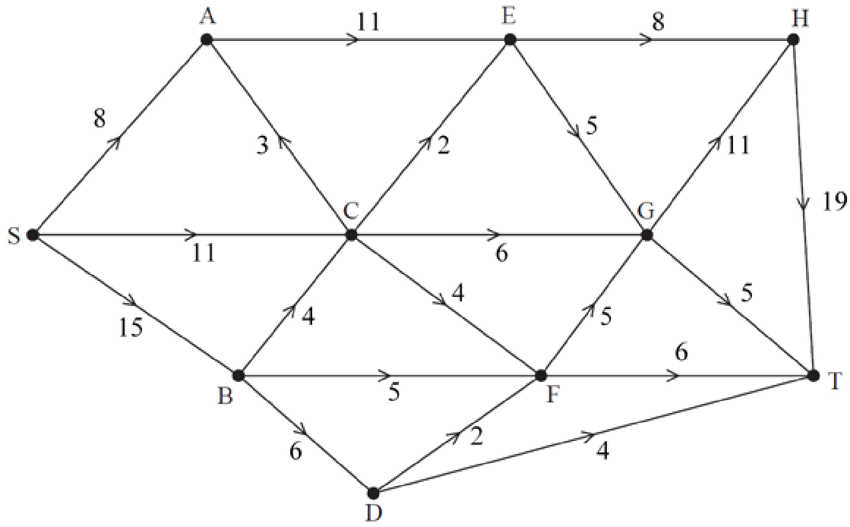
This question has a heavy problem solving and reasoning emphasis. As a result, it was found to be challenging by many students despite being a familiar area of the specification.
Part (a) and (b) are very standard questions and as such were the most successful for students. Nonetheless, the number of incorrect answers in (a) – particularly (a)(ii) was worrying at this level. Part (b) was almost always correct. Some responses which focussed on only the maximum flow into E or the maximum flow out of E rather than comparing the two values were insufficient for the mark here.

Part (c) was challenging and was often not attempted. Very few students correctly identified a cut of capacity 32 and deduced that a flow of 31 was not possible. Most students focussed instead on the flow into individual nodes or the flow at the source or sink and were unable to make progress.

Part (d) required a good understanding of flow in networks and required students to draw together their work in (a) and (c) to deduce the minimum and maximum flows. This was only very rarely achieved completely and indeed many students did not attempt this part of the question. However, a significant proportion of the students that did attempt part (d) were able to earn some marks for drawing consistent flows of 32 or 34 even if the justification for these flow values was missing. Unfortunately though, often the diagrams were left only partially complete or a number of errors were present such as multiple numbers on some (or all) of the arcs (even though, as in previous years, only a single number can be accepted on a flow diagram) and some students either left one arc blank or had an inconsistent flow pattern.

Mark Scheme

Question	Scheme	Marks	AOs
6 (a)(i) (ii)	$C_1 = 9 + 11 + 4 + 5 + 7 = 36$ $C_2 = 8 + 5 - 2 - 3 + 11 + 4 + 5 + 2 + 4 = 34$	1B1 2B1	1.1b 1.1b
		(2)	
(b)	If AE and CE were both full to capacity then 12 + 4 = 16 litres per second would flow though E but the maximum capacity of the two arcs out of E (EH and EG) is only 8 + 5 = 13 so AE and CE cannot both be full to capacity	1B1	2.4
		(1)	
(c)	The minimum flow through the arcs AE, CE, CG, FG, FT and DT (which provides a cut for the network) is 10 + 2 + 5 + 5 + 6 + 4 = 32 so a minimum of 32 must be flowing through the system so 31 is not possible.	1B1	2.4
		(1)	
(d)	Attempt to find a flow of 32 using the answer to (c)	1M1	3.4
	E.g. Minimum flow of 32 	1A1	1.1b
	Attempt to augment minimum flow and recognise that from (a) the maximum flow is less than or equal to 34	1B1	2.1

	E.g. Maximum flow of 34 	2B1	1.1b
		(4)	
			(8 marks)

Notes:

(a)(i) 1B1: CAO

(ii) 1B1: CAO

(b) 1B1: Calculates maximum capacity entering E and compares with maximum capacity leaving E. Concludes that maximum capacity into E exceeds maximum capacity out of E.

Condone statements such as ‘flow into E = 12 + 4 < 13 which is the flow out of E’.

(c) 1B1: valid reason why the flow in the network cannot be 31 litres per second.

Note: If smallest of C_1 and C_2 in a) is less than 31 then DO NOT allow this mark for deducing that ‘31 > {smallest of answers from a} hence, flow of 31 is not possible’.

