# Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8MA0)

# Pearson Edexcel Level 3 Advanced GCE in Mathematics (9MA0)

## Sample Assessment Materials Exemplar answers with examiner comments – Pure Mathematics

First teaching from September 2017
First certification from June 2018

# Contents

# About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Level 3 Advanced GCE in Mathematics specification (8MA0 & 9MA0).

The booklet looks at questions from the AS and A level Sample Assessment Materials, which was used in the trial undertaken in summer 2017. It shows real student responses to questions, and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

## How to use this booklet

Our examining team have selected student responses to all questions from the trial of the Sample Assessment Materials. Following each question you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

### Student Response B

Student response

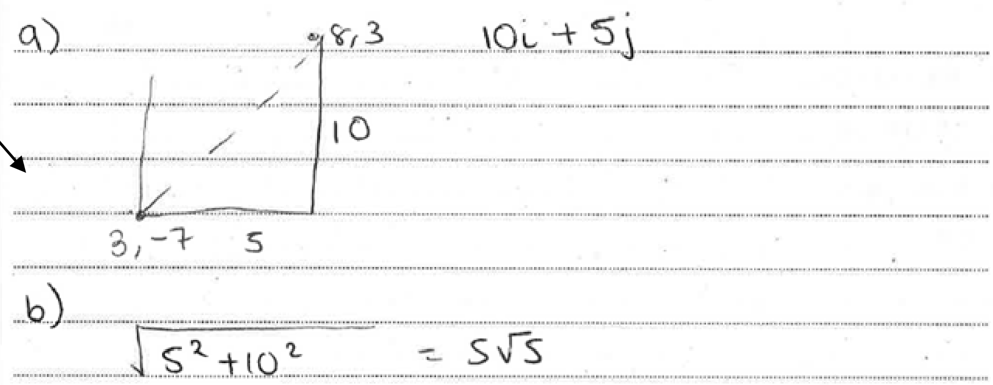
**3.** Given that the point $A$ has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)




3/4

Examiner Comments:

Part (a)

M1: The diagram suggests that the candidate has subtracted the vectors (or at least knows how to approach the problem). The error is notational, in respect that there is a mix up between the **i** and **j** components.

A0: Answer incorrect

Part (b)

M1: For the attempt to use Pythagoras implied by $\sqrt{5^2 + 10^2}$

A1: For the correct answer

Examiner commentary on the student response

Marks awarded for the question or question parts

# AS Mathematics Paper 1 (Pure)

## Exemplar question 1

**1.** The line $l$ passes through the points $A$ (3, 1) and $B$ (4, – 2).

Find an equation for $l$.

**(3)**

**(Total for Question 1 is 3 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1** **Way 1** | Uses $y = mx + c$ with both (3, 1) and (4, −2) and attempt to find $m$ or $c$ | M1 | 1.1b |
| | $m = -3$ | A1 | 1.1b |
| | $c = 10$ so $y = -3x + 10$ o.e. | A1 | 1.1b |
| | | **(3)** | |
| **Or** **Way 2** | Uses $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ with both (3, 1) and (4, −2) | M1 | 1.1b |
| | Gradient simplified to $-3$ (may be implied) | A1 | 1.1b |
| | $y = -3x + 10$ o.e. | A1 | 1.1b |
| | | **(3)** | |
| **Or** **Way 3** | Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find $a$, $b$ or $k$ in terms of one of them | M1 | 1.1b |
| | Obtains $a = 3b$, $k = -10b$ or $3k = -10a$ | A1 | 1.1b |
| | Obtains $a = 3$, $b = 1$, $k = -10$ or writes $3x + y - 10 = 0$ o.e. | A1 | 1.1b |
| | | **(3)** | |
| | | **(7 marks)** | |

**Notes:**

**M1:** Need correct use of the given coordinates

**A1:** Need fractions simplified to $-3$ (in ways 1 and 2)

**A1:** Need constants combined accurately

**N.B.** **Answer left in the form $(y - 1) = -3(x - 3)$ or $(y - (-2)) = -3(x - 4)$ is awarded M1A1A0 as answers should be simplified by constants being collected**

*Note that a correct answer implies all three marks in this question*

## Student Response A

**1.** The line $l$ passes through the points $A$ (3, 1) and $B$ (4, −2).

Find an equation for $l$.

**(3)**

$$\frac{-2-4}{1-3} = 3$$

$y = 3x + c$

$1 = 3(3) + c$

$1 = 9 + c \qquad c = -8$

$\therefore y = 3x - 8$

***0/3***

Examiner Comments:

The candidate attempts to solve the question by Way 2.

M0: For the attempt to find the gradient by calculating $\frac{y_2 - x_2}{y_1 - x_1}$. This is an incorrect method.

As M0 has been awarded, A0, A0 follow.

## Student Response B

1. The line $l$ passes through the points $A$ (3, 1) and $B$ (4, –2).

Find an equation for $l$.

**(3)**

$y = mx + c$

length of $l$: $\frac{-3}{1}$

m

$m = -3$

$y = -3x + c$

sub (3, 1):

$1 = (-3 \times 3) + c$

$1 = -9 + c$

$c = -8$

$y = -3x - 8$

***2/3***

Examiner Comments:

This candidate also attempts to solve the question by Way 2.

M1 A1: For writing down $m = -3$ which scores first two marks as the method can be implied by the correct answer.

A0: The answer is incorrect due to a slip on the penultimate line.

## Student Response C

**1.** The line $l$ passes through the points $A$ (3, 1) and $B$ (4, −2).

Find an equation for $l$.

**(3)**

grdt $= \frac{-2-1}{4-3} = \frac{-3}{1} = -3$

$y = -3x + c$

$1 = -9 + c$

$10 = c$

$y = -3x + 10$

***3/3***

Examiner Comments:

Also attempted by Way2. (This will probably be the most popular method)

M1: Awarded for gr $= \frac{-2-1}{4-3}$

A1: It is simplified correctly to $-3$ scoring A1.

A1: The candidate proceeds to a correct answer in the correct form. For $y = -3x + 10$

## Exemplar question 2

**2.** The curve C has equation

$$y = 2x^2 - 12 + 16.$$

Find the gradient of the curve at the point $P(5, 6)$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(4)**

**(Total for Question 2 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2** | Attempt to differentiate | M1 | 1.1a |
| | $\frac{dy}{dx} = 4x - 12$ | A1 | 1.1b |
| | Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = \ldots$ | M1 | 1.1b |
| | $\Rightarrow \frac{dy}{dx} = 8$ | A1ft | 1.1b |
| | | | **(4 marks)** |

**Notes:**

**M1:** Differentiation implied by one correct term

**A1:** Correct differentiation

**M1:** Attempts to substitute $x = 5$ into their derived function

**A1ft:** Substitutes $x = 5$ into **their** derived function correctly i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

## Student Response A

**2.** The curve $C$ has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P$ (5, 6).

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(4)**

$y = 2x^2 - 12x + 16$

$\frac{dy}{dx} = 2x - 12$

$x = 6$ gradient $= 2 \times 6 - 12$

$= 0$

***1/4***

Examiner Comments:

M1: Awarded as the candidate differentiates and has one correct term.

A0: $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 4x - 12$

M0: For substituting $x = 6$ (which is the $y$ coordinate).

A0: Follows M0

## Student Response B

**2.** The curve $C$ has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P$ (5, 6).

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(4)**

$\frac{dy}{dx} = 4x^2 - 12$

$4(5)^2 - 12$

$= 4 \times 25 - 12 = 88$

***3/4***

Examiner Comments:

M1: Awarded as the candidate differentiates and has one correct term.

A0: $\frac{dy}{dx} \neq 4x - 12$

M1: For substituting $x = 5$ into their $\frac{dy}{dx}$

A1ft: For a correct follow through. (This can be awarded following a slip on differentiation)

## Student Response C

**2.** The curve $C$ has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P$ (5, 6).

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(4)**

$y = 2x^2 - 12x + 16$

$\frac{dy}{dx} = 4x - 12$

$4(5) - 12$

$= 20 - 12 = 8$

**4/4**

Examiner Comments:

M1: Differentiating implied by one correct term.

A1: Fully correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 12$

M1: For substituting $x = 5$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: For a correct answer.

## Exemplar question 3

**3.** Given that the point $A$ has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

**(2)**

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

**(2)**

**(Total for Question 3 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar | M1 | 1.1b |
| | $\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Finds length using 'Pythagoras' $\lvert AB\rvert = \sqrt{(5)^2 + (10)^2}$ | M1 | 1.1b |
| | $\lvert AB\rvert = 5\sqrt{5}$ | A1ft | 1.1b |
| | | **(2)** | |
| | | | **(4 marks)** |

**Notes:**

**(a)**

**M1:** Attempts subtraction but may omit brackets

**A1:** cao (allow column vector notation)

**(b)**

**M1:** Correct use of Pythagoras theorem or modulus formula using their answer to (a)

**A1ft:** $\lvert AB\rvert = 5\sqrt{5}$ ft from their answer to (a)

*Note that the correct answer implies M1A1 in each part of this question*

## Student Response A

3. Given that the point $A$ has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

**(2)**

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

**(2)**

a. $3i - 7j$

$8i + 3j$

$(3i - 7j) + (8i + 3j)$

$= 11i - 4j$

b. $\sqrt{11i - 4j}$

***0/4***

Examiner Comments:

Part (a)

M0: The candidate adds the vectors. Subtraction must be attempted for the mark to be awarded.

A0: Follows M0

Part (b)

M0: There is no attempt to use Pythagoras on their answer to part (a).

A0: Follows M0

## Student Response B

3. Given that the point $A$ has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j}$,

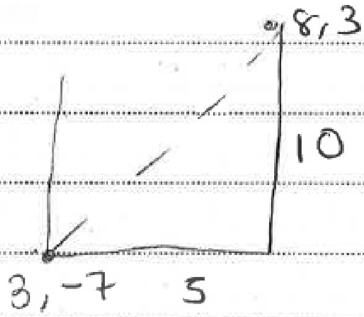
(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

a) $10i + 5j$




b) $\sqrt{5^2 + 10^2} = 5\sqrt{5}$

**3/4**

Examiner Comments:

Part (a)

M1: The diagram suggests that the candidate has subtracted the vectors (or at least knows how to approach the problem). The error is notational, in respect that there is a mix up between the **i** and **j** components.

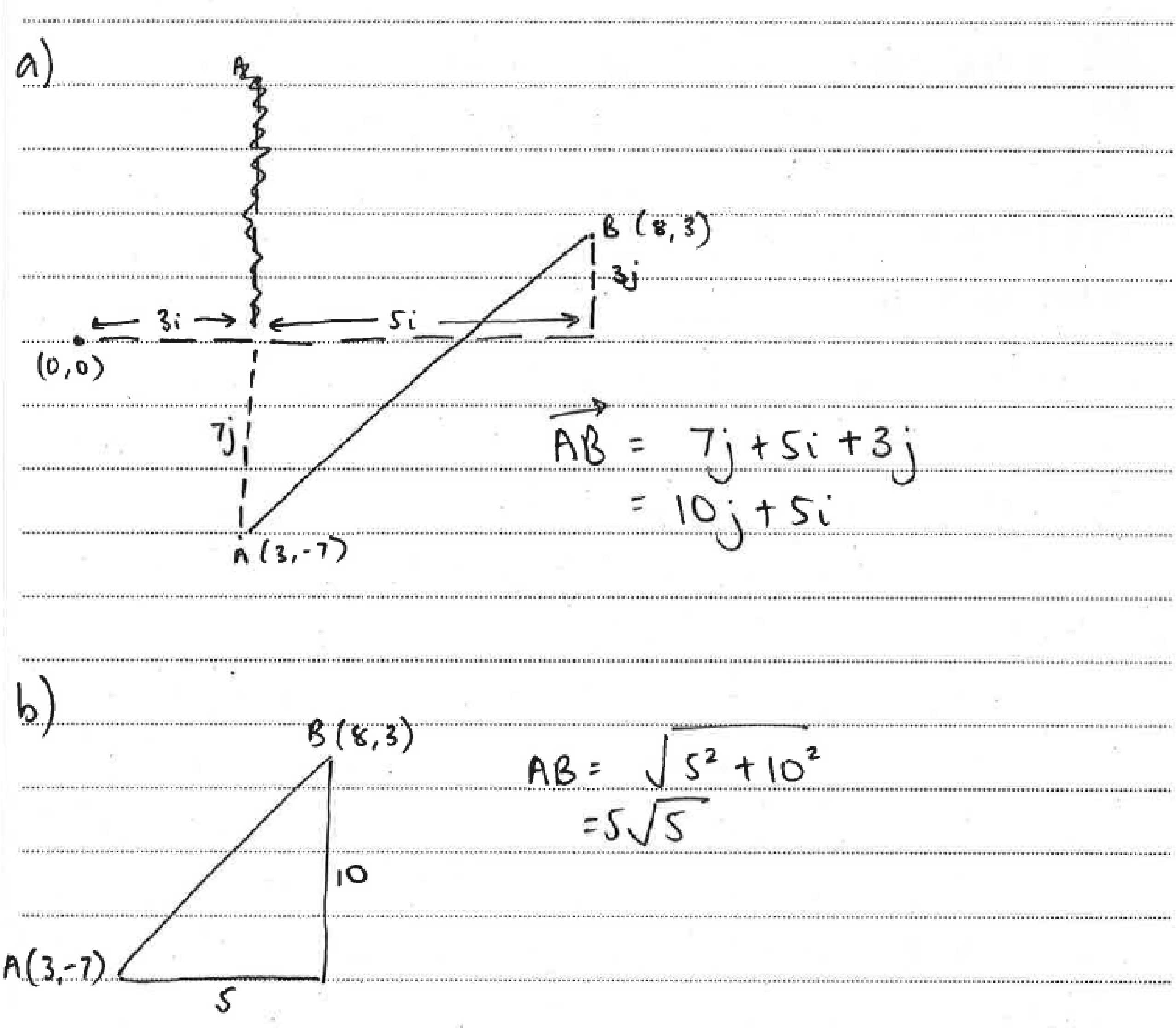
A0: Answer incorrect

Part (b)

M1: For the attempt to use Pythagoras implied by $\sqrt{5^2 + 10^2}$

A1: For the correct answer

## Student Response C




**4/4**

Examiner Comments:

Part (a)

M1: A good diagram clearly helps in understanding the problem. This candidate can be awarded this mark for either component and it is scored at the point where $\overrightarrow{AB} = 7\mathbf{j} + 5\mathbf{i} + 3\mathbf{j}$ is seen.

A1: Answer correct

Part (b)

M1: For the attempt to use Pythagoras implied by $\sqrt{5^2 + 10^2}$

A1: For the correct answer

## Exemplar question 4

**4.** $$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

**(2)**

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$.

**(4)**

**(Total for Question 4 is 6 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | States or uses $f(+3) = 0$ | M1 | 1.1b |
| | $4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Begins division or factorisation so $x$<br>$4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + \ldots)$ | M1 | 2.1 |
| | $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + 2)$ | A1 | 1.1b |
| | Considers the roots of their quadratic function using completion of square or discriminant | M1 | 2.1 |
| | $(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all $x$<br>So $x = 3$ is the only real root of $f(x) = 0$ * | A1* | 2.4 |
| | | **(4)** | |
| | | | **(6 marks)** |

**Notes:**

**(a)**
**M1:** States or uses $f(+3) = 0$
**A1:** See correct work evaluating and achieving zero, together with correct conclusion

**(b)**
**M1:** Needs to have $(x - 3)$ and first term of quadratic correct
**A1:** Must be correct – may further factorise to $2(x - 3)(2x^2 + 1)$
**M1:** Considers their quadratic for no real roots by use of completion of the square or consideration of discriminant then
**A1*:** A correct explanation

## Student Response A

**4.**

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

**(2)**

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

**(4)**

4 a) $f(3) = 4(3)^3 - 12(3)^2 + 2(3) - 6 = 0$

b) $x - 3 \overline{\smash{)}\,4x^3 - 12x^2 + 2x - 6}$ (quotient $4x^2$)

$4x^3 - 12x$

$0 + 2x - 6$

$0$

***2/6***

Examiner Comments:

Part (a)

M1: Scored for attempting to calculate $f(3)$.

A0: Although $f(3) = 0$ stated, there is no conclusion, so this mark cannot be awarded.

Part (b)

M1: For attempting to divide and achieves the correct first term $4x^2$

A0: Incomplete division.

M0: There is no quadratic term so the candidate cannot consider the roots.

A0: Follow M0

## Student Response B

a)

if $(x-3)$ is factor $r=0$

$f(3)=0$

$f(3) = 4(3)^3 - 12(3)^2 + 2(3) - 6$

$f(3) = 108 - 108 + 6 - 6$

$f(3) = 0$

$\therefore$ it is a factor

b)

$$(x-3)\overline{)\,4x^3 - 12x^2 + 2x - 6\,}$$ quotient $4x^2 + 2$

$- \; 4x^3 - 12x^2$

$0 + 2x$

$- \; 2x - 6$

$0$

1.) $4x^3 \div x$

$= 4x^2$

$4x^2(x-3)$

$4x^3 - 12x^2$

2.) $2x \div x = 2$

$2(x-3) = 2x - 6$

$\therefore (x-3)\;4x^2 + 2$

$4x^2 + 0x + 2$

$b^2 - 4ac < 0 \;\therefore$ no solutions

**(Total for Question 4 is 6 marks)**

***4/6***

Examiner Comments:

Part (a)

M1: Scored for attempting to calculate $\mathrm{f}(3)$.

A1: For stating $\mathrm{f}(3)=0$ and a valid conclusion "it is a factor".

Part (b)

M1: The candidate attempts to divide and achieves the correct first term $4x^2$

A1: Correct division with factor $4x^2+2$

M0: Whilst the candidate does state $'b^2-4ac'<0$ , it is important that it is shown how it applies to their quadratic. A simple statement such as $'b^2-4ac'=0^2-4\times4\times2$ would suffice.

A0: Follow M0

## Student Response C

4a) $f(3) = 4(3^3) - 12(3^2) + 2(3) - 6$

$= 108 - 108 + 6 - 6 = 0 \quad \therefore (x-3)$ is a factor of $f(x)$

b)
$$\begin{array}{c|ccc} & 4x^2 & +0 & +2 \\ \hline x & 4x^3 & +0x^2 & +2x \\ -3 & -12x^2 & +0 & -6 \end{array}$$

$\Rightarrow (x-3)(4x^2+2) = 0$

$4x^2 + 2 \Rightarrow a = 4, b = 0, c = 2$

$b^2 - 4ac = 0 - (4 \times 4 \times 2) = -32$

$\therefore (4x^2+2)$ has no real solutions so 3 is the only real root of $f(x) = 0$.

***5/6***

Examiner Comments:

Part (a)

M1: Scored for attempting to calculate $f(3)$.

A1: For stating $f(3) = 0$ and a valid conclusion "$(x-3)$ is a factor of f($x$)".

Part (b)

M1: The candidate attempts to divide and achieves the correct first term $4x^2$

A1: Correct division with factor $4x^2 + 2$

M1: This candidate attempt to find '$b^2 - 4ac$', for their quadratic.

Scored for '$b^2 - 4ac$' $= 0^2 - 4 \times 4 \times 2$ .

A0: Although a correct statement '$b^2 - 4ac$' $= -32$ and a correct conclusion are written down, the candidate do not have the reason. There must be a statement pointing out that there is no roots to the quadratic because $b^2 - 4ac < 0$

# Exemplar question 5

**5.** Given that $f(x) = 2x + 3 + \dfrac{12}{x^2}, \quad x > 0,$

show that $\displaystyle\int_1^{2\sqrt{2}} f(x)\ dx = 16 + 3\sqrt{2}.$

**(5)**

**(Total for Question 5 is 5 marks)**

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# Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5** | $f(x) = 2x + 3 + 12x^{-2}$ | B1 | 1.1b |
| | Attempts to integrate | M1 | 1.1a |
| | $\displaystyle\int\left(+2x+3+\frac{12}{x^2}\right)dx = x^2+3x-\frac{12}{x}$ | A1 | 1.1b |
| | $\left((2\sqrt{2})^2+3(2\sqrt{2})-\dfrac{12(\sqrt{2})}{2\times 2}\right)-(-8)$ | M1 | 1.1b |
| | $=16+3\sqrt{2}$ * | A1* | 1.1b |
| | | | **(5 marks)** |

**Notes:**

**B1:** Correct function with numerical powers

**M1:** Allow for raising power by one. $x^n \to x^{n+1}$

**A1:** Correct three terms

**M1:** Substitutes limits and rationalises denominator

**A1*:** Completely correct, no errors seen

## Student Response A

$$f(x) = 2x + 3 - 12x^{-2}$$

$$\int 2x + 3 - 12x^{-2}\, dx = x^{2} + 3x + 12x^{-1}$$

$$\left[x^{2} + 3x + 12x^{-1}\right]_{1}^{2\sqrt{2}} =$$

$$= 2\sqrt{2}^{2} + 3(2\sqrt{2}) + 12(2\sqrt{2})^{-1} - (1 + 3 + 24)$$

$$= 8 + 6\sqrt{2} + 3\sqrt{2} - 28$$

$$= 16 + 3\sqrt{2}$$

**2/5**

Examiner Comments:

B0: The candidate makes an error on the coefficient of the $x^{-2}$ term, hence B0.

M1: For attempting to integrate by raising the power of an index by one. This can be awarded for any of the three terms.

A0: This is incorrect due to the earlier error. (It is not a follow through mark)

M1: For substituting in the limits and rationalising the denominator (implied by the sight of $3\sqrt{2}$ )

A0*: Although $16+3\sqrt{2}$ is written down, this mark cannot be scored due to the earlier error.

## Student Response B

$$\int 2x + 3 + 12x^{-2} \, dx$$

$$= \frac{2x^2}{2} + 3x + 12x^{-1} + c$$

$$= \left[ x^2 + 3x + \frac{12}{x} \right]_1^{2\sqrt{2}}$$

$$= \left( (2\sqrt{2})^2 + 3(2\sqrt{2}) + \frac{12}{(2\sqrt{2})} \right) - \left( 1^2 + 3 + \frac{12}{1} \right)$$

$$= 8 + 6\sqrt{2} + 3\sqrt{2} - (16)$$

$$= 16 + 3\sqrt{2}$$

**3/5**

Examiner Comments:

B1: For the correct function with numerical powers.

M1: For attempting to integrate by raising the power of an index by one. This can be awarded for any of the three terms.

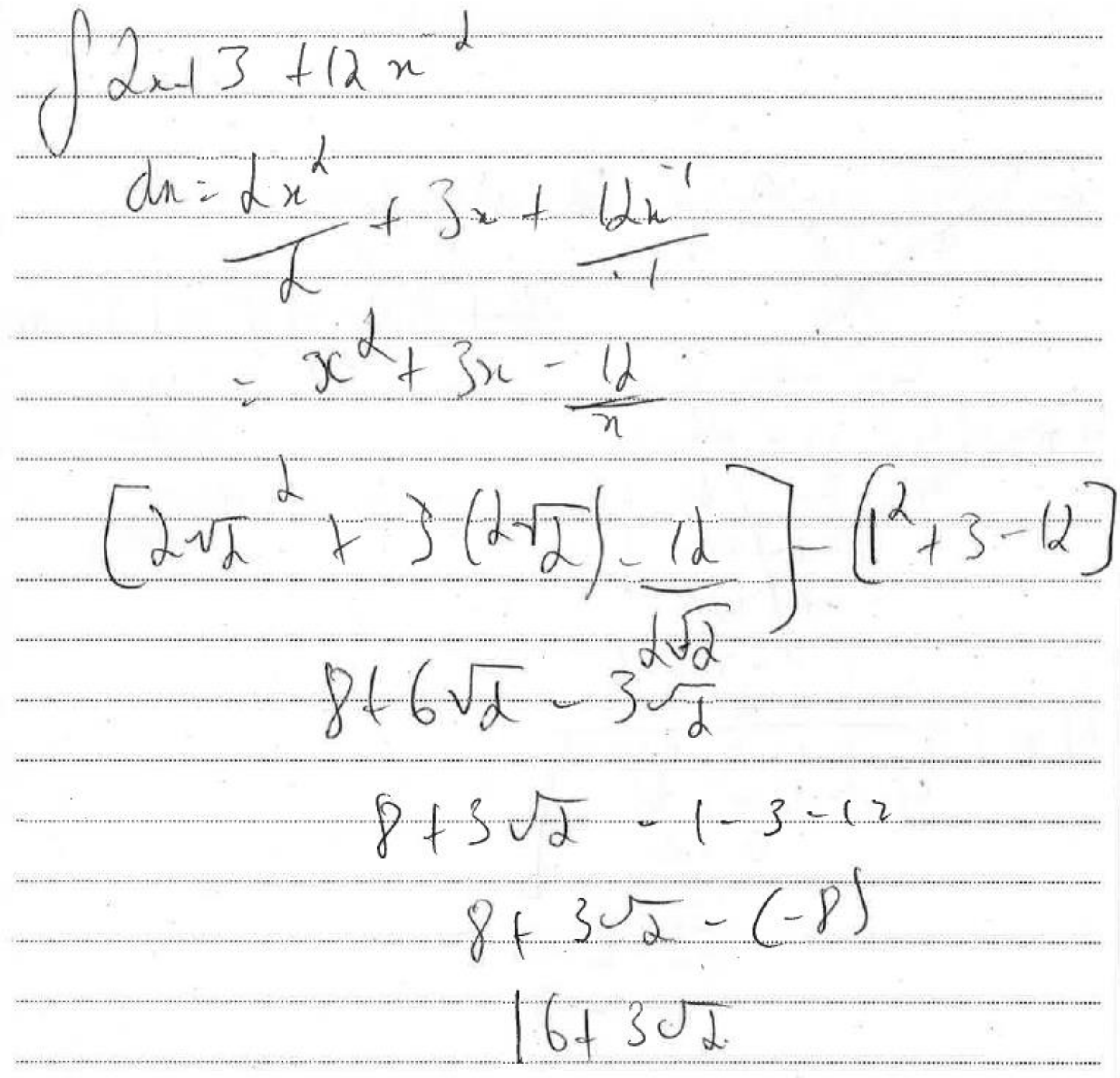
A0: This is incorrect due to the sign of the last term.

M1: For substituting in the limits and rationalising the denominator (implied by the sight of $3\sqrt{2}$).

NB. Calculators are allowed on this paper and it is acceptable for this to be just written down.

A0*: Although $16+3\sqrt{2}$ is written down, this mark cannot be scored due to the earlier error.

## Student Response C



**4/5**

Examiner Comments:

B1: For the correct function with numerical powers.

M1: For attempting to integrate by raising the power of an index by one. This can be awarded for any of the three terms.

A1: Correct three terms.

M1: For substituting in the limits and rationalising the denominator (implied by the sight of $3\sqrt{2}$).

A0*: Although $16+3\sqrt{2}$ is written down, this mark cannot be scored due to the error on the third last line. This should read $8+3\sqrt{2}-1-3+12$. In a "show that " question it is important that all lines are correct.

## Exemplar question 6

**6.** Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**

**(Total for Question 6 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6 | Considers $\frac{3(x+h)^2 - 3x^2}{h}$ | B1 | 2.1 |
| | Expands $3(x + h)^2 = 3x^2 + 6xh + 3h^2$ | M1 | 1.1b |
| | So gradient $= \frac{6xh + 3h^2}{h} = 6x + 3h$ or $\frac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$ | A1 | 1.1b |
| | States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ * | A1* | 2.5 |
| | | | **(4 marks)** |

**Notes:**

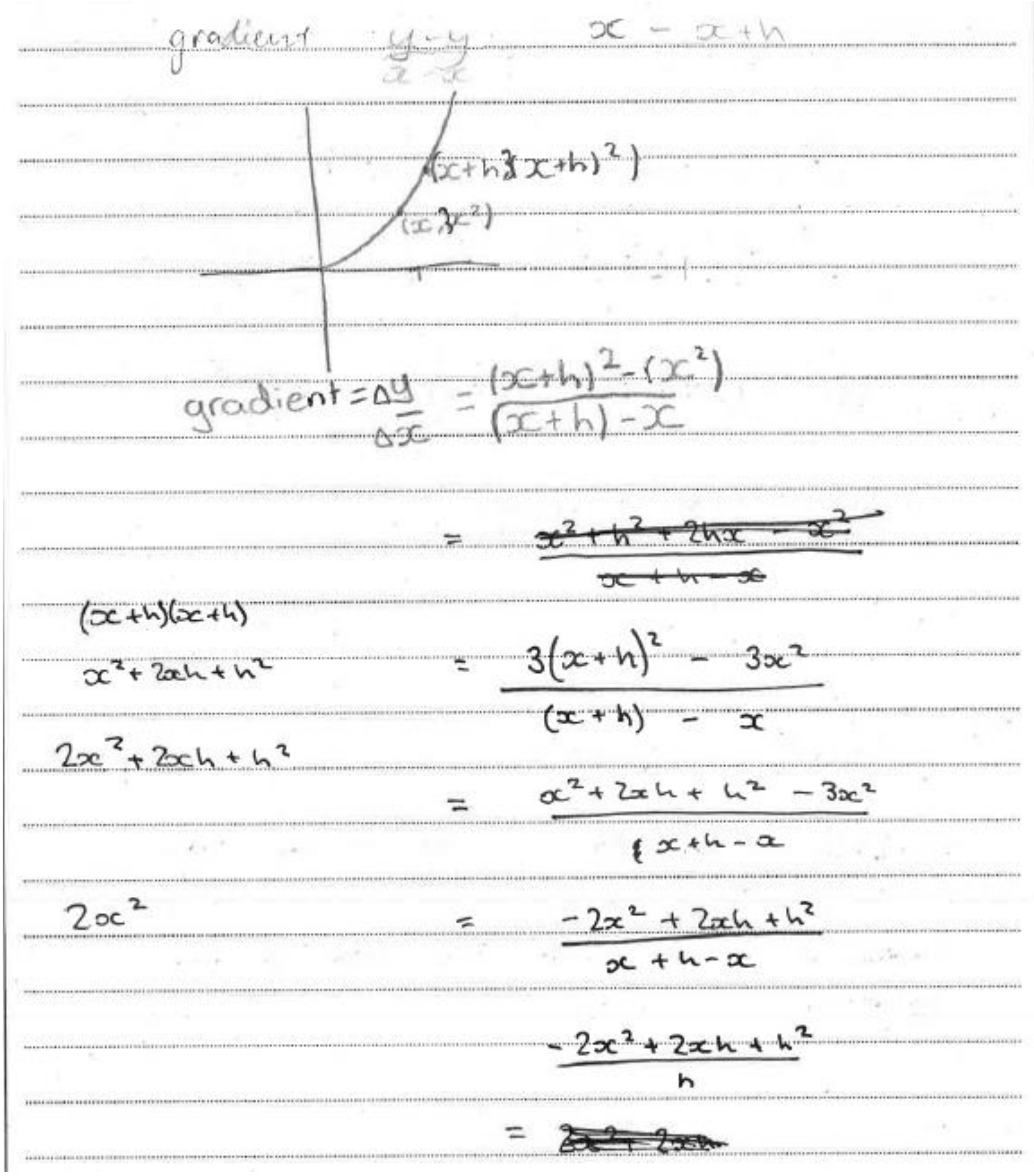
**B1:** Gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2 - 3x^2}{\delta x}$

**M1:** Expands the bracket as above or $3(x+\delta x)^2 = 3x^2 + 6x\delta x + 3(\delta x)^2$

**A1:** Substitutes correctly into earlier fraction and simplifies

**A1*:** Uses Completes the proof, as above ( may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

## Student Response A



gradient $\frac{y-y}{x-x}$ $x \to x+h$

$(x+h, (x+h)^2)$

$(x, x^2)$

gradient $= \frac{\Delta y}{\Delta x} = \frac{(x+h)^2-(x^2)}{(x+h)-x}$

$=$ ~~$\frac{x^2+h^2+2hx-x^2}{x+h-x}$~~

$(x+h)(x+h)$

$x^2+2xh+h^2$ $\quad = \frac{3(x+h)^2 - 3x^2}{(x+h) - x}$

$2x^2+2xh+h^2$ $\quad = \frac{x^2+2xh+h^2-3x^2}{x+h-x}$

$2x^2$ $\quad = \frac{-2x^2+2xh+h^2}{x+h-x}$

$\frac{-2x^2+2xh+h^2}{h}$

$=$ ~~$2x^2+2xh$~~

***1/4***

Examiner Comments:

B1: Scored where this candidate writes $\frac{3(x+h)^2-3x^2}{(x+h)-x}$

M0: The mark is not scored as it is required for $3(x+h)^2$ to be expanded. This candidate expands $(x+h)^2$

A0: Follows a previous M0.

A0*: Must follow any error.

## Student Response B

6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**

$y = 3x^2 \quad (x, 3x^2)$

$(x+h, 3(x+h)^2)$

$\lim_{h \to 0} \dfrac{3(x+h)^2 - 3x^2}{x+h-x} = \dfrac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$

$\dfrac{6xh + 3h^2}{h} = 6x + 3h$

set $h$ to $0$

$6x + 3(0) = 6x$

*3/4*

Examiner Comments:

B1: Scored where this candidate writes $\dfrac{3(x+h)^2 - 3x^2}{x+h-x}$

M1: $3(x+h)^2$ is expanded correctly within the expression $\dfrac{3(x+h)^2 - 3x^2}{x+h-x}$

A1: Having expanded $3(x+h)^2$ correctly, the candidate correctly simplifies $\dfrac{3(x+h)^2 - 3x^2}{x+h-x}$ to $6x + 3h$

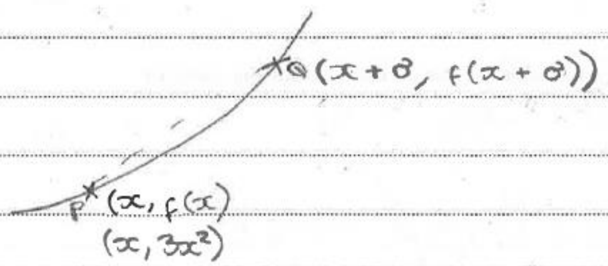
A0*: Final mark is not (quite) achieved. The candidate merely states that when $h$ is set to 0, $6x + 3(0) = 6x$

There must be some reference to the fact that the gradient of the chord tends to the gradient of the curve, meaning that "the derivative of $3x^2$ is $6x$."

## Student Response C

**6.** Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**




gradient of PQ $= \dfrac{f(x+\delta) - f(x)}{x+\delta - x}$

$= \dfrac{3(x+\delta)^2 - 3x^2}{\delta}$

$= \dfrac{3(x^2+2\delta x+\delta^2) - 3x^2}{\delta}$

$= \dfrac{3x^2 + 6\delta x + 3\delta^2 - 3x^2}{\delta}$

$= \dfrac{6\delta x + 3\delta^2}{\delta}$

$= 6x + 3\delta$    $\delta$ converges towards zero until it gets so small that it can be ignored therefore the derivative is $6x$

$= 6x$

***4/4***

Examiner Comments:

B1: Scored where this candidate writes $\dfrac{3(x+\delta)^2 - 3x^2}{x+\delta - x}$

Using $\delta$ may be considered unusual but it is consistent with how the candidate annotated their diagram.

M1: $3(x+\delta)^2$ is expanded correctly within the expression $\dfrac{3(x+\delta)^2 - 3x^2}{x+\delta - x}$

A1: Having expanded $3(x+\delta)^2$ correctly, the candidate correctly simplifies $\dfrac{3(x+\delta)^2 - 3x^2}{\delta}$ to $6x + 3\delta$

A1*: Final mark is achieved. The candidate work is correct and statement that when $\delta$ converges to zero, the gradient of *PQ* = 6*x* and the derivative is (therefore) 6*x* is seen.

## Exemplar question 7

**7.** (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

**(4)**

(b) Explain how you would use your expansion to give an estimate for the value of $1.995^7$.

