Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

June 2019 – Exemplar

Student answers with examiner comments

First teaching from September 2017
First certification from June 2018

Contents

About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced GCE in Mathematics specification (9MA0). The booklet looks at questions from the A Level Mathematics June 2019 Examination Papers. It shows student responses to questions, and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to all questions from the June 2019 Examination Papers. Following each question, you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student Response

4a. $3{\cdot}28 - 2{\cdot}35 = 0{\cdot}93$ change over 3 year
$0{\cdot}31$ every year
$H = 0{\cdot}31t +$ original height

b. Supports the same as doesn't make the growth exponential, its still linear and gives a value for original height.

1/5

Examiner Comments: (a) M1 M0 A0 (b) B0 B0

In part (a) this candidate finds the rate of growth per year by using both pieces of information. The second M mark cannot be awarded as there is no attempt to find the intercept or original height.

There is no merit in their answer for part (b).

Examiner commentary on the student response

Marks awarded for the question

A Level Mathematics Paper 1 (Pure) 9MA0 01

Exemplar Question 1

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1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

Mean Score 2.6 out of 3

Examiner Comments:

This proved to be a very suitable start to the paper. Most candidates used the factor theorem and produced an equation in a by setting $f(-3) = 0$. Most could then solve their linear equation in a to find its value. Candidates who used this method generally went on to score full marks. Common mistakes tended to be arithmetic or sign errors, for example not cubing (-3) accurately in the first term.

Other methods were seen but were generally less efficient and not as effective. Those candidates that started with $(x+3)(ax^2+bx+c)$ and equated coefficients were generally successful, however those candidates that tried to solve this problem using long division generally made errors and did not score highly. It was rare to see students scoring zero on this question.

Mark Scheme

Question	Scheme	Marks	AOs
1	Attempts $f(-3) = 3\times(-3)^3 + 2a\times(-3)^2 - 4\times -3 + 5a = 0$	M1	3.1a
	Solves linear equation $23a = 69 \Rightarrow a = ...$	M1	1.1b
	$a = 3$ cso	A1	1.1b
		(3)	
			(3 marks)

Notes

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3) = 0$ leading to a correct equation in a.

Missing brackets may be recovered.

Other methods may be seen but they are more demanding

If division is attempted must produce **a correct equation** in a similar way to the $f(-3) = 0$ method

$$\begin{array}{r l}
 & 3x^2 + (2a-9)x + 23 - 6a \\
x+3 \,\big) & 3x^3 + 2ax^2 - 4x + 5a \\
 & \underline{3x^3 + 9x^2} \\
 & \quad (2a-9)x^2 \quad - 4x \\
 & \quad \underline{(2a-9)x^2 \quad + (6a-27)x} \\
 & \qquad\qquad (23-6a)x + 5a \\
 & \qquad\qquad (23-6a)x + 69 - 18a
\end{array}$$

So accept $5a = 69 - 18a$ or equivalent, where it implies that the remainder will be 0

You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a - 27 + \frac{5a}{3} = -4$$

M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm 3) = 0$. Allow for a linear equation in a leading to $a =$…. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{\big)3x^3 + 2ax^2 - 4x + 5a}$ with quotient $3x^2...$ followed by a remainder in a set $= 0 \Rightarrow a = ...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a + 12 - 81$oe

A1: $a = 3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Student Response A

$$0 = 3(3)^3 + 2a(3)^2 - 4(3) + 5a$$
$$0 = 81 + 18a - 12 + 5a$$
$$0 = 69 + 23a$$
$$-69 = 23a$$
$$-3 = a$$

1/3

Examiner Comments: M0 M1 A0

This candidate is awarded M0 as they set $\mathrm{f}(3)$ rather than $\mathrm{f}(-3) = 0$. The next mark is scored however as they solve a linear equation in a. The accuracy mark cannot be scored due to the earlier error.

Student Response B

$$f(x) = x = -3$$

$$f(x+3) = 3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a$$

$$0 = -81 + 18a + 12 + 5a$$

$$0 = -81 + 12 + 18a + 5a$$

$$0 = -69 - 13a$$

$$\frac{+69}{-23} = a$$

$$+3 = a$$

2/3

Examiner Comments: M1 M1 A0

The first M1 is awarded on line 3 where we see an attempt to set $f(-3) = 0$. The next mark is scored when they solve a linear equation in a. The accuracy mark cannot be scored due to errors on line 5 and 6 of this solution.

Student Response C

$$p(-3) = 3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a$$

$$0 = 3(-3)^3 + 2a(-3)^2 + 12 + 5a$$

$$= -81 + 18a + 12 + 5a$$

$$0 = -69 + 23a$$

$$69 = 23a$$

$$\underline{a = 3}$$

3/3

Examiner Comments: M1 M1 A1

A completely correct solution.

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Exemplar Question 2

2.

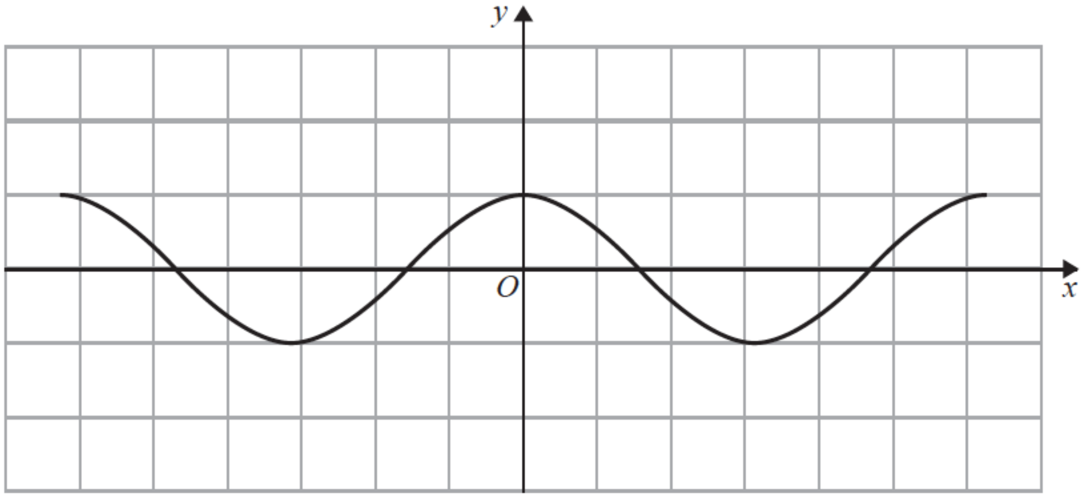


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Copy and use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

(Total for Question 2 is 5 marks)

Mean Score: 2.5 out of 5

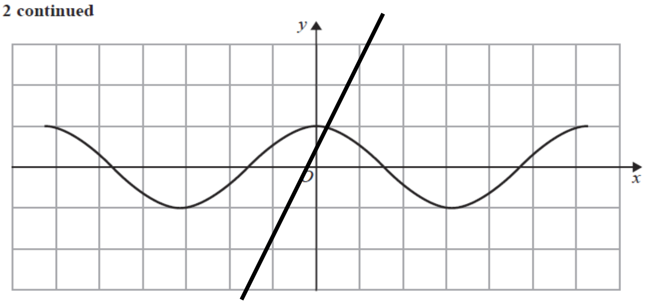
Examiner Comments:

This question proved to be more challenging for a number of candidates although many were still able to score full marks.

For part (a) it was important for candidates to pay attention to the scale of the graph. Most candidates who did draw a straight-line graph were able to make the link between one point of intersection and one real root to the equation, however, a significant minority of these were unable to construct the correct straight-line graph with enough accuracy to gain both marks for this part of the question.

Part (b) almost all candidates were able to write down the small angle approximation for $\cos x$ even if they were unable to use it in order to answer the question. Most candidates did substitute correctly into the equation in order to gain a quadratic equation, although some did contain a mixture of variables. Sign errors sometimes occurred at this stage and the weakest candidates were unable to deal correctly with resulting fractions in order to simplify the equation. Many candidates appear to be using calculators in order to solve quadratics and in this question, this usually resulted in full marks. The final mark was lost by some candidates who correctly solved the quadratic equation but gave both roots as their final answer.

Mark Scheme

Question	Scheme	Marks	AOs
2(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2 + \sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

Notes

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"
Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$
OR gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator.
Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2 + \sqrt{5}$ or awrt 0.236.
Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

Student Response A

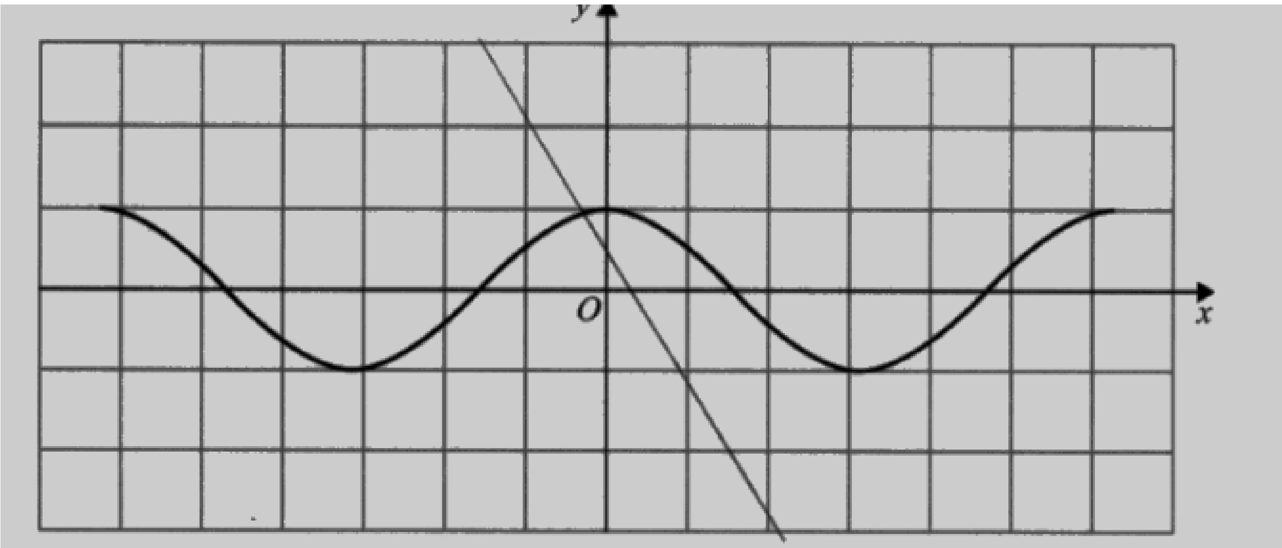


Diagram 1

a) ~~Try~~ $y = \cos x - 2x - \frac{1}{2} = 0$ only intersects $y = \cos x$ once therefore there is only 1 real root.

b) $\cos x - 2x - \frac{1}{2} = \cos x$ when $\cos x$ is small $\approx$

$1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 1 - \frac{x^2}{2}$

$\frac{1}{2} - \frac{x^2}{2} - 2x = 1 - \frac{x^2}{2}$

1/5

Examiner Comments: (a) B0 B1 (b) M0 M0 A0

In part (a) the first mark is not scored as it requires a straight line with equation $y = 2x + \frac{1}{2}$ to be drawn on Diagram 1. The second mark is awarded however, as an allowable line is drawn (the intercept is $\frac{1}{2}$) and a reason (intersects once) is given.

No marks are scored in part (b). Although $\cos x = 1 - \frac{x^2}{2}$ is seen, it is not used correctly within the given equation.

Student Response B

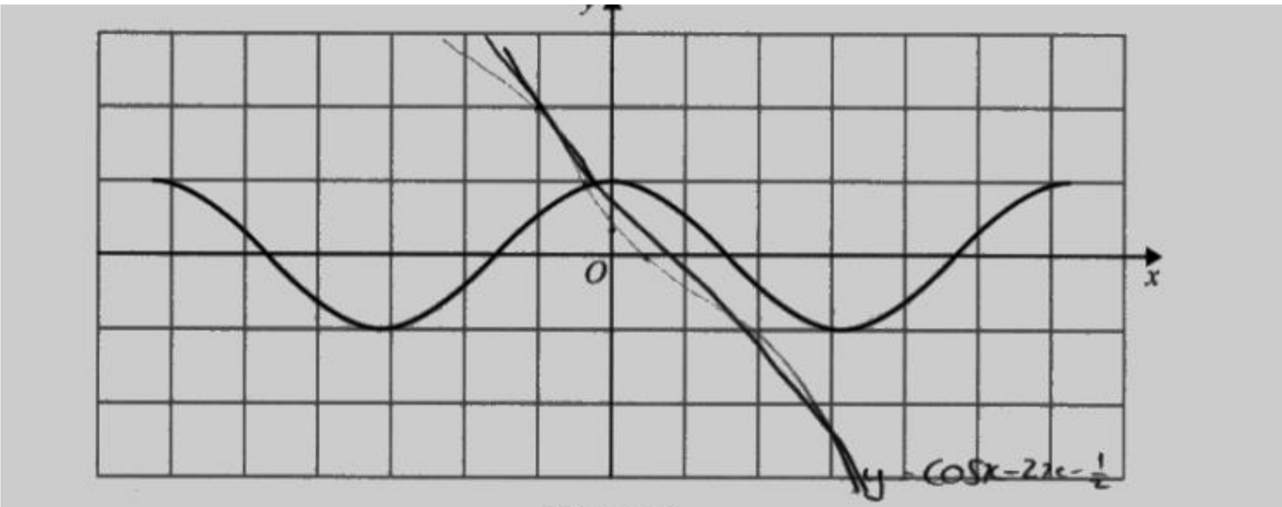


Diagram 1

a. The curves intersect only once

b. $\cos x \approx 1 - \frac{\theta^2}{2}$

$1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$

$2 - x^2 - 4x - 1 = 0$

$-x^2 - 4x + 1 = 0$

$x^2 + 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$

$x = -2 \pm \sqrt{5}$

$x < 0$

$x = 0.236$

3/5

Examiner Comments: (a) B0 B0 (b) M1 M1 A1

For part (a) no marks are awarded as a straight line must be drawn for both marks. This is not just a slip as the reason mentions "curves" intersecting.

Part (b) is fully correct with 0.236 being seen to be the chosen solution of $x^2 + 4x - 1 = 0$

Student Response C

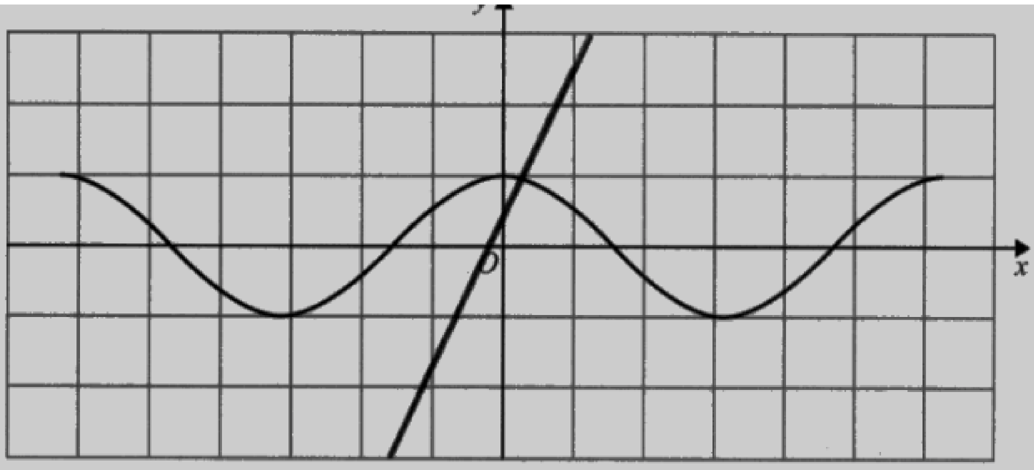


Diagram 1

(3)

a) $\cos x - 2x - \frac{1}{2} = 0.$

$\cancel{\cos x = 2x + \frac{1}{2}}$

$\cos x = 2x + \frac{1}{2}$

They only cross at one point.

b) $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$\cos x - 2x - \frac{1}{2} = 0$

$1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$

$-\frac{x^2}{2} - 2x + \frac{1}{2} = 0$

$x = 0.236, -4.236$

4/5

Examiner Comments: (a) B1 B1 (b) M1 M1 A0

In part (a) both marks are awarded as the graph is correct and a minimal reason is given.

In part (b) the solution loses the accuracy mark as the candidate is required to choose 0.236 as the only root of the equation.

Exemplar Question 3

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3.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$

(1)

(Total for Question 3 is 5 marks)

Mean Score: 2.8 out of 5

Examiner Comments:

This was an accessible question for most students, but the second half of part (a) did discriminate at all levels.

In part (a) students generally recognised the need to use the quotient or product rule and were successful at differentiating for the first 2 marks. The most common mistake was forgetting to square the denominator, with incorrect order of terms in the numerator being less common. Students should also be reminded to quote a formula before attempting to use it, as they may still gain method marks for this, even if there are slips in its application. Omission of brackets in terms in the numerator was also common, but often recovered in working. However, it would benefit students to be careful in the use of brackets.
Having achieved a derivative, attempting to simplify to the required form proved difficult for many students. Many did not realise that they could factorise the $(x+1)$ out early and instead expanded fully, simplified then factorised again to cancel in the last step. This wasted valuable time and it was during this process that errors usually occurred.
The alternative method of simplifying the fraction before differentiating was less common, with usually poor attempts at partial fractions being applied leading to error. Those who successfully simplified the fraction almost always went to achieve the correct answer.

Nearly all students who were correct in part (a) achieved the mark for part (b), including many who had failed to cancel the $(x+1)$. Indeed, there were many cases where a correct answer to part (b) was seen following incorrect work in (a), presumably due to the use of a calculator to plot and inspect the graph.

Mark Scheme

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\frac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\frac{dy}{dx}=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\frac{dy}{dx}=\frac{(x+1)\times(10x+10)-(5x^2+10x)\times2}{(x+1)^3}=\frac{A}{(x+1)^3}$	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ but follow through on their $\frac{dy}{dx}=\frac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
		(5 marks)	

Notes

(a)

M1: Attempts to use a correct rule to differentiate E.g.: Use of quotient (& chain) rules on

$$y=\frac{5x^2+10x}{(x+1)^2}$$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

A1: A correct (unsimplified) answer

E.g. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times-2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

You may see candidates expanding terms in the numerator.

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Student Response A

(1)

a) $u = 5x^2 + 10x \qquad v = (x+1)^2$

$u' = 10x + 10 \qquad v' = 2(x+1)$

$$\frac{dy}{dx} = \frac{(10x+10)(x+1)^2 - (5x^2+10)(2x+2)}{(x+1)^4}$$

$$= \frac{(10x+10)(x^2+2x+1) - (10x^3+10x^2+20x+20)}{(x+1)^4}$$

$$= \frac{\cancel{10x^3} + 20x^2 + 10x \cancel{+10x^2} \cancel{+20x} + 10 \cancel{-10x^3} \cancel{-10x^2} \cancel{-20x} - 20}{(x+1)^4}$$

$$\cancel{= \frac{20x^2+10x+20}{(x+1)^4}}$$

$$= \frac{10x^3 - 10x^3 + 20x^2 + 10x^2 - 10x^2 + 10x + 20x - 20x + 10 - 20}{(x+1)^4}$$

$$= \frac{20x^2 + 10x - 10}{(x+1)^4}$$

1/5

Examiner Comments: (a) M1 A0 M0 A0 (b) B0

In part (a) there is an allowable attempt at the quotient rule. A slip in writing $5x^2 + 10x$ as $5x^2 + 10$ means that the accuracy mark is not scored. As a result, it would be difficult to proceed to the form given in the question and so no more marks are scored.

Part (b) is not attempted.

Student Response B

$u = 5x^2 + 10x$ $\quad v = (x+1)^2$

$\frac{du}{dx} = 10x + 10$ $\quad \frac{dv}{dx} = 2(x+1) \times 1 = 2(x+1)$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)^2(10x+10) - (5x^2+10x)(2(x+1))}{((x+1)^2)(x+1)^2}$$

$(x+1)(x+1)$

$x^2 + x + x + 1$

$5x^3$

$(x^2+2x+1)(10x+10)$

$10x^3 + 10x^2 + 20x^2$ ~~$10x$~~

$+ 20x + 10x + 10$

$$\frac{[(x^2+2x+1)(10x+10)] - [(5x^2+10x)(2x+2)]}{(x+1)^2(x+1)^2}$$

$[(x+1)^2(10x+1)] - [(5x^2+10x)(2x+2)]$

$(x^2+2x+1)(10x+10)$

$10x^3 + 10x^2 + 20x^2 + 20x + 10x + 10$

$$\frac{(10x^3 + 30x^2 + 30x + 10) - (10x^3 + 10x^2 + 20x^2 + 20x)}{(x+1)^2(x+1)^2} \quad ((x+1)^2)^2$$

$$= \frac{10x+10}{(x+1)^4}$$

b) $\frac{dy}{dx} < 0$

$\frac{10x+10}{(x+1)^4} < 0$

$10x + 10 < 0$

$10x < -10$

$x < -1$

3/5

Examiner Comments: (a) M1 A1 M0 A0 (b) B1

Part (a) shown was typical of many responses to this question. The candidate clearly knows the quotient rule and can apply it accurately in this question. Unfortunately, they fail to spot the factor of $(x+1)$ and waste much time and energy in an attempt to simplify their fraction. They never reach the required form and so fail to score the last two marks.

Part (b) is fully correct.

Student Response C

(a) $y = (5x^2+10x)(x+1)^{-2}$

$\frac{dy}{dx}$ $\quad u' = 10x+10 \quad v' = -2(x+1)^{-3}$

$$\therefore \frac{dy}{dx} = -\frac{2(5x^2+10x)}{(x+1)^3} + \frac{(10x+10)}{(x+1)^2}$$

$$= \frac{10(x+1)^2 - 2(5x^2+10x)}{(x+1)^3}$$

$$= \frac{10x^2+20x+10-10x^2-20x}{(x+1)^3}$$

$$= \frac{10}{(x+1)^3}$$

$A=10$

$n=3$

(b) $\frac{10}{(x+1)^3} < 0 \qquad \boxed{x<-1}$

5/5

Examiner Comments: (a) M1 A1 M1 A1 (b) B1

This part (a) is an excellent solution. The candidate uses the product rule and uses a common denominator to combine the separate fractions. They reach the required form very quickly and efficiently.

Part (b) is fully correct.

Exemplar Question 4

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4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

(Total for Question 4 is 6 marks)

Mean Score: 3.3 out of 6

Examiner Comments:

The vast majority of candidates did well in part (a), securing most or all of the marks available. They appreciated the need to rewrite the expression with a negative power and dealt confidently with the factorisation. The binomial expansion itself was also correctly executed and careful working ensured that there were relatively few slips in powers or signage.
Common errors included:

bracketing errors seen with $-\frac{x^2}{4}$ in the third term rather than $\left(-\frac{x}{4}\right)^2$

failing to correctly combine the factored out $4^{-\frac{1}{2}}$ with the expansion $\left(1+\frac{x}{8}+\frac{3x^2}{128}\right)$

Part (b) caused more difficulty.
In part (i), many candidates appeared to realise that the question was linked to the range of valid x values and the correct answer, $x=-14$, was selected most often. To gain this mark however, the correct answer and a valid reason needed to be given. Examples of valid reasons were, "the expansion is only valid for $|x|<4$", "the expansion is not valid for $x=-14$ as $|-14|>4$".

In part (ii), only a small proportion of the candidates realised that the expansion is more accurate for values of x closest to 0. Many resorted to using the three values given and comparing the results to the exact value of $\sqrt{2}$ or else stating that the answer was 2 as it was the closest value to $\sqrt{2}$.

Mark Scheme

Question	Scheme	Marks	AOs
4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. E.g. explains that it is closest to zero	B1	2.4
		(1)	
		(6 marks)	

Notes

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1\pm ax)^n=1\pm nax\pm\frac{n(n-1)}{2}a^2x^2+$

Condone sign slips and the "a" not being squared in term 3. Condone $a=\pm1$

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

A1: $\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

(b)(i)

B1: Requires $x=-14$ with a suitable reason.

E.g. $x=-14$ as the expansion is only valid for $\lvert x\rvert<4$ or equivalent. Accept ‘ $x=-14$ as $\lvert-14\rvert>4$", ' I cannot use $x=-14$ as $\left\lvert\frac{-14}{4}\right\rvert>1$ ' or ‘ $x=-14$ as is outside the range $\lvert x\rvert<4$

Do not allow ‘ -14 is too big’ without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x=-\frac{1}{2}$ with a suitable reason such as' it is the smallest/smaller value’

Student Response A

4a) $(4-x)^{-1/2}$

$4^{-1/2}\left((1-x)^{-1/2}\right)$

$\frac{1}{2}\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-x)^2\right)$

$\frac{1}{2}\left(1+\frac{x}{2}+\frac{3x^2}{8}\right)$

$\frac{1}{2}+\frac{x}{4}+\frac{3x^2}{16}$

bi) $x = 2$.

ii)

2/6

Examiner Comments: (a) M1 M1 A0 A0 (b) B0 B0

Part (a) the first method mark is awarded for a correct strategy. The candidate has $4^{-\frac{1}{2}}$ outside the bracket and an attempt at a binomial expansion. They make an error when taking out the factor of 4 but can still score the second method mark for a correct attempt at expanding $(1-x)^{-\frac{1}{2}}$. Due to this, neither accuracy mark can be scored.

Part (b) Requires both a value to be chosen and a reason. In (i) the value chosen is incorrect. Part (ii) is not attempted.

Student Response B

4a.

$$\frac{1}{\sqrt{4-x}} = \frac{1}{(4-x)^{\frac{1}{2}}}$$

$$= (4-x)^{-\frac{1}{2}}$$

$$= 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$\frac{1}{2}\left(1 + \left(-\frac{x}{4}\right)\left(-\frac{1}{2}\right) + \frac{\left(-\frac{x}{4}\right)^2\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\right)$$

$$= \frac{1}{2}\left(1 + \frac{x}{8} + \frac{3x^2}{128}\right)$$

$$= \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{128}$$

b(i). a ~~$|x|<4$~~

$\cancel{|x|<}-4 < x < 4$

$x = -14$ should not be used because it does not fall in the ~~range~~ range of x.

(ii). $x = 2$ because it requires ~~multiplication~~ multiplication by smaller numbers which increases accuracy.

4/6

Examiner Comments: (a) M1 M1 A1 A0 (b) B1 B0

A slip from the penultimate to the final line means that the final accuracy mark in withheld in part (a).

(b) (i) $x = -14$ is chosen and a suitable reason is given
(ii) $x = 2$ is the incorrect value

Student Response C

a) $\frac{1}{\sqrt{4-x}} = (4-x)^{-1/2} = \left(4\left(1-\frac{x}{4}\right)\right)^{-1/2}$

$= \frac{1}{2}\left(1-\frac{x}{4}\right)^{-1/2}$

$\left(1-\frac{x}{4}\right)^{-1/2} = 1 + \frac{-1}{2}\left(\frac{-x}{4}\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2}\left(\frac{-x}{4}\right)^2$

$= 1 + \frac{x}{8} + \frac{3}{128}x^2$

$\frac{1}{\sqrt{4-x}} = \frac{1}{2}\left(1 + \frac{1}{8}x + \frac{3}{128}x^2\right)$

$= \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$

b) (i) $x = -14$, because -14, is a large number, and binomial expansion approximations, should only be used for with small numbers.

(ii) $x = -\frac{1}{2}$, because $-\frac{1}{2}$ is the smallest number

5/6

Examiner Comments: (a) M1 M1 A1 A1 (b) B0 B1

A completely correct solution is shown in part (a).

(b) (i) $x = -14$ is chosen but the reason is unacceptable.

(ii) $x = -\frac{1}{2}$ is chosen with an acceptable reason.

Exemplar Question 5

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5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f (x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = \mathrm{f}(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = 2(x-2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$\mathrm{h}(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

(Total for Question 5 is 10 marks)

Mean Score: 6.3 out of 10

Examiner Comments:

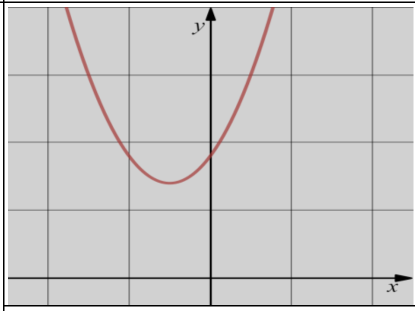
This question on completing the square and the quadratic function was accessible to all, especially the first 6 marks. Part (c) was more demanding but there were some excellent responses here.

Part (a) was very straightforward and many students were able to write down the answer without any difficulty. Although errors were rare, one common incorrect answer was $2(x+1)^2+8$.

Part (b) was equally straightforward, even for candidates who did not achieve the correct result in part (a). Many used the form $a(x+b)^2+c$ or their graphical calculators to produce accurate and well drawn curves. Marks were lost here when candidates drew V shaped curves or had incorrectly placed turning points.

Part (c) As stated earlier, this part was a lot more demanding. Rather strangely part (i) was found more accessible than part (ii). Many candidates sketched the new curve and compared the positions of the turning points noting that $(-1,7)$ had "moved" to $(1,3)$. Disappointingly a rather sizeable majority could only describe this "movement" as a "transformation" rather than "translation". Fewer students could use the form $\mathrm{h}(x)=\dfrac{21}{2(x+1)^2+7}$ to find the range of the function. Many resorted to substituting $x=0$ producing a range with $y=\dfrac{21}{9}$ as one of the limits. Common incorrect answers for the range of h scoring one mark were $\mathrm{h}(x) \le 3$ or $0 < \mathrm{f}(x) \le 3$.

Mark Scheme

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\mathrm{h}(x)\leq 3$	A1ft	1.1b
		(4)	
			(10 marks)

Notes

(a)

See scheme: Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

See scheme. Note that B3 is ft on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix}2\\-4\end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $\mathrm{g}(x)=\mathrm{f}(x-2)-4$ can score M1

A1: Requires both 'translate' and '$\begin{pmatrix}2\\-4\end{pmatrix}$' Allow 'shift' or move' instead of translate.

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

A1ft: $0<\mathrm{h}(x)\leq 3$ Allow for $0<\mathrm{h}(x)\leq 3$ $(0,3]$ and $0<y\leq 3$ but not $0<\mathrm{h}(x)\leq 3$

Follow through on their $a(x+b)^2+c$ so award for $0<\mathrm{h}(x)\leq\dfrac{21}{c}$

Student Response A

a) $f(x) = 2x^2+4x+9$ in the form $a(x+b)^2+c$

$2(x^2+4x)+9$

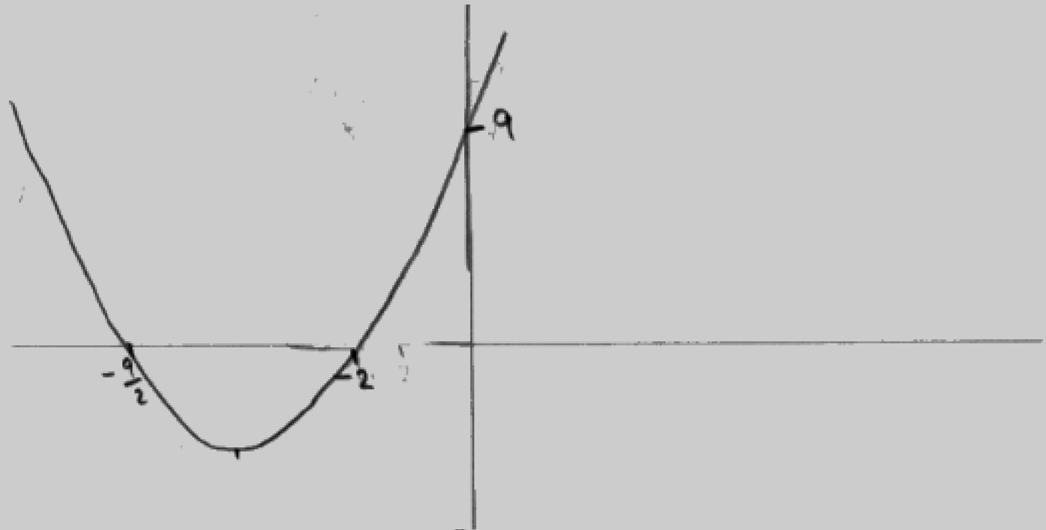
$2(x+2)^2-8+9$

$2(x+2)^2+1$

$\therefore a = 2$

$b = 2$

$c = 1$



c i) $2x^2+4x+9 = 2(x-2)^2+4x-3$

$2x^2+4x+9 = 2x^2-8x+8+4x-3$

$\cancel{2x^2}+4x+9 = \cancel{2x^2}-4x+5$

$8x = -4$

$x = -\frac{1}{2}$ transformation is by $x = -\frac{1}{2}$

(ii) $\frac{21}{2x^2+4x+9} = 0$??

3/10

Examiner Comments: (a) B1 M0 A0 (b) B1 B1 B0 (c) (i) M0 A0 (ii) M0 A0

For part (a) only the first mark is awarded for $2(x \pm k)^2$... The values of b and c are incorrect.

In part (b) the curve is U shaped (1st B1) with an intercept at 9 (2nd B1). To score the follow through mark here the minimum point would have needed to be at $(-2,1)$.

There is no work worthy of any marks in part (c).

Student Response B

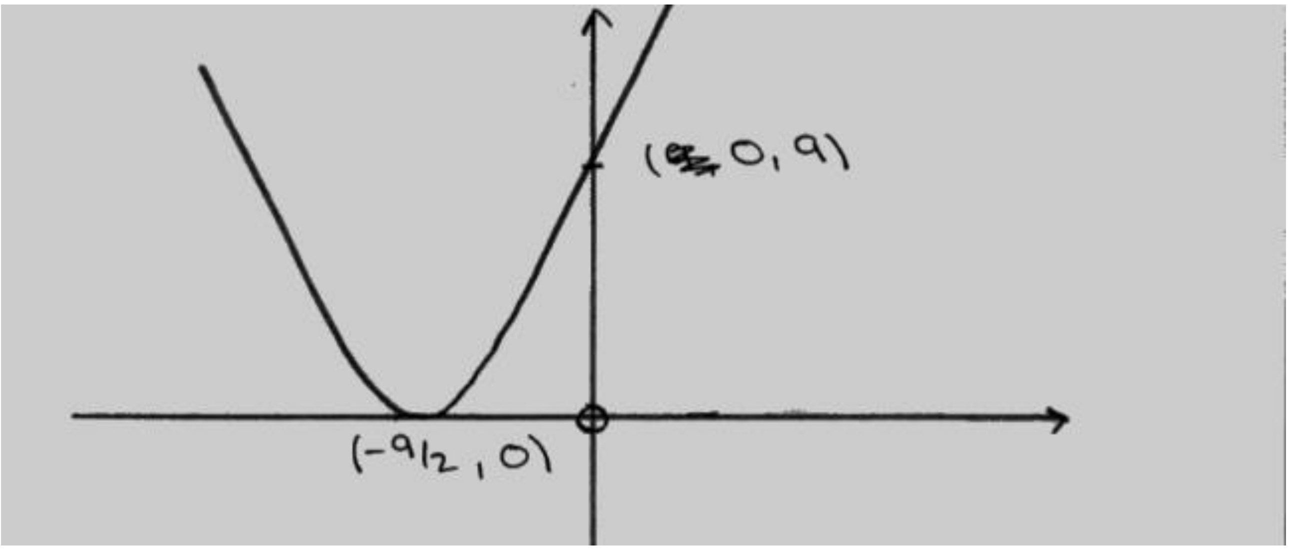
5. a) $f(x) = 2x^2 + 4x + 9$

$= 2(x^2 + 2x + 4.5)$

$= 2((x+1)^2 - 1 + 4.5)$

$= 2(x+1)^2 + 7.$

b) $a = 2 \quad b = 1 \quad c = 7$



c) i. $2(x-2)^2 + 4x - 3$

$= 2(x^2 - 4x + 4) + 4x - 3$

$= 2x^2 - 8x + 8 + 4x - 3$

$= 2x^2 - 4x + 5.$

so the graph is shifted down 6 units and to the right 2 units.

ii. so $2x^2 + 4x + 9 \neq 0.$

6/10

Examiner Comments: (a) B1 M1 A1 (b) B1 B1 B0 (c) (i) M1 A0 (ii) M0 A0

Part (a) is a completely correct solution.

In part (b) the curve is U shaped (1st B1) with an intercept at 9 (2nd B1). To score the follow through mark here the minimum point would have needed to be at $(-1, 7)$.

Part (c) (i) the candidate deduces that $f(x) \to g(x)$ via a translation (condone the word "shift" here) and has one aspect correct (2 to the right). The accuracy mark is withheld as they think that it shifts 6 down rather than 4.

For part (c)(ii) there is no progress here.

Student Response C

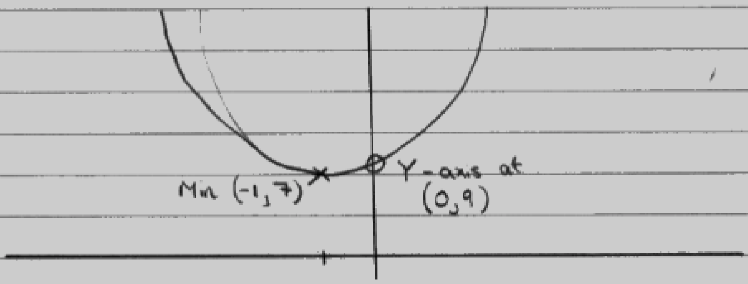
a) $2x^2 + 4x + 9$

$2\left(x^2 + 2x + \frac{9}{2}\right)$

$2\left((x+1)^2 - 1 + \frac{9}{2}\right)$

$\underline{2(x+1)^2 + 7}$ $\quad a=2 \quad b=1 \quad c=7$

b)



c) $\cancel{g(x)}$

The function has

$g(x)$ $2x^2 - \cancel{8x} + 8 + 4x - 3$

$\cancel{2x^2 - 4x + 5}$ $\cancel{2\left(x^2 - 2x + \frac{5}{2}\right)}$

$\cancel{2((x-2)^2 - 5}$

$2x^2 - 4x + 5$

$2\left(x^2 - 2x + \frac{5}{2}\right)$

$2\left((x-1)^2 - 1 + \frac{5}{2}\right)$

$2(x-1)^2 + 3$

The graph has been shifted down 4 on the y axis so the maximum is (1, 3) and to the right 2 on the x axis.

ii) $h(x) = \dfrac{21}{2x^2 + 4x + 9}$ $\quad \cancel{x}$

$\cancel{x}$ let $y = 2x^2 + 4x + 9$

min $-1, 7$

$\frac{21}{9} \leq h(x) \leq 3$

9/10

Examiner Comments: (a) B1 M1 A1 (b) B1 B1 B1 (c) (i) M1 A1 (ii) M1 A0

Part (a) is a completely correct solution.

Part (b) is a completely correct solution.

Part (c)(i) is an acceptable response for two marks. The candidate compares both minima and deduces that it shifts (allowed here for translation) 4 down and 2 to the right.

In part (c)(ii) the method mark can be awarded as 3 is found as their maximum value. The accuracy mark is withheld as this would require $0 < \mathrm{h}(x) \leq 3$.

Exemplar Question 6

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6. (a) Solve, for $-180° \leq \theta \leq 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50°) = 9 \tan (x - 25°)$$

(2)

(Total for Question 6 is 8 marks)

Mean Score: 4.4 out of 8

Examiner Comments:

This was generally a well attempted question with most candidates able to score over half of the marks.

In part (a) the correct identities were well known and efficiently used in order to obtain an equation in one function. Almost all candidates who reached this stage were then able to proceed to find at least one correct angle. It was clear from the graphs and diagrams sketched by most candidates that they had been taught to look for all angles in the required range and many were able to give all four of the angles needed. Only a minority of candidates identified $0, \pm 180$ as solutions to the equations. Most candidates divided through by $\sin\theta$ or $\cos\theta$ and failed to identify any of these values.

In part (b) a significant number used $2x - 50 = \theta$ rather than $x - 25 = \theta$. Of those who used one of their solutions to part (a), the majority then proceeded to the correct answer, although a significant number did not give the smallest positive solution to the equation. This was usually due to the fact that they used their smallest positive solution to part (a) (i.e. 18.4) in the equation, not realising that they needed to use – 18.4

Mark Scheme

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\frac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\frac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

Notes

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (E.g. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \frac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

Student Response A

a) $\sin 2\theta = 2\sin\theta\cos\theta \qquad \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$5(2\sin\theta\cos\theta) = \dfrac{\sin\theta}{\cos\theta}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$10\sin\theta\cos\theta = \dfrac{\sin\theta}{\cos\theta}$

$10\sin\theta\cos^2\theta = \sin\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$

$10\cos^2\theta = 1 \qquad \frac{1}{2}\cos 2\theta + \frac{1}{2} = \cos^2\theta$

$10\left(\frac{1}{2}\cos 2\theta + \frac{1}{2}\right) = 1$

$5\cos 2\theta + 5 = 1$

$5\cos 2\theta = -4$

$\cos 2\theta = -\frac{4}{5} \qquad -360° \leq 2\theta \leq 360°$

$2\theta = 143.13,\ 216.869898,\ -143.13,\ -216.87$

$\theta = 71.6°,\ 108.4°,\ -71.6°,\ -108.4°$

b) Smallest solution is when $5\sin(2x - 50°) = -1$

$\sin(2x - 50°) = -\frac{1}{5}$

$2x - 50° = -1$

2/8

Examiner Comments: (a) M1 A0 M1 A0 A0 B0 (b) M0 A0

In part (a) the first M mark can be awarded for the correct strategy of solving this equation. This candidate 'loses' the 9 on the rhs of the equation and so cannot be awarded accuracy marks in part (a). The second M is also scored for a correct method in finding θ from their equation in $\cos 2\theta$.

To score the method marks in part (b) this candidate would have had to solve $x - 25 = k$ where k is any of their solutions from (a).

Student Response B

$5\sin 2x = 9\tan x$

$5(2\sin x\cos x) = 9\tan x$

$= 10\sin x\cos x = \frac{9\sin x}{\cos x}$

$\frac{10\cancel{\sin x}\cos x}{\sin x} = \frac{9}{\cos x}$

$10\cos x = \frac{9}{\cos x}$

$10\cos^2 x = 9$

$(\cos x)^2 = \frac{9}{10}$

$\cos x = \pm\sqrt{\frac{9}{10}}$

S | A
T | C

$\cos x = \sqrt{\frac{9}{10}}$

$x = 18.4, 341.6$

$x = 18.4, -161.6$

$\cos x = -\sqrt{\frac{9}{10}}$

S | A
T | C

$x = 161.6$

$x = 341.6, -18.4$

$\therefore x = -161.6, -18.4, 18.4$

b) $\cos(x-25) = \pm\sqrt{\frac{9}{10}}$

$x - 25 = -161.6, -18.4, 18.4$

$x = \cancel{+136.6}\ -161.6 + 25$

$x = -136.6$

5/8

Examiner Comments: (a) M1 A1 M1 A1 A0 B0 (b) M1 A0

Part (a) was a typical response. This candidate uses the identities for $\sin 2x$ and $\tan x$ to set up a correct equation in $\cos x$. The first accuracy mark is scored for one correct value but all four values are required for the second A mark. As in this case, most candidates failed to realise that $\sin x = 0$ should be considered.

In part (b) the M mark is awarded for an attempt at solving $x - 25 = -161.6$. Although a ft mark, their value is not in the appropriate range to score the accuracy mark.

Student Response C

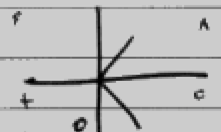
a) $5\sin 2\theta = 9\tan\theta$

$10\sin\theta\cos\theta = 9\dfrac{\sin\theta}{\cos\theta}$

$10\sin\theta\cos^2\theta = 9\sin\theta$

$\sin\theta(10\cos^2\theta - 9) = 0 \quad \therefore \sin\theta = 0, \ \theta = 0, 180°, -180°$

$10\cos^2\theta = 9 \quad \cos\theta = \dfrac{\sqrt{9}}{\sqrt{10}}, \ \theta = 18.4, -18.4$



$\therefore \theta = 0, -180°, 180°, 18.4°, -18.4°$

b) $5\sin 2x = 5\sin(2x - 90) \quad \therefore x = (x - 25)$

$x - 25 = 0, -180, 180, 18.4, -18.4$

$\therefore x = -18.4 + 25 = \dfrac{33}{5} = 6.6$

7/8

Examiner Comments: (a) M1 A1 M1 A1 A0 B1 (b) M1 A1

In part (a) this candidate uses the identities for $\sin 2\theta$ and $\tan\theta$ to set up a correct equation in $\cos\theta$. Crucially they factorise out the $\sin\theta$ term rather than cancelling it out. As a result, they not only pick up the accuracy mark for a correct solution of $\cos\theta = \dfrac{\sqrt{9}}{\sqrt{10}}$ but also the B marks for solutions of $\sin\theta = 0$. The penultimate mark is withheld as they fail to write down the solutions $\pm 161.6°$.