(d) 1M1: Award this mark for either:

- Indicates ‘minimum flow > 31 so minimum flow could be 32’, OR
- Attempts to find flow of 32 in which: the sum of flows along arcs from S is 32 **and** 7 ≤ flow along SA ≤ 9, 8 ≤ flow along SC ≤ 11 **and** 13 ≤ flow along SB ≤ 17, OR
- Attempts to find flow of 32 in which: the sum of flows along arcs into0 T is 32 **and** flow along DT = 4, 6 ≤ flow along FT ≤ 8, 4 ≤ flow along GT ≤ 4 **and** 17 ≤ flow along HT ≤ 20
- Identifies both ‘min flow = 32’ **AND** ‘max flow = 34’

Note: Only need consider arcs incident to S for the second bullet point above, or arcs incident to T for the third bullet point. Flows along other arcs may be incorrect or missing.

1A1: CAO for consistent flow of 32. One number per arc. Check for consistency at each node.

1B1: States that maximum flow must be 34 and makes some reference to (smallest cut in) part a).

1B1: CAO for consistent flow of 34. One number per arc. Check for consistency at each node.

Student Response A

6.

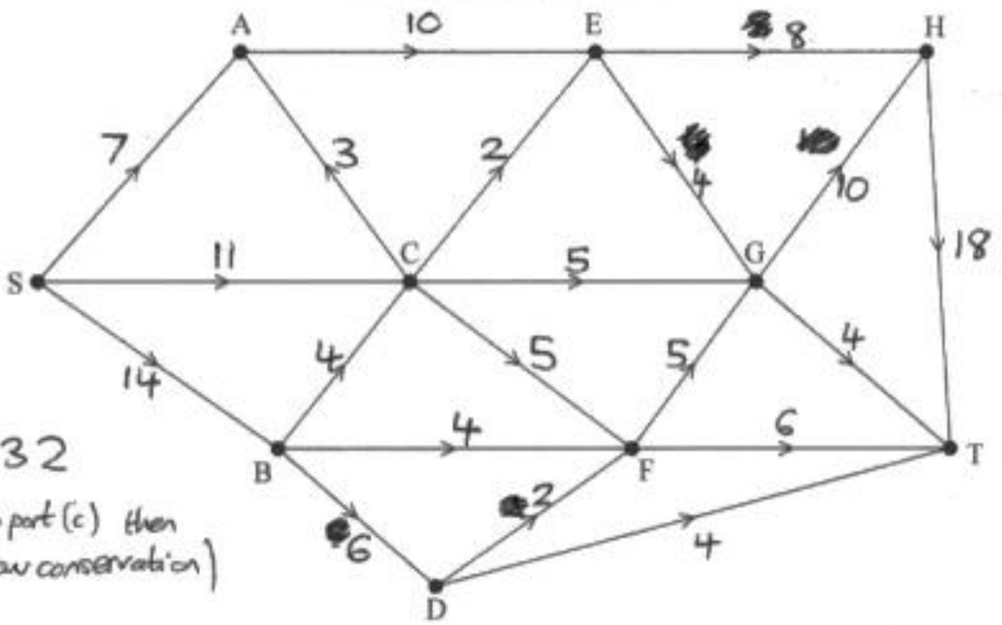
(a) (i) Capacity of cut C_1 = ~~[illegible]~~ 9 + 11 + 4 + 5 + 7 = 36

(ii) Capacity of cut C_2 = 8 + 5 − 2 − 3 + 11 + 4 + 5 + 2 + 4 = 34

6b) If both at upper capacities, 12 + 4 = 16 is flowing into E.
However the most that can come out of E is 8 + 5 = 13 16 > 13 so this isn't feasible

6c) The cut through ~~EH, GH, GT, FI, DT~~.
AE, CE, CG, FG, FT, DT
It must have at least 10 + 2 + 5 + 5 + 6 + 4 = 32 flowing through it so 31 isn't possible