**(1)**

**(Total for Question 7 is 5 marks)**

---

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right)+\binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2+\ldots$ | M1 | 1.1b |
| | $\left(2-\frac{x}{2}\right)^7 = 128+\ldots$ | B1 | 1.1b |
| | $\left(2-\frac{x}{2}\right)^7 = \ldots -224x +\ldots$ | A1 | 1.1b |
| | $\left(2-\frac{x}{2}\right)^7 = \ldots +\ldots + 168x^2 \ (+\ldots)$ | A1 | 1.1b |
| | | **(4)** | |
| (b) | Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for $x$ into the expansion | B1 | 2.4 |
| | | **(1)** | |
| | | **(5 marks)** | |

**Notes:**

**(a)**

**M1:** Need correct binomial coefficient with correct power of 2 and correct power of $x$. Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent

**B1:** Correct answer, simplified as given in the scheme

**A1:** Correct answer, simplified as given in the scheme

**A1:** Correct answer, simplified as given in the scheme

**(b)**

**B1:** Needs a full explanation i.e. to state $x = 0.01$ and that this would be substituted and that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

## Student Response A

7. (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

**(4)**

(b) Explain how you would use your expansion to give an estimate for the value of $1.995^7$

**(1)**

a) $(2)^7\left(-\frac{x}{2}\right)^0 + 7(2)^6\left(-\frac{x}{2}\right)^1 + 21(2)^5\left(-\frac{x}{2}\right)^2$

$128 + 448(-x^{-2})^1 + 672(-x^{-2})^2$

$128 + -448x^{-2} + 672x^{-4}$

$= 128 - 448x^{-2} + 672x^{-4}$

b) ~~Work out what fraction $\frac{x}{2}$ is equal to for 0.01 hence work out $x$~~

substitute $x = 0.01$ into the expansion of $\left(2-\frac{x}{2}\right)^7$

***3/5***

Examiner Comments:

Part (a)

M1: This mark is scored on line one where the correct binomial coefficients (1, 7, 21) are combined with the correct terms $(2)^7\left(-\frac{x}{2}\right)^0, (2)^6\left(-\frac{x}{2}\right)^1, (2)^5\left(-\frac{x}{2}\right)^2$

B1: For 128

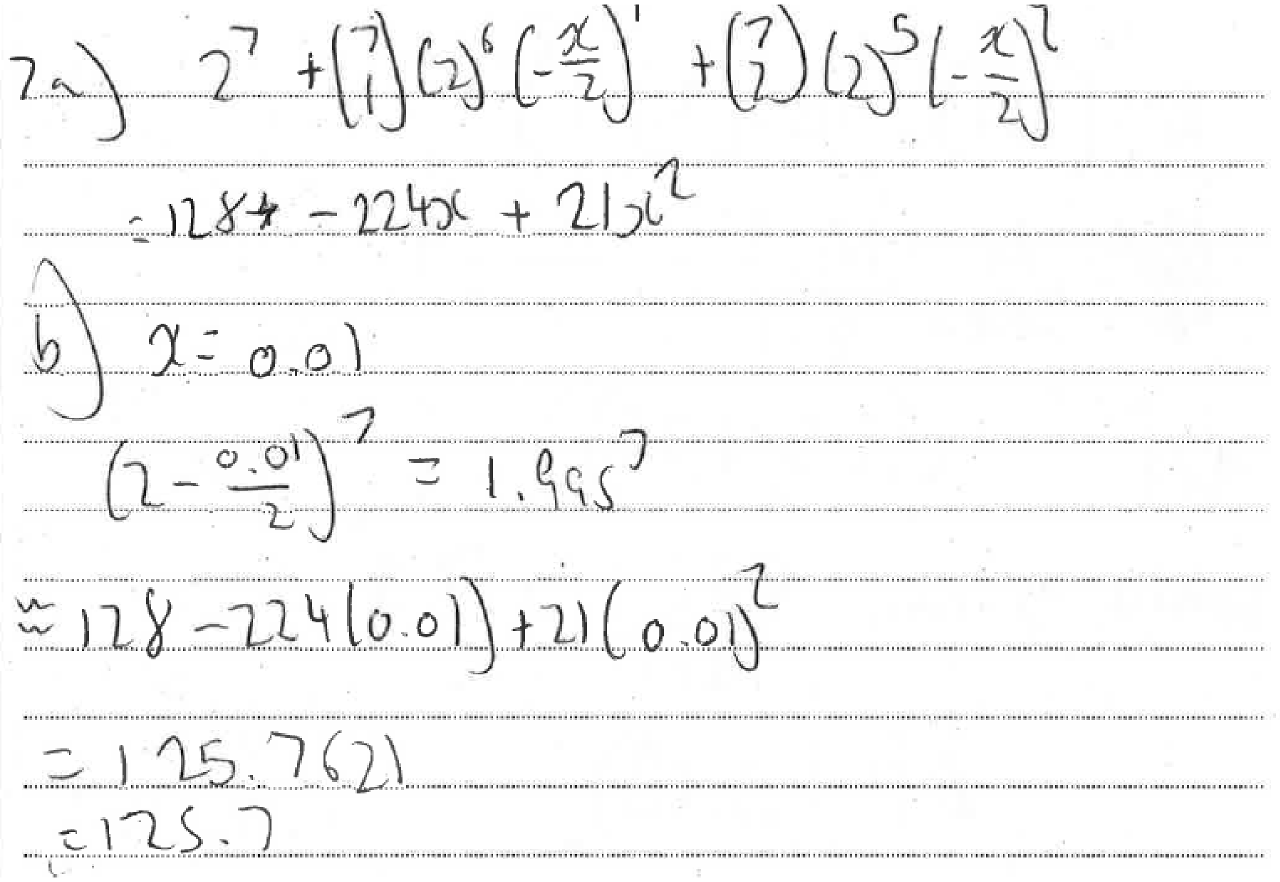
A0: For $-448x^{-2}$

A0: For $+672x^{-4}$

Part (b)

B1: Scored for stating the correct value of $x$ (0.01) and adding that it needs to be substituted into the expansion.

## Student Response B



7a) $2^7 + \binom{7}{1}(2)^6\left(-\frac{x}{2}\right)^1 + \binom{7}{2}(2)^5\left(-\frac{x}{2}\right)^2$

$= 128 - 224x + 21x^2$

b) $x = 0.01$

$\left(2 - \frac{0.01}{2}\right)^7 = 1.995^7$

$\approx 128 - 224(0.01) + 21(0.01)^2$

$= 125.7621$

$= 125.7$

**4/5**

Examiner Comments:

Part (a)

M1: This is scored on line one for $(2)^7 + \binom{7}{1}(2)^6\left(-\frac{x}{2}\right)^1 + \binom{7}{2}(2)^5\left(-\frac{x}{2}\right)^2$ The correct binomial coefficients $\left(\text{Eg.}\binom{7}{1}\right)$ are combined with the correct terms $\left(\text{Eg.}\,(2)^6\left(-\frac{x}{2}\right)^1\right)$.

B1: For 128

A1: For $-224x$

A0: For $+21x^2$

Part (b)

B1: Scored for stating the correct value of $x$ (0.01) and showing that it needs to be substituted into the expansion. Although the question states "explain" we felt justified in this award as the candidate shows how it would be attempted.

## Student Response C

$(2 - \frac{x}{2})^7$

a) $= 2^7 + 2^6(-\frac{x}{2}) + 2^5(-\frac{x}{2})^2 + 2^4(-\frac{x}{2})^3 + 2^3(-\frac{x}{2})^4 +$
$2^2(-\frac{x}{2})^5 + 2(-\frac{x}{2})^6 + (-\frac{x}{2})^7$

$128 - 32x + 8x^2 - 2x^3 + \frac{1}{2}x^4 - \frac{1}{8}x^5 + \frac{1}{32}x^6 - \frac{1}{128}x^7$

$64 \times -\frac{x}{2} = \frac{-64x}{2} = -32x$

$32 \quad (-\frac{x}{2})^2 \quad -\frac{1}{2}x^2 = \frac{1}{4}x^2$

$2^4 = 16 \qquad (-\frac{1}{2}x)^3 = -\frac{1}{8}x^3$

$2^3 = 8 \qquad (-\frac{1}{2}x)^4 = \frac{1}{16}x^4$

$8 \times \frac{1}{16} = \frac{1}{2}x^4$

$4 \qquad (-\frac{1}{2}x)^5 = -\frac{1}{32}x^5 \times 4 = -\frac{1}{8}x^5$

$(-\frac{1}{2})^6 = \frac{1}{64}x^6 \times 2 = \frac{1}{32}x^6$

$(-\frac{1}{2})^7 = -\frac{1}{128}x^7$

$(1)128 - (7)32x + 21(8x^2)$

$128 - 224x + 168x^2$

1 (0)
1 1 (1)
1 2 1 (2)
1 3 3 1 (3)
1 4 6 4 1 (4)
1 5 10 10 5 1 (5)
1 6 15 20 15 6 1 (6)
1 7 21 35 35 21 7 1 (7)

1, 7, 21

b) equate 1.995 to $2 - \frac{x}{2}$ to find $x$

$1.995 = 2 - \frac{x}{2}$

$3.99 = 4 - x$

$3.99 + x = 4$

$x = 0.01$

and sub in to the binomial expansion.

**5/5**

Examiner Comments:

Part (a)

M1: This is not scored until we see the candidate combining their binomial coefficients (1, 7, 21) produced from Pascal's triangle with the correct terms 128, $-32x$ and $+8x^2$. Evidence for this award can be seen on the line to the left of the "1 2 1 " of Pascal's triangle

B1: For 128

A1: For $-224x$

A1: For $+168x^2$

Part (b)

B1: Calculates $x = 0.01$ and states "sub in to the binomial expansion".

## Exemplar question 8

**8.**

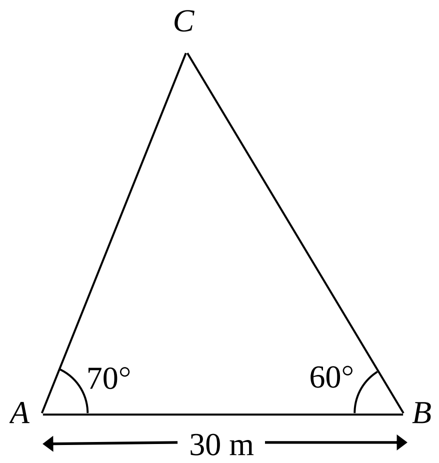



**Figure 1**

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 70° and angle *ABC* = 60°,

(a) calculate the area of the lawn to 3 significant figures.

**(4)**

(b) Why is your answer unlikely to be accurate to the nearest square metre?

**(1)**

**(Total for Question 8 is 5 marks)**

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## Mark scheme

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **8(a)** | **Way 1:** Finds third angle of triangle and uses or states $\frac{x}{\sin 60°} = \frac{30}{\sin"50°"}$ | **Way 2:** Finds third angle of triangle and uses or states $\frac{y}{\sin 70°} = \frac{30}{\sin"50°"}$ | M1 | 2.1 |
| | So $x = \frac{30\sin 60°}{\sin 50°}$ $(= 33.9)$ | So $y = \frac{30\sin 70°}{\sin 50°}$ $(= 36.8)$ | A1 | 1.1b |
| | Area = $\frac{1}{2}\times 30\times x\times \sin 70°$ or $\frac{1}{2}\times 30\times y\times \sin 60$ | | M1 | 3.1a |
| | = 478 m$^2$ | | A1ft | 1.1b |
| | | | **(4)** | |
| **(b)** | Plausible reason e.g. because the angles and the side length are not given to four significant figures<br>Or e.g. The lawn may not be flat | | B1 | 3.2b |
| | | | **(1)** | |
| | | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** Uses sine rule with their third angle to find one of the unknown side lengths

**A1:** Finds expression for, or value of either side length

**M1:** Completes method to find area of triangle

**A1ft:** Obtains a correct answer for their value of $x$ or their value of $y$

**(b)**

**B1:** As information given in the question may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate

## Student Response A

**8.**

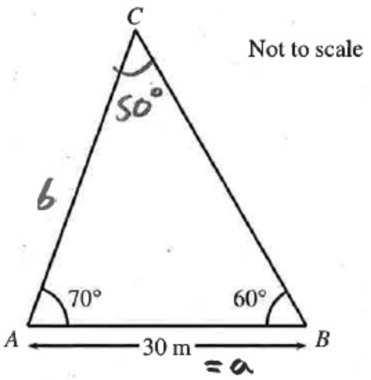


**Figure 1**

A triangular lawn is modelled by the triangle $ABC$, shown in Figure 1. The length $AB$ is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

a) Area $= \frac{1}{2} \times a \times b \times \sin C \qquad a = 30, \quad C = 50°$

$b \Rightarrow \frac{b}{\sin 60} = \frac{30}{\sin 50} \Rightarrow b = \frac{30}{\sin 50} \times \sin 60 = 33.92$

Area $= \frac{1}{2} \times 30 \times 33.92 \times \sin 50 = 390 m^2$

b) The answer has been rounded.

**2/5**

Examiner Comments:

Part (a)

M1: Scored for using the sine rule to find side "$b$".

A1: Can be scored for either the expression $b = \frac{30}{\sin 50} \times \sin 60$ or the value $b = 33.92$

M0: This mark is not awarded as the candidate does not use the correct angle and combination of sides within the formula $\frac{1}{2} ab \sin C$. A correct attempt would have been $\frac{1}{2} \times 33.9 \times 30 \sin 70$

A0: This follows the previous M0

Part (b)

B0: "The **answer** has been rounded" is too vague. The candidate could have made a statement about the initial values or that the lawn was unlikely to be flat.

## Student Response B

**8.**

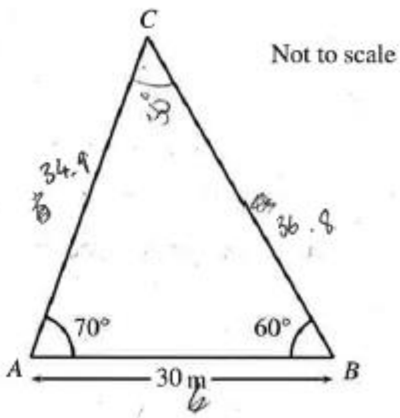


Figure 1

(1)

(a) $\frac{1}{2}ab\sin C$ $\angle ACB = 180-(70+60)$

$= 50°$

$\frac{30}{\sin 50} = \frac{B}{\sin 60}$ $B = \frac{30}{\sin 50} \times \sin 60$

$= 34.9\text{m}$

$\frac{30}{\sin 50} = \frac{A}{\sin 70}$ $A = \frac{30}{\sin 50} \times \sin 70$

$= 36.8\text{m}$

$\frac{1}{2} \times 34.9 \times 36.8 \times \sin 50$

$= 492\ \text{m}^2$

(b) Because the other side values were rounded to 1d.p.

**3/5**

Examiner Comments:

Part (a)

M1: Scored for either using the sine rule to find side "*b*" or using the sine rule to find side "*a*".

A1: Can be given for either correct expression or the value $a = 36.8$

M1: This mark is awarded as the candidate does use the correct angle and combination of sides within the formula $\frac{1}{2}ab\sin C$. This is implied by $\frac{1}{2}\times 34.9\times 36.8\sin 50$

A0: Answer is incorrect following the error on side *b*.

Part (b)

B0: "Because the other side values were rounded to 1 dp" is not a valid answer as that was the candidates' choice. The candidate should have made a statement about the initial values or that the lawn was unlikely to be flat.

## Student Response C

**8.**

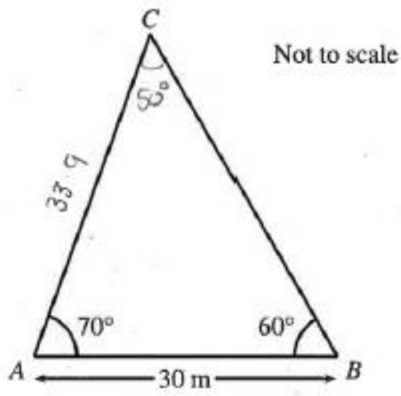


**Figure 1**

a. $180 - 70 - 60 = 50$ $\quad A\hat{C}B = 50°$

area $= \frac{1}{2} a b \sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\quad \therefore \frac{30}{\sin 50} = \frac{x}{\sin 60}$

$\frac{30}{\sin 50} \times \sin 60 = x = 33.9\text{m}$

$\frac{1}{2} \times 30 \times 33.9 \times \sin 70 = 447.83\ldots \text{ m}^2$

$= 448 \text{ m}^2$ (3sf)

b because the level of accuracy of the previous numbers is too low.

**4/5**

Examiner Comments:

Part (a)

M1: Scored for using the sine rule to find side *AB*.

A1: Scored for either the correct expression or the value 33.9

M1: This mark is awarded as the candidate does use the correct angle and combination of sides within the formula $\frac{1}{2}ab\sin C$. This is implied by $\frac{1}{2}\times 30\times 33.9\sin 70$

A0: The answer is incorrect.

Part (b)

B0: "Because the level of accuracy of the **previous** answers is too low " is not a valid answer as that was the candidates' choice. The candidate should have made a statement about the initial values or that the lawn was unlikely to be flat.

## Exemplar question 9

**9.** Solve, for $360° \leq x < 540°$,

$$12 \sin^2 x + 7 \cos x - 13 = 0.$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

**(Total for Question 9 is 5 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9** | Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$ | M1 | 3.1a |
| | $\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$ | A1 | 1.1b |
| | Uses solution of quadratic to give $\cos x =$ | M1 | 1.1b |
| | Uses inverse cosine on their values, giving two correct follow through values (see note) | M1 | 1.1b |
| | $\Rightarrow x = 430.5°,\ 435.5°$ | A1 | 1.1b |
| | | | **(5 marks)** |

**Notes:**

**M1:** Uses correct identity

**A1:** Correct three term quadratic

**M1:** Solves their three term quadratic to give values for $\cos x$ (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

**M1:** Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

**A1:** Two correct answers in the given domain

## Student Response A

**9.** Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(5)**

$$12(1 + \cos^2 x) + 7\cos x - 13 = 0$$

$$12\cos^2 x + 7\cos x - 1 = 0$$

$$\cos x = \frac{7 \pm \sqrt{49 - 4 \times 12 \times -1}}{2 \times 12}$$

$$= 0.702, \; -0.118$$

$$x = 45.4°, \; 96.8°$$

***1/5***

Examiner Comments:

M0: Candidate uses an incorrect identity.

A0: Follows the previous M0

M0: The expression written down suggests that an incorrect quadratic formula $\left(\frac{b \pm \sqrt{b^2 - 4ac}}{2a}\right)$ has been used.

M1: This is awarded as the candidate uses inverse cosine on their values. We follow through on their values and accept answers outside the given domain.

A0: Incorrect.

## Student Response B

**9.** Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(5)**

9. ~~$12x^2 + 7x - 13 = 0$~~

$12(1 - \cos^2 x) + 7\cos x - 13 = 0$

$12 - 12\cos^2 x + 7\cos x - 13 = 0$

$0 = 12\cos^2 x - 7\cos x + 1$

$0 = 12x^2 - 7x + 1$

$0 = 12x^2 - 6x - x + 1$

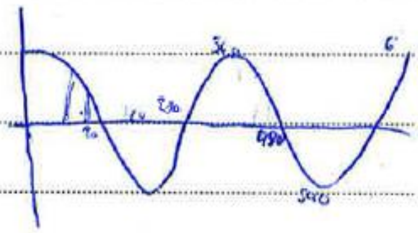
~~[illegible]~~

$0 = 12x(x - 0.5) - 1(x - 0.5)$

$0 = (12x - 1)(x - 0.5)$

$x = \frac{1}{12}$ $\quad$ $x = 0.5$

$\cos x = \frac{1}{12}$ $\quad$ $\cos x = 0.5$



$x = 445.22, 420$

**3/5**

Examiner Comments:

M1: Candidate uses a correct identity.

A1: For $0 = 12\cos^2 x - 7\cos x + 1$

M0: The candidate attempts to factorise their quadratic in cos $x$ but the method used is incorrect.

M1: This is awarded as the candidate uses inverse cosine on their values. We follow through on their values and accept the answers (360 + 85.22) and (360 + 60).

A0: Incorrect due to earlier error.

## Student Response C

**9.** Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(5)**

$12(1 - \cos^2 x) + 7\cos x - 13 = 0$

$12 - 12\cos^2 x + 7\cos x - 13 = 0$

$-12\cos^2 x + 7\cos x - 1 = 0$

$12\cos^2 x - 7\cos x + 1 = 0$

$12y^2 - 7y + 1 = 0$

$12y^2 - 4y - 3y + 1 = 0$ $\quad \left.\begin{matrix} \times : 12 \\ + : -7 \end{matrix}\right] -4, -3$

$4y(3y - 1) - 1(3y - 1) = 0$

$(4y - 1)(3y - 1) = 0$

$y = \frac{1}{4}$ or $y = \frac{1}{3}$

$\cos x = \frac{1}{4}$ $\qquad\qquad$ $\cos x = \frac{1}{3}$

$\cos^{-1}\left(\frac{1}{4}\right)$ $\qquad\qquad$ $\cos^{-1}\left(\frac{1}{3}\right) =$

$= 75.5$ $\qquad\qquad$ $= 70.5$

**4/5**

Examiner Comments:

M1: Candidate uses a correct identity.

A1: For $12\cos^2 x - 7\cos x + 1 = 0$

M1: The candidate factorises their quadratic in cos $x$ and correctly gives values for $\cos x$.

M1: This is awarded as the candidate uses inverse cosine on their (correct) values.

A0: Incorrect due to their values not being in the given domain.

## Exemplar question 10

**10.** The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that $0 \leq k < \frac{3}{4}$.

**(4)**

**(Total for Question 10 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10** | Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction) | B1 | 3.1a |
| | (For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$ | M1 | 2.4 |
| | $4k(4k - 3) < 0$ with attempt at solution | M1 | 1.1b |
| | So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leq k < \frac{3}{4}$ * | A1* | 2.1 |
| | | | **(4 marks)** |

**Notes:**

**B1:** Explains why $k = 0$ gives no real roots

**M1:** Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

**M1:** Attempts solution of quadratic inequality

**A1*:** Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

## Student Response A

**10.** The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

**(4)**

$\therefore b^2 - 4ac < 0$

$a =$ ~~k~~ $k$

$b = 4k$

$c = 3$

$\therefore (4k)^2 - (4 \times k \times 3) < 0$

$16k^2 - 12k < 0$

~~k~~ $8k^2 - 6k < 0$

$4k^2 - 3k < 0$

~~$k(4k-3) < 0$~~

$4k^2 < 3k$

$4k < 3$

$k < \frac{3}{4}$

***1/4***

Examiner Comments:

B0: The candidate fails to consider $k = 0$ so cannot be awarded this mark.

M1: The discriminant of $kx^2 + 4kx + 3 = 0$ is seen by the statement $(4k)^2 - (4 \times k \times 3) < 0$

M0: Only one solution is considered and therefore this mark is withheld.

A0: Incorrect.

## Student Response B

**10.** The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}.$$

**(4)**

$b^2 - 4ac < 0$

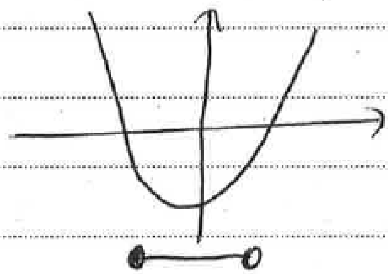
$(4k)^2 - 4(k)(3) < 0$

$16k^2 - 12k < 0$

$4k(4k - 3) < 0$

$4k = 0 \qquad 4k - 3 = 0$

$k = 0 \qquad k = \frac{3}{4}$



$0 \leq k < 3/4$

***2/4***

Examiner Comments:

B0: The candidate fails to explain why $k = 0$ gives no roots.

M1: For considering the discriminant of $kx^2 + 4kx + 3 = 0$ seen by the statement $(4k)^2 - 4(k)(3) < 0$

M1: For attempting the solution of the quadratic inequality using both critical values.

A0: Incomplete proof due to lack of explanation for $k = 0$

## Student Response C

**10.** The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

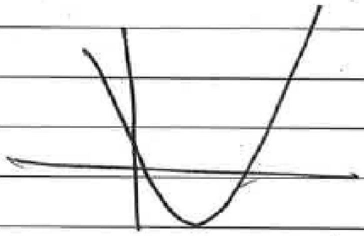
**(4)**

Real roots $b^2 - 4ac > 0$

$(4k)^2 - 4k \times 3 > 0$

$16k^2 - 12k > 0$

$4k(4k-3) > 0$



$k < 0 \quad k > 3/4$

No Real roots $0 < k < 3/4$

When $k=0$ $0x^2 + 0x + 3 = 0$

impossible X

**3/4**

Examiner Comments:

B1: Candidate explains why $k = 0$ gives no roots. (The "impossible $X$ is sufficient)

M1: For considering the discriminant of $kx^2 + 4kx + 3 = 0$ seen by the statement $(4k)^2 - 4k \times 3 > 0$

M1: For attempting the solution of the quadratic inequality using both critical values.

A0: Incomplete proof. The candidate fails to combine $0 < k < \frac{3}{4}$ with $k = 0$

## Exemplar question 11

**11.** (a) Prove that for all positive values of $x$ and $y$,

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

**(2)**

(b) Prove by counterexample that this is not true when $x$ and $y$ are both negative.

**(1)**

**(Total for Question 11 is 3 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11 (a) Way 1** | Since $x$ and $y$ are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$ | M1 | 2.1 |
| | $\therefore\ 2\sqrt{xy} \leq x+y$ provided $x$ and $y$ are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ * | A1* | 2.2a |
| | | **(2)** | |
| **Way 2 Longer method** | Since $(x-y)^2 \geq 0$ for real values of $x$ and $y$, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$ | M1 | 2.1 |
| | $\therefore\ 2\sqrt{xy} \leq x+y$ provided $x$ and $y$ are positive and so $\sqrt{xy} \leq \frac{x+y}{2}$ * | A1* | 2.2a |
| | | **(2)** | |
| **(b)** | Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= $-4$ so as $\sqrt{15} > -4$ result does not apply | B1 | 2.4 |
| | | **(1)** | |
| | | **(3 marks)** | |

**Notes:**

**(a)**

**M1:** Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging

**A1*:** Need all three stages making the correct deduction to achieve the printed result

**(b)**

**B1:** Chooses two negative values and substitutes, then states conclusion

## Student Response A

**11.** (a) Prove that for all positive values of $x$ and $y$

$$\sqrt{xy} \leq \frac{x+y}{2}$$

**(2)**

(b) Prove by counter example that this is not true when $x$ and $y$ are both negative.

**(1)**

a)

b $\sqrt{-1 \times -1} \leq \frac{-1 + -1}{2}$

$\sqrt{1} \leq \frac{-2}{2}$

$\sqrt{1} \nleq -1$

∴ The reverse is not true.

***1/3***

Examiner Comments:

Part (a)

M0 A0: Not attempted.

Part (b)

B1: Sets $x$ and $y$ negative, calculates both sides correctly and states that "it is not true".

## Student Response B

(a)

$$\sqrt{xy} \leq \frac{x+y}{2}$$

$$xy \leq \frac{x^2 + 2xy + y^2}{4}$$

$$4xy \leq x^2 + 2xy + y^2$$

$$x^2 - 2xy + y^2 \geq 0$$

$$(x-y)^2 \geq 0$$

(b) $x = -4 \quad y = -5$

$$\sqrt{-4 \times -5} \leq \frac{-4-5}{2}$$

$$\sqrt{20} \leq -\frac{9}{2}$$

$$4.47 \leq -4.5 \quad \times$$

NOT TRUE

***2/3***

Examiner Comments:

Part (a)

M1: This candidate starts from the premise that it must be true. This approach may be quite popular. The process followed is to square, rearrange and finish with a statement that is also true.

A0*: The proof lacks a deduction.

Part (b)

B1: Sets $x$ and $y$ negative, calculates both sides correctly and states "NOT TRUE".

## Student Response C

a. $\sqrt{xy} \leq \frac{x+y}{2}$

$xy \leq \left(\frac{x+y}{2}\right)^2$

$xy \leq \frac{x^2+2xy+y^2}{4}$

$4xy \leq x^2+2xy+y^2$

$0 \leq x^2-2xy+y^2$

$0 \leq (x-y)^2$

$\therefore (x-y)^2$ must be equal / bigger than 0 anything squared is positive $\therefore$ it must be true

b. $\sqrt{xy} \leq \frac{x+y}{2}$

$\sqrt{-1 \times -2} \leq \frac{-1+-2}{2}$

$\sqrt{2} > -1.5$

$\therefore$ not true for negative numbers

**3/3**

Examiner Comments:

M1: This candidate also starts from the premise that it must be true. The process followed is to square, rearrange and finish with a statement that is also true.

A1*: The proof is similar to candidate B but also gives a deduction. (We can condone the fact that it is not stated that "If $x$ and $y$ are positive" at the start $\sqrt{xy}$ exists" and the omission of $\Leftrightarrow$ symbols).

(b)

B1: Sets $x$ and $y$ negative, calculates both sides correctly and states "NOT TRUE".

## Exemplar question 12

**12.** A student was asked to give the exact solution to the equation $2^{2x+4} - 9(2^x) = 0$.

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$
$$2^{2x} + 2^4 - 9(2^x) = 0$$
$$\text{Let } 2^x = y$$
$$y^2 - 9y + 8 = 0$$
$$(y - 8)(y - 1) = 0$$
$$y = 8 \text{ or } y = 1$$
$$\text{So } x = 3 \text{ or } x\ 0$$

(a) Identify the two errors made by the student.

**(2)**

(b) Find the exact solution to the equation.

**(2)**

**(Total for Question 12 is 4 marks)**

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## Mark scheme

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| 12(a) | $2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$ | | B1 | 2.3 |
| | In line 4, $2^4$ has been replaced by 8 instead of by 16 | | B1 | 2.3 |
| | | | **(2)** | |
| (b) | **Way 1:**<br>$2^{2x+4}-9(2^x)=0$<br>$2^{2x}\times 2^4-9(2^x)=0$<br>Let $2^x=y$<br>$16y^2-9y=0$ | **Way 2:**<br>$(2x+4)\log 2-\log 9-x\log 2=0$ | M1 | 2.1 |
| | $y=\frac{9}{16}$ or $y=0$<br>So $x=\log_2\left(\frac{9}{16}\right)$ or $\frac{\log\left(\frac{9}{16}\right)}{\log 2}$<br>o.e. with no second answer | $x=\frac{\log 9}{\log 2}-4$ o.e. | A1 | 1.1b |
| | | | **(2)** | |
| | | | **(4 marks)** | |

**Notes:**

**(a)**
**B1:** Lists error in line 2 (as above)
**B1:** Lists error in line 4 (as above)

(b)
**M1:** Correct work with powers reaching this equation
**A1:** Correct answer here – there are many exact equivalents

## Student Response A

a) $2^4 = 16$ not 8.

$2^{2x} = 4^x$ $\therefore = 2y$ not $y^2$

b) $2^{2x+4} - 9(2^x) = 0$.

$2^{2x} + 2^4 - 9(2^x) = 0$

let $2^x = y$

$=$ ~~$y^2$~~ $2y + 16$ ~~$+8$~~ $- 9y = 0$

***1/4***

Examiner Comments:

Part (a)

B0: The candidate does not identify that $2^{2x+4} \neq 2^{2x} + 2^4$ Their (second) statement is incorrect.

B1: Correctly stated that $2^4 = 16$ not 8, thereby identifying the error.

Part (b)

M0: Cannot be scored from a linear equation.

A0: Follows M0

## Student Response B

a) $2^{2x} \times 2^{4} - 9(2^{x}) = 0$

$y^2 - 9y + \underline{\underline{16}} = 0$

b) $y^2 - 9y + 16 = 0$

×: 16
+: −9

$y = \dfrac{9 \pm \sqrt{(-9)^2 - 4(1)(16)}}{2} = \dfrac{9 \pm \sqrt{17}}{2}$

**2/4**

Examiner Comments:

Part (a)

B1: The expression $2^{2x} \times 2^{4}$ is sufficient for this award.

B1: The $\underline{\underline{+16}}$ is sufficient justification to award this mark.

Part (b)

M0: This mark is not awarded as the candidate writes an incorrect quadratic.

A0: Follows M0

## Student Response C

12) a)

$2^{2x+4} \neq 2^{2x} + 2^4$

$2^{2x+4} = 2^{2x} \times 2^4$

~~$y = 1$~~ ~~$y = 8$~~

~~$2^x = 1$~~ ~~$2^x = 8$~~

$2^4 \neq 8$

$2^4 = 16$

b) $2^{2x+4} - 9(2^x) = 0$

$(2^{2x} \times 2^4) - 9(2^x) = 0$

$(2^{2x} \times 16) - 9(2^x) = 0$

let $2^x = y$

$(y^2 \times 16) - 9y = 0$

$16y^2 - 9y = 0$

$y(16y - 9)$

$y = 0$ $\quad$ $16y = 9$

$y = 9/16$

~~tore~~ $y = 2^x$ $\quad$ $\log_b a = c \quad a = b^c$

$y = 2^x$

$2^x = 0$ $\quad$ $2^x = 9/16$

no solution $\quad$ $x = -0.83$

**3/4**

Examiner Comments:

Part (a)

B1: Either statement is sufficient for this award.

B1: For $2^4 \neq 8$

Part (b)

M1: For correct work leading to $16y^2 - 9y = 0$

A0: An exact answer is required.

## Exemplar question 13

**13.** (a) Factorise completely $x^3 + 10x^2 + 25x$.

**(2)**

(b) Sketch the curve with equation $y = x^3 + 10x^2 + 25x$, showing the coordinates of the points at which the curve cuts or touches the $x$-axis.

**(2)**

The point with coordinates (–3, 0) lies on the curve with equation

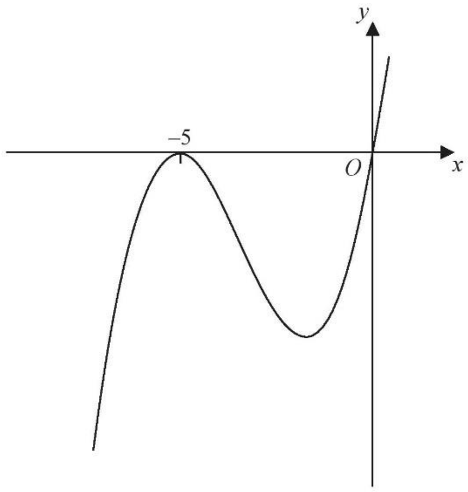
$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$, where $a$ is a constant.

(c) Find the two possible values of $a$.

**(3)**

**(Total for Question 13 is 7 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 13(a) | $x^3+10x^2+25x=x(x^2+10x+25)$ | M1 | 1.1b |
| | $=x(x+5)^2$ | A1 | 1.1b |
| | | **(2)** | |
| (b) |  A cubic with correct orientation | M1 | 1.1b |
| | Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft) | A1ft | 1.1b |
| | | **(2)** | |
| (c) | Curve has been translated $a$ to the left | M1 | 3.1a |
| | $a = -2$ | A1ft | 3.2a |
| | $a = 3$ | A1ft | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**Notes:**

**(a)**

**M1:** Takes out factor $x$

**A1:** Correct factorisation – allow $x(x + 5)(x + 5)$

**(b)**

**M1:** Correct shape

**A1ft:** Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

**(c)**

**M1:** May be implied by one of the correct answers for $a$ or by a statement

**A1ft:** ft from their cubic as long as it meets the $x$-axis only twice

**A1ft:** ft from their cubic as long as it meets the $x$-axis only twice

## Student Response A

a) $x^3 + 10x^2 + 25x$

$x(x^2 + 10x + 25)$ $x^2 + 5x + 5x + 25$

$x(x + 5)(x + 5)$ $x^2 + 10x + 25$

$x = -5$ $x = 0$

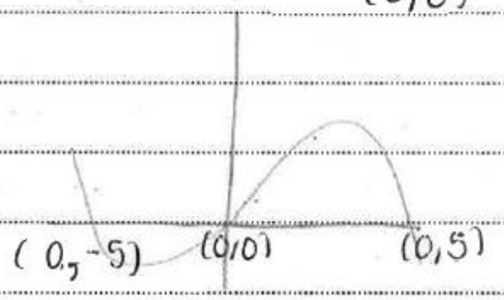
b) $y = (-5)^3 + 10(-5)^2 + 25(-5)$

$y = -125 + 250 - 125$

$y = 0$ $(0, -5)$

$y = (0)^3 + 10(0)^2 + 25(0) = 0$

$(0, 0)$ $(0, 5)$




2/7

Examiner Comments:

Part (a)

M1: For taking out a factor of $x$.

A1: Fully correct.

Part (b)

M0: Incorrect shape (orientation).

A0ft: Follows M0.

Part (c)

M0 A0 A0: Not attempted.

## Student Response B

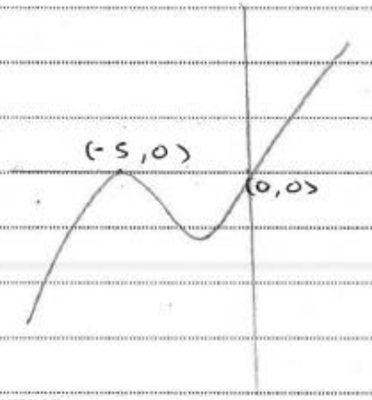
(3)

a) $x(x^2+10x+25)$

$x(x+5)(x+5)$

$= x(x+5)^2$

b)




$(-3,0)$

$y = (x+a)^3 + 10(x+a)^2 + 25(x+a)$

$0 = (-3+a)^3 + 10(-3+a)^2 + 25(-3+a)$

~~$-5 = x+a$~~

$(-3+a)(-3+a)(-3+a)$

$9 - 6a + a^2 \; (-3+a)$

$-6a$

$-27 + 9a + 18a^2 - 3a^2 + a^3$

$10(-3+a)(-3+a)$

$10(9-6a+a^2)$

$90 - 60a + 10a^2$

$25(-3+a) = -75+25a$

$-27 + 9a + 18a - 6a - 3a^2 + a^3 + 90 - 60a + 10a^2$
$-75+25a$

~~$-14a - 12a^2 - 12$~~

$12a^2 - 14a - 12$

$a^3 + 7a^2 - 14a - 12$

4/7

Examiner Comments:

Part (a)

M1: For taking out a factor of $x$.

