Pearson Edexcel
Level 3 Advanced Subsidiary GCE in Mathematics (8MA0)
Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

Sample Assessment Materials Model Answers – Mechanics

First teaching from September 2017
First certification from June 2018

Sample Assessment Materials Model Answers – Mechanics

Contents

Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Advanced GCE in Mathematics (8MA0 and 9MA0) specifications for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS and A level Mathematics qualifications, specifically at mechanics questions, and is intended to offer model solutions with different methods explored.

Content of Mechanics

Content	AS level content	A level content
Forces • in static situations • in dynamic situations (using kinematic acceleration formulae) • their application to Newton's Laws	Simple 2D situations Resolution of forces is not required	In 2D situations including friction and resolving forces into components
Kinematics • the equations of motion for constant acceleration • displacement velocity and acceleration time graphs • variable acceleration problems using calculus	1D problems or simple 2D problems and can include particles connected over a pulley	2D problems e.g. involving resolving components and projectiles
Moments • their applications to solving static problems	Not in AS	Equilibrium of rigid bodies such as ladder problems
Dynamics • the accelerations as a result of forces linking back to using kinematics	Newton's laws of motion Including connected particles and 1D situations	Newton's laws of motion in 2D situations including particles on an inclined plane
Vector Techniques • applied to Dynamics and Kinematics	Including unit vectors **i** and **j** and column vectors	Including unit vectors **i** and **j**, column vectors and equations such as $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

AS Level Question 6

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

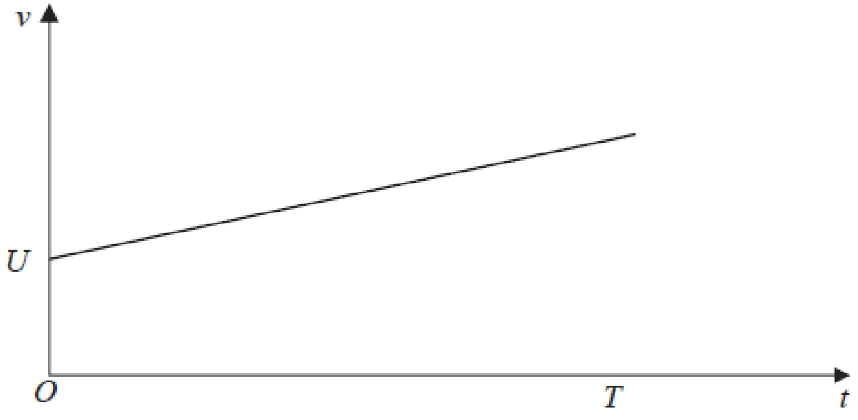


Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leq t \leq T$.

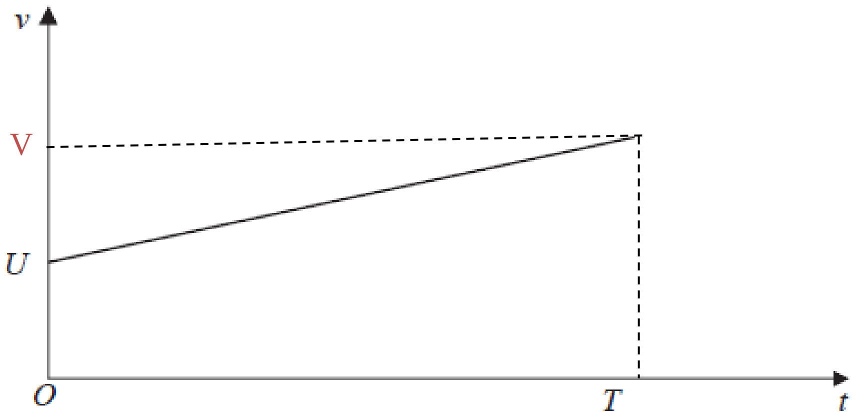
Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(4)

The area underneath a velocity – time graph represents the distance travelled.

M1



Let the final velocity be V.
The area under the graph is given by the area of the trapezium:

$$D = \frac{(U+V)}{2}T \qquad (1)$$

A1

The gradient of a velocity – time graph represents the acceleration:

$$a = \frac{V-U}{T} \qquad (2)$$

M1

Re-arranging (2) $V = U + aT$

Substitute into (1) $D = \frac{(U+U+aT)}{2}T$

Expand and simplify $D = UT + \frac{1}{2}aT^2$

A1

AS level Question 7

A car is moving along a straight horizontal road with constant acceleration. There are three points *A*, *B* and *C*, in that order, on the road, where *AB* = 22 m and *BC* = 104 m. The car takes 2 s to travel from *A* to *B* and 4 s to travel from *B* to *C*.