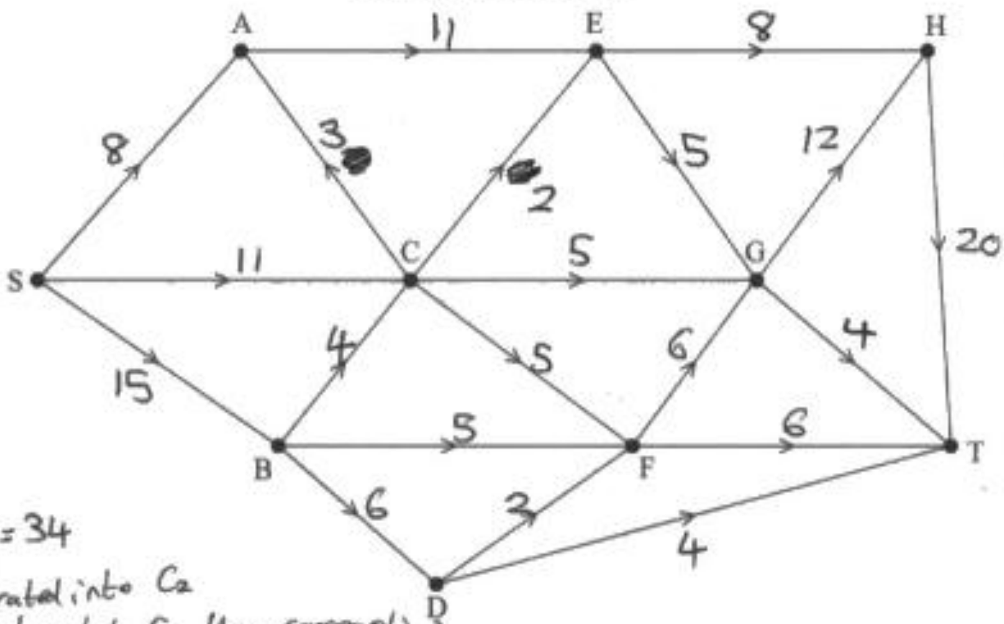
Minimum feasible flow



Flow = 32

(Cut from part (c) then flow conservation)

Maximum feasible flow



Flow = 34

(Saturated into C_2 ~~Empty~~ out of C_2 then conservation)

Lower caps

8/8

Examiner Comments

A rare completely correct response for this question.
In part (a), the correct value for each cut is seen.

In part (b), a correct comparison of the maximum capacity entering E and the maximum capacity leaving E is seen.

In part (c), a correct consideration of the value of the minimum cut through AE, CE, CG, FG, FT and DT together with correct reasoning.

In part (d), a consistent flow of 32 in the first network earns the first two marks. A sufficient argument for why the maximum flow must be 34 together with a consistent flow of 34 drawn on the second network earns the final two marks. Note, a reference to cut 2 in part (a) constitutes a sufficient argument for the maximum flow.

Student Response B

6.

(a) (i) Capacity of cut C_1 = 4 + 11 + 4 + 5 + 7 = 36.

(ii) Capacity of cut C_2 = 9 + 5 + 2 + 3 + 11 + 4 + 5 + 2 + 4 = 34.

b) The max flow into E must equal max flow out of E.
Max flow out of E: 8 + 5 = 13.
Since 13 < 16 (= 12 + 4), AE and CE cannot be at their maximum capacity simultaneously.

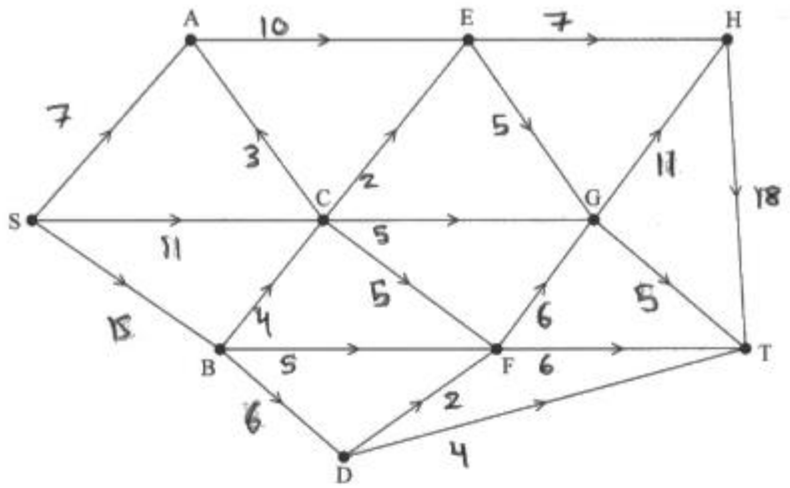
d). Minimum flow applying the conservation and feasibility condition at every vertex gives us this solution.

Min flow = 33.

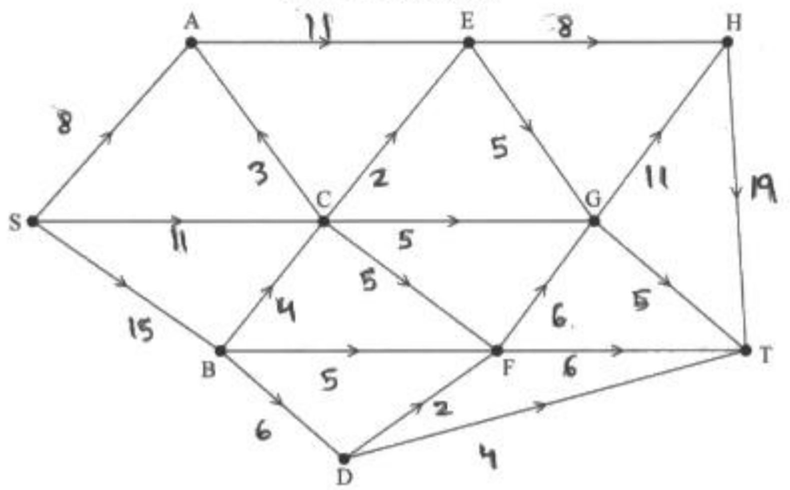
Max flow = 34.
~~By mi~~
cut C_1 = 34.
∴ By min cut – max flow theorem
34 = 34.
∴ flow is maximal.

c) A minimum of 7 must flow from SA to ensure AD ≥ 10,
SC = 11, so that flow in C = flow out of C
and SB = 15 so flow in B = flow out of B.
15 + 11 + 7 = 33
33 > 31 so a flow of 31 is not possible.