A1: Fully correct.

Part (b)

M1: For a cubic graph with the correct orientation.

A1: Fully correct.

Part (c)

M0: Potentially this method could succeed. This mark is not awarded as the candidate does not proceed with a correct method to find at least one value for $a$.

A0 A0: Must follow M0.

## Student Response C

a. $x^3 + 10x^2 + 25x$

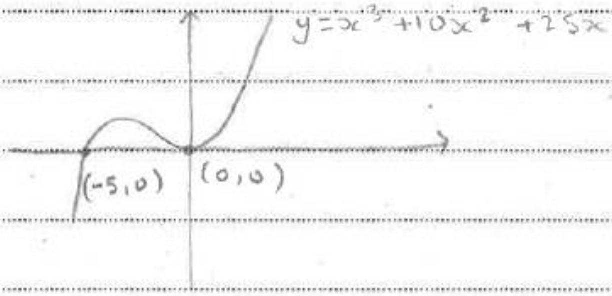
$x(x^2 + 10x + 25)$

$x(x + 5)(x + 5)$

$x(x+5)^2$

b $y = 0$ $\quad 0 = x(x+5)(x+5)$ $\quad$ (where $x = 0$, $y = 0$)

$x = 0, -5, -5$




c $y = (x+a)^3 + 10(x+a)^2 + 25(x+a)$ , $(-3, 0)$

$0 = (-3+a)^3 + 10(-3+a)^2 + 25(-3+a)$

$(-3+a) = 0$ or $-5$

$\therefore a = 3$ or $-2$

**6/7**

Examiner Comments:

Part (a)

M1: For taking out a factor of *x*.

A1: Fully correct.

Part (b)

M1: For a cubic graph with the correct orientation.

A0: Incorrect.

Part (c)

M1: Implied by one correct answer.

A1 A1: Both correct answers are given.

## Exemplar question 14

**14.**

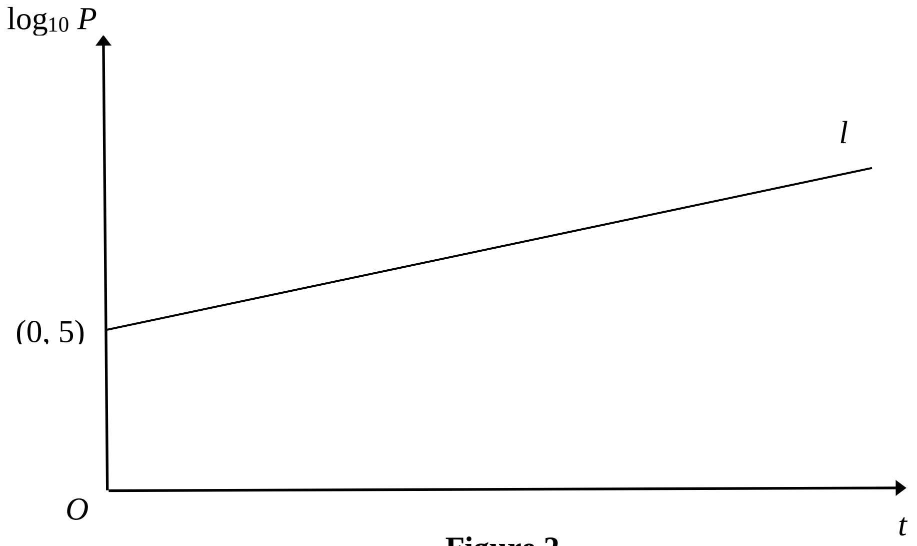



**Figure 2**

A town's population, $P$, is modelled by the equation $P = ab^t$, where $a$ and $b$ are constants and $t$ is the number of years since the population was first recorded. The line $l$ shown in Figure 2 illustrates the linear relationship between $t$ and $\log_{10} P$ for the population over a period of 100 years.

The line $l$ meets the vertical axis at (0, 5) as shown. The gradient of $l$ is $\frac{1}{200}$.

(a) Write down an equation for $l$.

**(2)**

(b) Find the value of $a$ and the value of $b$.

**(4)**

(c) With reference to the model, interpret

(i) the value of the constant $a$,

(ii) the value of the constant $b$.

**(2)**

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

**(3)**

(e) State two reasons why this may not be a realistic population model.

**(2)**

**(Total for Question 14 is 13 marks)**

Pearson

## Mark scheme

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **14(a)** | $\log_{10} P = mt + c$ | | M1 | 1.1b |
| | $\log_{10} P = \frac{1}{200}t + 5$ | | A1 | 1.1b |
| | | | **(2)** | |
| **(b)** | **<u>Way 1</u>:**<br>As $P = ab^t$ then<br>$\log_{10} P = t\log_{10} b + \log_{10} a$ | **<u>Way 2</u>:**<br>As $\log_{10} P = \frac{t}{200} + 5$ then<br>$P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$ | M1 | 2.1 |
| | $\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$ | $a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$ | M1 | 1.1b |
| | so $a$ = 100 000 **or** $b$ = 1.0116 | | A1 | 1.1b |
| | both $a$ = 100 000 **and** $b$ = 1.0116 (awrt 1.01) | | A1 | 1.1b |
| | | | **(4)** | |
| **(c)** | (i) The initial population | | B1 | 3.4 |
| | (ii) The proportional increase of population each year | | B1 | 3.4 |
| | | | **(2)** | |
| **(d)** | (i) 300000 to nearest hundred thousand | | B1 | 3.4 |
| | (ii) Uses $200000 = ab^t$ with their values of $a$ and $b$ or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t$ = | | M1 | 3.4 |
| | 60.2 years to 3sf | | A1ft | 1.1b |
| | | | **(3)** | |
| **(e)** | Any two valid reasons- e.g.<br>• 100 years is a long time and population may be affected by wars and disease<br>• Inaccuracies in measuring gradient may result in widely different estimates<br>• Population growth may not be proportional to population size<br>• The model predicts unlimited growth | | B2 | 3.5b |
| | | | **(2)** | |

| **Question 14 continued** |
|---|
| **Notes:** |
| **(a)**<br>**M1:** Uses a linear equation to relate log $P$ and $t$<br>**A1:** Correct use of gradient and intercept to give a correct line equation |
| **(b)**<br>**M1:** **Way 1:** Uses logs correctly to give log equation; **Way 2:** Uses powers correctly to "undo" log equation and expresses as product of two powers<br>**M1:** **Way 1**: Identifies log $b$ or log $a$ or both; **Way 2:** identifies $a$ or $b$ as powers of 10<br>**A1:** Correct value for $a$ or $b$<br>**A1:** Correct values for both |
| **(c)(i)**<br>**B1:** Accept equivalent answers e.g. The population at $t = 0$<br><br>**(c)(ii)**<br>**B1:** So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year |
| **(d)(i)**<br>**B1:** cao<br><br>**(d)(ii)**<br>**M1:** As in the scheme<br>**A1ft:** On their values of $a$ and $b$ with correct log work |
| **(e)**<br>**B2:** As given in the scheme – any two valid reasons |

## Student Response A

a) $y - 5 = \frac{1}{200}(x - 0)$ $\quad \log_{10} P = mt + c$

$y - 5 = \frac{1}{200}x$ $\quad \log_{10} P = \frac{t}{200} + 5$

$y = \frac{x}{200} + 5$

b)

$\log_{10} P = 5$

$8 = a$ $\quad 10^5 = P$

$P = 100\,000$

$100\,000 = ab^0$

$a = 100\,000$

***4/13***

Examiner Comments:

Part (a)

M1: For using a linear equation to link $\log_{10} P$ with $t$.

A1: For $\log_{10} P = \frac{t}{200} + 5$

Part (b)

M0: The candidate fails to link $\log_{10} P = \frac{t}{200} + 5$ with $P = ab^t$

M1: For identifying $\log_{10} P = 5$ from presumably putting $t = 0$ and using this as the value for $a$.

A1: For $a = 100\,000$

A0: This mark requires both $a$ and $b$.

Parts (c), (d), (e) No further progress.

## Student Response B

14a) $y = \frac{1}{200}x + 5$ $\log_{10} P = \frac{1}{200}t + 5$

14b) $\log_{10} P = 5$ $40^{\frac{1}{200}t+5} = P$

$10^5 = P$

$P = 100{,}000$

$P = ab^t$

$100{,}000 = ab^0$ $P = ab^t$

$= a \times 1$

$a = 100{,}000$

$b = 1$

c) $a$ is initial population

ii) $b$ is the rate of growth

d)

i) $P = 100{,}000 \times$

***6/13***

Examiner Comments:

Part (a)

M1: For using a linear equation to link $\log_{10} P$ with $t$.

A1: For $\log_{10} P = \frac{t}{200} + 5$

Part (b)

M0: The candidate fails to link $\log_{10} P = \frac{t}{200} + 5$ with $P = ab^t$

M1: For identifying $\log_{10} P = 5$ from presumably putting $t = 0$ and using this as the value for $a$.

A1: For $a = 100\,000$

A0: This mark requires both $a$ and $b$.

Part (c)(i) B1: For stating that '$a$' is the initial population.

Part (c)(ii) B1: For stating that '$b$' is the rate of growth. (We felt that this was acceptable)

Parts (d), (e) No further progress.

## Student Response C

(a) $\log_{10} P = \frac{1}{200}t + 5$

(b) $P = 10^{\frac{1}{200}t+5}$

$P = 10^{\frac{1}{200}t} \times 10^5$

$P = 1.22^t \times 10\,000$

$a = 10\,000 \quad b = 1.22$

(c) (i) the starting value

(ii) the population rises by 22% a year

(d) (i) $P = 1.22^{100} \times 10\,000 = 4 \times 10^{12}$

(ii) $200\,000 = 10\,000 \times 1.22^t$

$20 = 1.22^t$

$t = \log_{1.22} 20$

$= 15$

(e) The population is unlikely to continue rising at the same amount each year

The population will rise by different amounts each year.

***8/13***

Examiner Comments:

Part (a)

M1A1: For $\log_{10} P = \frac{t}{200} + 5$

Part (b)

M1: The candidate links $\log_{10} P = \frac{t}{200} + 5$ with $P = ab^t$ by way two in the mark scheme.

M1: Scored for $a = 10^5$

A0: Both values are incorrect $a \neq 10\,000$ and $b \neq 1.22$

A0: This mark requires both *a* and *b* The value of *b* is incorrect.

Part (c)(i)

B0: Stating that it is the starting value does make reference to the model. Population need to be mentioned. The starting population would be fine.

Part (c)(ii)

B1: For stating that 'the population rises by 22% a year" is acceptable.

Part (d)

B0: Incorrect.

M1: For using $200\,000 = 10\,000 \times 1.22^t$

A1ft: 15 is the correct follow through for their values.

Part (e)

B1: For one valid reason.

B0: The second reason is really the same as the first and cannot be accepted.

## Exemplar question 15

**15.**

Diagram not drawn to scale

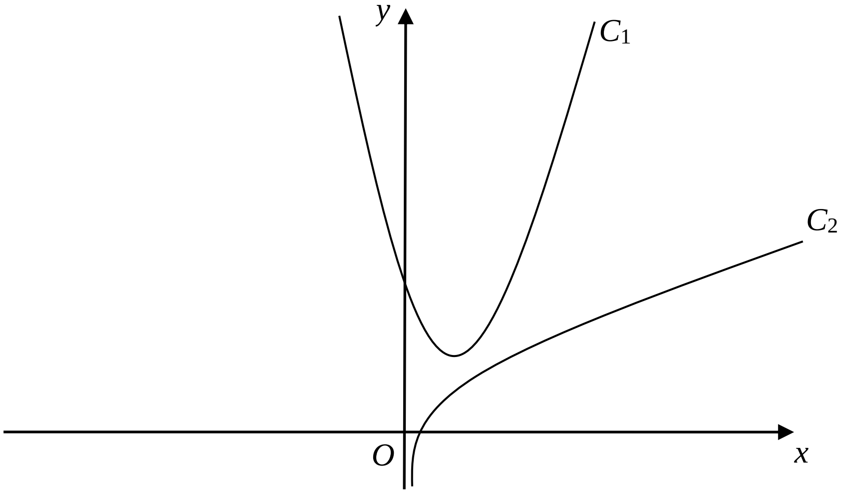



**Figure 3**

The curve $C_1$, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on $C_1$.

The curve $C_2$, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$. The normal to $C_1$ at the point $P$ meets $C_2$ at the point $Q$.

Find the exact coordinates of $Q$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(8)**

**(Total for Question 15 is 8 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15** | Finds $\frac{dy}{dx} = 8x - 6$ | M1 | 3.1a |
| | Gradient of curve at $P$ is –2 | M1 | 1.1b |
| | Normal gradient is $-\frac{1}{m} = \frac{1}{2}$ | M1 | 1.1b |
| | So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$ | A1 | 1.1b |
| | Eliminates $y$ between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in $x$ | M1 | 3.1a |
| | Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$ | M1 | 1.1b |
| | Substitutes to give value for $y$ | M1 | 1.1b |
| | Point $Q$ is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$ | A1 | 1.1b |
| | | | **(8 marks)** |

**Notes:**

**M1:** Differentiates correctly

**M1:** Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

**M1:** Uses negative reciprocal gradient

**A1:** Correct equation for normal

**M1:** Attempts to eliminate $y$ to find an equation in $x$

**M1:** Attempts to solve their equation using exp

**M1:** Uses their $x$ value to find $y$

**A1:** Any correct exact form

## Student Response A

$$\frac{dy}{dx} = 8x - 6$$

$$= 8\left(\tfrac{1}{2}\right) - 6 = -2$$

$$\text{so} \quad y - 2 = -2\left(x - \tfrac{1}{2}\right)$$

$$y = -2x + 3$$

$$-2x + 3 = \tfrac{1}{2}x + \ln(2x)$$

$$2.5x = 3 - \ln(2x)$$

$$x = \frac{3 - \ln(2x)}{2.5}$$

**3/8**

Examiner Comments:

M1: For $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - 6$

A1: For substituting $x = \frac{1}{2}$ and finding the gradient.

M0: Does not find the gradient of the normal.

A0: Follows M0.

M1: For eliminating $y$ to find an equation in $x$

M0: Cannot be awarded as the candidate don't use exponentials to find $x$

M0: A value of $y$ is not attempted.

A0: Incorrect.

## Student Response B

**15.**

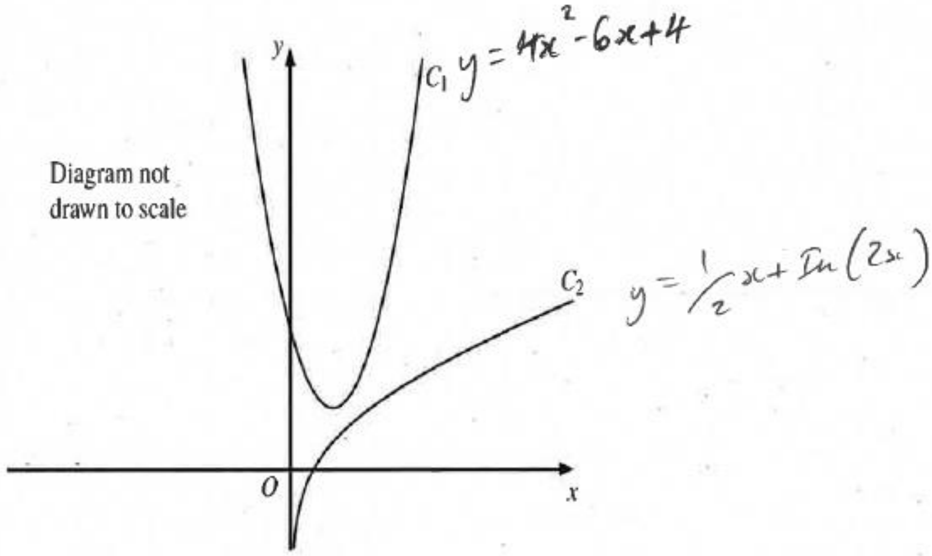



**Figure 3**

**(8)**

$P\left(\frac{1}{2}, 2\right)$ $\quad y = 4x^2 - 6x + 4$

$\frac{dy}{dx} = 8x - 6$

$8\left(\frac{1}{2}\right) - 6 = -2$

$\frac{y-2}{x-\frac{1}{2}} = \frac{1}{2}$

$y - 2 = \frac{1}{2}\left(x - \frac{1}{2}\right)$

$y = \frac{x}{2} - \frac{1}{4} + 2$, $\boxed{y = \frac{x}{2} + 1.75}$

$\frac{1}{2}x + \ln(2x) = \frac{1}{2}x + 1.75$

**5/8**

Examiner Comments:

M1: For $\frac{dy}{dx} = 8x - 6$

A1: For substituting $x = \frac{1}{2}$ and finding the gradient.

M1: For finding the gradient of the normal by using the negative reciprocal.

A1: For $y = \frac{x}{2} + 1.75$

M1: For eliminating $y$ to find an equation in $x$

M0M0A0: No further progress.

## Student Response C

$\frac{dy}{dx} = 8x - 6$

$= 8(\frac{1}{2}) - 6$

$= -2$

gradient of normal $= \frac{1}{2}$ at $(\frac{1}{2}, 2)$

$y - 2 = \frac{1}{2}(x - \frac{1}{2})$

$y = \frac{1}{2}x - \frac{1}{4} + 2 = y = \frac{1}{2}x + \frac{7}{4}$

$\frac{1}{2}x + \frac{7}{2} = \frac{1}{2}x + \ln(2x)$

$\frac{7}{4} = \ln(2x)$

$e^{7/4} = e^{\ln(2x)}$

$e^{7/4} = 2x$

$\frac{e^{7/4}}{2} = x$

~~2.887~~ =

2.877 =

~~y = 2.5~~ $y = \frac{1}{2}(2.887) + \ln(2 \times 2.887)$

$= 3.189$

Q (2.8~~7~~7, 3.189)

**7/8**

Examiner Comments:

M1: For $\frac{dy}{dx} = 8x - 6$

A1: For substituting $x = \frac{1}{2}$ and finding the gradient.

M1: For finding the gradient of the normal by using the negative reciprocal.

A1: For $y = \frac{x}{2} + \frac{7}{4}$

M1: For eliminating $y$ to find an equation in $x$.

M1: For attempting to solve their equation using exponentials.

M1: For substituting their $x$ value to find $y$.

A0: An exact form of the answer is required for this mark.

## Exemplar question 16

**16.**

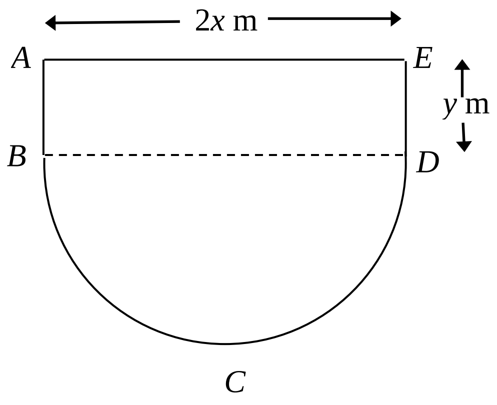



**Figure 4**

Figure 4 shows the plan view of the design for a swimming pool. The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semi-circular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m$^2$,

(a) show that the perimeter, $P$ metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}.$$

**(4)**

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$.

**(2)**

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

**(4)**

**(Total for Question 16 is 10 marks)**

Pearson

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 16(a) | Sets $2xy+\frac{\pi x^2}{2}=250$ | B1 | 2.1 |
| | Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into $P$ | M1 | 1.1b |
| | Use $P=2x+2y+\pi x$ with their $y$ substituted | M1 | 2.1 |
| | $P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | $x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e. | M1 | 2.4 |
| | As $x$ and $y$ are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ * | A1* | 3.2a |
| | | **(2)** | |
| **(c)** | Differentiates $P$ with negative index correct in $\frac{dP}{dx}$; $x^{-1}\to x^{-2}$ | M1 | 3.4 |
| | $\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$ | A1 | 1.1b |
| | Sets $\frac{dP}{dx}=0$ and proceeds to $x=$ | M1 | 1.1b |
| | Substitutes their $x$ into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8 m | A1 | 1.1b |
| | | **(4)** | |
| | | | **(10 marks)** |

**Question 16 continued**

**Notes:**

**(a)**

**B1:** Correct area equation

**M1:** Rearranges **their** area equation to make $y$ the subject of the formula and attempt to use with an expression for $P$

**M1:** Use correct equation for perimeter with their $y$ substituted

**A1*:** Completely correct solution to obtain and state printed answer

**(b)**

**M1:** States $x > 0$ and $y > 0$ and uses their expression from (a) to form inequality

**A1*:** Explains that $x$ and $y$ are positive because they are distances, and uses correct expression for $y$ to give the printed answer correctly

**(c)**

**M1:** Attempt to differentiate $P$ (deals with negative power of $x$ correctly)

**A1:** Correct differentiation

**M1:** Sets derived function equal to zero and obtains $x =$

**A1:** The value of $x$ may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$)

Need to see awrt 59.8m with units included for the perimeter

## Student Response A

(4)

a.) area = $250 = 2xy + \frac{\pi x^2}{2}$

$2xy = 250 - \pi x^2/2$

$y = \frac{250 - \frac{1}{2}\pi x^2}{2x}$

perimeter = $2x + 2y + \pi 2x$

$= 2\pi x + 2\pi + 2\left(\frac{250 - \frac{1}{2}\pi x^2}{2x}\right)$

$= 2x + 250/x + \pi x/2$

b. $x$ can't be negative because you can't have a negative perimeter, and it can't be over $\sqrt{\frac{500}{x}}$ because

c. $P = 2x + 250x^{-1} + \frac{1}{2}\pi x$

$dP/dx = 2 - 250x^{-1} + \frac{1}{2}\pi$

$250/x = 2 + \frac{1}{2}\pi$

$250 = 2x + \frac{1}{2}\pi x$

$250 = x(2 + \frac{1}{2}\pi)$

$x = \frac{250}{2 + \frac{1}{2}\pi} = 70.0\text{m}$

**3/10**

Examiner Comments:

Part (a)

B1: Scored on the first line for correct area equation.

M1: For rearranging their (correct) area equation, makes $y$ the subject and substitutes into an expression for $P$.

M0: This mark requires a correct equation for the perimeter. This candidate written $\mathbf{2}\pi x$ which is incorrect.

A0: Follows M0

Part (b)

M0: The candidate needs to consider $\frac{250-\frac{\pi x^2}{2}}{2x} > 0$

A0*: Follows M0

Part (c)

M0: The negative index is not correctly dealt with. We need to see $x^{-1} \to x^{-2}$

A0: Follows M0

M1: Scored for setting $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$ and using a correct method to proceed to a value for $x$.

A0: Incorrect.

## Student Response B

a) $2xy + \frac{\pi x^2}{2} = 250$

$4xy + \pi x^2 = 500$

$4xy = 500 - \pi x^2$

$y = \frac{500}{4x} - \frac{\pi x^2}{4x}$

$y = \frac{125}{x} - \frac{\pi x}{4}$

$P = 2x + 2y + \frac{2\pi x}{2}$

$P = 2x + 2y + \pi x$

$P = 2x + 2\left(\frac{125}{x} - \frac{\pi x}{4}\right)$

$P = 2x + \frac{250}{x} - \frac{2\pi x}{4}$

$P = 2x + \frac{250}{x} - \frac{\pi x}{2}$ $\qquad P = 2x + 250x^{-1} - \frac{1}{2}\pi x$

b) ~~$\frac{dy}{dx} = 2 - 250x^{-2} - \frac{\pi}{2}$~~

c) $\frac{dy}{dx} = 2 - 250x^{-2} - \frac{\pi}{2}$

$2 - 250x^{-2} - \frac{\pi}{2} = 0$

$4 - 500x^{-2} - \pi = 0$

$500x^{-2} = 4 - \pi$

$\frac{500}{x^2} = 4 - \pi$

$x^2 = \frac{500}{4 - \pi}$

$x = 24.13$

$P = 2(24.13) + \frac{250}{(24.13)} - \frac{\pi(24.13)}{2}$

$P = 20.72\text{cm}$

**5/10**

Examiner Comments:

Part (a)

B1: Scored on the first line for correct area equation.

M1: For rearranging their (correct) area equation, makes $y$ the subject and substitutes into an expression for $P$.

M1: For using the correct equation for $P$ with their $y$ substituted. We felt that the omission of the $\pi x$ can be regarded as a slip and not an error in method.

A0: Follows the accuracy slip. The correct answer is not achieved.

Part (b)

M0: The candidate needs to consider $\dfrac{250-\dfrac{\pi x^2}{2}}{2x} > 0$

A0*: Follows M0

Part (c)

M0: The negative index is correctly dealt with. We see $x^{-1} \to x^{-2}$.

A0: Incorrect $\dfrac{\mathrm{d}P}{\mathrm{d}x}$.

M1: Scored for setting $\dfrac{\mathrm{d}P}{\mathrm{d}x} = 0$ and using a correct method to proceed to a value for $x$.

A0: Incorrect.

## Student Response C

**16.**

$2x : 2 = x$

$\frac{\pi x^2}{2}$

$\pi x$

$\frac{\pi 2x}{2}$

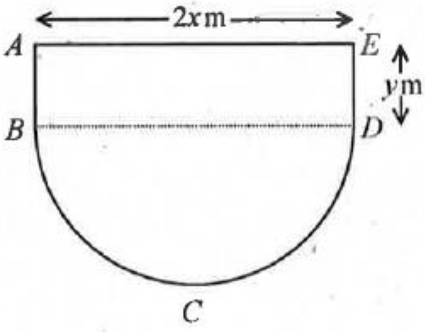



**Figure 4**

**(4)**

a. $\text{Area} = 250 = 2xy + \frac{\pi x^2}{2}$

$P = 2x + 2y + \pi x$

$250 - \frac{\pi x^2}{2} = 2xy$

$\frac{500 - \pi x^2}{4x} = y$

**Question 16 continued**

$$P = 2x + 2\left(\frac{500 - \pi x^2}{4x}\right) + \pi x$$

$$P = 2x + 250x^{-1} - \frac{\pi x}{2} + \pi x$$

$$P = 2x + \frac{250}{x} - \frac{\pi x}{2} + \frac{2\pi x}{2}$$

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

b. $0 = 2x + \frac{250}{x} + \frac{\pi x}{2}$ $\quad 4x^2 + 500 + \pi x^2 = 0$

c. $\frac{dy}{dx} = 2 - 250x^{-2} + \frac{\pi}{2}$ $\quad \frac{250}{x^2} = 2 + \frac{\pi}{2}$

$\frac{250}{3.57} = x^2$ $\quad x = \sqrt{\frac{250}{3.57}} = 8.368$

$$P = 2(8.368) + \frac{250}{(8.368)} + \frac{\pi(8.368)}{2}$$

$$P = 59.8 \text{ m}^2$$

***8/10***

Examiner Comments:

Part (a)

B1: Scored on the first line for correct area equation.

M1: For rearranging their (correct) area equation, makes $y$ the subject and substitutes into an expression for $P$.

M1: For using the correct equation for $P$ with their $y$ substituted.

A1: For a completely correct solution proceeding to the printed answer.

Part (b)

M0: The candidate needs to consider $\dfrac{250-\dfrac{\pi x^2}{2}}{2x} > 0$

A0*: Follows M0

Part (c)

M1: The negative index is correctly dealt with. We see $x^{-1} \to x^{-2}$.

A1: Correct. We condone the fact that $\dfrac{dP}{dx}$ is incorrectly labelled as $\dfrac{dy}{dx}$.

M1: Scored for setting $\dfrac{dP}{dx} = 0$ and using a correct method to proceed to a value for $x$.

A1: Correct answer achieved by a correct method.

# Exemplar question 17

**17.** A circle $C$ with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle $C$ also passes through the point $(10, 1)$.

**(3)**

The tangent to the circle C at the point $(10, 11)$ meets the $y$-axis at the point $P$ and the tangent to the circle $C$ at the point $(10, 1)$ meets the $y$-axis at the point $Q$.

(b) Show that the distance $PQ$ is 58, explaining your method clearly.

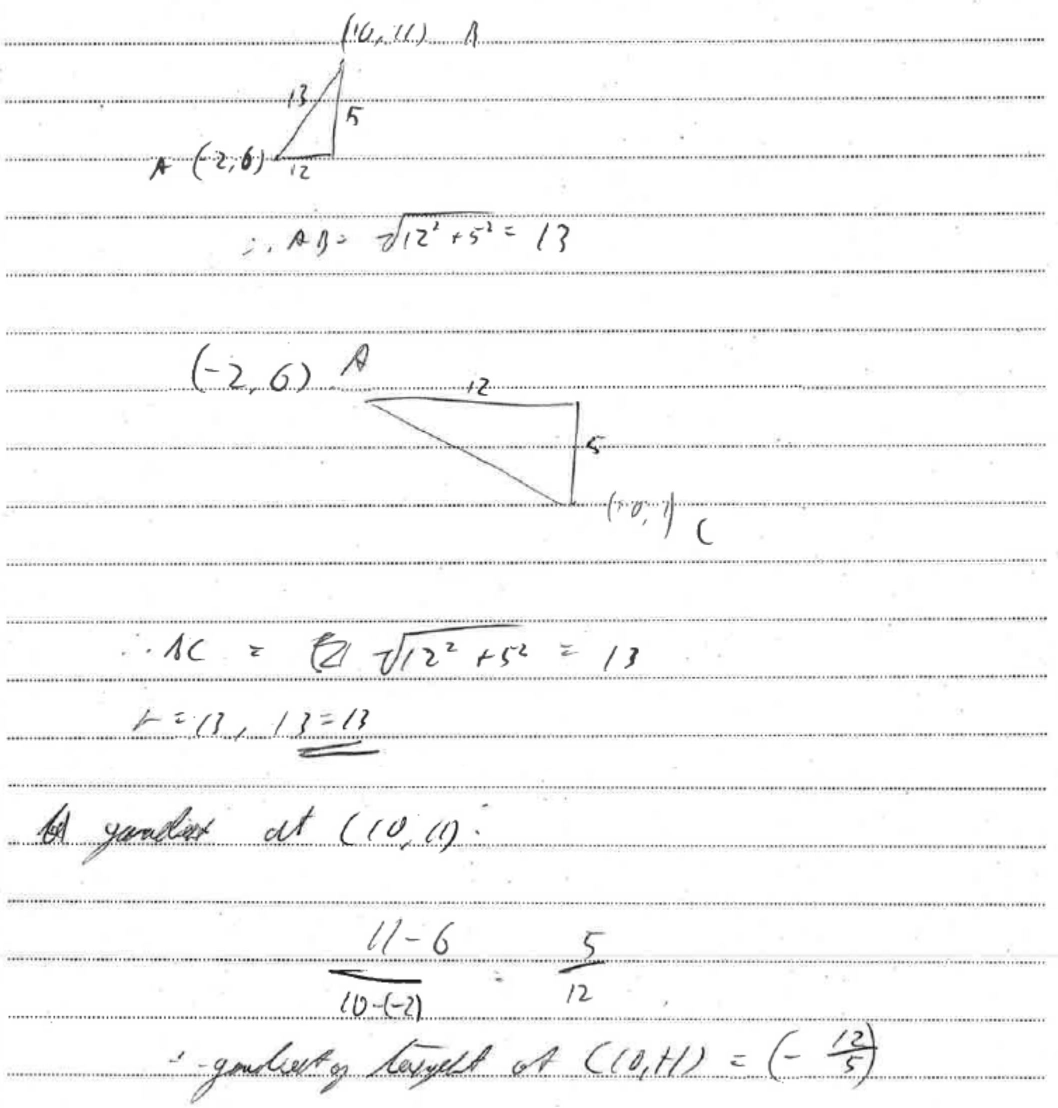
**(7)**

**(Total for Question 17 is 10 marks)**

## Mark scheme

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">17 (a)</td><td><u>Way 1:</u><br>Finds circle equation<br>$(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$</td><td><u>Way 2:</u><br>Finds distance between $(-2, 6)$ and $(10, 11)$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Checks whether (10, 1) satisfies their circle equation</td><td>Finds distance between $(-2, 6)$ and $(10, 1)$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that (10, 1) lies on $C$ *</td><td>Concludes that as distance is the same (10, 1) lies on the circle $C$ *</td><td>A1*</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="8">(b)</td><td colspan="2">Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ $(m)$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Finds gradient perpendicular to their radius using $-\frac{1}{m}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Finds (equation and ) $y$ intercept of tangent (see note below)</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Obtains a correct value for $y$ intercept of their tangent i.e. 35 or –23</td><td>A1</td><td>1.1b</td></tr>
<tr><td><u>Way 1</u>: Deduces gradient of second tangent</td><td><u>Way 2</u>: Deduces midpoint of $PQ$ from symmetry (0, 6)</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Finds (equation and ) $y$ intercept of second tangent</td><td>Uses this to find other intercept</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">So obtains distance $PQ$ = 35 + 23= 58*</td><td>A1*</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(7)</td><td></td></tr>
<tr><td colspan="5" align="right">(10 marks)</td></tr>
</table>

## Student Response A




***4/10***

Examiner Comments:

Part (a) Attempted by Way Two shown on the mark scheme.

M1: For attempting to find the distance between $(-2,6)$ and $(10,11)$ using a correct method.

M1: For attempting to find the distance between $(-2,6)$ and $(10,1)$ using a correct method.

A0*: Some written comment is required. This candidate would have needed to say "As $13=13$, both distances are the same and $(10,1)$ lies on the circle".

Part (b)

M1: For finding the gradient of the radius as seen by $\dfrac{11-6}{10-(-2)}$

M1: For finding the gradient of the tangent implied by $-\dfrac{12}{5}$ (following their $\dfrac{5}{12}$ ).

M0 A0 M0 M0 A0*: No further progress.

## Student Response B

(7)

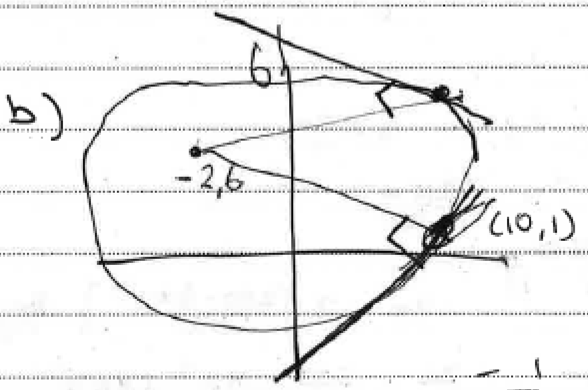
a) $\sqrt{(11-6)^2 + (10+2)^2} = 13$

$(x+2)^2 + (y-6)^2 = 169$

$(10+2)^2 + (1-6)^2 = 169$

$\therefore$ (10, 1) lies on circle

b)




$4 + y^2 - 12y + 36 = 169$

$y^2 - 12y - 129 = 0$

$\frac{11-6}{10+2} = \frac{5}{12}$ $\left(\frac{-1}{\frac{5}{12}}\right) = -\frac{12}{5} = \frac{y-11}{x-10}$

$-12x + 120 = 5y - 55$

$5y + 12x - 175 = 0$

$\frac{1-6}{10+2} = \frac{-5}{12}$ $\left(\frac{-1}{\frac{-5}{12}}\right)$

$= \frac{12}{5} = \frac{y-1}{x-10}$

$12x - 120 = 5y - 5$

$5y - 12x + 115 = 0$

~~$-12x + 175 = 12x - 115$~~

~~$24x = 290$~~

~~$x = 12.083$~~

$175 - 5y = 5y + 115$

$10y = 60$

$y = 6$

$12x - 120 = 5y - 5$

$5y = 12x - 115$

$y = \frac{12}{5}x - 23$

$\frac{12}{5} \times m = -1$

$m = -\frac{5}{12}$

$1 - y = -\frac{5}{12}(10 - x)$

$1 - y = \frac{-50}{12} + \frac{5x}{12}$

$12 - 12y = -50 + 5x$

***6/10***

Examiner Comments:

Part (a) Attempted by Way One shown on the mark scheme.

M1: For attempting to find the equation of the circle implied by $(x+2)^2 + (y-6)^2 = 169$

M1: For attempting to check whether $(10,1)$ satisfies their circle equation.

A1*: For a correct circle equation, correct calculations and a statement that $(10,1)$ lies on the circle.

