Pearson Edexcel
Level 3 Advanced
GCE in Mathematics (9MA0)

Sample Assessment Materials Exemplification – Pure Mathematics

First teaching from September 2017
First certification from June 2018

Contents

About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced GCE in Mathematics (9MA0) (first assessment summer 2018).

The booklet provides additional information on all the questions in the Sample Assessment Materials, accredited by Ofqual in 2017. It details the content references and Assessment Objectives being assessed in each question or question part.

How to use this booklet

Callouts have been added to each question in the accredited Sample Assessment Materials. In the callouts, the following information has been presented, as relevant to the question:

- **Specification References**;
- **Assessment Objectives**.

Where content references or Assessment Objectives are being assessed across all the parts of a question, these are referred to by a single callout at the end of the question rather than by a callout for each question part.

A level Mathematics – Paper 1 (Pure Mathematics)

1. The curve C has equation $y = 3x^4 - 8x^3 - 3$.

(a) Find (i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

(b) Verify that C has a stationary point when $x = 2$

(c) Determine the nature of this stationary point, giving a reason for your answer.

Specification reference (1.1, 7.1, 7.2, 7.3):

Understanding and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion.

Understand and use the second derivative as the rate of change of gradient.

Differentiate x^n for rational values of n, and related constant multiples, sums and differences.

Apply differentiation to find gradients, tangents and normal; maxima and minima and stationary points.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)

AO2.2a: Make deductions and inferences: make deductions (1 mark)

(Total for Question 1 is 7 marks)

2.

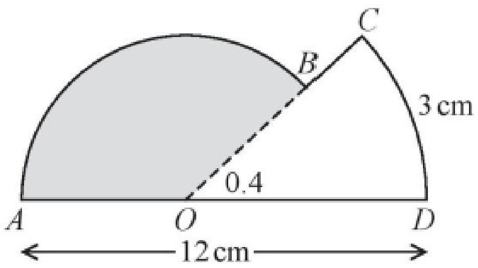

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, ∠*COD* = 0.4 radians and *AOD* is a straight line of length 12 cm, find

(a) the length of *OD*,

(b) the area of the shaded sector *AOB*.

Specification reference (5.1): Work with radian measure including use for arc length and area of sector.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks) **AO1.2:** Accurately recall facts, terminology and definations (1 mark) **AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 2 is 5 marks)

3. A circle C has equation $x^2 + y^2 - 4x + 10y = k$, where k is a constant.

(a) Find the coordinates of the centre of C.

(b) State the range of possible values for k.

Specification reference (2.3, 2.5, 3.2):
Completing the square
Solve linear and quadratic inequalities in single variable and interpret such inequalities graphically
Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x-a)^2 + (y-b)^2 = r^2$; completing the square to find the centre and radius of a circle.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.2a: Make deductions and inferences; make deductions (1 mark)

(Total for Question 3 is 4 marks)

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4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7,$$

show that $a = \ln k$, where k is a constant to be found.

Specification reference (6.4, 8.2, 8.3):
Understand and use the laws of logarithms:
$\log_a x + \log_a y = \log_a(xy)$
$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$
Integrate x^n (excluding n = -1) and related sums, differences and constant multiples.
Evaluate definite integrals; use definite integral to find the area under the curve.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)

(Total for Question 4 is 4 marks)

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5. A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x+1}, \qquad x \neq -1,$$

where a and b are integers to be found.

Specification reference (2.6, 3.3):

Manipulate polynomials algebraically, including factorisation and simple algebraic division.

Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (1 mark)

AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)

(Total for Question 5 is 3 marks)