Minimum feasible flow



Maximum feasible flow



5/8

Examiner Comments

In part (a), the correct value for each cut is seen.

In part (b), a correct comparison of the maximum capacity entering E and the maximum capacity leaving E is seen.

In part (c), no valid argument is seen.

In part (d), no valid attempt to find a flow of 32 in the first diagram (flows along arcs incident to S sum to 33 and flows along arcs incident to T sum to 33). Nor is there any statement that the minimum flow is 32. In fact, this candidate believes that the minimum flow is 32. For the maximum flow, a statement that the maximum flow must be 34 is seen together with a reference to the cut of 34 in part (a). We will give benefit of doubt that the reference to C_1 is a slip and refers to C_2. A consistent flow of 34 on the second network earns the final mark.

Student Response C

6.

(a) (i) Capacity of cut C_1 = $9 + 11 + 4 + 5 + 7 = 36$

(ii) Capacity of cut C_2 = $8 + 5 + 11 + 4 + 5 + 2 + 4 = 39$

b) ~~The max capacity of CH these to~~

AE and CE both enter EH, EG max capacity of EH = 8 EG = 5

$12 + 4 = 16$ $8 + 5 = 13$

~~+0=8 ∴ AE and CE cannot~~

$16 > 13$

∴ AE and CE cannot be full.

c) Min value to T is 31 ∴ 31 can't be flow

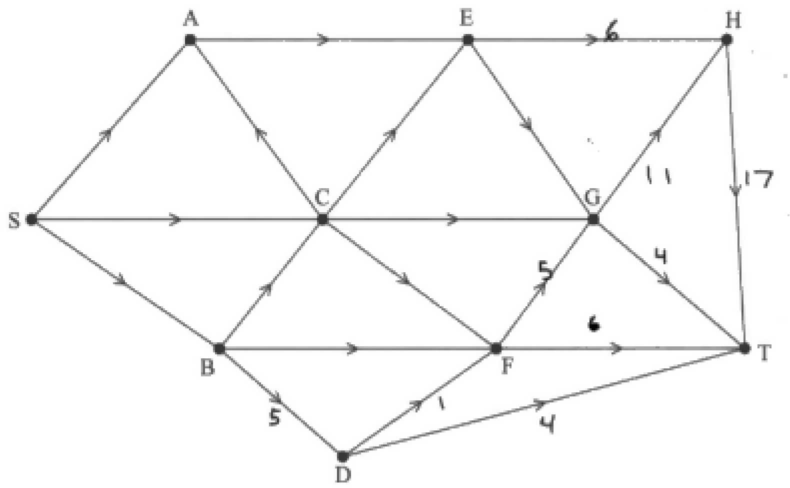
$T = 17 + 4 + 6 + 8 = 31$

$GH + GT = 14$

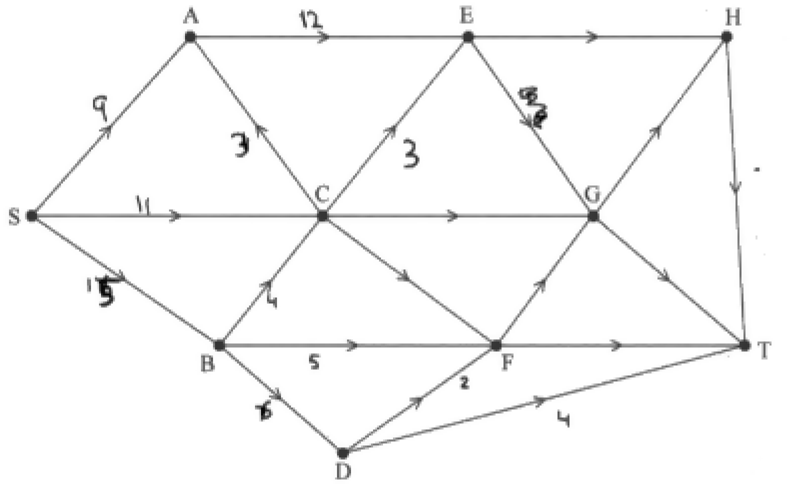
$EG + CG + FG = 13$

$13 < 14$ ∴ ~~14 can't flow through GH to~~ 14 cant flow through GH and GT ∴ 31 cant flow.

Minimum feasible flow



Maximum feasible flow



2/8

Examiner Comments

In part (a), the capacity of cut 1 is correct. The capacity of cut 2 is incorrect.

In part (b), a valid comparison of the maximum capacity entering E and the maximum capacity leaving E is seen.

In part (c), no valid argument is seen.