Part (b) is correct.

Exemplar Question 7

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7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

(Total for Question 7 is 7 marks)

Mean Score: 4.2 out of 7

Examiner Comments

It was pleasing to see that most candidates attempted to form an exponential model using a suitable equation. This is one of the new topics in the specification and candidates who didn't gain any marks on this question, generally did so because they did not attempt it, or that they tried to use a non-exponential model.

In part (a) $V = Ar^t$ or $V = Ae^{kt}$ were the most popular types of correct models that candidates opted for. However, some methods were often muddled or poorly shown, but many candidates who used a correct model were able to proceed to obtain correct values for their constants. Those candidates who used the model $V = Ae^{kt}$ frequently gave the constant k as a log and errors sometimes occurred with the sign of this constant. Errors in calculating constants with the model $V = Ar^t$ often occurred when candidates used $V = Ar^{t-1}$ instead.

In part (b) candidates who correctly answered part (a) usually understood that for this section they needed to substitute $t = 10$ into their model, and most were able to do this. Many candidates then proceeded to make a sensible comparison of the value gained by their model with the actual value in order to comment on the model's reliability (sometimes calculating and commenting on % error). The final mark, however, was frequently lost because answers were too vague, without a clear comparison or assessment of reliability being made. Some candidates appeared to think that a model was only acceptable if it gave the exact real value.

In part (c) most candidates who attempted, understood which of their two constants would need to be altered, although marks were lost either because the answer given just said the constant needed to be changed or because they wrongly identified whether the constant needed to be decreased/increased. This was more of a difficulty for those candidates who had used the model $V = Ae^{kt}$, because candidates were often dealing with negative constants and/or constants given as logs so the phrases used were not always mathematically correct. An example was often useful to convey what the candidate meant where there was ambiguity. Candidates who had gone with model $V = Ar^t$ were less likely to make an error in judging whether their constant should increase or decrease.

Mark Scheme

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	E.g. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	E.g. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\ 2150)$	M1	3.4
	E.g. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
		(7 marks)	

Notes

(a) Option 1

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, e.g. $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf.

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. E.g. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. E.g. states that the value of their '−0.233' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their −0.233 doesn't lie in the range (−0.233, 0)

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of *V* when *t* = 10. E.g. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares $(£)\,2147$ with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. E.g. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of *r* in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Student Response A

a) a_1 a_2

20,000 16000

$$\frac{20000-16000}{20000} = \frac{1}{5}$$

$y = ab^t$

depreciates by 0.2

$\therefore V = 20{,}000\,(0.2)^t$

b) $V = 20{,}000\,(0.5)^{10}$

$= 2.048 \times 10^{-3}$

this shows the cars value would have diminished extremely over the period. this is unrealistic and therefore isn't reliable for this time of 10 years.

c) $V = 20\,000$

~~you could change~~

the value of 0.5 which represents how much the car depreciates.

You would make it smaller to match the depreciation rate

2/7

Examiner Comments: (a) M1 M1 M0 A0 (b) M0 A0 (c) B0

In part (a) this candidate uses an exponential model of the form $v = Ar^t$ with $(0, 20000)$ to produce an equation of the correct form. Hence M1 M1 is awarded. The method to find r is incorrect however, with $\dfrac{20000-16000}{20000}$ being used rather than $\dfrac{16000}{20000}$

In part (b) the candidate's model is incorrect and here is confusion over which version has been used in the substitution, therefore no marks can be awarded.

Part (c) is incorrect. The value of 0.2 (or 0.5) should be increased.

Student Response B

a) $V = V_0 e^{-kt}$

whe $t=0$ $V=20000$ $\therefore V_0 = 20000$

$V = 20000e^{-kt}$

when $t=1$ $V=16000$

$16000 = 20000e^{-k}$

$\frac{4}{5} = e^{-k}$

$\ln\frac{4}{5} = -k$

$k = -\ln\frac{4}{5}$

$\therefore V = 20000e^{-\ln(4/5)t}$

b) $V = 20000e^{-\ln(4/5)t}$

when $t=10$ model gives V to be...

$V = 2000e^{-\ln 4/5 \times 10}$

$= 186264.5149$

Cannot reliable as it is very incorrect as it is 184264.5149 off the actual price

c) change the rate of depreciation i.e. in the power of e

4/7

Examiner Comments: (a) M1 M1 M1 A0 (b) M1 A0 (c) B0

In part (a) this candidate uses an exponential model of the form $V=V_0e^{-kt}$ with (0,20000) to produce an equation of the correct form on line 3. Hence M1 M1 is awarded. The method to find k is also correct, but in attempting to write the equation of the model the candidate has become confused between k and $-k$ and ends up writing down an incorrect equation linking V and t.

In part (b) the candidate scores the method mark for substituting $t=10$ into the equation of their exponential model but the accuracy mark cannot be awarded from an incorrect model.

Part (c) is too vague and not worthy of the mark.

Student Response C

a) $A = A_0 e^{-kt}$

$16\,000 = 20\,000\, e^{-k \times 1}$

$\boxed{A=V}$ $\quad \frac{4}{5} = e^{-k}$

$-\ln \frac{4}{5} = k$

$k = 0.22314$

$V = 20000\, e^{-0.223t}$

b) $V = 20000\, e^{-0.223 \times 10}$

$V = £1946$

My model is fairly reliable as 1945 is close to 2000

c) decrease the constant value k (0.223), so the exponential decay occurs more slowly.

6/7

Examiner Comments: (a) M1 M1 M1 A1 (b) M1 A0 (c) B1

In part (a) this is acceptable for all 4 marks. The accuracy of k, to 3sf, is sufficient here.

In part (b) the candidate scores the method mark for substituting $t = 10$ into the equation of their exponential model, but the accuracy mark cannot be awarded as their answer is incorrect. For this mark to be awarded we would need to see V =£2150 with either "it is accurate as £2150 is close to £2000" or "it is not very accurate as there is over a 7% error. So long as the candidate makes a comment on the reliability of the model and attempts to justify it, we can award the mark.

Part (c) is correct.

Exemplar Question 8

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8.

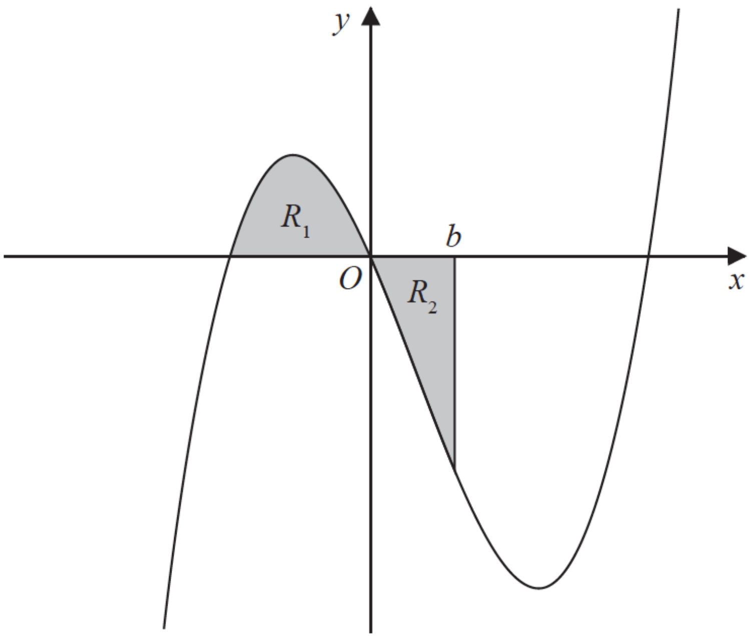


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\dfrac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for Question 8 is 10 marks)

Mean Score: 4.9 out of 10

Examiner Comments:

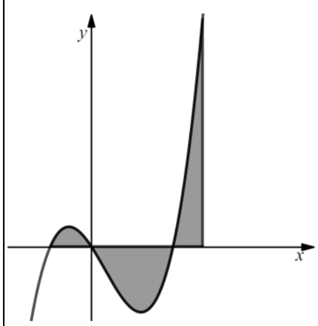
The first six marks in this question relating the definite integral with the area under the curve were straightforward. Only the best candidates were able to access the last four with many struggling to explain the significance of the root 5.442

In part (a) most students gained all 4 marks. Some unnecessarily complicated their proof by attempting integration by parts but in the main, all calculations were accurately performed.

As with all proofs, part (b) proved to be very discriminating. Many candidates picked up the first mark for setting $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$ but there was much confusion as to the sign of $\frac{20}{3}$ on the right hand side of the equation. Most students attempted to proceed by the method shown in the scheme. It should be noted that division by $b^2 + 4b + 4$ was much easier and faster than dividing by $(b+2)$ twice. Fully correct solutions to (b) were rare with many picking up 2 out of the 4 marks.

In part (c) many candidates realised that 5.442 was to the right of 4 on the *x*- axis and shaded an appropriate area, scoring one mark. Full explanations as to its significance were rare and confined to the best candidates. Any suitable statement that alluded to the fact that the area above the curve is equal to the area below the curve (between -2 and 5.442) was acceptable.

Mark Scheme

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, \mathrm{d}x \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, \mathrm{d}x$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

Notes

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.
Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1
It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function. The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

E.g. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients. It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm 80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection $3b^4-8b^3-48b^2\pm 80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out $3b^4-8b^3-48b^2\pm 80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have $=0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 ($x = 5.442$ may not be labelled.)

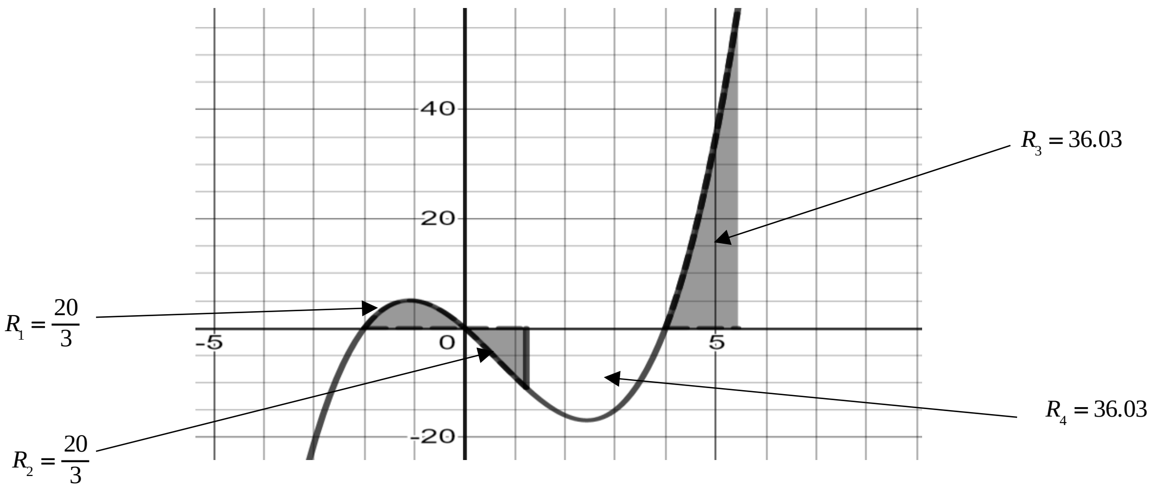
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x-axis with appropriate areas shaded or labelled.

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

E.g. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



Student Response A

a) $y = x(x+2)(x-4)$

$\therefore$ roots $= 0, -2, 4$

R, limits $= -2, 0$

$\int_{-2}^{0} y \, dx$

$= \int_{-2}^{0} x(x+2)(x-4)\,dx = \int_{-2}^{0} (x^2+2x)(x-4)\,dx$

$= \int_{-2}^{0} x^3 + 2x^2 - 4x^2 - 8x \;\, dx$

$= \int_{-2}^{0} x^3 - 2x^2 - 8x \;\, dx$

$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-2}^{0}$

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$

$= 0 - \left[\frac{1}{4}(-2)^4 - \frac{2}{3}(-2)^3 - 4(-2)^2\right]$

$=$

3/10

Examiner Comments: (a) B1 M1 M1 A0 (b) M0 A0 M0 A0 (c) B0 B0

This was a common response for weaker students.

In part (a) the candidate correctly expands and integrates $x(x+2)(x-4)$ using the limits 0 and -2 the correct way around. All elements of the proof are there apart from the given answer.

There is no work for (b) and (c)

Student Response B

(a) $x(x+2)(x-4)=0$

$x=-2,\ 0,\ 4$

so $R_1 = \displaystyle\int_{-2}^{0} x(x+2)(x-4)\,dx$

$R_1 = \displaystyle\int_{-2}^{0} (x^3-2x^2-8x)\,dx$

$= \left[\frac{1}{\cancel{3}4}x^4 - \frac{2}{3}x^3 - \cancel{8}4x^2\right]_{-2}^{0} = 0-\left(4+\frac{16}{3}-16\right)$

$= \frac{20}{3}$

(b) Area of $R_2 = \displaystyle\int_{0}^{b}(x^3-2x^2-8x)\,dx = \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_0^b$

$= \frac{1}{4}b^{\cancel{3}4} - \frac{2}{3}b^3 - 4b^2$

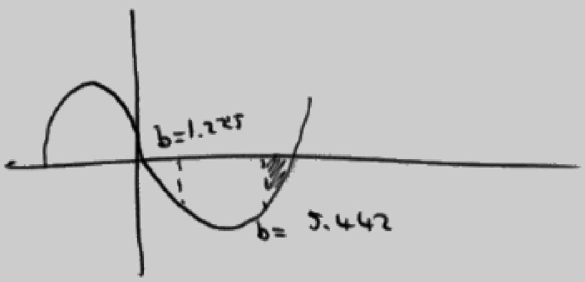
This area is $= \frac{20}{3}$ however as it is underneath the ~~graph~~ axis, we must change the sign.

Hence. $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = -\frac{20}{3}$

$3b^4 - 8b^3 - 48b^2 + 80 = 0$

$(b+2)^2(3b^2-20b+20)=0$

(c)



(c) The root $b=5.442$ gives the shaded area once again due to the symmetry of the curve. This area is equal to that of R_2 and R_1

6/10

Examiner Comments: (a) B1 M1 M1 A1 (b) M1 A1 M0 A0 (c) B0 B0

(a) This is completely correct and shows all the necessary steps required to show that the area of $R_1 = \dfrac{20}{3}$

(b) The first two marks are awarded for correctly deducing $3b^4 - 8b^3 - 48b^2 + 80 = 0$
Unfortunately, no more marks can be awarded as the candidate simply writes down the given solution. To score these marks they could have expanded $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ and shown that the two equations are the same or, more frequently seen, attempted to factorise $3b^4 - 8b^3 - 48b^2 + 80$ to the form $(b^2 + 4b + 4)\left(*b^2 + *b + *\right) = 0$ by inspection or otherwise.

(c) This requires 5.442 to be placed in the correct position so no marks can be awarded.

Student Response C

$$y = x^2 + 2x(x-4)$$
$$= x^3 - 4x^2 + 2x^2 - 8x$$
$$y = x^3 - 2x^2 - 8x$$

$$\int_{-2}^{0} x^3 - 2x^2 - 8x \, dx$$

$$\left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$$

$$0 - \left[4 + \tfrac{16}{3} - 16\right]$$

$$= \frac{20}{3}$$

b) $$\int_{0}^{b} x^3 - 2x^2 - 8x \, dx$$

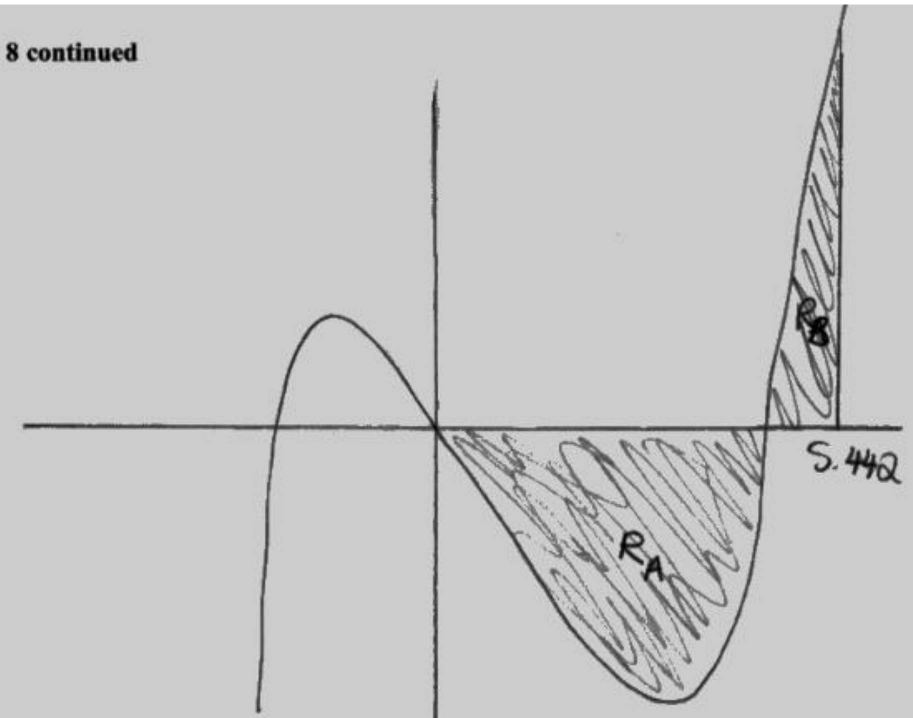
$$\left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - 4x^2\right]_{0}^{b}$$

$$\tfrac{1}{4}b^4 - \tfrac{2}{3}b^3 - 4b^2 = -\tfrac{20}{3}$$
$$\tfrac{3}{4}b^4 - 2b^3 - 12b^2 + 20 = 0$$
$$3b^4 - 8b^3 - 48b^2 + 80 = 0$$
$$(b+2)(b+2)(3b^2 - 20b + 20) = 0$$
$$(b^2 + 4b + 4)(3b^2 - 20b + 20) = 0$$
$$3b^4 - 20b^3 + 20b^2 + 12b^3 - 80b^2 + 80b + 12b^2 - 80b + 80$$
$$= 3b^4 - 8b^3 - 48b^2 + 80 = 0$$

RHS = LHS

Question 8 continued

c)



~~It~~ is outside the range $0 < b < 4$
5.442

but the significance of it is that it would still give the same area as regions below the x-axis, when you integrate them, come out as negative so the integral would be $R_B - R_A$ which would come out as $-20/3$ still

9/10

Examiner Comments: (a) B1 M1 M1 A1 (b) M1 A1 M1 A0 (c) B1 B1

This is an excellent solution.

(a) This is completely correct and shows all the necessary steps required to show that the area of $R_1 = \frac{20}{3}$

(b) The first two marks are awarded for correctly deducing $3b^4 - 8b^3 - 48b^2 + 80 = 0$
The candidate then expands $(b+2)^2(3b^2 - 20b + 20) = 0$ correctly. To score the A1* mark they were required to state that the two equations were the same. LHS= RHS was too vague.

(c) The candidate demonstrates via the graph and within the text that they understand the significance of 5.442

Exemplar Question 9

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9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

(Total for Question 9 is 5 marks)

Mean Score: 2.6 out of 5

Examiner Comments

There were many good attempts to this question, though complete answers to part (b) were achieved only by the very best.

In part (a) most students gained the B mark by writing $\log a - \log b = \log\left(\frac{a}{b}\right)$, though a few did attempt to rearrange first and used other equivalents. Students who gained this mark generally were successful at 'undoing' the logs and making a the subject. Students who failed to recognise the logarithmic rule for subtraction or addition often made no progress with the question.

Part (b) was more challenging with many only scoring the first mark. Many students focused solely on the exclusion of $b = 1$ from the denominator and paid no heed to the condition that $a > 0$. Few students were able to provide a full explanation including that as $a > 0$, $\frac{b^2}{b-1} > 0$ and so $b > 1$.
Most candidates knew that the denominator of a fraction cannot be zero, though few of them used the words “undefined” or “infinity” in their answers, but would more often refer to 'math error' or 'can’t didide by 0'.

Mark Scheme

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

Notes

(a)

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make 'a' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in a are on the same side of the equation and the term in b is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$.

They may state that b cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as a cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

Student Response A

a. $\log a - \log b = \log\left(\frac{a}{b}\right) = \log(a-b)$

$e^{\log\left(\frac{a}{b}\right)} = e^{\log(a-b)}$

$\frac{a}{b} = a - b$

$a = ab - b^2$

$b^2 = ab - b$

$b^2 = b(a-1)$

$b = a - 1$

$b + 1 = a$

b. $b \neq 1$ as that would make the denominator 0 which is undifined

2/5

Examiner Comments: (a) B1 M0 A0 (b) B1 B0

In part (a) B1 is scored for stating $\log a - \log b = \log \frac{a}{b}$. The M mark is not awarded as the candidate fails to proceed to a form where both terms in "a" are on the same side of the equation. When working from line 4 to line 5 it looks as though they have replaced the term in "b" with "a".

The candidates answer in part (b) is acceptable for one of the two marks. Such a response was common.

Student Response B

(2)

$$\log a - \log b = \log(a-b)$$

$$\log \frac{a}{b} = \log(a-b)$$

$$\frac{a}{b} = a - b \qquad a = b(a-b)$$

$$a = ba - b^2$$

$$a - ba = -b^2$$

$$a(1-b) = -b^2$$

$$a = -\frac{b^2}{1-b} \quad \times -1$$

$$a = \frac{b^2}{b-1}$$

b) $a > b > 0$ because $a > b$ notpossible and $b > 0$ also not possible in fraction $a = \frac{b^2}{b-1}$

3/5

Examiner Comments: (a) B1 M1 A1 (b) B0 B0

Part (a) is a completely correct solution.

In part (b) no work that merits the award of any marks.

Student Response C

a) $\log\left(\frac{a}{b}\right) = \log(a-b)$

$\therefore \frac{a}{b} = a - b \quad \therefore \quad a - \frac{a}{b} = b$

$a\left(1 - \frac{1}{b}\right) = b$

$\therefore a\left(\frac{b-1}{b}\right) = b \rightarrow \therefore \boxed{a = \frac{b^2}{b-1}} \quad \therefore \text{QED}$

b) $b > 1$, so $a > b > 1$

Since $a = \frac{b^2}{b-1}$, if $b < 1$, a becomes $-ve$

Since the original equation has the term $(\log a)$, a cannot be negative.

$\therefore b > 1$

5/5

Examiner Comments: (a) B1 M1 A1 (b) B1 B1

This is an excellent response.

In part (a) the candidate shows all necessary steps leading to the given answer.

In part (b) the candidates states that $b > 1$ and gives a suitable explanation of the reasons why it is.

Exemplar Question 10

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10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 10 is 6 marks)

Mean Score: 1.7 out of 6

Examiner Comments:

The idea of proof is another new one in this updated specification. It was not well known and a disproportionate number of blank or barely started solutions indicated a lack of expertise in this topic.

Candidates who scored full marks for part (i) most often did so via algebraic proof, letting n=2m (for even numbers) and then n=2m+1 (for odd numbers), although several candidates missed out on the final accuracy mark as they failed to insert a conclusion for all n. Weaker candidates tried to use $m + 1$ instead of $2m + 1$ to represent odd numbers. Also many candidates only considered either odd or even numbers, but not both.
Candidates who set out to use proof via contradiction tended to score few marks as they did not show an understanding of how to use that method of proof. Proofs via logic were rare and tended to appear as part of a mix of different methods where the candidates seemed to be unclear of the best way to proceed. Some candidates seemed to think that using random numbers to show that the expression is not divisible by 4 amounted to proof (receiving 0 marks). Students need more practice on this style of proof.

In part (ii) more candidates approached the problem algebraically than graphically. They were generally able to set up the two equations / inequalities required, but errors often occurred in their use of algebra when attempting to solve them. Some candidates only set up one of the two required inequalities. Many students attempted the question by just substituting numbers in to the expression, with most of these deducing that the statement was always true. Where candidates drew a graph, they often did not describe why their graph indicated that the statement is sometimes true. Many graphs showed a line and a V shape, but some were incorrect in the relative positioning of these shapes.

Mark Scheme

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2 + 2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2 + 2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2 + 2 = 4k \Rightarrow k = \frac{1}{4}n^2 + \frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up
(i)
M1: Awarded for setting up the proof for either the even or odd numbers.
A1: Concludes correctly with a reason why $n^2 + 2$ cannot be divisible by 4 for either n odd or even.
dM1: Awarded for setting up the proof for both even and odd numbers
A1: Fully correct proof with valid explanation and conclusion for all n
Example of an algebraic proof

For $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m+1$, $n^2 + 2 = (2m+1)^2 + 2 = \ldots$ FYI $(4m^2 + 4m + 3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of proof via contradiction

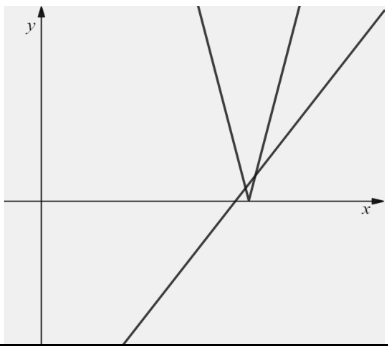
Sets up the contradiction 'Assume that $n^2 + 2$ is divisible by 4 $\Rightarrow n^2 + 2 = 4k$ '	M1	2.1
$\Rightarrow n^2 = 4k - 2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then $n^2 + 2$ cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

Question 10 (ii)	Scheme	Marks	AOs

(ii)
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.
A1: and gives an example where it is true,
Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\quad \lvert 3x-28 \rvert = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\quad \lvert 3x-28 \rvert = 8$ $\quad x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof

	States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\lvert 3x-28 \rvert < x - 9$ and points where $\lvert 3x-28 \rvert > x - 9$ oe in words like 'above' and 'below' or 'dips below at one point'		A1	2.4
		(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x - 28 < x - 9$ and $-3x + 28 < x - 9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x - 28 \geqslant x - 9$ and $-3x + 28 \geqslant x - 9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Student Response A

10i) n^2+2 not divisible by 4

Proof by contradiction

Assume: n^2+2 is ~~divible~~ divisible by 4

$$\frac{n^2+2}{4} = \frac{n^2}{4} + \frac{2}{4} = \frac{n^2}{4} + \frac{1}{2} \qquad \frac{n^2}{4} = \frac{1}{4}n^2$$

However, $\frac{n^2}{4}$ never gives a number which has a decimal of a half so a $\frac{1}{2}$ can't be added to make it a whole number

Therefore n^2+2 is not divisible by 4

10ii) $|3x-28| \geq (x-9)$

It's sometimes true just not for the value $x = \frac{28}{3}$

$|3(\frac{28}{3})-28| = 0$

$((\frac{28}{3})-9) = \frac{1}{3}$

$\frac{1}{3} > 0$

1/6

Examiner Comments: (i) M0 A0 M0 A0 (ii) M1 A0

Although the candidate sets up the contradiction in part (i), they would need to state that there exists a 'k' ($k \in \mathbb{N}$) such that $\frac{n^2+2}{4} = k$. No progress is made and so no marks are awarded.

For part (ii) the candidate states that it is "sometimes true" and finds a value where it is not true, that is $\frac{28}{3}$. Via this method they are required, for the A mark, to find a value where it is true.

Student Response B

ai) n^2+2 not $\div 4$

odd

~~$2n+1=n$~~ $n=2n+1$

$(2n+1)^2+2$

$=4n^2+4n+1+2$

$=4n^2+4n+3$

multiple of 4 here

makes any number odd

Multiples of 4 are always even.

even

$n=2n$

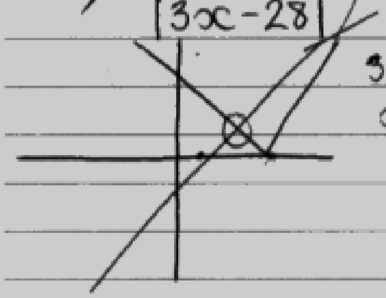
$(2n)^2+2$

$=4n^2+2$

adding 2 still makes the number even but ~~its~~ the number is always a multiple of 4 before you add 2. Adding the 2 means it's now not divisible by 4.

aii)

$|3x-28|$ $(x-9)$



$3x=28$

$x=28/3$

$-3x+28=x-9$

$4x=37$

$x=37/4$

$x=9.25$

$3x-28=x-9$

$2x=19$

$x=19/2$

$x=9.5$

$9.25<x<9.5$ so sometimes true

4/6

Examiner Comments: (i) M1 A1 M1 A0 (ii) M1 A0

This is potentially an excellent solution. With a little more care this could have been full marks.

In part (i) the candidate fails to give a conclusion that would imply that they have proven it for all $n \in \mathbb{N}$. A statement such as "hence true for odd and even numbers, so true for all n" would have sufficed.

In part (ii) the candidate should have either stated....it is only false when $9.25 < x < 9.5$ or mentioned that there are points when the graph of $y = x - 9$ is above and below the graph of $y = |3x - 28|$

Student Response C

(i) when n is even we can say $n = 2x$

$(2x)^2 + 2 = 4x^2 + 2$ this is not ~~a factor~~ divisible by 4 because not all terms have a factor of 4

When n is odd we can say $n = 2x + 1$

$(2x+1)^2 + 2 = 4x^2 + 4x + 1 + 2 = 4x^2 + 4x + 3$

again this isn't divisible by 4 because not all terms have a factor of 4

So if neither odd nor even numbers work then no numbers can be true for that expression.

(ii) $|3x - 28| \geqslant (x - 9)$ This is only sometimes true

$x = 2$ $|(3(2) - 28)| = 22$ $22 > 2 - 9$ $22 > -7$ this is true

but when $x = 28/3$ $|3(28/3) - 28| = 0$ $28/3 - 9 = 1/3$ $0 \geqslant 1/3$ isn't true

so it can only be true some of the time

6/6

Examiner Comments: (i) M1 A1 M1 A1 (ii) M1 A1

In part (i) note that in addition to the work on both odd and even numbers that we have concluding statement

In part (ii) the candidate states that it is sometimes true and gives two examples, one where it is false and one where it is true.

Exemplar Question 11

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11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

(Total for Question 11 is 7 marks)

Mean Score: 3.1 out of 7

Examiner Comments:
Overall very good attempts to this question were spoiled by a lack in accuracy.

Part (a) was routine for most and candidates were thorough in showing their method. The most frequent and concise method involved calculations of the form $24+6\times1.05+6\times1.05^2$. Occasionally the work was overcomplicated by converting to seconds though the end result was generally satisfactory.

Part (b) was found to be challenging with some candidates choosing not to answer. The most straightforward way was to use some values and spot the patterns. This is a skill taught at GCSE yet seemed forgotten to many.

	Time taken	Using the pattern on the left the time taken for the r th kilometre would be $6\times1.05^{r-4}$
5th kilometre	6×1.05^1	
6th kilometre	6×1.05^2	
7th kilometre	6×1.05^3	

Although most candidates knew that for part (c) they needed to sum a sequence of terms, errors were frequent and many. A sizeable minority lacked any appreciation of the model and resorted to the calculation $\dfrac{6\left(1.05^{20}-1\right)}{1.05-1}$. The majority of errors however were mainly due to a failure to link up the correct values for a and n in the formula for a geometric series. The most common of these was to use the strategy $24+\dfrac{6\left(1.05^{16}-1\right)}{1.05-1}$, forgetting that the geometric sequence in this case should start at 6.3 instead of 6.
It is often a good idea in such a question to write out the first few terms to gain an appreciation of the model and how the sequence is formed. (See below).

Km	1st	2nd	3rd	4th	5th	6th	7th		20th
Time	6	6	6	6	6×1.05	6×1.05^2	6×1.05^3		6×1.05^{16}

From here it should be a fairly straightforward task to write down $24+\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}$

Mark Scheme

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** $^{\text{th}}$ km is $6\times1.05=6\times1.05^1$ **6**$^{\text{th}}$ km is $6\times1.05\times1.05=6\times1.05^2$ **7**$^{\text{th}}$ km is $6\times1.05\times1.05\times1.05=6\times1.05^3$ Hence the time for the r^{th} km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = E.g. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum E.g. Time for 5 $^{\text{th}}$ to 20$^{\text{th}}$ km $=\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** E.g. Total time = 24 + $\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

Notes

(a)

M1: For using model to calculate the total time. E.g. 24+ $6\times1.05+6\times1.05^2$

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the r th km and the index of 1.05
Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a(r^n-1)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.
Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum 6\times1.05^n$ or $24+\frac{6(1.05^{20}-1)}{1.05-1}$

The geometric sequence formula must be used with $r=1.05$ oe but condone slips on *a* and *n*

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum
The value of *r* must be 1.05 oe such as 105% but you should allow a slip on the value of *n* used for their value of *a* (See below: We are going to allow the correct value of *n* or one less)
If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a=6,\quad n=17 \text{ or } 16$ E.g. $\frac{6(1.05^{17}-1)}{1.05-1}=(155.0)$ or $\frac{6(1.05^{16}-1)}{1.05-1}=(141.9)$

Allow for $a=6.3, n=16 \text{ or } 15$ Eg $\frac{6.3(1.05^{16}-1)}{1.05-1}=(149.0)$ or $\frac{6.3(1.05^{15}-1)}{1.05-1}=(135.9)$

Allow for $a=6.615,\ n=15 \text{ or } 14$ Eg $\frac{6.615(1.05^{15}-1)}{1.05-1}=(142.7)$ or $\frac{6.615(1.05^{14}-1)}{1.05-1}=(129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24+\frac{6.3(1.05^{16}-1)}{1.05-1}$, $18+\frac{6(1.05^{17}-1)}{1.05-1}$, $30.3+\frac{6.615(1.05^{15}-1)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds
This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

Student Response A

a) 6 min/km for 4km

5km = 6 × 1.05 = 6.3 min/km

6km = 6.3 × 1.05 = 6.615 min/km

= 6 + 6 + 6 + 6 + 6.3 + 6.615

= 36.915 mins

= 36 mins 54.9 secs

= 36 mins 55 secs (nearest second)   QED

b) $U_n = ar^{n-x}$

$a = 6$ min/km, $r = 1.05$, $x = 4$

$\therefore U_n = 6 \times 1.05^{n-4}$ QED

Question 11 continued

c) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$n = 20$, $a = 6$, $r = 1.05$

$S_{20} = \dfrac{6(1.05^{20} - 1)}{1.05 - 1}$

= 198.3957… mins

= 198 mins 23.74 secs

= 198 mins 24 secs (nearest second)

2/7

Examiner Comments: (a) M1 A1 (b) B0 (c) M0 M0 A0 A0

Part (a) gives a correct proof showing all necessary steps.

Part (b) there is no work here worthy of merit.

Part (c) was a common type of response. The question required the candidate to add a constant to the sum of a geometric sequence. This candidate finds the sum of a geometric sequence with 20 terms and is equivalent to finding $6 + 6 \times 1.05 + 6 \times 1.05^2 + \ldots + 6 \times 1.05^{19}$ which gained no marks via this scheme.

Student Response B

(4)

11a) $s=\frac{d}{t}$ $t=$ $4\times 6 = 24$ mins for first 4km

$24+(6\times 1.05)+(6.3\times 1.05)$ = ~~…~~ $= 36.915$ mins

= ~~…~~ = 36 mins 55 secs

b) $a=6$ $r=1.05$

$U_n = ar^{n-1}$ ~~When n=…~~ She has already run 4 km ∴ $r-4$

~~When n=5 1.05'~~

$U_n = 6\times 1.05^{r-4}$

c) $S_n = \frac{a(r^n-1)}{r-1}$ $S_{16} = \frac{6(1.05^{16}-1)}{1.05-1}$

S_{16} = ~~14.9449 mins~~ 141.945 mins

$141.945+36.915 = 178.86$ mins

178 mins 52 secs

4/7

Examiner Comments: (a) M1 A1 (b) B0 (c) M1 M1 A0 A0

Part (a) gives a correct proof showing all necessary steps.

In part (b) there is no work here worthy of merit.

In part (c) the candidate adds a constant (36.915) to the sum of a geometric sequence. Their S_{16} is an allowable sum to find and so the second M mark is awarded. As 36.915 + S_{16} is not a correct calculation to find the total time no accuracy marks can be awarded.

Student Response C

11a) first 4 kilometers at 6 minutes each = 24 minutes

Next 1 kilometer at $6 \times 1\cdot05 = 6\cdot3$ 6min 18

Nex 1 kilometer at $6 \times 1\cdot05^2 = 6\cdot615$ = 6min 37

24min + 6min 18secs + 6 min 37 secs = 36 mintes 55 seconds

11b) First four are run at 6 mintes the 5% slower each time so that gives $(r - 4)$ is needed so that the first four are not counted

$6 \times 1\cdot05^{r-4}$

11c) $S_{16} = \dfrac{6\cdot3(1 - 1\cdot05^{16})}{-0\cdot05}$ = 149 minutes 2 seconds

+ 24 minutes

173 minutes 2 seconds

6/7

Examiner Comments: (a) M1 A1 (b) B1 (c) M1 M1 A1 A0

Part (a) gives a correct proof showing all necessary steps.

In part (b) the candidate gives an allowable explanation of how the formula works.

In part (c) the candidate adds a constant (24) to the sum of a geometric sequence. Their S_{16} is an allowable sum to find and so the second M mark is awarded. As $24 + S_{16}$ is a correct calculation to find the total time A1 is awarded but the final accuracy mark is withheld as we are required to see 173 minutes and 3 seconds.

Exemplar Question 12

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12. $$f(x) = 10e^{-0.25x} \sin x, \quad x \geq 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

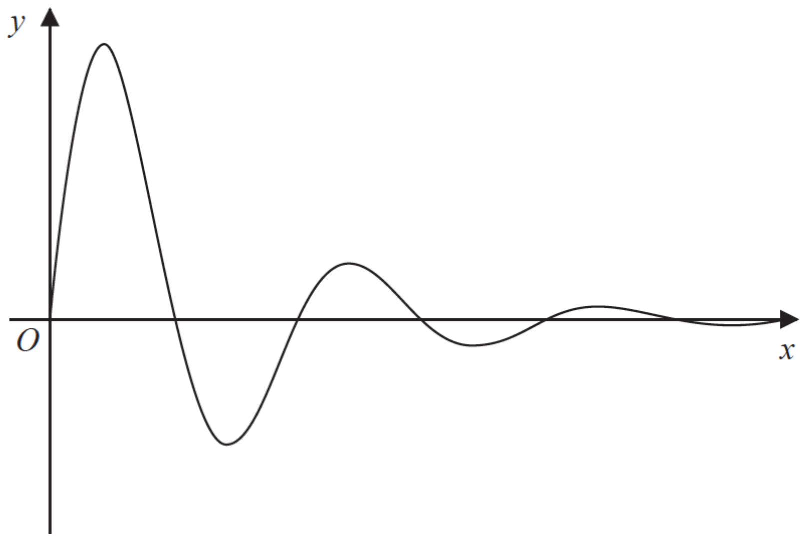


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = |10e^{-0.25x} \sin t| \qquad t \geq 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

(Total for Question 12 is 10 marks)

Mean Score: 4.1 out of 10

Examiner Comments:

This question discriminated well between students of all abilities with part (a) and (b) more accessible to students compared to part (c) and (d).

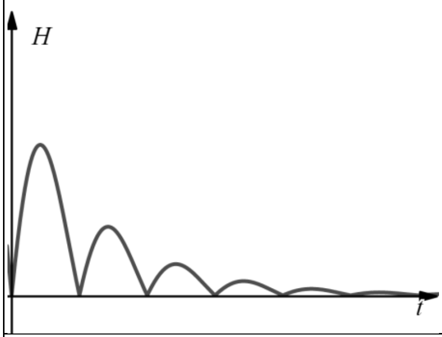
In part (a) most students were successful at using the product rule and setting dy/dx equal to 0 (though the latter was implied rather than being explicit in many cases). Missing the −0.25 factor when differentiating the $e^{(-0.25x)}$ was fairly common, but the differentiation was mostly correctly carried out. A few students failed to use the product rule correctly, instead multiplying the derivatives of each term, but these were in the minority.
Having attempt the derivative the majority did set the result equal to zero (possibly implied) and proceeded to cancel the exponentials. Reasoning for the cancellation was usually not given, but in this instance there was no requirement for such reasoning. However it might be well noted that such explanations may be required under the new specification. Many students lost the final accuracy mark as they did not state or show division by cos x. Students need to be clear that in a 'show that' question all steps need to be shown.

Part (b) was the most successfully answered part of the question. Students realised that the graph of $|f(x)|$ should be above the x-axis. Where students did not gain full credit, this was usually due to poor drawing of loops (not decreasing heights, or rounding at the cusps). A few students chose to draw over the original figure which made it difficult in many cases to determine which pieces formed their graph. They would be well advised to sketch separate graphs in similar questions.

There was less success in part (c). Attempts at this part were varied, with a sizeable proportion not making the connection with part (a) at all and attempting to find the value of $H(t)$ at π /2 or some other value, or failing to substitute into $H(t)$ at all. Many solved tan x = 4 to find the acute angle but did not then look for the further solution which was needed for the height between the first and second bounces and so achieved only the first mark. Also, use of degrees instead of radians was a common error, but this was allowed the first method mark. The magnitude of their resulting answer should have alerted them to the fact that something had gone very wrong.

Part (d) was another part with a varied array of answers. Common incorrect responses referred to negative time, and the times between each bounce would be longer or that the ball would not bounce forever. Many students gave a rote answer of "the effect of air resistance has not been taken into account" or "energy loss" or similar vague reasons rather than relate to the context of the question. Since the model may have taken air resistance into account (the contributing factors to the model were not listed in the question), such an answer was unacceptable.

Mark Scheme

Question	Scheme	Marks	AOs
12 (a)	$\mathrm{f}(x) = 10\mathrm{e}^{-0.25x} \sin x$		
	$\Rightarrow \mathrm{f}'(x) = -2.5\mathrm{e}^{-0.25x} \sin x + 10\mathrm{e}^{-0.25x} \cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x) = 0 \Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}} \sin x + 10\cancel{\mathrm{e}^{-0.25x}} \cos x = 0$	M1	2.1
	$\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5} \Rightarrow \tan x = 4*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \left\|10\mathrm{e}^{-0.25\times4.47} \sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

Notes

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .
So for example score expressions of the form $\pm ...\mathrm{e}^{-0.25x} \sin x \pm ...\mathrm{e}^{-0.25x} \cos x$ M1
Sight of $vdu - udv$ however is M0

A1: $\mathrm{f}'(x) = -2.5\mathrm{e}^{-0.25x} \sin x + 10\mathrm{e}^{-0.25x} \cos x$ which may be unsimplified

M1: For clear reasoning in setting their $\mathrm{f}'(x) = 0$, factorising/ cancelling out the $\mathrm{e}^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$
Do not allow candidates to substitute $x = \arctan 4$ into $\mathrm{f}'(x)$ to score this mark.