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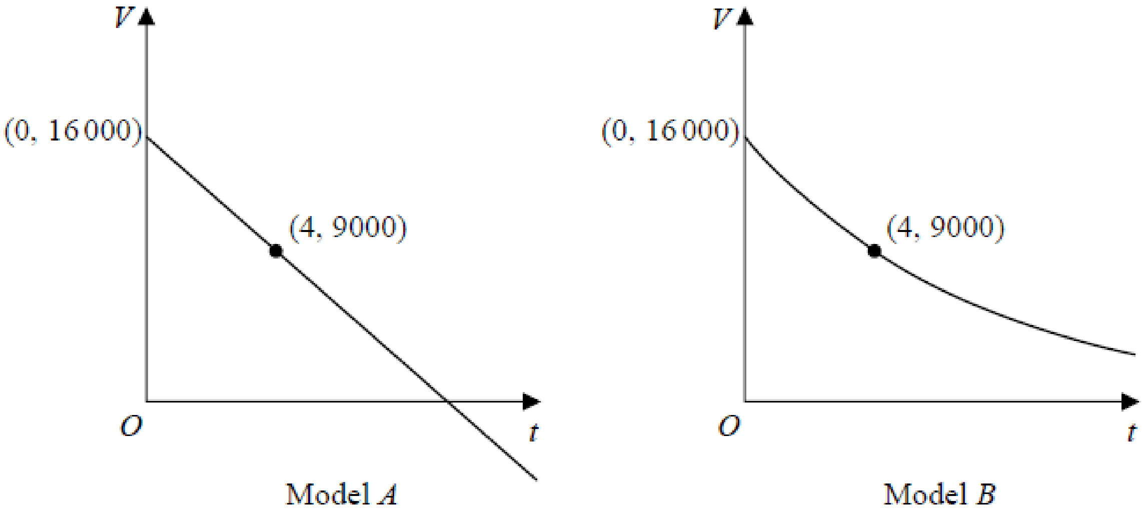
6. A car dealer wishes to model the value of a certain type of car.

The initial price of the car is £16 000 and the value after 4 years is expected to be £9 000.

In a simple model, the value of the car, £V, depends only on the age of the car, t years.

The diagram below shows the graphs of two possible models over 12 years.



(a) Explain why Model A is an unrealistic model for cars over 10 years of age.

It is given that the equation for Model B is of the form $V = pe^{kt}$, where p and k are constants.

(b) Find the equation for Model B.

Saima wants to know the value of her car when it is 3 years old.

(c) (i) Use Model A to predict the value of Saima's car.

(ii) Use Model B to predict the value of Saima's car.

(d) Write down one possible refinement of either Model A or Model B.

Specification reference (2.11, 3.1 6.1, 6.3):

Use of functions in modelling, including consideration of limitations and refinements of the models

Understand and use the equation of a straight line. Be able to use straight line models in a variety of contexts.

Know and use the function e^x and its graph.

Know and use ln x as the inverse function of e^x

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)

AO3.1b: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (1 mark)

AO3.3: Translate situations in context into mathematical models (1 mark)

AO3.4: Use mathematical models (2 marks)

AO3.5b: Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: recognise the limitations of models (1 mark)

(Total for Question 6 is 7 marks)

7.

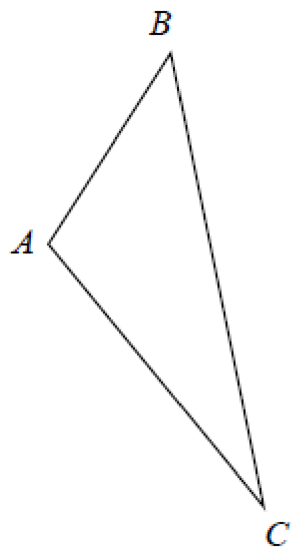


Figure 2

Figure 2 Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$, show that $\angle BAC = 105.9°$ to one decimal place.

Specification reference (5.1, 10.1, 10.2, 10.3):
Understand and use the definitions of the sine and cosine rules.
Use vectors in two dimensions.
Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.
Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

(Total for Question 7 is 5 marks)

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8. $f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4].

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(c) Show that α is the only root of $f(x) = 0$.

Specification reference (2.7, 9.1, 9.2):
Understand and use graphs of functions; sketch curves defined by simple equations including polynomials. Use intersection points of graphs to solve equations.
Locate roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x on which $f(x)$ is sufficiently well behaved.
Solve equations approximately using simple iterative methods.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.4: Explain their reasoning (2 marks)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

(Total for Question 8 is 6 marks)

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9. (a) Prove that $\tan\theta + \cot\theta \equiv 2\,\text{cosec}\,2\theta$, $\theta \neq \dfrac{n\pi}{2}$, $n \in \mathbb{Z}$.

(b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions.

Specification reference (1.1, 5.4, 5.5, 5.6, 5.8):
Understand and use the structure of mathematical proof.
Understand and use the definitions of secant, cosecant and cotangent; their ranges and domains.
Understand and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
Understand and use double angle formula.
Construct proofs involving trigonometric functions and identities.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.4: Explain their reasoning (2 marks)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

(Total for Question 9 is 5 marks)

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10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that, as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$.