In part (d), there is no statement that the minimum flow is 32, nor is there any valid attempt to find a minimum flow of 32. The arcs incident to S are blank and the flows on the arcs incident to T sum to 31. For the maximum flow, there is no statement which suggests that maximum flow must be 34 and the final mark can only be earned for a consistent flow of 34. As this attempt has arcs without flows assigned to them, this mark has not been achieved.

Exemplar Question 7

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7. Aisha is deciding whether or not to play a game.

The game involves rolling three fair six-sided dice, which have faces numbered from 1 to 6

If the total score on the three dice is 16 or more then she wins a prize. If the total score is 15 or
less then she loses and will have to pay the person running the game £3

(*a*) Given that the prize is £15

(i) draw a decision tree to model Aisha's possible decisions and the possible outcomes

(ii) determine Aisha's optimal EMV and state the optimal strategy indicated by the decision tree.

(6)

The utility function of the game to Aisha is $\mathrm{u}(m) = 1 - \mathrm{e}^{-\frac{m}{500}}$ where m is the amount of money that Aisha has available. Given that Aisha has exactly £3 and that the prize is now £x

(*b*) find the expected utility to Aisha of playing the game in the form

$$\frac{a}{b}\left(1-\mathrm{e}^{-\frac{(x-c)}{500}}\right)$$

where a, b and c are integers to be found.

(2)

Aisha decides to use the expected utilities to determine whether she should play the game or not.

(*c*) Find the minimum prize for which Aisha would consider playing the game.

(4)

(Total of Question 7 is 12 marks)

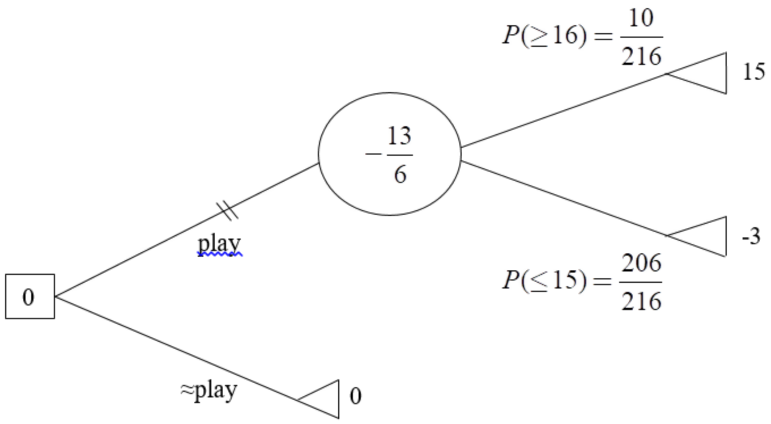
Mean Score 4.2 out of 12

Examiner Comments

This question was found to be the most challenging question on the paper. The majority of students demonstrated an understanding of decision trees to the extent that they could construct an appropriate tree starting with a decision node, including a chance node and concluding with three end pay-offs. An unexpected stumbling block presented in the form of the calculation of the probability of scoring 16 or more on three dice. This ultimately lost some marks but students were nonetheless able to earn later follow-through marks for the application of a correct method. In the majority of cases, the chance node EMV for playing the game was correctly calculated albeit following on from a, sometimes incorrect, stated probability of success. Unfortunately, some students did not fully complete their decision tree, and lost marks for omitting one or more of the key components including ‘play’ and ‘not play’ labels, pay-offs at the tree ends, double lines through the inferior option or ‘0’ in the decision node. In (a)(i), students were asked to determine the optimal EMV and state the optimal strategy. However, often students lost this straightforward mark for stating only the EMV of playing the game rather than the optimal EMV of 0. Some students only stated the EMVs and did not conclude that the optimal strategy was ‘Not Play’.

Parts (b) and (c) provided a challenge for the majority of candidates and many did not make much progress here. Of those students that did make some headway, some spread of marks was observed. It seems that students are unfamiliar with the application of the utility function and problems most commonly arose from an incorrect substitution of $m = 3$ into the function rather than the correct $m = x + 3$; demonstrating a lack of understanding that the utility function is defined in terms of the total money Aisha has, rather than her winnings or losses. Furthermore, some candidates attempted to evaluate the utility function alone rather than calculating the *expected* utility as requested: thus, omitting the probability multiple(s). In (c), only a minority of students were able to make progress. Most seemed unclear that they should be comparing the expected utility of playing with the expected utility of not playing. Instead some students simply set the expected utility of playing greater than zero. Others wrote down a correct inequality but seemed unable to make progress in solving the inequality for x which is, perhaps, surprising at this level. Unfortunately, a handful of students who were able to navigate the algebra here slipped at the final hurdle by interpreting their solution for the minimum prize by incorrectly stating $x >$ £66.18 rather than $x =$ £66.18 or $x \geq$ £66.18 (which was condoned).