Part (b)

M1: For finding the gradient of the radius as seen by $\frac{11-6}{10+2}$

M1: For finding the gradient of the tangent implied by $-\frac{12}{5}$ (following their $\frac{5}{12}$).

M0: A (correct) tangent equation is found $5y+12x-175=0$, but there is no attempt to find its *y*-intercept. The candidate do find a value for *y*, but this is after combining both equations.

A0: Follows M0

M1: For finding the gradient of the second tangent implied by $\frac{12}{5}$ (following their $-\frac{5}{12}$).

M0: A (correct) second tangent equation is found $5y-12x+115=0$, but there is no attempt to find its *y*-intercept.

A0*: No further progress.

## Student Response C

a. $(x+2)^2+(y-6)^2=r^2$

$\sqrt{(11-6)^2+(10+2)^2}=13$

$(x+2)^2+(y-6)^2=169$

$(10+2)^2+(1-6)^2=169$

b. $\frac{11-6}{10+2}=\frac{5}{12}$ $\quad -\frac{12}{5}=-2.4=m$

$y=-\frac{12}{5}x+c$ $\quad 11=-\frac{12}{5}(10)+c=35$

$P(0,35)$

$\frac{1-6}{10+2}=\frac{-5}{12}$ $\quad \frac{12}{5}=2.4=m$

$y=2.4x+c$

$1=2.4(10)+c$ $\quad c=-23$

$Q(0,-23)$

$35--23=58$

**9/10**

Examiner Comments:

Part (a) Attempted by Way One shown on the mark scheme.

M1: For attempting to find the equation of the circle implied by $(x+2)^2+(y-6)^2=169$

M1: For attempting to check whether $(10,1)$ satisfies their circle equation.

A0*: The candidate has a correct circle equation, correct calculations but missing a statement that $(10,1)$ lies on the circle as $169=169$

Part (b)

M1: For finding the gradient of the radius as seen by $\frac{11-6}{10+2}$

M1: For finding the gradient of the tangent implied by $-\frac{12}{5}$ (following their $\frac{5}{12}$).

M1: A (correct) method is used to find the $y$-intercept. The candidate substitutes $(10,11)$ into $y=-\frac{12}{5}x+c$ and finds $c$.

A1: For one correct $y$ -intercept (35).

M1: For finding the gradient of the second tangent implied by 2.4 (following their $-\frac{5}{12}$).

M1: A (correct) method is used to find the second $y$-intercept. The candidate substitutes $(10,1)$ into $y=2.4x+c$ and finds $c$.

A1*: The candidate obtains the given answer (58) by a valid method.

# A level Mathematics – Paper 1 (Pure)

## Exemplar question 1

**1.** The curve $C$ has equation $y = 3x^4 - 8x^3 - 3$.

(a) Find (i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

**(3)**

(b) Verify that $C$ has a stationary point when $x = 2$.

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

**(Total for Question 1 is 7 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | (i) $\frac{dy}{dx} = 12x^3 - 24x^2$ | M1<br>A1 | 1.1b<br>1.1b |
| | (ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$ | A1ft | 1.1b |
| | | **(3)** | |
| **(b)** | Substitutes $x = 2$ into their $\frac{dy}{dx} = 12 \times 2^3 - 24 \times 2^2$ | M1 | 1.1b |
| | Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" | A1 | 2.1 |
| | | **(2)** | |
| **(c)** | Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36 \times 2^2 - 48 \times 2$ | M1 | 1.1b |
| | $\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum" | A1ft | 2.2a |
| | | **(2)** | |
| | | **(7 marks)** | |

**Question 1 notes:**

**(a)(i)**

**M1:** Differentiates to a cubic form

**A1:** $\frac{dy}{dx} = 12x^3 - 24x^2$

**(a)(ii)**

**A1ft:** Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$

**(b)**

**M1:** Substitutes $x = 2$ into their $\frac{dy}{dx}$

**A1:** Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct

**(c)**

**M1:** Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of $C$ either side of $x = 2$

**A1ft:** For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$

## Student Response A

**1.** The curve $C$ has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

**(3)**

(b) Verify that $C$ has a stationary point when $x = 2$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

(a) (i) $\frac{dy}{dx} = 12x^2 - 8x^2 - 3$

(ii) $\frac{d^2y}{dx^2} = 24x - 16x - 3$

(b) $x = 2$ $\quad \frac{dy}{dx} = 13$

(c) $x = 2$ $\quad \frac{d^2y}{dx^2} = 13 > 0$ max

2/7

Examiner Comments:

Part (a)(i)

M0: The candidate fails to differentiate into a cubic form.

A0: Follows M0. Incorrect, $\frac{dy}{dx} \neq 12x^3 - 24x^2$

Part (a)(ii)

A0ft: Incorrect follow through. The $-3 \to -3$

Part (b)

M1: For substituting $x = 2$ into $\frac{dy}{dx}$. This is implied as their $\left.\frac{dy}{dx}\right|_{x=2} = 13$

A0: Incorrect.

Part (c)

M1: For substituting $x = 2$ into $\frac{d^2y}{dx^2}$ This is implied as their $\left.\frac{d^2y}{dx^2}\right|_{x=2} = 13$

A0ft: For having an incorrect conclusion. If the candidate had "Minimum" they would have been awarded this mark

# Student Response B

**1.** The curve $C$ has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

**(3)**

(b) Verify that $C$ has a stationary point when $x = 2$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

(a) $\frac{dy}{dx} = 9x^3 - 24x^2$

$\frac{\partial^2 y}{\partial x^2} = 27x^2 - 48x$

(b) $27 \times 2^2 - 48 \times 2$

$= 8$

(c) As $\frac{\partial^2 y}{\partial x^2} = 8 > 0$ it is a minimum

**4/7**

Examiner Comments:

Part (a)(i)

M1: Differentiates to a cubic form.

A0: Incorrect $\frac{dy}{dx} \neq 12x^3 - 24x^2$

Part (a)(ii)

A1ft: Correct follow through $\frac{d\left(9x^3 - 24x^2\right)}{dx} = 27x^2 - 48x$

Part (b)

M0: For failing to substitute $x = 2$ into $\frac{dy}{dx}$.

A0: Follows M0

Part (c)

M1: This is implied as the candidate states $\frac{d^2y}{dx^2} = 8$ (and the working seen in (b) confirms this).

A1: For a correct calculation, a valid reason and a correct conclusion for their $\left.\frac{d^2y}{dx^2}\right|_{x=2}$

## Student Response C

(2)

1 a)(i) $\dfrac{d}{dx}(3x^4 - 8x^3 - 3)$

$= 12x^3 - 24x^2$

(ii) $\dfrac{d^2y}{dx^2} = 36x^2 - 48x$.

b) $\dfrac{d}{dx}$ at $x = 2 = 12 \times 8 - 24 \times 4$.

$= 96 - 96$.

$= 0$.

c) $\dfrac{d^2y}{dx^2}$ at $x = 2,\ = 36 \times 4 - 48 \times 2$

$= 144 - 96$.

$= 48$.

$\Rightarrow$ ~~St~~ Minimum point $\because \dfrac{d^2y}{dx^2} > 0$.

**6/7**

Examiner Comments:

Part (a)(i)

M1: Differentiates to a cubic form.

A1: Correct $\dfrac{dy}{dx} = 12x^3 - 24x^2$

Part (a)(ii)

A1: Correct $\dfrac{d^2y}{dx^2} = 24x^2 - 48x$

Part (b)

M1: For substituting $x = 2$ into their $\dfrac{dy}{dx}$

A0: Although the candidate achieved $\dfrac{dy}{dx} = 0$, no conclusion is given.

Part (c)

M1: For substituting $x = 2$ into their $\dfrac{d^2y}{dx^2}$

A1: For a correct calculation, a valid reason and a correct conclusion for their $\left.\dfrac{d^2y}{dx^2}\right|_{x=2}$

## Exemplar question 2

**2.**

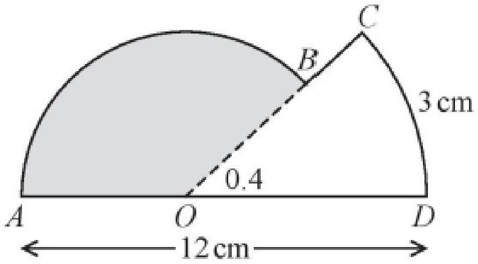


**Figure 1**

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, ∠*COD* = 0.4 radians and *AOD* is a straight line of length 12 cm, find

(a) the length of *OD*,

**(2)**

(b) the area of the shaded sector *AOB*.

**(3)**

**(Total for Question 2 is 5 marks)**

# Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | Uses $s = r\theta \Rightarrow 3 = r \times 0.4$ | M1 | 1.2 |
| | $\Rightarrow OD = 7.5\text{ cm}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm | M1 | 3.1a |
| | Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$ | M1 | 1.1b |
| | $= 27.8\text{cm}^2$ | A1ft | 1.1b |
| | | **(3)** | |
| | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

**A1:** $OD$ = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

**(b)**

**M1:** $AOB = \pi - 0.4$ may be implied by the use of $AOB$ = awrt 2.74 or uses radius is (12 – their '7.5')

**M1:** Follow through on their radius (12 – their $OD$) and their angle

**A1ft:** Allow awrt 27.8 cm$^2$. (Answer 27.75862562). Follow through on their (12 – their '7.5')
Note: Do not follow through on a radius that is negative.

## Student Response A

**2.**

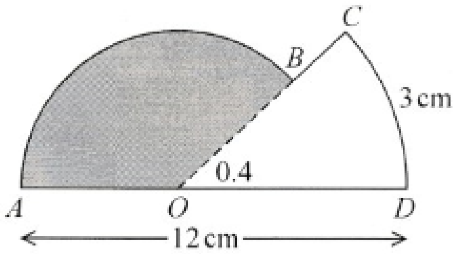


**Figure 1**

The shape $ABCDOA$, as shown in Figure 1, consists of a sector $COD$ of a circle centre $O$ joined to a sector $AOB$ of a different circle, also centre $O$.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and $AOD$ is a straight line of length 12 cm,

(a) find the length of $OD$,

**(2)**

(b) find the area of the shaded sector $AOB$.

**(3)**

(A) $S = r\theta$

$3 = r \times 0.4$

$r = 1.2$

(B) $A = r^2\theta$

$= 10.8^2 \times 5.88$

$= 686$

**2/5**

Examiner Comments:

Part (a)

M1: For attempting to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A0: Incorrect answer.

Part (b)

M1: Uses a radius of 10.8 which is (12 – their $OD$).

M0: Incorrect formula used.

A0: Follows M0

## Student Response B

(a) $3 = r\,0{\cdot}4$

$r = \dfrac{3}{0{\cdot}4} = 7{\cdot}5 \text{ cm}$

(b) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 4{\cdot}5^2 \times 177{\cdot}6$

$= 1800 \text{ cm}^2$

***3/5***

Examiner Comments:

Part (a)

M1: For attempting to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: Correct answer 7.5 cm

Part (b)

M1: Scored for using a radius of 4.5 cm

M0: This is not awarded as the candidate attempts the correct formula but the angle has been found incorrectly. It seems like they used 180 – 0.4, mixing up degrees and radians.

A0: Incorrect answer.

## Student Response C

a i) $y = 3x^4 - 8x^3 - 3$

$\frac{dy}{dx} = 12x^3 - 24x^2$

ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$

b) stationary point at $x=2$ if $\frac{dy}{dx} = 0$.

at $x=2$, $\frac{dy}{dx} = 12(2^3) - 24(2^2)$

$= 96 - 96$

$= 0$ ∴ turning point

c) at $x=2$, $\frac{d^2y}{dx^2} = 48 > 0$ ∴ min

**4/5**

Examiner Comments:

Part (a)

M1: For the implied use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: Correct answer 7.5 (cm). We are condoning the lack of units here in this question.

Part (b)

M1: For using a radius of 4.5 cm

M1: Awarded for a correct area formula with a correct radius and $\theta$.

A0: Inaccurate answer.

Pearson

## Exemplar question 3

**3.** A circle $C$ has equation $x^2 + y^2 - 4x + 10y = k$, where $k$ is a constant.

(a) Find the coordinates of the centre of $C$.

**(2)**

(b) State the range of possible values for $k$.

**(2)**

**(Total for Question 3 is 4 marks)**

---

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Attempts $(x-2)^2+(y+5)^2=....$ | M1 | 1.1b |
| | Centre (2, −5) | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Sets $k+2^2+5^2>0$ | M1 | 2.2a |
| | $\Rightarrow k>-29$ | A1ft | 1.1b |
| | | **(2)** | |
| | | **(4 marks)** | |

**Notes:**

**(a)**

**M1:** Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

**A1:** States the centre is at (2, −5). Also allow written separately $x=2, y=-5$

(2, −5) implies both marks

**(b)**

**M1:** Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is $>0$ or $\geq 0$

**A1ft:** $k>-29$ Also allow $k \geq -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

## Student Response A

**3.** A circle $C$ has equation

$$x^2 + y^2 - 4x + 10y = k$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$.

**(2)**

(b) State the range of possible values for $k$.

**(2)**

$(x-2)^2 - 4 + (y+5)^2 - 10 = k$

$(x-2)^2 + (y+5)^2 = k + 14$

(a) $(2, 5)$

(b) $r^2 = k^2 + 14^2 = k^2 + 196$

***1/4***

Examiner Comments:

Part (a)

M1: For attempting to complete the square $(x-2)^2 + (y+5)^2 = ..$

A0: Incorrect answer. Centre $\neq (2, -5)$.

Part (b)

M0: The candidate fails to deduce, in their case, that $k + 14 > 0$

A0: Follows M0

## Student Response B

**3.** A circle $C$ has equation

$$x^2 + y^2 - 4x + 10y = k$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$. **(2)**

$(x-2)^2 + (y+5)^2 - 29$

(b) State the range of possible values for $k$. **(2)**

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$

a) $(2, -5)$

b) $29$ $\quad k \geqslant 29$

***2/4***

Examiner Comments:

Part (a)

M1: Attempts to complete the square $(x-2)^2 + (y+5)^2 = ..$

A1: Correct answer. Centre = (2, –5).

Part (b)

M0: The candidate fails to deduce, in their case, that $k + 29 > 0$ If the candidate had written $k > -29$ both marks would be gained as the method would have been implied.

A0: Follows M0

# Student Response C

**3.** A circle $C$ has equation

$$x^2 + y^2 - 4x + 10y = k$$

where $k$ is a constant.

(a) Find the coordinates of the centre of $C$.

**(2)**

(b) State the range of possible values for $k$.

**(2)**

$x^2 - 4x + y^2 + 10y = k$

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$

$(x-2)^2 + (y+5) = k+29$

(a) centre $(2, -5)$

(b) $k + 29 > 0$

$k > 29$

***3/4***

Examiner Comments:

Part (a)

M1: Attempts to complete the square $(x-2)^2 + (y+5)^2 = ..$

A1: Correct answer. Centre = (2, –5).

Part (b)

M1: The candidate deduces that $k + 29 > 0$

A0: Incorrect answer.

## Exemplar question 4

**4.** Given that $a$ is a positive constant and

$$\int_{a}^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7,$$

show that $a = \ln k$, where $k$ is a constant to be found.

**(4)**

**(Total for Question 4 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4** | Writes $\int \frac{t+1}{t}\,\mathrm{d}t = \int 1 + \frac{1}{t}\,\mathrm{d}t$ and attempts to integrate | M1 | 2.1 |
| | $= t + \ln t \ (+c)$ | M1 | 1.1b |
| | $(2a + \ln 2a) - (a + \ln a) = \ln 7$ | M1 | 1.1b |
| | $a = \ln \frac{7}{2}$ with $k = \frac{7}{2}$ | A1 | 1.1b |
| | | | **(4 marks)** |

**Notes:**

**M1:** Attempts to divide each term by $t$ or alternatively multiply each term by $t^{-1}$

**M1:** Integrates each term and knows $\int \frac{1}{t}\,\mathrm{d}t = \ln t$. The + $c$ is not required for this mark

**M1:** Substitutes in both limits, subtracts and sets equal to ln7

**A1:** Proceeds to $a = \ln \frac{7}{2}$ and states $k = \frac{7}{2}$ or exact equivalent such as 3.5

## Student Response A

**4.** Given that $a$ is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where $k$ is a constant to be found.

**(4)**

$$\int_a^{2a} t + \frac{1}{t}\,dt = \left[\frac{t^2}{2} + \ln t\right]_a^{2a}$$

$$= \frac{2a^2}{2} + \ln 2a - \frac{a^2}{2} - \ln a$$

$$= \frac{a^2}{2} + \ln a$$

***1/4***

Examiner Comments:

M0: The first term has not been divided by $t$.

M1: For integrating each term and knowing that $\int \frac{1}{t}\,dt = \ln t$

M0: The candidate substitutes in both limits, subtract but don't set equal to ln7.

A0: Follows any M0

## Student Response B

**4.** Given that $a$ is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where $k$ is a constant to be found.

**(4)**

$$\int 1 + \frac{1}{t} = t + \ln t$$

~~$2a + \ln 2a - a - \ln a$~~

$$2a + \ln 2a - a - \ln a = \ln 7$$

***3/4***

Examiner Comments:

M1: For dividing both terms by $t$.

M1: For integrating each term and knowing that $\int \frac{1}{t}\,dt = \ln t$

M1: The candidate substitutes in both limits, subtract and set equal to ln7.

A0: No further progress.

## Student Response C

**4.** Given that $a$ is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where $k$ is a constant to be found.

**(4)**

$$\int \frac{t+1}{t}\,dt = \int \frac{t}{t} + \frac{1}{t}\,dt$$

$$= \int 1 + \frac{1}{t}$$

$$= \left[t + \ln t\right]_a^{2a} = \ln 7$$

$$2a - a + \ln 2a - \ln a = \ln 7$$

$$a + \ln 2 = \ln 7$$

$$a = \ln 7 - \ln 2 = \ln 3\tfrac{1}{2}$$

***4/4***

Examiner Comments:

M1: For dividing both terms by $T$. We can condone the use of $t$ and $T$ in this solution.

M1: For integrating each term and knowing $\int \frac{1}{T}\,dt = \ln T$

M1: The candidate substitutes in both limits, subtract and set equal to ln7.

A1: Acceptable answer of $a = \ln 3\frac{1}{2}$ is seen.

## Exemplar question 5

**5.** A curve $C$ has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve $C$ can be written in the form

$$y = \frac{2x^2 + ax + b}{x+1}, \qquad x \neq -1,$$

where $a$ and $b$ are integers to be found.

**(3)**

**(Total for Question 5 is 3 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5 | Attempts to substitute $= \frac{x+1}{2}$ into $y \Rightarrow y = 4\left(\frac{x+1}{2}\right) - 7 + \frac{6}{(x+1)}$ | M1 | 2.1 |
| | Attempts to write as a single fraction $y = \frac{(2x-5)(x+1)+6}{(x+1)}$ | M1 | 2.1 |
| | $y = \frac{2x^2 - 3x + 1}{x+1} \qquad a = -3, b = 1$ | A1 | 1.1b |
| | | **(3 marks)** | |
| **Notes:** | | | |

**M1:** Score for an attempt at substituting $t = \frac{x+1}{2}$ or equivalent into $y = 4t - 7 + \frac{3}{t}$

**M1:** Award this for an attempt at a single fraction with a correct common denominator.

Their $4\left(\frac{x+1}{2}\right) - 7$ term may be simplified first

**A1:** Correct answer only $y = \frac{2x^2 - 3x + 1}{x+1} \qquad a = -3, b = 1$

## Student Response A

**5.** A curve $C$ has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve $C$ can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where $a$ and $b$ are integers to be found.

**(3)**

$$t = \frac{1}{2}x + \frac{1}{2}$$

$$y = 4\left(\frac{1}{2}x + \frac{1}{2}\right) - 7 + \frac{3}{\frac{1}{2}x + \frac{1}{2}}$$

$$y = 2x + 2 - 7 + 3 \times \left(\frac{2}{x} + \frac{2}{1}\right)$$

$$y = 2x + 2 - 7 + \frac{6}{x} + \frac{6}{1}$$

$$y = \frac{2x^2 + 2x - 7x + 6}{x + 1}$$

$$y = \frac{2x^2 - 5x + 6}{x + 1}$$

**1/3**

Examiner Comments:

M1: For substituting $t = \frac{x}{2} + \frac{1}{2}$ into $y = 4t - 7 + \frac{3}{t}$

M0: An incorrect attempt. The work on the $\dfrac{3}{\frac{x}{2} + \frac{1}{2}}$ term is clearly incorrect.

A0: Incorrect answer.

## Student Response B

**5.** A curve $C$ has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0$$

Show that the Cartesian equation of the curve $C$ can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1$$

where $a$ and $b$ are integers to be found.

**(3)**

$$t = \frac{x+1}{2}$$

$$y = 4\left(\frac{x+1}{2}\right) + 7 + \frac{3}{\left(\frac{x+1}{2}\right)}$$

$$= 2x + 2 + 7 + \frac{6}{(x+1)}$$

$$= \frac{(2x+9)(x+1) + 6}{(x+1)} = \frac{2x^2 + 11x + 15}{x+1}$$

***2/3***

Examiner Comments:

M1: For attempting to substitute $t = \frac{x}{2} + \frac{1}{2}$ into $y = 4t - 7 + \frac{3}{t}$

The error on the sign of 7 is an accuracy slip rather than an error in method.

M1: This is a correct attempt at a single fraction with a common denominator of $(x+1)$.

A0: Incorrect answer due to earlier slip.

## Student Response C

**5.** A curve $C$ has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve $C$ can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where $a$ and $b$ are integers to be found.

**(3)**

$$y = 4\left(\frac{x+1}{2}\right) - 7 + 3 \times \frac{2}{(x+1)}$$

$\times(x+1)$ $\qquad (x+1)y = 2(x+1)^2 - 7(x+1) + 6$

$$(x+1)y = 2x^2 + 4x + 2 - 7x - 7 + 6$$

$$y = \frac{2x^2 - 3x + 1}{x+1}$$

***3/3***

Examiner Comments:

M1: Awarded on line one for $t = \dfrac{x+1}{2}$ being substituted into $y = 4t - 7 + \dfrac{3}{t}$

M1: The method mark is not awarded until the last line. What is important here is that all terms on line two are multiplied by $(x+1)$.

A1: Correct answer.

## Exemplar question 6

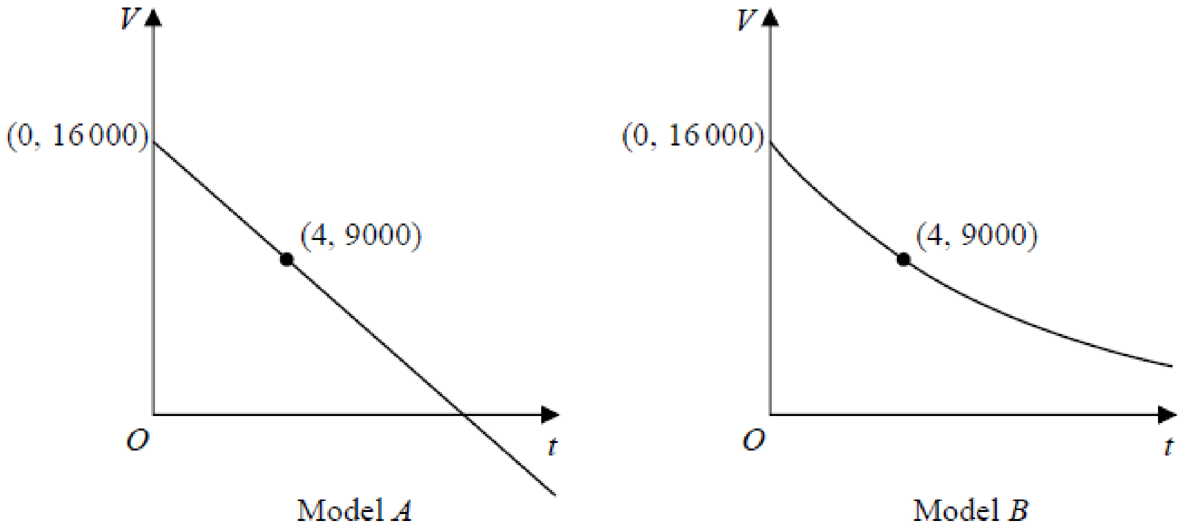
**6.** A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.




(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling. **(2)**

(ii) Write down a limitation of using model A.

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

**(5)**

**(Total for Question 6 is 7 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6 (a)(i)** | 10750 barrels | B1 | 3.4 |
| **(ii)** | Gives a valid limitation, for example<br>• The model shows that the daily volume of oil extracted would become negative as $t$ increases, which is impossible<br>• States when $t=10, V=-1500$ which is impossible<br>• States that the model will only work for $0 \le t \le \frac{64}{7}$ | B1 | 3.5b |
| | | **(2)** | |
| **(b)(i)** | Suggests a suitable exponential model, for example $V = Ae^{kt}$ | M1 | 3.3 |
| | Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$ | dM1 | 3.1b |
| | $\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt $-0.144$ | M1 | 1.1b |
| | $V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$ | A1 | 1.1b |
| **(ii)** | Uses their exponential model with $t = 3 \Rightarrow V =$ awrt 10 400 barrels | B1ft | 3.4 |
| | | **(5)** | |
| | | | **(7 marks)** |

**Notes:**

**(a)(i) B1:** 10750 barrels

**(a)(ii) B1:** See scheme

**(b)(i)**

**M1:** Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for $b$.

**dM1:** Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where $b$ is given as a positive constant and $A + b = 16000$ .

**M1:** Uses a correct method to find all constants in the model.

**A1:** Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0.144t}$ $\quad V = 16000 \times (0.866)^t$ $\quad V = 15800e^{-0.146t} + 200$

**(b)(ii) B1ft:** Follow through on their exponential model

## Student Response A

**(5)**

(ai) ~~[illegible]~~ 10750 ~~11833~~ barrels

(ii) ~~Assumes constant~~ eventually goes to 0 and this is unlikely as they would stop before it reaches 0.

(bi) $V = 16000e^{-t}$

***1/7***

Examiner Comments:

Part (a)(i)

B1: For 10750 barrels.

Part (a)(ii)

B0: "Going down to zero" isn't a limitation of the model. The limitation is that you cannot extract a negative number of barrels of oil.

Part (b)(i)

M0: The candidate suggests an exponential model $V = 16000e^{-t}$ but it isn't suitable as it would not work at (4, 9000).

dM0 A0

Part (b)(ii)

B0 ft No further progress.

## Student Response B

(a) (i) 11000 barrels

(ii) You cannot extract a negative amount of oil

(b) (i) $V = Ae^{-kt}$

$t=0,\ V=16000 \qquad 16000 = A$

$t=4,\ V=9000 \qquad 9000 = Ae^{-k\times 4}$

$9000 = 16000e^{-4k}$

$\frac{\ln 9000}{\ln 16000} = -4k$

$k = -0.235$

$V = 16000e^{-0.235t}$

(ii) $t=3 \qquad V = 7900$ barrels

**4/7**

Examiner Comments:

Part (a)(i)

B0: For 11000 barrels.

Part (a)(ii)

B1: For a correct limitation.

Part (b)(i)

M1: The candidate suggests an exponential model $V = Ae^{-kt}$

dM1: Both $(0, 16000)$ and $(4, 9000)$ are used correctly within the equation.

M0: The method of finding $k$ is incorrect.

A0: Follows M0

Part (b)(ii)

B1 ft: For using their model correctly to find the daily volume after 3 years.

## Student Response C

(3)

a i) 10,750

ii) It is unlikely the extractors will ever be able to extract all the oil => model H goes below 0 suggesting they will

~~b i) y = 9000 = (x − 4)~~

$V = Ae^{-kt}$

$A = 16,000$

$\Rightarrow V = 16000e^{-kt}$

$9000 = 16000 \; e^{-4k}$

$e^{-4k} = 9/16$

$-4k = \ln 9/16$

$4k = \ln 16/9$

$k = \frac{\ln 16/9}{4}$

$\Rightarrow V = 16,000 \; e^{-\frac{1}{4}\ln(16/9)t}$

ii) at $t = 3$,

$V = 16,000 \times e^{-\frac{1}{4}\ln(16/9)\times 3}$

$= 10,392.30485$

$= 10,400$ (3 s.f.)

6/7

Examiner Comments:

Part (a)(i)

B1: For 10750 barrels.

Part (a)(ii)

B0: Both reasons don't give a limitation of the model. The first one is really a statement about the inability to get all oil from an oil field and the second states that going below zero isn't an issue. The limitation is that you cannot extract a negative amount of oil from an oil field.

Part (b)(i)

M1: The candidate suggests an exponential model $V = Ae^{-kt}$

dM1: Both $(0,16000)$ and $(4,9000)$ are used correctly within the equation.

M1: For a correct method to get both constants.

A1: For a correct equation.

Part (ii)

B1 ft: For using their model correctly to find the daily volume after 3 years.

## Exemplar question 7

**7.**

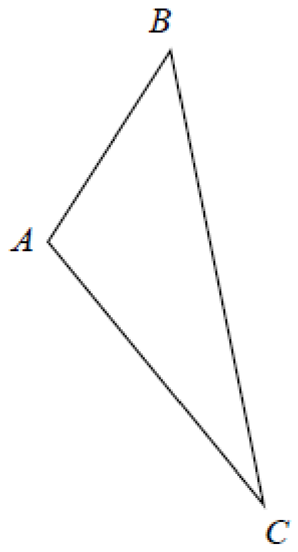


**Figure 2**

Figure 2 Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$, show that $\angle BAC = 105.9°$ to one decimal place.

**(5)**

**(Total for Question 7 is 5 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7 | Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k} = 3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$ | M1 | 3.1a |
| | Attempts to find any one length using 3-d Pythagoras | M1 | 2.1 |
| | Finds all of $\lvert AB\rvert = \sqrt{14},\quad \lvert AC\rvert = \sqrt{61},\quad \lvert BC\rvert = \sqrt{91}$ | A1ft | 1.1b |
| | $\cos BAC = \dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$ | M1 | 2.1 |
| | angle $BAC = 105.9°$ * | A1* | 1.1b |
| | | **(5)** | |
| | | | **(5 marks)** |

**Notes:**

**M1:** Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

**M1**: Attempts to find any one length by use of Pythagoras' Theorem

**A1ft:** Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

**M1**: Attempts to find $BAC$ using $\cos BAC = \dfrac{\lvert AB\rvert^2 + \lvert AC\rvert^2 - \lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

**A1***: This is a show that and all aspects must be correct. Angle $BAC$ = 105.9

## Student Response A

**7.**

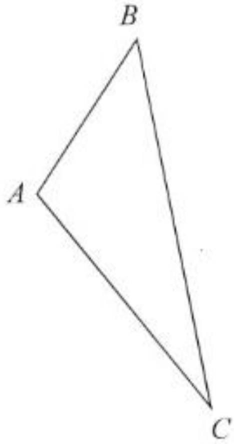



**Figure 2**

Figure 2 shows a sketch of a triangle $ABC$.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

**(5)**

$|\overrightarrow{AB}| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{1^2+9^2+3^2} = \sqrt{91}$

$AC^2 = 91 - 14 = 77$

$AC = \sqrt{77}$

$\cos\theta = \frac{14+91-77}{2 \times 14 \times 91} \Rightarrow \theta = 89.4°$

***1/5***

Examiner Comments:

M0: No attempt to find $\overrightarrow{AC}$.

M1: For attempting to find one length by use of Pythagoras' Theorem. Can be awarded on either $|\overrightarrow{AB}|$ or $|\overrightarrow{BC}|$.

A0ft: There is no $\overrightarrow{AC}$ so this cannot be scored. Using Pythagoras to find $|\overrightarrow{AC}|$ from $|\overrightarrow{BC}|$ and $|\overrightarrow{AB}|$ is incorrect.

M0: There is an attempt at the cosine rule, but the expression suggests that an incorrect formula has been used. Hence the method mark is not awarded.

A0: Incorrect.

## Student Response B

**7.**

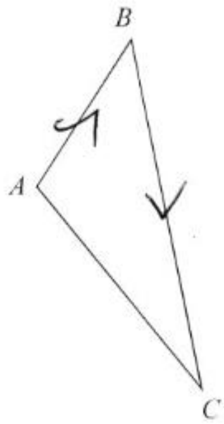



**Figure 2**

Figure 2 shows a sketch of a triangle $ABC$.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

**(5)**


$\overrightarrow{AC} = 3i - 6j + 3k$

$|AB| = \sqrt{14}$ $\quad |\overrightarrow{BC}| = \sqrt{91}$

$|\overrightarrow{AC}| = \sqrt{54}$

$BAC = \cos^{-1}\left(\dfrac{14+54-91}{\sqrt{14}\sqrt{54}}\right) = 105.9$

***3/5***

Examiner Comments:

M1: For attempting to find $\overrightarrow{AC}$. The method can be implied as two of the three components are correct.

M1: For attempting to find one length by use of Pythagoras' Theorem. Can be awarded on $|\overrightarrow{AB}|$ $|\overrightarrow{BC}|$ or $|\overrightarrow{AC}|$

A1ft: For correctly finding all three lengths. We follow through on $\overrightarrow{AC}$.

M0: There is an attempt at the cosine rule, but the expression suggests that an incorrect formula has been used. Hence the method mark is not awarded.

A0: Incorrect.

## Student Response C

7.

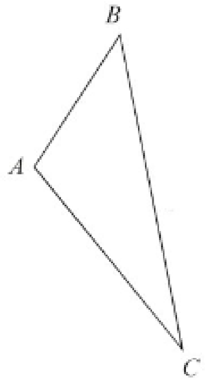


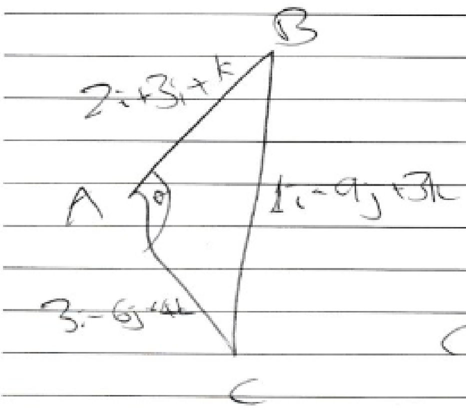

**Figure 2**

Figure 2 shows a sketch of a triangle $ABC$.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

**(5)**




$AB^2 = 2^2 + 3^2 + 1^2 = 14$

$BC^2 = 1^2 + 9^2 + 3^2 = 91$

$AC^2 = 3^2 + 6^2 + 4^2 = 61$

$\cos\theta = \dfrac{14 + 61 - 91}{2\sqrt{14}\sqrt{61}}$

$\cos\theta = -0.27\ldots$

$BAC = 105.9°$

**5/5**

Examiner Comments:

M1: For attempting to find $\overrightarrow{AC}$ This can be seen on the diagram.

M1: For attempting to find one length by use of Pythagoras' Theorem. Can be awarded on $\left|\overrightarrow{AB}\right| \left|\overrightarrow{BC}\right|$ or $\left|\overrightarrow{AC}\right|$

A1ft: For correctly finding all three lengths.

M1: There is an attempt at the cosine rule and the expression suggests that a correct formula has been used. Hence the method mark is awarded.

A1: Correct. We can condone the $\cos\theta = -0.27\ldots$ on the penultimate line.

## Exemplar question 8

**8.** $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that $f(x) = 0$ has a root $\alpha$ in the interval [3.5, 4].

**(2)**

A student takes 4 as the first approximation to $\alpha$.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for $\alpha$, giving your answer to 3 significant figures.

**(2)**

(c) Show that $\alpha$ is the only root of $f(x) = 0$.

**(2)**

**(Total for Question 8 is 6 marks)**

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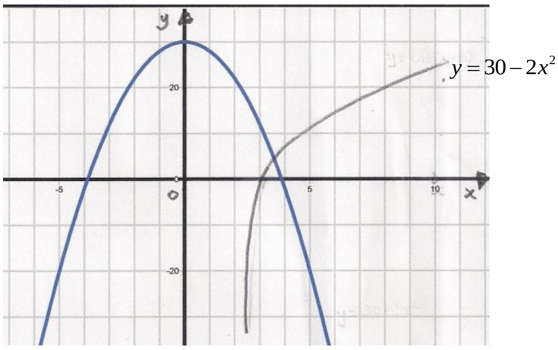
## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8 (a)** | $\mathrm{f}(3.5) = -4.8,\ \mathrm{f}(4) = (+)3.1$ | M1 | 1.1b |
| | Change of sign and function continuous in interval [3.5, 4] $\Rightarrow$ Root * | A1* | 2.4 |
| | | **(2)** | |
| **(b)** | Attempts $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} \Rightarrow \quad x_1 = 4 - \dfrac{3.099}{16.67}$ | M1 | 1.1b |
| | $x_1 = 3.81$ | A1 | 1.1b |
| | $y = \ln(2x - 5)$ | **(2)** | |
| **(c)** |  Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ | M1 | 3.1a |
| | States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow \mathrm{f}(x) = 0$ has just one root | A1 | 2.4 |
| | | **(2)** | |
| | | | **(6 marks)** |

| **Notes:** | |
|---|---|
| **(a)** | |
| **M1:** | Attempts f($x$) at both $x$ = 3.5 and $x$ = 4 with at least one correct to 1 significant figure |
| **A1*:** | f (3.5) and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $\text{f}(3.5)\times\text{f}(4)<0$ or similar with f($x$) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval' |
| **(b)** | |
| **M1:** | Attempts $x_1 = x_0 - \dfrac{\text{f}(x_0)}{\text{f}'(x_0)}$ evidenced by $x_1 = 4 - \dfrac{3.099}{16.67}$ |
| **A1:** | Correct answer only $x_1 = 3.81$ |
| **(c)** | |
| **M1:** | For a valid attempt at showing that there is only one root. This can be achieved by<br>• Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes<br>• Showing that $\text{f}(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points<br>• Sketching a graph of $\text{f}(x) = \ln(2x - 5) + 2x^2 - 30$ |
| **A1:** | Scored for correct conclusion |

## Student Response A

**8.** $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root $\alpha$ in the interval $[3.5, 4]$

**(2)**

A student takes 4 as the first approximation to $\alpha$.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for $\alpha$, giving your answer to 3 significant figures.

