Pearson Edexcel
Level 3 Advanced Subsidiary
GCE in Mathematics (8MA0)

Pearson Edexcel
Level 3 Advanced
GCE in Mathematics (9MA0)

Sample Assessment Materials Exemplar answers with examiner comments – Statistics and Mechanics

First teaching from September 2017
First certification from June 2018

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About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Level 3 Advanced GCE in Mathematics specification (8MA0 & 9MA0).

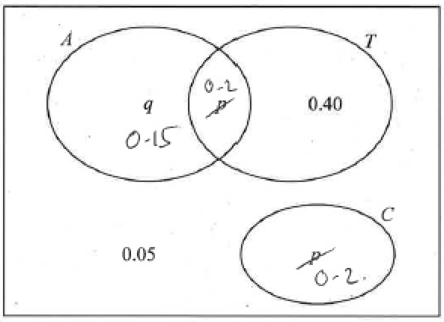
The booklet looks at questions from the AS and A level Sample Assessment Materials, which was used in the trial undertaken in summer 2017. It shows real student responses to questions, and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to all questions from the trial of the Sample Assessment Materials. Following each question you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student Response A

Student response



(a) $p + 0.05 = 0.25 \quad \therefore \quad p = 0.2$

(b) $q = 0.15$

$P(A) + P(T) = 0.95 \neq P(A \cup T) = 0.75$

So not independent.

(c) $P(A' \cup C') = P(A') + P(C') - P(A') \times P(C')$

$= 0.65 + 0.8 - 0.8 \times 0.65 = 0.93$

2/5

Examiner Comments: (a) B1 (b) B1M0A0 (c) B0

Part (a) Correct answer with clear working shown.

Part (b) The value of q is clearly given but then the formulae for independent and mutually exclusive events is confused so no further marks are awarded.

Part (c) Working suggests a misunderstanding of the question asked as this is a B mark, no credit can be given here.

Examiner commentary on the student response

Marks awarded for the question or question parts

AS Mathematics – Paper 2 (Statistics)

Exemplar question 1

1. Sara is investigating the variation in daily maximum gust, t kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(1)

(b) From your knowledge of the large data set, explain why this process may not generate a sample of size 20.

(1)

The data Sara collected are summarised as follows

$$n = 20 \qquad \sum t = 374 \qquad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

(2)

(Total for Question 1 is 4 marks)

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Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Systematic (sample) cao	B1	1.2
(b)	In LDS some days have gaps because the data was not recorded	B1	2.4
(c)	$\left[\bar{t} = \frac{374}{20} = 18.7\right]$ $s_t = \sqrt{\frac{7600}{20} - \bar{t}^2}$ $[=\sqrt{30.31}]$	M1	1.1a
	$= 5.5054\ldots$ awrt **<u>5.51</u>** (Accept use of $s_t = \sqrt{\frac{7600 - 20\bar{t}^2}{19}} = 5.6484\ldots$)	A1	1.1b
	(4 marks)		

Notes:

(b)

B1: A correct explanation

(c)

M1: For a correct expression for $\bar{t}$ and σ_t or s_t . Ft an incorrect evaluation of $\bar{t}$

A1: For σ_t = awrt 5.51 or s_t = awrt 5.65

Student Response A

a) ~~random sampling~~ interval sampling

b) Because she takes the data every third value there may not be enough values to get to a sample size of 20 if she picks one every third value.

c) $\cancel{\frac{\Sigma t}{n} - \frac{(\Sigma t)^2}{n} \quad \frac{374}{20} - \frac{(374)^2}{20} = 6975.1}$

$\cancel{\sqrt{6975.1} = 83.517664...}$

$$\sqrt{\frac{\Sigma t^2}{n} - \frac{(\Sigma t)^2}{n}}$$

$$\sqrt{\frac{7600}{20} - \frac{(374)^2}{20}} = 5.5.$$

1/4

Examiner Comments: (a) B0 (b) B0 (c) M1A0

Part (a) The candidate needs to use the correct technical term.

Part (b) No indication of the relevant knowledge of the data in the large data set (some data values being missing).

Part (c) Correct working is seen but the answer to only 2 significant figures is given, need awrt 5.51 In general candidates should always give rounded answers to at least 3 significant figures unless specifically directed otherwise.

Student Response B

(a) Sistematic sampling

(b) Some values are recorded as n/a because the data is missing

(c) $\sigma = \sqrt{7600 - \left(\frac{374}{20}\right)^2} = \sqrt{7250.31}$

$= \underline{85.14}$

2/4

Examiner Comments: (a) B1 (b) B1 (c) M0A0

Part (a) Correct answer, incorrect spelling is ignored.

Part (b) Correct.

Part (c) An incorrect formula is used. The formula for standard deviation needs to be known.

Student Response C

(a) Systematic

(b) You might run out of days

(c)

$$S_{tt} = 7600 - \frac{374^2}{20} = 606.2$$

$$\sigma_t = \sqrt{\frac{S_{tt}}{20}} = \sqrt{30.31} = 5.5054\ldots$$

$$= 5.50$$

3/4

Examiner Comments: (a) B1 (b) B0 (c) M1A1

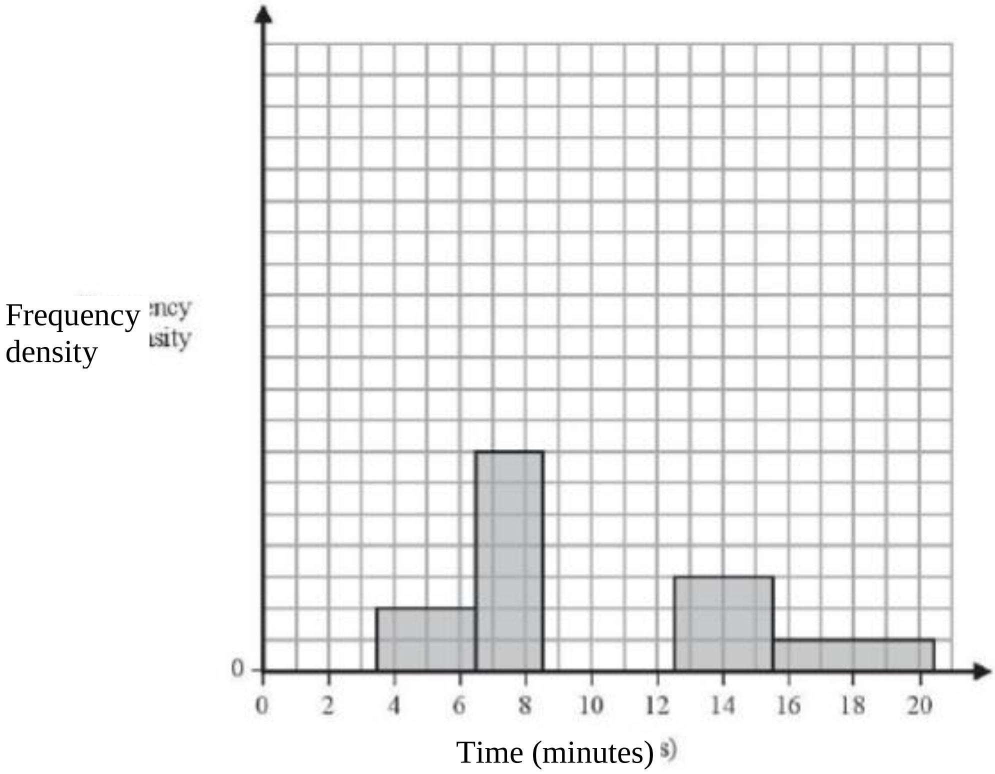
Part (a) Correct answer.

Part (b) No indication of relevant knowledge of the large data set is shown.

Part (c) Correct answer calculated but then subsequently the candidate rounds incorrectly. As a value of 5.5054… was given the 5.50 is treated as isw.

Exemplar question 2

2. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists were delayed by roadworks on a stretch of motorway.



Delay (minutes)	Number of motorists
4 – 6	6
7 –8	
9	17
10 – 12	45
13 – 15	9
16 – 20	

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

(Total for Question 2 is 5 marks)

Mark Scheme

Question	Scheme	Marks	AOs
2	$17 + 45 + \frac{1}{3} \times 9$ [= 65]	M1	2.2a
	(7 – 8) $\underline{\mathbf{14}}$ <u>or</u> (16 – 20) $\underline{\mathbf{5}}$ [Values may be seen in the table]	M1 A1	3.1a 1.1b
	Percentage of motorists is $\frac{"65"}{6+"14"+17+45+9+"5"}\times 100$	M1	3.1b
	= $\underline{\mathbf{67.7\%}}$	A1	1.1b
			(5 marks)

Notes:

M1: For a fully correct expression for the number of motorists in the interval

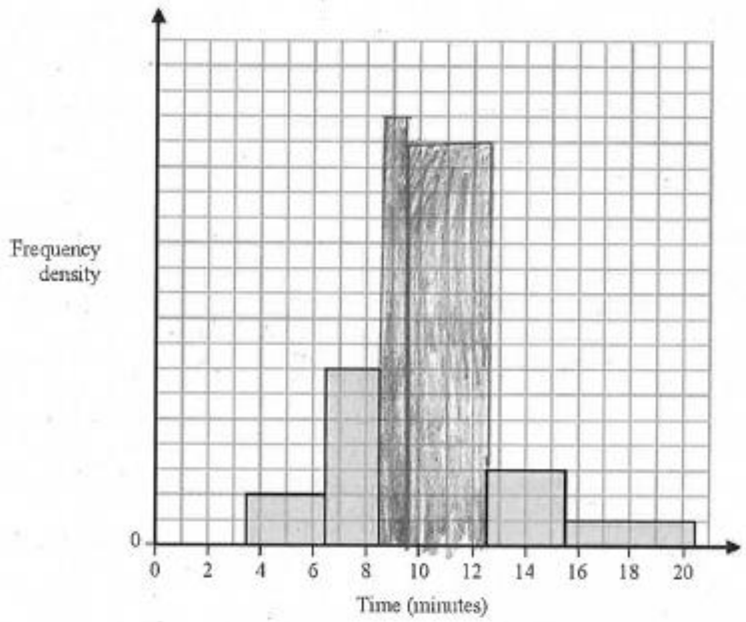
M1: For clear use of frequency density in (4- 6) or (13- 15) cases to establish the fd scale. Then use of area to find frequency in one of the missing cases

A1: For both correct values seen

M1: For realising that total is required and attempting a correct expression for %