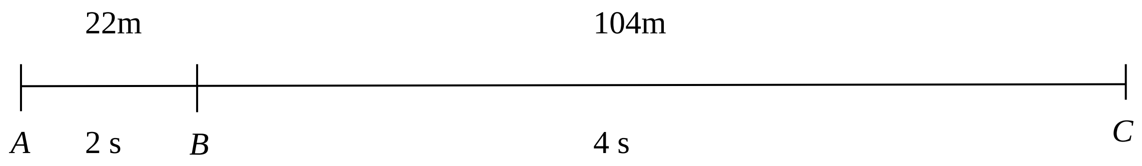
Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes *A*.

(7)

(i)



Let u m s^{-1} = velocity at *A*, a m s^{-2} = the constant acceleration.

Using $s = ut + \frac{1}{2}at^2$

Strategy to set up two equations in *a* and *u* — M1

For *AB*: $s = 22$, $u = u$, $t = 2$, $a = a$. M1

$$22 = 2u + \frac{1}{2}a \times 4$$
$$11 = u + a \quad (1)$$

A1

Now use *AC* rather than *BC* to allow a common use of *u* and *a*.

For *AC*: $s = 126$, $u = u$, $t = 6$, $a = a$. M1

$$126 = 6u + \frac{1}{2}a \times 36$$
$$21 = u + 3a \quad (2)$$

A1

(2) – (1) $\quad 10 = 2a$

$$a = 5$$

The acceleration is 5 m s^{-2}

A1

(ii) Substitute $a = 5$ into (1)

$$11 = u + 5$$
$$u = 6$$

A1ft

The speed at *A* is 6 m s^{-1}

Alternative

(i) As the acceleration is constant, the average speed for $AB = \frac{22}{2} = 11$ m s^{-1}

[This corresponds to the speed at the midtime of AB i.e. $t = 1$.]

and the average speed for $BC = \frac{104}{4} = 26$ m s^{-1}

[This corresponds to the speed at the midtimeof BC i.e. $t = 2 \times 2 = 4$.]

The acceleration between $t = 1$ and $t = 4$, a m s^{-2} is given by:

$$a = \frac{v - u}{t}$$

$$a = \frac{26 - 11}{3} = 5$$

The acceleration is 5 m s^{-2}

(ii) For A to the midpoint of AB

Using $v = u + at$, $v = 11$, $a = 5$, $t = 1$

$$u = 11 - 5 \times 1$$
$$u = 6$$

The speed at A is 6 m s^{-1}

AS level Question 8

A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \text{ where } 0 \leq t \leq 10.$$

(a) Explain the restriction $0 \leq t \leq 10$.

(3)

Substitute both values $t = 0$ and $t = 10$ into $s = \frac{1}{10}(t^4 - 20t^3 + 100t^2)$

M1

Gives $s = 0$ for both $t = 0$ and $t = 10$, which correspond with the start and end of the flight.

A1

The factorisation of $s = \frac{t^2}{10}(t-10)^2$ shows that $s > 0$ for $0 < t < 10$

A1

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

The bird will be at rest when $v = 0$

$$v = \frac{ds}{dt}, \text{ the rate of change of distance}$$

$$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$$

M1A1

When at rest, $v = 0$,

$$\frac{1}{10}(4t^3 - 60t^2 + 200t) = 0$$

M1

$$t^3 - 15t^2 + 50t = 0$$
$$t(t^2 - 15t + 50) = 0$$
$$(t - 5)(t - 10) = 0$$
Therefore, $t = 0, 5, 10$

A1

First at intantaneous rest when t = 5 (when the bird turns back)

When t = 5, $s = \frac{5^2}{10}(5-10)^2$ or $s = \frac{1}{10}\left(5^4 - 20\times 5^3 + 100\times 5^2\right)$

M1

$$s = \frac{1}{10}25\times 25$$

Distance = 62.5 m.