**(2)**

(c) Show that $\alpha$ is the only root of $f(x) = 0$

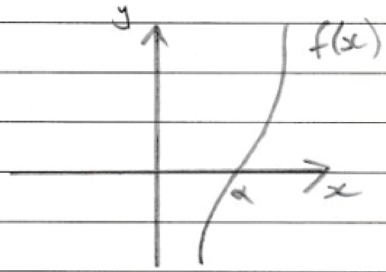
**(2)**

(a) $f(3.5) = -4.8$ $\quad f(4) = 3.1$

Hence Root

(b) $\alpha = 4 + \frac{3.099}{16.67} = 4.19$

(c)




α is only root

**2/5**

Examiner Comments:

Part (a)

M1: For attempting f(3.5) and f(4) with at least one correct to 1 sf.

A0*: This mark cannot be awarded as no reason is given for a root in the interval. The candidate needs to add "as the function is continuous and there is a change of sign, a root must lie in the interval."

Part (b)

M0: Incorrect formula implied by $\alpha = 4 + \frac{3.099}{16.67}$

A0: Follows M0

Part (c)

M1: There is an attempt to sketch $y = f(x)$. It is labelled $f(x)$ and it is the correct shape.

A0: "$\alpha$ is the only root" merely restates the question. A reason must be offered. In this case the candidate should have stated "as $y = f(x)$ only cuts the $x$-axis once, there is only one root, hence $\alpha$ is the only root"

## Student Response B

**8.** $\mathrm{f}(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$

(a) Show that $\mathrm{f}(x) = 0$ has a root $\alpha$ in the interval [3.5, 4]

**(2)**

A student takes 4 as the first approximation to $\alpha$.

Given $\mathrm{f}(4) = 3.099$ and $\mathrm{f}'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for $\alpha$, giving your answer to 3 significant figures.

**(2)**

(c) Show that $\alpha$ is the only root of $\mathrm{f}(x) = 0$

**(2)**

a) $f(3.5) = -4.8$ (1 d.p.)
$f(4) = 3.1$ (1 d.p)

Change of sign ∴ root in interval [3.5, 4]

b) $4 - \frac{3.099}{16.67} = 3.814097781$

$= 3.81$ (3s.f.)

*3/5*

Examiner Comments:

Part (a)

M1: For attempting f(3.5) and f(4) with at least one correct to 1 sf.

A0*: This cannot be awarded as the reason given for a root in the interval is not quite correct. The candidate needs to add about the function being continuous.

Part (b)

M1: Correct formula implied by $\alpha = 4 - \frac{3.099}{16.67}$

A1: 3.81

Part (c)

M0 A0 No attempt.

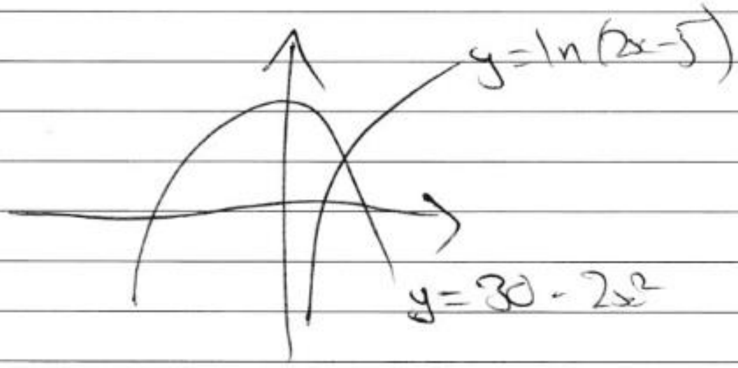
## Student Response C

(a) $f(3.5) = -5$. $f(4) = +3$

Due to a change of sign and function is continuous there is a root in between

(b) $\alpha = 4 - \frac{3.099}{16.76} = 3.82$

(c)




$\ln(2x-5) = 30 - 2x^2$ has one root (as they meet in one place)

hence $\ln(2x-5) + 2x^2 - 30 = 0$ has only one root

**4/5**

Examiner Comments:

Part (a)

M1: For attempting f(3.5) and f(4) with at least one correct to 1 sf.

A1*: Fully correct values with valid reason and conclusion.

Part (b)

M1: Correct formula implied by $\alpha = 4 - \dfrac{3.099}{16.76}$ The slip on the denominator is an accuracy error.

A0: Incorrect answer due to slip.

Part (c)

M1: There is an attempt to sketch both $y = 30 - 2x^2$ and $y = \ln(2x - 5)$. They are both correct.

A1: The reason and conclusion are acceptable for the award of this mark.

## Exemplar question 9

**9.** (a) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta,\ \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$.

**(4)**

(b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions.

**(1)**

**(Total for Question 9 is 5 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 9(a) | $\tan\theta + \cot\theta \equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ | M1 | 2.1 |
| | $\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ | A1 | 1.1b |
| | $\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$ | M1 | 2.1 |
| | $\equiv 2\text{cosec}2\theta$ * | A1* | 1.1b |
| | | **(4)** | |
| (b) | States $\tan\theta + \cot\theta = 1 \Rightarrow \sin 2\theta = 2$ AND no real solutions as $-1 \le \sin 2\theta \le 1$ | B1 | 2.4 |
| | | **(1)** | |
| | | **(5 marks)** | |

**Notes:**

**(a)**

**M1:** Writes $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

**A1:** Achieves a correct intermediate answer of $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

**M1:** Uses the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

**A1*:** Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta + \cot\theta \equiv \tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{\tan^2\theta + 1}{\tan\theta} \equiv \dfrac{\sec^2\theta}{\tan\theta} \equiv \dfrac{1}{\cos^2\theta \times \dfrac{\sin\theta}{\cos\theta}} \equiv \dfrac{1}{\cos\theta \times \sin\theta}$ then as the main scheme.

**(b)**

**B1:** Scored for sight of $\sin 2\theta = 2$ and a reason as to why this equation has no real solutions.

Possible reasons could be $-1 \le \sin 2\theta \le 1$........and therefore $\sin 2\theta \ne 2$

or $\sin 2\theta = 2 \Rightarrow 2\theta = \arcsin 2$ which has no answers as $-1 \le \sin 2\theta \le 1$

## Student Response A

a) $\tan\theta + \cot\theta = \tan\theta + \frac{1}{\tan\theta}$

$= \tan^2\theta + 1$

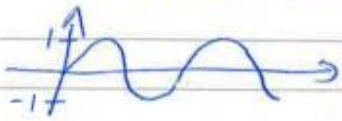
$= \operatorname{cosec}^2\theta$

$= 2\operatorname{cosec} 2\theta$

b) $2\operatorname{cosec} 2\theta = 1 \Rightarrow \operatorname{cosec} 2\theta = 1/2$

$\Rightarrow \sin 2\theta = 2$

No answers max $\sin 2\theta = 1$




***1/5***

Examiner Comments:

Part (a)

M0: This mark is not awarded. The first method mark is awarded when the candidate gets an expression in just $\sin\theta$ and $\cos\theta$.

A0M0A0: No further marks are possible.

Part (b)

B1: The candidate makes a correct deduction that $\sin 2\theta = 2$ A correct explanation is offered and a conclusion is written down.

## Student Response B

a) $LHS = \tan\theta + \cot\theta$

$= \tan\theta + \frac{1}{\tan\theta}$

$= \frac{\tan^2\theta + 1}{\tan\theta}$

$= \frac{\sec^2\theta}{\tan\theta} = \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} = \frac{1}{\sin\theta\cos\theta}$

$RHS = 2\operatorname{cosec}2\theta = \frac{2}{\sin 2\theta} = \frac{2}{2\sin\theta\cos\theta}$

b) $\tan\theta + \cot\theta = 1$

$\frac{\tan^2\theta + 1}{\tan\theta} = 1$

$\tan^2\theta + 1 = \tan\theta$

$\tan^2\theta - \tan\theta + 1 = 0$

$b^2 - 4ac = -3$

**3/5**

Examiner Comments:

Part (a)

M1: This is awarded for using suitable identities to get an expression in just $\sin\theta$ and $\cos\theta$.

A1: For achieving a correct intermediate expression.

M1: The candidate then tackles the proof from the rhs and arrives at the same expression.

A0*: Cannot be awarded as the candidate fails to link up the two expressions and give a conclusion.

Part (b)

B0: "Hence" suggests that the candidate should use part (a). They do not and so B0 is awarded. If they had gone on to say that (as $b^2 - 4ac < 0$) $\tan\theta$ has no roots and hence there are no values for $\theta$ it would be harsh to withhold the mark.

## Student Response C

a) $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\frac{1}{2}\sin 2\theta} = \dfrac{2}{\sin 2\theta}$

$= 2\operatorname{cosec} 2\theta$

b) If $\tan\theta + \cot\theta = 1$

then $2\operatorname{cosec} 2\theta = 1$

$\operatorname{cosec} 2\theta = \frac{1}{2}$

$\dfrac{1}{\sin 2\theta} = \dfrac{1}{2}$

$\sin 2\theta = 2$

↑

Not possible

**4/5**

Examiner Comments:

Part (a)

M1: This is awarded for using suitable identities to get an expression in just $\sin\theta$ and $\cos\theta$.

A1: For achieving a correct intermediate expression

M1: For using the double angle formula $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: The candidate completes the proof with no errors or omissions.

Part (b)

B0: Although the candidate reaches $\sin 2\theta = 2$, "Not possible" is not a sufficient reason to award this mark. The question "Why is it not possible?" must be addressed.

# Exemplar question 10

**10.** Given that $\theta$ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

**(5)**

**(Total for Question 10 is 5 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10** | Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$ | B1 | 2.1 |
| | Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$ | M1 | 1.1b |
| | Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ | A1 | 1.1b |
| | $=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$ | M1 | 2.1 |
| | Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$<br>Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ * | A1* | 2.5 |
| | | **(5 marks)** | |

**Notes:**

**B1:** States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small $h$ or $\delta\theta$

**M1:** Uses the compound angle identity for $\sin(A + B)$ with $A=\theta,\ B=h$ or $\delta\theta$

**A1:** Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

**M1:** Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

**A1*:** Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1,\ \dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10alt** | Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$ | B1 | 2.1 |
| | Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for sin(*A* + *B*) and sin(*A* – *B*) with $A=\theta+\frac{h}{2},\quad B=\frac{h}{2}$ | M1 | 1.1b |
| | Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$ | A1 | 1.1b |
| | $=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$ | M1 | 2.1 |
| | Uses $h\to0,\ \frac{h}{2}\to0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ <br> Therefore the $\text{limit}_{h\to0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$ * | A1* | 2.5 |
| **(5 marks)** | | | |
| **Additional notes:** | | | |
| **A1*:** Uses correct language to explain that $\frac{dy}{d\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to0,\ \frac{h}{2}\to0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$ | | | |

## Student Response A

**10.** Given that $\theta$ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

$$\frac{\Delta Y}{\Delta X} = \frac{\sin(x+h) - \sin x}{h}$$

$$= \frac{\sin x + h\cos x - \sin x}{h}$$

$$= \frac{h\cos x}{h}$$

$$= \cos x$$

***1/5***

Examiner Comments:

B1: A correct form for the gradient (of the chord).

M0: Incorrect attempt at a compound angle identity.

A0: Follows M0

M0: No attempt to write their expression in an appropriate form.

A0*: Incorrect proof.

## Student Response B

**10.** Given that $\theta$ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

**(5)**

$$Gr = \frac{\sin(\theta + \delta\theta) - \sin\theta}{\delta\theta}$$

$$= \frac{\sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta - \sin\theta}{\delta\theta}$$

$$= \frac{\sin\theta\cos\delta\theta}{\delta\theta} + \frac{\cos\theta\sin\delta\theta}{\delta\theta} - \frac{\sin\theta}{\delta\theta}$$

$$= \frac{\sin\theta}{\delta\theta} + \cos\theta - \frac{\sin\theta}{\delta\theta}$$

$$= \cos\theta \qquad \text{QED}$$

***3/5***

Examiner Comments:

B1: A correct form for the gradient (of the chord).

M1: Correct attempt at a compound angle identity for $\sin(\theta + \delta\theta)$.

A1: For a correct expression.

M0: No attempt to write their expression in terms of $\frac{\sin\delta\theta}{\delta\theta}$ and $\frac{\cos\delta\theta - 1}{\delta\theta}$

A0*: Incorrect proof.

## Student Response C

10 $\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$.

$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}$

$= \lim_{h \to 0} \sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right)$

$= \lim_{h \to 0} \sin x \times 0 + \cos x \times 1$

$= \lim_{h \to 0} 0 + \cos x$

**4/5**

Examiner Comments:

B1: A correct form for the gradient (of the chord).

M1: Correct attempt at a compound angle identity for $\sin(x+h)$.

A1: For a correct expression.

M1: For writing their expression in terms of $\frac{\sin h}{h}$ and $\frac{\cos h - 1}{h}$

A0*: Incorrect variable used, $x$ instead of $\theta$, and it lacks a conclusion.

# Exemplar question 11

**11.** An archer shoots an arrow. The height, $H$ metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where $d$ is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

**(3)**

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

**(1)**

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form $A - B(d - C)^2$, where $A$, $B$ and $C$ are constants to be found.

**(3)**

It is decided that the model should be adapted for a different archer. The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

**(2)**

**(Total for Question 11 is 9 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11(a)** | Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$ | M1 | 3.4 |
| | Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times -0.002}$ | dM1 | 1.1b |
| | Distance = awrt 204(m) only | A1 | 2.2a |
| | | **(3)** | |
| **(b)** | States the initial height of the arrow above the ground. | B1 | 3.4 |
| | | **(1)** | |
| **(c)** | $1.8+0.4d-0.002d^2=-0.002\left(d^2-200d\right)+1.8$ | M1 | 1.1b |
| | $=-0.002\left((d-100)^2-10000\right)+1.8$ | M1 | 1.1b |
| | $=21.8-0.002(d-100)^2$ | A1 | 1.1b |
| | | **(3)** | |
| **(d)** | (i) 22.1 metres | B1ft | 3.4 |
| | (ii) 100 metres | B1ft | 3.4 |
| | | **(2)** | |
| | | **(9 marks)** | |

**Notes:**

**(a)**

**M1:** Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$

**M1:** Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator

**A1:** Awrt 204 m only

**(b)**

**B1:** States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

**(c)**

**M1:** Score for taking out a common factor of $-0.002$ from at least the $d^2$ and $d$ terms

**M1:** For completing the square for their $\left(d^2-200d\right)$ term

**A1:** $=21.8-0.002(d-100)^2$ or exact equivalent

**(d)**

**B1ft:** For their '21.8+0.3' = 22.1m

**B1ft:** For their 100m

## Student Response A

**(2)**

(a) $0 = 1.8 + 0.4d - 0.002d^2$

$0.002d^2 - 0.4d = 1.8$

$\times 1000$ $2d^2 - 400d = 1800$

$2d(d - 200) = 1800$

$d = 2000$

(b) $d = 0$ $H = 1.8$

(c) $1.8 + 0.4d - 0.002d^2$

$\times 1000$ $1800 + 400d - 2d^2$

$2d^2 - 400d - 1800$

$\div 2$ $d^2 - 200d - 900$

$(d - 100)^2 - 10900$

(d) (i) $-10900$ (ii) $100$

**2/9**

Examiner Comments:

Part (a)

M1: For setting $H = 0$

dM0: Incorrect method of solving a quadratic.

A0: Incorrect answer.

Part (b)

B0: This answer does not reference the model. It should describe in words what 1.8 represents in the formula.

Part (c)

M0: The candidate does not take out a common factor of $-0.002$

M1: The candidate does complete the square for their $d^2 - 200d$

A0: Incorrect answers.

Part (d)(i)

B0: Incorrect. Even if this candidate had added 0.3 onto their – 10900 it could not have been the maximum height of the arrow.

Part (d)(ii)

B0: Correct value but this question asks for a distance and units are required.

## Student Response B

(a) $0 = 1.8 + 0.4d - 0.002d^2$

$d = 204.4$ m

(b) It is the height of the archer

(c) $1.8 + 0.4d - 0.002d^2$

$= -0.002(d^2 - 20d - 900)$

$= -0.002((d-10)^2 - 1000)$

$= 2 - 0.002(d-10)^2$

(d) $2.3 - 0.002(d-10)^2$

(i) 2.3 m

(ii) 10 m

**_6/9_**

Examiner Comments:

Part (a)

M1: For setting $H = 0$

dM1: Correct method of solving a quadratic implied by awrt 204

A1: Correct answer awrt 204 m.

Part (b)

B0: Incorrect response. The equation models the height of **the arrow** above the ground.

Part (c)

M0: The candidate attempts taking out a common factor of $-0.002$ but it is incorrect for the term in *d*.

M1: The candidate does complete the square for $d^2 - 20d$ which is their $d^2 - 200d$

A0: Incorrect answer.

Part (d)(i)

B1ft: Correct follow through.

Part (d)(ii)

B1ft: Correct follow through.

## Student Response C

**(2)**

$H = 1.8 + 0.4d - 0.002d^2$

(a) $H = 0$ $\quad d = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad a = -0.002$, $b = 0.4$, $c = 1.8$

$= 204.4\text{ m}$ (1dp)

(b) 1.8 is the height in metres from which the arrow was shot.

(c) $-0.002d^2 + 0.4d + 1.8$

$= -0.002(d^2 - 200d) + 1.8$

$= -0.002((d-100)^2 - 10000) + 1.8$

$= 21.8 - 0.002(d-100)^2$

(d) (i) 21.8 m

(ii) 100 m

***8/9***

Examiner Comments:

Part (a)

M1: For setting $H = 0$

dM1: Correct method of solving a quadratic implied by awrt 204

A1: Correct answer awrt 204 m.

Part (b)

B1: Correct response.

Part (c)

M1: The candidate correctly attempts taking out a common factor of $-0.002$

M1: The candidate does complete the square for $d^2 - 200d$

A1: Correct answer.

Part (d)(i)

B0: Incorrect.

Part (d)(ii)

B1: Correct.

## Exemplar question 12

**12.** In a controlled experiment, the number of microbes, $N$, present in a culture $T$ days after the start of the experiment, were counted.

$N$ and $T$ are expected to satisfy a relationship of the form $N = aT^b$, where $a$ and $b$ are constants.

(a) Show that this relationship can be expressed in the form $\log_{10} N = m \log_{10} T + c$, giving $m$ and $c$ in terms of the constants $a$ and/or $b$.

**(2)**

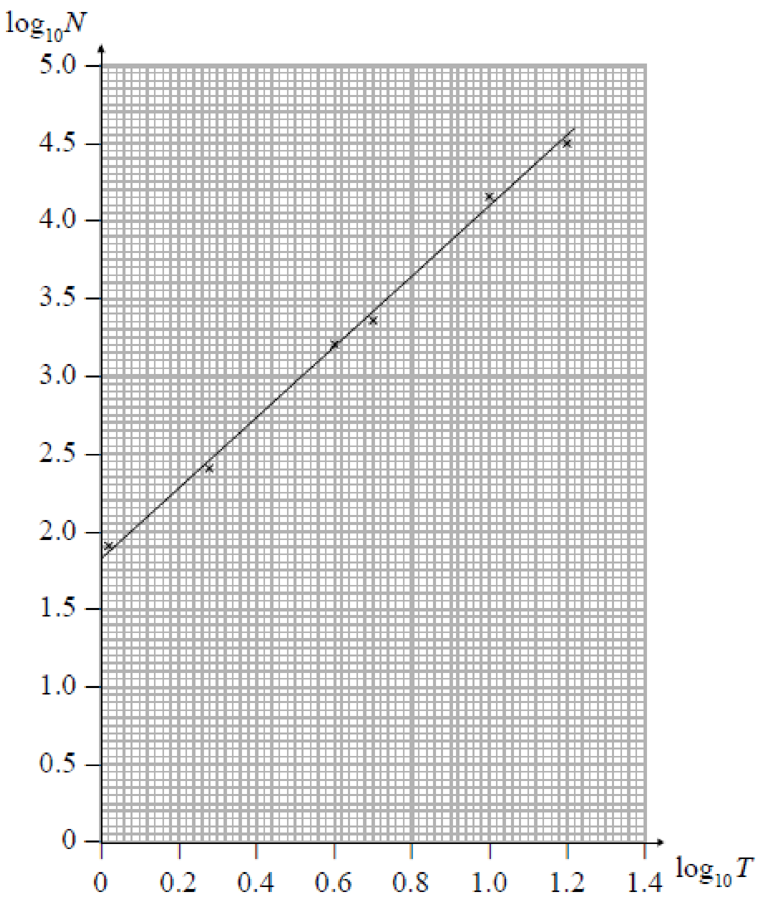



**Figure 3**

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

**(4)**

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

**(2)**

(d) With reference to the model, interpret the value of the constant $a$.

**(1)**

**(Total for Question 12 is 9 marks)**

Pearson

# Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12 (a)** | $N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$ | M1 | 2.1 |
| | $\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Uses the graph to find either $a$ or $b$ $a = 10^{\text{intercept}}$ **or** $b$ = gradient | M1 | 3.1b |
| | Uses the graph to find both $a$ and $b$ $a = 10^{\text{intercept}}$ **and** $b$ = gradient | M1 | 1.1b |
| | Uses $T = 3$ in $N = aT^b$ with their $a$ and $b$ | M1 | 3.1b |
| | Number of microbes $\approx 800$ | A1 | 1.1b |
| | | **(4)** | |
| **(c)** | $N = 1000000 \Rightarrow \log_{10} N = 6$ | M1 | 3.4 |
| | We cannot 'extrapolate' the graph and assume that the model still holds | A1 | 3.5b |
| | | **(2)** | |
| **(d)** | States that '$a$' is the number of microbes 1 day after the start of the experiment | B1 | 3.2a |
| | | **(1)** | |
| | | | **(9 marks)** |

| **Question 12 continued** |
|---|
| **Notes:** |
| **(a)**<br>**M1:** Takes logs of both sides and shows the addition law<br>**M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$ |
| **(b)**<br>**M1:** Uses the graph to find either $a$ or $b$ $a = 10^{\text{intercept}}$ **or** $b$ = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$<br>**M1:** Uses the graph to find both $a$ and $b$ $a = 10^{\text{intercept}}$ **and** $b$ = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$<br>**M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their $a$ and $b$. This is implied by an attempt at $63\times 3^{2.3}$<br>**A1:** Accept a number of microbes that are approximately 800. Allow $800 \pm 150$ following correct work.<br>There is an alternative to this using a graphical approach.<br>**M1**: Finds the value of $\log_{10} T$ from $T$ =3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$<br>**M1**: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48"<br>Accept $\log_{10} N \approx 2.9$<br>**M1:** Finds the value of $N$ from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$<br>**A1:** Accept a number of microbes that are approximately 800. Allow $800 \pm 150$ following correct work |
| **(c)**<br>**M1** For using $N$ = 1000000 and stating that $\log_{10} N = 6$<br>**A1:** Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate"<br>There is an alternative approach that uses the formula.<br>**M1:** Use $N$ = 1000000 in their $N = 63\times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$.<br>**A1:** The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds |
| **(d)**<br>**B1:** Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving $a$ is the value of $N$ at $T$ =1 |

## Student Response A

a) $N = aT^b$

$\log N = \log a + \log T^b$

$\log N = \log a + b\log T$

b) Equation of the line

$\log N = 2{\cdot}3\log T + 1{\cdot}8$

$T = 3 \quad \log N = 2{\cdot}3\log 3 + 1{\cdot}8 = 2.89$

$N = 10^{2.89} = 790$

c) It cannot be used because it isn't accurate

d) The starting number of microbes

**3/9**

Examiner Comments:

Part (a)

M1: For taking logs of both sides and using the addition law.

A0: The candidate needs to use $\log_{10}$ as well as stating $m = b$ and $c = \log_{10} a$

Part (b)

M1: For using the graph to find $b = 2.3$

M0: The error in the value of $a$ means that this mark is not awarded $a = 10^{1.8} \approx 63$ not 1.8

M1: For using their equation with $T = 3$ to find $N$.

A0: Although the answer is in a suitable range, the working is incorrect.

Part (c)

M0: Unsuitable answer. This mark requires some calculation.

A0: Follows M0

Part (d)

B0: Incorrect answer.

## Student Response B

(a) $N = aT^b$

$\log_{10} N = \log_{10}(aT^b)$

$\log_{10} N = \log_{10} a + \log_{10}(T^b)$

$\log_{10} N = b\log_{10} T + c$

(b) when $T=3$, $\log_{10} T = 0.477\ldots$

$\log_{10} N \approx 2.9$

$\therefore N = 794$ (3.sf)

(c) $\log_{10} N$ values only go up to 5

($\log_{10} N = 5 \Rightarrow N = 100,000$)

(d) $\log_{10} a = 1.85$

$a = 70.8$ (3.sf)

**6/9**

Examiner Comments:

Part (a)

M1: For taking logs of both sides and using the addition law.

A0: The candidate needs to state that $m = b$ and $c = \log_{10} a$

Part (b)

M1: For finding the value of $\log_{10} T$ from $T = 3$

M1: For using the line of best fit to find $\log_{10} N \approx 2.9$

M1: For finding $N$ from $\log_{10} N \approx 2.9$

A1: 794 is a suitable answer and it follows correct work.

Part (c)

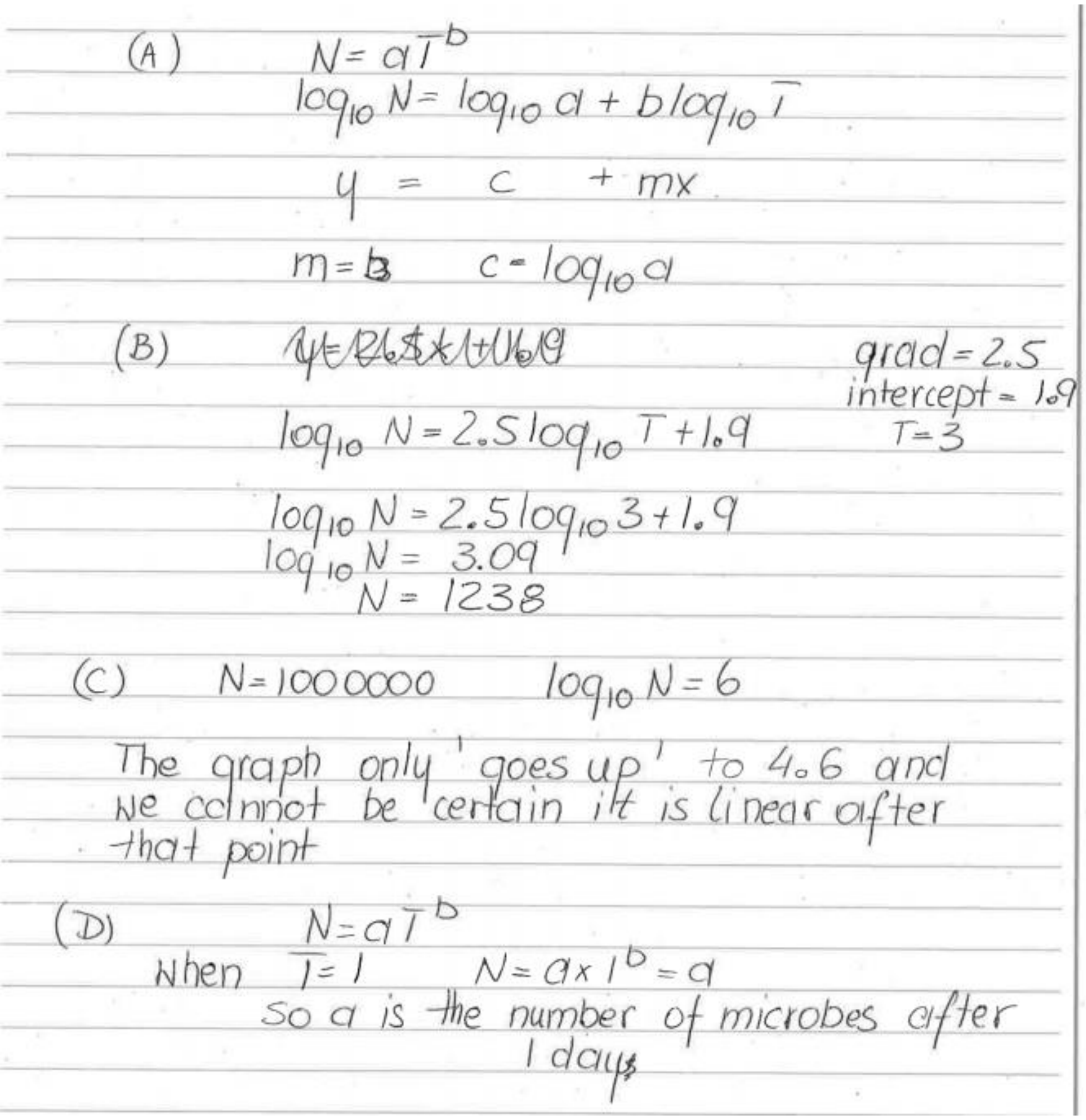
M1: $\log_{10} N$ values only go up to 5 followed by $N = 100\,000$ implies the correct method.

A0: There needs to be a statement that suggests that you cannot assume that the model holds for values greater than shown on the graph.

Part (d)

B0: Incorrect answer. This requires the value to be interpreted, not calculated.

## Student Response C



(A) $N = aT^b$

$\log_{10} N = \log_{10} a + b\log_{10} T$

$y = c + mx$

$m = b \quad c = \log_{10} a$

(B) grad = 2.5, intercept = 1.9, T = 3

$\log_{10} N = 2.5\log_{10} T + 1.9$

$\log_{10} N = 2.5\log_{10} 3 + 1.9$

$\log_{10} N = 3.09$

$N = 1238$

(C) $N = 1000000 \quad \log_{10} N = 6$

The graph only 'goes up' to 4.6 and we cannot be certain it is linear after that point

(D) $N = aT^b$

When $T = 1 \quad N = a \times 1^b = a$

So a is the number of microbes after 1 days

**7/9**

Examiner Comments:

Part (a)

M1: For taking logs of both sides and using the addition law.

A1: For correct work and stating that $m = b$ and $c = \log_{10} a$

Part (b)

M1: This work is equivalent to using the graph to find the gradient or intercept to find an equation linking $N$ with $t$. Intercept = 1.9 suggests a correct method with poor accuracy.

M0: We cannot guess how the gradient has been calculated. As it is incorrect M0 must be awarded. If a correct method was shown on how to find the gradient, this too could have been awarded.

M1: For using $T = 3$ in their $\log_{10} N = 2.5 \log_{10} T + 1.9$ in an attempt to find $N$.

A0: Answer incorrect.

Part (c)

M1: $N = 100\,000 \rightarrow \log_{10} N = 6$ implies the correct method.

A1: Acceptable statement.

Part (d)

B1: Acceptable answer.

## Exemplar question 13

**13.** The curve $C$ has parametric equations

$$x = 2\cos t,\ \ y = \sqrt{3}\cos 2t, \qquad 0 \le t \le \pi.$$

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of $t$.

**(2)**

The point $P$ lies on $C$ where $t = \dfrac{2\pi}{3}$.

The line $l$ is the normal to $C$ at $P$.

(b) Show that an equation for $l$ is $2x - (2\sqrt{3})y - 1 = 0$.

**(5)**

The line $l$ intersects the curve $C$ again at the point $Q$.

(c) Find the exact coordinates of $Q$.
You must show clearly how you obtained your answers.

**(6)**

**(Total for Question 13 is 13 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **13(a)** | Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ | M1 | 1.1b |
| | $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} \quad \left(= 2\sqrt{3}\cos t\right)$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} = \left(-\sqrt{3}\right)$ | M1 | 2.1 |
| | Uses gradient of normal = $-\frac{1}{dy/dx} = \left(\frac{1}{\sqrt{3}}\right)$ | M1 | 2.1 |
| | Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$ | B1 | 1.1b |
| | Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$ | M1 | 2.1 |
| | Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ * | A1* | 1.1b |
| | | **(5)** | |
| **(c)** | Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ | M1 | 3.1a |
| | Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$ | M1 | 3.1a |
| | $\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$ | A1 | 1.1b |
| | Finds $\cos t = \frac{5}{6}$, ~~$-\frac{1}{2}$~~ | M1 | 2.4 |
| | Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t$, $y = \sqrt{3}\cos 2t$, | M1 | 1.1b |
| | $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$ | A1 | 1.1b |
| | | **(6)** | |
| | | **(13 marks)** | |

| **Question 13 continued** |
|---|
| **Notes:** |
| **(a)**<br>**M1:** Attempts $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t\cos t}{\sin t}$<br>**A1:** Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$ |
| **(b)**<br>**M1:** For substituting $t=\frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of $t$<br>**M1:** Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of *l*.<br>**B1:** States or uses (in their tangent or normal) that $P=\left(-1,-\frac{\sqrt{3}}{2}\right)$<br>**M1:** Uses their numerical value of $-1/\frac{dy}{dx}$ with their $\left(-1,-\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at *P*<br>**A1*:** This is a proof and all aspects need to be correct. Correct answer only $2x-2\sqrt{3}y-1=0$ |
| **(c)**<br>**M1:** For substituting $x$ = 2cos $t$ and $y=\sqrt{3}\cos 2t$ into $2x-2\sqrt{3}y-1=0$ to produce an equation in $t$. Alternatively candidates could use $\cos 2t=2\cos^2 t-1$ to set up an equation of the form $y=Ax^2+B$.<br>**M1:** Uses the identity $\cos 2t=2\cos^2 t-1$ to produce a quadratic equation in cos$t$<br>In the alternative method it is for combining their $y=Ax^2+B$ with $2x-2\sqrt{3}y-1=0$ to get an equation in just one variable<br>**A1:** For the correct quadratic equation $12\cos^2 t-4\cos t-5=0$<br>Alternatively the equations in $x$ and $y$ are $3x^2-2x-5=0$ $12\sqrt{3}y^2+4y-7\sqrt{3}=0$<br>**M1:** Solves the quadratic equation in $\cos t$ (or $x$ or $y$) and rejects the value corresponding to *P*.<br>**M1:** Substitutes their $\cos t=\frac{5}{6}$ or their $t=\arccos\left(\frac{5}{6}\right)$ in $x$ = 2cos $t$ and $y=\sqrt{3}\cos 2t$<br>If a value of $x$ or $y$ has been found it is for finding the other coordinate.<br>**A1:** $Q=\left(\frac{5}{3},\frac{7}{18}\sqrt{3}\right)$. Allow $x=\frac{5}{3}, y=\frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents. |

## Student Response A

a) $\frac{dx}{dt} = -2\sin t \qquad \frac{dy}{dt} = -2\sqrt{3}\sin 2t$

$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t}$

b) $\frac{dy}{dx} = -\sqrt{3}$

$y = -\sqrt{3}x + c \quad @ \left(-1, -\frac{\sqrt{3}}{2}\right)$

$-\frac{\sqrt{3}}{2} = \sqrt{3} + c$

$c = -\frac{3\sqrt{3}}{2}$

$y = -\sqrt{3}x - \frac{3\sqrt{3}}{2}$

$2y = -2\sqrt{3}x - 3\sqrt{3}$

$2\sqrt{3}y = -6x - 3$

***4/13***

Examiner Comments:

Part (a)

M1: For a correct attempt and achieving $\frac{dy}{dx} = k\frac{\sin 2t}{\sin t}$

A1: For $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t}$

Part (b)

M1: Correct method implied by $\frac{dy}{dx} = -\sqrt{3}$

M0: The candidate does not use the gradient of the normal.