A1: For awrt 67.7%

Student Response A



Delay (minutes)	Number of motorists	
		CF
4.5 - 6.5 4 – 6	6	
6.5 - 8.5 7 – 8	14	
8.5 - 9.5 9	17 ÷ 1 = 17	17 cf
9.5 - 12.5 10 – 12	45 ÷ 3 = 15	13 cf
12.5 - 15.5 13 – 15	9	
15.5 - 20 16 – 20	5	total = 96

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

39.5

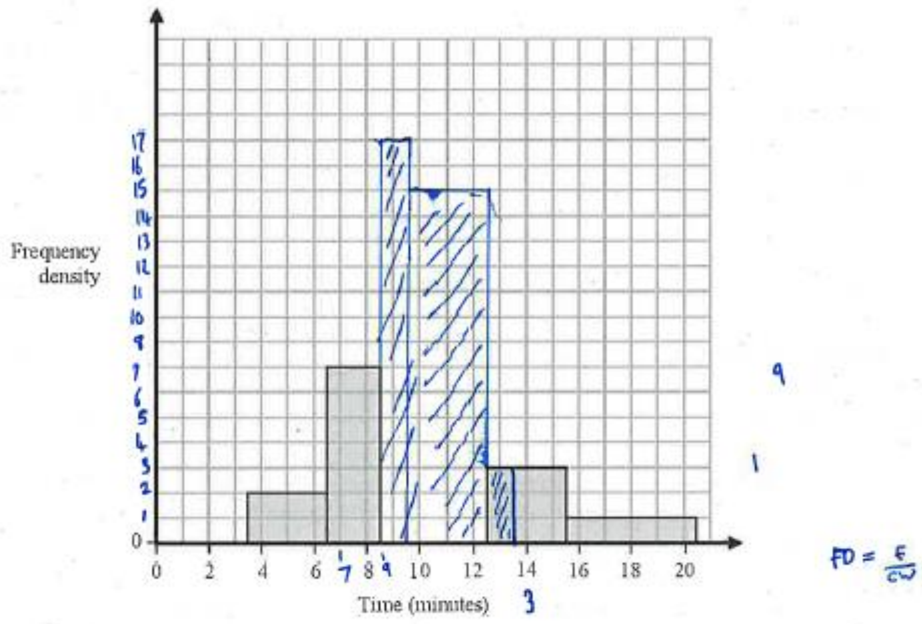
$\frac{96}{100} = 0.96$ 0.96 × 39.5 = 37.92 %

2/5

Examiner Comments: M0M1A1M0A0

This candidate was able to use the properties of the histogram to find the missing frequencies in the table correctly, picking up 2 marks. Whilst the 96 (total number of motorists) is seen in the script it is not used correctly and there is no visible attempt to find the number delayed for between 8.5 and 13.5 minutes so no further marks are scored.

Student Response B



CW	Delay (minutes)	Number of motorists	FD 1 = $\overline{6}$
3	3.5 - 6.5 4 – 6	6	2
2	6.5 - 8.5 7 – 8	14	7
1	8.5 - 9.5 9	17	17
3	9.5 - 12.5 10 – 12	45	15
3	12.5 - 15.5 13 – 15	9	3
6	15.5 - 21.5 16 – 20	6	1

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

Total = 97 tourists is the sample.

Percentage = 17 + 45 + (Number in [illegible] 12.5 - 15.5)

= 62 + 3 = 65 $\quad$ 65/97 ≈ 67%.

3/5

Examiner Comments: M1M1A0M1A0

In this case the candidate uses correct methods and shows good structure in their working picking up all the method marks. The boundary values for each class are shown but an error with the upper boundary for the top group is seen, perhaps an error in reading the scale, which results in the loss of the A marks.

Student Response C

Delay (minutes)	Number of motorists
4 – 6	6
7 – 8	14
9	17
10 – 12	45
13 – 15	9
16 – 20	5

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(5)

Missing values. Need f.density.

4-6 group f.density $= \frac{6}{3} = 2$ ∴ scale 1 square = 1 unit.

7-8 group f.den = 7 ∴ area $= 2 \times 7 = 14$

16-20 group f.den = 1 ∴ area $= 5 \times 1 = 5$

Number delayed for 8.5 to 13.5 mins

$= 17 + 45 + \frac{1}{3} \times 9$

∴ % delayed $= \dfrac{17 + 45 + \frac{1}{3} \times 9}{6 + 14 + 17 + 45 + 9 \times 5} \times 100$

$= \dfrac{65}{98} = 66\%$

4/5

Examiner Comments: M1M1A1M1A0

A strong response with clear well explained working. The candidate finds the frequency density scale and uses it to calculate the missing frequencies and then uses the correct method to find the number of motorists with the required time delay. Working picks up the 3rd method mark but an arithmetic error, total number of motorists is incorrect, means that the final answer is wrong. Even had this been correct giving their final answer to only 2 significant figures would also have cost the last A mark.

Exemplar question 3

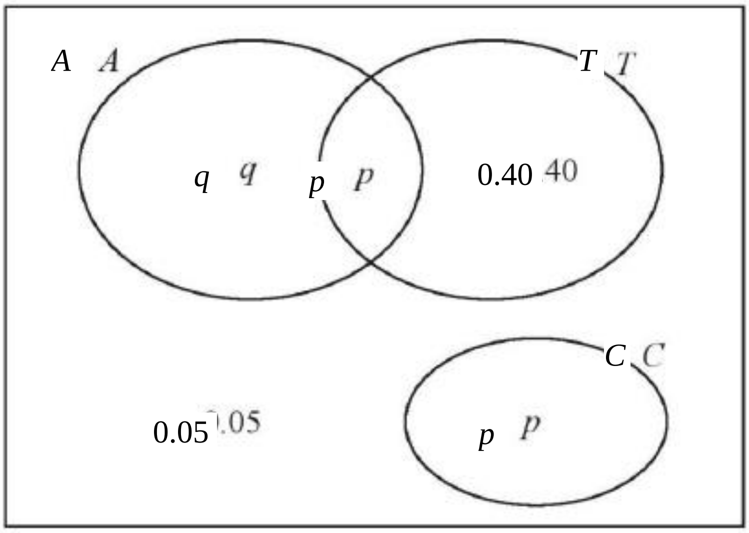
3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.

T represents the event that a student takes part in Tennis.

C represents the event that a student takes part in Cricket.

p and *q* are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75.

(a) Find the value of *p*.

(1)

(b) State, giving a reason, whether or not the events *A* and *T* are statistically independent. Show your working clearly.

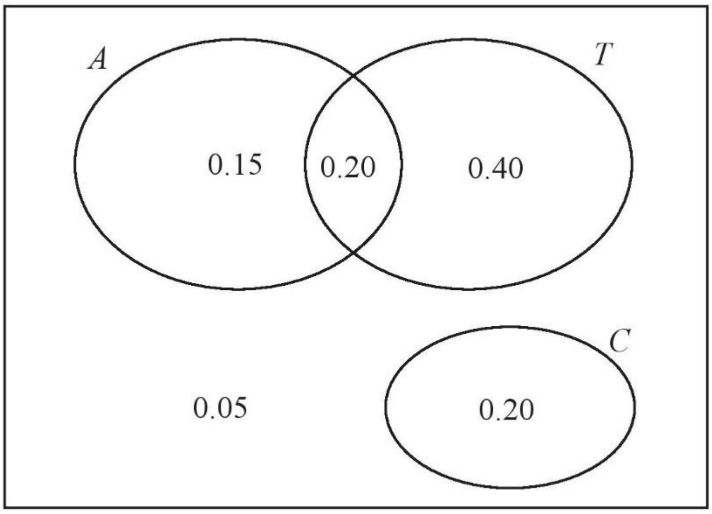
(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

(Total for Question 3 is 5 marks)

Mark Scheme

Question	Scheme	Marks	AOs
3 (a)	p = [1 – 0.75 – 0.05 =] **0.20**	B1	1.1b
		(1)	
(b)	q = **0.15**	B1ft	1.1b
	P(A) = 0.35 P(T) = 0.6 P(A and T) = 0.20 P(A)' × P(T) = 0.21	M1	2.1
	Since 0.20 ≠ 0.21 therefore A and T are **not** independent	A1	2.4
		(3)	
			
(c)	P(not [A or C]) = **0.45**	B1	1.1b
		(1)	
			(5 marks)

Notes:

(a)

B1: cao for p = 0.20

(b)

B1: ft for use of their p and P(A or T) to find q i.e. 0.75 – "p" – 0.40 or q = 0.15

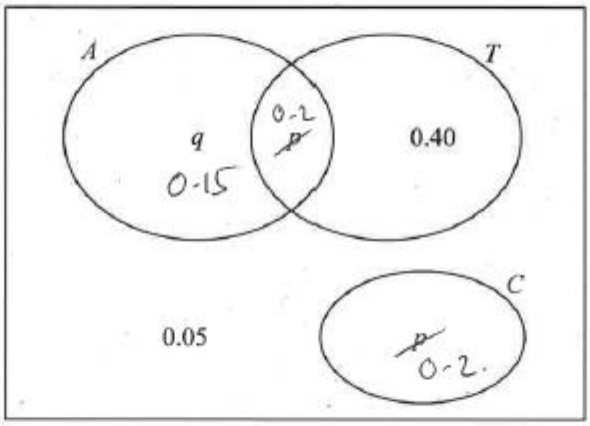
M1: For the statement of all probabilities required for a suitable test and sight of any appropriate calculations required

A1: All probabilities correct, correct comparison and suitable comment

(c)

B1: cao for 0.45

Student Response A



(a) $p + 0.05 = 0.25 \quad \therefore \quad \underline{p = 0.2}$

(b) $q = \underline{0.15}$

$P(A) + P(T) = 0.95 \neq P(A \cup T) = 0.75$

So not independent.

(c) $P(A' \cup C') = P(A') + P(C') - P(A') \times P(C')$

$= 0.65 + 0.8 - 0.8 \times 0.65 = \underline{0.93}$

2/5

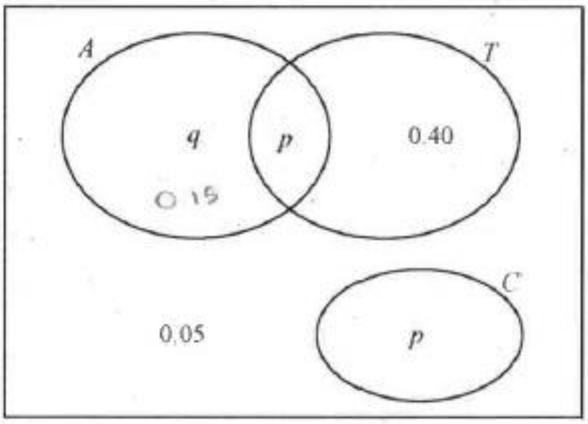
Examiner Comments: (a) B1 (b) B1M0A0 (c) B0

Part (a) Correct answer with clear working shown.