Mark Scheme

Question	Scheme	Marks	AOs
7(a)(i)		1M1 1B1 1A1 2M1 2A1	3.3 1.1b 1.1b 3.4 1.1b
(ii)	Optimum EMV is (£)0 and Aisha should not play the game	1B1ft	3.2a
		(6)	
(b)	Expected utility is $\frac{10}{216}\left(1 - e^{\frac{-(x+3)}{500}}\right)$	1M1 1A1	3.4 2.2a
		(2)	
(c)	If Aisha doesn't play she will have 3 $\Rightarrow 1 - e^{-\frac{3}{500}}$	1B1	3.1a
	For the prize to be worthwhile $\frac{10}{216}\left(1 - e^{\frac{-(x+3)}{500}}\right) > 1 - e^{\frac{-3}{500}}$	1M1	1.1b
	Correct order of operations and use of logs to find x	2dM1	1.1b
	$x > 66.178...$ so (minimum prize amount should be) £66.18	1A1	3.2a
		(4)	
			(12 marks)

Notes:

(a)(i) 1M1: tree diagram with at least three end pay-offs, one (rectangular) decision node and one (circular) chance node

1B1: Correct probability for rolling 16 or more (accept equivalent fractions)

1A1: Correct structure of tree diagram with each arc labelled correctly. The following must be seen for this mark:

- Probabilities on branches leading from chance node. Probabilities may not be correct but must sum to 1

- 'Play' and 'Not Play' (o.e.) labelled correctly. (If one branch is labelled correctly then condone lack of label on the other branch).
- Pay offs: 15, –3, 0 placed at tree ends (do not condone '3' for '–3')

2M1: Correct chance node. ft from their probabilities: $15p - 3(1-p)$ where p is their probability for rolling 16 or more. If given as a decimal allow for correct (or truncated) to 2dp

2A1: CAO (must be exact) for chance and decision node including double line through inferior option

Note must see 0 at the decision node for this mark.

(ii) 1B1ft: correct optimal EMV (clearly indicated) **and** analysis in context.

So: If *their* EMV for playing game < 0 then 'Optimum EMV = *0*, together with corresponding conclusion: 'Aisha should play' o.e. would earn B1.

Whereas, if *their* EMV for playing game > 0 then 'Optimum EMV = *their* EMV of playing game' together with corresponding conclusion: 'Aisha should not play' o.e. would earn B1.

BUT If their EMV for playing game does not ft from their probabilities then B0.

(b) 1M1: $p\left(1 - e^{\frac{-(x+3)}{500}}\right)$ with their p from (a)

Note: isw after a correct expression for M mark. May see:

$p\left(1 - e^{\frac{-(x+3)}{500}}\right) + (1-p)\left(1 - e^{\frac{-(-3+3)}{500}}\right)$ o.e.

1A1: CAO for expected utility. Must be of the correct form as is specified in the question.

So: $\frac{10}{216}\left(1 - e^{\frac{-(x+3)}{500}}\right)$ but allow fractions equivalent to $\frac{10}{216}$

(c) 1B1: CAO. May be in decimal form: 0.00598…. May be embedded in inequality/equation.

Note: Look out for missing minus sign in the power if stated exactly.

Note: Do not award this mark if seen in only part (b).

1M1: Sets up an inequality or equation with their expected utility of playing the game (from part b).

2dM1: Solving for x (dependent on previous M mark). Requires correct order of operations and log work.

For example,

May see: $e^{\frac{-(x+3)}{500}} \square 1 - 'p'\left(1 - e^{\frac{-(3)}{500}}\right)$ o.e. [Isolates exponential term]

Or $e^{-\frac{(x+3)}{500}} \square 0.870\ldots$

Followed by: $\frac{(x+3)}{500} \square -ln\left(1 - 'p'\left(1 - e^{\frac{-(3)}{500}}\right)\right)$ o.e. [Applies logs correctly]

Or $\frac{(x+3)}{500} \square$ 0.138…

Where □ is any inequality or equals sign.

Note: May not get to $x = \ldots$

Note: Can be implied by correct answer

1A1: CAO with units

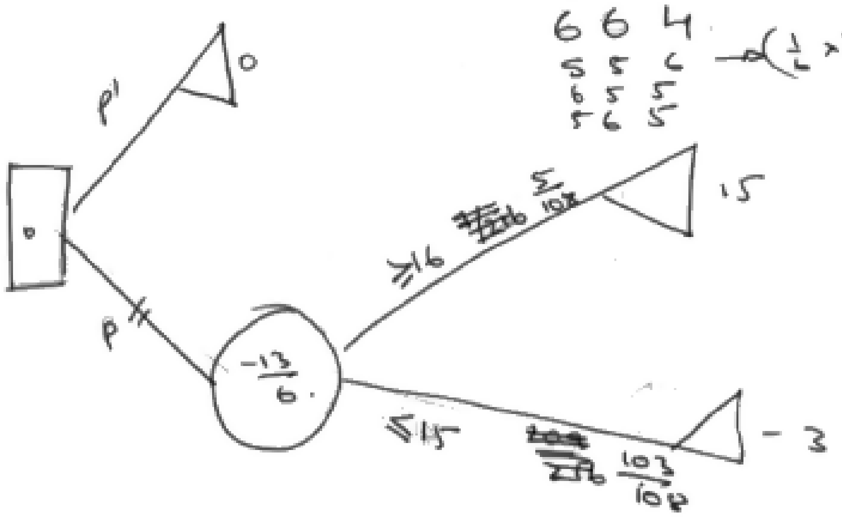
Note: For this mark, condone '£66.18' or 'x = £66.18' or '$x \geq$ £66.18', but not '$x >$ £66.18'.