Specification reference (1.1, 5.6, 7.1):
Understand and use the structure of mathematical proof.
Understand and use double angle formula; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$
Differentiation from first principles for small positive integer powers of x and for $\sin x$ and $\cos x$.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)
AO2.5: Uses mathematical language and notation correctly (1 mark)

(Total for Question 10 is 5 marks)

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11. An archer shoots an arrow. The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form $A - B(d - C)^2$, where A, B and C are constants to be found.

It is decided that the model should be adapted for a different archer. The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

Specification reference (2.3, 2.11):
Work with quadratic functions and their graphs. Completing the square.
Use of functions in modelling, including consideration of limitations and refinements of the models.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)
AO2.2a: Make deductions and inferences: make deductions (1 mark)
AO3.4: Uses mathematical models (4 marks)

(Total for Question 11 is 9 marks)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment, were counted.

N and T are expected to satisfy a relationship of the form $N = aT^b$, where a and b are constants.

(a) Show that this relationship can be expressed in the form $\log_{10} N = m \log_{10} T + c$, giving m and c in terms of the constants a and/or b.

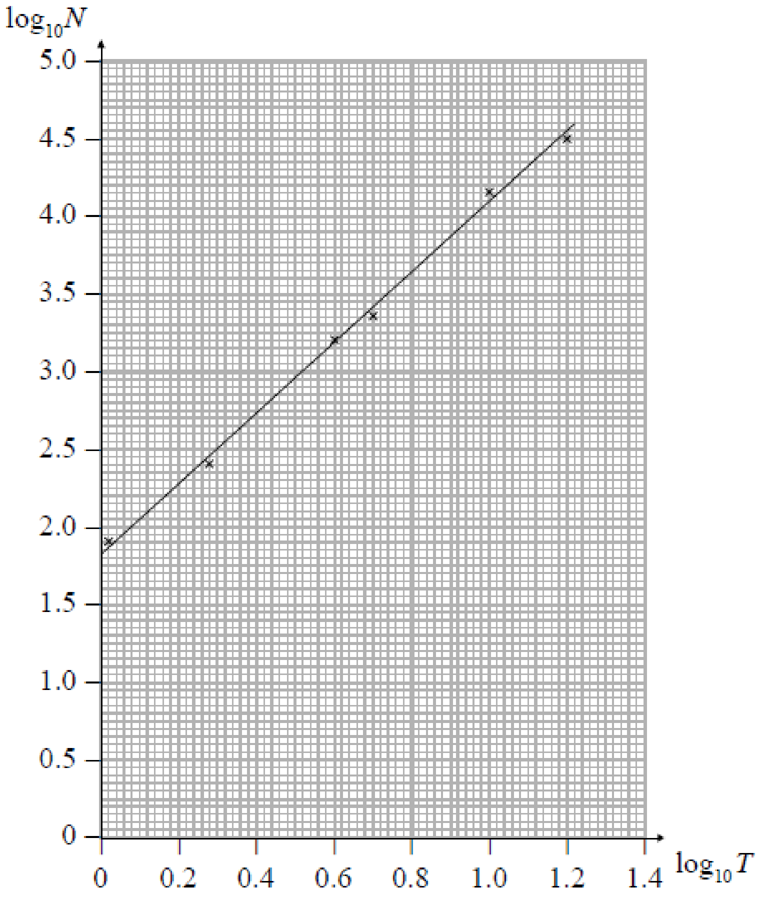

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(d) With reference to the model, interpret the value of the constant a.

Specification reference (6.1, 6.3, 6.4, 6.6, 6.7):

Know and use the function *ax* and its graph, where *a* is positive. Know and use the function e^x and its graph.

Know and use the definition of log*a* *x* as the inverse of a^x, where *a* is positive and $x \geq 0$. Know and use ln *x* as the inverse function of e^x

Understand and use the laws of logarithms:

$\log_a x + \log_a y = \log_a(xy)$

$\log_a x - \log_a y = \log_a(\frac{x}{y})$

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for *x* and *y*

Understand and use exponential growth and decay; use in modelling.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)

AO3.1b: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (2 marks)

AO3.2a: Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)

AO3.4: Uses mathematical models (1 mark)

AO3.5b: Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: recognise the limitations of models (1 mark)

(Total for Question 12 is 9 marks)

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13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \leq t \leq \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is the normal to C at P.

(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

Specification reference (2.2, 2.3, 3.1, 3.3, 5.3, 5.6, 7.2, 7.3, 7.5):

Use and manipulate surds, including rationalising the denominator.