Part (b) The value of *q* is clearly given but then the formulae for independent and mutually exclusive events is confused so no further marks are awarded.

Part (c) Working suggests a misunderstanding of the question asked as this is a B mark, no credit can be given here.

Student Response B



a. $q + p + 0.40 = 0.75$

$0.75 + 0.05 + p = 1$

$p = 0.20$

b. A and T are not independent

as $P(A) \times P(T) \neq P(A \cap T)$ $\quad 0.40 \times 0.15 \neq 0.20$

C $0.40 + 0.05 = 0.45$

3/5

Examiner Comments: (a) B1 (b) B1M0A0 (c) B1

Part (a) Correct answer using the known probability of $A \cup B$

Part (b) B1 mark is awarded as the value of q can be seen, the candidate writes it into the Venn diagram. In the working on independence the correct equation is quoted however the probabilities used are not P(A) and P(T) so M0 given.

Part (c) Correct.

Student Response C

$q + p + 0.4 + p + 0.05 = 1$

$2p + q = 0.55$ $\therefore 2p + (0.35 - p) = 0.55$

$p(A \cup T) = 0.75$ $p + 0.35 = 0.55$

$q + p + 0.4 = 0.75$ $p = 0.2.$

$q + p = 0.35$ $q = 0.35 - p$ $q = 0.35 - 0.2 =$

$q = 0.15$

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$p(A) \times p(B) =$

$(0.4 + 0.2) \times (0.15 + 0.2) = 0.11$

$0.11 \neq 0.2$

$\therefore$ not statistically independant

c) $0.4 + 0.05 = 0.45$

4/5

Examiner Comments: (a) B1 (b) B1M1A0 (c) B1

Part (a) A longer method (effectively solving simultaneous equations) is used but achieved the correct answer, B1.

Part (b) B1 mark for the value of q is achieved in part (a) and then the values is used correctly to establish whether A and T are independent, an arithmetic error results in loss of the A mark.

Part (c) Correct.

Exemplar question 4

4. Sara was studying the relationship between rainfall, r mm, and humidity, h %, in the UK.
She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

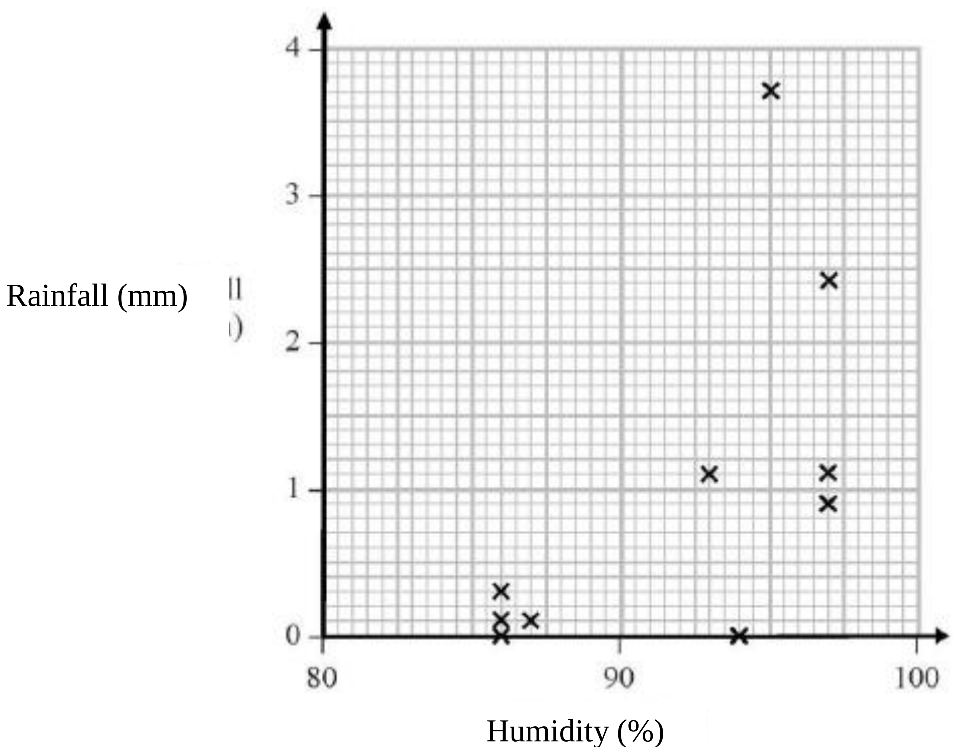
(b) Give a reason why Sara might

(i) include

(ii) exclude this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of *r* and *h*.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$.

(d) Give an interpretation of the gradient of this regression line.

(1)

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(2)

(Total for Question 4 is 7 marks)

Mark Scheme

Question	Scheme	Marks	AOs
4(a)	IQR = 2.3 and $20.6 \geq 2.4 + 1.5 \times 2.3$ (= 5.85) (Compare correct values)	B1	1.1b
		(1)	
(b)(i)	e.g. it is a piece of data and we should consider all the data o.e.	B1	2.4
(ii)	e.g. it is an extreme value and could unduly influence the analysis <u>or</u> it could be a mistake	B1	2.4
		(2)	
(c)	e.g. "as humidity increases rainfall increases"	B1	2.2b
		(1)	
(d)	e.g. a 10% increase in humidity gives rise to a 1.5 mm increase in rainfall **<u>or</u>** represents 0.15mm of rainfall per percentage of humidity	B1	3.4
		(1)	
(e)(i)	Not a good method since only uses 11 days from one location in one month	B1	2.4
(ii)	e.g. she should use data from more of the UK locations and more of the months **<u>or</u>** using a spreadsheet or computer package she could use all of the available UK data	B1	2.4
		(2)	
		(7 marks)	

Continued question 4
Notes:
(a) **B1:** For sight of the correct calculation and suitable comparison with 20.6
(b)(i) **B1**: For a suitable reason for including the data point
(b)(ii) **B1:** For a suitable reason for excluding the data point
(c) **B1:** For a suitable interpretation of positive correlation mentioning humidity and rainfall
(d) **B1:** For a suitable description of the rate: rainfall per percentage of humidity including reference to values
(e)(i) **B1:** For a comment that supports the idea that her sampling method was not a good one
(e)(ii) **B1:** For some sensible suggestions that would give a better representation of the data across the UK. Must show some awareness of the fact that LDS has different locations and more months of data available but must be clear they are NOT using any overseas locations
N.B. B0 for a comment that says use more than one location without specifying that only UK locations are required

Student Response A

4) a) $IQR = 2.4 - 0.1 = 2.3$

$1.5 \times 2.3 = 3.45$

$2.4 + 3.45 = 5.85$

$20.6 > 5.85 \therefore$ outlier.

b) i) because this result was anyway used to calculate Q_1, Q_2, Q_3 and the IQR

ii) because its value is far greater than any of the others and is well above ~~the other~~ 5.85 which is the maximum number excluding outliers.

4c) There is a weak positive correlation between humidity and rainfall.

d) the gradient is 0.15 which shows the is a positive gradient which means there will be a ~~positive~~ weak positive correlation as 0.15 is a small number.

e) i) It was not suitable as the samples picked may have been to close to each other, there needs to be a more definite spread when picking data. ~~and equal distribution, some day of the data must have been picked the d.~~

ii) She could have selected every 3rd value inorder to have a wider spread of data which is also less bias.

3/7

Examiner Comments: (a) B1 (b) B0B1 (c) B0 (d) B0 (e) B1B0

Part (a) Good clear answer showing full working.

Part (b)(i) The argument that all data should be included unless there is a good reason to suspect it is not a valid reading is needed.

Part (b)(ii) Answer is just about sufficient, referring to this value as being 'well above' the limit for outliers.

Part (c) The correlation shown in the scatter diagram is described but no interpretation is given as requested - which needs to refer to the context of the question – hence B0.

Part (d) No interpretation in context of the 0.15 value is given. This answer also shows a lack of understanding of the gradient, confusing this with the correlation coefficient.

Part (e) (i) The benefit of the doubt is given as the candidate has the idea of obtaining a wider range of data but ideally would have expanded on this in terms of dates and locations.

Part (e) (ii) The second B1 mark is a tougher mark requiring the candidate to refer to there being more than one UK location available.

Student Response B

a) IQR = 24 - 0.1 = ~~0.2~~ 23.9

$1.5 IQR + Q_3 = 1.5 \times 23.9 + 24 = 5.85$

$5.85 < 20.6 \therefore 20.6$ is an outlier

b) i) it is a statistical data which matters - this day happened and she cannot just pretend like this never happened

ii) outlier can effect, e.g., mean

c) the correlation between rainfall and humidity is very weak since data is very scattered but it is still clearly shown that it is positive

d) as the rainfall increases by 1 mm, the humidity increases by 92%

e) i) her sampling frame is small so the predictions and conclusions that come from her calculations aren't very reliable

ii) she could use more samples and then code the data to be easier to work with

4/7

Examiner Comments: (a) B1 (b) B1B1 (c) B0 (d) B0 (e) B1B0

Part (a) Good response showing all required working.

Part (b) Both marks can be awarded, in part (ii) is a little thin but just about enough.

Part (c) Comments on the correlation can be seen in the scatter diagram but no attempt to interpret it is made, ‘interpret’ is the key word in this question.

Part (d) Incorrect answer. This also highlights however that candidates should be aware which is the independent and which the dependent variable.

Part (e) The candidate has the right idea but does not give sufficient detail in the context of the large data set to pick up the second B mark.

Student Response C

a) IQR = 2.4 − 0.1 = 2.3
∴ Upper limit = 2.4 + 1.5×2.3 = 5.85
20.6 is much bigger than 5.85 ∴ is an outlier

b)(i) Its a real value so should be used, we have no reason to doubt it is real
(ii) It would increase the quartiles so shouldn't be used

c) Graph suggests a weak positive correlation, so in general as the humidity gets higher, the rainfall will increase too.

d) We have a weak positive gradient. For every mm of rainfall the humidity increases by 0.15

e) (i) Sara is studying rainfall and humidity in the UK but takes all her data from one month and one location so this is not a good method of data collection.
(ii) More data would be better, especially if spread around different months and different places, e.g all the places in the UK from the data set.