Student Response A

7.

6 6 6 → $\frac{1}{6\times6\times6}$

5 6 6 $\left(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\right)\times3$
6 5 6
6 6 5 → $\left(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\right)\times3$
6 4 6
4 6 6
6 6 4 → $\left(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\right)\times3$
5 5 6
6 5 5
5 6 5



(ii) The optimal EMV = 0
it is not worth playing the game.
as $-\frac{13}{6} < 0$ //

b). if she wins $m =$ ~~15+3 = 18~~ $3+x = x+3$
if she loses $m =$ ~~3−3 = 0~~ 3+x−... 0.
if she does not play $m = 3$.

expected utility $= \frac{5}{108}\left(1-e^{\frac{-x-3}{500}}\right) + \frac{103}{108}\left(1-e^{\frac{-3+x}{500}}\right)$.

$= \frac{5}{108}\left(1-e^{\frac{-x}{500}}\cdot e^{\frac{-3}{500}}\right) + \frac{103}{108}\left(1-e^{\frac{-3}{500}}\cdot e^{\frac{x}{500}}\right)$.

$\frac{1}{108}\left(5\left(1-e^{\frac{-x}{500}}\right) + 103\, e^{\frac{x}{500}}\right)$.

$\frac{1}{108}e^{\frac{3}{500}}$ $\quad 1-e^{\frac{0}{500}} = 0$

b expected utility = $\frac{5}{108}\left(1 - e^{\frac{-(x+3)}{500}}\right) - 0$

c) $u(3) = 5.92035946 \times 10^{-3}$.

$\frac{5}{108}\left(1 - e^{\frac{-(x+3)}{500}}\right) + \frac{103}{108}\left(1 - e^{\frac{0}{500}}\right)$

$= 0$.

$\frac{5}{108}\left(1 - e^{\frac{-(x+3)}{500}}\right) > 5.920359 \times 10^{-3}$.

$1 - e^{\frac{-(x+3)}{500}} > 0.12922\ldots$

$0.870788 > e^{\frac{-(x+3)}{500}}$.

$\ln 0.870788 = \frac{-(x+3)}{500}$.

$-0.1383567 = \frac{-(x+3)}{500}$

$69.178 < x + 3$.

$x > £66.18$

$x = £66.18$.

12/12

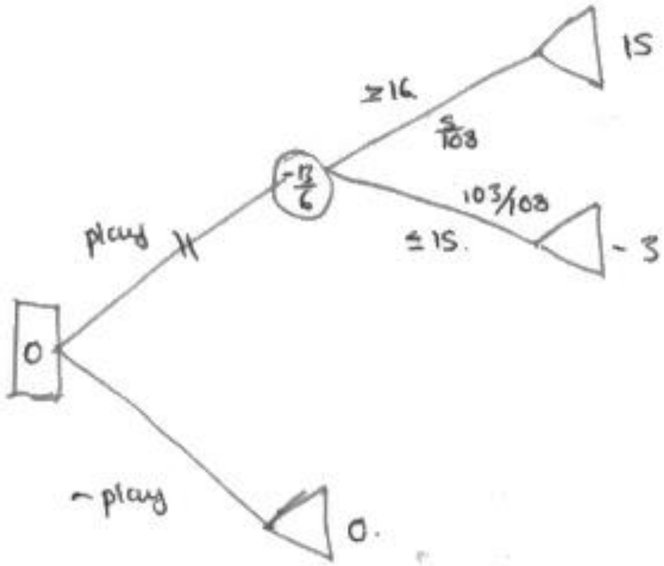
Examiner Comments

A rare, full-mark response.
In part (a), the correct decision tree structure is seen with correct probability for rolling 16 or more. All elements of the tree are present and correct. The correct value of the EMV is written in the chance node. The optimal strategy and optimal EMV are stated.

In part (b), the expected utility is stated correctly and given in the required form.

In part (c), the expected utility of not playing the game is found correctly (in decimal form) and an inequality is set up between the expected utility of playing and the expected utility of not playing. The inequality is solved correctly to obtain the correct value for *x* which is given in context with units.