A1*: Shows the steps $\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4*$. $\dfrac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop. Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.
The intention should be that the graph should 'sit' on the x -axis but be tolerant.
It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$.
This can be awarded for an attempt to substitute $t = \text{awrt } 1.33$ or $t = \text{awrt } 4.47$ into $H(t)$
$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t = \text{awrt } 76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t = \text{awrt } 4.47$ into $H(t) = \left|10e^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been <u>chosen</u>
It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.
Look for " time (or gap) between the bounces will change''
'bounces would not be equal times apart'
'bounces would become more frequent'
But do not accept 'the times between each bounce would be longer or slower'
Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Student Response A

(1)

a. turning point when $\frac{dy}{dx}=0$ or $f'(x)=0$ S $-c$ c $-s$

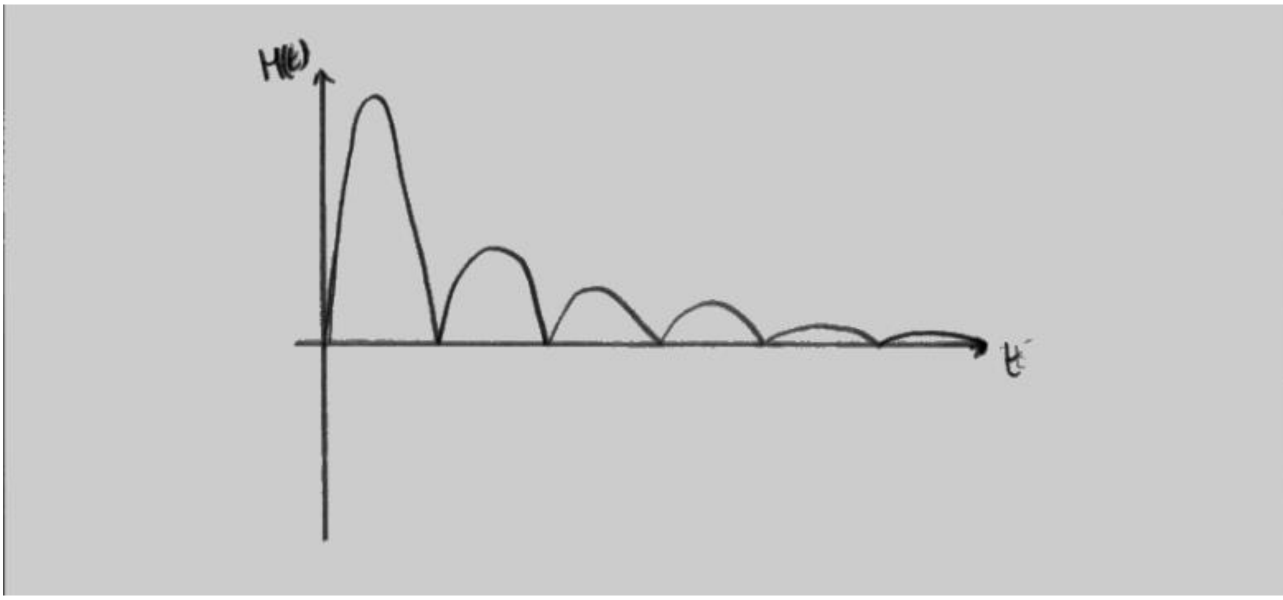
$f(x)=10e^{-0.25x}\sin x$ ~~f(x)~~

$u=\sin x$ $v=e^{-0.25x}$ $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

$\frac{du}{dx}=\cos x$ $\frac{dv}{dx}=-0.25e^{-0.25x}$

$=\sin x(-0.25e^{-0.25x})+e^{-0.25x}(\cos x)$

$\frac{dy}{dx}=-0.25x\sin xe^{-0.25x}+\cos xe^{-0.25x}$



3/10

Examiner Comments: (a) M1 A0 M0 A0 (b) M1 A1 (c) M0 M0 A0 (d) B0

In part (a) this candidate attempts the product rule but the missing "10" means that the accuracy mark cannot be awarded. There is no more progress in (a).

In part (b) both marks can be awarded for a correct and accurate graph.

In part (c) and (d) no attempt was made.

Student Response B

(1)

a. $f'(x) = 0$ at turning point

Product rule $f(x) = 10e^{-0.25x}$ $\quad g(x) = \sin x$

$f'(x) = -2.5e^{-0.25x}$ $\quad g'(x) = \cos x$

$\Rightarrow \frac{dy}{dx}$ $10e^{-0.25x} \cos x$ $-2.5e^{-0.3x} \sin x = 0$

$10e^{-0.25x} \cos x = 2.5e^{-0.25x} \sin x$

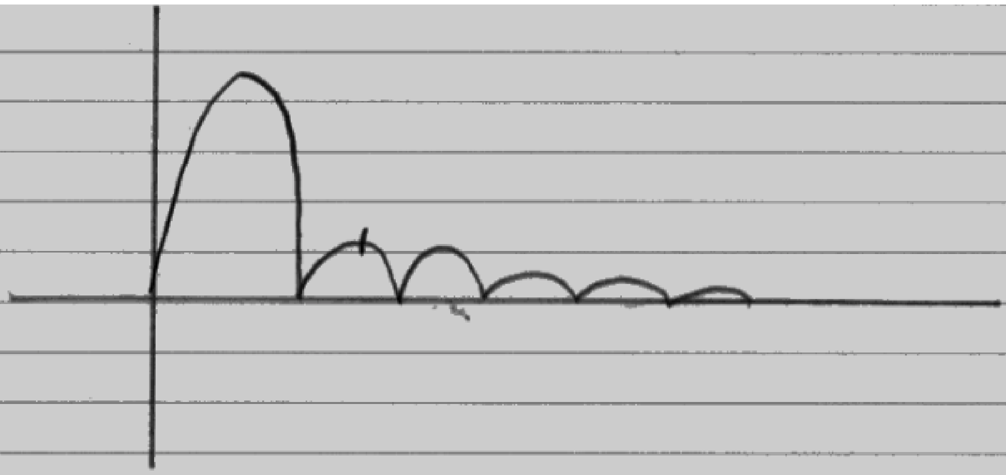
$4e^{-0.25x} \cos x = e^{-0.25x} \sin x$

$0 = \frac{e^{-0.25x} \sin x}{4e^{-0.25x} \cos x}$

$0 = \frac{1 \sin x}{4 \cos x}$

$0 = \frac{1}{4} \tan x = \frac{dy}{dx}$

b.



c. $\frac{dy}{dx}$ $\tan x = 4$ $\quad$ second value where $\tan x = 4$

$x = \cancel{75.96}, \cancel{255.96}$ $1.33,$ 4.47

Sub into equation: $\left|10e^{-0.25(4.47)} \sin 4.47\right| = 0.25 m$

7/10

Examiner Comments: (a) M1 A1 M1 A0 (b) M1 A1 (c) M1 M1 A0 (d) B0

In part (a) the product rule is accurately applied scoring M1 A1. $f'(x)$ is then set equal to 0, $e^{-0.25x}$ cancelled leading to an equation in $\sin x$ and $\cos x$. The A1* mark is withheld due to incorrect work.

In part (b) both marks can be awarded for a correct and accurate graph.

In part (c) the candidate substitutes the correct time, 4.47 seconds into H and finds the result. The accuracy mark is withheld as the height should be awrt 3.18 metres.

For part (d) there was no attempt.

Student Response C

a) $\cancel{10e^{-0.25x}\sin x =}$

$f(x) = 10e^{-0.25x}\sin x$

$f'(x) = \cos x(10e^{-0.25x}) + \sin x(-2.5e^{-0.25x})$

$\cos x(10e^{-0.25x}) + \sin x(-2.5e^{-0.25x}) = 0$

$e^{-0.25x}(\cancel{\cos}\ 10\cos x - 2.5\sin x) = 0$

$e^{-0.25x} \neq 0$

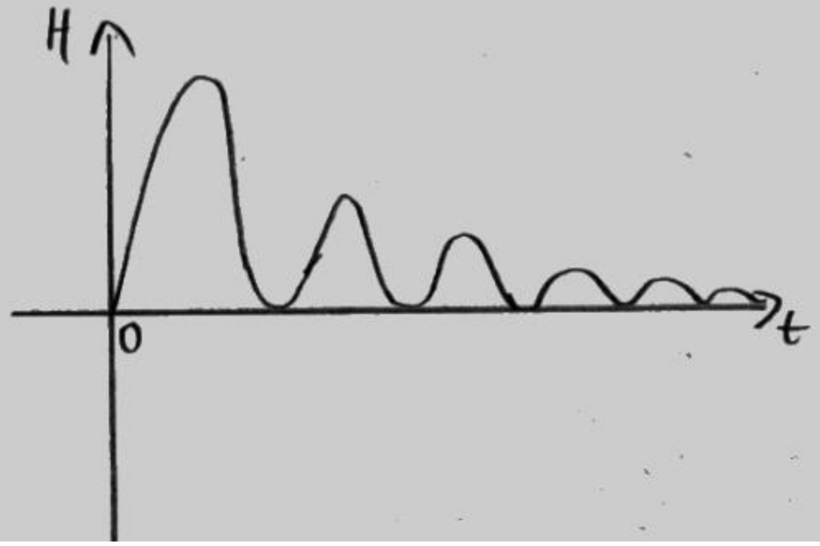
$10\cos x - 2.5\sin x = 0$

$10\cos x = 2.5\sin x$

$\cancel{4 = \tan x}$

$4 = \dfrac{\sin x}{\cos x} = \tan x$

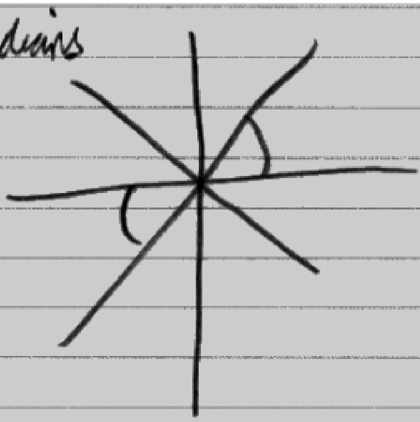
b)



tan in radians

c) $\tan t = 4$

$t = 1.33$ ~~7.609~~, 4.467, 7.609



Max height occurs at $t =$ ~~7.609~~ 4.467

$H(\cancel{7.609}\ 4.67)$

$=$ ~~1.45 m~~ $|-3.18|$

$= 3.18$

3.18m

d) ~~There is uneven amounts of time between bounces~~ There are equal amounts of time between bounces according to the model which isn't realistic for a ball bouncing.

9/10

Examiner Comments: (a) M1 A1 M1 A1 (b) M1 A0 (c) M1 M1 A1 (d) B1

In part (a) the candidate produces a correct and careful proof with all necessary steps leading to the given result.

In part (b) the sketch scores only 1 of the 2 marks due to the rounding at the cusps.

Part (c) is fully correct.

Part (d) gives an acceptable explanation.

Exemplar Question 13

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13. The curve C with equation

$$y = \frac{p-3x}{(2x-q)(x+3)} \qquad x \in \mathbb{R}, \quad x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

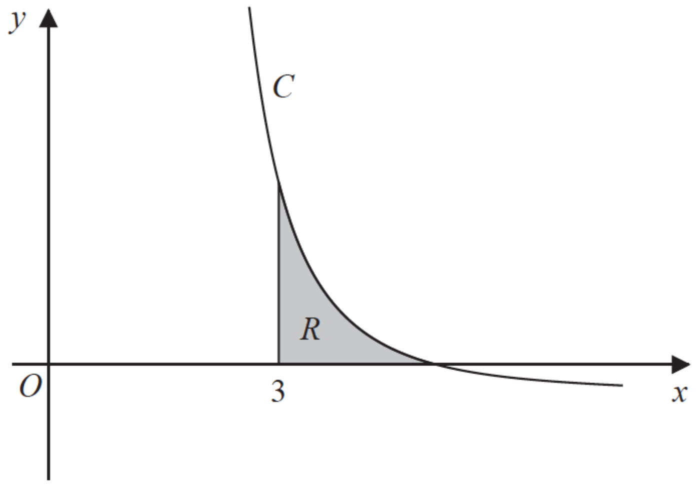

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8)

(Total for Question 13 is 11 marks)

Mean Score: 5.5 out of 11

Examiner Comments:

Only the strongest candidates scored all 11 marks here, with the majority losing at least 1 mark, usually the first mark in (a) or the last mark in (b).

In part (a) (i) many candidates realised that $x = 2$ was linked to establishing that $q = 4$ but they found difficulty explaining this by referencing the asymptote, or the denominator being zero.
Almost all candidates provided good explanations in (a)(ii) even when they had provided inadequate explanations on part (i).

In part (b) the vast majority of students knew they had to use partial fractions to attempt the question with many achieving the correct expression and correct integration. The most common error was in simplifying the final logarithmic expression with many candidates leaving some of their answers in terms of ln (3/4) or ln (4/3) for example. It was uncommon therefore to see a fully correct solution in the required form.

Mark Scheme

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15*$	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	I = $\int\frac{15-3x}{(2x-4)(x+3)}\,dx=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	I = $\int\frac{15-3x}{(2x-4)(x+3)}\,dx=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\,dx$ Or $[.............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

Notes

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q = 4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal e.g. ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area R = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)$

OR $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(x-2)+n\ln(x+3)$

Note that $\int \frac{l}{(x-2)}\,dx \to l\ln(kx-2k)$ and $\int \frac{m}{(x+3)}\,dx \to m\ln(nx+3n)$

A1ft: $=\int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}(dx)$ or $[..............]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once e.g. $\ln 6=\ln 2+\ln 3$, $\ln 8 = 3\ln 2$ or $m\ln 6k - m\ln 2k = m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3 - 4.8\ln 2$ oe

Student Response A

$$y = \frac{(p-3x)}{(2x-q)(x+3)} \qquad \left(3, \tfrac{1}{2}\right)$$

a.) $2x - q = 0$ $\quad x = 2$ $\quad x+3=0$

$4 - q = 0$ $\quad q = 4$ $\quad x = -3$

$-3+3=0$

$$\frac{p = 3x}{(2x-4)(x+3)}$$

$x = 3$

$$\frac{p-9}{(2)(6)} = \frac{1}{2}$$

$$\frac{p-9}{12} = \frac{1}{2}$$

$\cancel{p = 9}$ 4

$p - 9 = 6$

$p = 15$

b) $$\frac{15-3x}{(2x-4)(x+3)}$$

$15 - 3x = A(x+3) + B(2x-4)$ $\quad$ when $x = -3$

$\cancel{15}$ $24 = 0 + -8B$ $\quad B = -3$

when $x = 2$

$9 = 5A + 0$ $\quad A = \frac{9}{5}$

$$\int_0^3 \frac{9}{5(6x-5)} \cancel{+} \frac{-3}{x+3}$$

$= [\ \cancel{A} \quad - 3\ln x+3$

4/11

Examiner Comments: (a) B0 M1 A1 (b) M1 M1 A0 M0 A0 B0 M0 A0

In part (a) (i), although this candidate substitutes $x = 2$ into $2x - q = 0$ to reach $q = 4$ the mark required some kind of explanation with "asymptote" or "the curve being undefined" being alluded to.

Part (a)(ii) gives an acceptable proof that $p = 15$

In part (b) the strategy mark can be awarded as the candidate uses partial fractions and integrates using logs. There is an attempt at using partial fractions, scoring the next M mark, but the accuracy mark is denied as it is incorrect. As the limits are from 0 to 3 the B mark cannot be awarded.

Student Response B

(8)

a $(2x-4)(x+3) = 2x^2+2x-12$

ii $\dfrac{p-3x}{(2x-4)(x+3)} = \dfrac{1}{2}$ when $x=3$

$\dfrac{p-9}{12} = \dfrac{1}{2}$ $\quad 6 = p-9$

$p = 15$

Question 13 continued

(b) $\int \dfrac{15-3x}{2x^2+2x-12}$

$\int \dfrac{A}{(2x-4)} + \dfrac{B}{(x+3)} = (x+3)A + (2x-4)B = 15-3x$

when $x=-3$

$-10B = 24$

$B = \cancel{-2.4}\ \dfrac{24}{-10}\ \dfrac{12}{5}$

when $x = 2$

$5A = 9$

$A = \dfrac{5}{9}$

$\int \dfrac{5}{9(2x-4)} + \dfrac{12}{5(x+3)}\, dx$

$= \dfrac{5}{9}\left(\dfrac{1}{2}\ln|2x-4|\right) + \dfrac{12}{5}\ln|x+3|+c$

$\dfrac{5}{18}\ln|2x-4| + \dfrac{12}{5}\ln|x+3|+c$

between 3 and $x = 5$

$\left[\dfrac{5}{18}\ln|10-4| + \dfrac{12}{5}\ln|5+3|\right]$

$-\left[\dfrac{5}{18}\ln|6-4| + \dfrac{12}{5}\ln|3+3|\right]$

$= \dfrac{5}{18}\ln 6 + \dfrac{12}{5}\ln 8$

$-\dfrac{5}{18}\ln 2 - \dfrac{12}{5}\ln 6$

7/11

Examiner Comments: (a) B0 M1 A1 (b) M1 M1 A0 M1 A1 B1 M0 A0

In part (a) there is no merit in part (i) but part (ii) is correct.

In part (b) the first method mark can be awarded as the candidate uses partial fractions and integrates using logs. There is an attempt at using partial fractions, scoring the next M mark, but the accuracy mark is denied due to the missing – sign. The next M1 A1 are awarded as the integration is carried out correctly. As the limits are from 3 to 5 the B mark is also awarded. No appropriate log work is seen so no more marks are available.

Student Response C

i) Vertical asymptotes means denominator = 0 for that value

$\therefore (2x - q) = 0$ when $x = 2$

$\therefore 4 - q = 0$

$\therefore q = 4$

(8)

a) ii) If C passes through $(3, \frac{1}{2})$:

$\frac{1}{2} = \frac{p-9}{(6-q)(3)} \rightarrow 3(6-q) = p - 9$

$18 - 3q = p - 9$

$3q + p = 27$

ii) $\therefore 3 \times 4 + p = 27 \longrightarrow p = 27 - 12 = 15$ $\therefore$ QED

b) Root $\Rightarrow p - 3x = 0 \rightarrow \therefore 3x = 15$

$x = 5$

$$\therefore \int_3^5 \frac{15 - 3x}{2(x-2)(x+3)}\,dx = \frac{3}{2}\int_3^5 \frac{5 - x}{(x-2)(x+3)}\,dx$$

$A(x+3) + B(x-2) \equiv \cancel{5x}\ 5 - x$

$(x = 2): \quad 5A = 3 \longrightarrow A = \frac{3}{5}$

$(x = -3): \quad -5B = 8 \longrightarrow B = -\frac{8}{5}$

$$\therefore \frac{3}{2}\int_3^5 \frac{3}{5(x-2)} - \frac{8}{5(x+3)}\,dx$$

$$= \frac{3}{2}\left[\frac{3}{5}\ln|x-2| - \frac{8}{5}\ln|x+3|\right]_3^5$$

$$= \frac{3}{2}\left[\frac{3}{5}\ln 3 - \frac{8}{5}\ln 8 - \frac{3}{5}\ln 1 + \frac{8}{5}\ln 6\right]$$

$$= \frac{3}{2}\left[\frac{3}{5}\ln 3 + \frac{8}{5}\ln\frac{3}{4}\right]$$

$$= \frac{9}{10}\ln 3 + \frac{24}{10}\ln\frac{3}{4} = \frac{9}{10}\ln 3 + \frac{24}{10}\ln 3 - \frac{24}{10}\ln 4$$

$\cancel{\neq \frac{9}{10}\ln 3 + \frac{18}{5}\ln 2}$ $\quad = \frac{33}{10}\ln 3 - \frac{18}{5}\ln 4$

$$= 3.3\ln 3 - \frac{36}{5}\ln 2$$

10/11

Examiner Comments: (a) B1 M1 A1 (b) M1 M1 A1 M1 A1 B1 M1 A0

This is an excellent solution.

Note that (a)(i) includes a reference to the vertical asymptotes of the curve.

Also worth pointing out is that the error in part (b) occurred two lines from the end where $\frac{24}{10}\ln 4$ was simplified to $\frac{18}{5}\ln 2$ rather than $\frac{24}{5}\ln 2$

Exemplar Question 14

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14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-p}{4} < y < \frac{p}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

(Total for Question 14 is 7 marks)

Mean Score: 2.4 out of 7

Examiner Comments:

A number of candidates did not attempt this question. Whether this was due to lack of understanding or time constraints is unknown.

Part (a) was generally well done with a surprising proportion using implicit differentiation. A good number who found $\frac{\mathrm{d}y}{\mathrm{d}x}$ correctly failed to substitute in y =0 to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin. Candidates who tried to expand sin 2y using the double angle formulae and then differentiating using the product rule often went wrong.

The small angle approximation in (b) part (i) was also well handled. Most settled for the form $x = 8y$ although some rearranged to y = . Marks were generally lost when $\sin 2y$ was replaced by 2θ instead of 2y. Many candidates failed to see the link between (a) and (b)(i) even though they had both answers correct. They were unable to see that the gradients were the same or else failed to explain themselves coherently.

Only a small percentage of candidates were able to secure full marks for part (c) and it was not uncommon to see the whole question left unanswered. This type of question was common on Core 3 and usually well answered.

Mark Scheme

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
		(7 marks)	

Notes

(a)

M1: Attempts to differentiate $x = 4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy} = k\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$ or $1 = k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = ...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8\sin y\cos y \Rightarrow \frac{dx}{dy} = \pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen

Watch for candidates who reach this answer via $\frac{dx}{dy} = 8\cos 2x \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x = 8y$ oe such as $x = 4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$,

attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ E.g. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = \ldots\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

Student Response A

a) $x = 4\sin 2y$

$\frac{dx}{dy} = 8\cos 2y$

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$

bi) $x = 8y$

bii) proportional

c) $\frac{dy}{dx} = \frac{1}{8\cos 2y}$

$\cos 2y = \cos^2 y + \sin^2 y$

$\sin 2y = 2\sin y\cos y$

$\cancel{\sin^{-1}\left(\frac{x}{4}\right) = 2y}$

$\cancel{2\sin^2 2y = 4\sin^2 y\cos^2 y}$

$x^2 = 16\sin^2 2y$

$x^2 = 32\sin^2 y\cos^2 y$

$\sin^{-1}\left(\frac{x}{4}\right) = 2y$

$\frac{dy}{dx} = \frac{1}{8\cos\left(\sin^{-1}\left(\frac{x}{4}\right)\right)}$

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Examiner Comments: (a) M1 A0 (b) B1 B0 (c) M0 A0 A0

In part (a) the differentiation is correct, but the candidate does not attempt to find the value of $\frac{dy}{dx}$ at the origin.

Part (b)(i) is correct but the answer offered to part (ii) does not answer the question.

In part (c) there is no merit in this kind of solution.

Student Response B

14. a) $\frac{dx}{dy} = 8\cos 2y$

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$= \frac{1}{8\cos 2y}$

$= \frac{1}{8}\sec 2y$

At $y = 0$, $\frac{dy}{dx} = \frac{1}{8}\sec 0$

$= \frac{1}{8}$

b) i) $\sin 2y \approx 2y$

$x = 4 \times 2y$

$x = 8y$

$y = \frac{1}{8}x$

ii) The equation of the line for points close to the origin is $y = \frac{1}{8}x$. This is the gradient at $x = 0$.

c) $\frac{dx}{dy} = 8\cos 2y$

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$

$= \cancel{\frac{1}{8(1-2\sin^2 y)}}$

$= \frac{1}{8\sqrt{1-\sin^2 2y}}$

$= \frac{1}{\frac{1}{2}\sqrt{16-16\sin^2 2y}}$

$= \frac{1}{\frac{1}{2}\sqrt{16-x^2}}$

$a = \frac{1}{2}$ $b = 16$

$16\sin^2 2y + 16\cos^2 2y = 16$

$16\cos 2y = \sqrt{16-16\sin^2 2y}$

$8\cos 2y = \frac{1}{2}\sqrt{16-16\sin^2 2y}$

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Examiner Comments: (a) M1 A1 (b) B1 B0 (c) M1 A0 A0

Part (a) is correct

Part (b)(i) is correct

In part (b)(ii) the statement offered is too vague

In part (c) the method used by this candidate is sound and although their third line is correct, their fourth is not. So, neither accuracy mark may be awarded.

Student Response C

14. $x = 4\sin 2y$

a. $\frac{dy}{dx}$ $\quad \frac{dx}{dy} = 8\cos 2y$

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$ $\quad$ At $(0,0)$ $\frac{dy}{dx} = \frac{1}{8\cos 0} = \frac{1}{8}$

bi. $\sin 2y \approx 2y$ $\quad x = 8y$ $\quad y = \frac{x}{8}$

ii. The answer for part (a) is the gradient of the equation linking x and y at points close to the origin.

c. $\frac{dy}{dx} = \frac{1}{8\cos\frac{x}{4}} = \frac{1}{8\cos 2y}$

$\cos^2 2y + \sin^2 2y = 1$

$\cos^2 2y = 1 - \sin^2 2y$

$\cos 2y = \sqrt{1 - \sin^2 2y}$

$\frac{x}{4} = \sin 2y$ $\quad 2y = \arcsin\frac{x}{4}$

$\cos 2y = \sqrt{1 - \frac{x^2}{16}}$

$\frac{dy}{dx} = \frac{1}{8} \times \frac{1}{\sqrt{1 - \frac{x^2}{16}}} = \frac{1}{8\left(\sqrt{1 - \frac{x^2}{16}}\right)}$

$= \frac{1}{8\sqrt{16 - x^2}}$

$a = 8 \quad b = 16$

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Examiner Comments: (a) M1 A1 (b) B1 B1 (c) M1 A1 A0

An excellent solution.
Points to note are:
Part (b)(ii) is an acceptable explanation of the link between (a) and (b)(i)

In part (c) the penultimate line is correct, just not in the form required by the question, so the final accuracy mark is withheld as $\dfrac{1}{8\sqrt{16-x^2}}$ is incorrect.

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Exemplar Question 1

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1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

(Total for Question 1 is 3 marks)

Mean Score 1.4 out of 3

Examiner Comments

This question proved challenging for a significant number of candidates. This was probably because of the more open-ended nature of the question, allowing candidates to select their own problem-solving strategy to express y as a function of x. It was not apparent to most candidates that y could be expressed as a linear function of x.

More able candidates, however, were able to provide a concise solution. Other candidates were still able to achieve full marks but expressed y as a more complicated function of x. A few candidates attempted to express x as a function of y, suggesting a lack of understanding of the terminology.

It appeared that most candidates failed to realise that this question could be solved purely using indices. Instead, most candidates chose to immediately take logarithms as their first step towards a solution. A correct first step, e.g. $\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$, was rewarded by the mark scheme. Unfortunately, some candidates combined taking logarithms with an incorrect application of the addition law for logarithms as their first step, therefore gaining no marks – for example $2^x \times 4^y \to \log 2^x \times \log 4^y$. Most candidates who had a correct first step of work went on to achieve full marks, with some only failing to do so by making an arithmetic error.

This question highlighted some candidates' weaker algebraic, indices and logarithms skills. Common errors included $2^x \times 4^y \to 8^{xy}$, $2^x \times 4^y \to 8^{x+y}$, $\frac{1}{2^x\, 2\sqrt{2}} \to \frac{1}{4^x \sqrt{2}}$, or $2^x \times 4^y = \frac{1}{2\sqrt{2}} \to 4^y = \frac{2^x}{2\sqrt{2}}$.

The most common correct answers were $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$, $y = \log_4\left(\frac{1}{2^x\, 2\sqrt{2}}\right)$ or $y = \frac{\ln\left(\frac{1}{2^x\, 2\sqrt{2}}\right)}{\ln 4}$. Only a minority of candidates obtained a correct $y = -\frac{1}{2}x - \frac{3}{4}$.

Mark Scheme

Question	Scheme	Marks	AOs
1	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \quad \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\frac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = \ldots$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = \ldots$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = \ldots$	M1	2.1
	$y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 5	$4^{\frac{1}{2}x} \times 4^y = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 1

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain y written as a function of x.
A1:	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \frac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where y is written as a function of x
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \frac{1}{2\sqrt{2}} \Rightarrow 4^y = \frac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

Student Response A

$2^x 4^y = \frac{1}{2\sqrt{2}}$ $\quad 2^x 2^{2y} = \frac{1}{2\sqrt{2}}$ $\quad 2\sqrt{2} = 2 \times 2^{1/2} = 2^{3/2}$

$2^x = \frac{1}{4^y 2\sqrt{2}}$ $\quad 2^x 2^{2y} = 2^{-3/2}$

$x \times 2y = -3/2$

$x = \frac{-1.5}{2y}$

$x = \frac{-3}{4y}$ $\quad x4y = -3$

$y = \frac{-3}{4x}$

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Examiner Comments

B1 M0 A0

B1: Writes a correct equation $2^x 2^{2y} = 2^{-\frac{3}{2}}$ which is in powers of 2 only.

M0: Does not use correct index laws to write y as a function of x. This is evidenced by the step from a correct $2^x 2^{2y} = 2^{-\frac{3}{2}}$ to an incorrect $x \times 2y = -\frac{3}{2}$.

A0: Follows M0

Student Response B

(3)

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}.$$

$$2^x \times (2^2)^y = \frac{1}{2\sqrt{2}}.$$

$$2^x \times 2^{2y} = 2^{-\frac{3}{2}}.$$

$$x + 2y = -\frac{3}{2}.$$

$$2y = -\frac{3}{2} + x.$$

$$y = -\frac{3}{2} + \frac{x}{2}.$$

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Examiner Comments

B1 M1 A0

B1 M1: Writes a correct equation in powers of 2 only and uses correct index laws (which are implied by the correct step from $2^x \times 2^{2y} = 2^{-\frac{3}{2}}$ to $x + 2y = -\frac{3}{2}$) to obtain *y* as a function of *x*.

A0: Incorrect answer.

Student Response C

$$2^x \times 4^y = \frac{\sqrt{2}}{4}$$

$$4^y = \frac{\sqrt{2}}{4} \times \frac{1}{2^x}$$

$$4^y = \frac{\sqrt{2}}{4(2^x)}$$

$$\log 4^y = \log\left(\frac{\sqrt{2}}{4(2^x)}\right)$$

$$y = \frac{\log\left(\frac{\sqrt{2}}{4(2^x)}\right)}{\log 4}$$

$$y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right).$$

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Examiner Comments

B1 M1 A1

B1 M1: Complete process of writing a correct equation involving logarithms (e.g. $\log 4^y = \log\left(\frac{\sqrt{2}}{4(2^x)}\right)$) and using correct log laws to obtain y as a function of x.

A1: Obtains a correct $y = \frac{\log\left(\frac{\sqrt{2}}{4(2^x)}\right)}{\log 4}$. Note that $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ is also correct.

Exemplar Question 2

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2. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in m s^{-1}.

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

Using all of this information,

(a) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

(Total for Question 2 is 4 marks)

Mean Score 1.4 out of 4

Examiner Comments

This question proved to be challenging, with a significant number of candidates struggling to realise that the length of the runway could be estimated by the area under the speed-time curve generated from the table of values given in the question.

In part (a), only a minority of candidates estimated the length of the runway by applying the trapezium rule, which was considered the most appropriate method, with many of these giving the correct answer of 415 m. While the table of values clearly shows an interval width $h=5$, the application of the formula $h=\frac{b-a}{n}$ with $n=6$ instead of $n=5$ led some candidates to use an incorrect strip width $h=\frac{25}{6}$. Other candidates made calculation errors, bracketing errors or transcription errors.

Some candidates used the equations of motion with constant acceleration (i.e. the *suvat* equations) to estimate the length. Many of the candidates who used this method applied the calculation $\left(\frac{2+42}{2}\right)(25)$ to obtain an estimate of $550\,\text{m}$. A few used $v=u+at$ to find the acceleration, followed by $s=ut+\frac{1}{2}at^2$ to estimate the length. Some candidates used $u=0$ in their *suvat* calculations, in contradiction to the table of values which indicated an initial speed of 2 m s^{-1}. A few candidates used a method which was equivalent to the trapezium rule, of applying *suvat* to each of the 5 time-intervals, to give the correct answer of $415\,\text{m}$.

Other methods seen included: summing up the area of rectangles which enclosed the area above the curve for each of the 5 time intervals; summing up areas of rectangles which enclosed the area below the curve for each of the 5 time intervals; and applying (average speed)$\times$(total time).

The most common methods which were deemed incorrect included $25\times 42=1050$; $\frac{1}{2}(25)(42)=525$; $\sum(\text{speed})(\text{time})=(0)(2)+(5)(5)+(10)(10)+\ldots+(25)(42)=2005$; or a 6 rectangle approach (instead of 5 rectangles).

As for part (b), while many candidates who used a trapezium rule or *suvat* method in Q02(a) identified their estimate of the length as an overestimate, some found difficulty in articulating a reason for this. Those who were most successful drew a diagram which showed clearly the extra area generated by the trapezium rule in relation to the curve. Some candidates sketched the speed-time curve and explained that the curve was convex or that the acceleration was continually increasing. Only a few candidates who used a rectangle method were able to give a correct reason.

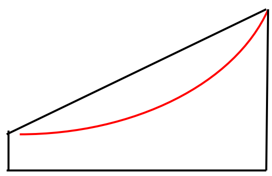
Some candidates answered part (b) by finding an alternative estimate and comparing it with their estimate in part (a). For example, some found 415 in part (a) and 550 in part (b), and then concluded that ‘415 must be an underestimate’. Other candidates, who also received no credit, gave physical explanations about friction, air resistance or the shape of the aircraft, while others stated that ‘the jet accelerated smoothly’.

Mark Scheme

Question	Scheme	Marks	AOs
2	Time (s): 0, 5, 10, 15, 20, 25 Speed (m s^{-1}): 2, 5, 10, 18, 28, 42		
(a)	Uses an allowable method to estimate the area under the curve. E.g. **Way 1:** an attempt at the trapezium rule (see below) **Way 2:** $\{s=\}\left(\frac{2+42}{2}\right)(25)\ \{=550\}$ **Way 3:** $42=2+25(a)\Rightarrow a=1.6\Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$ **Way 4:** $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \{=63(5)=315\}$ **Way 5:** $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \{=103(5)=515\}$ **Way 6:** $\{d=\}\ \frac{315+515}{2}\ \{=415\}$ **Way 7:** $\{d=\}\left(\frac{2+5+10+18+28+42}{6}\right)(25)\ \{=437.5\}$	M1	3.1a
	$\frac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\frac{1}{2}\times[\text{"315"}+\text{"515"}]$	M1	1.1b
	$=415\ \{\text{m}\}$	A1	1.1b
		(3)	
(b) Alt 1	<u>**Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)**</u>		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{\text{d}^2y}{\text{d}x^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5)	B1ft	2.4
		(1)	
(b) Alt 2	<u>**Uses a Way 4 method in (a)**</u>		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
			(4 marks)

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42);\ \lambda>0$ for 1st M1
	Way 2: Uses $s=\left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed)$\times$(time)

Notes for Question 2 Continued	
(a)	***continued***
M1:	Correct trapezium rule method with $h=5$. Condone a slip on one of the speeds. The '2' and '42' should be in the correct place in the [......].
A1:	415
Note:	Units do not have to be stated
Note:	Give final A0 for giving a final answer with incorrect units. e.g. give final A0 for 415 km or $415\,\mathrm{m\,s}^{-1}$
Note:	Only the 1st M1 can only be scored for Way 2, Way 3, Way 4, Way 5 and Way 7 methods
Note:	Full marks in part (a) can only be scored by using a Way 1 or a Way 6 method.
Note:	Give M0 M0 A0 for $\{d=\}\ 2(5)+5(5)+10(5)+18(5)+28(5)+42(5)\ \ \{=105(5)=525\}$ (i.e. using too many rectangles)
Note	Condone M1 M0 A0 for $\left[\frac{(2+10)}{2}(10)+\frac{(10+18)}{2}(5)+\frac{(18+28)}{2}(5)+\frac{(28+42)}{2}(5)\right]=395$ m
Note:	Give M1 M1 A1 for $5\left[\frac{(2+5)}{2}+\frac{(5+10)}{2}+\frac{(10+18)}{2}+\frac{(18+28)}{2}+\frac{(28+42)}{2}\right]=415$ m
Note:	Give M1 M1 A1 for $\frac{5}{2}(2+42)+5(5+10+18+28)=415$ m
Note:	**<u>Bracketing mistake:</u>**
	Unless the final calculated answer implies that the method has been applied correctly
	give M1 M0 A0 for $\frac{5}{2}(2)+2(5+10+18+28)+42\ \ \{=169\}$
	give M1 M0 A0 for $\frac{5}{2}(2+42)+2(5+10+18+28)\ \ \{=232\}$
Note:	Give M0 M0 A0 for a Simpson's Rule Method
(b)	**Alt 1**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Allow the explanation "curve concaves upwards"
Note:	Do not allow explanations such as "curve is concave" or "curve concaves downwards"
Note:	Do not allow explanation "gradient of the curve is positive"
Note:	Do not allow explanations which refer to "friction" or "air resistance"
Note:	The diagram opposite is sufficient as an explanation. It must show the top of a trapezium lying above the curve. 

(b)	**Alt 2**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Do not allow explanations which refer to "friction" or "air-resistance"

Student Response A

$S = ut + \frac{1}{2}at^2$ $a = \frac{v-u}{\Delta t}$ $a = \frac{42-2}{25}$ (1)

$a = 1.6 ms^{-2}$

$S = (2)(25) + \frac{1}{2}(1.6)(25)^2$

$= 550m$

b) an overestimate as resistive forces are not taken into account

1/4

Examiner Comments

(a) M1 M0 A0

M1: Uses Way 3 whereby a value for uniform acceleration is found and then applied to the equation for distance.

M0 A0: Way 3 cannot score these marks.

(b) B0

B0: Reference to "resistive forces" is not an allowable explanation.

Student Response B

2) ~~$s = ut +$~~ $\quad s = \frac{1}{2}(v+u)t \quad S = \frac{1}{2}(2+42) \times 25$ **(1)**

~~$v^2 = u^2 + 2a$~~ $\quad S = 550$ meters

b) overestimate as acceleration is ~~changing~~ increasing so value would be less

2/4

Examiner Comments

(a) M1 M0 A0
M1: Uses Way 2 whereby the initial speed, final speed and time taken are applied to the equation for distance.
M0 A0: Way 2 cannot score these marks.

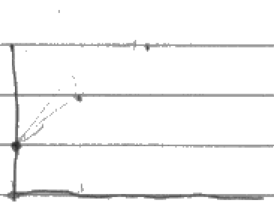
(b) B1
B1: States "Overestimate" which is correct for Way 2 and gives an acceptable explanation of "acceleration is increasing".

Student Response C

distance is area under graph.

$$\frac{1}{2}(5)\left((2+42)+2(5+10+18+28)\right)$$

$= 415$m.



b) Underestimate as curve is convex.

3/4

Examiner Comments

(a) M1 M1 A1
M1 M1 A1: Uses a fully correct trapezium rule method (Way 1) to give 415.

(b) B0
B0: States "Underestimate" which is incorrect for Way 1.

Exemplar Question 3

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3.

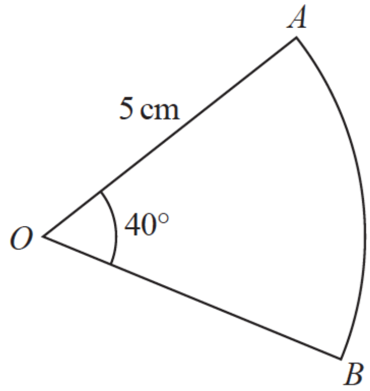

Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\begin{aligned}\text{Area of sector} &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2} \times 5^2 \times 40 \\ &= 500 \text{ cm}^2\end{aligned}$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

(Total for Question 3 is 3 marks)

Mean Score 2.5 out of 3

Examiner Comments

This question proved to be the most accessible question on this paper with many candidates obtaining full marks.

In part (a), many candidates explained that the angle 40° should have been converted to its equivalent in radians in order for it to be applied to the formula $\frac{1}{2}r^2\theta$. Some candidates' explanations lacked sufficient detail, e.g. they stated 'the angle is in degrees' but failed to mention that the angle used in the formula should have been expressed in radians. A few candidates did not give a reason in part (a), but correctly calculated the area of sector *AOB* in part (b).

Part (b) was also well-answered. Most candidates converted $40°$ to radians and applied a correct $\frac{2\pi}{9}$ to the given formula $\frac{1}{2}r^2\theta$. Some candidates applied a correct formula $\pi r^2\left(\frac{\theta}{360}\right)$ with $\theta = 40°$. Errors included converting $40°$ to either $\frac{40\pi}{360}$ or $\frac{9\pi}{2}$; applying $\frac{1}{2}(5^2)\left(\frac{2}{9}\right)$; and applying incorrect formulae such as $\frac{1}{2}r^2\sin\theta$ or $\frac{1}{2}\pi r^2\theta$. Most candidates gave the correct answer in exact form as $\frac{25}{9}\pi$, and some gave an answer 8.73 which was rounded to 3 significant figures, while others gave both exact and rounded answers. Most candidates gave the units cm^2 in their answer to part (b). On this occasion, a lack of units in their final answer was condoned.

Mark Scheme

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$=\frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$=\frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
		(3 marks)	

Notes for Question 3	
(a)	
B1:	Explains that the formula use is only valid when angle *AOB* is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta=\frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right)=\frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

Student Response A

a) θ should be in radians when using that formula

b) $40^\circ = \frac{2}{9}^c$

Area of Sector $= \frac{1}{2} \times 5^2 \times \frac{2}{9}$

$= \frac{25}{9}$ cm^2

1/3

Examiner Comments

(a) B1

B1: Correct explanation.

(b) M0 A0

M0: Has converted incorrectly from degrees to radians. Note: A correct angle in radians must be used in the formula $\frac{1}{2}r^2\theta$.

A0: Follows M0.

Student Response B

a) The area should be multiplied by pi not half

b) $A = \pi r^2 \frac{\theta}{360}$

$= \pi \times 5^2 \times \frac{40}{360}$

$A = \frac{25}{9}\pi = 8.73\text{cm}^2$

2/3

Examiner Comments

(a) B0

B0: Incorrect explanation.

(b) M1 A1

M1 A1: Uses a correct method to find a correct answer $\frac{25}{9}\pi$. Note that 8.73 is also correct.

Student Response C

(a) She has used the angle in degrees not radians ~~to the diameter~~

(b) $40° \Rightarrow \frac{40}{180} = \frac{2}{9}$

$\therefore \theta = \frac{2\pi}{9}$

$\therefore A = \frac{1}{2} \times 5^2 \times \frac{2\pi}{9} = \frac{25\pi}{9}\text{ cm}^2$

$= 8.7\text{ cm}^2$

3/3

Examiner Comments

(a) B1
B1: Correct explanation.

(b) M1 A1

M1 A1: Uses a correct method to find a correct answer $\frac{25\pi}{9}$. Note: 8.7 which does not round to 8.73 is ignored because candidate obtains the correct exact answer.

Exemplar Question 4

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4.

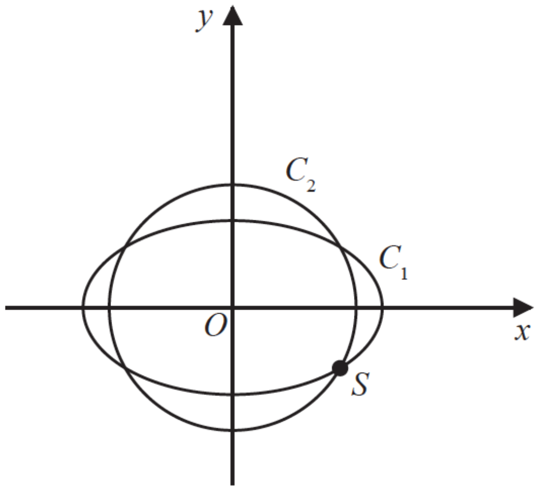


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \qquad y = 4\sqrt{2}\sin t, \qquad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

(Total for Question 4 is 6 marks)

Mean Score 4.0 out of 6

Examiner Comments

This question was well attempted by both medium ability and higher ability candidates. Lower ability candidates struggled to make progress, with most of them scoring no more than one mark.

There were two common methods that were used by candidates. The first method, covered by Way 1 in the mark scheme, was the substitution of the parametric equations of C_1 into the Cartesian equation of C_2 to give an equation in t only; the trigonometric identity $\sin^2 t + \cos^2 t \equiv 1$ being used to obtain an equation in $\sin^2 t$ (or $\cos^2 t$) only, and a value for $\sin t$ (or $\cos t$) found. The second method, covered by Way 2 in the mark scheme, was the application of the trigonometric identity $\sin^2 t + \cos^2 t \equiv 1$ to the parametric equations of C_1 to give $\left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1$, the Cartesian equation of C_2 being used to obtain an equation in x^2 (or y^2) only, and a value for x (or y) found.

A few candidates used a correct method of progressing from $(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$ to either $\tan^2 t = 1$ or $\cos 2t = 0$. In all these methods, substituting back yielded their coordinates for S, with most candidates realising that $x_S > 0$ and $y_S < 0$. Some candidates incorrectly stated S as $(5\sqrt{2}, 4)$. In the first method, successful candidates used either $t = \frac{7\pi}{4}$ or $t = \frac{\pi}{4}$, and then applied symmetry to find the correct coordinates for S as either $(5\sqrt{2}, -4)$ or $(7.07, -4)$.