5/7

Examiner Comments: (a) B1 (b) B1B0 (c) B1 (d) B0 (e) B1B1

Part (a) Good response showing all necessary working.

Part (b)(i) Good response is seen but part (ii) is incorrect as removing the outlier would not significantly influence the quartiles.

Part (c) Good response which comments on the correlation and then an interpretation in context.

Part (d) The answer suggests confusion between correlation and the gradient of the regression line but also has the two variables the wrong way around.

Part (e) Good response referring to the large data set and suggesting an awareness of the data available.

Exemplar question 5

5. The discrete random variable $X \sim B(40, 0.27)$.

(a) Find $P(X \geq 16)$.

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

(Total for Question 5 is 9 marks)

Mark Scheme

Question	Scheme	Marks	AOs
5(a)	$\mathrm{P}(X) \geq 16 = 1 - \mathrm{P}(x \leq 15)$	M1	1.1b
	$= 1 - 0.949077\ldots$ $=$ awrt **0.0509**	A1	1.1b
		(2)	
(b)	$\mathrm{H}_0 : p = 0.3$ $\mathrm{H}_1 : p \neq 0.3$ (Both correct in terms of p or π)	B1	2.5
		(1)	
(c)	[$Y \sim \mathrm{B}(20, 0.3)$] sight of $\mathrm{P}(Y \leq 2) = 0.0355$ **or** $\mathrm{P}(Y \leq 9) = 0.9520$	M1	2.1
	Critical region is $\{Y \leq \mathbf{2}\}$ or o.e.	A1	1.1b
	$\{ Y \geq \mathbf{10}\}$ o.e.	A1	1.1b
		(3)	
(d)	[0.0355 + (1 – 0.9520)] = 0.0835 or **8.35%**	B1ft	1.1b
		(1)	
(e)	(Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager's suspicion is supported	B1ft	3.2a
		(1)	
(f)	e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid)	B1	3.5a
		(1)	
			(9 marks)

Student Response A

a) $X \sim B(40, .27)$ $X \sim N(10.8, \sqrt{7.884}^2)$

$P(X \geq 16$

$$\frac{16 - 10.8}{\sqrt{7.884}} = 1.85$$

$= 0.9676$

b) $H_0 : P = .3$ $X \sim B(20, .3)$

$H_1 : P \neq .3$

c) 0.05 $P(X \leq 2) = 0.0355$

0.95 $P(X \leq 8) = 0.952$

$X < 2$ $X > 10$

c) $0.0355 + 0.048 = 0.0835$

e) $P(X \neq 12)$ $= 0.9987 = 0.0013$

$0.0013 \leq 0.05$

30% of $20 = 6$ $12 = 60\%$. So most likly right

f) This is an anomaly So distorts the validity of the model

3/9

Examiner Comments: (a) M0A0 (b) B1 (c) M1A0A0 (d) B1 (e) B0 (f) B0

Part (a) It appears that the candidate tries to use a Normal approximation rather than the Binomial cumulative distribution function. Students should be encouraged to be familiar with the functions available on their calculators.

Part (b) Answer is correct.

Part (c) Starts well correctly quoting the relevant probabilities but then the candidate muddles converting less than or equal to statements into less than or greater than statements. Errors at both ends of the range are seen.

Part (d) Correct.

Part (e) Reference to the critical region compared to the observed value needs to be made.

Part (f) There is a realisation that the scout group may have an impact but there needs to be a statement in context as to why this might be the case.

Student Response B

a) $X \sim B(40, 0.27)$ $40 \times 0.27 = 10.8 \rightarrow Y \sim Po(11)$

$P(X \geq 16) = 1 - 0.8\cancel{1694} 8369 = \cancel{0.0983}\ 0.1631$

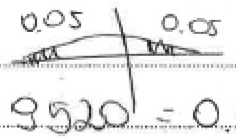
~~$P(X \geq 16) = 1 - P(X \leq 15)$~~ ← using $p = 0.25$ instead of $p = 0.27$

b) $X \sim B(20, 0.3)$

$H_0: p = 0.3$

$H_1: p \neq 0.3$

Question 5 continued



c) $P(X \leq 2) = 0.0355$

$P(X \leq 9) = 0.9520$ $1 - 0.9520 = 0.048$

critical region: $X \leq 2$, $X \geq 10$

d) $0.0355 + 0.048 = 0.0835$

e) $\frac{12}{20} = 0.6$

the probability of customers buying baked beans in single tins may be higher than 0.3

f) groups of scouts aren't regular customers on daily baises so the manager should pay much attention to his observation

6/9

Examiner Comments: (a) M1A0 (b) B1 (c) M1A1A1 (d) B1 (e) B0 (f) B0

Part (a) Attempt to use a valid method for finding $1 - P(X \leq 15)$ is seen but the appropriate calculator function is not used. Candidates should be encouraged to be familiar with the probability distribution functions available on their calculators and to use them whenever possible.

Part (b) Answer is correct.

Part (c) Also correct including clear statement of the relevant probabilities, leading to a correct answer to part (d).

Part (e) Correct, the use of the earlier parts of the question is required to give a comment with some justification in the context of the manager's suspicion.

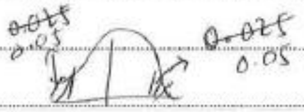
Part (f) The candidate needs to make it clear what impact the group of scouts may have had on the sample and why.

Student Response C

$p(Y \geq 16)$

$= 1 - p(Y \leq 15)$

I would use tables, but not available.



b) $H_0 = p = 0.3$

$H_1 = p \neq 0.3$ $\quad p(X \leq C_1) = 0.05$

$n = 20$ $\quad p(X \geq C_2) = 0.05$

10% significance $\quad p(X \leq C_2 - 1) = 0.95$

$\therefore \quad X \sim B(20, 0.30)$

$= \quad p(X \leq 3) = 0.1071$

$p(X \leq 2) = 0.0355$ → closer to 5% ∴

$C_1 = 2.$

$p(Y \leq 9) = 0.9520$ → closer to 95%

$p(Y \leq 8) = 0.8867$

$\therefore \quad C_2 - 1 = 9$

$C_2 = 10$

∴ critical region =

$\left\{ p(X \leq 2) \cap \cancel{p(X \geq 10)} \; p(X \geq 10) \right\}$

d) actual significance

$(1 - 0.9520) + 0.0355 = 0.0835$

$= 8.35\%$

e) reject H_0 as 12 lies in the critical region [12 > 10] ∴ and we have suffecient Evidence to prove that there is a change in the proportion of costumers Buying Baked Beans.

f) Not valid as it is Bias

[campers Eat Baked Beans mainly when they are camping]

(Total for Question 5 is 9 marks)

8/9

Examiner Comments: (a) M1A0 (b) B1 (c) M1A1A1 (d) B1 (e) B1 (f) B1

Part (a) This candidate knows what probability to find but the tables are being used rather than the functions on the calculator. Questions will be set that expect candidates to use the cumulative distribution functions on the calculator, the Binomial tables are provided for questions where critical regions need to be found.

Part (b) Answer is correct.

Part (c) Very good answer as the candidate finds the correct critical regions and not only quotes the relevant probabilities but shows in working that they also checked the adjacent values.

Part (d) Answer is correct.

Part (e) Good answer, the result of the hypothesis test are given by referring to the critical region. A conclusion in context is seen.

Part (f) Good answer in the context to provide a relevant comment as to the possible validity.

AS Mathematics – Paper 2 (Mechanics)

Exemplar question 6

6.

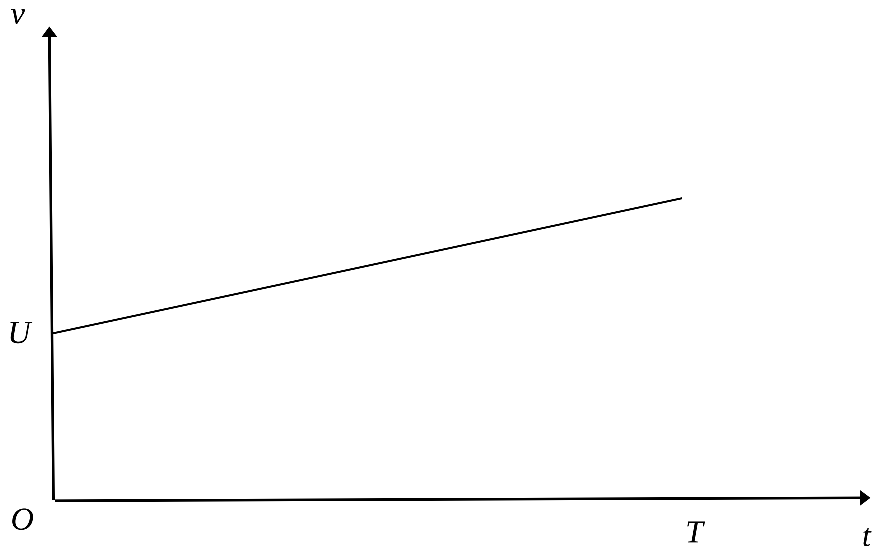


Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leq t \leq T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(4)

(Total for Question 6 is 4 marks)

Pearson

Mark Scheme

Question	Scheme	Marks	AOs
6.	Using distance = total area under graph (e.g. area of rectangle + triangle or trapezium or rectangle – triangle)	M1	2.1
	e.g. $D = UT + ½\, Th$, where h is height of triangle	A1	1.1b
	Using gradient = acceleration to substitute $h = aT$	M1	1.1b
	$D = U\,T + ½\, aT^2$ *	A1 *	1.1b
		(4)	
			(4 marks)

Notes:

M1: For use of distance = total area to give an equation in D, U, T and one other variable

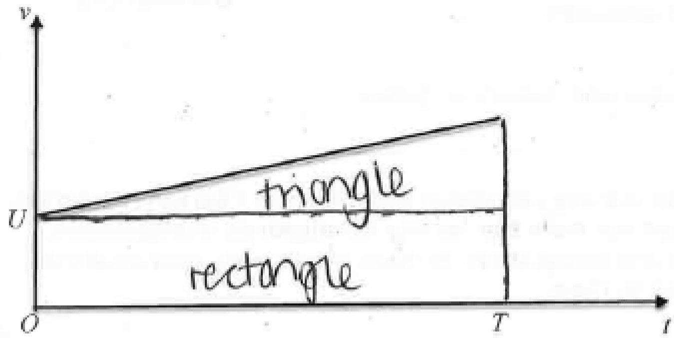
A1: For a correct equation

M1: For using gradient = a to eliminate the other variable to give an equation in D, U, T and a *only*

A1*: For a correct given answer

Student Response A

6.