Work with quadratic functions and their graphs, including solving quadratic equations in a function of the unknown.

Be able to use straight line models in a variety of contexts.

Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.

Know and use exact values of sin and cos for 0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and π and multiples thereof, and exact values of tan for 0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, π and multiples thereof.

Understand and use double angle formulae; use of formulae for sin $(A \pm B)$, cos $(A \pm B)$, and tan $(A \pm B)$, understand geometrical proofs of these formulae.

Differentiate e^{kx} and a^{kx}, sin kx, cos kx, tan kx and related sums, differences and constant multiples.

Apply differentiation to find gradients, tangents and normals.

Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (7 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (3 marks)

AO2.4: Explain their reasoning (1 mark)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (2 marks)

(Total for Question 13 is 13 marks)

14.

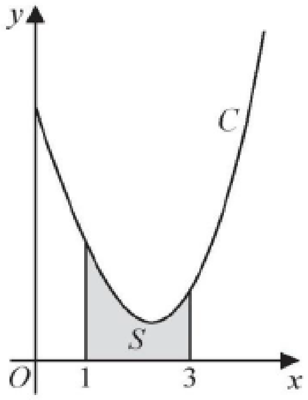

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)

Specification reference (6.4, 8.3, 8.5, 9.3):

Understand and use the laws of logarithms:

$\log_a x + \log_a y = \log_a(xy)$

$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$

Evaluate definite integrals; use a definite integral to find the area under a curve.

Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively.

Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (6 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (3 marks)

AO2.4: Explain their reasoning (1 mark)

(Total for Question 14 is 10 marks)

15.

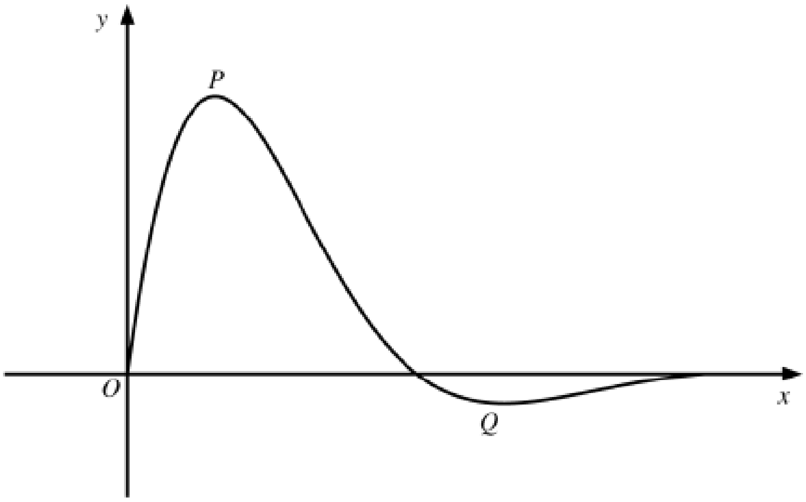

Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leq x \leq \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure 5.

(a) Show that the x-coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{2}$.

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = f(2x)$,

(ii) $y = 3 - 2f(x)$.

Specification reference (2.9, 5.4, 7.2, 7.3, 7.4):

Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$, including sketching associated graphs: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$ and combinations of these transformations.

Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.

Differentiate e^{kx} and a^{kx}, sin kx, cos kx, tan kx and related sums, differences and constant multiples.

Apply differentiation to find gradients, tangents and normals; maxima and minima and stationary points.

Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (2 marks)

(Total for Question 15 is 8 marks)

A level Mathematics – Paper 2 (Pure Mathematics)

1. $$f(x) = 2x^3 - 5x^2 + ax + a.$$

Given that $(x + 2)$ is a factor of f (x), find the value of the constant a.

Specification reference (2.6):

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 1 is 3 marks)

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation $\cos\theta = 2\sin\theta$.

The attempts of two of the students are shown below.

Student A	Student B
$\cos\theta = 2\sin\theta$	$\cos\theta = 2\sin\theta$
$\tan\theta = 2$	$\cos^2\theta = 4\sin^2\theta$
$\theta = 63.4°$	$1 - \sin^2\theta = 4\sin^2\theta$
	$\sin^2\theta = \frac{1}{5}$
	$\sin\theta = \pm\frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(a) Identify an error made by student A.

Student B gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

Specification reference (5.5, 5.7):

Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$; Understand and use $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.