An area of concern centred around arithmetical slips or errors in elementary algebra. Examples of the former included $(4\sqrt{2})^2$ becoming 8 or 64; $\sin^2 t = \frac{1}{2}$ becoming $\sin t = \frac{1}{4}$; and $\frac{x^2}{100} + \frac{y^2}{32} = 1$ becoming $32x^2 + 100y^2 = 1$ or 100 or 32. Examples of the latter included the invalid methods of $y^2 = 66 - x^2$ becoming $y = \sqrt{66} - x$ and $\sin^2 t = 1 - \frac{x^2}{100}$ becoming $\sin t = 1 - \frac{x}{10}$. Candidates who made no creditable progress included those who differentiated the parametric equations for C_1, and those who obtained, e.g. $y = 4\sqrt{2}\sin\left(\arccos\left(\frac{x}{10}\right)\right)$ leading to $x^2 + 32\sin^2\left(\arccos\left(\frac{x}{10}\right)\right) = 66$.

Mark Scheme

Question	Scheme	Marks	AOs
4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$		
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$ $100\cos^2 t + 32(1-\cos^2 t) = 66$	M1	2.1
		A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = \ldots$ $68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$	M1	3.1a
	$\frac{x^2}{100} + \frac{66-x^2}{32} = 1$ $\frac{66-y^2}{100} + \frac{y^2}{32} = 1$	M1	2.1
		A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = \ldots$ $2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
Way 3	$\{C_2: x^2 + y^2 = 66 \Rightarrow\}\ \ x = \sqrt{66}\cos\alpha,\ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ \ 10\cos t = \sqrt{66}\cos\alpha,\ \ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$	M1	3.1a
	then continue with applying the mark scheme for Way 1		
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right) + 32\left(\frac{1-\cos 2t}{2}\right) = 66$	M1	2.1
		A1	1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = \ldots$	dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$		
		(6 marks)	

Notes for Question 4	
	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \le \sin t \le 1$ or $\cos t = ...$ where $-1 \le \cos t \le 1$ Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2 \alpha + \sin^2 \alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs
4	$C_1: \ x = 10\cos t\ , \ y = 4\sqrt{2}\sin t, \quad 0 \le t < 2\pi; \quad C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \ \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \ \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Student Response A

$\cancel{\text{cos}}\ \sin^2 t + \cos^2 t = 1$

$x = 10\cos t$

$\frac{x^2}{100} = \cos^2 t$

$y = 4\sqrt{2}\sin t$

$y^2 = 32\sin^2 t$

$\frac{y^2}{32} = \sin^2 t$

$\frac{y^2}{32} + \frac{x^2}{100} = 1$

C_1:

$\frac{y^2}{32} + \frac{x^2}{100} - 1 = 0$

C_2:

$x^2 + y^2 = 66$

$x^2 + y^2 - 66 = 0$

$\frac{y^2}{32} + \frac{x^2}{100} - 1 = x^2 + y^2 - 66$ $(+1)$

$\frac{y^2}{32} + \frac{x^2}{100} = x^2 + y^2 - 65$ $(\times 100)$

$\frac{100y^2}{32} + x^2 = 100x^2 + 100y^2 - 6500$

$(-x^2)$

$(-100y^2)$

$\frac{100y^2}{32} - 100y^2 = 99x^2 - 6500$

$y^2\left(\frac{100}{32} - 100\right) = 99x^2 - 6500$

$y^2 = \frac{99x^2 - 6500}{\frac{100}{32} - 100}$

$y = \sqrt{\left(\frac{99x^2 - 6500}{\frac{100}{32} - 100}\right)}$

1/6

Examiner Comments

M1 M0 A0 M0 M0 A0 [Uses Way 2]
M1: Uses $\sin^2 t + \cos^2 t \equiv 1$ to convert the parametric equations for C_1 into a Cartesian equation for C_1.
M0 A0 dM0 M0 A0: Makes no further creditable progress.

Student Response B

$x^2 + y^2 = 66$

$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$

$100\cos^2 t + 16\sin^2 t = 66$

$84\cos^2 t + 16(\cos^2 t + \sin^2 t) = 66$

$84\cos^2 t + 16(1) = 66$

$84\cos^2 t = 50$

$\cos^2 t = 25/42$

$\cos t = \pm 5\sqrt{42}/42 \quad \Rightarrow \quad x = 10\cos t$

$= 10(\pm 5\sqrt{42}/42)$

$= 7.715, -7.715$

We want +ve x

$\Rightarrow y = 4\sqrt{2}\left(\sin\left(\cancel{\cos}\ \arccos(5\sqrt{42}/42)\right)\right)$

$= 3.599$

Want -ve of answer

$\Rightarrow s = (7.715, -3.599)$

4/6

Examiner Comments

M1 M1 A0 M1 M1 A0 [Uses Way 1]

M1: Substitutes the parametric equations of C_1 into the Cartesian equation C_2 to obtain an equation in *t* only.

M1: Uses $\sin^2 t + \cos^2 t \equiv 1$ to obtain an equation which is in $\cos^2 t$ only.

A0: Their equation $84\cos^2 t + 16(1) = 66$ is incorrect.

dM1 (dependent on the 2 previous M marks): Rearranges to make $\cos t = \ldots$ where $-1 \leq \cos t \leq 1$.

M1: Substitutes their value for *t* back into the original parametric equations to find corresponding values for *x* and *y*.

A0: Incorrect answer.

Student Response C

$y = 4\sqrt{2}\sin(t) \qquad x = 10\cos(t)$

when subbed in

$(10\cos(t))^2 + (4\sqrt{2}\sin(t))^2 = 66$

$100\cos^2(t) + 32\sin^2(t) = 66$

$100 - 100\sin^2(t) + 32\sin^2 t = 66$

$100 - 68\sin^2 t = 66$

$100 = 66 + 68\sin^2 t$

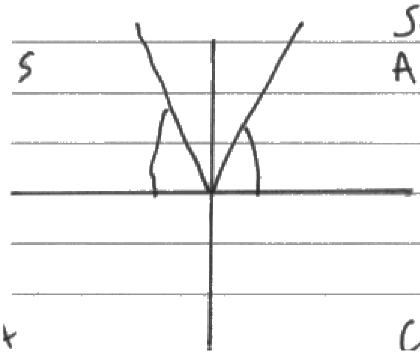
$44 = 68\sin^2 t$

$\frac{11}{17} = \sin^2 t \quad c\sqrt{\ })$

$\frac{\sqrt{187}}{17} = \sin t$

$\sin^{-1}\left(\frac{\sqrt{187}}{17}\right) = 0.9346$ rads

S A



T C

$t = 0.9346, 2.207$

when $t = 2.207$
$y = 4.550131$
$x = -5.941$

when $t = 0.9346$
$y = 4.55\,0155$
$x = 5.941$

in quadrant 4 x +ve y −ve

some know that $t = 0.9346$

when $t = 0.9346$
$S_y = -4.55$
$S_x = 5.941$

$S = (5.94, -4.55)$

5/6

Examiner Comments

M1 M1 A1 M1 M1 A0 [Uses Way 1]

M1: Substitutes the parametric equations of C_1 into the Cartesian equation C_2 to obtain an equation in *t* only.

M1 A1: Uses $\sin^2 t + \cos^2 t \equiv 1$ to obtain a correct equation $100 - 100\sin^2 t + 32\sin^2 t = 66$ which is in $\sin^2 t$ only.

dM1 (dependent on the 2 previous M marks): Rearranges to make $\sin t = \ldots$ where $-1 \leq \sin t \leq 1$.

M1: Substitutes their value for *t* back into the original parametric equations to find corresponding values for *x* and *y*.

A0: Incorrect answer.

Exemplar Question 5

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5.

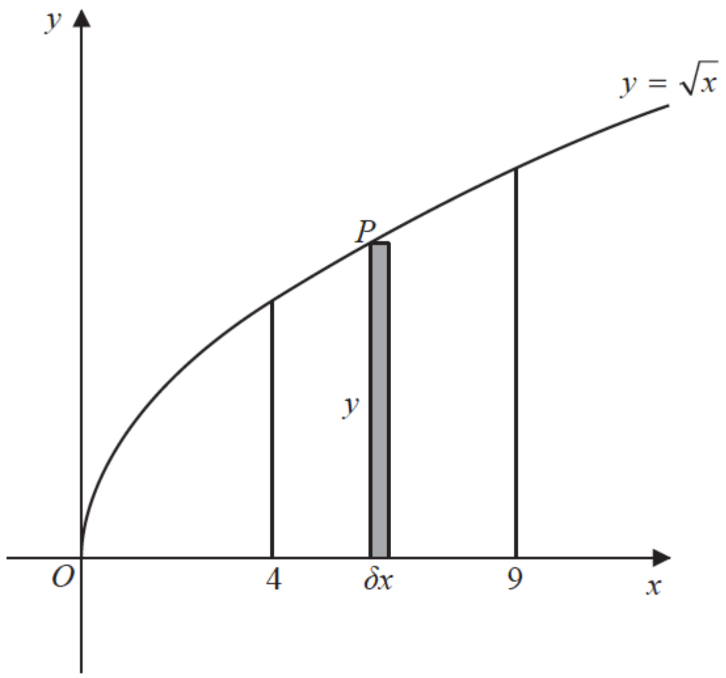


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$$

(3)

(Total for Question 5 is 3 marks)

Mean Score 0.8 out of 3

Examiner Comments

This question proved challenging for many candidates and some candidates were unfamiliar with the notation that was used. Most candidates scored either all three marks or zero marks in this question. There were a substantial number of blank responses.

It is noted that section 8.4 (guidance) of the Pure Specification states 'Recognise $\int_a^b \mathrm{f}(x)\,\mathrm{d}x = \lim_{\delta x \to 0} \sum_{x=a}^{b} \mathrm{f}(x)\,\delta x$'. It is also noted that a similar question had not appeared on the 9MA0 June 2018 papers, the SAMs, the specimen papers or the mock papers.

Those candidates who recognised that $\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$ was another way of writing $\int_4^9 \sqrt{x}\,\mathrm{d}x$ generally produced a correct solution to score all 3 marks. Only a few made errors in their integration of $\sqrt{x}$ or in the application of the limits 4 and 1.

Most candidates, however, made no creditable progress. Some applied the trapezium rule with $h = 1$, but did not link their trapezium rule attempt with a stated $\int_4^9 \sqrt{x}\,\mathrm{d}x$. Other candidates attempted to sum $\sqrt{x}$ using integer values from $x = 4$ to $x = 9$, resulting in $\sqrt{4} + \sqrt{5} + \sqrt{6} + \ldots + \sqrt{9}$ $(=15.1597\ldots)$. Some candidates mistakenly interpreted 'δx' as a trigger to differentiate $\sqrt{x}$, with a few of these candidates attempting to differentiate $\sqrt{x}$ from first principles.

A few candidates applied the incorrect method $\left(\sqrt{4} + \sqrt{5} + \sqrt{6} + \ldots + \sqrt{9}\right) - \int_4^9 \sqrt{x}\,\mathrm{d}x$ to give an answer of 2.49 to 3 significant figures.

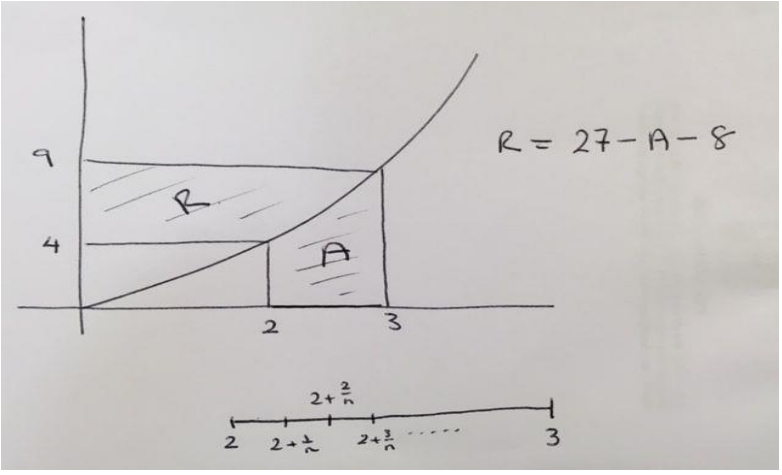
Mark Scheme

Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x \text{ is} \right\}$ $\int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}} \right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x - \int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3} x^{\frac{3}{2}} + c \right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question 5 Continued	
Alt	The following method is correct:



$$\text{Area }(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\displaystyle\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\;\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or} \quad \text{awrt } 12.7$$

Student Response A

$$\lim_{\delta x \to 0} \frac{f(x+\delta x) + f(x)}{\delta x}.$$

$$\lim_{h \to 0} \frac{f(x+h) + f(x)}{h}.$$

$$\lim_{\delta x \to 0} \frac{\sqrt{x+\delta x} + \sqrt{x}}{\delta x}$$

$$= \int_4^9 \sqrt{x} = \left[\tfrac{1}{2}x^{-1/2}\right]_4^9$$

$$= \left(\tfrac{1}{2}(9)^{-1/2}\right) - \left(\tfrac{1}{2}(4)^{-1/2}\right)$$

$$= \tfrac{1}{6} - \tfrac{1}{4}$$

1/3

Examiner Comments

B1 M0 A0

B1: States $\int_4^9 \sqrt{x}\,dx$, where the missing 'dx' is condoned.

M0: Does not integrate $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$.

A0: Follows M0.

Student Response B

(3)

$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$ δx tends to 0.

$\therefore \int_4^9 \sqrt{x}$ as $\lim \delta x \to 0$

$\therefore \int_4^9 x^{1/2}\,dx$

$\therefore \left[2x^{3/2}\right]_4^9$

$\therefore 2(9)^{3/2} - 2(4)^{3/2}$

$= 54 - 16 = 38$

2/3

Examiner Comments

B1 M1 A0

B1: States $\int_4^9 \sqrt{x}\,dx$.

M1: Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$.

A0: Incorrect answer.

Student Response C

(3)

as $\delta x \to 0$, $\sum_{x=4}^{9} \sqrt{x}\,\delta x \to \int_4^9 x^{\frac{1}{2}}\,dx$

$\int_4^9 x^{\frac{1}{2}}\,dx = \left[\frac{x^{\frac{3}{2}}}{3/2}\right]_4^9 = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$

$= \left(\frac{2}{3}(9^{\frac{3}{2}})\right) - \left(\frac{2}{3}(4^{\frac{3}{2}})\right) = 18 - \frac{16}{3} = \frac{38}{3}$ sqr units

3/3

Examiner Comments

B1 M1 A1

B1 M1 A1: Correct solution leading to a correct answer $\frac{38}{3}$.

Exemplar Question 6

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6.

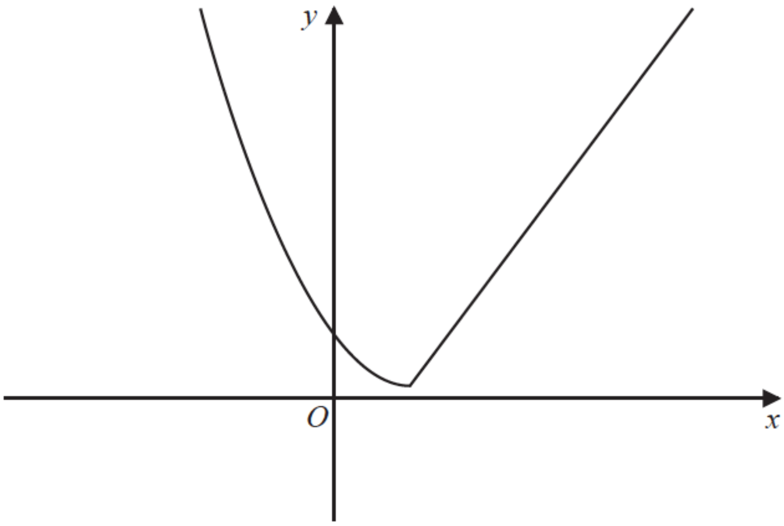

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for Question 6 is 10 marks)

Mean Score 5.6 out of 6

Examiner Comments

This question discriminated well between candidates of all abilities, with only a few candidates gaining full marks. There were some marks in parts (b) and (d) which were accessible to the majority of candidates.

In part (a), many candidates struggled with the split domain for $g(x)$ (i.e. for $x \le 2$, $g(x) = (x-2)^2 + 1$ and for $x > 2$, $g(x) = 4x - 7$) and only a minority gained both marks. Many candidates found a correct $g(0) = 5$, but a significant number used the incorrect method of substituting '5' to give $gg(0) = (5-2)^2 + 1 = 10$. Only a minority used a correct method and found $gg(0) = 4(5) - 3 = 13$. Occasionally candidates found an algebraic expression for $gg(x)$, although this was often incorrect. Very few obtained a correct answer of 13 by substituting $x = 0$ into $4((x-2)^2 + 1) - 7$ or $4(x-2)^2 - 3$.

In part (b), many candidates attempted to solve the inequality $g(x) > 28$ by finding the critical values for *x*. Most solved the linear equation to find a correct critical value $x = \frac{35}{4}$, and many solved the quadratic equation to find $x = 2 \pm 3\sqrt{3}$. A minority rejected $x = 2 + 3\sqrt{3}$ and deduced that $x = 2 - 3\sqrt{3}$ was the second critical value. Only a few candidates used the critical values and the diagram in Q6 to write down the correct solution $x < 2 - 3\sqrt{3} \cup x > \frac{35}{4}$. Some candidates, however, incorrectly gave $x = \frac{35}{4}$, $x = 2 \pm 3\sqrt{3}$ or gave $x < 2 + 3\sqrt{3} \cup x > \frac{35}{4}$ as their final answer. A few candidates made no creditable progress in Q06(b) by using a method of equating both parts of $g(x)$ and attempting to solve the equation $(x-2)^2 + 1 = 4x - 7$.

Although part (c) required a comment for both functions, f and g, some candidates only wrote a comment about one of the two functions. A significant number of candidates believed that g did not have an inverse due to its being defined in 'two parts'. Candidates who were successful in part (c) usually gave a reason such as, 'g is a many-one function and h is a one-one function'. A few correctly stated that the inverse of g is one-many, which is not a function, whereas the inverse of h is one-one, which is a function. A few candidates incorrectly described g as a 'one-many function'.

In part (d), only a few candidates applied the method of finding *x* by applying $h\left(-\frac{1}{2}\right)$. As this method was less prone to error, most candidates who used it scored full marks. Most candidates applied a complete method of finding their inverse $h^{-1}(x)$, followed by using their inverse to form and solve the equation $h^{-1}(x) = -\frac{1}{2}$. Many candidates who found a correct answer $x = 7.25$ lost the final mark in Q06(d). This was because they used the incorrect inverse $h^{-1}(x) = 2 + \sqrt{x-1}$ (instead of the correct $h^{-1}(x) = 2 - \sqrt{x-1}$) which led to their 'solving' the incorrect $\sqrt{x-1} = -\frac{1}{2}$ (which has no solutions) to give $x = 7.25$. A few candidates in part (d) incorrectly believed that $h^{-1}(x)$ referred to the reciprocal of h, or even to the first derivative of h with respect to *x*.

Mark Scheme

Question	Scheme	Marks	AOs
6 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	(4)	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $\text{h}^{-1}(x)$ } $=\pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 6

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Notes for Question 6 Continued	
(b)	***continued***
Note:	Give final A0 for $\{ x \in \mathbb{R} : x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of " $x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$ "
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for " h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \mathrm{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \mathrm{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\mathrm{h}^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

Student Response A

a) $\cancel{gg(x) = (x-2)^2+1}$

$\cancel{= ((4x-7)-2)^2+1}$

$\cancel{= (4x-9)^2+1}$

$\cancel{gg(0) = (81)+1} = 82$

$gg(x) = 4((x-2)^2+1) - 7$

$= 4(x-2)^2 - 3$

$gg(0) = 4(0-2)^2 - 3$

$= 13$

$\therefore gg(0) = 13$

b) $4(x-2)^2 - 3 > 28$

$4(x-2)^2 > 31$

$(x-2)^2 > 7.75$

$(x-2) > \sqrt{31}/2$

$x > \frac{4+\sqrt{31}}{2}$

$x > 4.783882181$

c) $g(x)$ cannot be made inverse as there are two separate equations.

d) $h(x) = (x-2)^2 + 1$ Let $y = x$

$x = (y-2)^2 + 1$

$x - 1 = (y-2)^2$

$\sqrt{x-1} = y - 2$

$\sqrt{x-1} + 2 = y$

$\therefore h^{-1}(x) = \sqrt{x-1} + 2$

$h^{-1}(x) = (x-1)^{1/2} + 2$

$h^{-1}(x) = -\frac{1}{2}$

$-\frac{1}{2} = (x-1)^{\frac{1}{2}} + 2$

$-\frac{5}{2} = (x-1)^{\frac{1}{2}}$

$\frac{25}{4} = x - 1$

$\frac{29}{4} = x$

$7.25 = x$

4/10

Examiner Comments

(a) M1 A1

M1 A1: Substitutes $x = 0$ into a correct $4(x-2)^2 - 3$ to obtain a correct $\mathrm{gg}(0) = 13$.

(b) M0 A0 M0 A0

M0: Attempts to solve $\mathrm{gg}(x) > 28$ instead solving $\mathrm{g}(x) > 28$.

A0 M0 A0: Follows M0.

(c) B0

B0: Incorrect explanation for g and no explanation for h.

(d) M1 M1 A0

M1: Obtains an inverse for h which is in an acceptable form $\mathrm{h}^{-1}(x) = \pm 2 \pm \sqrt{x \pm 1}$.

M1: Proceeds from an acceptable $2 + \sqrt{x-1} = -\frac{1}{2}$ to give an acceptable $\sqrt{x-1} = ...$

A0 cso: Although $x = 7.25$ is the correct answer, it has not been obtained from solving a correct equation $2 - \sqrt{x-1} = -\frac{1}{2}$. Note: Candidate's equation $-\frac{5}{2} = (x-1)^{\frac{1}{2}}$ has no real solutions.

Student Response B

a) $(0-2)^2+1$
$= 3$
$(4\times3)-7$
$= 5$

b)
$4x-7 = 28$
$x > 8.75$

$(x-2)^2+1 = 28$
$(x-2)^2 = 27$
$(x-2) = -3\sqrt{3}$

$x = +2 - 3\sqrt{3}$

$x < 2-3\sqrt{3}$

$x > 8.75 \quad , \quad x < 2-3\sqrt{3}$

c) because the function g depends on the number you put in whereas h doesn't

d) $y = (x-2)^2+1$
$\sqrt{(y-1)} = x-2$
$x = 2+\sqrt{y-1}$
$h^{-1}(x) = 2\pm\sqrt{x-1}$
$x = -\frac{1}{2}$
$2\pm\sqrt{-\frac{1}{2}-1}$
$h^{-1}(x) = 2 \pm\frac{\sqrt{6}}{2}i$

6/10

Examiner Comments

(a) M1 A0

M1: Substitutes $x=0$ into $(x-2)^2+1$ and substitutes their result (in this case '3') into the relevant part of $\mathrm{g}(x)$.

A0: Incorrect answer.

(b) M1 A1 M1 A1

M1 A1 M1 A1: Correct solution leading to a correct $x>8.75$, $x<2-3\sqrt{3}$.

(c) B0

B0: Incorrect explanation for g and an incorrect explanation for h.

(d) M1 M0 A0

M1: Obtains an inverse for h which is in an acceptable form $\mathrm{h}^{-1}(x)=\pm 2\pm\sqrt{x\pm 1}$.

M0: Does not attempt to solve their '$2+\sqrt{x-1}$' $=-\frac{1}{2}$ or their '$2-\sqrt{x-1}$' $=-\frac{1}{2}$.

A0: Follows previous M0.

Student Response C

(a). $g(0) = (0-2)^2+1 = 5.$

$gg(0) = 4\times5-7 = 13.$

(b) ① when $x \le 2$. $g(x) = (x-2)^2+1 > 28$

$(x-2)^2 > 29$ ~~$x-2$~~ $x-2 > \sqrt{29}$ or $x-2 < -\sqrt{29}$.

~~$x>$~~ $x > 2+\sqrt{29}$ (reject)

or $x < 2-\sqrt{29}$

$\therefore$ $x < 2-\sqrt{29}$.

② when $x > 2$. $g(x) = 4x-7 > 28$.

$4x > 35$ $x > 8.75$

$\therefore$ when $g(x) > 28$, $x < 2-\sqrt{29}$ or $x > 8.75$.

(c). $h(x)$ is 1-1 mapping. $g(x)$ is many to 1 mapping.

(d). $h(x) = (x-2)^2+1$ $x \le 2$.

$y = h(x) = (x-2)^2+1$.

$y = (x-2)^2+1$.

$(x-2)^2 = y-1$

$x ~~-2~~ = \sqrt{y-1} +2$

or $x = -\sqrt{y-1} + 2$.

~~$\therefore h^{-1}(x) = \sqrt{x-1}$~~

$h^{-1}(x) = 2 \pm \sqrt{x-1}$

$h^{-1}(x) = -\frac{1}{2}$.

~~$2\pm\sqrt{x-1} =$~~

① $2+\sqrt{x-1} = -\frac{1}{2}$.

$\sqrt{x-1} = -\frac{5}{2}$ (reject)

② $2-\sqrt{x-1} = -\frac{1}{2}$.

$\sqrt{x-1} = \frac{5}{2}$

$x-1 = \frac{25}{4}$

$x = \frac{29}{4}$

the solution of

$h^{-1}(x) = -\frac{1}{2}$

is $x = \frac{29}{4}$

9/10

Examiner Comments

(a) M1 A1

M1 A1: Correct solution leading to a correct $\mathrm{gg}(0) = 13$.

(b) M1 A1 M1 A0

M1: Correct method for solving both $(x-2)^2 + 1 > 28$ leading to a critical value(s) for *x* **or** for solving $4x - 7 > 28$ leading to a critical value for *x*.

A1: Obtains a correct critical value $x = 8.75$.

M1: Correct method for solving both $(x-2)^2 + 1 > 28$ leading to a critical value(s) for *x* **and** for solving $4x - 7 > 28$ leading to a critical value for *x*.

A0: Incorrect answer.

(c) B1

B1: Correct explanation for h and correct explanation for g.

(d) M1 M1 A1

M1 M1 A1: Solves the correct equation $2 - \sqrt{x-1} = -\frac{1}{2}$ to give a correct answer $x = \frac{29}{4}$.

Exemplar Question 7

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7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

(Total for Question 7 is 7 marks)

Mean Score 3.9 out of 7

Examiner Comments

This question discriminated well between the medium and higher ability candidates. Lower ability candidates struggled to make much progress with this modelling question and most of them scored no more than one mark.

In part (a), most candidates wrote down a correct equation for the model in the form $y = kx + c$ with constants k and c. A minority struggled to give a suitable form of the linear equation, with some omitting the fixed cost constant, c. Some candidates gave incorrect answers, such as $y \propto x$; equations involving exponential models; or differential equations.

Part (b) proved challenging for those candidates who struggled to grasp the concept '$\text{profit} = \text{sales} - \text{cost}$', with many of them incorrectly assuming that the model from part (a) was for the profit made. Many of these candidates used $(800, 500)$ and either $(300, 80)$ or $(300, -80)$ to form two linear equations, with some erroneously finding a gradient of 0.84.

Those candidates who understood the link between profit and cost usually found the total cost to the factory of £1100 and £680 for 800 and 300 bars of soap respectively. Most used $(800, 1100)$ and $(300, 680)$ to form two linear equations and proceeded to find the gradient, 0.84, and y-intercept, 428. A large proportion concluded by stating the given equation $y = 0.84x + 428$. A few candidates, however, did not complete their analysis with a suitable conclusion, and so lost the final mark. Some candidates used $(800, 1100)$ and $(300, 680)$ to verify the model $y = 0.84x + 428$. Many of these were successful but often failed to give a suitable conclusion stating that the given model was true.

Part (c) gave a significant number of candidates their only creditable access to Q07. Some candidates gave a correct interpretation such as 0.84 represented 'the cost of making an extra bar of soap'. Incorrect explanations included that 0.84 represented 'the selling price of a bar of soap' or 'the profit per bar of soap made'.

There was a mixed response to part (d). The most popular approach was to form and solve a correct $2n = 0.84n + 428$ or $2n > 0.84n + 428$, which mostly led to the correct answer $n = 369$. A final answer left as an inequality such as $n > 368$ or $n \geq 369$, without reference to the statement that 369 was the number of bars required, was not sufficient for the final mark. Some candidates used trials of both $n = 368$ and $n = 369$ in a complete trial and error method leading to a correct conclusion of $n = 369$.

Mark Scheme

Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) Way 1	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies $(800$, their $1100)$ and $(300$, their $680)$ to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 2	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \dfrac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) Way 3	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} **(**Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss = 1.12}	M1	3.4
	• Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit = 0.04} leading to an answer of 369 {bars}	A1	3.2a
		(2)	
			(7 marks)

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points $(800, \text{their } 1100)$ and $(300, \text{their } 680)$
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c$, $80 = 300k + c \Rightarrow k = \dfrac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	<u>**Special Case:**</u> Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Student Response A

a) ~~graph~~

$y = C + kx$. where C is the fixed cost

b) $500 = C + 800k$

$-80 = C + 300k$

$500k = 580$

$k = 1.16$

c) when one bar of soap is sold, the total cost to the factory increases by 0.84.

d) $0.84x + 428 > 0$

2/7

Examiner Comments

(a) B1
B1: Correct form of the equation.

(b) M0 M0 A0
M0: Does not use the profit and selling price information given to find the cost to the factory of selling either 800 bars of soap or 300 bars of soap.
dM0 A0*: Follows M0.

(c) B1
B1: Acceptable explanation.

(d) M0 A0
M0: Does not use the model to construct an argument which would lead to a critical value for the least number of bars of soap required to make a profit.
A0: Follows M0.

Student Response B

a) $y = c + kx$

where c is a fixed cost
where kx is the cost proportionate to the number of bars of soap made

b) $y = 0.84x + 428$

~~500 = 800~~
~~(500 = 0.84(800) + 428)~~

~~1500 = 0.84(800) + 428~~

800 bars = 800x2 = £1600 → profit of £500
∴ costs of making bars of soap = £1100

$\frac{1100}{800} = 1.375$ to make a bar of soap.

c) for every bar of soap that's made, the ~~pro~~ ~~###~~ amount of money made increases by 0.84

b) 800 bars = (800x2) = £1600 money made
↳ £1600 − profit(500) = £1100

£1100 to make 800 bars of soap.
£ 11/8 to make one bar of soap

300 bars = (300x2) = £600 money made

£680 to make 300 bars of soap

$y = 0.84x + 428$

@ $x = 800$, $y = (0.84 \times 800) + 428$
$y = 1100$

@ $x = 300$, $y = (0.84 \times 300) + 428$
$y = 680$

d) $y = 0.84x + 428 > 2x$

$0.84x + 428 > 2x$
$428 > 1.16x$
$368.9655\ldots > x$

@ $x = 369$, $y = (0.84 \times 369) + 428$
$= 737.96$

total costs $= £737.96$
money made $= £738$

$\therefore$ profit $= £0.04$

least ~~minimum~~ number of bars of soap ~~[illegible]~~ is 369 bars for profit to be made

5/7

Examiner Comments

(a) B1
B1: Correct form of the equation.

(b) M1 M1 A0 [Uses Way 3]
M1: Uses the profit and selling price information given to find the cost to the factory of selling either 800 bars of soap or 300 bars of soap.
dM1: Uses the model to test both points $(800, 1100)$ and $(300, 680)$.
A0*: No conclusion given after they have tested their points.

(c) B0
B0: Incorrect explanation. 'Amount of money made' alludes to profit and not cost to the factory.

(d) M1 A1
M1: Uses the model to construct an argument leading to a critical value for the least number of bars of soap required to make a profit.
A1: Correct answer of 369 found. Note: Inequality symbols in (d) are ignored.

Student Response C

7a. $y = kx$

b. cost for 800 bars: $500(2\times800) - 500$

$= \underline{\underline{£1100}}$

cost for 300 bars: $(2\times300) + 80 = \underline{\underline{£680}}$

Point on line are:

$(800, 1100), (300, 680)$

$m = \dfrac{1100 - 680}{500} \Rightarrow m = 0.84$

~~Or~~ $\therefore y = 0.84x + c$

sub $(800, 1100)$ in.

$1100 = 0.84(800) + c$

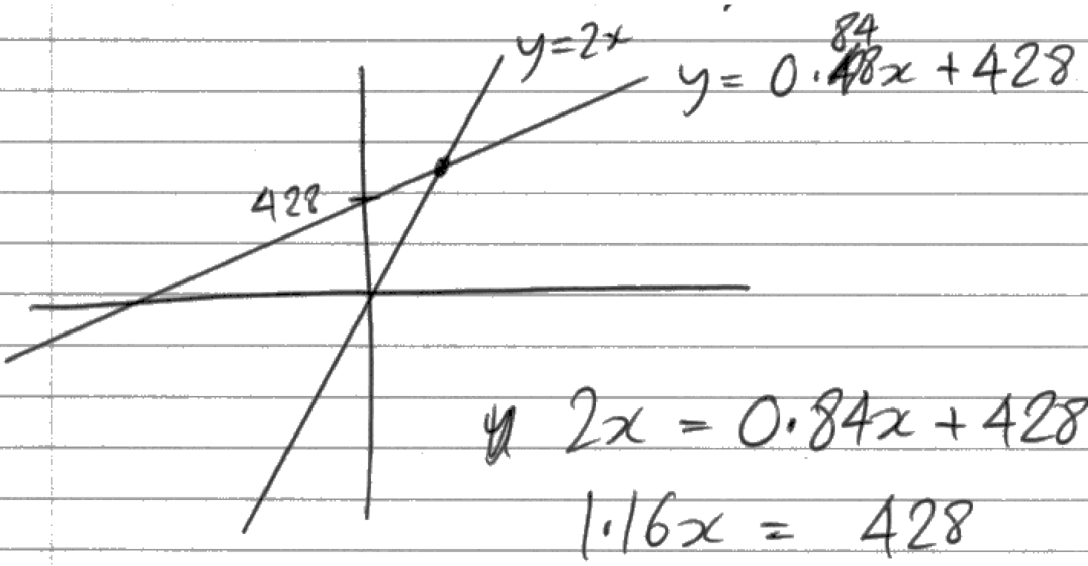
$1100 = 672 + c$

$c = 428.$

$\therefore \underline{\underline{y = 0.84x + 428.}}$

c. The total cost to the factory increases by £0.84 for ~~every bar~~ each bar of soap they make.

d.



$$2x = 0.84x + 428$$

$$1.16x = 428$$

$$x = 368.9655....$$

369 bars of soap are built so that must be made for their to be a profit.

6/7

Examiner Comments

(a) B0
B0: Incorrect form of the equation seen in part (a).

(b) M1 M1 A1 [Uses Way 2]
M1: Translates the problem into the model shown by either $(2\times800)-500$ or $(2\times300)+80$.
dM1: A complete method for finding both '0.84' and '428'.
A1*: Correct solution followed by a correctly stated equation.

(c) B1
B1: Acceptable explanation.

(d) M1 A1
M1 A1: Uses the model in a correct solution leading to a correct answer of 369.

Exemplar Question 8

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8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

(Total for Question 8 is 6 marks)

Mean Score 2.1 out of 6

Examiner Comments

This question was found challenging by many candidates, with part (i) more successfully answered than part (ii). The most successful candidates were the ones who listed the first few terms of the series. In other words, those candidates that wrote $20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$ and $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + \ldots$ gained a better understanding of the series that they were required to sum. There were, however, many attempts that made no creditable progress and a significant number of blank responses to either one or both parts of Q08.

In part (i), many candidates identified that a sum to infinity was required in their solution and attempted to apply the formula $\frac{a}{1-r}$. Most used $r = \frac{1}{2}$ in this formula, although some used incorrect values such as $r = \frac{1}{4}$ or $r = -\frac{1}{2}$. Disappointingly, a significant number of candidates used $r = 4$ from the expression given in the question and applied it to the formula $\frac{a}{1-r}$, even though the formula only works when $-1 < r < 1$. There were a number of correct strategies that candidates could use. The most successful strategy was to apply $a = 20\left(\frac{1}{2}\right)^4$, $r = \frac{1}{2}$ to $\frac{a}{1-r}$, and many candidates who used this strategy achieved the correct answer of 2.5.

Other complete strategies included applying $\sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$ or $\sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$ which led to $(20-17.5)$ or $(40-37.5)$ respectively. Some candidates applied an incorrect overall strategy by subtracting an incorrect number of terms from their sum to infinity. These candidates gave incorrect answers such as $(20-18.75)$, $(40-38.75)$, $(40-35)$ or $(40-17.5)$. A few candidates obtained the correct answer of 2.5 by using their calculator.

Only a few candidates produced a correct solution to Q08(ii). There were two main correct methods that were used by candidates in equal measure. One was to list the first few terms and the last few terms of the series and use the addition law of logarithms to achieve $\log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$. Many candidates used cancelling to complete their proof correctly by writing $\log_5\left(\frac{50}{2}\right)=\log_5(25)=2$.

The other method was to use the subtraction law of logarithms to give $\sum_{n=1}^{48}(\log_5(n+2)-\log_5(n+1))$ and list terms to give $(\log_5 3+\log_5 4+\ldots\ldots+\log_5 50)-(\log_5 2+\log_5 3+\ldots\ldots+\log_5 49)$. Again many candidates used cancelling and completed their proof by writing $\log_5(50)-\log_5(2)=\log_5(25)=2$. A few candidates progressed by writing $\log_5(3\times 4\times\ldots\ldots\times 50)-\log_5(2\times 3\times\ldots\ldots\times 49)$ and used factorials to give $\log_5\left(\frac{50!}{2}\right)-\log_5(49!)=\log_5\left(\frac{50!}{2(49!)}\right)=\log_5(25)=2$.

Some candidates incorrectly assumed an arithmetic series and attempted to find its sum by using $\frac{n}{2}(a+l)$ or $\frac{n}{2}[2a+(n-1)d]$ with $n=48$, $a=\log_5\left(\frac{3}{2}\right)$, $l=\log_5\left(\frac{50}{49}\right)$ and $d=\log_5\left(\frac{4}{3}\right)-\log_5\left(\frac{3}{2}\right)$. A few candidates incorrectly assumed a geometric series and attempted to find its sum. Some candidates, who made no creditable progress, resorted to listing their terms as decimals, or attempted to add all 48 terms as decimals by writing $0.2519\ldots+0.1787\ldots+0.1386\ldots+\ldots\ldots+0.0125\ldots=2$.

Mark Scheme

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20 \times \left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20 \times \left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20 \times \left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20 \times \left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2} \times \frac{4}{3} \times \ldots \times \frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \right\} \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \ldots\ldots + \log_5 50) - (\log_5 2 + \log_5 3 + \ldots\ldots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
		(6 marks)	

Notes for Question 8	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $\frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \ldots\ldots + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times \ldots \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}*** • $\log_5\left(\frac{3}{2} \times \ldots \times \frac{49}{48} \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}***
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	<u>**Listing all 48 terms**</u> Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519\ldots + 0.1787\ldots + 0.1386\ldots + \ldots\ldots + 0.0125\ldots = 2$ {all terms in decimals}

Notes for Question 8	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2)-\log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3+\log_5 4+......+\log_5 50)-(\log_5 2+\log_5 3+......+\log_5 49)$ • $(\log_5 3+......+\log_5 49+\log_5 50)-(\log_5 2+......+\log_5 48+\log_5 49)$ • $(\log_5 3+\log_5 4+......+\log_5 50)-(\log_5 2+\log_5 3+......+\log_5 49)$ • $(\log_5 3-\log_5 2)+(\log_5 4-\log_5 3)+......+(\log_5 50-\log_5 49)$ • $\log_5 3-\log_5 2,\ ......, \log_5 49-\log_5 48,\ \log_5 50-\log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48}(\log_5(n+2)-\log_5(n+1))$ $= \log_5(3\times 4\times ....\times 50) - \log_5(2\times 3\times ....\times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25\times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

Student Response A

i) $\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$ $\qquad S_\infty = \frac{a}{1-r}$ $\qquad U_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$ $\qquad a = 20$

$r = 1/2$

$= \sum_{r=1}^{\infty} 20 \times (1/2)^r - \sum_{r=1}^{3} 20 \times (1/2)^r$ $\qquad n = 4$

$= \left(\frac{20}{1-1/2}\right) - \left(\frac{20(1-1/2^3)}{1-1/2}\right)$

$= (40) - (35)$

$= 5$ $\qquad$ for $|r| < 1$

ii) $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$

$= \log_5\left(\frac{1+2}{1+1}\right) + \log_5\left(\frac{2+2}{2+1}\right) + \log_5\left(\frac{3+2}{3+1}\right) \ldots$

$= \log_5(3/2) + \log_5(4/3) + \log_5(5/4)$

$= \log_5(3/2 \times 4/3 \times 5/4 \times 6/5 \times 7/6 \ldots)$ $\qquad a = 3/2$

$r =$

2/6

Examiner Comments

(i) M1 M0 A0 [Uses Way 2]

M1: Applies $\frac{a}{1-r}$ for their value of r (where $-1 <$ their $r < 1$) and for their value of a.

M0: Does not apply the correct strategy $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$ due to their incorrect method for finding the first term, a.

A0: Follows previous M0.

(ii) M1 M0 A0 [Uses Way 1]

M1: Correctly applies the addition law of logarithms as part of a valid proof.

M0: Writes down the first two terms but does not write down the last term of the series.

A0*: Follows from previous M0.

Student Response B

(3)

i) $\sum_{r=4}^{\infty} (20) \times \left(\frac{1}{2}\right)^r$

$= 20 \sum_{r=4}^{\infty} \left(\frac{1}{2}\right)^r \qquad S_\infty = \frac{a}{1-r}$

$= 20 \left(\frac{\frac{1}{16}}{1-\frac{1}{2}} \right)$

$= 20 \left(\frac{1}{8} \right)$

$= \frac{20}{8}$ //

ii) $\sum_{n=1}^{48} \log_5 \left(\frac{n+2}{n+1}\right)$

$= \log_5 \left(\frac{1+2}{1+1}\right) + \log_5 \left(\frac{2+2}{2+1}\right) + \log_5 \left(\frac{3+2}{3+1}\right) + \ldots \log_5 \left(\frac{48+2}{48+1}\right)$

$= \log_5 \left(\frac{3}{2}\right) + \log_5 \left(\frac{4}{3}\right) + \log_5 \left(\frac{5}{4}\right) + \ldots \log_5 \left(\frac{50}{49}\right)$

$= \log_5 \left(\frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \ldots \frac{50}{49}\right)$

4/6

Examiner Comments

(i) M1 M1 A1 [Uses Way 1]

M1 M1 A1: Applies a correct Way 1 strategy leading to a correct equivalent answer of $\frac{20}{8}$.

(ii) M0 M1 A0 [Uses Way 1]

M0: Neither the addition law nor the subtraction law of logarithms is correctly applied as part of a valid proof.

M1: Writes down the first two terms and the last term of the series.

A0*: Follows from earlier M0.

Student Response C

ai) $S_\infty = \dfrac{20}{0\cdot5} = 40$

$S_4 = \dfrac{20\left(1-\left(\frac{1}{2}\right)^4\right)}{0.5}$

$= 37.5$

$S_\infty - S_4 = 40 - 37.5$

$= \underline{2.5}$

ii) $\displaystyle\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right)$

$\Rightarrow \log_5 \frac{3}{2}, \log_5 \frac{4}{3}, \log_5 \frac{5}{4} \ldots$

$= \log_5 \frac{3}{2} + \log_5 \frac{4}{3} + \log_5 \frac{5}{4} + \ldots + \log_5 \frac{50}{49}$

$= \log_5 \frac{\cancel{3}}{2} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{4}} \times \ldots \times \frac{50}{\cancel{49}}$

$= \log_5 \frac{50}{2}$

$= \log_5 25$

$= \underline{2}$

6/6

Examiner Comments

(i) M1 M1 A1 [Uses Way 3]
M1 M1 A1: Applies a correct Way 3 strategy leading to a correct answer of 2.5.

(ii) M1 M1 A1 [Uses Way 1]
M1 M1 A1*: Correct proof.