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(4)

Area of rectangle $= UT$

Area of triangle $= \left(\frac{1}{2} \times a \times T\right)T$

$= \frac{1}{2}aT^2$

Distance $=$ area under graph $=$ triangle $+$ rectangle

$= UT + \frac{1}{2}aT^2$

1/4

Examiner Comments:

Given that there is a printed answer, it is essential that sufficient working and explanation is given. The candidate uses the fact that the distance travelled is equal to the total area under the graph to produce an equation but fails to explain where the '*aT*' term has come from.

Student Response B

area under graph = distance

area of trapezium = $\frac{U+V}{2} \times T$

~~Bt~~

$V = U + at$

$D = \frac{UT}{2} + \frac{VT}{2}$

$D = \frac{UT}{2} + \frac{UT}{2} + \frac{aT \times T}{2}$

$D = UT + \frac{1}{2}aT$

2/4

Examiner Comments:

The candidate earns the first M1A1 but then uses a *suvat* formula (see bracket at bottom of question) to obtain the answer and so scores no further marks.

Student Response C

$$\frac{(u+v)t}{2} = D = \text{area of trapezium}$$

gradient $= \dfrac{v-u}{t} = a$

$v = u + at$

$$v = \frac{2D}{t} - u$$

$$\frac{2D}{t} - u = u + at$$

$$\frac{2D}{t} = 2u + at$$

$$2D = 2ut + at^2$$

$$D = ut + \tfrac{1}{2}at^2$$

2/4

Examiner Comments:

The candidate uses the area under graph = distance travelled and gradient of graph = acceleration to obtain two equations for v and then eliminating to obtain the printed answer. However, the equations and the result should be given in terms of T not t, so both A marks have been deducted.

Exemplar question 7

7. A car is moving along a straight horizontal road with constant acceleration. There are three points *A*, *B* and *C*, in that order, on the road, where $AB = 22$ m and $BC = 104$ m.
The car takes 2 s to travel from *A* to *B* and 4 s to travel from *B* to *C*.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes *A*.

(7)

(Total for Question 7 is 7 marks)

Mark Scheme

Question	Scheme	Marks	AOs
7(i)(ii)	Using a correct strategy for solving the problem by setting up two equations in *a* and *u* only and solving for either	M1	3.1b
	Equation in *a* and *u* only	M1	3.1b
	$22 = 2u + \frac{1}{2} a\, 2^2$	A1	1.1b
	Another equation in *a* and *u* only	M1	3.1b
	$126 = 6u + \frac{1}{2} a\, 6^2$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1ft	1.1b
			(7 marks)

Notes:

M1: For solving the problem by setting up two equations in *a* and *u* only and solving for either
M1: Use of (one or more) *suvat* formulae to produce an equation in *u* and *a* only
A1: For a correct equation
M1: Use of (one or more) *suvat* formulae to produce another equation in *u* and *a* only
A1: For a correct equation
A1: For correct accln 5 m s^{-2}
A1: For correct speed 6 m s^{-1} (The second of these A marks is an ft mark, following an incorrect value for *u* or *a*, depending on which has been found first)
N.B. Do not award the ft mark for absurd answers e.g. $a > 15$, $u > 50$

See alternative on the next page

ALTERNATIVE

Question	Scheme	Marks	AOs
7(i)(ii)	Using a correct strategy for solving the problem by obtaining actual speeds at two times and using a = change in speed / time taken.	M1	3.1b
	Actual speed at $t = 1 =$ Average speed over interval	M1	3.1b
	22/2 = 11	A1	1.1b
	Actual speed at $t = 4 =$ Average speed over interval	M1	3.1b
	104/4 = 26	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1ft	1.1b
			(7 marks)

Notes:

M1: For solving the problem by obtaining two actual speeds and use of $a = (v - u)/ t$

M1: Use of speed at half-time = av speed over interval to produce a speed at $t = 1$

A1: For a correct speed

M1: Use of speed at half-time = av speed over interval to produce a speed at $t = 4$

A1: For a correct speed

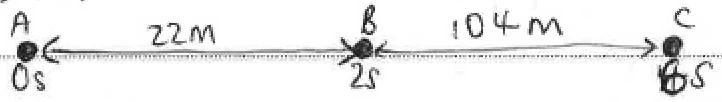
A1: For correct accln 5 m s^{-2}

A1: ft for correct speed 6 m s^{-1} (This is an ft mark, following an incorrect value of a)

N.B. Do not award the ft mark for absurd answers e.g. $a > 15$, $u > 50$

Student Response A

7\. i) $AB = 22m \quad BC = 104m$



$s = 126m \quad t = 6s \qquad 126 = 6u + \frac{1}{2}a6^2$

$s = 104m \quad t = 4s \qquad 104 = 4u + \frac{1}{2}a4^2$

$126 = 6u + 18a$

$104 = 4u + 8a$

$u = 21 - 3a \qquad u = 26 - 2a$

$21 - 3a = 26 - 2a \qquad a = -5m/s^2$

ii) $a = -5 \quad t = 2 \quad s = 22$

$s = ut + \frac{1}{2}at^2$

$22 = u(2) + \frac{1}{2}(-5)(2)^2$

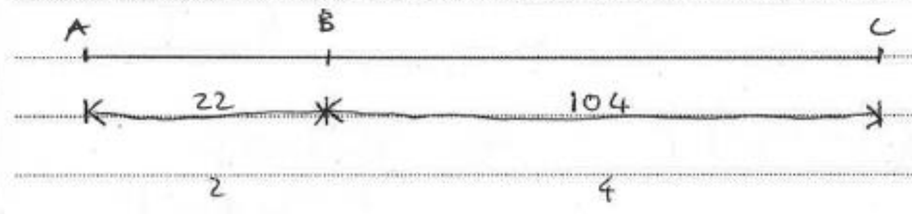
$32 = 2u \qquad u = 16ms^{-1}$

2/7

Examiner Comments:

The candidate uses the same letter (u) to represent two different speeds, namely the speed at A and the speed at B. Consequently, only the M1A1 for one equation can be awarded. Note that the A1 ft can only be earned if the first M mark is earned (i.e. if two equations in two unknowns have been *correctly* obtained) which is not the case here.

Student Response B



AB	s	22	BC	s	104
	u	x		u	y
	v	y		v	2
	a			a	a
	t	2		t	4

$\cancel{v = u + at}$

$\cancel{y =}$

$s = vt - \frac{1}{2}at^2$

$22 = 2y - 8a$

$s = ut + \frac{1}{2}at^2$

$104 = 4y + 8a$

& $4y + 8a = 104$

$2y - 8a = 22$ +

$6y = 126$

$y = 21$

$8a = 104 - 4y$

$8a = 20$

$a = \underline{2.5 \text{ ms}^{-2}}$

s	22
u	
v	21
a	2.5
t	2

$v = u + at$

$21 = u + 5$

$16 = u$

$\underline{16 \text{ ms}^{-1}}$

5/7

Examiner Comments:

The candidate uses a slightly different method from that on the mark scheme by first setting up two equations in a and ‘y’, where y is defined as the speed at B, then solving for a and using $v = u + at$ to find u. The candidate has a correct equation for the motion from B to C but makes a slip in the A to B equation and loses an A mark. The equations are then solved simultaneously (M1) but the value of a is incorrect (A0). The value of u found is a correct follow through value (A1ft).

Pearson

Student Response C

7) A $22m$ B 104 C

$t=2$ $t=6$

i) $s = 22$, $u = ?\ u$, $v = ?\ v$, $a = \checkmark$, $t = 2$

$s = 104$, $u = v$, $v = ?$, $a = \checkmark$, $t = 4$

~~[illegible]~~

$S = vt - \frac{1}{2}at^2$

$22 = 2v - \frac{1}{2} \times a \times 4$

$22 = 2v - 2a$

$2v = 22 + 2a$

$\times 2$

$= 4v = 44 + 4a$

$104 = ut + \frac{1}{2}at^2$

$104 = \overset{4v}{\cancel{6t}} + \cancel{4}8a$

$104 = 44 + 4a + 8a$

$12a = 60$

$\boxed{a = 5\ m/s^2}$

$s = 22$, $u = \cancel{v}$, $v =$, $a = 5$, $t = 2$

$22 = 2u + 10$

$2u = 12$

$\boxed{u = 6\ m/s}$

7/7

Examiner Comments:

The candidate uses the same method as in the previous response but this time all working is correct and full marks are achieved.

Exemplar question 8

8. A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \tfrac{1}{10}(t^4 - 20t^3 + 100t^2), \text{ where } 0 \leq t \leq 10.$$

(a) Explain the restriction $0 \leq t \leq 10$.

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for Question 8 is 9 marks)

Mark scheme

Question	Scheme	Marks	AOs
8(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t - 10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t - 5)(t - 10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance $= 62.5$ m	A1ft	1.1b
		(6)	
			(9 marks)

Notes:

(a)

M1: For substituting $t = 0$ and $t = 10$ into s expression

A1: For noting that $s = 0$ at both times

A1: Since s is a perfect square, $s > 0$ for all other t- values

(b)

M1: For differentiating s w.r.t. t to give v (powers of t reducing by 1)

A1: For a correct v expression in any form

M1: For equating v to 0 and factorising

A1: For correct t values

M1: For substituting their intermediate t value into s

A1: ft following an incorrect t-value

Student Response A

a) 0 seconds is the lower limit because time cannot be negative. 10 seconds is the upper limit as when $t = 10$ $(10^4 - 20 \times 10^3 + 100 \times 10^2) = 0$
$t = 10$ is when the displacement is 0 so the bird has returned to its nest.

b) $v = 0$.

$s = \frac{1}{10}t^4 - 2t^3 + 10t^2$. differentiating:

$v = \frac{4}{10}t^3 - 6t^2 + 20t$.

$0 = \frac{4}{10}t^3 - 6t^2 + 20t$

$t\left(\frac{4}{10}t^2 - 6t + 20\right) = 0$ $\qquad \frac{6 \pm \sqrt{36 - 4 \cdot \frac{4}{10} \cdot 20}}{2 \times \frac{4}{10}}$

$t = 10$ or $t = 5$ $\qquad \boxed{t = 5 \text{ seconds}}$

4/9

Examiner Comments:

Part (a) To earn the M mark the candidate needs to substitute *both* $t = 0$ and $t = 10$ into the s expression.

Part (b) The first 4 marks are earned but the candidate forgets to substitute in their $t = 5$ value to find the s value at that time.