AO2.3: Assess the validity of mathematical arguments (1 mark)
AO2.4: Explain their reasoning (2 marks)

(Total for Question 2 is 3 marks)

3. Given $y = x(2x + 1)^4$, show that $\frac{dy}{dx} = (2x + 1)^n (Ax + B)$, where n, A and B are constants to be found.

Specification reference (7.2, 7.4):

Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.

Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)

(Total for Question 3 is 4 marks)

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R},$$

$$g(x) = 3 \ln x, \quad x > 0, x \in \mathbb{R},$$

(a) find an expression for gf (x), simplifying your answer.

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$.

Specification reference (1.1, 2.8, 6.3):

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion.

Understand and use composite functions; inverse functions and their graphs.

Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$. Know and use $\ln x$ as the inverse function of e^x

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

AO2.2a: Make deductions and inferences: make deductions (1 mark)

(Total for Question 4 is 5 marks)

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5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}.$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

Specification reference (6.2, 6.7):
Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.
Understand and use exponential growth and decay; use in modelling.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)
AO3.4: Use mathematical models (1 mark)

(Total for Question 5 is 4 marks)

6. For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case. The first one has been done for you.

> The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has 2 real roots.
>
> Sometimes true.
>
> It only has 2 real roots when $b^2 - 4ac > 0$.
> When $b^2 - 4ac < 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.

(i) When a real value of x is substituted into $x^2 - 6x + 10$, the result is positive.

(ii) If $ax > b$ then $x > \dfrac{b}{a}$.

(iii)The difference between consecutive square numbers is odd.

Specification reference (1.1, 2.3, 2.5):

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: disproof by counter example.

The discriminant of a quadratic function, including the conditions for real and repeated roots.

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically.

AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)

AO2.2a: Make deductions and inferences: make deductions (2 marks)

AO2.3: Assess the validity of mathematical arguments (1 mark)

AO2.4: Explain their reasoning (1 mark)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

Specification reference (4.1):
Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}^nC_r$ link to binomial probabilities.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)
AO2.4: Explain their reasoning (1 mark)

(Total for Question 7 is 5 marks)

8.

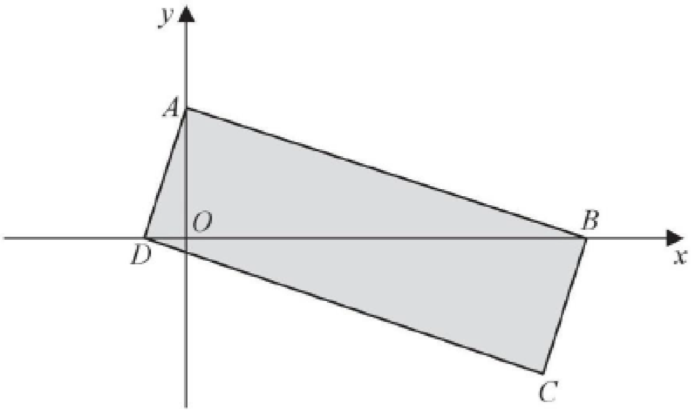

Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$,

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$,

(b) find the area of the rectangle *ABCD*.

Specification reference (3.1):
Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$
Gradient conditions for two straight lines to be parallel or perpendicular.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)
AO2.2a: Make deductions and inferences: make deductions (1 mark)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 8 is 7 marks)

9. Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A)\ \mathrm{d}x = 2A^2,$$

show that there are exactly two possible values for A.

Specification reference (2.3, 8.2, 8.3):
Solution of quadratic equations.
Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.
Evaluate definite integrals; use a definite integral to find the area under a curve.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.4: Explain their reasoning (1 mark)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 9 is 5 marks)

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10. In a geometric series the common ratio is r and sum to n terms is S_n.

Given $S_\infty = \frac{8}{7} \times S_6$, show that $1 = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

Specification reference (4.5):
Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$; modulus notation

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)

(Total for Question 10 is 4 marks)

11.

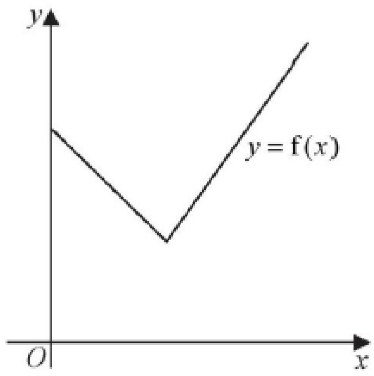

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geq 0$.

(a) State the range of f.

(b) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 30$.

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

Specification reference (2.5, 2.7, 2.9):

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically.