Exemplar Question 9

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9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

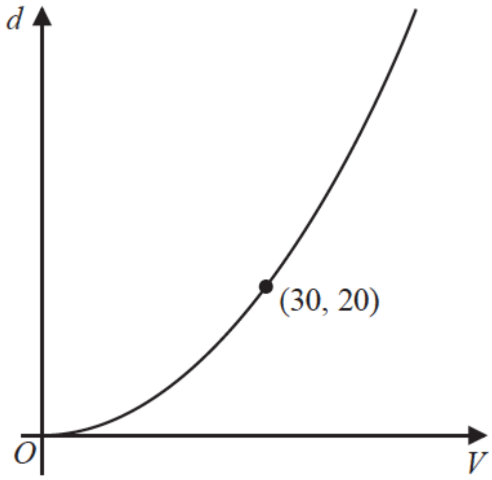

Figure 5

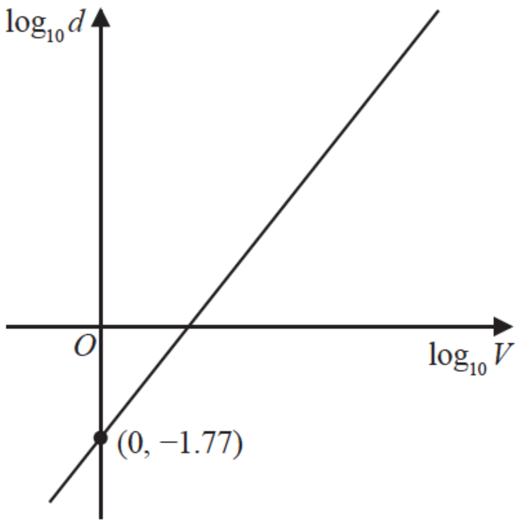

Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with k » 0.017

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

(Total for Question 9 is 9 marks)

Mean Score 4.6 out of 9

Examiner Comments

In general, this question discriminated well between candidates of all abilities. Part (b) and part (c) were more accessible to candidates than part (a). Some candidates left part (a) blank while others struggled to make any creditable progress.

In part (a), many candidates wrote down a valid log equation as part of their explanation. Those who used mathematical reasoning to progress from $\log_{10} d = \log_{10} k + n\log_{10} V$ to $d = kV^n$ (or vice versa) usually showed an intermediate step in their working and so were successful in their explanation. A few candidates made errors in progressing between these two equations, such as an incorrect method of combining logarithms (e.g. $\log_{10} d = n\log_{10} kV$), or an incorrect use of powers (e.g. $d = 10^c + V^n$). Those who made a direct comparison of their log equation with $y = mx + c$ usually did so clearly enough but sometimes did not allude to $d = kV^n$ (if using -1.77 rather than $\log_{10} k$ in their log equation). A significant number of candidates did not apply $10^{-1.77}$ or use logarithms to show that $k = 0.017$.

In part (b), the majority of candidates substituted $V = 30$ and $d = 20$ into $d = kV^n$ and applied logarithms correctly, leading to the correct value of $n = 2.08$. A few candidates used the incorrect method of simplifying $20 = 0.017(30)^n$ to give $20 = (0.51)^n$. Some candidates substituted in V and d the wrong way around, usually leading to $n = 2.50$, and a few used both V and d as 20. A small minority substituted V = 30 and d = 20 into a correct log equation, while a few others correctly found n by finding the gradient between the known points $(0, -1.77)$ and $(\log_{10} 30, \log_{10} 20)$. The most common mistake was not writing a complete equation $d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ after finding a correct $n = 2.08$.

In part (c), most candidates applied V = 60 to their exponential model and found the braking distance. Some then used the incorrect method of comparing this breaking distance with the overall stopping distance. The majority attempted to calculate the thinking distance and either added it to their breaking distance d or subtracted it from 100. The main error was due to the units. Some candidates did not attempt to convert $60\ \mathrm{km\,h^{-1}}$ to $\mathrm{m\,s^{-1}}$ and simply multiplied 60 by the thinking time of 0.8 seconds to obtain a thinking distance of 48 metres. Others only managed a partial conversion, with some using $\frac{1}{75}$ metres $\left(\text{instead of } 13.3\,\mathrm{m} \approx \frac{1}{75}\,\mathrm{km}\right)$ as their thinking distance.

In some of the answers, candidates used an incorrect method of combining their values to give a dimensionally incorrect quantity. For example, they might add their breaking distance to either a speed or a time. Candidates who obtained a thinking distance of $13.3\,\mathrm{m}$ usually reached a correct answer and conclusion. A few compared the maximum speed needed to stop at the puddle with the given 60, while even fewer compared the maximum value of n needed to stop at the puddle with $n = 2.08$; and with correct conclusions these were valid solutions.

Mark Scheme

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = \ldots$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = \ldots$		
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

Notes for Question 9

Note:	In their solution to (a) and/or (b) condone writing log in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their *d* to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of *d*
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their *d* to find a total stopping distance and gives a correct ft conclusion **or** finds their *d* and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of *d* in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Student Response A

(a) $d = kV^n$ shows an exponential (non-linear) equation
Because in figure 6 log to both variables means
$\log_{10} d = \log_{10} k + n \log_{10} V$ is plotted. This turns the
exponential graph into a straight line.

(b) $\log_{10} d = n \log_{10} V + \log_{10} k$

$\log_{10} d = n \log_{10} V + \log_{10} 0.017.$

$d = kV^n \quad = \quad 30 = 0.017(20)^n.$

$30 = 0.017 \times 20^n.$

$\log_{10} 30 = n \log_{10} 20 + \log_{10} 0.017.$

$\frac{\log_{10} 30 - \log_{10} 0.017}{\log_{10} 20} = n.$

$n = 2.50 \; (3sf)$

$s = \frac{D}{T}$

$\therefore \log_{10} d = 2.50 \log_{10} V + \log_{10} 0.017.$

(c) $\log_{10} d = 2.50 \log_{10}(60) + \log_{10} 0.017.$

$\log_{10} d = 2.675827047...$

$10^{2.67} = d.$

$d = 474.1 km.$

3/9

Examiner Comments

(a) M1 A0 B0 [Uses Way 2]
M1: Writes $\log_{10} d = \log_{10} k + n\log_{10} V$.
A0: Starts from $d = kV^n$ but does not show an intermediate step in their working to give $\log_{10} d = \log_{10} k + n\log_{10} V$.
B0: Does not show that $k \approx 0.017$.

(b) M0 M1 A0
M0: Does not apply $V = 30$ and $d = 20$ to the model.
M1: Apples $(V, d) = (20, 30)$ to the model and applies logarithms correctly leading to $n = ...$
A0: Incorrect model equation.

(c) M1 M0 A0
M1: Applies $V = 60$ to their model.
M0 A0ft: Makes no further progress.

Student Response B

a) $\log_{10} d = n\log_{10} V - 1.77$

$10^{\log_{10} d} = 10^{n\log_{10} V} + 10^{-1.77}$

$d = V^n \times 0.017$

$d = 0.017V^n$

As figure 6 is the data coded from figure 5, meaning that an exponential increase would be seen - represented through the equation $d = kV^n$.

b) $d = 0.017V^n$ $d = 20$

$V = 30$

$20 = 0.017(30)^n$

$(30)^n = \frac{20000}{17}$

$\log_{30} \frac{20000}{17} = n$

$n = 2.08$

c) ~~[illegible]~~ $V = 60$ $n = 2.08$ ~~[illegible]~~ $k = 0.017$

$d = 0.017 \times 60^{2.08}$

$= 84.92\text{m}$

$d = st$

$t = \frac{\Delta d}{\Delta s}$ so n is gradient = time.

$d = st$

$d = 60 \times 0.8$

$d = 48\text{m}$

$84.92 + 48$

$= 132.92\text{m}$ to stop.

Meaning Sean would not be able to stop before reaching the puddle.

6/9

Examiner Comments

(a) M1 A0 B1 [Uses Way 3]
M1: Writes $\log_{10} d = n\log_{10} V - 1.77$.
A0: Follows from the incorrect line $10^{\log_{10} d} = 10^{n\log_{10} V} + 10^{-1.77}$ in their working.
B1: For replacing $10^{-1.77}$ with 0.017.

(b) M1 M1 A0
M1: Applies $V = 30$ and $d = 20$ to the model.
M1: Apples $(V, d) = (30,\ 20)$ to the model and applies logarithms correctly leading to $n = \ldots$.
A0: Achieves a correct $n = 2.08$ but does not state the model $d = 0.017V^{2.08}$.

(c) M1 M1 A0
M1: Applies $V = 60$ to their model.
M1: Adds their acceptable ‘thinking distance’ to their *d* to find an overall stopping distance.
A0ft: Follows from an incorrect ‘thinking distance’ of 48 metres.

Student Response C

a) as $\log_{10} d$ against $\log_{10} V$ is a straight line,

$y = mx + c$

$\log_{10} d = m \log_{10} V + c$ where $c = -1.77$

$\log_{10} d = \log_{10} V^m - 1.77$

$10^{\log_{10} d} = \cancel{\log}\ 10^{\log_{10} V^m - 1.77}$

$\cancel{\log}\ d = 10^{\log_{10} V^m} \times 10^{-1.77}$

$d = V^m \times 10^{-1.77}$ $m = n$

$d = V^n \times 10^{-1.77}$

$10^{-1.77} = 0.016982436 52$
$= 0.017$
$k = 0.017$

$d = 0.017 V^n$
$d = kV^n$

b) $d = 0.017 V^n$ (30, 20

$20 = 0.017 \times 30^n$

$\frac{20000}{17} = 30^n$ $n = \log_{30}\left(\frac{20000}{17}\right)$ $n = 2.08$
$n = 2.07876\ldots$ (3sf)

c) $V = 60$

$d = kV^n$

$d = 0.017 \times 60^{2.08}$

$d = 84.9187362$

$d = 85\text{m}$

60 km h^{-1} $\quad \frac{60}{60 \times 60} = \frac{1}{60} \text{ km s}^{-1}$

$0.8 \times \frac{1}{60} = \frac{1}{75} \text{ km}$

$100 - \left(\frac{1}{75} \times 10^3\right)$

$100 - \frac{40}{3} = \frac{260}{3}$ m he has to stop before reaching puddle

$= 86.\dot{6}$

$= 87\text{m}$

So he does stop in time to not reach the puddle.

8/9

Examiner Comments

(a) M1 A1 B1 [Uses Way 3]
M1: Writes $\log_{10} d = m\log_{10} V + c$.
A1: Then uses algebra to show that $d = kV^n$ with no errors seen in their working.
B1: Shows that $k = 10^{-1.77} = 0.017$.

(b) M1 M1 A0
M1: Applies $V = 30$ and $d = 20$ to the model.
M1: Apples $(V, d) = (30, 20)$ to the model and applies logarithms correctly leading to $n = ...$
A0: Achieves a correct $n = 2.08$ but does not state the model $d = 0.017V^{2.08}$.

(c) M1 M1 A1
M1: Applies $V = 60$ to their model.
M1: Applies 100 – 'an acceptable thinking distance' and finds their value of *d*.
A1ft: Obtains a correct '87' and $d = 85$ which is followed by a correct conclusion.

Exemplar Question 10

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10.

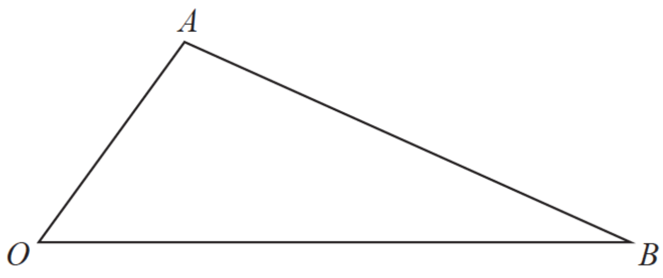


Figure 7

Figure 7 shows a sketch of triangle *OAB*.

The point *C* is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point *M* is the midpoint of *AB*.

The straight line through *C* and *M* cuts *OB* at the point *N*.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{CM} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)

(Total for Question 10 is 6 marks)

Mean Score 2.0 out of 6

Examiner Comments

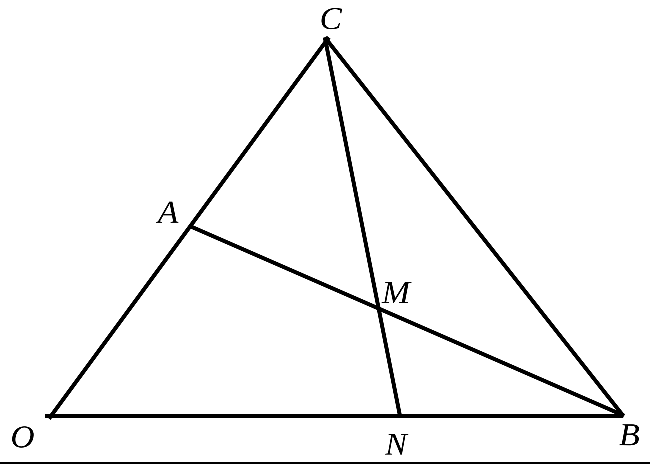
This question discriminated well between the medium and higher ability candidates, with lower ability candidates struggling to gain any creditable access. This question increased in difficulty as the candidates progressed through the question, and only a small proportion of candidates scored full marks.

In part (a), most candidates followed a correct method to achieve the correct answer $\overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$. Of the candidates who identified a suitable route from C to M, the most common approach was $\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$. Less efficient routes such as $\overrightarrow{CM} = \overrightarrow{CO} + \overrightarrow{OB} + \overrightarrow{BM}$ or even $\overrightarrow{CM} = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AM}$ were also observed. Some candidates struggled to apply the vectors **a** and **b** to the routes they had chosen. Incorrect methods included applying $\overrightarrow{CA} = \mathbf{a}$ (instead of $\overrightarrow{CA} = -\mathbf{a}$) or applying $\overrightarrow{AB} = \mathbf{a} - \mathbf{b}$ (instead of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$). There were a few candidates who incorrectly deduced the position of C to be in the opposite direction to $\overrightarrow{OA}$ and they were mostly able to apply the approach, $\overrightarrow{CM} = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{CO} + \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$, to gain some credit.

Part (b) proved to be demanding, with some candidates struggling to understand what the question required them to do, and many seemed to be confused by the introduction of the scalar multiple λ. Only a minority deduced that $\overrightarrow{CN} = \lambda\overrightarrow{CM}$ and applied this deduction to the route $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$. Most of these candidates progressed well to the given answer unless they had found an incorrect $\overrightarrow{CM}$ in Q10(a). Many candidates used $\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CM} + \overrightarrow{MB} + \overrightarrow{BN}$ or $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BN}$ and made no creditable progress. These candidates usually produced solutions which led to the elimination of λ from the **a** component of the resulting vector for $\overrightarrow{ON}$. A few candidates used $\overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN}$ and proceeded to obtain a correct expression $\overrightarrow{ON} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ in terms μ (or λ), but only a few deduced that this could lead to the stated solution by setting $\mu = \lambda - 1$ (or $\lambda' = \lambda - 1$).

Part (c) proved to be challenging with many candidates failing to make any creditable progress. The crucial step was to recognise that $\overrightarrow{ON}$ is parallel to $\overrightarrow{OB}$ and hence is a multiple of **b** with no **a** component. Only a few candidates set the **a** component $\overrightarrow{ON}$ to zero to give the equation $2 - \frac{3}{2}\lambda = 0$. Almost all of these candidates found $\lambda = \frac{4}{3}$, with many finding $\overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ and giving a correct explanation of the given ratio.

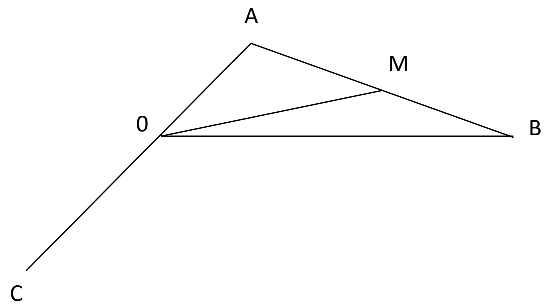
Mark Scheme

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA}=\mathbf{a},\ \overrightarrow{OB}=\mathbf{b}$		
(a)	$\left\{\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{AM}=\overrightarrow{CA}+\frac{1}{2}\overrightarrow{AB}\Rightarrow\right\}\ \overrightarrow{CM}=-\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM}=\overrightarrow{CB}+\overrightarrow{BM}=\overrightarrow{CB}+\frac{1}{2}\overrightarrow{BA}\Rightarrow\right\}\ \overrightarrow{CM}=(-2\mathbf{a}+\mathbf{b})+\frac{1}{2}(\mathbf{a}-\mathbf{b})$		
	$\Rightarrow\overrightarrow{CM}=-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ ***(needs to be simplified and seen in (a) only)***	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON}=\overrightarrow{OC}+\overrightarrow{CN}\Rightarrow\overrightarrow{ON}=\overrightarrow{OC}+\lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON}=2\mathbf{a}+\lambda\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)\Rightarrow\overrightarrow{ON}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) Way 1	$\left(2-\frac{3}{2}\lambda\right)=0\Rightarrow\lambda=\ldots$	M1	2.2a
	$\lambda=\frac{4}{3}\Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON}=\mu\mathbf{b}\Rightarrow\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}=\mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2-\frac{3}{2}\lambda\right)=0\Rightarrow\lambda=\ldots\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda=\mu\ \&\ \lambda=\frac{4}{3}\Rightarrow\mu=\frac{2}{3}\right\}$	M1	2.2a
	$\lambda=\frac{4}{3}$ or $\mu=\frac{2}{3}\Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2-\frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2-\frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$ $\left\{\mathbf{b}: 1 = \frac{1}{2}\lambda + K \ \ \& \ \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}$ & $\overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}$, $\mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ $\left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	<u>**Special Case**</u>
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $\left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	<u>**Alternative 1:**</u> Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow$ $\overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

<table>
<tr><th colspan="2">Notes for Question 10 Continued</th></tr>
<tr><td>Note:</td><td>Part (b) and part (c) can be marked together.</td></tr>
<tr><td rowspan="3">(a) Special Case</td><td><u>Special Case where the point C is believed to be below the origin O</u>

</td></tr>
<tr><td>Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$</td></tr>
<tr><td>$\left\{ \text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right\}$</td></tr>
</table>

Student Response A

a) $\overrightarrow{OC} = 2a$ $\qquad$ $\overrightarrow{AM} = \frac{1}{2}(b-a)$

$\overrightarrow{CO} = -2a$

$\overrightarrow{OB} = b$

$\overrightarrow{BM} = -b + a + \frac{1}{2}(b-a)$

$= -b + a + \frac{1}{2}b - \frac{1}{2}a$ $\qquad$ $\overrightarrow{CM} = -2a + b + \left(\frac{1}{2}a - \frac{1}{2}b\right)$

$= \frac{1}{2}a - \frac{1}{2}b$ $\qquad$ $\overrightarrow{CM} = -\frac{3}{2}a + \frac{1}{2}b$

b) $\overrightarrow{OC} = 2a$

$\overrightarrow{CM} = -\frac{3}{2}a + \frac{1}{2}b$

$\overrightarrow{OM} = 2a - \frac{3}{2}a + \frac{1}{2}b$

$\left(2 - \frac{3}{2}\right)a + \frac{1}{2}b$

$\overrightarrow{MN} = \lambda a + \lambda b$

$\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)a + \frac{1}{2}\lambda b$

c) $\overrightarrow{OB} = b$ $\qquad$ $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)a + \frac{1}{2}\lambda b$

$\overrightarrow{ON}$ is $\frac{2}{3}$ $\overrightarrow{OB}$

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Examiner Comments

(a) M1 A1

M1 A1: Applies $\overrightarrow{CM} = \overrightarrow{CO} + \overrightarrow{OB} + \overrightarrow{BM}$ and uses a correct method to achieve a correct $\overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$.

(b) M0 A0

M0: Does not attempt to apply $\overrightarrow{OC} + \lambda\overrightarrow{CM}$.

A0*: Follows M0.

(c) M0 A0 [Uses Way 1]

M0: Does not attempt to set the **a**-component of $\overrightarrow{ON}$ equal to 0.

A0*: Follows M0.

Student Response B

a. $\overrightarrow{OC} = 2\overrightarrow{OA}$

$\overrightarrow{OC} = 2a$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= b - a$

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= a + \frac{1}{2}(b-a)$

$= a + \frac{1}{2}b - \frac{1}{2}a$

$= \frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC}$

$= \frac{1}{2}a + \frac{1}{2}b - 2a$

$= \underline{\frac{1}{2}b - \frac{3}{2}a}$

b. $\lambda\overrightarrow{CM}$

$\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$

$= 2a + \lambda\left(\frac{1}{2}b - \frac{3}{2}a\right)$

$= 2a + \frac{\lambda}{2}b - \frac{3\lambda}{2}a$

$= \underline{\left(2 - \frac{3\lambda}{2}\right)a + \frac{\lambda}{2}b}$

c. $\overrightarrow{NB} = \overrightarrow{OB} - \overrightarrow{ON}$

$= b - \left(\left(2 - \frac{3\lambda}{2}\right)a + \frac{\lambda}{2}b\right)$

$= \left(1 + \frac{\lambda}{2}\right)b - \left(2 - \frac{3\lambda}{2}\right)a$

$\cancel{\frac{\lambda}{2}\ 2\ \ldots}$

$\cancel{\lambda\left(1 + \frac{\lambda}{2}\right) \div \frac{\lambda}{2}}$

$\cancel{1 + \frac{\lambda}{2} \times \frac{2}{\lambda}}$

$\cancel{24}$

$= b - \left(2 - \frac{3\lambda}{2}\right)a - \frac{\lambda}{2}b$

$= \left(1 - \frac{\lambda}{2}\right)b - \left(2 - \frac{3\lambda}{2}\right)a$

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Examiner Comments

(a) M1 A1

M1 A1: Applies $\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC}$ and uses a correct method to achieve a correct $\overrightarrow{CM} = \frac{1}{2}\mathbf{b} - \frac{3}{2}\mathbf{a}$.

(b) M1 A1

M1 A1: Applies $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$ to correctly show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{\lambda}{2}\mathbf{b}$.

(c) M0 A0

M0: Does not attempt to set the **a**-component of their $\overrightarrow{NB}$ equal to 0.
A0*: Follows M0.

Student Response C

(2)

a) $\overrightarrow{CM} = -a + \frac{1}{2}(\overrightarrow{AB})$

$\overrightarrow{AB} = -a + b$

$\therefore \overrightarrow{CM} = -a + \frac{1}{2}(-a+b)$

$\overrightarrow{CM} = -\frac{3}{2}a + \frac{1}{2}b$

$\therefore \boxed{\overrightarrow{CM} = -\frac{3}{2}a + \frac{1}{2}b}$

~~b) $\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{AB}) - \lambda b$~~ $\frac{1}{2}(\overrightarrow{AB})$

~~$\therefore \overrightarrow{CN} = \overrightarrow{CM} + \overrightarrow{MN} = -\frac{3}{2}a + \frac{1}{2}b + \frac{1}{2}(-a+b) - \lambda b$~~

~~$= -2a + b - \lambda b$~~

~~$\therefore \overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN}$~~

~~$= 2a + (-2a + b - \lambda b)$~~

~~$=$~~

~~$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN}$~~

~~$= a + \frac{1}{2}(\overrightarrow{AB}) + \frac{1}{2}(\overrightarrow{AB}) - \lambda b$~~

b) $\overrightarrow{ON} = \overrightarrow{OM} + \lambda(\overrightarrow{CM})$

$= a + \frac{1}{2}(\overrightarrow{AB}) + \lambda(-\frac{3}{2}a + \frac{1}{2}b)$

$= a + \frac{1}{2}(-a+b) + \lambda(-\frac{3}{2}a + \frac{1}{2}b)$

$= \frac{1}{2}a + \frac{1}{2}b + \lambda(-\frac{3}{2}a + \frac{1}{2}b)$

$= (\frac{1}{2} - \frac{3}{2}\lambda)a + (\frac{1}{2} + \frac{1}{2}\lambda)b$

~~c) $\overrightarrow{ON} = (2 - \frac{3}{2}\lambda)a + \frac{1}{2}\lambda b$~~

~~$\overrightarrow{NB} = -\overrightarrow{ON} + a + \overrightarrow{AB}$~~

~~$= (\frac{3}{2}\lambda - 2)a - \frac{1}{2}\lambda b + a - a + b$~~

~~$= (\frac{3}{2}\lambda - 2)a + (1 - \frac{1}{2}\lambda)b$~~

c) $\overrightarrow{ON} = \mu \overrightarrow{OB}$

$\overrightarrow{ON} = \mu b$

$\left(2-\frac{3}{2}\lambda\right)a + \frac{1}{2}\lambda b = \mu b$

$2-\frac{3}{2}\lambda = 0 \rightarrow \lambda = \frac{4}{3}$

$\frac{1}{2}\lambda = \mu$

$\mu = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$

$\therefore \overrightarrow{ON} = \frac{2}{3}\overrightarrow{OB}$

$\therefore ON:NB = \frac{2}{3} : \frac{1}{3} = 2:1$ as required.

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Examiner Comments

(a) M1 A1

M1 A1: Applies $\overrightarrow{CM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$ and uses a correct method to achieve a correct $\overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$.

(b) M1 A0 [See Special Case]

M1 A0*: Special case M1 A0 is given for achieving $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(-\mathbf{a}+\mathbf{b}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ via a correct method $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN}$. **Note:** They do not proceed via 'Alternative 1' to obtain the result given in the question.

(c) M1 A1 [Uses Way 2]

M1: Deduces that $2 - \frac{3}{2}\lambda = 0$ and finds a value for λ.

A1*: Correct work leading to the correct ratio $ON : NB = 2:1$.

Exemplar Question 11

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11.

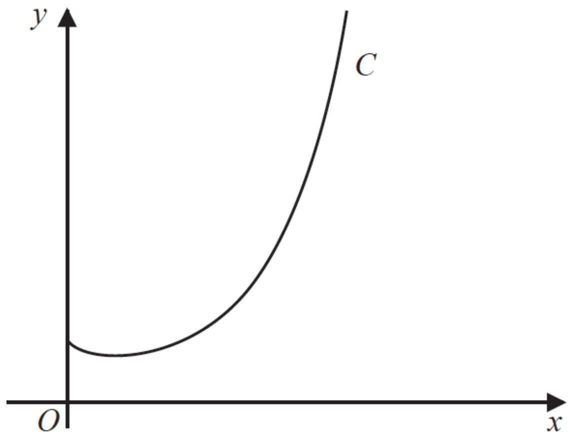

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

(Total for Question 11 is 11 marks)

Mean Score 5.4 out of 11

Examiner Comments

In general, this question discriminated well between candidates of all abilities. Part (a) was demanding with most candidates scoring no more than one mark. Parts (b), (c) and (d) proved accessible to candidates of all abilities.

In part (a), most candidates took logs, as advised by the question, and many obtained either $\ln y = x\ln x$ or $\log y = x\log x$. At this point a significant number of candidates made no further creditable progress. Some candidates used a complete method of applying the product rule on $x\ln x$ and implicit differentiation to obtain a correct $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$. Most of these candidates then applied $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to obtain $x = \mathrm{e}^{-1}$. Some candidates, who received partial credit, used a '$\log_{10}$' method to obtain $x = 0.1$, after finding $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log_{10} x$. A few candidates used logarithms without reference to a base. Those who proceeded to obtain $x = 0.1$ received partial credit and those who obtained $x = \mathrm{e}^{-1}$ were allowed full marks.

There were some alternative methods employed by a few candidates in part (a). A few candidates rewrote $y = x^x$ as $y = \mathrm{e}^{x\ln x}$ and applied $\frac{\mathrm{d}y}{\mathrm{d}x} = (1+\ln x)\mathrm{e}^{x\ln x} = 0$ to give a correct $x = \mathrm{e}^{-1}$. Others differentiated both sides of the equation $\ln y = x\ln x$ with respect to y and usually proceeded to find $x = \mathrm{e}^{-1}$. A very few candidates applied the quotient rule to the equation $x = \frac{\ln y}{\ln x}$, but were rarely successful with this method.

In part (b), most candidates evaluated either x^x or $x^x - 2$ at both $x = 1.5$ and $x = 1.6$. Many candidates compared their values with either 2 or 0 (depending on whether they evaluated x^x or $x^x - 2$). Although many of these concluded that $1.5 < \alpha < 1.6$, only a few made any reference to the curve being continuous in this interval. A few candidates, who made no creditable progress, used values of x outside the given range, e.g. $x = 1.4$ and $x = 1.6$. Very few candidates used values within the range e.g. $x = 1.51$ and $x = 1.59$, which meant that they were able to earn both marks.

In part (c), most candidates found a correct $x_4 = 1.673$, although a few did not state x_4 correct to 3 decimal places. Some wrote down the iterates x_1, x_2 and x_3 as part of their solution, while others stated x_4 with no intermediate work.

In part (d), some candidates gave correct descriptions such as 'sequence oscillates between 1 and 2' or 'the sequence is periodic with period 2'. Other descriptions such as 'alternates between 1 and 2', 'keep getting 1 and 2' or 'fluctuates between 1 and 2' were condoned. Some credit was given for partial explanations such as 'sequence is non-convergent', 'sequence oscillates' or 'sequence is divergent'.

Mark Scheme

Question	Scheme		Marks	AOs
11 (a) Way 1	$\{y = x^x \Rightarrow\}\ \ln y = x\ln x$		B1	1.1a
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$		M1	1.1b
			A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$		M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368		A1	1.1b
	Note: $k \neq 0$		**(5)**	
(a) Way 2	$\{y = x^x \Rightarrow\}\ y = \mathrm{e}^{x\ln x}$		B1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$		M1	1.1b
			A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$		M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368		A1	1.1b
	Note: $k \neq 0$		**(5)**	
(b) Way 1	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp		M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$		A1	2.1
			(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63		M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**		A1	1.1b
			(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone)	Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
		Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
			(2)	
			(11 marks)	

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$; $\lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\ \ k \neq 0$

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 11

(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1 + \ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question 11 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...,\ x_3 = 1.46626...,\ x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for “Converges in a cob-web pattern” or “Converges up and down to α ”

Student Response A

a) $\log y = \log x^x$

$\log y = x \log \cancel{x}$

$\log y = 0 \log 0$

$\log y = 0$

$\therefore y = 1$

$\frac{dy}{dx} = x x^{x-1}$

$0 = x^{2x-1}$

$\cancel{\log_x 0 = 2x-1}$

b) $y = 1.5^{1.5}$

$2 > 1.837$

~~close to 2~~

$y = 1.6^{1.6}$

$2 < 2.12$

~~close to~~

$\therefore 1.5 < \alpha < 1.6$

c) $x_{n+1} = 2 x_n^{\,1-x_n}$

$\cancel{x_{0+1} = 2}$

$x_{1+1} = 2(1.5)^{1-1.5}$
$_{(2)}$

$= 1.6329$

$x_3 = 2(1.6329)^{1-1.6329}$

$= 1.4662$

$x_4 = 2(1.4662)^{1-1.4662}$

$= 1.67313$

$= 1.673$

d) it keep increasing to 1.6.... and decreasing to 1.4...
therefore it keeps fluctuating between 1.4... and 1.6...

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Examiner Comments

(a) B1 M0 A0 M0 A0 [Referred to as 'A common solution' on the mark scheme]
B1: States $\log y = x\log x$.

M0: Neither $\log y \to \lambda \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}; \lambda \neq 0$ nor $x\log x \to \frac{x}{x} + \log x$.

A0: Follows previous M0.

M0: Does not attempt to solve $1 + \log x = 0$.

A0: Follows previous M0.

(b) M1 A0 [Uses Way 1]
M1: Attempts both $1.5^{1.5}$ and $1.6^{1.6}$ with at least one correct to awrt 1 dp.
A0: No mention of C being continuous.

(c) M1 A1
M1 A1: Correct calculations leading to $x_4 = 1.673$.

(d) B1 B0
B1: Allowed for "increasing to …" and "decreasing to …" OR for "fluctuates between …".
B0: No reference to 1 and 2.

Student Response B

a) $y = x^x$

$\log y = \log x^x$

$\log y = x \log x$

$\frac{dy}{dx} \frac{1}{y} = \log x + 1$

$\frac{dy}{dx} = y \log x + y$

$= x^x \log x + x^x$

$x^x \log x + x^x = 0$

$\log x + 1 = 0$

$\log x = -1 \qquad x = \frac{1}{10}$

$= 0 \cdot 1$

b) $P(a, 2)$

$2 = x^x$

when $x = 1 \cdot 5$

$1 \cdot 5^{1 \cdot 5} = 1 \cdot 837$

$1 \cdot 6^{1 \cdot 6} = 2 \cdot 12$

~~f(x)~~ ~~f(x)~~ $y(1.5) < 2 < y(1.6)$

$1.837 < 2 < 2.12$

The gradient between these two points don't change so $1.5 < a < 1.6$

c) $x_1 = 1 \cdot 5$

$x_2 = 1 \cdot 632993162$

$x_3 = 1 \cdot 466264596$

$x_4 = 1 \cdot 673135301$

d) x_n oscilates. ~~[illegible]~~ between 2 and 1

7/11

Examiner Comments

(a) B1 M1 A0 M1 A0 [Referred to as 'A common solution' on the mark scheme]
B1: States $\log y = x\log x$.

M1: Allowed for either $\log y \to \frac{1}{y}\frac{dy}{dx}$ or $x\log x \to 1+\log x$.

A0: Incorrect equation involving $\frac{dy}{dx}$.

M1: Sets $1+\log x = 0$ and proceeds to $x = ...$ via $\log x = k$; $k \neq 0$.

A0: Incorrect answer.

(b) M1 A0 [Uses Way 1]
M1: Attempts both $1.5^{1.5}$ and $1.6^{1.6}$ with at least one correct to awrt 1 dp.
A0: No mention of C being continuous.

(c) M1 A0
M1: Allowed for $x_1 = \text{awrt } 1.63$.

A0: x_4 is not given as 1.673.

(d) B1 B1
B1 B1: Allowed for "oscillates between 1 and 2".

Student Response C

a) $y = x^x$

$\ln y = \ln(x^x)$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$

$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$

$\frac{dy}{dx} = y(1 + \ln x)$

$= x^x(1 + \ln x)$

$\frac{dy}{dx} = 0$

$x^x(1 + \ln x) = 0$

From Figure 8, $x^x > 0$

$\rightarrow 1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1}$

x-coordinate of the turning point of C:

$x = \frac{1}{e}$

b) ~~$\alpha^{\alpha} = 2$~~ α ~~satif~~ satisfies:

~~$\alpha^{\alpha} - 2 = 0$~~ $x^x = 2$

$\rightarrow x^x - 2 = 0$

$x = 1.5 \rightarrow x^x - 2 = -0.16\ldots < 0$

$x = 1.6 \rightarrow x^x - 2 = 0.12\ldots > 0$

Continuous function and change of sign on the interval $[1.5, 1.6] \rightarrow$ root

$\Rightarrow 1.5 < \alpha < 1.6$

c) $x_{n+1} = 2x_n^{1-x_n}$

$x_1 = 1.5$

$x_2 = 1.63299$

$x_3 = 1.46626$

$x_4 = 1.673$ (3 dp)

d) x_n is getting further away from $x_1 = 1.5$ so it will diverge away from the value of α, failing to converge on the root.

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Examiner Comments

(a) B1 M1 A1 M1 A1 [Uses Way 1]

B1 M1 A1 M1 A1: Correct solution leading to a correct answer $x = \mathrm{e}^{-1}$.

(b) M1 A1 [Uses Way 2]

M1: Attempts both $1.5^{1.5} - 2$ and $1.6^{1.6} - 2$ with at least one correct to awrt 1 dp.

A1: $-0.16... < 0$ and $0.12... > 0$, C stated as continuous and a correct conclusion.

(c) M1 A1

M1 A1: Correct calculations leading to $x_4 = 1.673$.

(d) B1 B0

B1: Allowed for "diverge away …".

B0: Complete explanation not given.

Exemplar Question 12

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12. (a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

(Total for Question 12 is 7 marks)

Mean Score 3.5 out of 7

Examiner Comments

Part (a) discriminated well between the higher ability candidates and it was possible for candidates of all abilities to gain access to part (b).

In part (a), many candidates struggled to apply a complete strategy which would help them to make significant progress in proceeding from $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}$, via an intermediate stage of $\frac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}$, $\frac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or $\frac{\cos 2\theta}{\sin\theta\cos\theta}$, to the given result $2\cot 2\theta$.

Candidates who used Way 1, as described in the mark scheme, gained some credit for rationalising $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}$ to give $\frac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$. Of those candidates who correctly applied $\sin\theta\cos\theta=\frac{1}{2}\sin 2\theta$ to the denominator, only a few realised that the numerator could be written as $\cos 2\theta$. A common error in using Way 1 was to simplify $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}$ to give $\frac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ or $\frac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$.

Way 2, as described in the mark scheme (and variations of this), was the most popular approach among candidates who completed the proof successfully. Candidates only started to access the marks for Way 2 when they combined their fractions by rationalising the denominator. Errors using Way 2 included the incorrect expansion of $\cos(2\theta+\theta)$ or $\sin(2\theta+\theta)$, bracketing errors, manipulation errors or sign errors.

A few candidates started their proof from $2\cot 2\theta$. They usually progressed as far as writing either $2\cot 2\theta=\frac{2\cos 2\theta}{\sin 2\theta}$ or $2\cot 2\theta=\frac{2(1-\tan^2\theta)}{2\tan\theta}$, but could make no creditable progress until they applied a Way 3 method as described in the mark scheme.

Some candidates spent a considerable amount of time attempting to write $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}$ in terms of θ to give e.g. $\frac{4\cos^3\theta-3\cos\theta}{\sin\theta}+\frac{3\sin\theta-4\sin^3\theta}{\cos\theta}$. Candidates only started to access the marks for this method when they combined their fractions by rationalising the denominator.

In part (b), most candidates used the result given in part (a) to deduce the equation $2\cot 2\theta=4$. Most of these then attempted to solve $\tan 2\theta=\frac{1}{2}$, and some obtained the correct answer $\theta=103.3°$. Common errors included attempting to solve $\tan 2\theta=2$ and finding more than one value for θ. A few candidates obtained $\theta=103.3°$ by solving the equation $\frac{2(1-\tan^2\theta)}{2\tan\theta}=4$.

Mark Scheme

Question	Scheme	Marks	AOs
12	$\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS} = \}\ \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\quad \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} = \}\ \dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\sin\theta}+\dfrac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\cos\theta}$		
	$=\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}\quad \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} = \}\ \dfrac{2\cos 2\theta}{\sin 2\theta} = \dfrac{2\cos(3\theta-\theta)}{\sin 2\theta} = \dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$=\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k;\ k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \dfrac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \dfrac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)} = 4 \Rightarrow 2(1-\tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta+4\tan\theta-1=0 \Rightarrow \tan\theta = \dfrac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2\pm\sqrt{5}\ \} \Rightarrow \tan\theta = k;\ k\neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2-\sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 12	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(\ldots)\cos\theta+(\ldots)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta}+\dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of $\text{LHS} \to \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or $\text{LHS} \to \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or $\text{LHS} \to \cos 2\theta(\cot\theta+\tan\theta)$ or $\text{LHS} \to \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as $\text{LHS} = \ldots = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}=\dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}=2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \dfrac{1}{2}$
Note:	Condone 2nd M1 for applying $\dfrac{1}{2}\arctan\left(\dfrac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \dfrac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \dfrac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0$, and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta = \text{awrt } 103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta = \text{awrt } 103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

Student Response A

(a) $\dfrac{\cos 3\theta}{\sin \theta} + \dfrac{\sin 3\theta}{\cos \theta}$

$\dfrac{\cos^2 3\theta + \sin^2 3\theta}{\sin \theta \cos \theta}$

$\dfrac{1}{\frac{1}{2} \sin 2\theta}$

(b) $2 \cot 2\theta = 4 \qquad 180^\circ < \theta < 360^\circ$

$\cot 2\theta = 2$

$\tan 2\theta = \frac{1}{2}$

$2\theta = 2\,206.6^\circ,$

$\theta = 103.3^\circ$

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Examiner Comments

(a) M0 A0 M0 A0

M0: Incorrect method of forming a common denominator of $\sin\theta\cos\theta$.
A0 dM0 A0: Follows from 1st M0.

Note: In (a), the mark scheme states give 1st M0 for $\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos^2 3\theta + \sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta + \sin^2 3\theta}{\sin\theta\cos\theta}$.

(b) M1 M1 A1
M1 dM1 A1: Correct solution leading to a correct final answer of 103.3 only.

Student Response B

a) $$\frac{(\cos 3\theta)(\cos\theta) + (\sin 3\theta)(\sin\theta)}{\sin\theta \cos\theta}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$
$$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

$$\frac{(\cos 3\theta)(\cos\theta) + (\sin 3\theta)(\sin\theta)}{\frac{1}{2}\sin 2\theta}$$

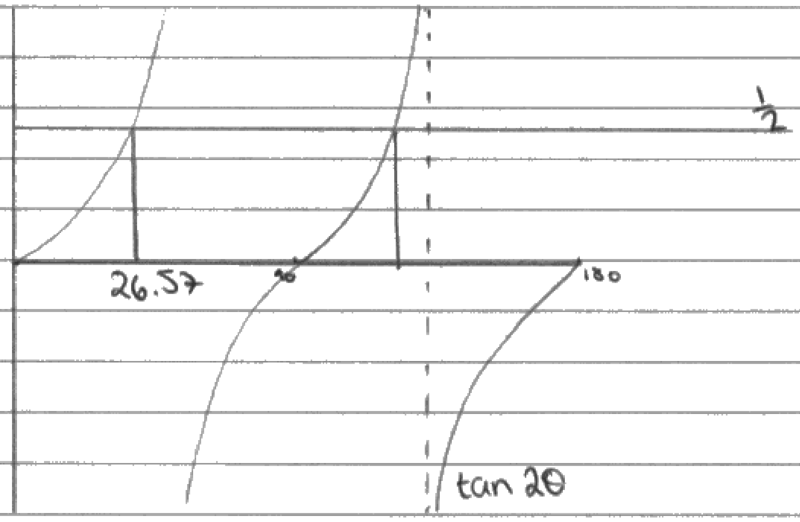
$$\frac{2(\cos\theta\cos 3\theta + \sin\theta\sin 3\theta)}{\sin 2\theta}$$

$$\frac{2(\cos\theta\cos 3\theta)}{\sin 2\theta} + \frac{2(\cancel{2}\sin\theta\sin 3\theta)}{\sin 2\theta}$$

$$\frac{2\cos\theta\cos 3\theta}{\sin 2\theta} + \frac{2\sin\theta\sin 3\theta}{\sin 2\theta}$$

$$\cot\theta = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\tan\theta}$$

b) $$2\cot 2\theta = 4$$
$$\cot 2\theta = 2$$
$$\frac{1}{\tan 2\theta} = 2$$
$$\tan 2\theta = \frac{1}{2}$$



$\tan^{-1}\left(\frac{1}{2}\right) = 26.57°$

Solution = $90° + 26.57°$
$= ~~9~~\ 116.57°$
$= 116.6°$ (1dp)

4/7

Examiner Comments

(a) M1 A0 M1 A0

M1: Correct valid method of forming a common denominator of $\sin\theta\cos\theta$.

A0: Does not achieve $\cos(3\theta-\theta)$ or $\cos 2\theta$ on the numerator.

dM1: Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$.

A0: Follows from earlier A0.

(b) M1 M1 A0

M1: Applies $\cot 2\theta = \dfrac{1}{\tan 2\theta}$.

dM1: Rearranges to give $\tan 2\theta = k$ and applies $\arctan k$. Note: their $k = \dfrac{1}{2}$.

A0: Incorrect answer.