A1ft

AS level Question 9

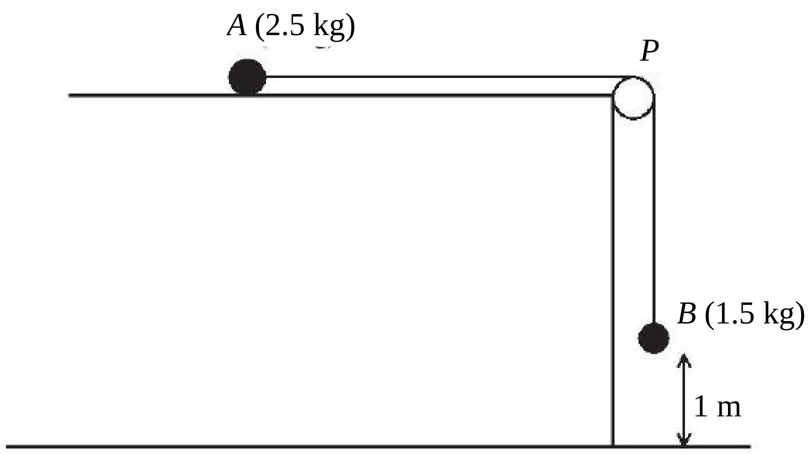


Figure 2

A small ball *A* of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley *P* which is fixed at the edge of the table. The other end of the string is attached to a small ball *B* of mass 1.5 kg hanging freely, vertically below *P* and with *B* at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of *A* from the rough table is modelled as having constant magnitude 12.7 N. Ball *B* reaches the floor before ball *A* reaches the pulley.

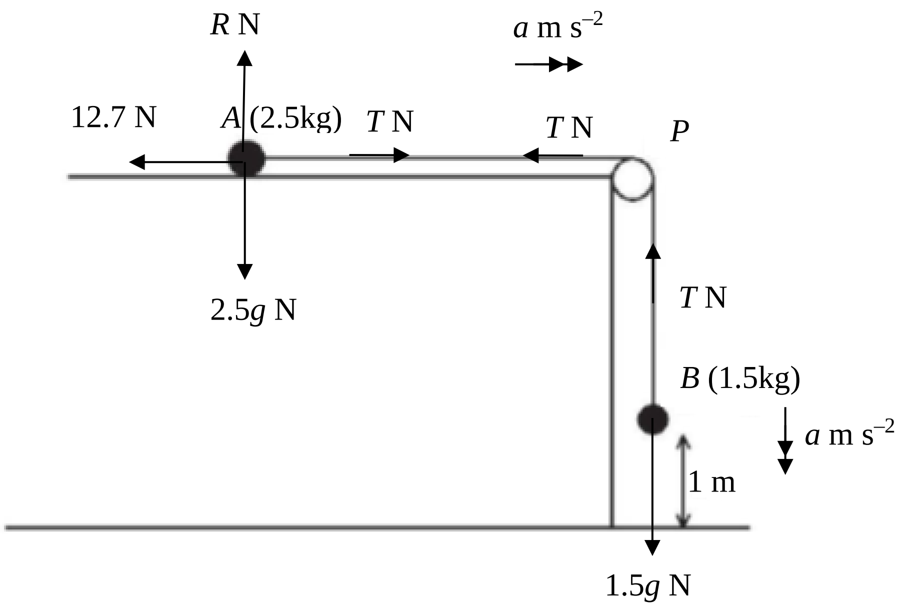
The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for *A*.

(ii) Write down an equation of motion for *B*.

(4)

As the pulley is smooth, the tension in the string will be the same in both parts of the string.
The system will have a common acceleration, a m s^{-2}.
A has no vertical component of motion so the normal reaction, *R* newtons, will be equal to the weight of *A*.
Friction (the constant resistance here) opposes motion so acts to the left.



Using $F = ma$

(i) For A $\quad T - 12.7 = 2.5a$ M1 A1

(ii) For B $\quad 1.5g - T = 1.5a$ M1 A1

(b) Hence find the acceleration of B. **(2)**

Adding (i) and (ii) $\quad 1.5g - 12.7 = 4a$ M1

$14.7 - 12.7 = 4a$

$2 = 4a$

$a = 0.5$

Acceleration of $B = 0.5$ m s^{-2}

A1

(c) Using the model, find the time it takes, from release, for B to reach the floor. **(2)**

Using $s = ut + \frac{1}{2}at^2$ with $a = 0.5$, $u = 0$, $s = 1$

$$1 = 0 \times t + \frac{1}{2} \times 0.5 \times t^2$$

M1

$$1 = \frac{1}{4}t^2$$
$$t^2 = 4$$
$$t = 2$$

Time for *B* to reach the floor is 2 s

A1

It was found that it actually took 2.3 seconds for ball *B* to reach the floor.

(d) Using this information,

(i) comment on the appropriateness of using the model to find the time it takes ball *B* to reach the floor, justifying your answer.