Understand and use graphs of functions; sketch curves defined by simple equations including polynomials. The modulus of a linear function.

Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$, including sketching associated graphs: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$ and combinations of these transformations.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)

AO2.2a: Make deductions and inferences: make deductions (1mark)

AO2.5: Use mathematical language and notation correctly (1 mark)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 11 is 6 marks)

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12. (a) Solve, for $-180° \leq x < 180°$, the equation $3\sin^2 x + \sin x + 8 = 9\cos^2 x$, giving your answers to 2 decimal places.

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°),$$

giving your answer to 2 decimal places.

Specification reference (2.3, 5.3, 5.5, 5.7):

Work with quadratic functions including solving quadratic equations in a function of the unknown.

Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

Understand and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; Understand and use $\sin^2\theta + \cos^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$

Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (6 marks)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (2 marks)

(Total for Question 12 is 8 marks)

13. (a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

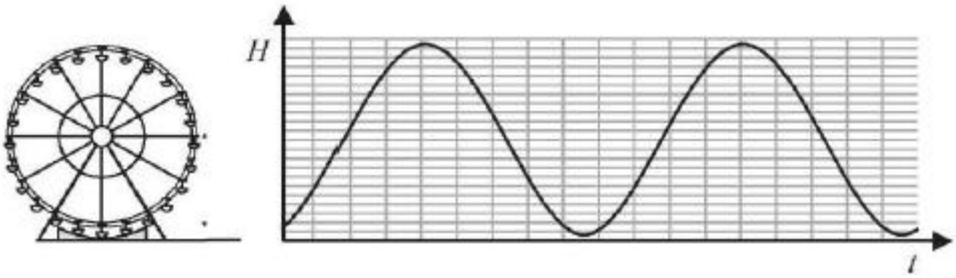

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation $H = a - 10 \cos (80t)° + 3 \sin (80t)°$, where a is a constant. Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model.

(ii) Hence find the maximum height of the passenger above the ground.

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

Specification reference (2.9, 5.3, 5.6, 5.9):

Understand the effect of simple transformations on the graph of $y = f(x)$, including sketching associated graphs.

Understand and use expressions for $a \cos\theta + b \sin\theta$ in the equivalent forms of $r \cos (\theta \pm \alpha)$ or $r \sin (\theta \pm \alpha)$

Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

AO3.3: Translate situations in context into mathematical models (2 marks)

AO3.4: Use mathematical models (2 marks)

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius *r* cm and height *h* cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \dfrac{1000}{r}$.

Given that *r* can vary,

(b) find the dimensions of a can that has minimum surface area.

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

Specification reference (1.1, 2.11, 7.2, 7.3):
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion.
Use of functions in modelling, including consideration of limitations and refinements of the models.
Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.
Apply differentiation to find gradients, tangents and normals; maxima and minima and stationary points. Identify where functions are increasing or decreasing.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (3 marks)
AO3.2a: Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)
AO3.4: Use mathematical models (1 mark)

(Total for Question 14 is 9 marks)

15.

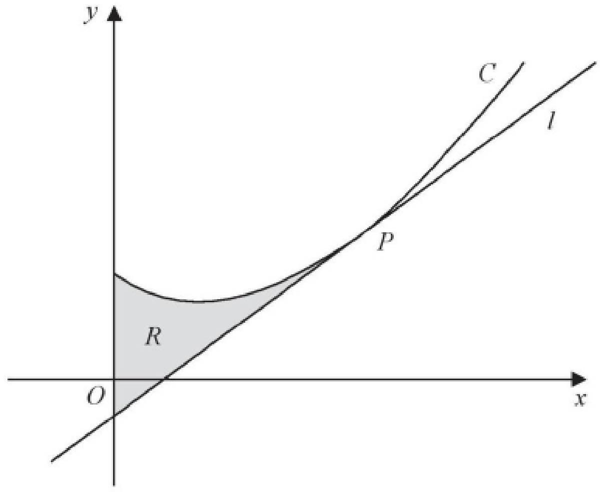

Figure 4

Figure 4 shows a sketch of the curve C with equation $y = 5x^{\frac{3}{2}} - 9x + 11, \quad x \geq 0$.

The point P with coordinates (4, 15) lies on C. The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Specification reference (3.1, 7.2, 7.3, 8.2, 8.3):

Understand and use mutually exclusive and independent events when calculating probabilities.

Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.

Apply differentiation to find gradients, tangents and normals; maxima and minima and stationary points. Identify where functions are increasing or decreasing.

Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (3 marks)

AO2.5: Use mathematical language and notation correctly (1 mark)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (2 marks)

(Total for Question 15 is 10 marks)

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16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}},$$

where A, B and C are integers to be found.

Specification reference (2.1, 6.3, 6.4, 8.6, 8.7, 8.8):

Understand and use the laws of indices for all rational exponents.

Know and use ln x as the inverse function of e^x

Understand and use the laws of logarithms:

$\log_a x + \log_a y = \log_a(xy)$

$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$

Integrate using partial fractions that are linear in the denominator.

Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)

Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

AO1.1a: Select and correctly carry out routine procedures: select routine procedures (1 mark)

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (3 marks)

AO3.2a: Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)

(Total for Question 16 is 12 marks)

A level Mathematics – Paper 3 (Applied)

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leq y < 5$	$5 \leq y < 8$	$8 \leq y < 11$	$11 \leq y < 12$	$12 \leq y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leq y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leq y < 5$ group.

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures.

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

Specification reference (2.1, 2.3, 2.4, 4.2, 4.3): Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency. Connect to probability distributions. Interpret measures of central tendency and variation, extending to standard deviation. Be able to calculate standard deviation, including from summary statistics. Select or critique data presentation techniques in the context of a statistical problem. Understand and use the Normal distribution as a model; find probabilities using the Normal distribution. Link to histograms, mean, standard deviation, points of inflection. Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate.

AO1.1a: Select and correctly carry out routine procedures: select routine procedures (1 mark) **AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (7 marks) **AO2.2b:** Make deductions and inferences: make inferences (1 mark) **AO2.4:** Explain their reasoning (1 mark) **AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark) **AO3.4:** Use mathematical models (1 mark) **AO3.5a:** Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: evaluate the outcomes of modelling in context (1 mark)

(Total for Question 1 is 13 marks)

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$.

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C.

(b) State what is measured by the product moment correlation coefficient.

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

Using the same 9 days, a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$.

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

Specification reference (2.2, 5.1):

Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded).

Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; extend to correlation coefficients as measures of how close data points lie to a straight line.

Be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded).

AO1.1a: Select and correctly carry out routine procedures: select routine procedures (1 mark)

AO1.2: Accurately recall facts, terminology and definitions (2 marks)

AO2.2b: Make deductions and inferences: make inferences (1 mark)

AO2.4: Explain their reasoning (1 mark)

AO2.5: Uses mathematical language and notation correctly (1 mark)

(Total for Question 2 is 6 marks)

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips cannot be used.

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm.

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm.

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm.

Specification reference (4.1, 4.2, 4.3, 5.3):

Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.

Understand and use the Normal distribution as a model; find probabilities using the Normal distribution.

Link to histograms, mean, standard deviation, points of inflection and the binomial distribution.

Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate.

Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

AO2.2b: Make deductions and inferences: make inferences (1 mark)

AO2.5: Uses mathematical language and notation correctly (1 mark)

AO3.3: Translate situations in context into mathematical models (2 marks)

AO3.4: Use mathematical models (3 marks)

(Total for Question 3 is 12 marks)

4. Given that

$$\mathrm{P}(A) = 0.35,\ \ \mathrm{P}(B) = 0.45\ \text{ and }\ \mathrm{P}(A \cap B) = 0.13$$

find

(a) $\mathrm{P}(A' \mid B')$

(b) Explain why the events A and B are not independent.

The event C has $\mathrm{P}(C) = 0.20$.

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(d) Find $\mathrm{P}([B \cup C]')$

Specification reference (3.1, 3.2):

Understand and use mutually exclusive and independent events when calculating probabilities.

Link to discrete and continuous distributions.

Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables.

Understand and use the conditional probability formula $\mathrm{P}(A / B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (6 marks)

AO2.4: Explain their reasoning (1 mark)

AO2.5: Uses mathematical language and notation correctly (1 mark)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (2 marks)

(Total for Question 4 is 10 marks)

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

Specification reference (4.1, 4.2, 5.2):

Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.

Understand and use the Normal distribution as a model; find probabilities using the Normal distribution.

Link to histograms, mean, standard deviation, points of inflection and the binomial distribution.

Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

AO1.2: Accurately recall facts, terminology and definitions (1 mark)

AO2.2b: Make deductions and inferences: make inferences (1 mark)

AO2.4: Explain their reasoning (1 mark)

AO3.3: Translate situations in context into mathematical models (1 mark)

AO3.4: Use mathematical models (1 mark)

(Total for Question 5 is 9 marks)

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6. At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}\,.$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

Specification reference (6.1, 7.1, 7.4):

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Use calculus in kinematics for motion in a straight line: $v = \frac{\mathrm{d}r}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$, $r = \int v\,\mathrm{d}t$, $v = \int a\,\mathrm{d}t$

Extend to 2 dimensions using vectors.

AO1.1a: Select and correctly carry out routine procedures: select routine procedures (1 mark)

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems into mathematical contexts into mathematical processes (1 mark)

(Total for Question 6 is 6 marks)

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

Specification reference (6.1, 7.1, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6):

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Understand the concept of a force; understand and use Newton's first law.

Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); extend to situations where forces need to be resolved (restricted to 2 dimensions).

Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy.

Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

Understand and use addition of forces; resultant forces; dynamics for motion in a plane.

Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)

AO1.2: Accurately recall facts, terminology and definitions (1 mark)

AO2.2a: Make deductions and inferences: make deductions (2 marks)

AO3.1b: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (3 marks)

(Total for Question 7 is 8 marks)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(b) Find **r** in terms of t.

(c) Find the value of t when the boat is north-east of O.

(d) Find the value of t when the boat is moving in a north-east direction.

Specification reference (6.1, 7.1, 7.3):

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Understand, use and derive the formulae for constant acceleration for motion in a straight line.

Extend to 2 dimensions using vectors.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

AO3.1b: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (5 marks)

(Total for Question 8 is 10 marks)

9.

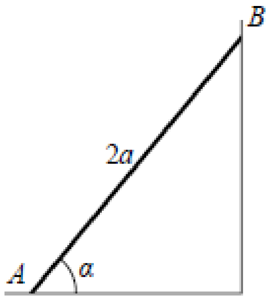

Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.
The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.
The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

Specification reference (6.1, 8.4, 8.5, 8.6, 9.1):
Understand and use fundamental quantities and units in the S.I. system: length, time, mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.
Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.
Understand and use addition of forces; resultant forces; dynamics for motion in a plane.
Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.
Understand and use moments in simple static contexts.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)
AO2.2a: Make deductions and inferences: make deductions (1 mark)
AO2.4: Explain their reasoning (3 marks)
AO2.5: Uses mathematical language and notation correctly (1 mark)
AO3.3: Translate situations in context into mathematical models (1 mark)
AO3.4: Use mathematical models (3 marks)

(Total for Question 9 is 13 marks)

10.

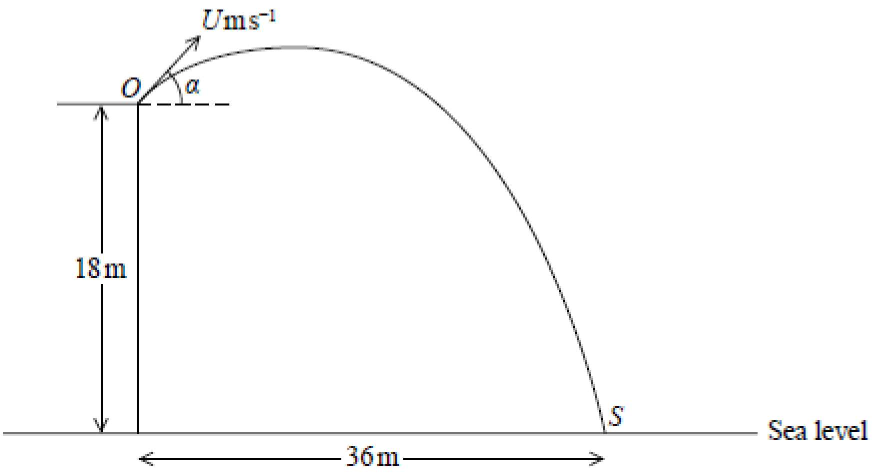

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with g = 10 m s^{-1}.

Find

(a) the value of U,

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(c) Suggest two improvements that could be made to the model.

Specification reference (6.1, 7.1, 7.5):
Understand and use fundamental quantities and units in the S.I. system: length, time, mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight, moment.
Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.
Model motion under gravity in a vertical plane using vectors; projectiles.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)
AO3.1b: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (2 marks)
AO3.4: Use mathematical models (4 marks)
AO3.5c: Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: where appropriate, explain how to refine models (2 marks)

(Total for Question 10 is 13 marks)