Student Response C

$\cos(3\theta) = \cos(2\theta + \theta)$ $\quad \cot\theta = \frac{1}{\tan\theta}$ or $\frac{\cos\theta}{\sin\theta}$ (3)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\therefore \cos(2\theta + \theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$\sin(3\theta) = \sin(2\theta + \theta)$, $\quad \sin(A+B) = \sin A\cos B + \cos A\sin B$

$\therefore \sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$$\therefore \frac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\sin\theta} + \frac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\cos\theta}$$

$$= \frac{\cos 2\theta\cos\theta}{\sin\theta} - \cancel{\sin 2\theta} + \cancel{\sin 2\theta} + \frac{\cos 2\theta\sin\theta}{\cos\theta}$$

$$= \frac{\cos 2\theta\cos\theta}{\sin\theta} + \frac{\cos 2\theta\sin\theta}{\cos\theta}$$

$$= \frac{(\cos 2\theta\cos\theta)\cos\theta + (\cos 2\theta\sin\theta)\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos 2\theta\cos^2\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos(2\theta)(\cos^2\theta + \sin^2\theta)}{\frac{1}{2}\sin 2\theta}$$

Since $\cos^2\theta + \sin^2\theta \equiv 1$

$$\Rightarrow \frac{\cos(2\theta)}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta \equiv \text{RHS} \;//$$

b) $2\cot 2\theta = 4$, $180 < 2\theta < 360$

$\therefore \cot 2\theta = 2 \Rightarrow \frac{1}{\tan 2\theta} = 2 \Rightarrow \tan 2\theta = \frac{1}{2}$

$\therefore \cancel{2\theta =}\ 2\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6$ (1dp)

$\therefore$ ~~$\theta = 13.3$ (1.d.p)~~

~~13.3 +~~ ~~13.3 + 180 = 193.3 (1dp)~~

$26.6 + 180 = 206.6$

$\therefore \theta = 103.3$ (1dp)

$206.6 + 180 = 386.6$

$386.6 \div 2$ 193.3

$\therefore \theta = 103.3$ and 193.3 (1dp)

6/7

Examiner Comments

(a) M1 A1 M1 A1
M1 A1 dM1 A1: Correct proof.

(b) M1 M1 A0

M1: Applies $\cot 2\theta = \frac{1}{\tan 2\theta}$.

dM1: Rearranges to give $\tan 2\theta = k$ and applies $\arctan k$. Note: their $k = \frac{1}{2}$.

A0: Correct answer 103.3 is given but other answers (whether inside or outside the given range of $90° < \theta < 180°$) need to be unambiguously rejected.

Exemplar Question 13

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13.

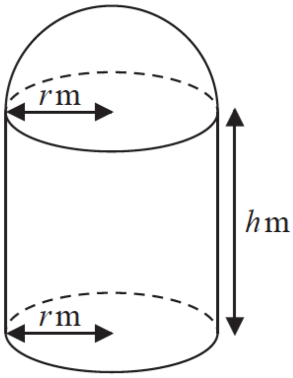

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

(Total for Question 13 is 10 marks)

Mean Score 4.8 out of 10

Examiner Comments

This question discriminated well between the medium and higher ability candidates. Lower ability candidates, however, struggled to gain any creditable progress. It was obvious that some candidates had been well prepared for this 'optimisation' question and the quality of their responses reflected this. Candidates who attempted this question generally found parts (b) and (c) accessible but some struggled to complete the proof in part (a). In a few cases, candidates who scored zero marks in part (a) went on to achieve full marks in the remainder of the question by using the given result for the surface area. There were, however, many instances where candidates who struggled with part (a) then gave up and did not attempt the remainder of the question.

In part (a), successful candidates followed a correct strategy of forming an equation for the volume of the tank, rearranging their equation to give an expression for h in terms of r and substituting their expression for h into their formula for the surface area of the tank. Some candidates did not halve the given sphere formulae so that they could use these for the hemispherical shell. This led to errors in both their volume equation and surface area expression. Other candidates did not include the area of the circular base in their surface area expression. These errors led to an incorrect volume equation $6 = \pi r^2 h + \frac{4}{3}\pi r^3$ and incorrect surface area expressions $A = 5\pi r^2 + 2\pi rh$ and $A = 2\pi r^2 + 2\pi rh$. In other cases, bracketing errors, manipulation errors, or using incorrect formulae for the curved surface area of a cylinder, prevented candidates from achieving the given result.

In part (b), many candidates who attempted this part applied a complete method to find their value of r for which $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$. Most successful candidates found $r = 1.05$ metres, with a few giving an exact answer $r = \sqrt[3]{\frac{18}{5\pi}}$ metres, which was condoned. Some candidates differentiated A incorrectly to give $\frac{\mathrm{d}A}{\mathrm{d}r} = 12\ln r + \frac{10}{3}\pi r$, while others used incorrect algebra when solving the correct equation $-\frac{12}{r^2} + \frac{10}{3}\pi r = 0$. A few candidates used the incorrect method of finding a value of r for which $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} = 0$.

In part (c), most candidates who attempted this part correctly substituted their value of r, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$, into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$, with many obtaining a correct minimum surface area of $17\,\mathrm{m}^2$. A few candidates, who made no creditable progress, substituted their value for r, found from solving $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} = 0$, into the model for A.

Mark Scheme

Question	Scheme	Marks	AOs
13 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \pi r h = \frac{6}{r} - \frac{2}{3}\pi r^2,\ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi r h + 2\pi r^2 \{\Rightarrow A = 3\pi r^2 + 2\pi r h\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2\ \Rightarrow\ A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\}$	M1	3.4
		A1	1.1b
	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r} = 0 \Rightarrow\right\}\ -\frac{12}{r^2} + \frac{10}{3}\pi r = 0 \Rightarrow -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \ldots \left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736... \Rightarrow r = 1.05\text{ (m) (3 sf) or awrt }1.05\text{ (m)}$	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min} = \frac{12}{(1.046...)} + \frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min} = 17.20... \Rightarrow\}\ A = 17\ (\mathrm{m}^2)$ or $A = \text{awrt }17\ (\mathrm{m}^2)$	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = \ldots$ or $\pi h = \ldots$ or $\pi r h = \ldots$ or $rh = \ldots$, where '...' $= \mathrm{f}(r)$ into an expression for the surface area which is of the form $A = \lambda \pi r^2 + \mu \pi r h;\ \lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\ 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = \ldots$ or $S = \ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r} =\right\}\ -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \rightarrow r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r = 1.05$ (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Notes for Question 13 Continued	
Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r\ >0\ \Rightarrow r>1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r\ >0\ \Rightarrow r>1.0464...\ \Rightarrow r=1.0464...$
(c)	
M1:	Substitutes their $r=1.046...$, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b), into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their *r* which has been found from solving $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=0$ or from using $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
A1ft:	$\{A=\}\,17$ or $\{A=\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of *r* for $0.6\le$ their $r\le 1.3$ (and where their *r* has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b))

r	***A***	***A* (nearest integer)**
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Note:	Give M1 A1 for $A=17\ (\mathrm{m}^2)$ or $A=$ awrt $17\ (\mathrm{m}^2)$ from no working

Student Response A

a) Volume of half sphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

Surface Area of half sphere $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

Volume of cylinder $=$ ~~[illegible]~~ $h \times \pi r^2$

Volume of tank $=$ ~~[illegible]~~ $h\pi r^2 + \frac{2}{3}\pi r^3 = 6$

Surface Area of cylinder $= h \times 2\pi r = 2\pi r h$

Volume of tank: $h\pi r^2 + \frac{2}{3}\pi r^3 = 6$

$h\pi r^2 = 6 - \frac{2}{3}\pi r^3$

$h = \dfrac{6 - \frac{2}{3}\pi r^3}{\pi r^2}$

$$\left[\frac{2\pi r^3}{3} \times \frac{1}{\pi r^2} \qquad \frac{2\pi r^3}{3\pi r^2}\right]$$

$h = \dfrac{6}{\pi r^2} - \dfrac{2}{3}r$

$\therefore S = 2\pi r\left(\dfrac{6}{\pi r^2} - \dfrac{2}{3}r\right)$

$S = \dfrac{12\pi r}{\pi r^2} - \dfrac{4\pi r^2}{3}$

$S = \dfrac{12}{r} - \dfrac{4}{3}\pi r^2$

b) Minimum when $\dfrac{d^2y}{dx^2} > 0$

* Using equation given in paper *

$S = 12r^{-1} + \frac{5}{3}\pi r^2$

$\dfrac{dS}{dr} = -\dfrac{12}{r^2} + \dfrac{10}{3}\pi r \rightarrow \dfrac{d^2S}{dr^2} = \dfrac{24}{r^3} + \dfrac{10}{3}\pi$

(Next page)

$$\frac{d^2S}{dr^2} = \frac{24}{r^3} + \frac{10}{3}\pi$$

$$\frac{24}{r^3} + \frac{10}{3}\pi = 0$$

$$24 + \frac{10}{3}\pi r^3 = 0$$

$$\frac{10}{3}\pi r^3 = -24$$

$$\pi r^3 = -\frac{36}{5}$$

$$r^3 = -\frac{\frac{36}{5}}{\pi}$$

$$r = \sqrt[3]{-\frac{36}{5}/\pi}$$

$r = -1.31844$

$r = 1.32$ (3sf) (~~As~~ Radius can't be negative).

c) Using $r = 1.32$ ~~cm~~, $S = \frac{12}{1.32} + \frac{5}{3}\pi(1.32)^2$

$S = 18.2$

$S = 18\,m^2$

3/10

Examiner Comments

(a) B1 M0 A0 A0

B1: States a correct $h\pi r^2 + \frac{2}{3}\pi r^3 = 6$.

M0: Their surface area expression is not in the form $S = \lambda\pi r^2 + \mu\pi rh;\ \lambda, \mu \neq 0$.

A0 A0: Follows M0.

(b) M1 A1 M0 A0

M1 A1: For a correct $\frac{dS}{dr} = -\frac{12}{r^2} + \frac{10}{3}\pi r$.

M0: Does not set their $\frac{dS}{dr} = 0$.

A0: Follows previous M0.

(c) M0 A0

M0: Incorrect method of substituting their r which has been found from solving $\frac{d^2S}{dr^2} = 0$ into the model with equation $S = \frac{12}{r} + \frac{5}{3}\pi r^2$.

A0: Follows M0

Student Response B

a) $V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$6 = \frac{4}{3}\pi r^3 + \pi r^2 h$

$6 = \pi r^2 \left(\frac{4}{3}r + h\right)$

$\frac{6}{\pi r^2} = \frac{4}{3}r + h$

$\frac{6}{\pi r^2} - \frac{4}{3}r = h$

$SA = 4\pi r^2 + 2\pi r h$

$= 4\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{4}{3}r\right)$

$= 24 \cancel{\times} -\frac{16}{3}\pi r^3 + \frac{12}{r} - \frac{8}{3}\pi r^2$

b) $A = \frac{12}{r} + \frac{5}{3}\pi r^2$

$A = 12r^{-1} + \frac{5}{3}\pi r^2$

$\frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$

$\frac{dA}{dr} = 0 \qquad -12r^{-2} + \frac{10}{3}\pi r = 0$

$\cancel{\frac{10}{3}\pi r = 12\pi}$

$\cancel{r\left(-12 + \frac{10}{3}\pi\right) = 0}$

$\cancel{r \neq 0 \text{ or } -12 + \frac{10}{3}\pi = 0}$

$\cancel{\frac{10}{3}\pi = 12}$

$\cancel{r \neq 0}$

$12r^{-2} = \frac{10}{3}\pi r$

$\frac{12}{r^2} = \frac{10}{3}\pi r$

$12 = \frac{10}{3}\pi r^3$

$\frac{18}{5\pi} = r^3$

$r = \sqrt[3]{\frac{18}{5\pi}} \qquad r = 1.0464$ 4dp

$$\frac{d^2A}{dr^2} = 24r^{-3} + \frac{10}{3}\pi$$

$$24\left(\sqrt[3]{\frac{18}{5\pi}}\right)^{-3} + \frac{10}{3}\pi = 10\pi$$

$$\frac{d^2A}{dr^2} > 0 \quad \text{Minimum when } r = 1.0464$$

$$r = 1.046 \text{ 3dp}$$

$$SA = \frac{12}{1.046} + \frac{5}{3}\pi(1.046)^2$$

$$= 17.201 \text{ m}^2$$

$$= 17 \text{ m}^2 \text{ nearest integer}$$

7/10

Examiner Comments

(a) B0 M1 A0 A0

B0: Does not state a correct $\pi r^2 h + \frac{2}{3}\pi r^3 = 6$.

M1: Complete process of substituting their $h = \mathrm{f}(r)$ into an expression for the surface area which is of the form $\mathrm{SA} = \lambda\pi r^2 + \mu\pi rh;\ \lambda, \mu \neq 0$.

A0 A0: Follows B0.

(b) M1 A1 M1 A1

M1 A1: For a correct $\frac{\mathrm{d}A}{\mathrm{d}r} = -12r^{-2} + \frac{10}{3}\pi r$.

M1 A1: Sets their $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ and solves to give a correct answer for *r* which rounds to 1.05.

(c) M1 A1

M1: Substitutes their *r* which has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ into the model with equation $\mathrm{SA} = \frac{12}{r} + \frac{5}{3}\pi r^2$.

A1ft: Obtains a correct answer of 17

.

Student Response C

a. surface area of hemisphere $= \frac{1}{2}4\pi r^2$

$= 2\pi r^2$

surface area of cylinder $= \pi r^2 + 2\pi r h$

overall surface area $= 2\pi r^2 + \pi r^2 + 2\pi r h$

$= 3\pi r^2 + 2\pi r h$

volume of tank $= 6m^3 = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$$6 = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$6 = \pi r^2\left(h + \frac{2}{3}r\right)$$

$$\frac{6}{\pi r^2} = h + \frac{2}{3}r$$

$$h = \frac{6}{\pi r^2} - \frac{2}{3}r$$

sub h into equation for overall surface area:

$$SA = 3\pi r^2 + 2\pi r h$$

$$= 3\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right)$$

$$= 3\pi r^2 + \frac{12\cancel{\pi r}}{\cancel{\pi} r^{\cancel{2}}} - \frac{4}{3}\pi r^2$$

$$= 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2$$

$= \frac{12}{r} + \frac{5}{3}\pi r^2$ as required

(S = surface area)

b. $\cancel{SA}\ S = \frac{12}{r} + \frac{5}{3}\pi r^2 = 12r^{-1} + \frac{5}{3}\pi r^2$

$$\frac{dS}{dr} = -12r^{-2} + \frac{10}{3}\pi r$$

$$= \frac{10}{3}\pi r - \frac{12}{r^2} = 0$$

$\frac{10\pi r}{3} = \frac{12}{r^2}$

$10\pi r^3 = 36$

$r^3 = \frac{36}{10\pi}$

$r = 1.07m \Rightarrow$ the surface area is a minimum when $r = 1.07m$

c. $r = 1.07m$, $S = \frac{12}{r} + \frac{5}{3}\pi r^2$

$= \frac{12}{1.07} + \frac{5}{3} \times \pi \times (1.07)^2$

$= 17.2m^2$

$= 17m^2$ (to the nearest integer)

9/10

Examiner Comments

(a) B1 M1 A1 A1

B1 M1 A1 A1: Correct proof.

(b) M1 A1 M1 A0

M1 A1: For a correct $\frac{dS}{dr} = -12r^{-2} + \frac{10}{3}\pi r$.

M1: Sets their $\frac{dS}{dr} = 0$ and rearranges to give $r^3 = k;\ k \neq 0$.

A0: Incorrect answer.

(c) M1 A1

M1: Substitutes their r which has been found from solving $\frac{dS}{dr} = 0$ into the model with equation $S = \frac{12}{r} + \frac{5}{3}\pi r^2$.

A1ft: Obtains an allowable value for the minimum surface area. Note this is a follow through mark, as long as $0.6 \leq$ their $r \leq 1.3$ (see table in the note on the mark scheme).

Exemplar Question 14

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14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{dh}{4-\sqrt{h}} = -8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant.

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

(Total for Question 14 is 15 marks)

Mean Score 5.1 out of 15

Examiner Comments

This question discriminated well between the higher ability candidates. Lower and medium ability candidates, however, found this question demanding with most of these candidates scoring no more than two marks. Most higher ability candidates made good progress in parts (a) and (c) but they often struggled to make progress in part (b).

In part (a), most candidates who attempted this part differentiated $u = 4 - \sqrt{h}$ to give a correct $\frac{\mathrm{d}u}{\mathrm{d}h} = -\frac{1}{2}h^{-\frac{1}{2}}$, with a few finding a correct $\frac{\mathrm{d}h}{\mathrm{d}u} = -2(4-u)$ and some differentiating incorrectly to give $\frac{\mathrm{d}u}{\mathrm{d}h} = \frac{1}{2}h^{-\frac{1}{2}}$. At this stage, some candidates failed to make further creditable progress. Many candidates struggled to apply the substitution completely to give an integral of the form $\int \frac{k(4-u)}{u}\,\mathrm{d}u$, with some making sign errors in the numerator of this integral. Those who progressed this far generally went on to divide each term in the numerator by *u* to give an integral of the form $\int \left(\frac{A}{u} + B\right)\mathrm{d}u$, with many of them obtaining a correct $\int \left(-\frac{8}{u} + 2\right)\mathrm{d}u$. Most integrated correctly and applied $u = 4 - \sqrt{h}$ to obtain a correct $-8\ln\left|4-\sqrt{h}\right| + 2(4-\sqrt{h}) + c$. Some candidates could not deal with the transition from $8 + c$ to k to achieve the given $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k$, and a large number of these candidates incorrectly stated $k = 8$. Those few candidates who applied integration by parts on $\int \frac{2u-8}{u}\,\mathrm{d}u$ rarely progressed to give a correct solution to this part.

In part (b), some candidates correctly set $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$ but only a few deduced the equation $4 - \sqrt{h} = 0$. Some candidates used $h = 16$ to state a correct allowable answer such as $0 < h < 16$, $0 \le h \le 16$ or $h < 16$. A common mistake was to solve $\sqrt{h} = 4$ to give $h = 2$.

Material for part (c) was sometimes seen in part (b). Most candidates who attempted part (c) separated the variables correctly. Some candidates integrated $t^{0.25}$ incorrectly to give $\lambda t^{0.25}$, $\lambda t^{-0.75}$ or $\frac{5}{4}t^{1.25}$ while others integrated $\frac{1}{(4-\sqrt{h})}$ without reference to the result from part (a). Many candidates applied $t = 0$, $h = 1$ to find their constant of integration. Common errors at this stage included not applying a constant of integration and using either $t = 0$, $h = 0$ or $t = 1$, $h = 1$ to find their constant of integration. Most candidates who progressed this far applied a complete process of substituting their constant of integration and $h = 12$ into their integrated equation and solving this equation to find their value for *t*. In many cases, sign errors, bracketing errors, manipulation errors or an incorrect method of solving a correct $t^{1.25} = 221.2795202...$, prevented candidates from obtaining the correct answer $t = 75.2$ years.

Mark Scheme

Question	Scheme	Marks	AOs
14 (a)	$\{u = 4 - \sqrt{h} \Rightarrow\}\ \frac{du}{dh} = -\frac{1}{2}h^{-\frac{1}{2}}$ or $\frac{dh}{du} = -2(4-u)$ or $\frac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{\int \frac{dh}{4-\sqrt{h}} = \right\} \int \frac{-2(4-u)}{u}du$	M1	2.1
	$= \int \left(-\frac{8}{u} + 2\right) du$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) + c = -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16,\ 0 \le h < 16,\ 0 < h \le 16,\ 0 \le h \le 16,$ $h < 16,\ h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \frac{1}{(4-\sqrt{h})}dh = \int \frac{1}{20}t^{0.25}dt$	B1	1.1b
	$-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \frac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t = 0,\ h = 1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h = 12 \Rightarrow\}\ -8\ln\lvert 4-\sqrt{12}\rvert - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\int_1^{12} \frac{20}{(4-\sqrt{h})}dh = \int_0^T t^{0.25}dt$	B1	1.1b
	$\left[20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h})\right]_1^{12} = \left[\frac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

Notes for Question 14	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h,\ \ \mathrm{d}h = -2(4-u)\mathrm{d}u,\ \ \mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: **Note:**	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u\ ;\ k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\};\ \ A,\ B \neq 0$
M1:	$\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \ \rightarrow\ D\ln u + Eu;\ \ A,\ B,\ D,\ E \neq 0;$ with or without a constant of integration
A1:	$\int\left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \ \rightarrow\ -8\ln u + 2u\,;$ with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ + k.$ Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ + k,$ with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k\ =\ -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}\ + k$
	<u>**Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark**</u>
	$\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u\ =\ \int \frac{2u-8}{u}\ \mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u\ =\ (2u-8)\ln u\ -\ 2(u\ln u - u)\ \{+\ c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{\mathrm{d}u\}\ ;\ \ A,\ B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu;\ D,\ E \neq 0;$ which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u\ -\ 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u\ -\ 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: **Note:**	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow\ 4-\sqrt{h} = 0.$ Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow\ 4-\sqrt{h}\ > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 14	
(c)	**Way 1**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = \ldots$ followed by a correct method to give $t = \ldots$; $t > 0$
Note:	$t^{\text{their } 1.25} = \ldots$ can be negative, but their '$t = \ldots$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = \ldots$ (usually $t^{0.25} = \ldots$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{\mathrm{d}h}{\mathrm{d}t}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1:	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working.
Note:	Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

Student Response A

a) $u = 4 - \sqrt{h}$ $\qquad \sqrt{h} = 4 - u.$

$u = 4 - h^{1/2}$ $\qquad h = (4-u)^2$

$\frac{du}{dh} = -\frac{1}{2}h^{-1/2}$

$dh = \frac{1}{-1/2 h^{-1/2}} du.$

$= \int \frac{\frac{1}{-1/2 h^{-1/2}}}{u} du$

$= \int \frac{\frac{1}{-1/2((4-u)^2)^{-1/2}}}{u} du$

$= \int \frac{\frac{1}{-1/2(4-u)^{-1}}}{u} du$

$= \int 1 \times \frac{u}{-1/2(4-u)^{-1}} du$

$= \int \frac{u}{-1/2(4-u)^{-1}} du$

$= \int \frac{\frac{u}{-1/2}}{4-u}$

$= \int \frac{u.4-u}{-1/2}$

$= \int \frac{4u-u^2}{-0.5}$

$$= \int -2u + \frac{1}{2}u^2 \quad du.$$

$$= \quad -8\ln|4-\sqrt{h}| - 2\sqrt{h} + k.$$

b)

c) ~~ka1~~ $\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20}$

$$\frac{dh}{dt} = \frac{t^{0.25}}{20}(4-\sqrt{h})$$

$$\int \frac{1}{4-\sqrt{h}} \, dh = \int \frac{t^{0.25}}{20} \, dt$$

$$\int \frac{1}{4-\sqrt{h}} \, dh = \frac{t^{5/4}}{25} = \frac{1}{25} t^{5/4}$$

$$\int \frac{1}{(4-\sqrt{h})} = \int (4-\sqrt{h})^{-1} \qquad -h^{1/2} = -\tfrac{1}{2}h^{-1/2}$$

$$\frac{dy}{dx} \; -1(4-\sqrt{h})^{-2} \cdot \quad -\cancel{4} \quad -\tfrac{1}{2}h^{-1/2}$$

$$= \tfrac{1}{2}h^{-1/2}(4-\sqrt{h})^{-2}$$

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Examiner Comments

(a) B1 M0 M0 M0 A0 A0

B1: $\frac{\mathrm{d}u}{\mathrm{d}h} = -\frac{1}{2}h^{-\frac{1}{2}}$ is correct.

M0: Does not obtain an expression of the form $\int \frac{k(4-u)}{u}\,\mathrm{d}u$; $k \neq 0$.

M0 M0 A0 A0: Makes no further creditable progress.

(b) M0 A0

M0 A0: Does not attempt this part.

(c) B1 M1 A0 M0 M0 M0 A0

B1: Separates the variables correctly.

M1: Integrates $t^{0.25}$ to give $\lambda t^{0.25}$; $\lambda \neq 0$.

A0: Does not obtain a correct integrated equation.

M0 dM0 M0 A0: Makes no further creditable progress.

Student Response B

14a) $\int \frac{1}{4-\sqrt{h}}\, dh$ $\quad U = 4 - \sqrt{h}$

$U = 4 - h^{\frac{1}{2}}$

$\frac{dU}{dh} = -\frac{1}{2}h^{-\frac{1}{2}}$

$\frac{dh}{dU} = -2h^{-\frac{1}{2}}$

$dh = 2h^{-\frac{1}{2}}\, dU$

$d\cancel{U}^{h} = 2\sqrt{h}\, dU$ $\quad U - 4 = -\sqrt{h}$

$4 + U = {}^{2}\sqrt{\cancel{h}}$

$2(4+U)\,dU\cancel{4} = {}^{2}\sqrt{h}$

$\int \frac{2(4+U)}{U}\, dU = \int \frac{8}{U} + 2\, dU$

$8\ln|U| + 2U + \cancel{C}\, K$

$\cancel{8}\ 8\ln|4-\sqrt{h}| + 2(4-\sqrt{h}) + K$ $\cancel{+K}$

$\cancel{-8\ln|4-\sqrt{h}| - 2\sqrt{h} + 8 + K}$

$8\ln|4-\sqrt{h}| + 2(4-\sqrt{h}) + K$

$8\ln|4-\sqrt{h}| + 8 - 2\sqrt{h}$

$-8(-8\ln|4-\sqrt{h}| - 2\sqrt{k}\ \cancel{+K}) + K$

14b) $0 \leq h \leq 16$

14c) $\dfrac{dH}{dt} = \dfrac{1}{20} t^{0.25}(4-\sqrt{h})$

$\displaystyle\int \frac{1}{4-\sqrt{h}}\, dH = \int \frac{1}{20} t^{0.25}$

$\displaystyle\int \frac{1}{4-h^{\frac{1}{2}}} = \cancel{8h} - 8\ln|4-\sqrt{h}| - 2\sqrt{h} = \frac{1}{25} t^{1.25} + K$

$-8\ln|4-1| - 2 = \dfrac{1}{25}\ \cancel{t^{1.25}} + K$

$-8\ln|3| = \frac{51}{25} = K$

$-8\ln|4-\sqrt{h}| - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln 3 - \frac{51}{25}$

$-8\ln|4-\sqrt{12}| - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln 3 - \frac{51}{25}$

$8.89 = \frac{1}{25}t^{1.25}$

$222.28 = t^{1.25}$

$\sqrt[1.25]{222.28} = t$

75.4 (3 sig fig) $= t$.

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Examiner Comments

(a) B1 M0 M1 M1 A0 A0

B1: $\frac{\mathrm{d}u}{\mathrm{d}h} = -\frac{1}{2}h^{-\frac{1}{2}}$ is correct.

M0: $\int \frac{2(4+u)}{u}\,\mathrm{d}u$ is not an expression of the form $\int \frac{k(4-u)}{u}\,\mathrm{d}u$; $k \neq 0$.

M1 M1: Splits expression and integrates to $8\ln u + 2u$, which is of the form $D\ln u + Eu$; $D, E \neq 0$.

A0: Does not correctly obtain $-8\ln u + 2u$.

A0*: Follows previous A0.

(b) M1 A1

M1 A1: $0 \leq h \leq 16$ is an acceptable answer.

(c) B1 M1 A1 M0 M0 M1 A0

B1: Separates the variables correctly. Condone the missing '$\mathrm{d}t$'.

M1 A1: Integrates to give a correct $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} + k = \frac{1}{25}t^{1.25}$.

M0 dM0 (dependent on the previous M mark): Does not apply $t = 0$ to their integrated equation. Note: Instead they apply $t = 1$.

M1: Rearranges their equation to make $t^{\text{their } 1.25} = \ldots$ followed by a correct method to give $t = \ldots$.

A0: Incorrect answer.

Student Response C

a) $u = 4 - \sqrt{h}$

$\frac{du}{dh} = -\frac{1}{2}h^{-\frac{1}{2}} = -\frac{1}{2\sqrt{h}}$

$\sqrt{h} = 4 - u$

$h = (4-u)^2$

$\frac{dh}{du} = -2(4-u)$

$\therefore \int \frac{-2(4-u)}{u} \, du$

$= \int \frac{-8}{u} + 2 \, du$

$= -8\ln|u| + 2u + c$

$= -8\ln|4-\sqrt{h}| + 2(4-\sqrt{h}) + c$

$= -8\ln|4-\sqrt{h}| + 8 - 2\sqrt{h} + c$

$\therefore 8 + c = k$

$= -8\ln|4-\sqrt{h}| - 2\sqrt{h} + k$

b) $\frac{dh}{dt} = 0$

$$\frac{t^{0.25}(4-\sqrt{h})}{20} = 0$$

$4 - \sqrt{h} = 0$

$h = 2m$

$\therefore h \geq 2$

c) $\frac{1}{4-\sqrt{h}}\,dh = \frac{t^{0.25}}{20}\,dt$

$-8\ln|4-\sqrt{h}| - 2\sqrt{h} + k = \frac{t^{\frac{5}{4}}}{25}$

$\therefore t=0,\ h=1$

$-8\ln|4-1| - 2(1) + k = 0$

$-8\ln(3) - 2 + k = 0$

$k = 8\ln 3 + 2$

Question 14 continued

$\therefore\ -8\ln|4-\sqrt{h}| - 2\sqrt{h} + 8\ln 3 + 2 = \frac{t^{\frac{5}{4}}}{25}$

for $h = 12$,

$-8\ln|4-\sqrt{12}| - 2\sqrt{12} + 8\ln 3 + 2 = \frac{t^{\frac{5}{4}}}{25}$

$-8\ln|4-2\sqrt{3}| - 4\sqrt{3} + 8\ln 3 + 2 = \frac{t^{\frac{5}{4}}}{25}$

$221.2795 = t^{\frac{5}{4}}$

~~log~~ $\sqrt[\frac{5}{4}]{221.2795} = t$

$t = 75.2$ years

14/15

Examiner Comments

(a) B1 M1 M1 M1 A1 A1
B1 M1 M1 M1 A1 A1: Correct proof.

(b) M1 A0
M1: Deduces that $4-\sqrt{h}=0$.
A0: Incorrect answer.

(c) B1 M1 A1 M1 M1 M1 A1
B1: Separates the variables correctly. Condone absence of integral signs.

M1 A1: Integrates to give a correct $-8\ln\left|4-\sqrt{h}\right|-2\sqrt{h}+k=\dfrac{t^{\frac{5}{4}}}{25}$.

M1: Complete process of applying both $t=0$ and $h=1$ to their integrated equation which contains a constant of integration.
dM1 (dependent on the previous M mark): Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation.
M1: Rearranges their equation to make $t^{\text{their } 1.25}=\ldots$ followed by a correct method to give $t=\ldots.$
A1: Obtains the correct answer of 75.2

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Exemplar Question 1

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1. Three Bags, A, B and C, each contain 1 red marble and some green marbles.

Bag A contains 1 red marble and 9 green marbles only
Bag B contains 1 red marble and 4 green marbles only
Bag C contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from Bag A.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag B.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag C.

(a) Draw a tree diagram to represent this information.

(2)

(b) Find the probability that Sasha selects 3 green marbles.

(2)

(c) Find the probability that Sasha selects at least 1 marble of each colour.

(2)

(d) Given that Sasha selects a red marble, find the probability that he selects it from Bag B.

(2)

(Total for Question 1 is 8 marks)

Mean Score: 6.0 out of 8

Examiner Comments

On the whole, this accessible question was answered very well by majority of candidates with nearly 40% going on to achieve full marks.

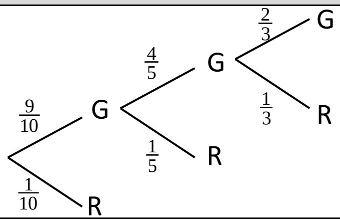
In part (a) most were able to draw the correct tree diagram and label both outcomes and probabilities clearly. However, a number of candidates made mistakes due to deciding Bag A contained only 9 marbles, B contained 4 marbles and C contained 2 marbles. Some candidates attempted a 3 branch tree diagram and made little progress. Others thought that the successive draws from Bag C could take place. All branches must be labelled as blank branches are not assumed to represent a probability of 0.

Part (b) was almost universally answered correctly, even if the tree diagram was incorrect, with the majority of candidates gaining full marks or at least follow through marks, although occasionally the three probabilities were added rather than multiplied together.

In part (c) many correct responses were seen here with the majority of candidates gaining full marks or at least follow through marks for the method. The most common error was the inclusion of 0.1 for P(red) in addition to the two correct products, perhaps indicating that the candidates failed to interpret ‘at least’ correctly.

Part (d) was somewhat less successfully answered. Many candidates were unable to recognise that this involved conditional probability, with the most common wrong answer of $\frac{9}{10} \times \frac{1}{5}$ often seen. A small number had the conditional probability the wrong way round, attempting to find $P(R \mid B)$ instead of $P(B \mid R)$.

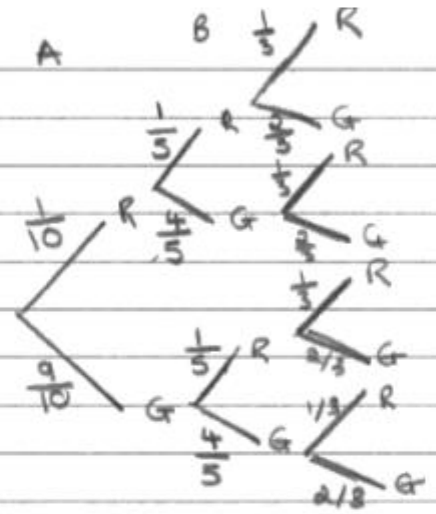
Mark Scheme

Question	Scheme	Marks	AOs
1(a)		B1 dB1	1.1b 1.1b
		(2)	
(b)	$\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}$	M1	1.1b
	$=\frac{12}{25}$ (= 0.48)	A1	1.1b
		(2)	
(c)	$\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}$ or $1-\left(\frac{1}{10}+\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}\right)$	M1	3.1b
	$=\frac{21}{50}$ (= 0.42)	A1	1.1b
		(2)	
(d)	[P(Red from B|Red selected)] $=\dfrac{\frac{9}{10}\times\frac{1}{5}}{\frac{1}{10}+\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}}\left[=\dfrac{\frac{9}{50}}{\frac{13}{25}}\right]$	M1	3.1b
	$=\frac{9}{26}$	A1	1.1b
		(2)	
		(8 marks)	

	Notes
	Allow decimals or percentages throughout this question.
(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\frac{12}{25}$ oe
(c)	**M1:** Either addition of two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-('\frac{1}{10}'+'(b)')$ **A1:** $\frac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\dfrac{'\frac{9}{10}'\times'\frac{1}{5}'}{1-'(b)'}$ or $\dfrac{'\frac{9}{10}'\times'\frac{1}{5}'}{'\frac{1}{10}'+'(c)'}$ with num < den **A1:** $\frac{9}{26}$ (allow awrt 0.346)

Student Response A

a)



RRR
RRG
RGR
RGG
GRR
GRG
GGR
GGG

b) $\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3} = \frac{12}{25}$

c) RRG, RGR, RGG, GRR, GRG, GGR

$\left(\frac{1}{10}\times\frac{1}{5}\times\frac{2}{3}\right)+\left(\frac{1}{10}\times\frac{4}{5}\times\frac{1}{3}\right)+\left(\frac{1}{10}\times\frac{4}{5}\times\frac{2}{3}\right)+\left(\frac{9}{10}\times\frac{1}{5}\times\frac{1}{3}\right)+\left(\frac{9}{10}\times\frac{1}{5}\times\frac{2}{3}\right)+\left(\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}\right)$

$= \frac{1}{75} + \frac{2}{75} + \frac{4}{75} + \frac{3}{50} + \frac{3}{25} + \frac{6}{25} = \frac{77}{150}$

d) $\left(\frac{9}{10} \times \frac{1}{5}\right) = \frac{9}{50}$

2/8

Examiner Comments

In part (a) the shape of the tree diagram is incorrect.

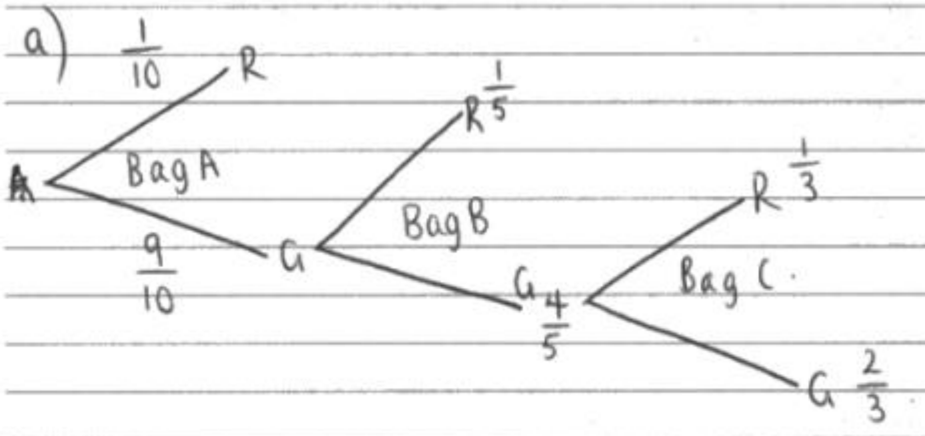
In part (b) the probability that all three marbles are green has been found correctly

In part (c) we require the addition of the product of two probabilities and the product of three probabilities for a follow through method mark to be awarded.

In part (d) there is no attempt at a conditional probability.

Student Response B

a)



b) $\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3} = \frac{12}{25}$

d) $\frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$.

c) $\left(\frac{9}{10} \times \frac{1}{5}\right) + \left(\frac{9}{10} \times \frac{4}{5} \times \frac{1}{3}\right) = \frac{21}{50}$

$\frac{9}{50} + \frac{6}{25} =$

6/8

Examiner Comments

In part (a) the tree diagram is drawn correctly with all correct labels and probabilities.

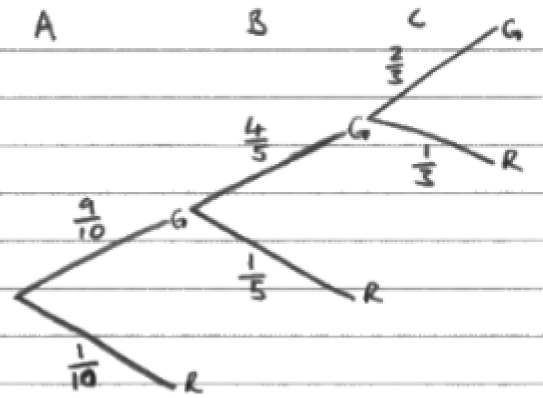
In part (b) the probability that all three marbles are green has been found correctly.

In part (c) the two required probabilities have been found and added together correctly.

Part (d) was a common mistake where there is no attempt at a conditional probability. Here no marks are awarded.

Student Response C

a)



b) $P(3G) = \frac{9}{10} \times \frac{4}{5} \times \frac{2}{3}$
$= \frac{12}{25} = 0.48$

c) $P(GR) = \frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$
$P(GGR) = \frac{9}{10} \times \frac{4}{5} \times \frac{1}{3} = \frac{6}{25}$
$\frac{9}{50} + \frac{6}{25} = \frac{21}{50} = 0.42$

d) $P(R) = \frac{1}{10}$
$P(GR) = \frac{9}{10} \times \frac{1}{5} = \frac{9}{50}$
$P(GGR) = \frac{9}{10} \times \frac{4}{5} \times \frac{1}{3} = \frac{6}{25}$
$\frac{1}{10} + \frac{9}{50} + \frac{6}{25} = \frac{13}{25}$
$P(B|R) = \dfrac{\frac{9}{50}}{\frac{13}{25}} = \frac{9}{26} = 0.346$ (3sf)

8/8

Examiner Comments

In part (a) the tree diagram is drawn correctly with all correct labels and probabilities.

In part (b) the probability that all three marbles are green has been found correctly.

In part (c) the two required probabilities have been found and added together correctly.

In part (d) the correct conditional probability has been found and the exact fraction has been given.

Exemplar Question 2

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2.

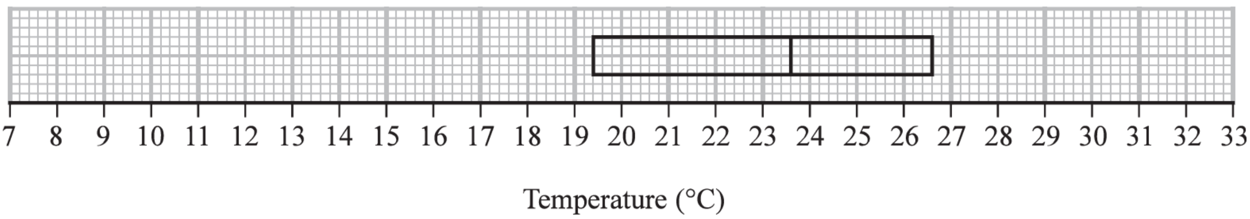


Figure 1

The partially completed box plot in Figure 1 shows the distribution of daily mean air temperatures using the data from the large data set for Beijing in 2015

An outlier is defined as a value
more than $1.5 \times IQR$ below Q_1 or
more than $1.5 \times IQR$ above Q_3

The three lowest air temperatures in the data set are 7.6 °C, 8.1 °C and 9.1 °C
The highest air temperature in the data set is 32.5 °C

(a) Complete the box plot in Figure 1 showing clearly any outliers

(4)

(b) Using your knowledge of the large data set, suggest from which month the two outliers are likely to have come.

(1)

Using the data from the large data set, Simon produced the following summary statistics for the daily mean air temperature, x °C, for Beijing in 2015

$$n = 184 \qquad \sum x = 4153.6 \qquad S_{xx} = 4952.906$$

(c) Show that, to 3 significant figures, the standard deviation is 5.19 °C

(1)

Simon decides to model the air temperatures with the random variable

$$T \sim \mathrm{N}(22.6, 5.19^2)$$

(d) Using Simon's model, calculate the 10th to 90th interpercentile range.

(3)

Simon wants to model another variable from the large data set for Beijing using a normal distribution.

(e) State two variables from the large data set for Beijing that are **not** suitable to be modelled by a normal distribution. Give a reason for each answer.

(2)

(Total for Question 2 is 11 marks)

Mean Score: 6.2 out of 11

Examiner Comments

In part (a) most candidates correctly read the quartile values from the diagram to obtain and use the interquartile range to find outlier boundaries (where the IQR was incorrect it was commonly given as 7.4). Surprisingly there were some inaccuracies in using the formula for outliers even though it was given. Not all candidates showed working, which meant marks could not be awarded if the box plot was not correct. The left hand whisker was commonly drawn to 9.1 (sometimes to 8.6) as required, but the right hand whisker was more occasionally incorrect, often extending off the grid instead of stopping at the stated maximum value of 32.5 Usually the two outliers were plotted although they were sometimes omitted by candidates who had shown the correct calculations.

In part (b) candidates are required to be familiar with the large data set. It was clear from responses in part (b) that many were not aware that for all locations the data is for the months of May to October only. Successful candidates often explained that Beijing was in the northern hemisphere and so low outliers for temperature were likely to be for winter, and so October. Some candidates perhaps did not read the question carefully enough and stated two months.

In part (c) most candidates scored well, usually using the given $\sqrt{\frac{S_{xx}}{n}}$ formula. A small number appeared unaware of this method instead choosing to work back from S_{xx} to find $\sum x^2$, to then use in the 'standard' formula. Some were successful here although there were a number who mistakenly took the given S_{xx} to be $\sum x^2$. Most candidates showed their calculation giving the result to at least 4 significant figures so that they could show it rounded to 5.19.

For part (d) the most popular approach seen here seemed to be use of the inverse Normal function on a calculator to find the 10th and 90th percentiles. This was usually successful although many candidates then went no further, hence gaining only the first mark. It was evident that some were not aware of what was meant by interpercentile range; it was not understood that the difference needed to be found (as when finding *IQR*), some stating a range for x as an inequality (e.g. $15.9 \le x \le 29.3$) with others instead reaching a probability answer of 0.8. Of those continuing to find the IPR some had prematurely rounded both percentile values to 3 significant figures before subtraction, leading to 13.4 as an inaccurate final answer.

Candidates need to be encouraged to work with accurate figures, only rounding their final answer. Fewer candidates used the more 'traditional' (and longer) approach of standardisation to obtain the percentiles needed, although this was often done successfully.