(ii) suggest one improvement that could be made in the model.

(2)

(i)

The model is inappropriate because the ball has taken longer to reach the floor as the model

- does not include air resistance
- does not include the roughness of the pulley

Or because of any other appropriate reason such as:

- the string will have mass which will affect the tension
- the string may stretch which will affect the acceleration.
- the pulley may not be smooth which will affect the tension

B1

(ii)

- Do not model *B* as a particle but give its dimensions so the distance it falls will be less.
- If the pulley is not modelled as small the tension in the string along the table would not be horizontal (parallel to the table).
- Do not model the resistance as being constant.

B1

A level Question 6

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration **a** m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

Rate of change of velocity = acceleration

Therefore $\quad \dfrac{\mathrm{dv}}{\mathrm{d}t} = \mathbf{a}$

Hence $\quad \mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$

M1

$$\mathbf{v} = \int 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}\,\mathrm{d}t$$

$$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - \frac{15t^{\frac{3}{2}}}{\frac{3}{2}}\mathbf{j} + \mathbf{c}$$

A1

$$\mathbf{v} = 2.5t^2\,\mathbf{i} - 10t^{\frac{3}{2}}\,\mathbf{j} + \mathbf{c}$$

To find the constant vector **c**, use $\mathbf{v} = 20\mathbf{i}$ when $t = 0$

$$\mathbf{c} = 20\mathrm{i}$$

Therefore, $\quad \mathbf{v} = \left(2.5t^2 + 20\right)\mathbf{i} - 10t^{\frac{3}{2}}\,\mathbf{j}$

A1

When $t = 4$ $\quad \mathbf{v} = \left(2.5 \times 16 + 20\right)\mathbf{i} - 10 \times 4^{\frac{3}{2}}\,\mathbf{j}$

$\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$

M1

Speed is the magnitude of the velocity vector

$$|\mathbf{v}| = \sqrt{60^2 + 80^2} = \sqrt{3600 + 6400} = \sqrt{10000}$$

M1

Speed = 100 m s^{-1}

A1ft

A level Question 7

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

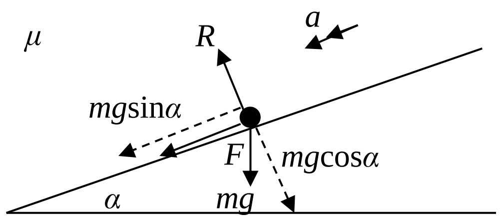
A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

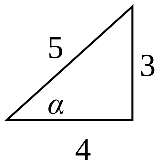
(a) Find the value of μ.

(6)



$\tan\alpha = \frac{3}{4}$

When the angle is given as a trig ratio, use a right-angled triangle to find the other ratios:



$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

The weight mg can be resolved into two components one parallel to and one perpendicular to the plane $mg\sin\alpha$ and $mg\cos\alpha$ as shown above.
Let the normal reaction to the plane be R and the force due to friction be F.

Then, resolving perpendicular to the plane;

$$R = mg\cos\alpha$$

B1

$$R = \frac{4}{5}mg \text{ (1)}$$

Using $F = ma$ parallel to the plane:

M1

$$F + mg\sin\alpha = \frac{4}{5}mg$$

A1

$$F + \frac{3}{5}mg = \frac{4}{5}mg$$

$$F = \frac{1}{5}mg \text{ (2)}$$

For friction, $F \le \mu R$. Since the particle is moving, F has reached its maximum value so

$$F = \mu R$$

M1

Or, $$\mu = \frac{F}{R}$$

M1

Hence, dividing (2) by (1) $$\mu = \frac{\frac{1}{5}mg}{\frac{4}{5}mg} = \frac{1}{4}$$

A1

> The particle comes to rest at the point A on the plane.
>
> (b) Determine whether the particle will remain at A, carefully justifying your answer.
>
> **(2)**

Once the particle stops moving up the plane, friction will act up the plane trying to prevent the particle moving back down the plane.

The particle will move back down the plane if the component of the weight of the particle down the plane exceeds the maximum force of friction.

M1

Component of weight down the plane: $$mg\sin\alpha = \frac{3}{5}mg$$

Maximum force of friction: $$F = \mu R$$

$$F = \frac{1}{4} \times \frac{4}{5}mg$$

$$F = \frac{1}{5}mg$$

Hence the particle will move back down the plane as $\frac{3}{5}m > \frac{1}{5}mg$

A1

A level Question 8

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

Using

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$10.5\mathbf{i} - 0.9\mathbf{j} = 0.6\mathbf{j} + 15\mathbf{a}$$

M1

$$10.5\mathbf{i} - 1.5\mathbf{j} = 15\mathbf{a}$$

$$\mathbf{a} = 0.7\mathbf{i} - 0.1\mathbf{j}$$

A1

(b) Find $\mathbf{r}$ in terms of t.