B1: For $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

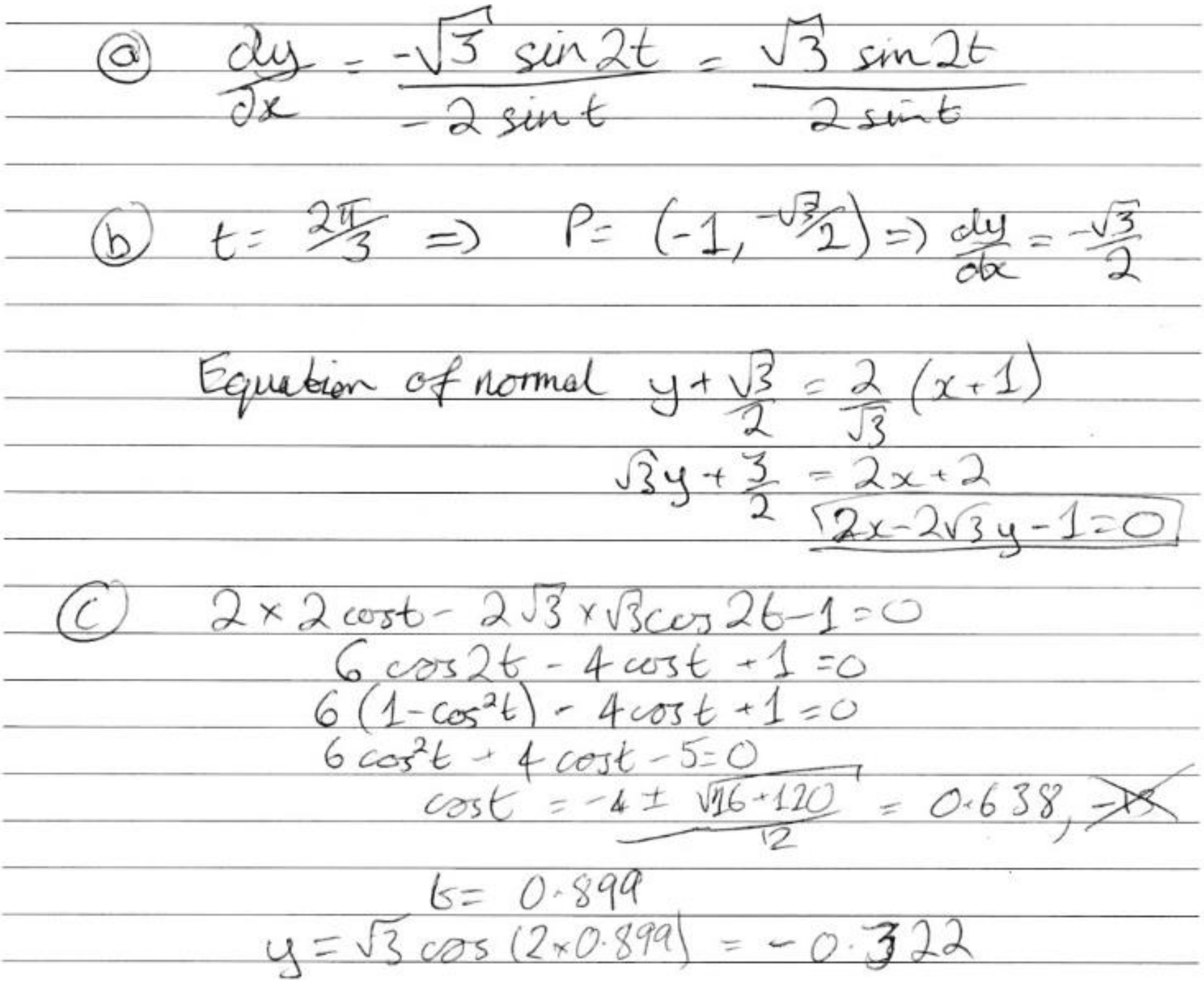
M0: Incorrect form of the normal as incorrect gradient is used.

A0*: Follows M0

Part (c)

No more work.

## Student Response B




***7/13***

Examiner Comments:

Part (a)

M1: For a correct attempt and achieving $\frac{dy}{dx} = k\frac{\sin 2t}{\sin t}$

A0: $\frac{dy}{dx} \neq \frac{\sqrt{3}\sin 2t}{\sin t}$

Part (b)

M1: Correct method implied by $\frac{dy}{dx} = -\frac{\sqrt{3}}{2}$

M1: Correct method implied with embedded gradient of the normal $\frac{2}{\sqrt{3}}$ within equation.

B1: For $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Correct form for the equation of the normal

A0*: Follows error in $\frac{dy}{dx}$

Part (c)

M1: Scored for combining both equations to form an equation in $t$.

M0: Incorrect identity used $\cos 2t = 1 - 2\sin^2 t$

A0: Incorrect equation following the use of the incorrect identity.

M1: For a correct attempt at finding $\cos t$ and rejecting the other value.

M0: No attempt to find the $x$ coordinate.

A0: Incorrect.

## Student Response C

**(6)**

(a) $\frac{dx}{dx} = \frac{+2\sqrt{3}\sin 2t}{-2\sin t}$ ~~(crossed out)~~ $= \frac{2\sqrt{3}\sin t\cos t}{\sin t}$

$= 2\sqrt{3}\cos t$

(b) when $t = \frac{2\pi}{3}$ grad $= 2\sqrt{3}\cos\left(\frac{2\pi}{3}\right) = 2\sqrt{3} \times -\frac{1}{2}$

$= -\sqrt{3}$

Point $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

Equation of normal is $\frac{1}{\sqrt{3}} = \frac{y + \frac{\sqrt{3}}{2}}{x+1}$

$x + 1 = \sqrt{3}y + \frac{3}{2}$

$\sqrt{3}y = x - \frac{1}{2}$  $\boxed{2\sqrt{3}y = 2x - 1}$

(c) Meets C when $2\sqrt{3} \times \sqrt{3}\cos 2t = 2 \times 2\cos t - 1$

$6\cos 2t - 4\cos t + 1 = 0$  $\cos t = 5/6$

$6(2\cos^2 t - 1) - 4\cos t + 1 = 0$  $t = 0.586$

$12\cos^2 t - 4\cos t - 5 = 0$  $Q = (1.66, 0.673)$

***11/13***

Examiner Comments:

Part (a)

M1: For a correct attempt and achieving $\frac{dy}{dx} = k\frac{\sin 2t}{\sin t}$

A1: $\frac{dy}{dx} = 2\sqrt{3}\cos t$

Part (b)

M1: For substituting $t = \frac{2\pi}{3}$ into $\frac{dy}{dx} = 2\sqrt{3}\cos t$

M1: Correct method implied with embedded gradient of the normal $\frac{1}{\sqrt{3}}$ within equation.

B1: For $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Correct attempt at the equation of the normal.

A0*: This mark is withheld as the normal is not in the form given in the question.

Part (c)

M1: Scored for combining both equations to form an equation in $t$.

M1: Correct identity used to form a quadratic equation in cos $t$

A1: Correct equation $12\cos^2 t - 4\cos t - 5 = 0$

M1: Implied by $\cos t = \frac{5}{6}$

M1: For attempting to find the coordinates of $Q$ by using $t$

A0: Incorrect. An exact form is required here.

## Exemplar question 14

**14.**

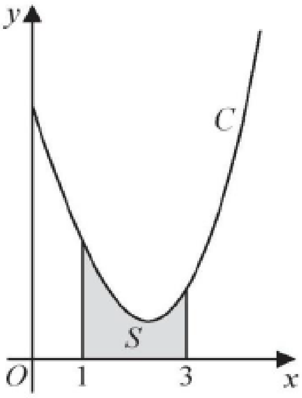



**Figure 4**

Figure 4 shows a sketch of part of the curve $C$ with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region $S$, shown shaded in Figure 4, is bounded by the curve $C$, the line with equation $x = 1$, the $x$-axis and the line with equation $x = 3$.

The table below shows corresponding values of $x$ and $y$, with the values of $y$ given to 4 decimal places as appropriate.

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 1 | 1.5 | 2 | 2.5 | 3 |
| $y$ | 3 | 2.3041 | 1.9242 | 1.9089 | 2.2958 |

(a) Use the trapezium rule, with all the values of $y$ in the table, to obtain an estimate for the area of $S$, giving your answer to 3 decimal places.

**(3)**

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of $S$.

**(1)**

(c) Show that the exact area of $S$ can be written in the form $\frac{a}{b} + \ln c$, where $a$, $b$ and $c$ are integers to be found.

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)

**(6)**

**(Total for Question 14 is 10 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14(a)** | Uses or implies $h = 0.5$ | B1 | 1.1b |
| | For correct form of the trapezium rule = | M1 | 1.1b |
| | $\frac{0.5}{2}\left\{3+2.2958+2\left(2.3041+1.9242+1.9089\right)\right\}=4.393$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Any valid statement reason, for example<br>• Increase the number of strips<br>• Decrease the width of the strips<br>• Use more trapezia | B1 | 2.4 |
| | | **(1)** | |
| **(c)** | For integration by parts on $\int x^2 \ln x \,\mathrm{d}x$ | M1 | 2.1 |
| | $=\frac{x^3}{3}\ln x-\int \frac{x^2}{3}\,\mathrm{d}x$ | A1 | 1.1b |
| | $\int -2x+5\,\mathrm{d}x=-x^2+5x \quad (+c)$ | B1 | 1.1b |
| | All integration attempted and limits used<br>Area of $S = \int_1^3 \frac{x^2 \ln x}{3}-2x+5\,\mathrm{d}x=\left[\frac{x^3}{9}\ln x-\frac{x^3}{27}-x^2+5x\right]_{x=1}^{x=3}$ | M1 | 2.1 |
| | Uses correct ln laws, simplifies and writes in required form | M1 | 2.1 |
| | Area of $S = \frac{28}{27}+\ln 27 \quad (a=28, b=27, c=27)$ | A1 | 1.1b |
| | | **(6)** | |
| | | | **(10 marks)** |

**Question 14 continued**

**Notes:**

**(a)**

**B1:** States or uses the strip width $h$ = 0.5. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule

**M1:** For the correct form of the bracket in the trapezium rule. Must be $y$ values rather than $x$ values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$

**A1:** 4.393

**(b)**

**B1:** See scheme

**(c)**

**M1:** Uses integration by parts the right way around.

Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$

**A1:** $\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$

**B1:** Integrates the $-2x+5$ term correctly $= -x^2 + 5x$

**M1:** All integration completed and limits used

**M1:** Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$

**A1:** Correct answer only $\frac{28}{27} + \ln 27$

## Student Response A

(a) $\frac{3-1}{5}\left\{3+2.2958+2\times(2.3041+1.9242+1.9089)\right\}$

$= 7.028$

(b) Use more decimal places

(c) Area $= \int_1^4 \left(\frac{x^2 \ln x}{3} - 2x + 5\right) dx$

$= \left[\frac{x^3 \ln x}{9} - x^2 + 5x\right]_1^4$

$= \left(\frac{64 \ln 4}{9} - 16 + 20\right) - \left(\frac{1}{9}\ln 1 - 1 + 5\right)$

$= \frac{64 \ln 4}{9}$

**2/10**

Examiner Comments:

Part (a)

B0: Incorrect strip width used.

M1: Correct form of the bracket in the trapezium rule.

A0: Answer incorrect.

Part (b)

B0: Invalid statement.

Part (c)

M0: No attempt to integrate by parts.

A0: Follows M0

B1: For $\int -2x+5\,dx \rightarrow -x^2+5x$

M0: Although the limits are used, this mark requires a correct attempt at integrating the first term.

M0: No attempt to use the log law $a\ln b = \ln b^a$

A0: Incorrect.

## Student Response B

Area $\approx \frac{1}{4}\left(3 + 2.2958 + 2\left(2.5041 + 1.9242 + 1.9079\right)\right]$

$\approx 4.393.$

c) Use more ordinates.

$\frac{1}{3}\int_1^3 x^2 \ln x \, dx.$

$u = \ln x \qquad \frac{dv}{dx} = x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{1}{3}x^3.$

$\frac{1}{3}\left[\frac{1}{3}x^3 \ln x\right]_1^3 - \frac{1}{3}\int_1^3 \frac{1}{3}x^2 \, dx$

$\frac{1}{9}\left[x^3 \ln x\right]_1^3 - \frac{1}{9}\int_1^3 x^2 \, dx.$

$\frac{1}{9}\left[x^3 \ln x\right]_1^3 - \frac{1}{9}\left[\frac{1}{3}x^3\right]_1^3$

$\frac{1}{9}(27\ln 3 - 0) - \frac{1}{9}\left(9 - \frac{1}{3}\right).$

$3\ln 3 - \left(1 - \frac{1}{27}\right)$

$= 3\ln 3 - \frac{26}{27}$

**6/10**

Examiner Comments:

Part (a)

B1: Correct strip width used.

M1: Correct form of the bracket in the trapezium rule.

A1: Correct answer.

Part (b)

B1: Valid statement. This is equivalent to using more strips.

Part (c)

M1: Correct attempt to integrate by parts.

A1: Correct answer.

B0: For not attempting $\int -2x+5\,dx \to -x^2+5x$

M0: Although the limits are used, this mark requires a correct attempt at integrating both terms.

M0: No attempt to use the log law $a\ln b = \ln b^a$

A0: Incorrect.

## Student Response C

a) $\frac{h}{2}(y_0+y_n+2(y_1+y_2 \ldots + y_{n-1}))$

$\frac{0.5}{2}(3+2.2958+2(2.3041+1.9242+1.9089))$

$= 4.393$

b) More strips could be used ~~(more y values)~~ (to give more y values to use).

$\int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \, dx$

$u = \ln x \quad v = \frac{x^3}{9}$

$u' = \frac{1}{x} \quad v' = \frac{x^2}{3}$

$\left[\frac{x^3 \ln x}{9} - \int \frac{x^2}{9}\right]_1^3 - x^2 + 5x$

$\left[\frac{x^3 \ln x}{9} - \frac{x^3}{27} - x^2 - 5x\right]_1^3$

$= \left(3\ln 3 - 1 - 9 - 15\right) - \left(-\frac{1}{27} - 1 - 5\right)$

$= 3\ln 3 - 25 + \frac{163}{27}$

$= \ln 27 - \frac{512}{27}$

$-\frac{512}{27} + \ln 27$

**9/10**

Examiner Comments:

Part (a)

B1: Correct strip width used.

M1: Correct form of the bracket in the trapezium rule.

A1: Correct answer.

Part (b)

B1: Awarded for a valid statement.

Part (c)

M1: Correct attempt to integrate by parts.

A1: Correct answer.

B1: For $\int -2x+5\,\mathrm{d}x \to -x^2+5x$

M1: All integration has been attempted and both limits have been used.

M1: Awarded for using the log law $a\ln b = \ln b^a$ and writing in the required form.

A0: Incorrect due to error on the sign of $5x$

## Exemplar question 15

**15.**

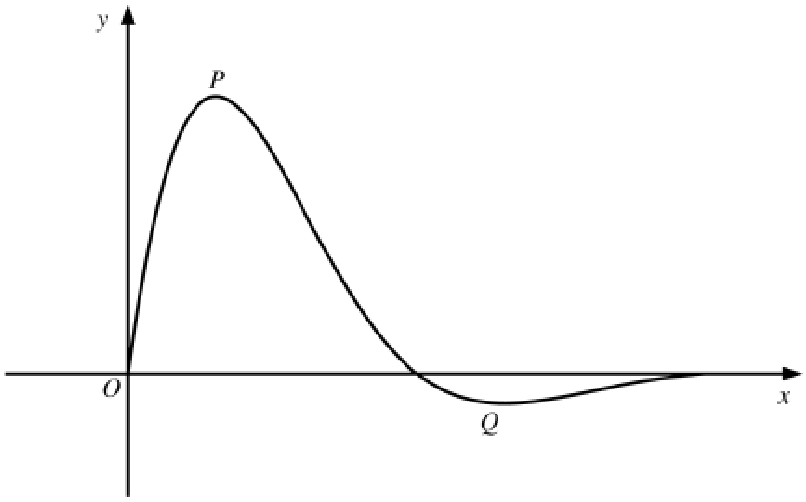



**Figure 5**

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leq x \leq \pi$$

The curve has a maximum turning point at $P$ and a minimum turning point at $Q$, as shown in Figure 5.

(a) Show that the $x$-coordinates of point $P$ and point $Q$ are solutions of the equation $\tan 2x = \sqrt{2}$.

**(4)**

(b) Using your answer to part (a), find the $x$-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$,

(ii) $y = 3 - 2\mathrm{f}(x)$.

**(4)**

**(Total for Question 15 is 8 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15(a) | Attempts to differentiate using the quotient rule or otherwise | M1 | 2.1 |
| | $f'(x)=\dfrac{e^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$ | A1 | 1.1b |
| | Sets $f'(x)=0$ and divides/ factorises out the $e^{\sqrt{2}x-1}$ terms | M1 | 2.1 |
| | Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow \tan 2x=\sqrt{2}$ * | A1* | 1.1b |
| | | **(4)** | |
| (b) | (i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2nd solution | M1 | 3.1a |
| | $x=1.02$ | A1 | 1.1b |
| | (ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1st solution | M1 | 3.1a |
| | $x=0.478$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes:**

**(a)**

**M1:** Attempts to differentiate by using the quotient rule with $u=4\sin 2x$ and $v=e^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u=4\sin 2x$ and $v=e^{1-\sqrt{2}x}$

**A1:** For achieving a correct $f'(x)$. For the product rule $f'(x)=e^{1-\sqrt{2}x}\times 8\cos 2x+4\sin 2x\times -\sqrt{2}e^{1-\sqrt{2}x}$

**M1:** This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

**A1*:** Proceeds to $\tan 2x=\sqrt{2}$. This is a given answer.

**(b) (i)**

**M1:** Solves $\tan 4x=\sqrt{2}$ attempts to find the 2nd solution. Look for $x=\dfrac{\pi+\arctan\sqrt{2}}{4}$

Alternatively finds the 2nd solution of $\tan 2x=\sqrt{2}$ and attempts to divide by 2

**A1:** Allow awrt $x=1.02$. The correct answer, with no incorrect working scores both marks

**(b)(ii)**

**M1:** Solves $\tan 2x=\sqrt{2}$ attempts to find the 1st solution. Look for $x=\dfrac{\arctan\sqrt{2}}{2}$

**A1:** Allow awrt $x=0.478$. The correct answer, with no incorrect working scores both marks

## Student Response A

(a) $y = \dfrac{4\sin 2x}{e^{\sqrt{2}x-1}}$

$\dfrac{dy}{dx} = \dfrac{8\cos 2x}{\sqrt{2}\,e^{\sqrt{2}x-1}}$

TURNING POINT $\dfrac{dy}{dx} = 0 \Rightarrow \tan 2x = \sqrt{2}$

(b) (i) $\tan 2x = \sqrt{2} \Rightarrow x = 0.478$

$f(2x) = \times \frac{1}{2}$ $\quad x = 0.239$

(ii) $3 - 2f(x)$ $\quad x = -0.956$

***0/8***

Examiner Comments:

Part (a)

M0: The candidate does not use a suitable rule to differentiate f($x$).

A0: Follows M0

M0: Does not find an equation in cos 2$x$ and sin 2$x$ from f `($x$) = 0

A0: Incorrect.

Part (b)(i)

M0: Does not attempt to find the second solution.

A0: Follows.

Part (b)(ii)

M0: Does not attempt to find the first solution.

A0: Follows.

## Student Response B

15)a) $f(x) = (4\sin 2x)(e^{1-\sqrt{2}x})$

$u = 4\sin 2x \qquad v = e^{1-\sqrt{2}x}$

$u' = 8\cos 2x \qquad v' = -\sqrt{2}\, e^{1-\sqrt{2}x}$

$\therefore f'(x) = \dfrac{-4\sqrt{2}\sin 2x}{e^{\sqrt{2}x-1}} + \dfrac{8\cos 2x}{e^{\sqrt{2}x-1}}$

b) $4\sqrt{2}\sin 2x = 8\cos 2x$

$\therefore \tan 2x = \sqrt{2}$

~~tan α: i) x = 0.955~~
~~ii) x =~~

i) $x = 4.0969 = 4.10$ (3sf)
ii) $x =$ ~~4.604~~ $4.05$ (3sf)

**4/8**

Examiner Comments:

Part (a)

M1: The candidate uses the product rule to differentiate f($x$).

A1: Correct answer.

M1: Sets f `($x$) = 0, cancels out the $e^{\sqrt{2}x-1}$ term and finds an equation in just cos 2$x$ and sin 2$x$

A1: Proceeds correctly to the given answer.

Part (b)(i)

M0: Attempts the second solution of tan 2$x$ = $\sqrt{2}$ but multiplies by 2

A0: Follows.

Part (b)(ii)

M0A0: Incorrect answer.

## Student Response C

a) $F(x) = \dfrac{4\sin 2x}{e^{\sqrt{2}x-1}}$ $\quad u = 4\sin 2x \quad v = e^{\sqrt{2}x-1}$ (4)

$u' = 8\cos 2x \quad v' = \sqrt{2}e^{\sqrt{2}x-1}$

$$F'(x) = \frac{8e^{\sqrt{2}x-1}\cos 2x - 4\sqrt{2}e^{\sqrt{2}x-1}\sin 2x}{(e^{\sqrt{2}x-1})^2}$$

$F'(x) = 0$

$8e^{\sqrt{2}x-1}\cos 2x - 4\sqrt{2}e^{\sqrt{2}x-1}\sin 2x = 0$

$8\cos 2x - 4\sqrt{2}\sin 2x = 0$

$8\cos 2x = 4\sqrt{2}\sin 2x$

$\tan 2x = \frac{2}{\sqrt{2}}$

$\tan 2x = \sqrt{2}$

~~$\tan 2x = \sqrt{2}$~~

b) i) $y = F(2x)$

$\tan 4x = \sqrt{2}$

$4x = 0.9553,\ 4.0969$

$x = $ ~~$0.239$~~$,\ 1.02$ ← Max

$x = 1.02$

ii) $y = 3 - 2F(x)$

$\tan 2x = \sqrt{2}$

$2x = 0.9553,\ 4.0969$

$x = $ ~~$0.478$~~$,\ 2.04$

↑ Max

$x = 2.04$

**7/8**

Examiner Comments:

Part (a)

M1: The candidate uses the quotient rule to differentiate f($x$).

A1: Correct answer.

M1: Sets f `($x$) = 0, cancels out the $e^{\sqrt{2}x-1}$ term and finds an equation in just cos 2$x$ and sin 2$x$

A1: Proceeds correctly to the given answer.

Part (b)(i)

M1: For attempting the second solution of tan 4$x$ = $\sqrt{2}$

A1: For 1.02

Part (b)(ii)

M1: For attempting to find the first solution of tan 2$x$ = $\sqrt{2}$ (This can be implied by the 0.478 which is subsequently crossed out).

A0: Incorrect answer given. If both answers were given it would still be A0.

# A level Mathematics – Paper 2 (Pure)

## Exemplar question 1

**1.** $$f(x) = 2x^3 - 5x^2 + ax + a.$$

Given that $(x + 2)$ is a factor of f $(x)$, find the value of the constant $a$.

**(Total for Question 1 is 3 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1 | Sets $f(-2)=0 \Rightarrow 2\times(-2)^3 - 5\times(-2)^2 + a\times -2 + a = 0$ | M1 | 3.1a |
| | Solves linear equation $2a - a = -36 \Rightarrow a =$ | dM1 | 1.1b |
| | $\Rightarrow a = -36$ | A1 | 1.1b |
| | | **(3 marks)** | |

**Notes:**

**M1:** Selects a suitable method given that $(x + 2)$ is a factor of $f(x)$
Accept either setting $f(-2) = 0$ or attempted division of $f(x)$ by $(x + 2)$

**dM1**: Solves linear equation in $a$. Minimum requirement is that there are two terms in '$a$' which must be collected to get $..a = .. \Rightarrow a =$

**A1**: $a = -36$

## Student Response A

**1.**

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant $a$.

**(3)**

$2x^2 - 9x + 18$

$x+2 \enclose{longdiv}{2x^3 - 5x^2 + ax + a}$

$2x^3 + 4x^2$

$-9x^2 + ax$

$-9x^2 - 18x$

$ax + 18x$

$36 + 18x$

$ax - 36 + a$

$ax - 36 + a = 0$

$x = \dfrac{a - 36}{a}$

***1/3***

Examiner Comments:

M1: For attempting to divide $f(x)$ by $(x+2)$ and sets the remainder = 0. To score this mark the first two terms, $2x^2 - 9x$ needed to be correct and the remainder had to be set equal to zero.

dM0: This could not be awarded for two reasons. Firstly there could not be a term in $x$ and secondly, the candidate does not reach $a = k$

A0: Follows dM0

## Student Response B

**1.**

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant $a$.

**(3)**

$x+2=0$

$x=-2$

$f(-2)=2(-8)-5(4)-2a-a$

$0=-16-20-3a$

$3a=-36$

$a=-12$

***2/3***

Examiner Comments:

M1: For attempting f (– 2) and settting = 0

dM1: The candidate solves linear equation (with two terms in $a$) $\Rightarrow a = ..$

A0: Incorrect answer.

# Student Response C

**1.**

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant $a$.

**(3)**

$f(-2) = 0$

$2(-2)^3 - 5(-2)^2 + a(-2) + a = 0$

$-36 - 2a + a = 0$

[illegible]

$a = -36$

***3/3***

Examiner Comments:

M1: For attempting f (– 2) and settting = 0

dM1: The candidate solves linear equation (with two terms in $a$) $\Rightarrow a = ..$

A1: Correct

# Exemplar question 2

**2.** Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation $\cos\theta = 2\sin\theta$.

The attempts of two of the students are shown below.

| Student A | Student B |
|---|---|
| $\cos\theta = 2\sin\theta$ | $\cos\theta = 2\sin\theta$ |
| $\tan\theta = 2$ | $\cos^2\theta = 4\sin^2\theta$ |
| $\theta = 63.4°$ | $1 - \sin^2\theta = 4\sin^2\theta$ |
| | $\sin^2\theta = \frac{1}{5}$ |
| | $\sin\theta = \pm\frac{1}{\sqrt{5}}$ |
| | $\theta = \pm 26.6°$ |

(a) Identify an error made by student A.

**(1)**

Student B gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

**(2)**

**(Total for Question 2 is 3 marks)**

# Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2(a) | Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$ It should be $\frac{\sin\theta}{\cos\theta} = \tan\theta$ | B1 | 2.3 |
| | | **(1)** | |
| **(b)** | (i) Shows $\cos(-26.6°) \neq 2\sin(-26.6°)$, so cannot be a solution | B1 | 2.4 |
| | (ii) Explains that the incorrect answer was introduced by squaring | B1 | 2.4 |
| | | **(2)** | |
| | | | **(3 marks)** |

**Notes:**

**(a)**

**B1:** Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta} = \tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta} = \tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta = \frac{1}{2}$ not $\tan\theta = 2$'

Accept also statements such as 'it should be $\cot\theta = 2$'

**(b)**

**B1:** Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta = 2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°) = +ve$ and $2\sin(-26.6°) = -ve$ and stating that they therefore cannot be equal.

**B1:** Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x = 5$ squared gives $x^2 = 25$ which has answers $\pm 5$

## Student Response A

(a) It should be $\cot\theta = 2$ not $\tan\theta = 2$

(b) (i) because you cannot square root a minus number.

(ii) they made a mistake.

***1/3***

Examiner Comments:

Part (a)

B1: For a correct statement.

Part (b)(i)

B0: This is an incorrect statement.

Part (b)(ii)

B0: This is also incorrect.

## Student Response B

a) $\frac{\cos\theta}{\sin\theta}$ is not $\tan\theta$. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **(2)**

b) i) $\cos(-26.6)$ does not equal $2\sin(-26.6)$

ii) When Student B squared both sides they increased the number of solutions.

***2/3***

Examiner Comments:

Part (a)

B1: For a correct statement.

Part (b)(i)

B0: This statement is not enough (It can be regarded as just restating the question in words). There needs to be some evidence to support this such as sight of the values or a response that refers to the different signs of each side.

Part (b)(ii)

B1: This is a correct statement.

## Student Response C

**(2)**

a) $\frac{\cos\theta}{\sin\theta}$ does not equal 2

Meant to be $\tan\theta = \frac{1}{2}$

b)(i) $\cos(-26.6) = 0.894$ $2\sin(-26.6) = -0.896$

Not equal so

~~$\cos\theta$~~ $\cos\theta = 2\sin\theta$ is

not true when $\theta = -26.6$

(ii) Because both sides were squared.

*3/3*

Examiner Comments:

Part (a)

B1: For a correct statement.

Part (b)(i)

B1: The work shown is sufficient to justify the award of this mark.

Part (b)(ii)

B1: This is a correct statement.

## Exemplar question 3

**3.** Given $y = x(2x + 1)^4$, show that $\frac{dy}{dx} = (2x + 1)^n (Ax + B)$, where $n$, $A$ and $B$ are constants to be found.

**(Total for Question 3 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3** | Attempts the product and chain rule on $y = x(2x+1)^4$ | M1 | 2.1 |
| | $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$ | A1 | 1.1b |
| | Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$ | M1 | 1.1b |
| | $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$ | A1 | 1.1b |
| | | | **(4 marks)** |

**Notes:**

**M1:** Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

**A1:** $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

**M1:** Takes out a common factor of $(2x+1)^3$

**A1:** The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

## Student Response A

**3.** Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x+1)^n(Ax+B)$$

where $n$, $A$ and $B$ are constants to be found.

**(4)**

$$y = x(2x+1)^4$$

$$\frac{dy}{dx} = 1 \times (2x+1)^4 + x(2x+1)^3$$

$$\frac{dy}{dx} = (2x+1)^3 \cdot (1+x)$$

***1/4***

Examiner Comments:

M1: The candidate applies the product rule and reaches a suitable form.

A0: Incorrect $\frac{dy}{dx}$

M0: Incorrect attempt to take out a common factor of $(2x+1)^3$ They should have written $(2x+1)^3\{2x+1+x\}$

A0: Incorrect.

## Student Response B

**3.** Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where $n$, $A$ and $B$ are constants to be found.

**(4)**

$$y = x(2x+1)^4$$

$$= (2x+1)^4 \times 1 + x\,4(2x+1)^3 \times 2$$

$$= (2x+1)^4 + 8x(2x+1)^3$$

$$= (2x+1)^3\left((2x+1) + 8x\right)$$

***3/4***

Examiner Comments:

M1: The candidate applies the product rule and reaches a suitable form.

A1: Correct $\frac{dy}{dx}$

M1: Correct attempt to take out a common factor of $(2x+1)^3$

A0: Fails to simplify to the required form.

## Student Response C

**3.** Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where $n$, $A$ and $B$ are constants to be found.

**(4)**

$$y = x(2x+1)^4$$

$$u = x \qquad v = (2x+1)^4$$

$$u = 1 \qquad v' = 8(2x+1)^3$$

$$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$$

$$= (2x+1)^3(2x+1+8x)$$

$$= (2x+1)^3(10x+1)$$

**4/4**

Examiner Comments:

M1: The candidate applies the product rule and reaches a suitable form.

A1: Correct $\frac{dy}{dx}$

M1: Correct attempt to take out a common factor of $(2x+1)^3$

A1: Correct expression reached. There is no need to state $n$, $A$ and $B$ in this question.

## Exemplar question 4

**4.** Given

$$f(x) = e^x, \quad x \in \mathbb{R},$$

$$g(x) = 3 \ln x, \qquad x > 0, x \in \mathbb{R},$$

(a) find an expression for gf (x), simplifying your answer.

**(2)**

(b) Show that there is only one real value of $x$ for which $gf(x) = fg(x)$.

**(3)**

**(Total for Question 4 is 5 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4 (a)** | $gf(x) = 3\ln e^x$ | M1 | 1.1b |
| | $= 3x, \ (x \in R)$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | $gf(x) = fg(x) \Rightarrow 3x = x^3$ | M1 | 1.1b |
| | $\Rightarrow x^3 - 3x = 0 \Rightarrow x =$ | M1 | 1.1b |
| | $\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$ | M1 | 2.2a |
| | | **(3)** | |
| | | **(5 marks)** | |

**Notes:**

**(a)**

**M1:** For applying the functions in the correct order

**A1:** The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$
An answer of $3x$ with no working would score both marks

**(b)**

**M1:** Allow the candidates to score this mark if they have $e^{3\ln x} =$ their $3x$

**M1:** For solving their cubic in $x$ and obtaining at least one solution.

**A1:** For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$
or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.
Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

## Student Response A

**4.** Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

**(2)**

(b) Show that there is only one real value of $x$ for which $gf(x) = fg(x)$

**(3)**

(a) $gf(x) = e^{3\ln x}$

$= x^3$

(b) $fg(x) = 3\ln e^x = 3x$

$gf(x) = fg(x)$

$x^3 = 3x$

$x = 3$ only one answer.

***1/5***

Examiner Comments:

Part (a)

M0: The candidate has applied the functions in the wrong order $x^3 = \text{fg}(x)$

A0: Follows M0

Part (b)

M1: The candidate sets $\text{gf}(x) = \text{fg}(x)$. This is allowed from incorrect orders.

M0: The correct cubic is seen but it is not solved appropriately, $x = 3$ is incorrect.

A0: Incorrect.

## Student Response B

**4.** Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

**(2)**

(b) Show that there is only one real value of $x$ for which $gf(x) = fg(x)$

**(3)**

(a) $3\ln e^x = 3x$

(b) $3x = \not{3}\, e^{3\ln x}$

$3x = e^{\ln x^3}$

$3\not{x} = x^{\not{3}2}$

$x^2 = 3$

$x = \sqrt{3}$

*4/5*

Examiner Comments:

Part (a)

M1: For applying the functions in the correct order.

A1: Correct simplified order. We condone the lack of a domain in this case.

Part (b)

M1: The candidate sets $gf(x) = fg(x)$

M1: The correct cubic is seen, which is solved to obtain one solution.

A0: The candidate fails to explain why there is just one answer.

## Student Response C

**4.** Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

**(2)**

(b) Show that there is only one real value of $x$ for which $gf(x) = fg(x)$

**(3)**

4)a) $gf(x) = g(e^x) = 3\ln e^x = 3x$

b) $fg(x) = e^{3\ln x} = e^{\ln x^3} = x^3$

$x^3 = 3x$

$x^2 = 3$ as $x > 0$

$x = \sqrt{3}$ as $x > 0$.

*5/5*

Examiner Comments:

Part (a)

M1: For applying the functions in the correct order.

A1: Correct simplified order. We condone the lack of a domain in this case.

Part (b)

M1: The candidate sets $gf(x) = fg(x)$

M1: The correct cubic is seen, which is solved to obtain one solution.

A1: This is an appropriate explanation as to why there is just one answer.

## Exemplar question 5

**5.** The mass, $m$ grams, of a radioactive substance, $t$ years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}.$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

**(2)**

(b) show that $\frac{dm}{dt} = km$, where $k$ is a constant to be found.

**(2)**

**(Total for Question 5 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times0.5}$ | M1 | 3.4 |
| | $\Rightarrow m = 24.4\text{g}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | States or uses $\frac{d}{dt}\left(e^{-0.05t}\right) = \pm C\,e^{-0.05t}$ | M1 | 2.1 |
| | $\frac{dm}{dt} = -0.05\times25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$ | A1 | 1.1b |
| | | **(2)** | |
| | | **(4 marks)** | |

**Notes:**

**(a)**

**M1:** Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times0.5}$

**A1:** $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks

**(b)**

**M1:** Applies the rule $\frac{d}{dt}\left(e^{kx}\right) = k\,e^{kx}$ in this context by stating or using $\frac{d}{dt}\left(e^{-0.05t}\right) = \pm C\,e^{-0.05t}$

**A1:** $\frac{dm}{dt} = -0.05\times25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$

## Student Response A

5. The mass, $m$ grams, of a radioactive substance, $t$ years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

**(2)**

(b) show that $\frac{dm}{dt} = km$, where $k$ is a constant to be found.

**(2)**

(a) $t = 0.6$ $m = 25e^{-0.05 \times 0.6} = 24.3g$

(b) $m = 25 \times e^{-0.05t}$

$\frac{dm}{dt} = -1.25te^{-0.05t}$ $\therefore k = -1.25$

***0/5***

Examiner Comments:

Part (a)

M0: The candidate does not substitute $t = 0.5$ or equivalent into $m = 25e^{-0.05t}$

A0: Follows the award of M0

Part (b)

M0: The candidate fails to differentiate into the form $\frac{dm}{dt} = \pm Ae^{-0.05t}$

A0: Incorrect value for $k$.

## Student Response B

**5.** The mass, $m$ grams, of a radioactive substance, $t$ years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

**(2)**

(b) show that $\frac{dm}{dt} = km$, where $k$ is a constant to be found.