Part (e) was very poorly done with only a handful of candidates scoring marks here. Depth of familiarity was lacking by many candidates. Some misunderstood the question entirely and cited the conditions for a Normal being a suitable approximation to a Binomial. Others thought this part was related to the variable under study and suggested the Normal was not suitable for modelling air temperature for a variety of reasons including the skew on the box plot.
Those who did suggest two other variables from the large data set rarely gave acceptable reasons for their choice; some gave no reasons at all even when their choice was a suitable one. The use of the Beaufort scale for wind speed was mentioned but it was wrongly said to be discrete, with only a few saying that it was non-numeric. Wind direction which was non-numeric was quite often seen but was not an acceptable answer. Some other wrong reasons

were: the data is non-uniform, the values are too variable; it is not possible to have values below zero. The most commonly suggested variable was cloud cover, presumably because it was used as the basis for Question 4 and possibly because it appeared in last summer's AS Mathematics paper 2.

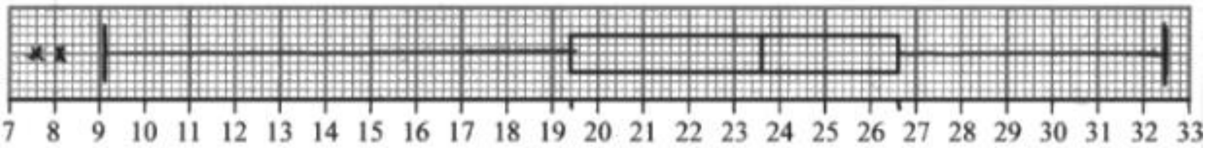
Mark Scheme

Question	Scheme	Marks	AOs
2(a)	IQR = 26.6 – 19.4 [= 7.2]	B1	2.1
	19.4 – 1.5 × ‘7.2’ [= 8.6] or 26.6 + 1.5 × ‘7.2’ [= 37.4]	M1	1.1b
	Plotting one upper whisker to 32.5 **and** one lower whisker to 8.6 or 9.1	A1	1.1b
	Plotting 7.6 and 8.1 as the only two outliers	A1	1.1b
		(4)	
(b)	October (since it is the month with the coldest temperatures between May and October in Beijing)	B1	2.4
		(1)	
(c)	$[\sigma =]\sqrt{\frac{4952.906}{184}}$ or e.g. $\sigma = \sqrt{\frac{S_{xx}}{n}} = 5.188...$ [=5.19*]	B1cso*	1.1b
		(1)	
(d)	$z = (\pm)\ 1.28(16)$ $[P_{90} =]29.251...$ or $[P_{10} =]15.948...$	B1	3.1b
	2 × 1.2816 × 5.19 ‘29.251…’ – ‘15.948…’	M1	1.1b
	= awrt **13.3**	A1	1.1b
		(3)	
(e)	Daily mean wind speed/Beaufort conversion since it is qualitative	B1	2.4
	Rainfall since it is not symmetric/lots of days with 0 rainfall	B1	2.4
		(2)	
		(11 marks)	

Notes

(a)	**B1:** for a correct calculation for the IQR (implied by 10.8 **or** 8.6 **or** 37.4 seen) **M1:** for a complete method for either lower outlier limit or upper outlier limit (allow ft on their IQR) (may be implied by the 1st A1) **A1:** both whiskers plotted correctly (allow ½ square tolerance) **A1:** only two outliers plotted, one at 7.6 and one at 8.1 (allow ½ square tolerance) **NOTE:** A fully correct box plot with no incorrect working scores 4/4
(c)	**B1cso*:** Correct expression with square root **or** correct formula and 5.188 or better Allow a complete correct method finding $\sum x^2 = \text{awrt } 98720$ and $\sigma = \sqrt{\frac{98715.9...}{184} - \left(\frac{4153.6}{184}\right)^2}$
(d)	**B1:** Identifying z-value for 10th or 90th percentile (allow awrt (±) 1.28) or for identifying $[P_{90} =]29.251...$ (awrt 29.3) or $[P_{10} =]15.948...$ (awrt 15.9) (This may be implied by a correct answer awrt 13.3) **M1:** for $2 \times z \times 5.19$ where $1 < z < 2$ or for their $P_{90} - P_{10}$ where $25 < P_{90} < 35$ and $10 < P_{10} < 20$ **A1:** awrt 13.3
(e)	**B1:** for one variable identified and a correct supporting reason **B1:** for two variables identified and a correct supporting reason for each Allow any two of the following: • Wind speed Beaufort since the data is non-numeric (o.e.). They need not mention Beaufort provided there is a description of the data as non-numeric (Do not allow wind direction/wind gust) • Rainfall as not symmetric/is skewed/is not bell shaped/lots of 0s /many days with no rain/mean≠mode or median • Date since each data value appears once/it is uniformly distributed • Daily mean pressure since it is not symmetric/is skewed/not bell shaped • Daily mean wind speed since it is not symmetric/is skewed/not bell shaped Do not allow ‘not continuous’ or ‘discrete’ as a supporting reason. Ignore extraneous non-contradicting statements

Student Response A



a) IQR = 26.6 − 19.4 = 7.2 $\quad Q_1 = 19.4$
$1.5 \times 7.2 = 10.8$ $\quad Q_2 = 26.6$

$19.4 - 10.8 = 8.6$
$26.6 + 10.8 = 37.4$

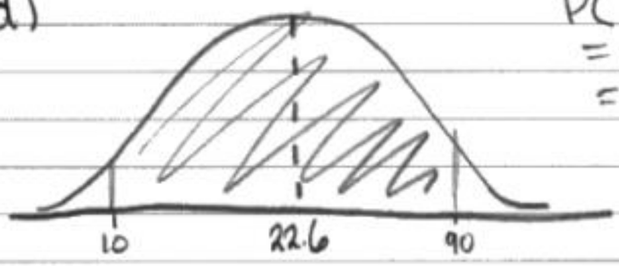
$7.6 < 8.6$ and $8.1 < 8.6$ ∴ they are both outliers
but
$9.1 > 8.6$ ∴ not an outlier
$32.5 < 37.4$ ∴ not an outlier

b) october and november

c) $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sigma$

$\sqrt{\frac{4952.906}{184} - \left(\frac{4153.6}{184}\right)^2} = 5.19$ (3sf)

d)



$P(X < 90) - P(X < 10)$
$= 0.9924$ (4dp)
$= 0.992$ (3sf)

e) cloud coverage ~~a~~ and daily amount of sunshine because they do not show a bell shape or symmetrical design.

4/14

Examiner Comments

In part (a) the *IQR* and the outlier limits are found correctly and the box plot is drawn accurately scoring full marks.

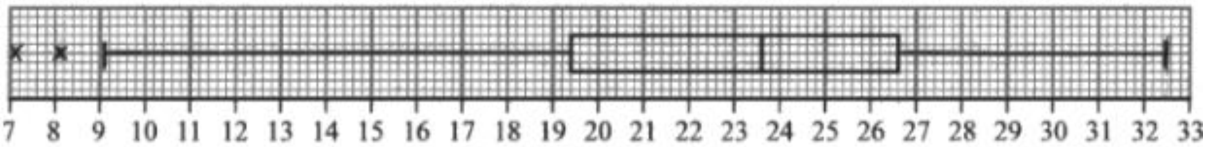
In part (b) two answers are given, so the mark cannot be awarded.

In part (c) the given answer has been found from an incorrect method so does not score.

In part (d) there is no valid attempt made to find the 10th or 90th percentiles.

In part (e) neither ‘cloud coverage’ nor ‘sunshine’ are variables on the large data set for Beijing, so these are not accepted.

Student Response B



(a) IQR = $Q_3 - Q_1$

= 26·6 − 19·4 = 7·2

26·6 ~~[illegible]~~ + (1·5 × 7·2) = 37·4 ∴ 32·5 not an outlier

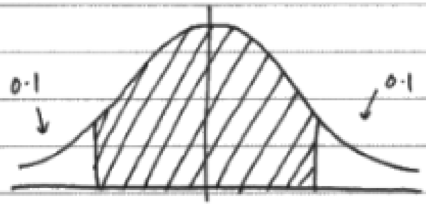
19·4 − (1·5 × 7·2) = 8·6 ∴ 7·1 & 8·1 are outliers but 9·1 is not.

(b) October.

(c) Standard deviation = $\sqrt{\frac{S_{xx}}{n}} = \sqrt{\frac{4952.906}{184}}$ = 5·188252...

= 5·19°C (3sf)

(d) $T \sim N(22.6, 5.19^2)$



Area : 0·9 = 29·251253

Area : 0·1 = 15·948747

29·25(...) − 15·95(...) = 13·3025...

= $\boxed{13.3}$ (3sf)

(e) Cloud cover and ~~[illegible]~~ wind speed because I presume they are modelled by discrete distributions (Beaufort scale and oktas)

11/14

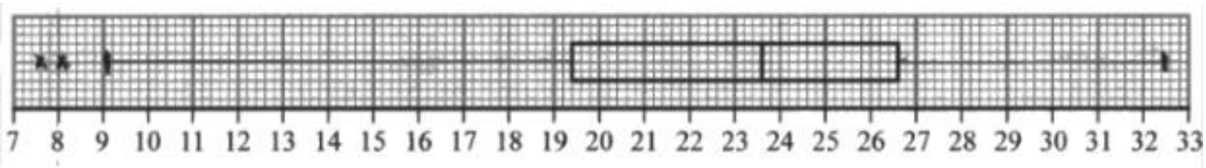
Examiner Comments

Although the outlier limits have been calculated correctly in part (a), the outliers have not been plotted accurately. The first outlier is plotted at 7.1 instead of 7.6.

Parts (b), (c) and (d) have been accurately completed.

Although ‘Beaufort wind speed’ has been identified in part (e), the supporting reason given is incorrect as they have confused discrete with qualitative. ‘Cloud cover’ is not a variable on the large data set for Beijing.

Student Response C



a) 1st Q = 19.4 2nd Q = 23.6 3rd Q = 26.6

IQR = 7.2 IQR × 1.5 = 10.8

19.4 − 10.8 = 8.6 26.6 + 10.8 = 37.4

so the outliers are 7.6°C and 8.1°C marked with X

b) October

c) $\sigma = \sqrt{\frac{S_{xx}}{n}}$ $\sigma = \sqrt{\frac{4952.906}{184}} = 5.188 = 5.19$ 3 s.f.

d) $T \sim N(22.6, 5.19^2)$

$P(X < \alpha) = 0.9$ $P(X < \beta) = 0.1$ Inv Normal

$\alpha = 29.251$ $\beta = 15.948$

10th to 90th interpercentile range $= \alpha - \beta$

$= 29.251 - 15.948 = 13.30$

e) One variable which is not suitable is daily mean windspeed as it is measured qualitatively e.g. moderate, high etc.

Another variable which is also not suitable is the rainfall as there are too many values for 0 when it doesn't rain, skewing the data.

11/11

Examiner Comments

In part (a) the box plot has been drawn accurately and the outlier limits have been worked out correctly.

Parts (b), (c) and (d) are fully correct.

In part (e) ‘windspeed’ is identified and accepted as ‘Beaufort’ and the supporting reason that it is ‘qualitative’ has been given. ‘Rainfall’ is the second variable accepted and it is given with a correct supporting reason that ‘there are too many values of 0’. It was very rare for two correct variables to be identified with correct supporting reasons.

Exemplar Question 3

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3. Barbara is investigating the relationship between average income (GDP per capita), x US dollars, and average annual carbon dioxide (CO_2) emissions, y tonnes, for different countries.

She takes a random sample of 24 countries and finds the product moment correlation coefficient between average annual CO_2, emissions and average income to be 0.446

(a) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the product moment correlation coefficient for all countries is greater than zero.

(3)

Barbara believes that a non-linear model would be a better fit to the data.
She codes the data using the coding $m = \log_{10} x$ and $c = \log_{10} y$ and obtains the model $c = -1.82 + 0.89m$

The product moment correlation coefficient between c and m is found to be 0.882

(b) Explain how this value supports Barbara's belief.

(1)

(c) Show that the relationship between y and x can be written in the form $y = ax^n$ where a and n are constants to be found.

(5)

(Total for Question 3 is 9 marks)

Mean Score: 5.8 out of 9

Examiner Comments

This was the second most successful question on the paper with nearly a quarter of candidates scoring full marks.
In part (a) the required symbol ρ was often written as p in the hypotheses and though this was condoned for the first mark. Candidates who lost the mark were mainly using r or pmcc instead of the parameter ρ. Occasional two-tailed hypotheses were stated.

The majority of candidates were successful in finding the critical value but some final conclusions were confused with candidates thinking that the critical region was where the product moment correlation coefficient (pmcc) was less than the critical value. A small number of candidates incorrectly tried to apply a Binomial distribution writing $X \sim \mathrm{B}(24, 0.446)$ or tried to standardise using $1.6449 = \frac{x - 24}{0.446}$ to find the critical value.

In part (b) many gave a correct response, using the idea that the correlation was stronger or the value closer to 1. Some candidates only compared the value of 0.822 to 0.446 (or 0 or 0.3438) without explaining how this supported Barbara's belief by showing a stronger correlation compared to the uncoded data. A common incorrect answer was to state that 0.882 was close to 0.89, the gradient of the regression equation.

For part (c) a variety of approaches were taken to answer this. Those that opted to take method 2 (working from the model) on the whole were usually more successful, as many could jump straight to $\log_{10} y = \log_{10} a + n \log_{10} x$. A common error was to give $n = 7.762\ldots$ ($10^{0.89}$). The candidates that struggled with this question were able to score the first mark, since many could state $\log_{10} y = -1.82 + 0.89(\log_{10} x)$ but then failed to make y the subject. Even those candidates that could deal with logarithms often lost the final mark since did not evaluate their value of a (too many left this as $10^{-1.82}$).

Mark Scheme

Question	Scheme		Marks	AOs
3(a)	$H_0 : \rho = 0 \qquad H_1 : \rho > 0$		B1	2.5
	Critical value 0.3438		M1	1.1a
	(0.446 > 0.3438) so there is evidence that the product moment correlation coefficient (pmcc) is greater than 0/there is positive correlation		A1	2.2b
			(3)	
(b)	The value is close(r) to 1 **or** there is strong(er) (positive) correlation		B1	2.4
			(1)	
(c)	$\log_{10} y = -1.82 + 0.89(\log_{10} x)$	$y = ax^n \rightarrow$ $\log_{10} y = \log_{10}(ax^n)$	M1	1.1b
	$y = 10^{-1.82+0.89(\log_{10} x)}$	$\log_{10} y = \log_{10} a + \log_{10} x^n$	M1	2.1
	$y = 10^{-1.82} \times 10^{0.89(\log_{10} x)}$ $[= 10^{-1.82} \times 10^{(\log_{10} x^{0.89})}]$	$\log_{10} y = \log_{10} a + n\log_{10} x$ $[\log_{10} a = -1.82, n = 0.89]$	M1	1.1b
	$y = 0.015x^{0.89}$	$y = 0.015x^{0.89}$	A1A1	1.1b 1.1b
			(5)	
				(9 marks)

Notes	
(a)	**B1:** for both hypotheses correct in terms of ρ **M1:** for the critical value: sight of 0.3438 or any cv such that $0.25 < \lvert cv \rvert < 0.45$ **A1:** a comment suggesting a significant result/ H_0 is rejected on the basis of seeing +0.3438 **and** which mentions "pmcc/correlation" and "greater than 0/positive" or an answer in context which includes "income"(o.e.) and "CO_2/emissions"(o.e.) A **contradictory** statement scores A0 e.g. 'Accept H_0, therefore positive correlation'
(b)	**B1:** for suitable reason e.g. *r* is close(r) to 1 **or** "strong(er)"/"near perfect" "correlation"
(c)	**For both methods, once an M0 is scored, no further marks can be awarded and condone missing base 10 throughout** **Method 1: (working to the model)** **M1:** Correct substitution for both *c* and *m* (may be implied by 2nd M1 mark) **M1:** Making *y* the subject to give an equation in the form $y = 10^{a+b(\log_{10} x)}$ (may be implied by 3rd M1 mark) **M1:** Correct multiplication to give an equation in the form $y = 10^a \times 10^{b(\log_{10} x)}$ (this line implies M1M1M1 provided no previous incorrect working seen) **Method 2: (working from the model)** **M1:** Taking the log of both sides (may be implied by 2nd M1 mark) **M1:** Correct use of addition rule (may be implied by 3rd M1 mark) **M1:** Correct multiplication of power (this line implies M1M1M1 provided no previous incorrect working seen) **A1:** $n = 0.89$ **or** a = awrt 0.015 **or** $y = ax^{0.89}$ **or** $y = \text{awrt}0.015x^n$ (**dep on M3**) **A1:** $n = 0.89$ **and** a = awrt 0.015 / $y = \text{awrt}0.015x^{0.89}$ (**dep on M3**)

Student Response A

a) $H_0: \rho = 0$

$H_1: \rho > 0$

$n = 24$

At 5% significance level

$0.3438 < 0.3446$

there is insufficient evidence to reject H_0.

b)

d) $c = -1.82 + 0.89m$

$\log_{10} y = -1.82 + 0.89 \log_{10} x$

3/9

Examiner Comments
In part (a) the hypotheses are stated correctly and the correct critical value is found. Though the null hypothesis is rejected, there is no conclusion given in context which is required for the final mark to be awarded. For part (b) no attempt is made. For part (c)a correct substitution is given but no further progress is made.

Student Response B

a) • $H_0: \rho = 0$ $H_1: \rho > 0$ $\alpha = 0.05$ $n = 24$

• Critical values: ± 0.3438 ∴ critical regions: $r < -0.3438$, $r > 0.3438$

• $r = 0.446 > 0.3438$ ∴ lies in critical region

• There is sufficient evidence to reject H_0 ∴ there is enough evidence to suggest the pmcc for all countries is greater than zero.

b) the pmcc $= 0.882$ isn't close enough to 1 ~~to~~ therefore doesn't show a strong +ve ~~cor~~ linear correlation

c) $c = -1.82 + 0.89m \rightarrow (\log_{10} y) = (-1.82 + 0.89 \log_{10} x)$
$\quad 10 \qquad\qquad 10$

$\Rightarrow 10^{\log_{10} y} = 10^{-1.82} \times 10^{\log_{10} x^{0.89}}$

$y = 10^{-1.82} \times x^{0.89}$ ∴ $y = (10^{-1.82})\, x^{0.89}$

where $a = 10^{-1.82}$ and $n = 0.89$

7/9

Examiner Comments

In part (a) both hypotheses are stated in terms of rho. The correct critical value is found and the conclusion given includes the required context that the 'pmcc is greater than 0'.

In part (b) an incorrect conclusion has been made that this correlation coefficient is not close to 1 and that is does not show a strong correlation. This type of response was surprisingly common.

In part (c) the correct substitution has been made. The attempt to make *y* the subject of the equation is correct and the addition of the powers has been turned into a correct multiplication. The final answer has a correct value of *n* but the final mark is not awarded as the value of *a* is not given to the required accuracy.

Student Response C

$H_0: \rho = 0$ pmcc $= 0.446$

$H_1: \rho > 0$

sample size $= 24$

one tailed test at 5% sig level.

$r > 0.3438$

because $0.446 > 0.3438$, there is enough evidence at the 5% significance level to reject H_0 in favour of H_1 and therefore ~~to say~~ ~~that~~ support that the pmcc (product moment correlation coefficient) for all countries is greater than zero.

b) because the pmcc is close to 1, ~~almost~~ strong positive correlation, there is a linear relationship with $\log_{10} y$ against $\log_{10} x$ and thus an exponential between US dollars (x) and CO_2 emissions (y).

c) ~~log~~ $y = ax^n$

$\log y = \log ax^n$

$\log y = \log a + \log x^n$

$\log y = \log a + n\log x.$

$\log a = -1.82$

$a = 10^{-1.82}$

$a = 0.015135612$ ... (0.0151) (3 s.f.)

$n = 0.89$

$y = 0.0151x^{0.89}$

9/9

Examiner Comments

In part (a) the hypotheses have been stated correctly and the correct critical value has been found. The null hypothesis is rejected, and the conclusion is in context.

In part (b) both correct answers are given ‘pmcc is close to 1’ and ‘strong correlation’ and either of these comments would have been sufficient for the mark.

Part (c) is method 2 on the mark scheme as the working is shown from the model. Both *a* and *n* are found to the required accuracy.

Exemplar Question 4

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4. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim \mathrm{B}(8, 0.76)$

Using Magali's model,

(b) (i) find $\mathrm{P}(X \geq 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(c) Explain whether or not your answers to part (*b*) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(e) Comment on Magali's model in light of your answer to part (*d*).

(2)

(Total for Question 4 is 9 marks)

Mean Score: 5.2 of 9

Examiner Comments

In part (a) most earned the mark in part (a) except the few candidates who did not give an answer to the required degree of accuracy.

In part (b)(i) most candidates scored both marks, usually using $1 - P(X \leq 5)$ and rounding to a minimum of three significant figures. Where mistakes were seen, they were generally down to the accuracy of the rounding (only to two significant figures) or else those who attempted $P(X = 6)$ or incorrectly assumed $P(X \geq 6) = 1 - P(X \leq 6)$.

While in (b)(ii) candidates rarely had difficulty finding $P(X = 7)$, they frequently went no further, despite the question's requirement for the expected value.

Part (c) was not particularly well attempted by candidates with many leaving it out completely. Where there was a clear attempt those failing to score the mark usually only compared one set of the values, generally the 51.7 and the 52.

For part (d) most who attempted scored the mark.

Part (e) was the least successfully answered part of the question with lots of blanks seen. Where candidates scored the two marks it was usually through stating that if there had been cloud cover the day before there was a higher chance of cloud cover the next day. Those going completely down the wrong route failed to realise that the data was from the same source and were mostly commenting on the size of the sample rather than what the statistics already demonstrated. In general there was far too much tendency to describe vague contextual knowledge rather than use the figures found previously. Of those who did try and compare figures an error often seen was to compare the result in part (d) to 0.76. The failure to focus on the fact that the model was Binomial, and whether therefore the conditions of independence had been met, demonstrated the general unfamiliarity with modelling.

Mark Scheme

Question	Scheme	Marks	AOs
4 (a)	$\frac{132}{184} = 0.71739...$ awrt **0.717**	B1	1.1b
		(1)	
(b)(i)	$\text{P}(X \geq 6) = 1 - \text{P}(X \leq 5)$ or $\text{P}([X=]6) + \text{P}([X=]7) + \text{P}([X=]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **0.703**	A1	1.1b
		(2)	
(b)(ii)	$184 \times \text{P}(X = 7) \quad [= 184 \times 0.2811\ldots]$	M1	1.1b
	$= 51.7385...$ awrt **51.7**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	
(d)	$\frac{23}{28} = 0.82142...$ awrt **0.821**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)

Notes	
	Allow decimals or percentages throughout this question.
(a)	Allow equivalent fraction, e.g. $\frac{33}{46}$
(b)(i)	**M1:** for writing or using $1 - \text{P}(X \leq 5)$ or $\text{P}(X=6) + \text{P}(X=7) + \text{P}(X=8)$ **A1:** awrt 0.703 (correct answer scores 2 out of 2)
(b)(ii)	**M1:** for $184 \times \text{P}(X=7)$ o.e. e.g., $184 \times [\text{P}(X \leq 7) - \text{P}(X \leq 6)]$ **A1:** awrt 51.7
(c)	**B1ft:** comparing '0.717' with '0.703' <u>and</u> '51.7 or '0.281' with 52 or 0.283 **and** concluding that Magali's model **is supported** (must be comparing prob. with prob. <u>and</u> days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b)
(e)	**B1:** Any bullet point **dB1:** (dep on previous B1) for Magali's model may not be suitable (o.e.) **SC:** part (d) is similar to part (a)/(b)(i) and a compatible conclusion (i.e. Magali's model is supported) to score B1B1.

Student Response A

a) $\frac{52}{184} + \frac{52}{184} + \frac{28}{184} = \underline{\underline{0.717}}$

b)i) $X \sim B(8, 0.76)$

a) $P(X \geqslant 6) = 1 - P(X < 7)$

$= \underline{\underline{0.111}}$ 3.s.f.

b) $P(X = 7) = 0.28118\ldots$
$\times 184 = \underline{\underline{51.7}}$ days

c) Yes because in the table the frequency of days with 7 oktas of cloud cover is 52 $\approx$ 51.7

$\frac{52}{184} = P(X=7)$ using her model.
$(= 0.28118)$.

d) $P(X = \cancel{28}\,8) = 28$ days

$\frac{28}{184} = 0.15217\ldots = P(X=8)$

$5 + 9\cancel{9} = \frac{23}{77}$ $= \underline{\underline{0.2987}}$

3/9

Examiner Comments

In part (a) the answer has been evaluated to the required 3 significant figure accuracy.

In part (b)(i) the probability expression is incorrect.
In part (b)(ii) the P(X =7) has been found correctly and this has been multiplied by 184 to give a correct answer.

In part (c) only one of the required comparisons has been made. Here we require both answers in (b) to be compared with the appropriate values.

In part (d) both 23 and 28 are seen but they have not been compared in a correct proportion.

For part (e) no attempt is made.

Student Response B

a) $52 + 52 + 28 = 132$
total = 184

$$\frac{132}{184} = \frac{33}{46}$$

b) i. $P(X \geq 6) = 1 - P(X \leq 5)$
$= 0.7032776593$
$= 0.703$ (3sf)

ii. $P(X = 7) = 0.2811877358$
$184 \times 0.2811877358 = 51.73854338$
$= 51.7$ (1dp)

c) my answers in part (b) support the use of Magali's model because they provide very similar answers ~~tested~~ when compared against the answers from the actual data.
E.g. $\frac{33}{46} = 0.717 \approx 0.703$

$51.7 \approx 52$.

d) $5 + 9 + 9 = 23$ $\frac{23}{28} = 0.8214285714$
total = 28

e) Magali's model has become less reliable and accurate for the data shown in the 2nd table because the model was based on other data and ∴ using Magali's model ~~es~~ to estimate ~~t~~ data from the 2nd table would be extrapolation.

7/9

Examiner Comments

In part (a) the fraction is correct.

In part (b)(i) there is a correct probability statement and a correct answer. Part (b)(ii) is also answered to the correct degree of accuracy.

In part (c) both comparisons are made and the correct conclusion that these support the use of Magali's model is given.

In part (d) the correct proportion has been found.

In part (e) the candidate has not understood that the data in the second table is a subset of the data in the first table. The comment about extrapolation is therefore incorrect and no marks are scored in this part.

Student Response C

a. $\frac{52+52+28}{184} = \boxed{\frac{33}{46}}$

$= 0.717$

b. $X \sim B(8, 0.76)$

c i) $P(X \geqslant 6) = 1 - P(X \leq 5)$

$= 1 - 0.297$

$= 0.7033$

$\boxed{= 0.703 \quad 3 s.f.}$

c ii) $P(X=7) = 0.281$

$0.281 \times 184 = 51.7 \quad 3 s.f. = 52 \quad 0dp.$ (whole number)

$\boxed{52 \text{ days}}$ expected cloud cover 7

c. probability of cloud cover being 6 or greater suggested by model is 0.703 which is close to actual probability of 0.717

Predicted number of days with cloud cover 7 is 52 which is the same as the actual number

Therefore model is suitable since predictions agree with reality.

d. $\frac{5+9+9}{28} = \boxed{\frac{23}{28} = 0.821}$

e. Magali's model may not be suitable as binomial distribution assumes are trials/outcomes are independant.

Information from the large data set suggests trials are not independent as the ~~table~~ probability of cloud cover being 6 or greater is significantly ~~much~~ higher after a day with cloud cover 8 than it usually would be.

$$\frac{23}{28} > \frac{33}{46}$$

9/9

Examiner Comments

In part (a) the fraction is correct.

In part (b)(i) there is a correct probability statement and a correct answer. Note that candidates are expected to have a calculator that calculates cumulative binomial probabilities.
(b)(ii) the subsequent rounding of 51.7 to 52 can be ignored and full marks are scored.

In part (c) both comparisons are made and the correct conclusion that these support the use of Magali's model is given.

In part (d) the correct proportion has been found.

In part (e) a correct comment is made about the fact that the independence assumption required for the binomial distribution does not apply here. The correct conclusion that Magali's model may not be suitable is given.

Exemplar Question 5

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5. A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

(c) Test Hannah's belief at the 5% level of significance.
You should state your hypotheses clearly.

(5)

(Total for Question 5 is 13 marks)

Mean Score: 5.6 out of 13

Examiner Comments

In part (a) candidates struggled to come up with a complete strategy to answer this multi-step question. Some candidates were unable to use the given information to find the missing standard deviation. Although many realised that they needed to standardise, they incorrectly used the given probability rather than the associated *z*-value. Many of the candidates who found the correct standard deviation went on to give a correct answer although a significant number then made mistakes manipulating the probability statements and therefore failed to score after this. Candidates should be reminded of the benefits of drawing a diagram that would have helped in this part.

It is worth noting that many candidates showed very little method once they had calculated the standard deviation, this meant that candidates who had an incorrect answer often lost 3 marks. With increased reliance on the calculator the use of diagrams and probability statements are essential to demonstrate what methods are being used. Candidates also need to get into the habit of checking their answers for example by substituting their answers into the original question. Although most candidates gave their answer to 2 decimal places, as requested, some gave answers to a smaller degree of accuracy.

Part (b) was one of the most discriminating parts of the entire paper. There were a range of errors seen. Many failed to set up the correct Normal approximation and many tried to use the model found in part (a). Some appeared to struggle with the phrase ‘fewer than half’. It was also quite common to see candidates ignore the request for an approximation and complete the question using the binomial distribution given in the question. Candidates need to be aware that they will not be awarded marks for using the binomial distribution to calculate a probability if they have been asked to use an approximation. Candidates who did manage to set up the correct Normal distribution often made errors with the continuity correction, either they made no attempt to use one or they used one in the wrong direction.

Part (c) was a standard hypothesis test for a sample mean from a Normal distribution that should have been relatively straight forward but in many cases candidates struggled. It was common for candidates to use the wrong parameter when defining the null and alternative hypotheses; they need to be fully aware that they must use the standard parameters when setting up hypotheses, in this case μ.

For those who did make progress failing to divide the variance by n was a common error and candidates often gave large probabilities that were nowhere near 0.05 which should be a warning sign that they likely have a mistake in their working. Some used $\sigma = 0.16$ or $\mu = 24.94$. Those using a critical regions approach were rare and these candidates often had issues when rounding the critical region to two decimal places, which meant it was equal to the test statistic. The majority of candidates made an attempt to interpret their results and give a contextual conclusion. Those candidates that opted to give their contextual conclusion in terms of ‘Hannah’s belief’ were usually more successful than those who tried to write a statement about the mean amount of liquid being less than 25 ml. Some candidates wrote ‘the mean is less than 25’ which did not mention the context and was therefore not acceptable.

Mark Scheme

Question	Scheme	Marks	AOs
5(a)	$\frac{24.63-25}{'\sigma'}=-1.0364$	M1	3.1b
	$[\sigma=]0.357$ (must come from compatible signs)	A1	1.1b
	$\mathrm{P}(D>k)=0.4$ or $\mathrm{P}(D<k)=0.6$	B1	1.1b
	$\frac{k-25}{'0.357'}=0.2533$	M1	3.4
	k = awrt **<u>25.09</u>**	A1	1.1b
		(5)	
(b)	$[Y\sim\mathrm{B}(200, 0.45)\rightarrow]\ W\sim\mathrm{N}(90, 49.5)$	B1	3.3
	$\mathrm{P}(Y<100)\approx\mathrm{P}(W<99.5)\left[=\mathrm{P}\left(Z<\frac{99.5-90}{\sqrt{49.5}}\right)\right]$	M1	3.4
	$=0.9115\ldots$ awrt **<u>0.912</u>**	A1	1.1b
		(3)	
(c)	$\mathrm{H}_0:\mu=25$ $\mathrm{H}_1:\mu<25$	B1	2.5
	$[\bar{D}\sim]\mathrm{N}\left(25, \frac{0.16^2}{20}\right)$	M1	3.3
	$\mathrm{P}(\bar{D}<24.94)[=\mathrm{P}(Z<-1.677...)]=0.046766...$	A1	3.4
	$p=0.047<0.05$ <u>or</u> $z=-1.677\ldots<-1.6449$ <u>or</u> $24.94<24.94115\ldots$ <u>or</u> reject H_0/in the critical region/significant **with no contradictions**	M1	1.1b
	There is sufficient evidence to support <u>Hannah's</u> <u>belief</u>.	A1	2.2b
		(5)	
			(13 marks)

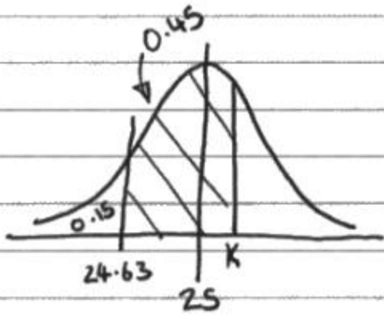
Notes	
(a)	**M1:** for standardising 24.63, 25 and 'σ' (ignore label) and setting = to z where $1<\lvert z\rvert<2$ **A1:** $[\sigma=]$ awrt 0.36. Do **not** award this mark if signs are not compatible. **B1:** for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram **M1:** for a correct expression with z = awrt 0.253 (may be implied by correct answer) **A1:** awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)
(b)	**B1:** setting up normal distribution approximation of binomial N(90, 49.5) (may be implied by a correct answer) Look out for e.g. $\sigma=\frac{3\sqrt{22}}{2}$ or $\sigma=$ awrt 7.04 **M1:** attempting a probability using a continuity correction i.e. $\mathrm{P}(W<100.5)$, $\mathrm{P}(W<99.5)$ or $\mathrm{P}(W<98.5)$ (The continuity correction may be seen in a standardisation). **A1:** awrt 0.912 [Note: 0.911299… from binomial scores 0 out of 3]
(c)	**B1:** for both hypotheses in terms of μ **M1:** selecting suitable model must see N(ormal), mean 25, sd = $\frac{0.16}{\sqrt{20}}$ (o.e.) or var = $\frac{4}{3125}$ (o.e.) **A1:** p value = awrt 0.047 <u>or</u> **test statistic** awrt –1.68 (**any of these correct values imply the M1**) <u>or</u> **CV** awrt 24.941 **M1:** a correct comparison (including compatible signs) **or** correct non-contextual conclusion (f.t. their p value, test statistic or critical value in the comparison)

	M1 may be implied by a correct contextual statement **NB** Any contradictory non contextual statements/comparisons score M0A0 e.g. '$p <$ 0.05, therefore not significant' **A1:** correct conclusion in context mentioning <u>Hannah's belief</u> <u>or</u> the mean <u>amount</u>/<u>liquid</u> in each bottle is now <u>less than 25</u>ml (**dep on M1A1M1**)

Student Response A

$D \sim N(25, \sigma^2)$

$P(D < 24.63) = 0.15$



a)

$Z\left(\dfrac{25 - 24.63}{\sigma}\right) \equiv D \sim N(0,1)$

-1.036432474

$\dfrac{24.63 - 25}{\sigma^2} = -1.0364$

$\sigma^2 = 0.35699$

~~$\sigma = 0.59749$ ml~~

$\sigma = 0.357$ (3dp)

b) $D \sim N(25, 0.357^2)$

$\bar{D} \sim N\left(25, \sqrt{\dfrac{0.375^2}{200}}\right)$

$\sqrt{\dfrac{0.375^2}{200}} = 0.265638\ldots$

$\sqrt{\dfrac{0.375}{200}} = 0.04334$

c) $\sigma = 0.16$

Sample size $= 20$

$H_0 = \mu = 25$ ml

$\alpha = 0.05$

1-tailed test

$H_1 : \mu < 25$ ml

$D \sim N(25, 0.16^2)$

$\bar{D} \sim N\left(25, \sqrt{\dfrac{0.16^2}{20}}\right)$

test statistic : 0.0894427191

$P(d < 24.94) = 0.251167$

$0.25 > 0.089$ $\therefore$ there is not sufficient evidence to reject H_0

3/13

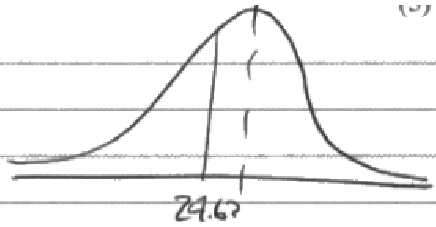
Examiner Comments

In part (a) we condone the incorrect label in the standardisation equation as it has later been corrected and the value of the standard deviation is correct. The response stops there and no further marks are gained in this part.

In part (b) there is an attempt at a normal distribution but this does not have the correct mean and standard deviation approximated from the binomial distribution.

In part (c) both hypotheses are stated correctly. The normal distribution for the sample mean contains the wrong variance. Although there is an attempt at a p-value it has been compared with 0.089 instead of 0.05 so the final method mark is not scored.

Student Response B



$\mu = 25$

To find σ

Using inverse function $\rightarrow$ 0.15 area ~~[illegible]~~

$\hookrightarrow Z = -1.036$ for standardised

$Z = \dfrac{X - \mu}{\sigma}$

$\rightarrow -1.036 = \dfrac{24.63 - 25}{\sigma}$

$-1.036\sigma = -0.37$

$\sigma = 0.357$

$X \sim N(25, 0.357^2)$

~~Using normal CD~~

~~Using inverse normal Z area [illegible]~~

Using normal CD $\rightarrow P(D < 24.63) = 0.15$.

$0.15 + 0.45 = 0.6$

Using inverse normal $\rightarrow (\mu = 25, \sigma = 0.357, \text{area} = 0.6)$

$\hookrightarrow X = 25.09$

$k = 25.09$

b) $200 = \text{sample size}$. $\mu = np = 200 \times 0.45 = 90$

$\sigma = \sqrt{np(1-p)} = \sqrt{200 \times 0.45 \times 0.55} = \frac{3\sqrt{22}}{2}$

Normal CD $\longrightarrow$ lower = 0, upper = 100, $\sigma = \frac{3\sqrt{22}}{2}$, $\mu = 90$

$\hookrightarrow P(X < 100) \longrightarrow = \underline{0.922}$

c) $\sigma = 0.16$

$H_0: \mu = 25$

$H_1: \mu < 25$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$\hookrightarrow \bar{X} \sim N\left(25, \frac{0.16^2}{20}\right)$

$P(\bar{D} < 24.9) = 0.0468$

$0.0468 < 0.05$ — thus it is significant

There is evidence to suggest that the μ is now less than 25.

10/13

Examiner Comments

In part (a) the standard deviation is found correctly. The correct value of *k* has been found and this implies the final method mark. Use of a calculator function generally does not count as a method, so candidates are encouraged to show all stages of working even when using a calculator. Here full marks can be awarded since the answer is correct and no incorrect working is seen.

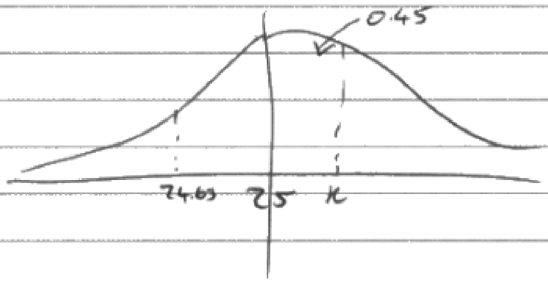
In part (b) the correct normal approximation to the binomial distribution is found. There is no attempt at the continuity correction and the final answer is therefore incorrect.

In part (c) both hypotheses are stated correctly in terms of mu. The distribution of the sample mean is also correct. The result is significant (and a correct comparison with 0.05 is given), but the conclusion is insufficient as it does not include the required context.

Student Response C

a) $D \sim N(25, \sigma^2)$

$P(24.63 < D < k) = 0.45$



$\Rightarrow 1 - 0.45 = 0.55 = P(D < 24.63) + P(D > k)$

~~P(D < 24.63)~~

$P(D < 24.63) = 0.15$

$Z = \dfrac{X - \mu}{\sigma} \Rightarrow Z = \dfrac{24.63 - 25}{\sigma}$

$P(Z < \cancel{24.63} c) = 0.15$

$c = \dfrac{24.63 - 25}{\sigma} = -1.03643$

$\sigma = 0.35699$

$\sigma \approx 0.357$ (3sf) $\Rightarrow D \sim N(25, 0.357^2)$

$0.55 = 0.15 + P(D > k)$

$P(D > k) = 0.40$

$k = 25.090$

$k = 25.09$ (2dp)

b) $n = 200 \quad p = 0.45$

$X \sim B(200, 0.45) \Rightarrow X \sim N(90, 49.5)$

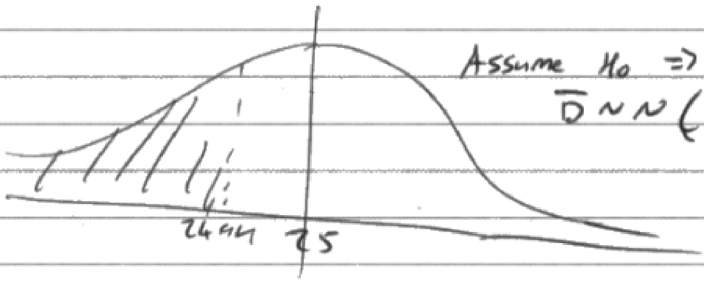
~~P(X ≤ 100)~~ $P(X < 100) \Rightarrow P(X \leq \cancel{100.5}\ 99.5)$

$= \underline{0.9115}$ (4sf)

c) $\bar{D} \sim N(\mu, \frac{0.16^2}{20})$

$n = 20$ Test statistic: $\mu = 24.94$ ml

$H_0: \mu = 25 \quad H_1: \mu < 25 \quad \alpha = 5\%$



Assume $H_0 \Rightarrow$
$\bar{D} \sim N(25, \frac{0.16^2}{20})$

Reject H_0 if $P(\bar{D} \leq 24.94) \leq 0.05$

$0.04677 \leq 0.05$

$\Rightarrow$ Reject H_0. Sufficient evidence to suggest mean amount of liquid put in each bottle is less than 25 ml.

13/13

Examiner Comments

In part (a) there is a correct standardisation equation with compatible signs leading to a correct value for the standard deviation. The correct probability statement P($D > k$) is given and the final result is found to the required 2 decimal place accuracy.

In part (b) the correct normal approximation to the binomial is found. A continuity correction is stated and the final answer is accurate.

In part (c) both hypotheses are stated in terms of mu. The correct normal distribution for the sample mean is stated and the test statistic and p-value are correct. The null hypothesis is rejected and the conclusion is given with the required context ‘liquid’ and ‘less than 25’.

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Exemplar Question 1

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1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geq 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

(Total for Question 1 is 6 marks)

Mean Score 4.2 out of 6

Examiner Comments

In the first part, the majority of candidates attempted to differentiate the given vector expression for velocity to find the acceleration as required. This was completed mostly successfully with just the occasional slip. Occasionally the expression was integrated or the fact that it was a vector just ignored with the coefficients of **i** and **j** being added; however, such instances were relatively rare. Those who completed the differentiation generally substituted $t = 4$ to obtain the acceleration and many correct answers were seen. Some went on to calculate the magnitude which was not actually required here but generally the marks had already been achieved for a correct vector. Some candidates worked in column vectors throughout and some in **i** and **j** components; both are equally acceptable. In part (b) integration was required in order to find the position vector with both powers of t increasing by 1. Mostly the indices were dealt with correctly although errors in the coefficients were more common. Some stopped there without either using the initial conditions or substituting $t = 4$. Many failed to achieve the final mark as a result of numerical/sign errors. A minority attempted to use ***suvat*** formulae to solve the problem despite having used calculus in part (a).