Student Response B

8a) It must be greater than (or equal to) 0 because if ~~it was~~ ≤ 0, ~~it~~ 'negative time' cannot exist. It must be less than 10 (or equal to) because at $t = 10$, ~~the~~ $s = 0$; therefore it has returned from the rest and has finished its journey. ~~the~~

b) $s = \frac{1}{10}(t^4 - 20t^3 + 100t^2)$

$v = \frac{ds}{dt} = \frac{2}{5}t^3 - 6t^2 + 20t$

$\therefore$ when at rest; $v = 0$

$\therefore \frac{2}{5}t^3 - 6t^2 + 20t = 0$

$t\left(\frac{2t^2}{5} - 6t + 20\right) = 0$

$\therefore \frac{2t^2}{5} - 6t + 20 = 0$

$2t^2 - 30t + 100 = 0$

$t^2 - 15t + 50 = 0$

$(t - 5)(t - 10) = 0$

$\therefore t = 5$

$\therefore s = \frac{1}{10}(5^4 - 20(5^3) + 100(5^2))$

$s = \frac{1}{10}(625)$

$\boxed{s = 62.5 \text{ m}}$

(Total for Question 8 is 9 marks)

6/9

Examiner Comments:

Part (s) The candidate does not do enough to earn the M mark (needs to explicitly show that $s = 0$ at both $t = 0$ and $t = 10$).

Part (b) Here the candidate, after a false start, earns all the marks.

Student Response C

8a) when $t = 0$, $s = \frac{1}{10}(0^4 \ldots) = 0$

$t = 10$, $s = \frac{1}{10}\left(10^4 - 20(10^3) + 100(10^2)\right) = 0$

when $t=0$, bird leaves the nest, displacement of 0, and when $t=10$, bird returns to the nest, displacement of 0, hence $0 \le t \le 10$ is the time of its flight

b) $v = \frac{ds}{dt}$ and $s = \left(\frac{1}{10}t^4 - 2t^3 + 10t^2\right)$

$\therefore v = \frac{4}{10}t^3 - 6t^2 + 20t$

$= \frac{2}{5}t^3 - 6t^2 + 20t$

$\therefore$ when $v = 0$, instantaneous rest

$\frac{2}{5}t^3 - 6t^2 + 20t = 0$

$\frac{2}{5}t\left[t^2 - 15t + 50\right] = 0$

$\frac{2}{5}t\,[t-5][t-10] = 0$

$\therefore$ when $t = 5$, at instantaneous rest

$\therefore s = \frac{1}{10}\left(5^4 - 20(5^3) + 100(5^2)\right)$

$= \frac{1}{10}(625)$

$s = 62.5$ m

8/9

Examiner Comments:

Part (a) The candidate earns the M mark and the first A mark. The key point is that s is the *distance*, which is non-negative by definition, of the bird from the nest and so, to earn the final A mark, the candidate needs to show that s is always positive for t values between 0 and 10.

Part (b) Full marks earned here for a neat and concise solution.

Exemplar question 9

9.

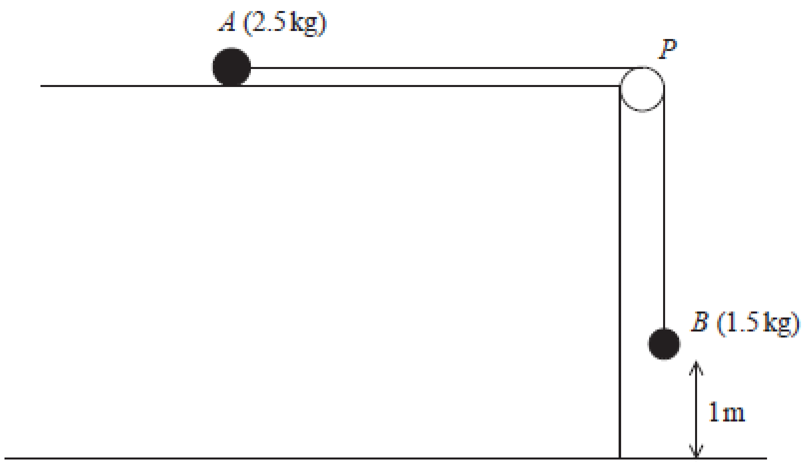


Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible, the pulley is modelled as being small and smooth and the acceleration due to gravity, g, is modelled as being 9.8 m s^{-2}

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)

(Total for Question 9 is 10 marks)

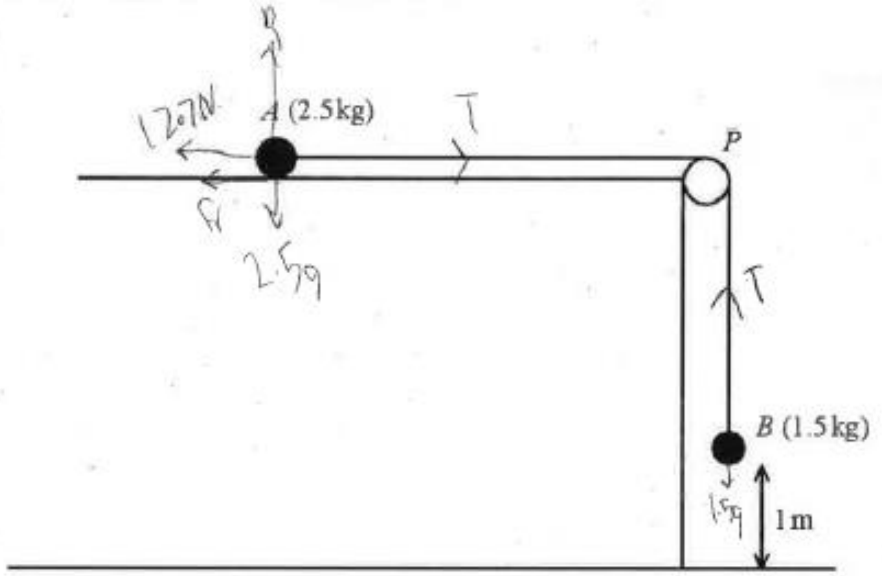
Mark scheme

Question	Scheme	Marks	AOs
9(a) (i)	Equation of motion for *A*	M1	3.3
	$T - 12.7 = 2.5a$	A1	1.1b
	Equation of motion for *B*	M1	3.3
(ii)	$1.5g - T = 1.5a$	A1	1.1b
		(4)	
	Solving two equations for *a*	M1	1.1b
	$a = 0.5$	A1	1.1b
(b)		**(2)**	
(c)	$1 = \frac{1}{2} \times 0.5\, t^2$	M1	3.4
	$t = 2$ seconds	A1ft	1.1b
		(2)	
(d)	Valid improvement, see below in notes	B1	3.5c
	Valid improvement, see below in notes	B1	3.5c
		(2)	
			(10 marks)

Continued question 9
Notes:
(a)(i) **M1:** For resolving horizontally for *A* **A1:** For a correct equation
(a)(ii) **M1:** For resolving vertically for *B* **A1:** For a correct equation
(b) **M1:** For complete correct strategy for solving the problem, setting up **two** equations in *a*, and then solving them for *a* **A1:** For $a = 0.5$
(c) **M1:** For a complete method (which could involve use of more than one *suvat* formula) to give an equation in *t* only **A1:** ft from their *a* to get time in seconds
(d) **B1, B1** For any two of e.g. Include the dimensions of the ball in the model so that the distance it falls changes e.g. Include the dimensions of the pulley in the model so string not parallel to table e.g. Include a variable resistance in the model instead of taking it to be constant e.g. Include a more accurate value for *g* in the model

Student Response A

9.



a) (i) A → $F = ma$

$T - 12.7 - F_r = 2.5a$

(ii) B ↓ $F = ma$

$1.5g - T = 1.5a$

b) (A+B) $1.5g - 12.7\cancel{g} - F_r = 4a$

$2 - F_r = 4a$?

B $\cancel{s = 1}$ $\cancel{u = 0}$ $\cancel{a = g}$ ↑ = ↓

$R = 2.5g = 24.5N$

2/10

Examiner Comments :

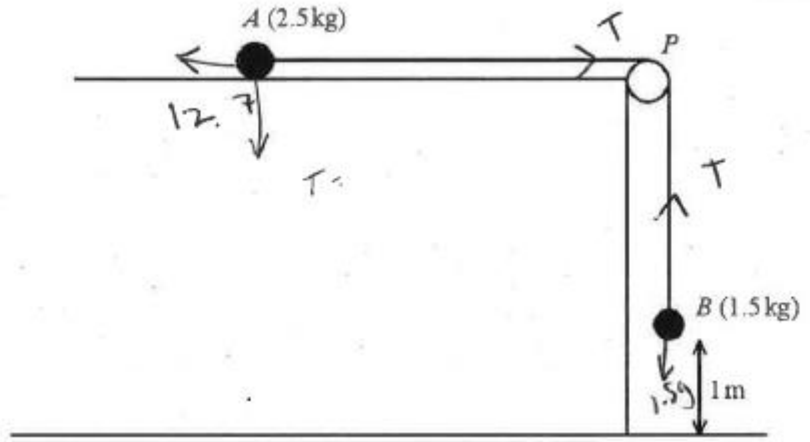
Part (a)(i) The candidate has included an extra friction force in the equation of motion and so loses the M mark (incorrect number of terms) and therefore the dependent A mark also.

Part (a)(ii) Full marks here.

Part (b) The candidate does not solve for *a*, so no marks awarded.

Student Response B

9.



a) i) $T - 12.7 = 2.5a$

$\Rightarrow a = \dfrac{T - 12.7}{2.5}$

ii) $1.5g - T = 1.5a$

$a = \dfrac{1.5g - T}{1.5}$

b) $T = -1.5a + 1.5g$

$T = 2.5a + 12.7$

$-1.5a + 1.5g = 2.5a + 12.7$

$1.5 \times 9.8 - 12.7 = 4a$

$a = 0.5\,ms^{-2}$

c) $s = 1, \quad a = 0.5$

$t =$

6/10

Examiner Comments :

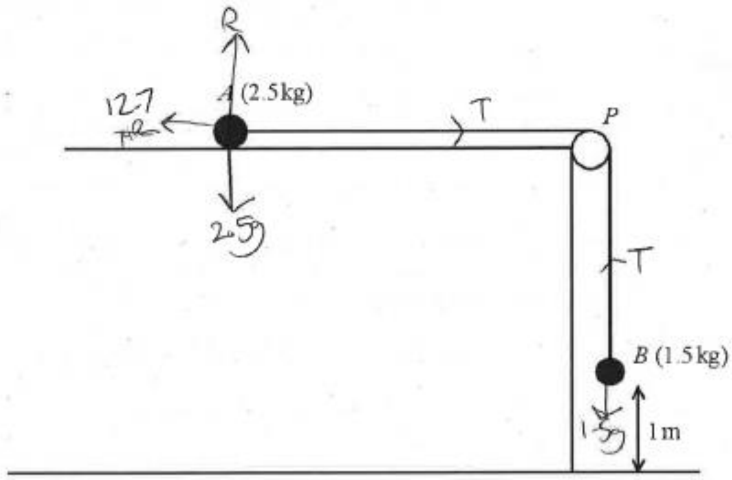
Part (a) Full marks for the two equations of motion.