**(2)**

(a) $t = 0.5 \quad M = 25e^{-0.05 \times 0.5} = 24g$

(b) $M = 25e^{-0.05t}$

$\frac{dM}{dt} = 25 \times -0.05e^{-0.05t}$

$= -1.25e^{-0.05}$

$k = -1.25$

**2/5**

Examiner Comments:

Part (a)

M1: For substituting $t = 0.5$ into $m = 25e^{-0.05t}$

A0: 24 g lacks the accuracy. As a specific accuracy is not stated in the question, 3sf must be used as stated in the rubric on the front of the paper.

Part (b)

M1: The candidate differentiates into an appropriate form $\frac{dm}{dt} = \pm Ae^{-0.05t}$

A0: Incorrect value for $k$.

## Student Response C

**5.** The mass, $m$ grams, of a radioactive substance, $t$ years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

**(2)**

(b) show that $\frac{dm}{dt} = km$, where $k$ is a constant to be found.

**(2)**

(a) $t = \frac{1}{2}$ $\quad m = 25e^{-0.025} = 24.38g$

(b) $m = 25e^{-0.05t}$

$\frac{dm}{dt} = 25 \times -0.05\, e^{-0.05t}$

$= -0.05 \times 25e^{-0.05t}$

$= -0.05m$

$k = -0.05$

**4/5**

Examiner Comments:

Part (a)

M1: For substituting $t = \frac{1}{2}$ into $m = 25e^{-0.05t}$

A1: Awrt 24.4 g

Part (b)

M1: The candidate differentiates into an appropriate form $\frac{dm}{dt} = \pm Ae^{-0.05t}$

A1: Correct answer and states $k = -0.05$

## Exemplar question 6

**6.** For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case. The first one has been done for you.

| |
|---|
| The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has 2 real roots.<br>Sometimes true.<br>It only has 2 real roots when $b^2 - 4ac > 0$.<br>When $b^2 - 4ac < 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots. |

(i) When a real value of $x$ is substituted into $x^2 - 6x + 10$, the result is positive.

**(2)**

(ii) If $ax > b$ then $x > \dfrac{b}{a}$.

**(2)**

(iii)The difference between consecutive square numbers is odd.

**(2)**

**(Total for Question 6 is 6 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6(i)** | $x^2 - 6x + 10 = (x-3)^2 + 1$ | M1 | 2.1 |
| | Deduces "always true"<br>as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2 + 1 \geqslant 1$ and so is always positive | A1 | 2.2a |
| | | **(2)** | |
| **(ii)** | For an explanation that it need not (always) be true<br>This could be if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$ | M1 | 2.3 |
| | States 'sometimes' and explains if $a > 0$ then $ax > b \Rightarrow x > \frac{b}{a}$<br>if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$ | A1 | 2.4 |
| | | **(2)** | |
| **(iii)** | Difference $= (n+1)^2 - n^2 = 2n+1$ | M1 | 3.1a |
| | Deduces "Always true" as $2n+1$ = (even +1) = odd | A1 | 2.2a |
| | | **(2)** | |
| | | | **(6 marks)** |

| **Notes:** | |
|---|---|
| **(i)** | |
| **M1:** | Attempts to complete the square or any other valid reason. Allow for a graph of $y = x^2 - 6x + 10$ or an attempt to find the minimum by differentiation |
| **A1:** | States always true with a valid reason for their method |
| **(ii)** | |
| **M1:** | For an explanation that it need not be true (sometimes). This could be if $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$ or simply $-3x > 6 \Rightarrow x < -2$ |
| **A1:** | Correct statement (sometimes true) and explanation |
| **(iii)** | |
| **M1:** | Sets up the proof algebraically.<br>For example by attempting $(n+1)^2 - n^2 = 2n+1$ or $m^2 - n^2 = (m-n)(m+n)$ with $m = n+1$ |
| **A1:** | States always true with reason and proof<br>Accept a proof written in words. For example<br>If integers are consecutive, one is odd and one is even<br>When squared $\text{odd} \times \text{odd} = \text{odd}$ and $\text{even} \times \text{even} = \text{even}$<br>The difference between odd and even is always odd, hence always true<br>Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent. |

## Student Response A

| Statement | Always True | Sometimes True | Never True | Reason |
|---|---|---|---|---|
| The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots. | | ✓ | | It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots. |
| (i) When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive. **(2)** | ✓ | | | $x^2 - 6x + 10$ $= (x-3)^2 + 1$ $> 0$ |
| (ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)** | ✓ | ~~✓~~ | | $2x > 4$ $x > 2$ |
| (iii) The difference between consecutive square numbers is odd. **(2)** | ✓ | | | 1, 4, 9, 16, 25 $4 - 1 = 3$ $9 - 4 = 3$ $16 - 9 = 7$ $25 - 16 = 9$ always odd |

**1/6**

Examiner Comments:

Part (a)

M1: For attempting to complete the square.

A0: The candidate states always true but fails to give a valid reason why.

Part (b)

M0: The candidate does not give any reason why it might not be true (sometimes).

A0: Follows M0

Part (c)

M0: No proof offered. Simply considering the first 5 odd numbers scores no marks.

A0: Follow M0

## Student Response B

| Statement | Always True | Sometimes True | Never True | Reason |
|---|---|---|---|---|
| The quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ has 2 real roots. | | ✓ | | It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots. |
| (i) When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive. **(2)** | ✓ | | | $\frac{d}{dx}(x^2 - 6x + 10)$ <br> $= 2x - 6$ <br> $2x - 6 = 0$ <br> $2x = 6$ <br> $x = 3$ $y = 1$ <br> Min $= 1$ ∴ always +ve |
| (ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)** | | ✓ | | If a is -ve it is not true. |
| (iii) The difference between consecutive square numbers is odd. **(2)** | ✓ | | | 1, 4, 9, 16, 25, 36 <br> Odd × Odd = Odd <br> Even × Even = Even <br> Even - Odd = Odd <br> Odd - Even = Odd <br> ∴ Difference is always odd. |

**3/6**

Examiner Comments:

Part (a)

M1: For attempting to find the minimum point.

A0: This is an incomplete proof. There must be some further work showing that $(3,1)$ is the minimum and hence $x^2 - 6x + 10 > 0$ for all $x$. Acceptable work could be to include a U shaped graph or use of the second derivative.

Part (b)

M1: The candidate suggests a reason why it might not be true (sometimes).

A0: A statement is required for $a > 0$ $ax > b \Rightarrow x > \frac{b}{a}$

Part (c)

M1: This is an example of a proof written in words. Two of the three lines are present.

A0: The proof lacks the explanation that in consecutive integers, one is odd and one is even.

## Student Response C

| Statement | Always True | Sometimes True | Never True | Reason |
|---|---|---|---|---|
| The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots. | | ✓ | | It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots. |
| (i) When a real value of $x$ is substituted into $x^2 - 6x + 10$ the result is positive. **(2)** | ✓ | | | $x^2 - 6x + 10$ <br> $= (x-3)^2 - 3^2 + 10$ <br> $= (x-3)^2 + 1$ <br> As $(x-3)^2 \geq 0$ <br> $x^2 - 6x + 10 \geq 1$ <br> Hence $x^2 - 6x + 10 > 0$ for all $x$ |
| (ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)** | | ✓ | | When a is positive <br> $ax > b \Rightarrow x > b/a$ <br> When a is negative <br> $ax > b \Rightarrow x < b/a$ |
| (iii) The difference between consecutive square numbers is odd. **(2)** | ✓ | | | The square numbers go odd, even, odd, even, odd, even... As the difference is always (odd – even) or (even – odd) the answer is always odd. |

***5/6***

Examiner Comments:

Part (a)

M1: For attempting to complete the square.

A1: Acceptable proof and explanation.

Part (b)

M1: The candidate suggests a reason why it might not be true (sometimes).

A1: Ticks sometimes true and provides both scenarios.

Part (c)

M1: This is an example of a proof written in words. Two of the three lines are present. It was decided that there was just enough evidence to suggest that consecutive numbers are odd followed by even.

A0: The proof lacks the explanation as to why the square numbers go up odd, even etc.

## Exemplar question 7

**7.** (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where $k$ is a rational constant to be found.

**(4)**

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of $x$.

**(1)**

**(Total for Question 7 is 5 marks)**

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| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$ | M1 | 2.1 |
| | $\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$ | M1 | 1.1b |
| | $\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$ | A1 | 1.1b |
| | $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used | B1 | 2.4 |
| | | **(1)** | |
| | | **(5 marks)** | |

**Notes:**

**(a)**

**M1:** Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

**M1:** For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

**A1:** Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

**A1:** $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

**(b)**

**B1:** The expansion is valid for $\lvert x\rvert < 4$, so $x=1$ can be used

## Student Response A

7. (a) Use the binomial expansion, in ascending powers of $x$, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where $k$ is a rational constant to be found.

**(4)**

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of $x$.

**(1)**

(a) $\sqrt{4-x} = 4^{\frac{1}{2}}(1-x)^{\frac{1}{2}}$

$= 2\left(1 - \frac{1}{2}x + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(-x)^2\right)$

$= 2\left(1 - \frac{1}{2}x - \frac{1}{4}x^2\right)$

$= 2 - \frac{1}{4}x - \frac{1}{8}x^2$

(b) Yes because when $x=1$ $\sqrt{4-1} = \sqrt{3}$

***1/5***

Examiner Comments:

Part (a)

M1: Scored for an attempt to take out a common factor of 4

M0: Incorrect attempt at the binomial expansion. (There should be a 2! on the denominator of the term in $x^2$).

A0: Incorrect expression inside the brackets.

A0: Incorrect expansion.

Part (b)

B0: There is no reference to the validity of the expansion.

## Student Response B

7. (a) Use the binomial expansion, in ascending powers of $x$, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where $k$ is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of $x$.

(1)

a) $\sqrt{4-x} = \sqrt{4}\left(1 - x/4\right)^{1/2}$

$= 2\left(1 - x/8 + \frac{(1/2)(-1/2)(-x/2)^2}{2!} + \ldots\right)$

$= 2 - \frac{1}{4}x + \frac{1}{128}x^2$

b) $x = 1$ $\quad \sqrt{3} = 2 - \frac{1}{4} - \frac{1}{128}$

$= 1\frac{95}{128}$

**3/5**

Examiner Comments:

Part (a)

M1: Scored for an attempt to take out a common factor of 4

M1: Correct attempt at the binomial expansion.

A1: Correct expression inside the brackets. There is no requirement to simplify this.

A0: Incorrect expansion.

Part (b)

B0: There is no reference to the validity of the expansion.

## Student Response C

$1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots$

**7.** (a) Use the binomial expansion, in ascending powers of $x$, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where $k$ is a rational constant to be found.

**(4)**

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of $x$.

**(1)**

7) a) $(4-x)^{\frac{1}{2}} = (4(1-\frac{x}{4}))^{\frac{1}{2}}$

$= 2(1-\frac{x}{4})^{\frac{1}{2}}$

$= 2\left(1 - \frac{x}{8} + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}\left(-\frac{x}{4}\right)^2\right)$

$= 2\left(1 - \frac{x}{8} - \frac{x^2}{128}\right)$

$= 2 - \frac{x}{4} - \frac{x^2}{64}$

$k = -\frac{1}{64}$

***5/5***

Examiner Comments:

Part (a)

M1: Scored for an attempt to take out a common factor of 4

M1: Correct attempt at the binomial expansion.

A1: Correct expression inside the brackets. There is no requirement to simplify this.

A1: Correct expansion with statement for *k*

Part (b)

B1: Yes as $|x| < 4$ is sufficient.

## Exemplar question 8

**8.**

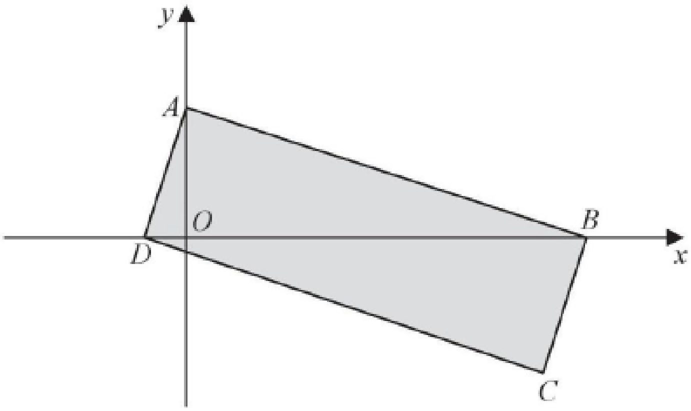



**Figure 1**

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$,

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$,

**(4)**

(b) find the area of the rectangle *ABCD*.

**(3)**

**(Total for Question 8 is 7 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 8 (a) | Gradient $AB = -\frac{2}{5}$ | B1 | 2.1 |
| | $y$ coordinate of $A$ is 2 | B1 | 2.1 |
| | Uses perpendicular gradients $y = +\frac{5}{2}x + c$ | M1 | 2.2a |
| | $\Rightarrow 2y - 5x = 4$ * | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | Uses Pythagoras' theorem to find $AB$ or $AD$<br>Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$ | M1 | 3.1a |
| | Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$ | M1 | 1.1b |
| | area $ABCD = 11.6$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

**Notes:**

**(a) It is important that the student communicates each of these steps clearly**

**B1:** States the gradient of *AB* is $-\frac{2}{5}$

**B1:** States that *y* coordinate of *A* = 2

**M1:** Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

**A1*:** Proceeds to given answer

**(b)**

**M1:** Finds the lengths of *AB* or *AD* using Pythagoras' Theorem. Look for $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$

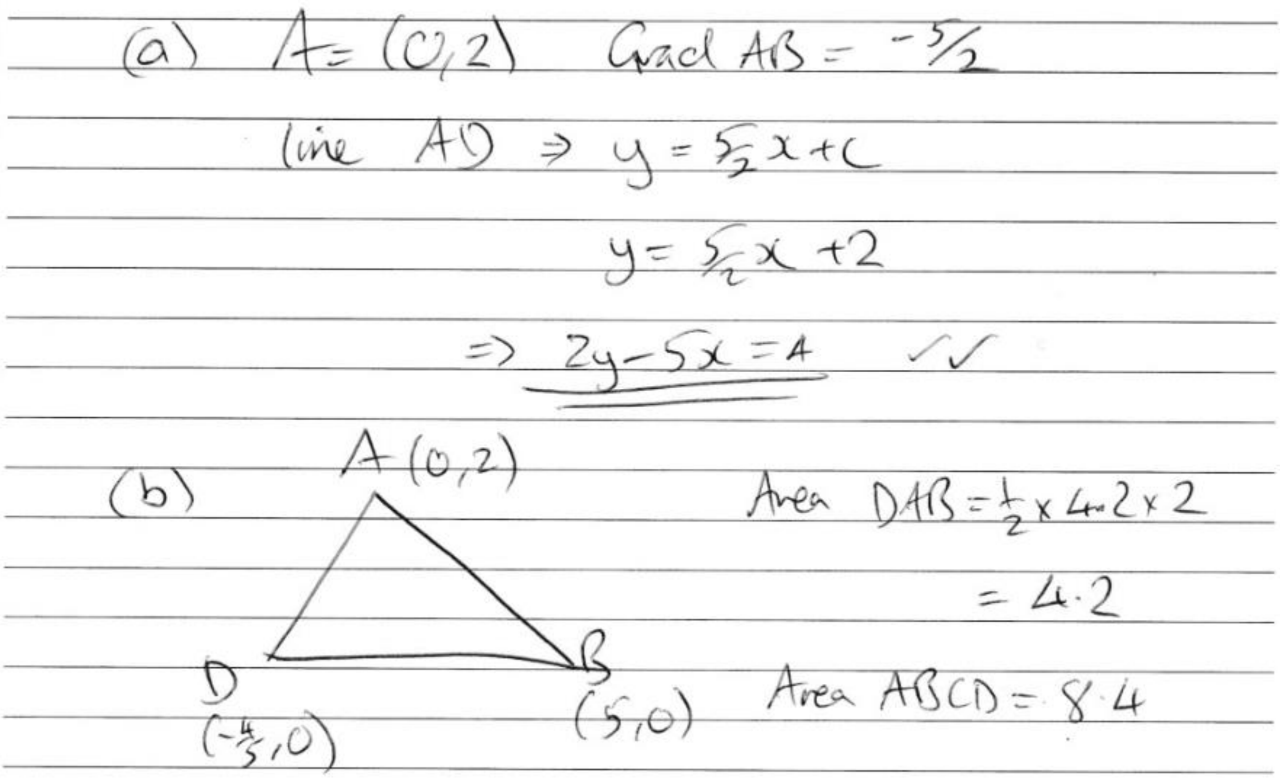
Alternatively finds the lengths *BD* and *AO* using coordinates. Look for $\left(5 + \frac{4}{5}\right)$ and 2

**M1:** For a full method of finding the area of the rectangle *ABCD*. Allow for $AD \times AB$

Alternatively attempts area *ABCD* = $2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}\,'5.8' \times '2'$

**A1:** Area *ABCD* = 11.6 or other exact equivalent such as $\frac{58}{5}$

## Student Response A




***2/7***

Examiner Comments:

Part (a)

B0: Gradient $AB \neq -\frac{2}{5}$

B1: Statement that $A = (0, 2)$ is seen.

M0: The candidate does not use a perpendicular gradient. For their *AB* us of a gradient of $\frac{2}{5}$ should be seen.

A0: Follows incorrect work.

Part (b)

M0: For the incorrect method of finding length *DB* (The 5 and the - 0.8 should be subtracted).

M1: Correct method for finding the area of a rectangle.

A0: Incorrect answer.

## Student Response B

(a) $5y + 2x = 10$

$5y = -2x + 10$

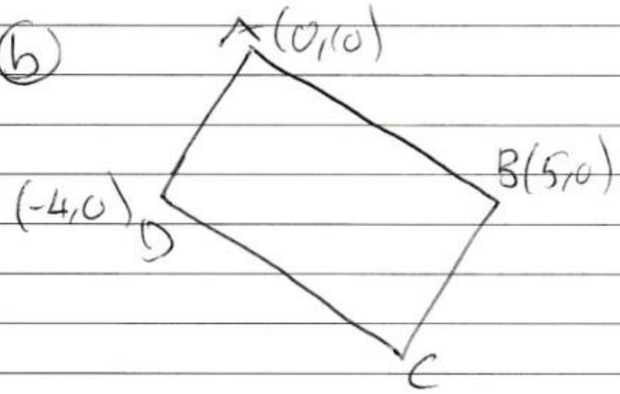
$y = -\frac{2x}{5} + 10$

Grad $AB = -\frac{2}{5}$ $\quad A = (0,0)$

Line $AD$ $\quad y = \frac{5}{2}x + 10$

$2y - 5x = 20$

(b)




$AD = \sqrt{116}$

$AB = \sqrt{125}$

$ABCD = \sqrt{116} \times \sqrt{125}$

**4/7**

Examiner Comments:

Part (a)

B1: Gradient $AB = -\frac{2}{5}$

B0: Incorrect. This candidate states that $A = (0,10)$.

M1: The use of a perpendicular gradient and $A = (0,10)$ to find the equation of line *AD* scores the mark.

A0: Follows incorrect work.

Part (b)

M1: A correct method of finding lengths *AD* and *AB* using Pythagoras' Theorem.

M1: Correct method for finding the area of a rectangle.

A0: Incorrect answer.

## Student Response C

(3)

(a) $5y + 2x = 10$

$y = -\frac{2}{5}x + 2$

$m_{AB} = -\frac{2}{5}$ $\quad m_{AD} = \frac{5}{2}$

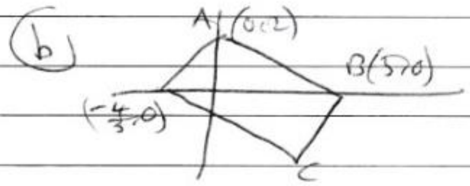
For line AD $m = \frac{5}{2}$, $c = 2$

$y = \frac{5}{2}x + 2$

$2y = 5x + 4$

$2y - 5x = 4$ QED

(b)




$AB = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$

$AD = \sqrt{2^2 + 0.8^2}$

$= \sqrt{5.6}$

Area ABCD $= \sqrt{29} \times \sqrt{5.6}$

$= 12.74$

**6/7**

Examiner Comments:

Part (a)

B1: Gradient $AB = -\frac{2}{5}$

B1: Correct. Implied by $c = 2$

M1: The candidate uses a perpendicular gradient and $c = 2$ to find the equation of line *AD*.

A1: Correct answer following correct work.

Part (b)

M1: A correct method of finding lengths *AD* and *AB* using Pythagoras' Theorem.

M1: Correct method for finding the area of a rectangle.

A0: Incorrect answer due to slip on the length of *AD*.

## Exemplar question 9

**9.** Given that $A$ is constant and

$$\int_1^4 (3\sqrt{x} + A)\ \mathrm{d}x = 2A^2,$$

show that there are exactly two possible values for $A$.

**(Total for Question 9 is 5 marks)**

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## Mark scheme

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **9** | $\int(3x^{0.5}+A)\,\mathrm{d}x = 2x^{1.5}+Ax(+c)$ | | M1<br>A1 | 3.1a<br>1.1b |
| | Uses limits and sets = $2A^2 \Rightarrow (2\times8+4A)-(2\times1+A)=2A^2$ | | M1 | 1.1b |
| | Sets up quadratic and attempts to solve | Sets up quadratic and attempts $b^2-4ac$ | M1 | 1.1b |
| | $\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots | States $b^2-4ac=121>0$ and hence there are two roots | A1 | 2.4 |
| | | | | **(5 marks)** |

**Notes:**

**M1:** Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax(+c)$ where $k$ is a non- zero constant

**A1:** Correct answer but may not be simplified

**M1:** Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

**M1:** Sets up quadratic equation in $A$ and either attempts to solve or attempts $b^2-4ac$

**A1:** Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

## Student Response A

**9.** Given that $A$ is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for $A$.

**(5)**

$$\int_1^4 \left(3x^{\frac{1}{2}} + A\right)dx = \left[\frac{3x^{1\frac{1}{2}}}{1\frac{1}{2}} + \frac{A^2}{2}\right]_1^4$$

$$= \left(2\times 8 + \frac{A^2}{2}\right) - \left(2\times 1 + \frac{A^2}{2}\right)$$

$$= 14$$

$$2A^2 = 14 \Rightarrow A^2 = 7 \Rightarrow A = \pm\sqrt{7}$$

Two Answers

***1/5***

Examiner Comments:

M0: For attempting to integrate but must achieve and answer of the form $kx^{1.5} + Ax$

A0: Follows M0

M0: This cannot be awarded if $\int A \to \frac{A^2}{2}$

M1: The candidate sets up a quadratic equation in $A$ and attempts to solve.

A0: Incorrect solution.

## Student Response B

**9.** Given that $A$ is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for $A$.

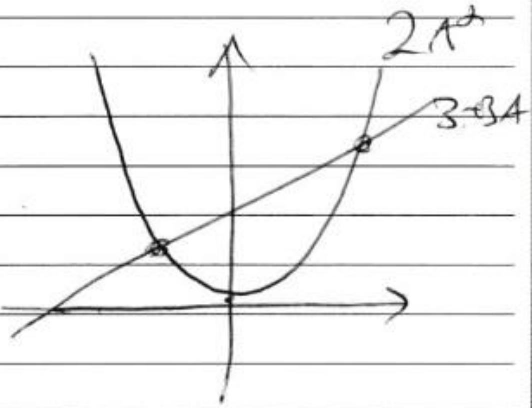
**(5)**

$$\int_1^4 (3x^{\frac{1}{2}}+A)\,dx = 2A^2$$

$$\left[3x^{\frac{1}{2}}+Ax\right]_1^4 = 2A^2$$

$$(6+4A)-(3+1A) = 2A^2$$

$$3+3A = 2A^2$$




two answers as there are two intersections

***2/5***

Examiner Comments:

M0: The candidate fails to integrates to a form $kx^{1.5}+Ax$

A0: Incorrect answr.

M1: For substituting in limits and subtracting. It is also a requirement that $\int A\mathrm{d}x \to Ax$

M1: The candidate sets up a quadratic equation in $A$ and attempts to show that there are two roots. This is an acceptable if alternative method.

A0: Incorrect solution.

## Student Response C

(5)

$$\int_1^4 3\sqrt{x} + A \, dx = 2A^2$$

$$\left[2x^{3/2} + Ax\right]_1^4 = 2A^2$$

$$\left(2(4)^{3/2} + 4A\right) - \left(2(1)^{3/2} + A\right) = 2A^2$$

$$16 + 4A - 2 - A = 2A^2$$

$$14 + 3A = 2A^2$$

$$2A^2 - 3A - 14 = 0$$

$a = 2$

$b = -3$ $\qquad b^2 - 4ac > 0$ if two values of $A$

$c = -14$

$$3^2 - 4 \times 2 \times -14 > 0$$

$$9 + 112 > 0$$

$$121 > 0 \quad \therefore \text{ two values of } A$$

***5/5***

Examiner Comments:

M1: The candidate integrates to a form $kx^{1.5} + Ax$

A1: Integration correct.

M1: For substituting in limits and subtracting. It is also a requirement that $\int A\,\mathrm{d}x \to Ax$

M1: The candidate sets up a quadratic equation in $A$ and attempts to find $b^2 - 4ac$

A1: Correct solution with correct statements.

## Exemplar question 10

**10.** In a geometric series the common ratio is $r$ and sum to $n$ terms is $S_n$.

Given $S_\infty = \frac{8}{7} \times S_6$, show that $1 = \pm\frac{1}{\sqrt{k}}$, where $k$ is an integer to be found.

**(Total for Question 10 is 4 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10** | Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$ | M1 | 2.1 |
| | $\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$ | M1 | 2.1 |
| | $\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$ | M1 | 1.1b |
| | $\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k = 2$) | A1 | 1.1b |
| | | | **(4 marks)** |
| **Notes:** | | | |

**M1:** Substitutes the correct formulae for $S_\infty$ and $S_6$ into the given equation $S_\infty = \frac{8}{7} \times S_6$

**M1:** Proceeds to an equation just in $r$

**M1:** Solves using a correct method

**A1:** Proceeds to $r = \pm\frac{1}{\sqrt{2}}$ giving $k = 2$

## Student Response A

**10.** In a geometric series the common ratio is $r$ and sum to $n$ terms is $S_n$

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where $k$ is an integer to be found.

**(4)**

$$\frac{a}{1-r} = \frac{8}{7}\,\frac{a(1-r^6)}{1-r}$$

$$7(1-r) = 8(1-r^7)$$

$$8r^7 - 7r - 1 = 0$$

$$r = 1$$

***1/4***

Examiner Comments:

M1: The candidate uses the correct formulae to form an equation in *a* and *r*

M0: An incorrect method has been used to create an equation in just *r*

M0: Although $r = 1$ is a solution of their equation in *r*, no method is shown and this mark cannot be awarded.

A0: Incorrect answer.

## Student Response B

**10.** In a geometric series the common ratio is $r$ and sum to $n$ terms is $S_n$

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where $k$ is an integer to be found.

**(4)**

$$S_\infty = \frac{8}{7} \times S_6$$

$$7 \times S_\infty = 8 \times S_6$$

$$\frac{7a}{1-r} = \frac{8a(1-r^6)}{1-r}$$

$$7 = 8 - 8r^6$$

$$8r^6 = 1$$

$$r^6 = \frac{1}{8}$$

[crossed out]

$$r = 0.707$$

***3/4***

Examiner Comments:

M1: The candidate uses the correct formulae to form an equation in *a* and *r*

M1: A correct method has been used to create an equation in just *r*

M1: The answer of $r = 0.707$ implies a correct method as this is the decimal equivalent of $\frac{1}{\sqrt{2}}$

A0: Incorrect answer.

## Student Response C

**10.** In a geometric series the common ratio is $r$ and sum to $n$ terms is $S_n$

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where $k$ is an integer to be found.

**(4)**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_6 = \frac{a(1-r^6)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

$$\frac{8}{7} \times \frac{a(1-r^6)}{1-r} = \frac{a}{1-r}$$

$$\frac{8a(1-r^6)}{7} = a$$

$$\frac{8}{7}(1-r^6) = 1$$

$$1-r^6 = \frac{7}{8}$$

$$\frac{1}{8} = r^6$$

$$\left(\frac{1}{2}\right)^3 = (r^2)^3$$

$$\frac{1}{2} = r^2$$

$$\pm\frac{1}{\sqrt{2}} = r$$

**4/4**

Examiner Comments:

M1: The candidate uses the correct formulae to form an equation in $a$ and $r$

M1: A correct method has been used to create an equation in just $r$

M1: For solving their equation in $r$ using a correct method.

A1: Correct answer. There is no need to state $k$.

# Exemplar question 11

**11.**

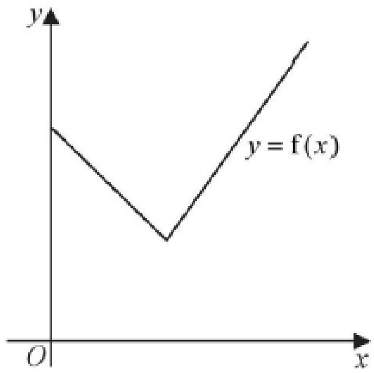



**Figure 2**

Figure 2 shows a sketch of part of the graph $y = \text{f}(x)$ where $\text{f}(x) = 2|3 - x| + 5, \quad x \geq 0$.

(a) State the range of f.

**(1)**

(b) Solve the equation $\text{f}(x) = \frac{1}{2}x + 30$.

**(3)**

Given that the equation $\text{f}(x) = k$, where $k$ is a constant, has two distinct roots,

(c) state the set of possible values for $k$.

**(2)**

**(Total for Question 11 is 6 marks)**

# Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11 (a)** | $\mathrm{f}(x) \geq 5$ | B1 | 1.1b |
| | | **(1)** | |
| **(b)** | Uses $-2(3-x)+5=\frac{1}{2}x+30$ | M1 | 3.1a |
| | Attempts to solve by multiplying out bracket, collect terms etc<br>$\frac{3}{2}x=31$ | M1 | 1.1b |
| | $x=\frac{62}{3}$ only | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | Makes the connection that there must be two intersections.<br>Implied by either end point $k>5$ or $k \leq 11$ | M1 | 2.2a |
| | $\{k : k \in \mathbb{R},\ 5<k \leq 11\}$ | A1 | 2.5 |
| | | **(2)** | |
| | | | **(6 marks)** |

**Notes:**

**(a)**

**B1:** $\mathrm{f}(x) \geq 5$Also allow $\mathrm{f}(x) \in [5, \infty)$

**(b)**

**M1:** Deduces that the solution to $\mathrm{f}(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

**M1:** Correct method used to solve their equation. Multiplies out bracket/ collects like terms

**A1:** $x=\frac{62}{3}$ only. Do not allow 20.6

**(c)**

**M1:** Deduces that two distinct roots occurs when $y=k$ intersects $y=\mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leq 11$

**A1:** Correct solution only $\{k : k \in \mathbb{R},\ 5<k \leq 11\}$

## Student Response A

(a) Range $f(x) \geq 5$

(b) $2(3-x)+5 = \frac{1}{2}x+30$

$6-2x+5 = \frac{1}{2}x+30$

$-19 = 2\frac{1}{2}x$

$x = \frac{38}{5}$

(c) $2|3-x|+5=k$

$6-2x+5=k$

$2x+k-11=0$

Two roots $b^2-4ac>0$

$k^2-4\times2\times-11>0$

$k^2>88$

**2/6**

Examiner Comments:

Part (a)

B1: Correct answer.

Part (b)

M0: Incorrect branch of the line chosen.

M1: Correct method used to solve their equation.

A0: Incorrect answer.

Part (c)

M0: Incorrect method.

A0: Follows M0

## Student Response B

(a) Range $(5, \infty)$ **(2)**

(b) Either $2(3-x)+5=\frac{1}{2}x+30$

$6-2x+5=\frac{1}{2}x+30$

$-19=2\frac{1}{2}x$

$x=-\frac{38}{5}$

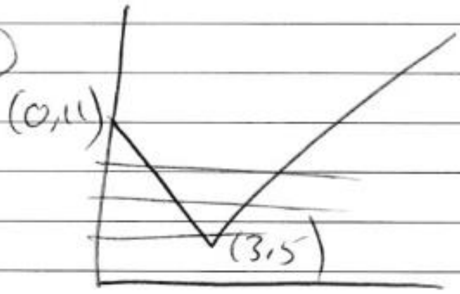
OR $-2(3-x)+5=\frac{1}{2}x+30$

$-6+2x+5=\frac{1}{2}x+30$

$1\frac{1}{2}x=31$

$x=\frac{62}{3}$

(c)




$5<k<11$

***3/6***

Examiner Comments:

Part (a)

B0: Incorrect answer.

Part (b)

M1: Correct branch of the line attempted (as well as the incorrect branch).

M1: Correct method used to solve their equation.

A0: Incorrect. This mark requires $x=\frac{62}{3}$ only.

Part (c)

M1: For one end of the inequality $k>5$

A0: Incorrect.

## Student Response C

a) $f(x) \geqslant 5$

b) $2|3 - x| + 5 = \frac{1}{2}x + 30$

$2(3 - x) + 5 = \frac{1}{2}x + 30$ $\quad$ $2(x - 3) + 5 = \frac{1}{2}x + 30$

$6 - 2x + 5 = \frac{1}{2}x + 30$ $\quad$ $2x - 6 + 5 = \frac{1}{2}x + 30$

$11 - 2x = \frac{1}{2}x + 30$ $\quad$ $2x - 1 = \frac{1}{2}x + 30$

$\frac{5}{2}x = -19$ $\quad$ $\frac{3}{2}x = 31$

~~$x = -\frac{38}{5}$~~ $\quad$ $x = \frac{62}{3}$

$x \geqslant 0$

c) $k \leqslant 11$

**5/6**

Examiner Comments:

Part (a)

B1: Correct answer.

Part (b)

M1: Correct branch of the line attempted (as well as the incorrect branch).

M1: Correct method used to solve their equation.

A1: Correct. Candidate gives the answers as $x = \frac{62}{3}$

Part (c)

M1: For one end of the inequality $k \leq 11$

A0: Incorrect.

## Exemplar question 12

**12.** (a) Solve, for $-180° \leq x < 180°$, the equation $3\sin^2 x + \sin x + 8 = 9\cos^2 x$, giving your answers to 2 decimal places.

**(6)**

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°),$$

giving your answer to 2 decimal places.

**(2)**

**(Total for Question 12 is 8 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12(a)** | Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9\left(1 - \sin^2 x\right)$ | M1 | 3.1a |
| | $\Rightarrow 12\sin^2 x + \sin x - 1 = 0$ | A1 | 1.1b |
| | $\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$ | M1 | 1.1b |
| | $\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$ | A1 | 1.1b |
| | Uses arcsin to obtain two correct values | M1 | 1.1b |
| | All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$ | A1 | 1.1b |
| | | **(6)** | |
| **(b)** | Attempts $2\theta - 30° = -19.47°$ | M1 | 3.1a |
| | $\Rightarrow \theta = 5.26°$ | A1ft | 1.1b |
| | | **(2)** | |
| | | **(8 marks)** | |

**Notes:**

**(a)**

**M1**: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

**A1**: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

**M1:** Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

**A1**: $\sin x = \frac{1}{4}, -\frac{1}{3}$

**M1**: Obtains two correct values for their $\sin x = k$

**A1:** All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

**(b)**

**M1:** For setting $2\theta - 30° =$ their '$-19.47°$'

**A1ft**: $\theta = 5.26°$ but allow a follow through on their '$-19.47°$'

## Student Response A

**12.** (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

**(6)**

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

**(2)**

(a) $3\sin^2 x + \sin x + 8 = 9\cos^2 x$

$3\sin^2 x + \sin x + 8 = 9(1 + \sin^2 x)$

$3\sin^2 x + \sin x + 8 = 9 + 9\sin^2 x$

$6\sin^2 x - \sin x + 1 = 0$

$(3\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -\frac{1}{3}, \frac{1}{2}$

$x = -19.5°, -160.5, 30°, 150°$

(b) $2\theta - 30 = 30$

$2\theta = 60$

$\theta = 30$

***2/8***

Examiner Comments:

Part (a)

M0: For substituting $\cos^2 x = 1 + \sin^2 x$

A0: Follows M0 (Incorrect quadratic).

M1: Scored for a correct attempt at solving their quadratic in sin $x$

A0: Incorrect values for sin $x$

M1: Scored for obtaining two correct values for their $\sin x = \frac{1}{2}$ or $-\frac{1}{3}$

A0: Incorrect answers.

Part (b)

M0: Incorrect method. They should be attempting to solve $2\theta - 30 = -19.5$

A0: Follows M0

## Student Response B

**12.** (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

**(6)**

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

**(2)**

(a)
$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$
$$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$$
$$12\sin^2 x + \sin x - 1 = 0$$
$$(4\sin x + 1)(3\sin x - 1) = 0$$
$$\sin x = -\frac{1}{4},\ +\frac{1}{3}$$
$$x = -14.5°, -165.5°, 19.5°, 160.5°$$

(b)
$$2\theta - 30 = -14.5$$
$$2\theta = 16.5$$
$$\theta = 8.25°$$

**5/8**

Examiner Comments:

Part (a)

M1: For substituting $\cos^2 x = 1 - \sin^2 x$ to create a quadratic in sin $x$

A1: Correct quadratic.