Mark Scheme

Question	Scheme	Marks	AO
1(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a} =) 6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$= 6\mathbf{i} - 15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
1(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r} =)(\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$= (-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j}) = 28\mathbf{i} - 44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

Marks		Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
1a	M1	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
1b	M1	Use of $\mathbf{r} = \int \mathbf{v}dt$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

Student Response A

t s
v
a } int diff

a) $v = 6ti - 5t^{\frac{3}{2}}j$

$a = 6i - \frac{15}{2}t^{\frac{1}{2}}j$

$\underset{t=1}{a} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$

$= \sqrt{6^2 + 15^2}$

$= 16.16 \text{ ms}^{-2}$

b) $s = 3t^2 i - \frac{25}{8}t^{\frac{8}{5}}j$

$\underset{t=1}{S} = 48i - 28.7j$

$= \begin{pmatrix} 48 \\ 28.7 \end{pmatrix}$

2/6

Examiner Comments

(a) M1: Attempt to differentiate with both powers decreasing by 1
A1: Correct differentiation
A0: The – sign on the 15j has been lost

(b) M0: Attempted to integrate but only one power has increased by 1
A0: Follows M0
A0: Follows M0

Student Response B

1a)

$\text{acc}^n = 6\underline{i} - \frac{15}{2} t^{\frac{1}{2}} \underline{j}$

when $t = 4$

$\text{acc}^n = 6\underline{i} - \frac{15}{2} \times \sqrt{4} \underline{j}$

$= 6\underline{i} - 15\underline{j}$

$\therefore$ magnitude $= \sqrt{6^2 + 15^2}$

$= 16.15549$

$= 16.2 \text{ ms}^{-2}$

b) ~~s = ? u = 0 v = a =~~

when $t = 4$, $v = 6 \times 4\underline{i} - 5 \times 4^{\frac{3}{2}}$

$= 24\underline{i} - 40\underline{j}$

$s = ?$ $u = 0$ $v = 24\underline{i} - 40\underline{j}$ $a = x$ $t = 4$

$$\cancel{v^2 = u^2 + 2as}$$

$$s = \frac{1}{2}(u + v)t$$

$$s = 2(24\underline{i} - 40\underline{j})$$

$$= 48\underline{i} - 80\underline{j}$$

$\therefore$ position vector of P when $t = 4$,

$$= 48\underline{i} - 26\underline{i} - 80\underline{j} + 26\underline{j}$$

$$= 28\underline{i} - 60\underline{j}$$

3/6

Examiner Comments

(a) M1: Attempt to differentiate with both powers decreasing by 1
A1: Correct expression for acceleration.
A1: Correct and simplified answer. Ignore the fact that they go on to find the magnitude.

(b) M0: No attempt to integrate to find an expression for r (using a suvat equation but a is not constant – this was a very common error)
A0: Following M0.
A0: Following M0

Student Response C

a) $v = 6t\underline{i} - 5t^{\frac{3}{2}}\underline{j}$

$a = 6\underline{i} - \frac{15}{2}t^{\frac{1}{2}}\underline{j}$

When $t = 4$

$a = 6\underline{i} - 15\underline{j}$

b) $v = 6t\underline{i} - 5t^{\frac{3}{2}}\underline{j}$

$s = 3t^{2}\underline{i} - 2t^{\frac{5}{2}}\underline{j}$

When $t = 4$

$s = 3(4)^{2}\underline{i} - 2(4)^{\frac{5}{2}}\underline{j}$

$s = 48\underline{i} - 64\underline{j}$

5/6

Examiner Comments

(a) M1: A valid attempt to differentiate correctly to find the acceleration.
A1: All correct.
A1: $t = 4$ has been correctly substituted to give the correct answer

(b) M1: A valid attempt to integrate correctly to find position vector.
A1: Correct integration. (not necessary for constant to be seen)
A0: Incorrect answer.(substitution $t = 4$ occurs but has not used initial position – this was a common error on this question)

Exemplar Question 2

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2. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})$ m s^{-1}

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

(Total for Question 2 is 8 marks)

Mean Score 3.4 out of 8

Examiner Comments

In part (a), although many candidates realised that finding an expression for the velocity was relevant to determining the direction of motion, only a minority could use the fact that it was specified as being in the direction of $3\mathbf{i} - 4\mathbf{j}$. The most common error was in equating the actual vectors rather than either equating the ratios of components or equating to a multiple of $3\mathbf{i} - 4\mathbf{j}$. A fair number attempted to use a displacement vector showing a lack of understanding of the situation. The second part was generally handled more successfully with many correct answers seen for the displacement vector (using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$) although not all proceeded to find the distance by calculating its magnitude. Although most used **suva***t* formulae throughout the question, some successfully integrated the given constant acceleration to find the velocity and displacement. Use of magnitudes of vectors throughout and some invalid multiplication/division of vectors were seen on occasion.

Mark Scheme

Question	Scheme	Marks	AO
2(a)	$(\mathbf{v} =)\mathbf{C} + (2\mathbf{i} - 3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v} =)(-\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} - 3\mathbf{j})t$	A1	1.1b
	$\dfrac{4 - 3T}{-1 + 2T} = \dfrac{-4}{3}$ oe	M1	3.1a
	$T = 8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s} =)\,\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 \quad (+\,\mathbf{D})$	M1	3.1a
	$(\mathbf{s} =)(-\mathbf{i} + 4\mathbf{j})t + \frac{1}{2}(2\mathbf{i} - 3\mathbf{j})t^2 \quad (+\,\mathbf{D})$	A1	1.1b
	$AB = \sqrt{12^2 + 8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2 + 12^2)}$ this is M0A0M0A0.	M1	3.1a
	$= 4\sqrt{13}\,(= 14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		Notes
2a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j)** for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in T only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get $T = 8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in T only, second M0 (ii) Can score M1A1 M1A1 if they get $T = 8$, using trial and error, but if they don't get $T = 8$, can only score max M1A1M0A0
2b	M1	Use of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i** + 3**j)** for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t=4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

Student Response A

2) $a = (2i - 3j)$ $\quad u = (-i + 4j)$

$\cancel{(3i - 4j) = (-i + 4j)T + \left(\tfrac{1}{2}\right)(2i - 3j)T^2}$

$\cancel{(3i - 4j) = (-i + 4j)T + \left(i - \tfrac{3}{2}j\right)T^2}$

$\cancel{s = T\sqrt{17} + \tfrac{\sqrt{13}}{2}T^2}$

$(3i - 4j) = (-i + 4j) + (2i - 3j)T$

$(4i - 8j) = (2i - 3j)\,T$ $\qquad T = 2i + \tfrac{8}{3}j$

$T = \tfrac{10}{3}$ seconds $\qquad |T| = \tfrac{10}{3}s \quad \sqrt{2^2 + \left(\tfrac{8}{3}\right)^2}$

b) $S = ut + \tfrac{1}{2}at^2$

$S = (3i - 4j)(4) + \left(\tfrac{1}{2}\right)(2i - 3j)(16)$

$S = (12i - 16j) + (16i - 24j)$

$S = (28i - 40j)$

$d = 48.83\,m$

3/8

Examiner Comments

(a) M1: **v** = **u** + **a**t has been used to give an expression of form **C** + (2**i** - 3**j**)*t* where **C** is non zero vector
A1: The expression is correct
M0: No use of ratios to give equation in *T* only - this was a very common error, where **v** was simply *equated* to (3**i** - 4**j**).
A0: Follows M0

(b) M0: Obtains an expression in the correct form but their **C** is not their non-zero constant vector from part (a), [(3**i** - 4**j**) has been used]
A0: Follows M0
M0: Although *t* = 4 is used, use of Pythagoras is not seen
A0: Follows M0

Student Response B

Suvat

$S = (3i - 4j)\ u = (-i + 4j)\ v = \ a = (2i - 3j)\ t = ?$

$S = ut + \frac{1}{2}at^2$

$(3i - 4j) = (-i + 4j)t + \frac{1}{2}(2i - 3j)t^2$

$0 = (i - \frac{3}{2}j)t^2 + (-i + 4j)t - (3i - 4j)$

i & j cancell as constants

$0 = -\frac{1}{2}t^2 + 3t + 1$

$t = 3 + \sqrt{11}$ or $t = 3 - \sqrt{11}$

can't be $t < 0$

$T = 3 + \sqrt{11}$

b) $S = ?\ u = (-i + 4j)\ v = \ a = (2i - 3j)\ t = 4$

$S = ut + \frac{1}{2}at^2$

$S = (-i + 4j)(4) + \frac{1}{2}(2i - 3j)(4)^2$

$S = (-4i + 16j) + (i - \frac{3}{2}j)(16)$

$= (-4i + 16j) + (16i - 24j)$

$S = (12i - 8j)$

2/8

Examiner Comments

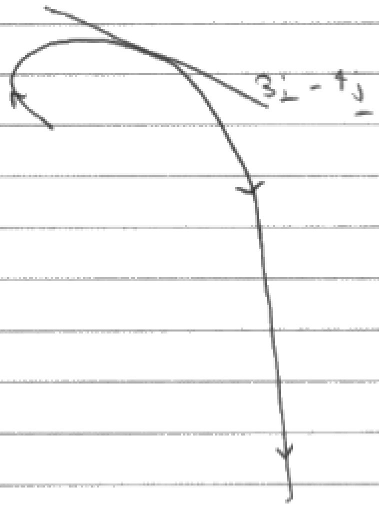
(a) The approach used here was very common and lost all 4 marks in (a)
M0: Instead of finding the velocity vector (which determines the direction of motion) the candidate has found the displacement vector
A0: After M0
M0: In order to obtain this mark, ratios had to be used on a velocity vector
A0: Follows M0

(b) M1: Obtains an expression in the correct form.
A1: A correct expression for **s** at time *t* has been obtained.
M0: No use of Pythagoras to find the distance is seen
A0: Follows M0

Student Response C

a) $v = u + at$

$k(3\underline{i} - 4\underline{j}) = (-\underline{i} + 4\underline{j}) + (2\underline{i} - 3\underline{j})t$



b) $t = 4 \Rightarrow$ $v = u + at$

$\underline{v} = (-\underline{i} + 4\underline{j}) + 4(2\underline{i} - 3\underline{j})$

$\underline{v} = 7\underline{i} - 8\underline{j}$

$\underline{s} = \frac{1}{2}(\underline{u} + \underline{v})t$

$\underline{s} = \frac{1}{2}(-\underline{i} + 4\underline{j} + 7\underline{i} - 8\underline{j}) \times 4$

$\underline{s} = \cancel{\frac{1}{2}}\ 12\underline{i} - 8\underline{j}$ $\sqrt{12^2 + 8^2} = 14.4$

$AB = 14.4\ m$

6/8

Examiner Comments

(a) M1: **v** = **u** + **at** has been used to give an expression of form **C** + (2**i** - 3**j**)t where **C** is non zero vector
A1: Correct unsimplified expression
M0: Has started correctly using a ratio, but needs to obtain an equation in T only, by equating coefficients and eliminating k.
A0: Follows M0

(b) M1: Complete method to find s in terms of t by finding **v** at $t = 4$ and then using s = t(**u**+ **v**)/2
A1: Correct unsimplified expression for s
M1: Use of $t = 4$ and Pythagoras
A1cso: Correct answer from correct working

Exemplar Question 3

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3.

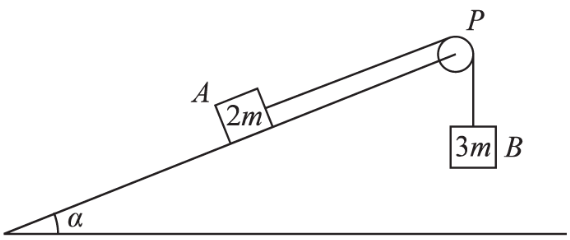


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle a to the horizontal ground, where $\tan \alpha = \dfrac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.
Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\dfrac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \dfrac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

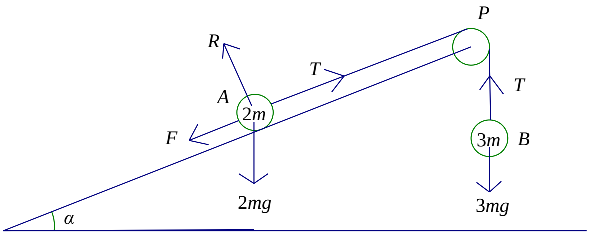
(Total for Question 3 is 12 marks)

Mean Score 6.5 out of 12

Examiner Comments

In part (a), many candidates earned the first mark for correctly finding R, but there were a number who resolved incorrectly, using $\sin\alpha$ instead of $\cos\alpha$. Almost all candidates obtained the second mark, very often from simply stating $F = \frac{2}{3}R$ somewhere on their paper. Most candidates who found the equations of motion correctly did proceed to find T. The most successful methods were either adding the two initial equations to eliminate T, finding a and substituting back into one of the equations or setting both equations to $6ma$ and finding T directly. Candidates who rearranged their equations to give a and then equated the quotients more frequently made errors when solving for T. Common errors were inconsistent use or loss of m, including an extra g in the acceleration term or forgetting to include g in their weight and, on the RHS of the equation, putting $3a$ and $2a$ rather than $3ma$ and $2ma$. A significant number of students set their equation of motion to zero, finding the tension which would keep A at rest on the slope and received little credit. It was surprising how many candidates simply wrote $T = \frac{12mg}{5}$ at the end of their incorrect working, assuming it would not be checked. In the second part, many candidates tried to describe what would happen without any supporting calculations. Many talked about the continued motion of A, ignoring the actual question. Others simply said that it would continue moving as B had more mass than A, ignoring the effects of the plane. Many who showed calculations did successfully compare the component of the weight acting down the slope with the maximum friction, although others continued to include tension or actually compared friction with tension, ignoring the weight altogether. A few candidates showed a deeper understanding of the problem and simply compared μ and $\tan\alpha$. Some candidates did not notice that the terms required were available from the equation of motion for A. Some candidates did not read the question carefully and considered what happened immediately after B reached the ground, rather than when block A came to rest. Some used an equation of motion rather than considering forces separately. A minority showed their lack of understanding by using tension in their explanation. The final part was generally well done, although candidates still persist in giving more answers than are required, which very often means that they include a wrong one and so lose marks. A few just restated the model – “light string” etc. Candidates who didn’t score well in parts (a) and (b) were often able to gain a mark or two in (c) with correct refinements of the model.

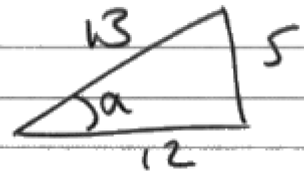
Mark Scheme

Question	Scheme	Marks	AO
3(a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for *A*:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for *B*:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in *T*, *m* and *g* only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{\max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so *A* will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance";"include rotational effects of forces on blocks i.e. spin"	B1 B1	3.5c 3.5c
		(2)	
		(12)	

Marks		Notes
3a	B1	Normal reaction between A and the plane seen or implied, $\cos\alpha$ does not need to be substituted.
	B1	$F = \frac{2}{3}R$ seen or implied anywhere, including part (b)
	M1	Form an equation of motion for A. Must include all relevant terms. Must be the correct mass but condone consistent missing m's. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$ **N.B**. If $T - 2mg = 2ma$ is seen with no working, M0A0 unless both B1 marks have been scored.
	M1	Form an equation of motion for B. Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$
		N.B. Allow the 'whole system' equation to replace the equation for A or B. $3mg - F - 2mg\sin\alpha = 5ma$ Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.
	M1	Complete method to give an equation in T, m and g only. **N.B.** Allow θ in the equation if they have defined what θ is: e.g. $\theta = \tan^{-1}(\frac{5}{12})$ This is an <u>independent</u> mark but they must have two simultaneous equations in T and a unless one of the equations is the whole system equation in which case one equation will be in T and a and the other equation will be in a only.
	A1*	Obtain the **given answer** from correct working using EXACT trig ratios. (not available if using a decimal angle)
3b	M1	Comparison of their F_{max} $(\frac{2}{3}R)$ and their component of weight down the slope, must be comparing numerical values. oe e.g. if they consider the difference **N.B.** Allow comparison of μ and $\tan\alpha$ with numerical values
	A1	Correctly justified conclusion and no errors seen **N.B.** If they equate their difference to an 'ma' term then A0
3c	B1 B1	Deduct 1 mark for each extra (more than 2) incorrect answer up to a maximum of 2 incorrect answers. Ignore extra correct answers. e.g. two correct, one incorrect B1 B0 one correct, one incorrect B1 B0 one correct, two incorrect B0 B0 Ignore incorrect reasons or consequences. Ignore any mention of wind or a general reference to friction.

Student Response A

a).



$\tan a = \frac{5}{12}$

$\sin a = \frac{5}{13}$

$\cos a = \frac{12}{13}$

$\cancel{f = ma}$

$\cancel{T - 2g\sin a = 2ma}$

$\cancel{T = 2ma \; \frac{45}{13} \; \frac{10g}{13}}$

$\cancel{f = ma}$

$\cancel{3g - T = (3m \times g)}$

$\cancel{T = 3g - 3mg^{a}}$

$\cancel{= 3g(1 = m)}$

$\cancel{2ma - \frac{10g}{13} = 3g(1 - m)}$

$\cancel{2ma - \frac{10g}{13} = 3g - 3ma}$

$\cancel{5ma = \frac{4g}{13} g}$

2m

$f = \frac{2}{3} \times 2g \frac{12}{13}$

$\cancel{f = 12}$

$f = \frac{16g}{13}$

$f = ma$

$T - \frac{10g}{13} - \frac{16g}{13} = 2ma$

$T - \frac{26g}{13} = 2ma$

$T - 2g = 2ma$

$T = 2ma + 2g$

3m

$f = ma$

$3g - T = 3ma$

$T = 3g - 3ma$

$3g - 3ma = 2ma + 2g$

$5g = 5ma$

b). ~~A will not remain at rest because 2gsinα will pull~~

$2g\sin\alpha = 7.55$ $\qquad$ $18.11 > 7.55$

$F = 2g\cos\alpha$

$= 18.11$

As friction > forces acting away from pulley, ~~A~~ A will remain at rest.

c). Air resistance acting on 3m

Resistance in pulley, P

2/12

Examiner Comments

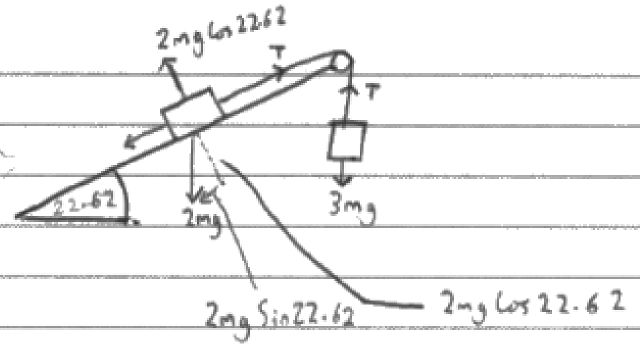
(a) B0: R is incorrect
B1: $F = 2/3\ R$ seen
M0: The equation for A has the correct number of terms but is dimensionally incorrect. The m is missing in both the Friction and the component of weight but not in the $2ma$ term so inconsistent omission of m.
A0: Follows M0:
M0: The equation for B has the correct number of terms but the m is missing in the weight but not in the $3ma$ term so inconsistent omission of m
A0: Follows M0.
M0: Candidate has no equation in T, m and g only
A0: Follows M0.

(b) M0: They are not comparing their F and the component of weight down the plane
A0: Follows M0

(c) B1: Air resistance is correct
B1: Resistance on the pulley is correct

Student Response B

a)



$\alpha = 22.62°$

$$\text{Friction} = \frac{2}{3} \times 2mg \cos 22.62 = \frac{4}{3} mg \cos 22.62$$

~~3Mgα~~

$$3mg - T = 3m\cancel{g}\, a$$

$$T - \frac{4}{3} mg \cos 22.62 = \cancel{3Mgα}\; 2m\cancel{g}\, a$$

$$\frac{3mg - T}{3m\cancel{g}} = \frac{T - \frac{4}{3} mg \cos 22.62}{2m\cancel{g}}$$

$$3mT - 4m^2 g \cos 22.62 = 6m^2 g - 2mT$$

$$3T - 4mg \cos 22.62 = 6mg - 2T$$

$$5T = 6mg - 4mg \cos 22.62$$

$$T = \frac{12mg}{5}$$

c) Including resistance over the pulley

Air resistance on the falling 3m block

7/12

Examiner Comments

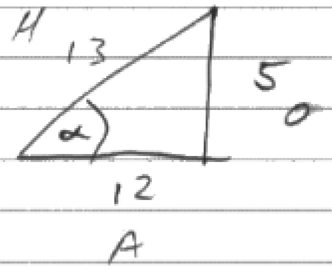
(a) B1: A correct expression for *R* can be seen on the diagram
B1: They have a correct expression for Friction
M0: The equation of motion for *A* has an incorrect number of terms. The component of weight is missing.
A0: Follows M0.
M1: The equation of motion for *B* has the correct number of terms and is dimensionally correct.
A1: The equation is correct
M1: Candidate has found equations of motion for *A* and *B* and formed an equation in *T*, *m* and *g* only.
A0: The given answer is not correctly obtained.

(b) M0: This question has not been attempted
A0: Follows M0

(c) B1: Resistance on the pulley is correct.
B1: Air resistance acting on the block is correct.

Student Response C

a)



$\tan\alpha = 5/12$

$\sin\alpha = 5/13$

$\cos\alpha = 12/13$

$B(\downarrow): \ 3mg - T = 3ma$

$\Rightarrow T = 3mg - 3ma \quad (1)$

$A(\nwarrow): \ R = 2mg\cos\alpha = 2mg\left(12/13\right) = \frac{24}{13}mg$

$A(\nearrow): \ T - \mu R - 2mg\sin\alpha = 2ma$

$\Rightarrow T = 2ma + \mu R + 2mg\sin\alpha$

$= 2ma + \frac{2}{3}\left(\frac{24}{13}mg\right) + 2mg\left(5/13\right)$

$= 2ma + \frac{16}{13}mg + \frac{10}{13}mg$

$= 2ma + 2mg \quad (2)$

Equating (1) and (2):

$\hookrightarrow \ 2\cancel{m}a + 2\cancel{m}g = 3\cancel{m}g - 3\cancel{m}a$

$\Rightarrow 5a = g$

$\Rightarrow a = \frac{1}{5}g \quad (3)$

Subbing (3) into (2):

$\hookrightarrow \ T = 2m\left(\frac{1}{5}g\right) + 2mg$

$= \frac{2}{5}mg + 2mg$

$= \frac{12mg}{5}$

b) ~~A~~: Tension $= 0$

$A(\rightarrow)$: $-2mg\sin\alpha - \mu R$
$= -2mg\left(\frac{5}{13}\right) - \frac{2}{3}\left(\frac{24}{13}mg\right)$
$= -\frac{10mg}{13} - \frac{16mg}{13} = -2mg$

$A(\nearrow)$: $2m(a) = 2m\left(\frac{1}{3}g\right) = \frac{2}{3}mg$

Upward force is ~~greater~~ less than the downward force
$\therefore$ A will start moving back down the plane.

c) · Take into account dimensions of the masses

· Take into account resistive forces such as air resistance

9/12

Examiner Comments

Examiner comment on student response
(a) B1: A correct expression for R can be seen
B1: They have a correct expression for Friction
M1: A valid attempt at the equation of motion for A
A1: A correct equation.
M1: The equation of motion for B has the correct number of terms and is dimensionally correct.
A1: The equation is correct
M1: Candidate has found equations of motion for A and B and formed an equation in T, m and g only.
A1: The given answer is correctly obtained.

(b) M0: The candidate is not comparing the friction with the weight component down the plane.
A0: Follows M0

(c) B1: Dimensions of blocks is a correct refinement.
B1: Air resistance acting on the block is a correct refinement.

Exemplar Question 4

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4.

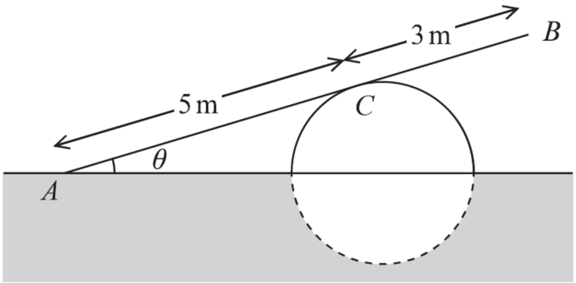


Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where AC = 5 m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(b) Find the magnitude of the resultant force acting on the ramp at A.

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

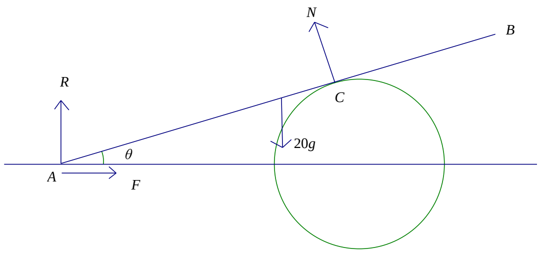
(Total for Question 4 is 11 marks)

Mean Score 2.7 out of 11

Examiner Comments

In part (a), the vast majority of candidates either produced a geometric reason for the reaction acting perpendicular to the ramp (such as the ramp being tangential to the circular drum) or gave an argument about action being equal and opposite to reaction. Neither of which were correct. <u>Many seemed to think that a reaction is, by definition, at right angles to the surface.</u> This is only true of a normal reaction. Very few recognised the significance of the drum being smooth with the consequence that there is no frictional force and hence only a normal component of the reaction. The second part involved finding the components of the resultant force on the ramp at the point of contact with the ground and hence its magnitude. This proved challenging for many and some made little valid progress. A clearly labelled diagram would have helped to identify forces and distances. Most used a normal reaction and a horizontal frictional force at *A*; however, the latter was sometimes omitted which significantly reduced the number of marks available for subsequent equations. Some employed components parallel and perpendicular to the ramp; such attempts were rare but generally successful. A variety of resolution and moments equations were attempted but sometimes components were omitted, forces not resolved or perpendicular distances not used. Working was often difficult to decipher. Those who attempted vertical and horizontal resolutions and a 'moments about *A*' equation tended to be the more successful although a significant number stopped when they had found the normal reaction only. Some introduced a coefficient of friction which was not necessary to answer the question (even giving it the value $\frac{2}{3}$ from the previous question). Occasionally those who completed a correct solution by squaring and adding components to find the magnitude of the resultant failed to achieve the final accuracy mark for not rounding to 2 or 3 significant figures following the use of $g = 9.8 \text{ m s}^{-2}$. Some who had achieved very few marks for the rest of the question managed to secure the mark in part (c) for stating that the reaction at *C* decreases if the centre of mass is assumed to be closer to *A* than to *B*. No reason was required although some chose to give one. A minority claimed it increased or remained unchanged (although the explanation sometimes implied that candidates were considering the reaction at *A* rather than at *C*).

Mark Scheme

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\mathrm{M}(A)$: $20g \times 4\cos\theta = 5N$ $\mathrm{M}(B)$: $3N + R\times 8\cos\theta = F\times 8\sin\theta + 20g\times 4\cos\theta$ $\mathrm{M}(C)$: $R\times 5\cos\theta = F\times 5\sin\theta + 20g\times\cos\theta$ $\mathrm{M}(G)$: $R\times 4\cos\theta = F\times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X = 20g\sin\theta$ $(\nwarrow): Y + N = 20g\cos\theta$ $(\uparrow): X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ $(\rightarrow): X\cos\theta = Y\sin\theta + N\sin\theta$ $\mathrm{M}(A): 20g\times 4\cos\theta = 5N$ $\mathrm{M}(B): 20g\times 4\cos\theta = 8Y + 3N$ $\mathrm{M}(C): 20g\times\cos\theta = 5Y$ $\mathrm{M}(G): 4Y = N\times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (*H*) and cpt perp to ramp (*S*) $(\nearrow): H\cos\theta = 20g\sin\theta$ $(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$ $(\uparrow): S\cos\theta + N\cos\theta = 20g$ $(\rightarrow): H = S\sin\theta + N\sin\theta$ $\mathrm{M}(A): 20g\times 4\cos\theta = 5N$ $\mathrm{M}(B): 20g\times 4\cos\theta + H\times 8\sin\theta = 8S + 3N$ $\mathrm{M}(C): 20g\times\cos\theta + H\times 5\sin\theta = 5S$ $\mathrm{M}(G): 4S = N\times 1 + H\times 4\sin\theta$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover <u>Enter marks on ePEN in order in which equations appear</u>. **N.B.** If reaction at *C* is not perpendicular to the ramp, can only score marks for M(*C*) Allow use of (μR) for *F*
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find *F* and *R* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find *X* and *Y* using only TWO equations, the 1^{st} and 7^{th} in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1

Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find *H* and *S* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark <u>but must use 3 equations (</u>unless it's the special case when 2 is sufficient<u>)</u>
	M1	Use Pythagoras to find magnitude (this is an <u>independent</u> M mark but must have found a value for *F* (or *X*) and a value for *R* (or *Y*)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for *H* and a value for *S*
	A1	Correct answer only
	B1	Ignore reasons

Student Response A

b)

$\sin\theta = \frac{7}{25}$ $\cos\theta = \frac{24}{25}$

$\circlearrowright A = \cancel{(20\times 4)} - \cancel{(R_C\times 5) = 0}$

$\cancel{80 = 5R_C}$

$\cancel{R_C = 16N}$

$(20g\times 4) - 5R_C = 0$

$\cancel{20}\ 80g = 5R_C$

$R_C = 156.8N$

↑ resolve vertically:

$20g = R_A + 156.8\cos\theta$

$20g = R_A + 150.528$

$R_A = 45.472N$

c) the normal reaction of the ramp and the drum at C would be smaller as the weight would be acting further away from C.

d) The normal reaction at C ~~prevents the drum from~~ is where friction acts and therefore is what stops the ramp from moving.

3/11

Examiner Comments

(a) B0: Answer at the end. Incorrect reason. No reference made to smoothness of drum

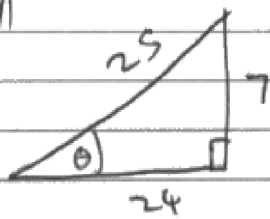
(b) M0: First equation seen - attempt at moments about *A*, but 20*g* x 4 is missing resolution (no sin/cos seen)
A0: Follows M0
M1: Second equation seen - vertical resolution with correct number of terms, dimensionally correct and resolution of their R_C (= 156.8 N)
A1: Correct unsimplified equation (in their R_C)
M0: No third equation seen
A0: Follows M0
M0: Has not got 3 equations to solve for both *F* and R_A
M0: Has not used Pythagoras
A0: Follows M0

(c) B1: Correct answer.

Student Response B

(1)

4)



a) As the reaction force always acts perpedicular to the plane

b) Moments about A

$4 \times 20g \cos\theta = 5 \times R_c$

$80g \times \frac{24}{25} = 5 \times Rc$

$76.6g = 5R_c$

$R_c = 15.36g$

$R_c = 150.528\,N$

Resolve Vertically

$R_A + R_c \cos\theta = 20g$

$R_A + 15.36g \times \frac{24}{25} = 20g$

$R_A + 14.7456g = 20g$

$R_A + 144.5 = 196$

$R_A = 51.5N$

c) The reaction between the ramp and the drum at C would be smaller

5/11

Examiner Comments

(a) B0: Incorrect reason. No reference made to smooth drum

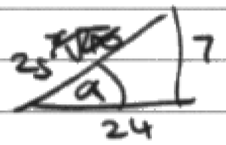
(b) M1: First equation seen - takes moments about *A* with correct number of terms, dimensionally correct (force x distance each term) and sin/cos with relevant force/distance
A1: Correct unsimplified equation
M1: Second equation seen - vertical resolution with correct number of terms, dimensionally correct and resolution of Rc
A1: Correct unsimplified equation
M0: No third equation seen
A0: Follows M0
M0: Has not got 3 equations to solve for both F and R_A
M0: Has not used Pythagoras
A0: Follows M0

(c) B1: Correct answer given

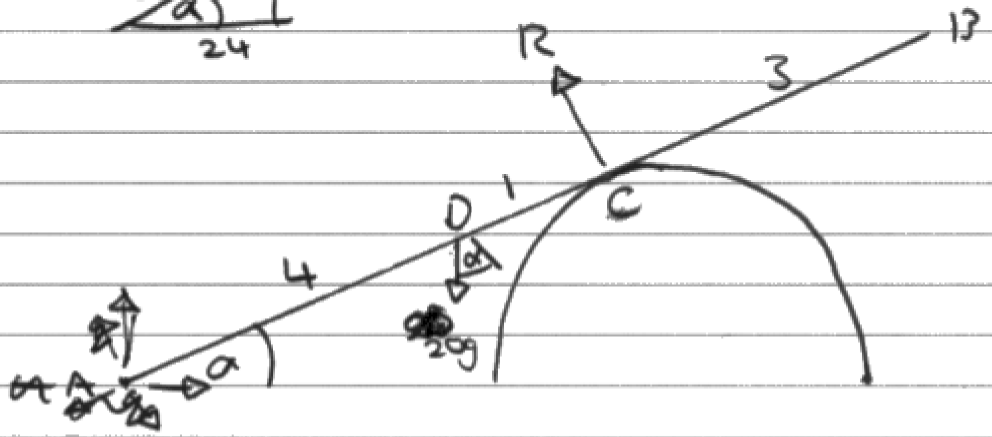
Student Response C

4a) Because the drum only contacts the ramp once and the cord between that point of contact and the centre of the drum is perpendicular to the ramp. Therefore, the reaction force will be perpendicular.

4b)



$\sqrt{24^2+7^2} = 25$



Moment at A: $4 \times 20g \cos\alpha = 5R$

$\cos\alpha = \frac{24}{25}$

$\therefore 4 \times 20g \times \frac{24}{25} = 5R$

$\frac{18816}{25} = 5R$

$R = \frac{18816}{125} = 150.528$

$R = 151$ (3sf)

Moments at C: $1 \times 20g \cos\alpha = 5A$

$4g \times \frac{24}{25} = A$

$A = \frac{4704}{125} = 37.632$

$A = 37.6$ (3sf)

Force $\overleftarrow{\cancel{AC}} = 20g \sin\alpha$ ~~F~~

$= 20g \times \frac{7}{25} = \frac{1372}{25} = 54.88$

$\therefore$ At $A\overrightarrow{F} = 54.88$

$A = \frac{1372}{25}\,i + \frac{4704}{125}\,j$

$|A| = \sqrt{\left(\frac{1372}{25}\right)^2 + \left(\frac{4704}{125}\right)^2} = 66.54(45)\ N$

4c) The weight would be ~~acting on A~~ closer to A then C compared to the uniform rod. Therefore the force applied on C would decrease so the reaction ~~force~~ force would decrease.

9/11

Examiner Comments

(a) B0: Does not mention drum is smooth

(b) <u>Mark as Alternative 1 (ignore the diagram)</u>
M1: Attempts to find equation by taking moments about *A*, dimensionally correct.
A1: Correct equation
M1: Attempts to find equation by taking moments about *C* (see Alt 1 scheme)
A1: Correct equation
M1: Resolves along the ramp
A1: Correct equation
M1: Attempts to solve equations and find two perpendicular components
M1: Uses Pythagoras to find their magnitude
A0: Answer over accurate

(c) B1: Correct answer

Exemplar Question 5

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5.

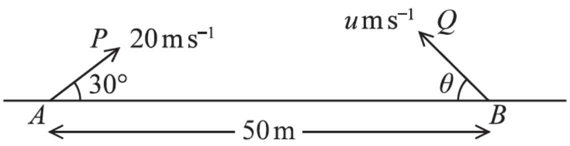

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

(Total for Question 5 is 13 marks)

Mean Score 5.2 out of 13

Examiner Comments

In part (a), the majority of candidates obtained the first mark for the horizontal component and most students also made a good attempt to find the vertical component, although sometimes it was seen as part of their working for part (b). Of those who found the horizontal and vertical components of the velocity, only a proportion proceeded to calculate the speed and of those, very few found the direction. This was despite the clear lead from the question which had given each velocity as a speed with a direction. A sizeable minority broke the universal rule that answers must be given in terms of quantities which are given in the question, giving their velocity in terms of **i** and **j**, which had not been defined in this question. The second part proved to be quite discriminating although a significant number of candidates did successfully find the correct values of both θ and u. The most common error was for candidates to simply equate both the horizontal and vertical displacements at the time of collision and ignore the 50 m horizontal distance altogether. Others assumed that the particles met halfway between A and B. However, only a minority of students made errors when resolving their velocities. In the final part, a substantial number of candidates correctly identified a limitation of the model with the size of the balls being the most common response. Wind, spin and an inaccurate value for g were also relatively common. A significant number lost the mark by referring to mass or weight and some referred to air resistance even though the question specified “other than air resistance’’.

Mark Scheme

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity at $t = 2$	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\frac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \frac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $(\mathrm{m\,s^{-1}})$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $= 50$ at $t = 2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \frac{g}{2}\times 4 = (u\sin\theta)\times 2 - \frac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		Notes
5a	B1	Seen or implied, possibly on a diagram
	M1	Use of $v = u + at$ or any other complete method using $t = 2$ Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation in v or v^2
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)
5b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.
5c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. '*AB* (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised

Student Response A

(1)

a) $S = \cancel{x}$
$U = 20\sin 30$
$V = \checkmark$
$A = -9.8$
$t = 2$
$\therefore$
↳ velocity $= \begin{pmatrix} -9.6 \\ 17.3 \end{pmatrix} ms^{-1}$

$V_v = u + at$
$= 20\sin 30 + (-9.8)(2)$
$= -9.6\ ms^{-1}$
$V_H = 20\cos 30$
$= 10\sqrt{3}$
$= 17.3\ ms^{-1}$

$\cancel{P}\ \cancel{Q}$

b) i $S = 0.4$
$U = u\sin\theta$
$V = \cancel{x}\ x$
$A = -9.8$
$t = 2$

For P
$S = \checkmark$
$u = 20\sin 30$
$V = x$
$A = -9.8$
$t = 2$

b)i $S = ut + \frac{1}{2}at^2$

$= (20\sin 30)(2) + \frac{1}{2}(-9.8)(2)^2$

$= 0.4m$

So →

$S = ut + \frac{1}{2}at^2$

$0.4 = U\sin\theta + \frac{1}{2}(-9.8)(2)^2$

$= U\sin\theta - 19.6$

$U\sin\theta = 20$ — ①

P

~~20 Cos 3~~ $S = \frac{d}{t}$

$d = S \times t$

$= 20\cos 30 \times 2$

$= 40\cos 30$

$= 20\sqrt{3}$

$U\cos\theta \times 2 = 20\sqrt{3}$

$2U\cos\theta = 20\sqrt{3}$ — ②

$U\cos\theta = 10\sqrt{3}$

$\underline{① \div ②}$

$\tan\theta = \frac{2\sqrt{3}}{3}$

$= 1.15$

$\theta = 49.1°$

b)ii $\underline{①^2 + ②^2}$

$U = \sqrt{(10\sqrt{3})^2 + (20)^2}$

$= 10\sqrt{7}$

$= 26.5 ms^{-1}$

c) The Force of weight of the balls

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Examiner Comments

(a) B1: 20 cos 30 seen
M1: Uses $v = u + at$ vertically at time $t = 2$
A1: Obtains correct (unsimplified) equation
M0: No further work (Gives answer as a vector in terms of **i** and **j** which are NOT defined in this question and so answer is inadmissible)
M0: No further work
A0: No further work

(b) M0: Does not use sum of horizontal distances = 50
A0: Follows M0
M1: Sets vertical distances equal and quotes a correct formula but makes a slip in applying it using $t = 2$
A0: Incorrect equation (2 missing from $u\sin\theta$ term)
M0: '50' not used earlier, so mark not available
A0: Follows M0

(c) B0: Incorrect limitation of the model.

Student Response B

a) $PR(\uparrow)$: s
$u = 70\sin\theta$
v
$A = -9.8$
$t = 2$

$PR(\rightarrow)$: s
$u = 70\cos\theta$
v
$a = 0$
t

$R(\uparrow)$ $v = 70\sin\theta - 9.8(2)$ $R(\rightarrow)$ $v = 70\cos\theta$

$v = -9.6$

$\therefore$ $|V|$ = Velocity $= \sqrt{9.6^2 + (70\cos 30)^2}$

$= 19.8 \text{ ms}^{-1}$

b) i) ~~$s = 70\sin\theta \times 2$~~ $- 9.8(2)^2 ($

$s = 70\sin\theta \times 2 - \frac{1}{2}(9.8)(2)^2$

s = height they meet

$= 8.4\text{m}$

$\therefore$ Q $R(\uparrow)$: $s = 0.4$
$u = U\sin\theta$
V
$A = -9.8$
$T = 2$

$R(\rightarrow)$: $s = 30$
$u = U\cos\theta$
v
A
$T = 2$

~~$0.4 = U\sin\theta \times 2$~~

$0.4 = U\sin\theta \times 2 - \frac{1}{2}(9.8)(2)^2$

$20 = 2U\sin\theta$ $\qquad \frac{10}{\sin\theta} = U$ (1)

(1,-7) for P $\qquad 20\sin 30 \times 2 = 20$

$\therefore$ 30 for Q

$30 = U\cos\theta \times 2$ $\qquad \frac{15}{\cos\theta} = \frac{10}{\sin\theta}$

$15 = U\cos\theta$ $\qquad \tan\theta = \frac{2}{3}$

$\frac{15}{\cos\theta} = U$ (2) $\qquad$ ~~$\theta = 218.43$~~

$\theta = 33.69$

ii) $\frac{10}{\sin(18.43)} = v = 31.62$

$\frac{15}{\cos(18.43)} = v =$

$\frac{10}{\sin(33.64)} = v = \cancel{\sin}\ v = 18.027$

$\cancel{v = 18}$

$v = 18.03\ ms^{-1}$

$\theta = 33.64$

c) The ~~Gark~~ smalls will not directly hit perfectly therefore the distance ~~a~~ covered by Q will not be exactly 30 as they are not particles ∴ impact will not be perfectly distributed.

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Examiner Comments

(a) B1: Correct horizontal speed
M1: $v = u + at$ used vertically with $t = 2$
A1: Correct equation
M0: Angle not attempted
M1: Pythagoras used to find the speed
A0: Correct speed but direction not found

(b) M1: Equation with two horizontal distances using $t = 2$ adding to 50 m implied by 30m used (50 - 20)
A0: Uses 20sin30 instead of 20cos30 for the horizontal speed of P
M1: Vertical distance travelled by P (0.4) is equated to the vertical distance travelled by Q
A1: Equation correct
M1: Solves for both u and θ with 50 m used
A0: Incorrect values of u and θ

(c) B1: Valid limitation of the model

Student Response C

a) vertically:

$s = ut + \frac{1}{2}at^2$

$s = 20\sin 30\, t - 4.9t^2$

$s = 10(2) - 4.9(4)$

$s = 0.4\text{m}$

vertical velocity at 2secs:

$v = u + at$

$v = 10 + (-9.8)(2)$

$v = -9.6$

horizontal velocity $= 20\cos 30 = 17.32\ldots$

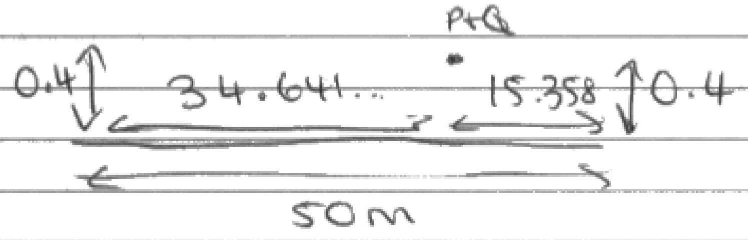
$s = ut = 20\cos 30\,(2) = 34.641\ldots$

$\therefore$ overall velocity $= \sqrt{(-9.6)^2 + (20\cos 30)^2}$

$v = -19.8 \text{ ms}^{-1}$

↳ as acting downward.

b) i) ~~at 2 secs & must also be at~~



At 2 secs:

Q's vertical height $= 0.4$

range $= 15.3589.8385$

horizontally:

$\frac{15.358..}{2} = U\cos\alpha$ ② $(s = ut)$

vertically:

$s = ut + \frac{1}{2}at^2$

$0.4 = U\sin\alpha(2) - 19.6$

$U\sin\alpha = 10$ ①

$\frac{①}{②}$: $\frac{U\sin\alpha}{U\cos\alpha} = \frac{10}{\frac{1}{2}(15.358..)} = \tan\alpha$

$\tan\alpha = 1.30$

$\alpha = 52.5°$ (3sf)

ii) "$s = ut$"

$U\cos\alpha = 7.67949...$

$U = \frac{7.67949}{\cos(52.4..)} = 12.6 \text{ ms}^{-1}$

(c) the balls are modelled as particles → means they experience no resistive forces, whereas in real life they would.

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Examiner Comments

(a) B1: Correct horizontal speed
M1: Uses $v = u + at$ vertically with $t = 2$
A1: Correct equation for vertical velocity component
M0: No attempt to find angle
M1: Uses Pythagoras to find the speed
A0: No angle found

(d) M1: Correct method for finding horizontal distances, adding and equating to 50, with $t = 2$
A1: Correct equation
M1: Correct method for finding and equating vertical distances at $t = 2$
A1: Correct equation
M1: Having found the necessary equations, solves for u and θ and has used 50 m
A1: Correct answers

(c) B1: Acceptable answer. Has mentioned not modelling the balls as particles. "Air resistance" only would be B0