Student Response B



i) 6 + 6 + 4/5/6. 5 + 6 + 5/6. 4 + 6 + 6.
6 + 5 + 5/6. 5 + 5 + 6.
6 + 4 + 6.

ii) optimal EMV: $\frac{5}{108}(15) + \frac{103}{108}(-3) = -13/6$.
Since $-13/6 < 0$, the optimal strategy is to not play the game.

b) $\frac{a}{b}$ $\frac{a}{b}\left(1 - e^{-\frac{(x+3)}{500}}\right) + \frac{a}{b}$

$\frac{5}{108}\left(1 - e^{-\frac{0}{500}}\right) + \left(1 - e^{-\frac{(x+3)}{500}}\right) \cdot \frac{103}{108}$

$= \frac{103}{108}\left(1 - e^{-\frac{(x+3)}{500}}\right)$
$a = 103, b = 108; c = 3$.

7c) $\frac{103}{108}\left(1 - e^{-\left(\frac{x+3}{500}\right)}\right) > \cancel{1-e}\ 1 - e^{-\frac{3}{500}}$

$\frac{103}{108} - \frac{103}{108}e^{-\left(\frac{x+3}{500}\right)} > 1 - e^{-\frac{3}{500}}$

$\cancel{\rightarrow}\ \frac{103}{108}e^{-\left(\frac{x+3}{500}\right)} < \frac{103}{108} - 1 + e^{-\frac{3}{500}}$

$\frac{103}{108}e^{-\left(\frac{x+3}{500}\right)} < 0.9477\ldots$

$e^{-\left(\frac{x+3}{500}\right)} < 0.9937\ldots$

$-\frac{(x+3)}{500} < -6.292\ldots \times 10^{-3}$

$(x+3) > 3.146\ldots$

$x > 0.146\ldots$

$\cancel{\rightarrow}$ for $\pounds 0.15$.

8/12

Examiner Comments

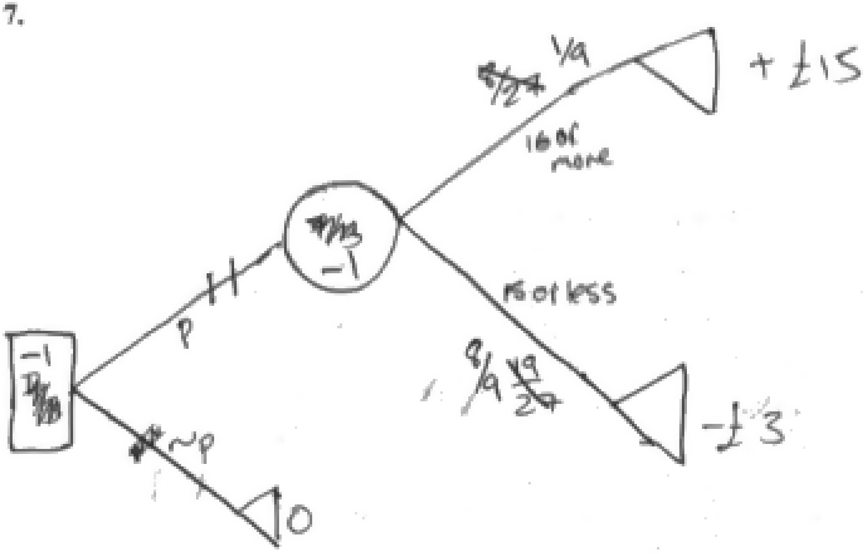
In part (a), the correct decision tree structure is seen with the correct probability for rolling 16 or more. All elements of the tree are present and correct. The correct value of the EMV is written in the chance node. The optimal strategy and optimal EMV are stated.

In part (b), the expression for the expected utility for playing the game is not stated correctly – the ‘p’ and ‘1 – p’ multipliers are the wrong way around.

In part (c), the expected utility of not playing the game is found correctly and an inequality is set up between the expected utility of playing (follow through from part (b)) and the expected utility of not playing. The inequality is solved correctly using correct log work and order of operations. The final value for *x* is incorrect due to earlier errors.

Student Response C

7.



Possible out comes = $6^3 = 216$

To get 16 or more : (6,6,6) (6,6,5) (6,6,4)
(6,5,5) = ~~4~~ 3 4
~~(5,5,6)~~

~~$4^3 = 64$~~

$4 \times 6 = 24$
ways to get 16
or more

1 2 3
2 1 3 2
2 1 3
2 3 2
3 1 2
3 2 1

Her EMV is ~~-£1~~ −£1 every game so
She should not play.
Her Optimal Strategy is not to play.

$U(m) = 1 - e^{-\frac{m}{500}}$

$m = x + c$

3/12

Examiner Comments

In part (a), a decision tree with three end pay-offs, one decision node and one chance node is seen. The probability for rolling 16 or more is incorrect. The structure of the tree diagram is correct however and contains all relevant components. The EMV in the play chance node is correct following through from the probability of rolling 16 or more. The optimal EMV is not explicitly stated.

No further progress made for parts (b) and (c).