(2)

Using

$$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

M1

$$\mathbf{r} = 0.6\mathbf{j}t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})t^2$$

A1

(c) Find the value of t when the boat is north-east of O.

(3)

When north-east of O, the **i** and **j** components will be equal

M1

$$0.35t^2 = 0.6t - 0.05t^2$$

A1

$0.4t^2 - 6t = 0$

$2t(2t - 3) = 0$

Therefore $t = 0$ or $t = 1.5$s

A1

$t = 0$ is the start when the boat is at O

Boat is north-east of O when $t = 1.5$

(d)	Find the value of t when the boat is moving in a north-east direction.	**(3)**

When it is moving north east, the components of velocity will be equal.

$\mathbf{v} = u + \mathbf{a}t$

$\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})t$

M1

Hence the boat is moving north-east when:

$0.7t = 0.6 - 0.1t$

M1

$0.8t = 0.6$

$t = 0.75$ s

A1

A level Question 9

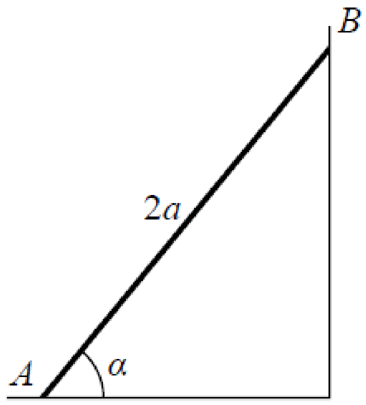


Figure 1

A uniform ladder *AB*, of length $2a$ and weight *W*, has its end *A* on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end *B* of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.

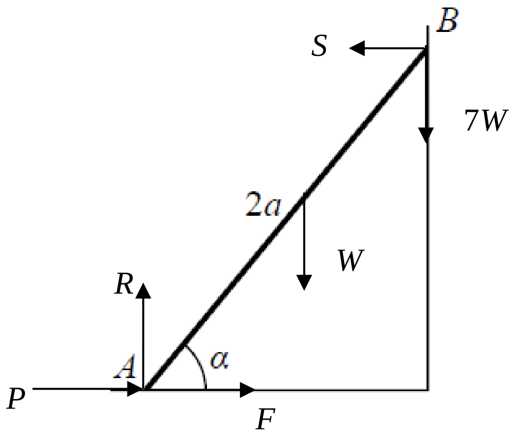
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude *P* to the ladder at *A*, towards the wall. The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at *B* has magnitude $3W$.

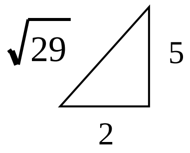
(5)



$\tan\alpha = \frac{5}{2}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



Since the ladder is modelled as a uniform rod, it weight can be assumed to act at the midpoint of the ladder.
Let the normal reactions at the ground and at the wall be R and S.

The moment of a force about a point = Force × perpendicular distance from the force to the pivot.

By taking moments about A, none of the forces acting through A have a moment as their distance from A is zero. Therefore:

M1

$$S(2a \sin\alpha) = 7W(2a \cos\alpha) + W(a \cos\alpha)$$

A1 A1

Dividing by cos α $\quad S(2a \tan\alpha) = 15aW$

M1

$$S \times 2a \times \frac{5}{2} = 15aW$$

$$S = 3W$$

A1

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

The force P stops the ladder from slipping.
If P is just preventing the ladder from slipping **down** the wall, friction will act to the right in the same direction as P as shown in the diagram.
A larger value of P would push the base of the ladder towards the wall and so if P is large enough cause slipping **up** the wall. Friction will act to the left opposing P.

These two situations lead to the range of values for P.

In equilibrium $F \leq \mu R$. When about to move (limiting equilibrium) or moving, F has its maximum value F_{max} , where $F_{max} = \mu R$

To find the friction force F_{max} when the ladder is at the point of slipping down or up the wall:

Resolving vertically $R = 8W$

B1

In limiting equilibrium $F_{max} = \mu R$

$F_{max} = \frac{1}{4} 8W$

$F_{max\ x} = 2W$

A1

Slipping down the wall:

Resolving horizontally. $P + F_{max} = 3W$

$P + 2W = 3W$

$P = W$

M1 A1

Slipping up the wall:

Resolving horizontally. $P - F_{max} = 3W$

$P - 2W = 3W$

$P = 5W$

M1 Either $P + F = 3W$ or $P - F = 3W$
A1 Either $P = W$ or $P = 5W$

(M1 A1)

Therefore $W \leq P \leq 5W$

A1

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

When the builder's assistants stands at the bottom of the ladder, there is a vertical force T acting downwards at A.