M1: Scored for a correct attempt at solving their quadratic in sin $x$ This type of error is regarded as an accuracy error.

A0: Incorrect values for sin $x$

M1: Scored for obtaining two correct values for their $\sin x = \frac{1}{3}$ or $-\frac{1}{4}$

A0: Incorrect answers.

Part (b)

M1: Correct method. This is an attempt to solve $2\theta - 30 = \text{their} -19.5$

A0ft: This is a follow through mark but there is a slip in the working so cannot be awarded.

## Student Response C

**12.** (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

**(6)**

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

**(2)**

12) a) $3\sin^2 x + \sin x + 8 = 9 - ~~9\cos^2 x~~ \; 9\sin^2 x$

$12\sin^2 x + \sin x - 1 = 0$

$(4\sin x - 1)(3\sin x + 1) = 0$

$\therefore \sin x = -\frac{1}{3}, \; \frac{1}{4}$

$x = -160.53°, -19.47°, 14.48°, 165.52°$ (2dp)

b) $2\theta - 30 = -19.47$

$2\theta = 10.53$

$\theta = 5.26°$ (2dp)

***8/8***

Examiner Comments:

Part (a)

M1: For substituting $\cos^2 x = 1 - \sin^2 x$ to create a quadratic in sin *x*

A1: Correct quadratic.

M1: Scored for a correct attempt at solving their quadratic in sin *x*

A1: Correct values for sin *x*

M1: Scored for obtaining two correct values for $\sin x = -\frac{1}{3}$ or $\frac{1}{4}$

A1: Correct answers with suitable accuracy.

Part (b)

M1: Correct method. There is an attempt to solve $2\theta - 30 = -19.5$

A1ft: Correct answer

## Exemplar question 13

**13.** (a) Express 10 cos $\theta$ – 3 sin $\theta$ in the form $R$ cos ($\theta + \alpha$), where $R > 0$ and $0 < \alpha < 90°$. Give the exact value of $R$ and give the value of $\alpha$, in degrees, to 2 decimal places.

**(3)**

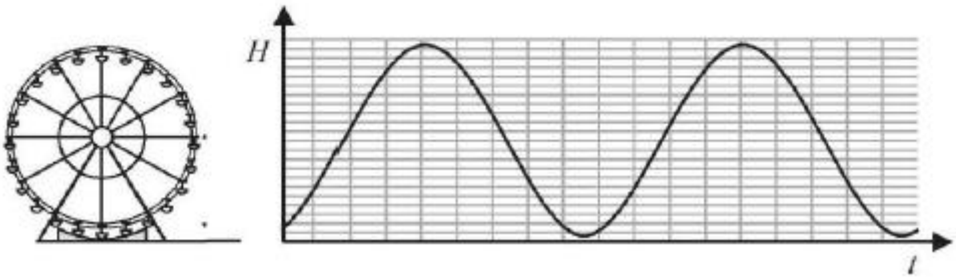



**Figure 3**

The height above the ground, $H$ metres, of a passenger on a Ferris wheel $t$ minutes after the wheel starts turning, is modelled by the equation $H = a - 10\cos(80t)° + 3\sin(80t)°$, where $a$ is a constant. Figure 3 shows the graph of $H$ against $t$ for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model.

(ii) Hence find the maximum height of the passenger above the ground.

**(2)**

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(3)**

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

**(1)**

**(Total for Question 13 is 9 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 13(a) | $R=\sqrt{109}$ | B1 | 1.1b |
| | $\tan\alpha=\frac{3}{10}$ | M1 | 1.1b |
| | $\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$ | A1 | 1.1b |
| | | **(3)** | |
| (b) | (i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$ | B1 | 3.3 |
| | (ii) $11+\sqrt{109}$ or 21.44 m | B1ft | 3.4 |
| | | **(2)** | |
| (c) | Sets $80t+"16.70"=540$ | M1 | 3.4 |
| | $t=\frac{540-"16.70"}{80}=(6.54)$ | M1 | 1.1b |
| | $t$ = 6 mins 32 seconds | A1 | 1.1b |
| | | **(3)** | |
| (d) | Increase the ‘80’ in the formula<br>For example use $H=11-10\cos(90t)°+3\sin(90t)°$ | | 3.3 |
| | | **(1)** | |
| | | | **(9 marks)** |

**Notes:**

**(a)**

**B1:** $R=\sqrt{109}$ Do not allow decimal equivalents

**M1:** Allow for $\tan\alpha=\pm\frac{3}{10}$

**A1:** $\alpha=16.70°$

**(b)(i)**

**B1:** see scheme

**(b)(ii)**

**B1ft:** their $11+$ their $\sqrt{109}$ Allow decimals here.

**(c)**

**M1:** Sets $80t+"16.70"=540$. Follow through on their 16.70

**M1:** Solves their $80t+"16.70"=540$ correctly to find $t$

**A1:** $t$ = 6 mins 32 seconds

**(d)**

B1: States that to increase the speed of the wheel the 80’s in the equation would need to be increased.

## Student Response A

(a) $\tan\alpha = \frac{3}{10}$ $\alpha = 16\cdot70°$

$R^2 = 10^2 + 3^2 \Rightarrow R = \sqrt{109}$

(b) $H = a - 10\cos(80t) + 3\sin(80t)$

$t = 0,\ H = 1$ $1 = a$

(i) $H = 1 - 10\cos(80t) + 3\sin(80t)$

(ii) $1 + 3 = 4\text{m}$

(c) $4 = 1 - 10\cos(80t) + 3\sin(80t)$

$10\cos(80t) - 3\sin(80t) = 3$

$80t = 270°$

$t =$ 3 minutes 37 seconds.

(d) make $t$ smaller.

**3/9**

Examiner Comments:

Part (a)

B1: Scored for $R = \sqrt{109}$

M1: Scored for $\tan\alpha = \dfrac{3}{10}$

A1: Scored for the correct value of $\alpha = (16.70°)$

Part (b)(i)

B0: $H = 1 - 10\cos(80t) + 3\sin(80t)$ is incorrect.

Part (b)(ii)

B0ft: 4 m is an incorrect follow through. It should be $1 + \sqrt{109}$

Part (c)

M0: Incorrect method. Part (a) has not been used.

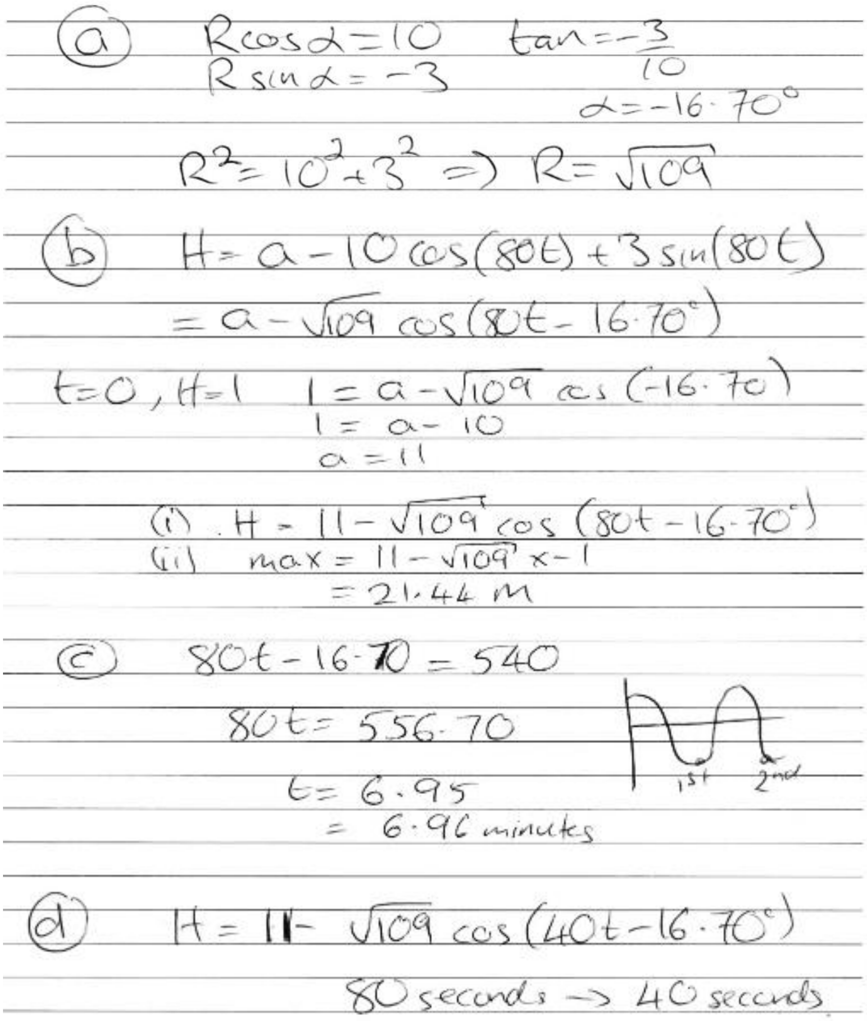
M0: Follows previous M0

A0: Incorrect.

Part (d)

B0: Incorrect statement.

## Student Response B



(a) $R\cos\alpha = 10$ $\tan = -\frac{3}{10}$

$R\sin\alpha = -3$

$\alpha = -16.70°$

$R^2 = 10^2 + 3^2 \Rightarrow R = \sqrt{109}$

(b) $H = a - 10\cos(80t) + 3\sin(80t)$

$= a - \sqrt{109}\cos(80t - 16.70°)$

$t = 0, H = 1$ $1 = a - \sqrt{109}\cos(-16.70)$

$1 = a - 10$

$a = 11$

(i) $H = 11 - \sqrt{109}\cos(80t - 16.70°)$

(ii) $\max = 11 - \sqrt{109} \times -1$

$= 21.44$ m

(c) $80t - 16.70 = 540$

$80t = 556.70$

$t = 6.95$

$= 6.96$ minutes



(d) $H = 11 - \sqrt{109}\cos(40t - 16.70°)$

80 seconds → 40 seconds

***5/9***

Examiner Comments:

Part (a)

B1: Scored for $R = \sqrt{109}$

M1: Scored for $\tan\alpha = -\frac{3}{10}$ The incorrect sign is an accuracy slip

A0: Incorrect value of $\alpha = (-16.70°)$

Part (b)(i)

B0: This is not a follow through mark. $H = 11 - \sqrt{109}\cos(80t - 16.70°)$ is incorrect.

Part (b)(ii)

B1ft: 21.44 m is a correct follow through for their function.

Part (c)

M1: Correct method.

M1: Correct attempt to solve $80t - 16.70° = 540°$

A0: Answer incorrect.

Part (d)

B0: Incorrect.

## Student Response C

(1)

13) a) $R = \sqrt{3^2+10^2} = \sqrt{109}$

$\tan\alpha = \frac{3}{10}$ $\therefore \alpha = 16.70$ (2dp)

b) i) $1 - a = -\sqrt{109}$

$1 + \sqrt{109} = a$

$H = 1+\sqrt{109} - \sqrt{109}\cos(~~80t~~ 16.70)$

ii) $1+\sqrt{109} + \sqrt{109} = 1 + 2\sqrt{109} = 21.88$ (2dp)

**Question 13 continued**

c) ~~$\cos(80t\ 16.7) = 1$~~

~~$+16.7 =$~~

$\cos(80t + 16.7) = -1$

$80t + 16.7 = 540$

$80t = 523.3$

$t = 6.54$ mins

$t = 392$ seconds (0dp)

d) Change it from $80t$ to $zt$, where $z > 80$

***8/9***

Examiner Comments:

Part (a)

B1: Scored for $R = \sqrt{109}$ M1: Scored for $\tan\alpha = \frac{3}{10}$

A1: Correct value and accuracy for $\alpha = (16.70°)$

Part (b)(i) B0: $H = 1 + \sqrt{109} - \sqrt{109}\cos(80t + 16.70°)$ is incorrect.

Part (b)(ii) B1ft: $1 + 2\sqrt{109}$ is a correct follow through for their function.

Part (c)

M1: Correct method. M1: Correct attempt to solve $80t + 16.70° = 540°$

A1: Answer is correct.

Part (d) B1: Scored for a correct statement.

## Exemplar question 14

**14.** A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius $r$ cm and height $h$ cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, $S$ cm$^2$, of the can is given by $S = 2\pi r^2 + \dfrac{1000}{r}$.

**(3)**

Given that $r$ can vary,

(b) find the dimensions of a can that has minimum surface area.

**(5)**

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

**(1)**

**(Total for Question 14 is 9 marks)**

---

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14(a)** | Sets $500 = \pi r^2 h$ | B1 | 2.1 |
| | Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$ | M1 | 2.1 |
| | Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ * | A1* | 1.1b |
| | | **(3)** | |
| **(b)** | Differentiates $S$ with both indices correct in $\dfrac{\mathrm{d}S}{\mathrm{d}r}$ | M1 | 3.4 |
| | $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \dfrac{1000}{r^2}$ | A1 | 1.1b |
| | Sets $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 0$ and proceeds to $r^3 = k, k$ is a constant | M1 | 2.1 |
| | Radius $= 4.30\,\text{cm}$ | A1 | 1.1b |
| | Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2}$ $\Rightarrow$ Height $= 8.60\,\text{cm}$ | A1 | 1.1b |
| | | **(5)** | |
| **(c)** | States a valid reason such as<br>• The radius is too big for the size of our hands<br>• If $r = 4.3\,\text{cm}$ and $h = 8.6\,\text{cm}$ the can is square in profile. All drinks cans are taller than they are wide<br>• The radius is too big for us to drink from<br>• They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans | B1 | 3.2a |
| | | **(1)** | |
| | | | **9 marks** |

| **Notes:** | |
|---|---|
| **(a)** | |
| **B1**: | Uses the correct volume formula with $V$ =500. Accept $500 = \pi r^2 h$ |
| **M1:** | Substitutes $h = \frac{500}{\pi r^2}$ or $rh = \frac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get $S$ as a function of $r$ |
| **A1*:** | $S = 2\pi r^2 + \frac{1000}{r}$ Note that this is a given answer. |
| **(b)** | |
| **M1:** | Differentiates the given $S$ to reach $\frac{dS}{dr} = Ar \pm Br^{-2}$ |
| **A1**: | $\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$ or exact equivalent |
| **M1:** | Sets $\frac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant |
| **A1**: | $R$ = awrt 4.30cm |
| **A1:** | $H$ = awrt 8.60 cm |
| **(c)** | |
| **B1:** | Any valid reason. See scheme for alternatives |

## Student Response A

(a)


$\pi r h = 500$

$h = \dfrac{500}{\pi r}$

$S = \pi r^2 + \pi r^2 + 2\pi h$

$S = 2\pi r^2 + \dfrac{1000}{r}$ ✓✓

(b) $S = 6.28r^2 + 1000r^{-1}$

$\dfrac{dS}{dr} = 12.56r - 1000r^{-2}$

$\dfrac{dS}{dr} = 0$ $\quad 12.56r = \dfrac{1000}{r^2}$

$12.56r^3 = 1000$

$2.32r = 10$

$r = 4.31$

(c) It could be too expensive.

**3/9**

Examiner Comments:

Part (a)

B0: Incorrect volume formula used.

M0: The candidate substitutes an incorrect $h$ into an incorrect $S$.

A0*: This mark cannot be scored from incorrect work.

Part (b)

M1: The candidate differentiates $S$ with both indices correct.

A1: This is a decimal equivalent and would be allowed (as it is an intermediate result).

M1: The candidate sets $\dfrac{dS}{dr} = 0$ and proceeds with a correct method (to produce an answer lacking accuracy).

A0: Requires $r = 4.30$ cm

A0: There is no attempt to find the height.

Part (c)

B0: This is not a suitable response.

## Student Response B

(1)

(a)

$V = \pi r^2 h$

$S = 2\pi r^2 + 2\pi r h$

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$

$S = 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right)$

$S = 2\pi r^2 + \frac{1000}{r}$

(b) $\frac{dS}{dr} = 4\pi r + \frac{1000}{r^2}$

$\frac{dS}{dr} = 0 \qquad 4\pi r + \frac{1000}{r^2} = 0$

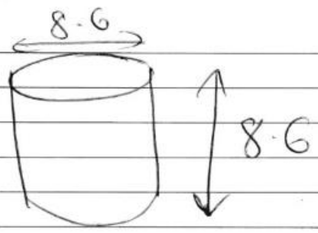
$4\pi r^3 = -1000$

$r^3 = \frac{-1000}{4\pi}$

$r = 4.30$

$h = 8.60$

(c)




***5/9***

Examiner Comments:

Part (a)

B1: Correct volume formula used with $V = 500$.

M1: Correct method.

A1*: Correct proof with all steps shown.

Part (b)

M1: The candidate differentiates $S$ with both indices correct.

A0: Incorrect $\frac{dS}{dr}$.

M1: The candidate sets $\frac{dS}{dr} = 0$ and proceeds with a correct method to $r^3 = k$.

A0: Although $r = 4.30$ cm is achieved, it is found from incorrect work so cannot be awarded.

A0: As above.

Part (c)

B0: This mark is only awarded when there is a suitable comment.

## Student Response C

(1)

14) a)

$$S = 2\pi r^2 + h\pi d = 2\pi r^2 + \frac{2h\pi r}{1}$$

$$\pi r^2 \times h = 500$$

$$h = \frac{500}{\pi r^2}$$

$$S = 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$$

$$S = 2\pi r^2 + \frac{1000}{r}$$

b)

$$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$$

$$4\pi r - \frac{1000}{r^2} = 0$$

$$4\pi r^3 - 1000 = 0$$

$$r^3 = 250 \div \pi$$

$$r = 9.23 \ (3sf)$$

$$\therefore h = \frac{500}{\pi r^2} = 1.87 \ (3sf)$$

c) It is ~~poorly designed~~ hard to drink out of & handle

***7/9***

Examiner Comments:

Part (a)

B1: Correct volume formula used with $V = 500$

M1: We can allow this method mark as (1) the expression for $S$ in terms of $r$ and $h$ is correct and (2) the correct answer is shown following a correct expression for $rh$.

A1*: Correct.

Part (b)

M1: The candidate differentiates $S$ with both indices correct.

A1: Correct $\frac{dS}{dr}$.

M1: The candidate sets $\frac{dS}{dr} = 0$ and proceeds with a correct method to $r^3 = 79.5...$

A0: $r \neq 4.30\,\text{cm}$

A0: $h \neq 8.60\,\text{cm}$

Part (c)

B1: This is a suitable response.

## Exemplar question 15

**15.**

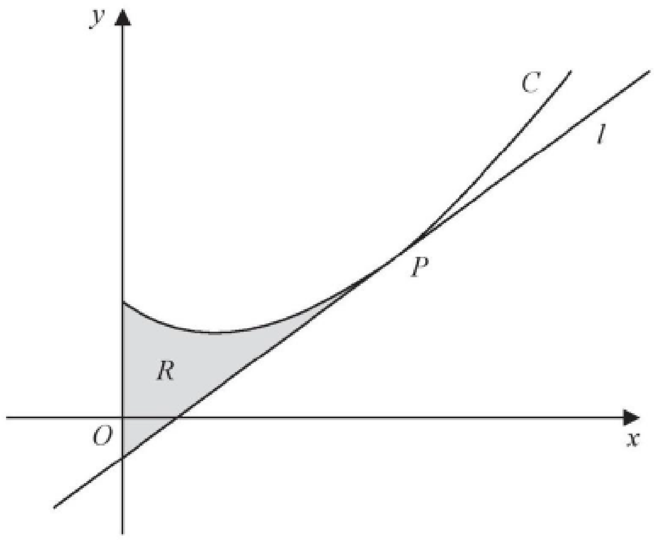



**Figure 4**

Figure 4 shows a sketch of the curve $C$ with equation $y = 5x^{\frac{3}{2}} - 9x + 11, \quad x \geq 0$.

The point $P$ with coordinates (4, 15) lies on $C$. The line $l$ is the tangent to $C$ at the point $P$.

The region $R$, shown shaded in Figure 4, is bounded by the curve $C$, the line $l$ and the $y$-axis.

Show that the area of $R$ is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(Total for Question 15 is 10 marks)**

## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15 | $\frac{dy}{dx}=\frac{15}{2}x^{\frac{1}{2}}-9$ | M1<br>A1 | 3.1a<br>1.1b |
| | Substitutes $x=4 \Rightarrow \frac{dy}{dx}=6$ | M1 | 2.1 |
| | Uses (4, 15) and gradient $\Rightarrow y-15=6(x-4)$ | M1 | 2.1 |
| | Equation of $l$ is $y=6x-9$ | A1 | 1.1b |
| | Area $R$ = $\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,dx$ | M1 | 3.1a |
| | $=\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x(+c)\right]_0^4$ | A1 | 1.1b |
| | Uses both limits of 4 and 0<br>$\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x\right]_0^4=2\times4^{\frac{5}{2}}-\frac{15}{2}\times4^2+20\times4-0$ | M1 | 2.1 |
| | Area of $R$ = 24 * | A1* | 1.1b |
| | Correct notation with good explanations | A1 | 2.5 |
| | | **(10)** | |
| | | | **(10 marks)** |

**Question 15 continued**

**Notes:**

**M1:** Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

**A1:** $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

**M1:** Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

**M1:** Uses their gradient and the point (4, 15) to find the equation of the tangent

**A1:** Equation of $l$ is $y = 6x - 9$

**M1:** Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

**A1:** $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

**M1:** Substitutes in both limits and subtracts

**A1*:** Correct area for $R = 24$

**A1:** Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of $l$. See scheme.
- Correct explanation in finding the area of $R$. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

**M1:** Area under curve = $\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

**A1:** $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

**M1:** This requires a full method with all triangles found using a correct method

Look for Area $R = \text{their } 36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

## Student Response A

$l \quad \frac{dy}{dx} = \frac{5}{2}x^{\frac{1}{2}} - 9$

At P gradient $= -4$

Equation of $l \qquad \frac{1}{4} = \frac{y-15}{x-4}$

$x - 4 = 4y - 60$

$y = \frac{1}{4}x + 14$

$$\text{Area} = \int_0^4 \left(5x^{3/2} - 9x + 11\right) - \left(\tfrac{1}{4}x + 14\right)\, dx$$

$$= \int 5x^{3/2} - 9\tfrac{1}{4}x + 25\, dx$$

$$= 2x^{2\frac{1}{2}} - \frac{9\frac{1}{4}x^2}{2} + 25x$$

$$= 24$$

***3/10***

Examiner Comments:

M1: The candidate differentiates to a form $\frac{dy}{dx} = Ax^{\frac{1}{2}} + B$

A0: $\frac{dy}{dx}$ incorrect.

M1: For substituting $x = 4$ into their $\frac{dy}{dx}$ implied by ‘gradient = – 4’

M0: Incorrect method for the equation of a tangent. The normal gradient is used.

A0: Equation of $l$ is incorrect.

M1: For attempting to find the area by integrating (curve-line).

A0: Incorrect.

M0: No use of the limits 4 and 0

A0*: This is a show that question and a candidate cannot simply write down 24

A0: This mark cannot be scored for an incorrect solution.

## Student Response B

$y = 5x^{3/2} - 9x + 11$

$= 5 \times \frac{3}{2} x^{1/2} - 9 \times 1$

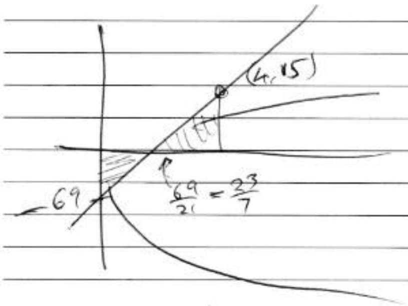
$= 5 \times \frac{3}{2} 4^{1/2} - 9$

$= 21$

$y = 21x + c$

$(4, 15) \quad 15 = 84 + c \quad \Rightarrow y = 21x - 69$

$c = -69$




$\frac{1}{2} \times \left(15 - \frac{23}{7}\right) \times 4 = 23.4$

$\frac{1}{2} \times \frac{23}{7} \times 69 = 113.4$

$A = \int 5x^{3/2} - 9x + 11 = \frac{5x^{2\frac{1}{2}}}{2\frac{1}{2}} - \frac{9x^2}{2} + \frac{11x^1}{1}$

$A = \left[2x^{2\frac{1}{2}} - 4\tfrac{1}{2}x^2 + 11x\right]_0^4 = 36$

$R = 36 - 23.4 + 113.4$

$= ?$

***7/10***

Examiner Comments:

M1: The candidate differentiates to a form $\frac{dy}{dx} = Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx}$ correct (unsimplified) on line two.

M1: For substituting $x = 4$ into their $\frac{dy}{dx}$ as seen on line three. The error is an accuracy error.

M1: Correct method for the equation of a tangent.

A0: The equation of $l$ is incorrect.

M1: For attempting to find the area under the curve by integration. (This requires correct index work)

A1: The answer 36 is correct.

M1: Scored for a full attempt at the area of $R$ using triangles.

A0*: The error on the tangent means that the correct answer cannot be found.

A0: This mark cannot be scored for an incorrect solution.

Student Response C

(Solutions based entirely on graphical or numerical methods are not acceptable.)

**(10)**

15) $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$

$\therefore$ at $x = 4$, $\frac{dy}{dx} = 6$ $\therefore$ $y = 6x + C$

$15 = 24 + C$

$-9 = C$

$y = 6x - 9$

$\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) dx = \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

at x intercept: $0 = 6x - 9 \therefore 9 = 6x \therefore x = \frac{3}{2}$

and $y = -9$ at y-intercept

$\Delta$①: $9 \times \frac{1}{2} \times \frac{3}{2} = 6.75$

$\Delta$②: $\left(4 - \frac{3}{2}\right) \times \frac{1}{2} \times 15 = 18.75$

$\therefore R = 36 + 6.75 - 18.75 = 24$

***9/10***

Examiner Comments:

M1: The candidate differentiates to a form $\frac{dy}{dx} = Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx}$ correct.

M1: For substituting $x = 4$ into their $\frac{dy}{dx}$ This is implied by $\frac{dy}{dx} = 6$

M1: Correct method for the equation of a tangent using $y = mx + c$

A1: The equation of $l$ is correct.

M1: For attempting to find the area under the curve by integration. (This requires correct index work)

A1: The answer 36 is correct.

M1: Scored for a full attempt at the area of $R$ using triangles.

A1*: 24 is achieved with no errors.

A0: This mark is now available to the candidate as they have produced a correct solution. It is awarded for:

- Correct notation. (Good ✓)
- Correct explanation for producing the equation of $l$. (Acceptable. It would have been better if this candidate had used some words along with the calculations by way of explanation, e.g. gradient of $l$ = 6 $\Rightarrow$ equation of $l$ is $y = 6x + c$ )
- Correct explanation in finding $R$ =24. (Incorrect ✗ There is no diagram or annotation of the diagram to explain $\Delta_1$ $\Delta_2$)

Hence this mark is not awarded.

# Exemplar question 16

**16.** (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

**(3)**

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where $P$, in thousands, is the population of meerkats and $t$ is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

**(6)**

(c) show that

$$P = \frac{A}{B + C\,e^{-\frac{1}{2}t}},$$

where $A$, $B$ and $C$ are integers to be found.

**(3)**

**(Total for Question 16 is 12 marks)**

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## Mark scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **16(a)** | Sets $\frac{1}{P(11-2P)}=\frac{A}{P}+\frac{B}{(11-2P)}$ | B1 | 1.1a |
| | Substitutes either $P=0$ or $P=\frac{11}{2}$ into $1=A(11-2P)+BP \Rightarrow A \text{ or } B$ | M1 | 1.1b |
| | $\frac{1}{P(11-2P)}=\frac{1/11}{P}+\frac{2/11}{(11-2P)}$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Separates the variables $\int \frac{22}{P(11-2P)}\,dP=\int 1\,dt$ | B1 | 3.1a |
| | Uses (a) and attempts to integrate $\int \frac{2}{P}+\frac{4}{(11-2P)}\,dP=t+c$ | M1 | 1.1b |
| | $2\ln P-2\ln(11-2P)=t+c$ | A1 | 1.1b |
| | Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c=(-2\ln 9)$ | M1 | 3.1a |
| | Substitutes $P=2 \Rightarrow t=2\ln 2+2\ln 9-2\ln 7$ | M1 | 3.1a |
| | $\text{Time}=1.89$ years | A1 | 3.2a |
| | | **(6)** | |
| **(c)** | Uses ln laws $2\ln P-2\ln(11-2P)=t-2\ln 9$ $\Rightarrow \ln\left(\frac{9P}{11-2P}\right)=\frac{1}{2}t$ | M1 | 2.1 |
| | Makes '$P$' the subject $\Rightarrow \left(\frac{9P}{11-2P}\right)=e^{\frac{1}{2}t}$ $\Rightarrow 9P=(11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P=f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P=f\left(e^{-\frac{1}{2}t}\right)$ | M1 | 2.1 |
| | $\Rightarrow P=\frac{11}{2+9e^{-\frac{1}{2}t}} \Rightarrow A=11, B=2, C=9$ | A1 | 1.1b |
| | | **(3)** | |
| | | **(12 marks)** | |

**Notes:**

**(a)**

**B1**: Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$

**M1:** Substitutes $P = 0$ or $P = \dfrac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$

Alternatively compares terms to set up and solve two simultaneous equations in $A$ and $B$

**A1:** $\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$ or equivalent $\dfrac{1}{P(11-2P)} = \dfrac{1}{11P} + \dfrac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

**(b)**

**B1:** Separates the variables to reach $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$ or equivalent

**M1:** Uses part (a) and $\displaystyle\int \frac{A}{P} + \frac{B}{(11-2P)}\mathrm{d}P = A\ln P \pm C\ln(11-2P)$

**A1:** Integrates both sides to form a correct equation including a '$c$' Eg $2\ln P - 2\ln(11-2P) = t + c$

**M1:** Substitutes $t$ = 0 and $P$ =1 to find $c$

**M1**: Substitutes $P$ = 2 to find $t$. This is dependent upon having scored both previous M's

**A1:** Time = 1.89 years

**(c)**

**M1:** Uses correct log laws to move from $2\ln P - 2\ln(11-2P) = t + c$ to $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d$ for their numerical '$c$'

**M1:** Uses a correct method to get $P$ in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\dfrac{P}{11-2P}\right) = \dfrac{1}{2}t + d \Rightarrow \dfrac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in $P$ (See scheme)

Alternatively uses a correct method to get $P$ in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example $\dfrac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d} \Rightarrow \dfrac{11-2P}{P} = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} - 2 = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \dfrac{11}{P} = 2 + \mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

**A1:** Achieves the correct answer in the form required. $P = \dfrac{11}{2+9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$ oe

## Student Response A

(a) $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$

$1 = A(11-2P) + BP$

$P = 11/2 \quad 11/2\, B = 1 \Rightarrow B = 2/11$

$P = 0 \quad 11A = 1 \Rightarrow A = \frac{1}{11}$

(b) $\frac{dP}{dt} = \frac{1}{22}(P)(11-2P)$

$P = \frac{1}{22} P(11-2P)t + C$

$t = 0 \quad P = 1000 \quad 1000 = \frac{1}{22} 1000(11-2000) \times 0 + C$

$C = 1000$

$P = \frac{1}{22} P(11-2P) \times t + 1000$

$P = 2000 \quad 2000 = \frac{1}{22} \times 2000(11-4000) \times t + 1000$

$22 = -3989t + 1000$

$t = 0.245$ years

(c) ?

**2/12**

Examiner Comments:

Part (a)

B1: Scored for setting $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

M1: Scored for setting either $P = 0$ or $P = \frac{11}{2}$ into $1 = A(11-2P) + BP$

A0: The candidate does not write in partial fractions as requested.

Part (b)

B0: Variables are not separated.

M0: The candidate does not use part (a) and use a correct method to integrate.

A0: Follows M0

M0: The candidate does not substitute $t = 0$ and $P = 1$ to find $c$

M0: Cannot be scored as the two previous M's have not been awarded.

A0: Incorrect.

Part (c)

M0, M0, A0 Not attempted.

## Student Response B

(a) $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$

$1 = A(11-2P) + BP$

$1 = 11A + BP - 2PA$

$A = \frac{1}{11}$ $\quad B - 2A = 0$

$B = \frac{2}{11}$

(b) $\frac{dP}{dt} = \frac{1}{22} P(11-2P)$

$\int \frac{dP}{P(11-2P)} = \int \frac{1}{22}\, dt$

$\int \frac{\frac{1}{11}}{P} + \frac{\frac{2}{11}}{11-2P} = \int \frac{1}{22}\, dt$

$\int \frac{2}{P} + \frac{4}{11-2P} = \int t$

$2\ln P - 2\ln(11-2P) = t + c$

when $t=0$, $P=1$ $\quad 2\ln 1 - 2\ln 9 = t$

$t = -2\ln 9$

$2\ln 9 - 2\ln(11-2P) = t - 2\ln 9$

$P=2$ $\quad 2\ln 2 - 2\ln 7 = t - 2\ln 9$

$2\ln 2 - 2\ln 7 + 2\ln 9 = t$

$t = 2\ln 4 = 2.77$

(c) $2\ln P - 2\ln(11-2P) = t - 2\ln 9$

$2P - 2(11-2P) = e^t - 18$

$6P = e^t + 4$

$P = \frac{e^{t+4}}{6}$

***6/12***

Examiner Comments:

Part (a)

B1: Scored for setting $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

M1: Scored for finding $A$ or $B$ by comparing coefficients.

A0: The candidate does not write in partial fractions as requested.

Part (b)

B1: For separating the variables.

M1: For using part (a) and using a correct method to integrate.

A1: For correct integration.

M1: For substituting $t = 0$ and $P = 1$ in order to find $c$

M0: The candidate substitutes $P = 2$ but uses an incorrect method to find $t$

A0: Incorrect.

Part (c)

M0: The candidate does not use correct ln laws.

M0: Cannot be scored from such an attempt. Some correct ln work must have been carried out.

A0: Incorrect.

## Student Response C

**(3)**

16) a)

$$1 = A(11-2p) + Bp$$

$$\text{sub } p=0, \quad 1 = 11A \quad \therefore \quad A = \frac{1}{11}$$

$$\text{sub } p = \frac{11}{2}, \quad 1 = \frac{11}{2}B \quad \therefore \quad \frac{2}{11} = B$$

$$\frac{1}{p(11-2p)} = \frac{1}{11p} + \frac{2}{11(11-2p)}$$

b)

$$\int \left(\frac{1}{11}p^{-1} + \frac{2}{11}(11-2p)^{-1}\right)dp = \int \frac{1}{22}\,dt$$

$$\int \left(p^{-1} + 2(11-2p)^{-1}\right)dp = \int \frac{1}{2}\,dt$$

$$\ln p - \ln|11-2p| = \frac{t}{2} + \ln k$$

$$\ln\left|\frac{p}{11-2p}\right| = \frac{t}{2} + \ln k$$

$$ke^{\frac{t}{2}} = \frac{p}{11-2p}$$

$$k = \frac{1}{11-2} = \frac{1}{9}$$

$$\therefore \frac{1}{9}e^{\frac{t}{2}} = \frac{2}{11-4} \quad \therefore \quad e^{\frac{t}{2}} = \frac{18}{7} \quad \therefore \quad \frac{t}{2} = \ln\frac{18}{7}$$

$$t = 2\ln\frac{18}{7}$$

$$t = 1.89 \ (2\text{dp})$$

c)

$$\frac{1}{9}e^{t/2} = \frac{p}{11-2p}$$

$$e^{t/2} = \frac{9p}{11-2p}$$

$$11e^{t/2} - 2pe^{t/2} = 9p$$

$$11e^{t/2} = p\left(9 + 2e^{t/2}\right)$$

$$\frac{11e^{t/2}}{9+2e^{t/2}} = p$$

***11/12***

Examiner Comments:

Part (a)

B1: Scored for setting $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$ This can be implied either by the second line or the answer line.

M1: Scored for finding $A$ or $B$ by comparing coefficients.

A1: Correct answer.

Part (b)

B1: For separating the variables.

M1: For using part (a) and using a correct method to integrate.

A1: For correct integration.

M1: For substituting $t = 0$ and $P = 1$ in order to find $k$

This is implied when $ke^{kt} = \frac{P}{11-2P} \rightarrow k = \frac{1}{11-2}$

M1: For substituting $P = 2$ and uses a correct method to find $t$

A1: For 1.89 years following correct work.

Part (c)

M1: For using correct ln laws to (effectively scored in (b))

M1: The candidate uses a correct method to get $P$ as a function of $e^{\frac{t}{2}}$

A0: Incorrect form of the answer.