Part (b) The candidate solves the two equations to obtain a correct value of *a*.

Part (c) No valid attempts.

Student Response C

9.



a) i) R(→) EqM: $T - 12.7 = 2.5a$

ii) R(↓) EqM: $1.5g - T = 1.5a$

b) $T - 12.7 = 2.5a$ +

$1.5 \times 9.8 - T = 1.5a$

$2 = 4a$ $a = 0.5 \text{ ms}^{-2}$

c) $s = 1$ $u = 0$ $v = ?$ $a = 0.5$ $t = ✓$

$s = ut + \frac{1}{2}at^2$ $1 = 0 + 0.25t^2$ $t^2 = 4$

$t = \sqrt{4} = 2s$

d) The model could be improved by taking air resistance (of the balls) into account or by including the string having mass) etc. ~~effect~~

10/10

Examiner Comments:

Parts (a), (b) and (c) are all fully correct.

Part (d) The candidate suggests two valid improvements (air resistance on balls and mass of string) and so both B marks are scored.

A level Mathematics – Paper 3 (Statistics)

Exemplar question 1

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \le y < 5$	$5 \le y < 8$	$8 \le y < 11$	$11 \le y < 12$	$12 \le y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \le y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \le y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (*b*) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

(Total for Question 1 is 13 marks)

Mark Scheme

Question	Scheme	Marks	AOs
1(a)	**Area** = $8\times1.5=12$ cm^2 **Frequency** = 8 so 1 cm^2 = $\frac{2}{3}$ **hour (o.e.)**	M1	3.1a
	Frequency of 12 corresponds to area of 18 so height = $18\div2.5$ = **7.2 (cm)**	A1	1.1b
	Width = 5×0.5 = **2.5 (cm)**	B1cao	1.1b
		(3)	
(b)	$[\bar{y}=]\ \frac{205.5}{31}=$ awrt 6.63	B1cao	1.1b
	$[\sigma_y=]\ \sqrt{\frac{1785.25}{31}-\bar{y}^2}\ =\sqrt{13.644641}$ **= awrt 3.69** allow $[s=]\ \sqrt{\frac{1785.25-31\bar{y}^2}{30}}$ = **awrt 3.75**	M1 A1	1.1a 1.1b
		(3)	
(c)	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		(2)	
(d)	$\bar{x}+\sigma\approx10.3$ so number of days is e.g. $\frac{(11-\text{"10.3"})}{3}\times8\ (+5)$	M1	1.1b
	= 6.86 so **7 days**	A1	1.1b
		(2)	
(e)	[H = no. of hours] P(H > 10.3) or P(Z > 1) = [0.15865…]	M1	3.4
	Predict $31\times0.15865...$ = <u>**4.9 or 5 days**</u>	A1	1.1b
		(2)	
(f)	(5 or) 4.9 days < (7 or) 6.9 days so model may **not** be suitable	B1	3.5a
		(1)	
			(13 marks)

Question 1 continued
Notes:
(a) **M1:** for clear attempt to relate the area to frequency. Can also award if their height $\times$ their width = 18 **A1:** for height = 7.2 (cm)
(b) **M1:** for a correct expression for σ or s, can ft their value for mean **A1:** awrt 3.69 (allow s = 3.75)
(c) **M1:** for a suitable comparison of standard deviations to comment on reliability. **A1:** for stating Hurn is south of Heathrow and a correct conclusion
(d) **M1:** for a correct expression – ft their $\bar{x} + \sigma \approx 10.3$ **A1:** for 7 days but accept 6 (rounding down) following a correct expression
(e) **M1**: for a correct probability attempted **A1:** for a correct prediction
(f) **B1:** for a suitable comparison and a compatible conclusion

Student Response A

a) $11 - 8 = 3$ $\quad 1.5 \div 3 = 0.5$

$5 \times 0.5 = 2.5$ $\quad$ width $= 2.5$ cm

height $= 12$ cm

b) $2.5 \times 12 + 6.5 \times 6 + 9 \times 8 + 3 \times 11.5 + 13 \times 2 = 201.5$

$\bar{y} = \frac{201.5}{31} = 6.5$

$12^2 \times 2.5 + 6.5 \times 6^2 + 9.5 \times 8^2 + 11.5 \times 3^2 + 13 \times 2^2 = 1357.5$

$\sigma^2 = \frac{1357.5}{31} - \left(\frac{201.5}{31}\right)^2 = 1.54\ldots$

$\Rightarrow \sigma = 1.24$ (3 s.f.)

c) The higher variance shows greater variability meaning evidence does not support Thomas' beliey

d) $\sigma + \mu = 7.74 \approx 8$ days.

e) $6.6 + 3.7 = 10.3 \approx 11$ days

f)

2/13

Examiner Comments: (a) M0A0B1 (b) B0M0A0 (c) M1A0 (d) M0A0 (e) M0A0 (f) B0

Part (a) The candidate has a correct value for the width but the height is incorrect. The height of the 8~11 class is the same as the frequency and the candidate just assumes that the same would be true in the case of the 0~5 class but of course this ignores the principle of histograms that the **area** is proportional to the frequency

Part (b) The mean is incorrect (the midpoint for 8~11 class was given as 9 not 9.5) and $\sum f^2 x$ rather than $\sum f\ x^2$ is used when finding the standard deviation.

Part (c) The candidate comments on the difference between the standard deviations meaning greater variability but the answer is incomplete because they do not mention that Hurn is south of Heathrow and that is the reason that Thomas' belief is not supported.

Parts (d) and (e) Calculations seen represents hours of sunshine but the question asks about number of <u>days</u>.

No attempt at part (f).

Student Response B

(1)

a) $8 \le y < 11$ $11-8=3$ $3 \div 2 = 1.5$ $\qquad 0 \le y < 5$

$W = 1.5 \rightarrow$ class width $\qquad W = 5 \div 2 = \cancel{}\, 2.5$

$h = 8 \rightarrow$ F.d. $\qquad F = 12$

$\therefore F = W \times h = 8 \times 1.5 = 12$ $\qquad h = 1\cancel{8}2$

~~12÷8~~ $\qquad$ The box is 2.5cm wide and 1~~8~~2 cm high.

~~∴ x = 12~~

~~x = 12×12 = 18~~

Question 1 continued

b)

# (m.p)	2.5	~~10.5~~ 6.5	9.5	11.5	13	
F	12	6	8	3	2	= 31
P	12/31	6/31	8/31	3/31	2/31	

$\overline{x^2} = (2.5^2 \times 12 + 6.5^2 \times 6 + 9.5^2 \times 8 + 11.5^2 \times 3 + 13^2 \times 2) \div 31 =$

$= 57.58870967741935483$

$\bar{x} = (2.5 \times 12 + 6.5 \times 6 + 9.5 \times 8 + 11.5 \times 3 + 13 \times 2) \div 31 = 6.6290322... \approx$

$\approx 6.63 \text{ (3sf)} = \frac{411}{62}$

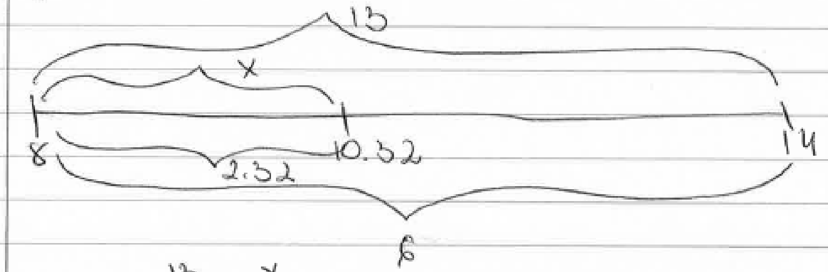
$Var(x) = \overline{x^2} - (\bar{x})^2 = 57.58... - \left(\frac{411}{62}\right)^2 = 13.644641$

$\sigma = \sqrt{Var(x)} = \sqrt{13.644641} = 3.69386532 \approx 3.69 \text{ (3sf)}$

c) I'm not good at geography but ~~the~~ my s.d. ~~and~~ is smaller than his so my data spread is smaller and my mean is bigger so that's means ~~the~~ ~~the~~ in Heathrow were more sunny days and it ~~the~~ Heathrow is more on the south than Hurn is. **(Total for Question 1 is 13 marks)**

1. c) (...) then that supports Thomas' believe.

d) $\bar{x} + \sigma = 6.63 + 3.69 = 10.32$



$$\frac{13}{6} = \frac{x}{2.32}$$

$30.16 = 6x$

$x = 5.026 \approx 5$ days

~~31-5=26 days~~

e) $X \sim N(6.6, 3.7^2)$

$P(X > 10.32) = P\left(Z > \frac{10.32 - 6.6}{3.7}\right)$

$= P(Z > 1.0054)$

$= 1 - P(Z \leq 1.01)$

$= 1 - 0.8438 = 0.1563$

$0.1563 \times 31 = 4.8422 \approx 5$ days

f) Helen's model isn't perfect but suitable enough

7/13

Examiner Comments: (a) M0A0B1 (b) B1M1A1 (c) M1A0 (d) M0A0 (e) M1A1 (f) B0

Part (a) The width is correct but there is no clear working for the height and an answer of 12 is incorrect.

Part (b) Fully correct. There is a clear expression given for the standard deviation which meant that, had the mean been incorrect, a method mark could have been awarded.

Part (c) The candidate comments that they don't know whether Hurn is north or south of Heathrow. It is a requirement of the specification that candidates are familiar with the large data set and so they should know from this that Hurn is south of Heathrow. The candidate uses the standard deviations to comment on the variability and so scores one mark.

Part (d) An attempt to interpolate is seen but the class boundary is incorrect and so the expression is incorrect. This approach works up from the bottom of the class interval and to answer the question one would then need to find 31 – (18 + their x) which some candidates did not appreciate.

Part (e) Fully correct.