By taking moments about A, it is seen that the reaction at B is unchanged.

M1

However the reaction, R at A, will increase. This is seen be resolving vertically as $R = 8W + T$.

M1

Since $F_{max} = \mu R$, F_{max} will increase as R increases

M1

This makes the ladder less likely to slip as the maximum force of friction is greater.

A level Question 10

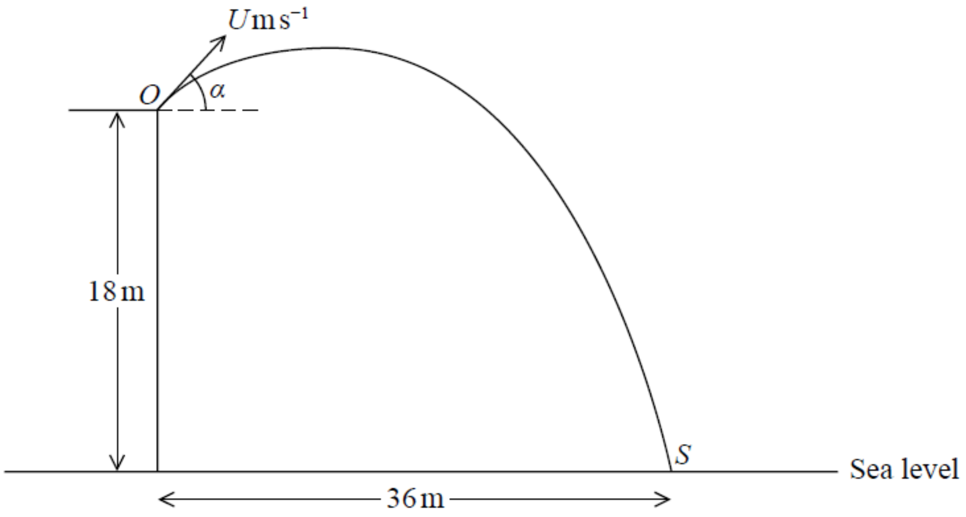


Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff. The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}.

Find

(a) the value of U,

(6)

Displacement O to S vertically using $s = ut + \frac{1}{2}at^2$ horizontally using $s = ut$

$\tan\alpha = \frac{3}{4}$
$\sin\alpha = \frac{3}{5}$
$\cos\alpha = \frac{4}{5}$

(Note $g = 10$ m s^{-2} in this question)

$$-18 = \frac{3}{5}Ut - 5t^2$$

$$\frac{4}{5}Ut = 36 \text{ or } Ut = 45$$

A1

$$\text{and } \frac{3}{5}Ut - 5t^2 = -18$$

A1

substituting for *Ut*

$$\frac{3}{5} \times 45 - 5t^2 = -18$$

M1

$$27 - 5t^2 = -18$$
$$5t^2 = 45$$
$$t^2 = 9$$
$$t = 3 \text{ as } t > 0$$

Since $Ut = 45$ and $t = 3$

$$U = 15$$

A1

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures. **(5)**

Using $v^2 = u^2 + 2as$, vertically

$v_y^2 = (U\sin\alpha)^2 + 2 \times (-10) \times (-7.2)$

M1

$v_y^2 = \frac{9}{25}15^2 + 144$

$v_y^2 = 225$

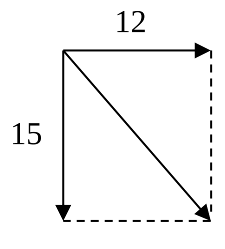
$v_y = 15$

A1

Horizontally

$v_x = 15 \times \frac{4}{5} = 12$

B1



$v = \sqrt{12^2 + 15^2}$

M1

$v = \sqrt{369}$

$v = 19.21$

Speed = 19 m s^{-1} to 2 significant figures

A1ft

(c) Suggest two improvements that could be made to the model.

(2)

Two valid reasons, for example:

- Include air resistance in the model of the motion
- Use a more accurate value for g in the model of the motion
- Include wind effects in the model of the motion
- Include the dimensions of the stone in the model of the motion

B1 B1