In the final part we would expect to see a comparison of their answers to parts (d) and (e) to support the conclusion. In this case a comment that "the number of days in parts (d) and (e) are similar" followed by the statement that Helen's model is therefore suitable would have been sufficient for the mark but the student's response here was not sufficient.

Student Response C

(1)

a) $8 \leq y < 11$: class width $= 3$

frequency density $= \frac{8}{3} =$

width: 1.5cm $=$ width 3 height: 8cm $= 8/3$ f.d.

0.5cm $=$ width 1 3cm $= 1$ f.d.

Question 1 continued

$0 \leq y < 5$: class width $= 5$

frequency density $= \frac{12}{5}$

width $= 0.5\text{cm} \times 5$ height $= 3\text{cm} \times 12/5$

$= 2.5$cm $= 7.2$cm

b) $\text{mean} = \frac{\Sigma fx}{\Sigma f} = \frac{(12\times2.5)+(6\times6.5)+(8\times9.5)+(3\times11.5)+(2\times13)}{12+6+8+3+2}$

$= \frac{205.5}{31}$

$= 6.63$ (3 s.f.)

$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \mu^2}$

$\Sigma fx^2 = (12\times2.5^2)+(6\times6.5^2)+(8\times9.5^2)+(3\times11.5^2)+(2\times13^2)$

$= 1785.25$

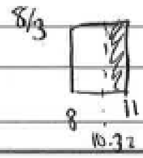
$\sigma = \sqrt{\frac{1785.25}{31} - \left(\frac{205.5}{31}\right)^2} = 3.69$ (3 s.f.)

c)

	Mean	S.d.
Heathrow	6.63	3.69
Hurn	5.98	4.12

larger s.d. so results more spread out ∴ Thomas wrong as Hurn more south than Heathrow

d) $6.63 + 3.69 = 10.32$



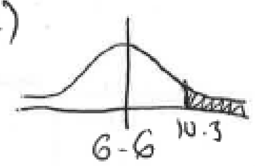
freq $= 8/3 \times 0.68$

$= 1.81\dot{3}$

$1.81\dot{3} + 3 + 2 = 6.81$ days

(Total for Question 1 is 13 marks)

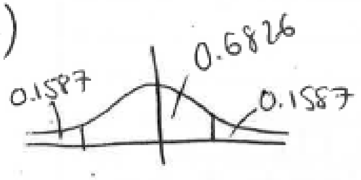
e)



$6.6 + 3.7 = 10.3$

$$P(N > 10.3) = 1 - P(N < 10.3)$$
$$= 1 - P\left(Z < \frac{10.3 - 6.6}{3.7}\right)$$
$$= 1 - P(Z < 1)$$
$$= 1 - 0.8413 = \underline{0.1587}$$

f)



approx $2/3$ of data lies within one s.d. of the mean

$\therefore$ model is suitable

11/13

Examiner Comments: (a) M1A1B1 (b) B1M1A1 (c) M1A1 (d) M1A1 (e) M1A0 (f) B0

Parts (a) and (b) Fully correct.

Part (c) Correct full response that compares standard deviations and mentions that Thomas' belief is not supported because Hurn is south of Heathrow.

Part (d) There is a correct interpolation and the candidate uses the correct tail to reach 6.8 days.

Part (e) Correct calculation for the probability is seen and score the method mark but the candidate do not go on to multiply this by 31 to obtain an estimate of the number of days so lose the accuracy mark.

Part (f) In the final part there is no comparison of the number of days between parts (d) and (e) so no mark is scored.

Exemplar question 2

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$.

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C.

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days, a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$.

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

(Total for Question 2 is 6 marks)

Mark Scheme

Question	Scheme	Marks	AOs
2(a)	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
(b)	e.g. Linear association between w and t	B1	1.2
		(1)	
(c)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
(d)	Higher $\bar{t}$ suggests overseas and not Perth…lower wind speed so perhaps not close to the sea so suggest **Beijing**	B1	2.4
		(1)	
			(6 marks)

Notes:

(a)
B1: for a correct statement (unreliable) with a suitable reason

(b)
B1: for a correct statement

(c)
B1: for both hypotheses in terms of ρ
M1: for selecting a suitable 5% critical value compatible with their H_1
A1: for a correct conclusion stated

(d)
B1: for suggesting Beijing with some supporting reason based on t or w
Allow Jacksonville with a reason based just on higher $\bar{t}$

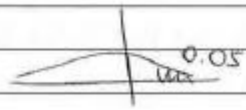
Student Response A

a) due to extrapolation - data set goes till 19.5°C

b) the relationship between one and another variable - how closely they are linked together; how they depend and effect each other

c) $H_0: r = 0$

$H_1: r > 0$



$X \sim B(9, 0.609)$

$P(X \geq 9) > 0.95$

$P(X \geq 9) = P\left(Z \geq \frac{9 - 5.481}{1.463923154}\right)$

$= 1 - P(Z \leq 2.40(354))$

$= 1 - 0.9918$

$= 0.0082$

$0.0082 < 0.05 \therefore$ accept H_1 & r cannot be equal to 0, it has to be greater

1/6

Examiner Comments: (a) B1 (b) B0 (c) B0M0A0 (d) B0

Part (a) The candidate mentions that extrapolation is involved which is sufficient to score the mark.

Part (b) No mention that the product moment correlation coefficient is a measure of the strength of the linear relationship between the variables and so no mark is scored.

Part (c) The candidate states the hypotheses but uses r rather than the population parameter ρ and so the 1st mark is lost. For tests involving correlation the critical values are given in the book of tables. This student attempts to use a normal distribution to carry out the test which is an incorrect method so no further marks are scored in this part.

There was no attempt at part (d).

Student Response B

a.) 24°C is outside the range of the data and so the model is unreliable

b.) pmcc measures the strength of the relationship between w and t

c.) $H_0 : \rho = 0 \qquad H_1 : \rho > 0$

From tables, critical value $\rho = 0.5822$

$0.609 > 0.5822$ so in critical region

$\therefore$ Reject H_0 and accept H_1

d.) The mean temperature is very high which means this place is not in the UK.

3/6

Examiner Comments: (a) B1 (b) B0 (c) B1M1A0 (d) B0

Part (a) The candidate mentions that 24°C is outside the range of the data, which is equivalent to mentioning extrapolation.

Part (b) There is a mention of the strength of the relationship but no mention that it is a <u>linear</u> relationship.

Part (c) The hypotheses are correctly written in terms of ρ and the critical value is stated correctly. The candidate states they would reject H_0 but no explanation what this means is given. We would expect a comment specifying that there is evidence that the correlation between the variables is greater than zero.

Part (d) In the final part the candidate uses a comparison of temperatures to state that the location is outside of the UK but their knowledge of the large data set is not used to suggest a specific location.

Student Response C

a) Extrapolation beyond range of data

$\therefore$ unreliable

b) Measures the strength of fit of w & t to a straight line.

c) Let ρ be the population pmcc

$\therefore$ Testing $H_0 : \rho = 0$ against $H_1 : \rho > 0$ (1-tail)

From tables, critical value is 0.6000 for $n=9$, 5%

Test stat $r = 0.609 > 0.6000$ $\therefore$ in critical region $\Rightarrow$ sufficient evidence to reject H_0

$\therefore$ evidence that population pmcc is greater than 0

d) The mean temperature is very high & the mean windspeed is low.

Could be an international location on large data set

$\therefore$ possibly Jacksonville.

5/6

Examiner Comments: (a) B1 (b) B1 (c) B1M1A0 (d) B1

Parts (a) and (b) The candidate mentions extrapolation and "the strength of fit …to a straight line" which conveys the idea of the strength of the linear relationship and scores the mark.

Part (c) The hypotheses are written correctly but the critical value quoted is from the Spearman's table. Care should be taken when using these tables to use the correct columns. We would allow this for the method mark but, even though the conclusion is stated correctly the accuracy mark cannot be given since the critical value is incorrect.

In the final part the candidate comments on the temperature and windspeed and selects Jacksonville which is an acceptable answer in the mark scheme.

Exemplar question 3

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips cannot be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm.

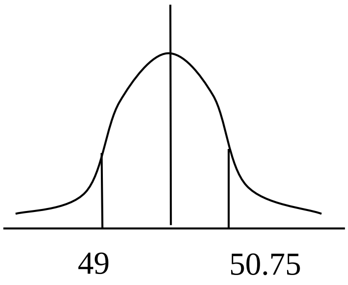
A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm.

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm.

(5)

(Total for Question 3 is 12 marks)

Mark Scheme

Question	Scheme	Marks	AOs
Q3(a)			
	P (L > 50.98) = 0.025	B1cao	3.4
	$\therefore \frac{50.98-\mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \quad \mu = 50$	A1cao	1.1b
	P(49 < L < 50.75)	M1	3.4
	= 0.9104… awrt **<u>0.910</u>**	A1ft	1.1b
		(5)	
(b)	S = number of strips that cannot be used so S~B(10, 0.090)	M1	3.3
	$= \text{P}(S \le 3) = 0.991166\ldots$ awrt 0.991	A1	1.1b
		(2)	
(c)	$\text{H}_0 : \mu = 50.1$ $\quad \text{H}_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim \text{N}\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$\text{P}(\bar{X} > 50.4) = 0.0264$	A1	3.4
	p = 0.0264 > 0.01 <u>or</u> z = 1.936… < 2.3263 and not significant	A1	1.1b
	There is insufficient evidence that the **<u>mean length</u>** of strips is **<u>greater than 50.1</u>**	A1	2.2b
		(5)	
			(12 marks)

Question 3 continued
Notes:
(a) **1st M1:** for standardizing with μ and 0.5 and setting equal to a z value ($\|z\| > 1$) **2nd M1:** for attempting the correct probability for strips that can be used **2nd A1ft:** awrt 0.910 (allow ft of their μ)
(b) **M1:** for identifying a suitable binomial distribution **A1:** awrt 0.991 (from calculator)
(c) **B1:** hypotheses stated correctly **M1:** for selecting a correct model (stated or implied) **1st A1:** for use of the correct model to find p = awrt 0.0264 (allow z = awrt 1.94) **2nd A1:** for a correct calculation, comparison and correct statement **3rd A1:** for a correct conclusion in context mentioning "mean length" and 50.1

Student Response A

a) $L \sim N(\mu, 0.25)$

$P(L > 50.98) = 0.025$

$\Rightarrow z = 1.9600$

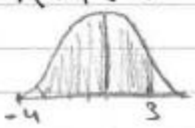
$z = \dfrac{50.98 - \mu}{0.5}$

$1.9600 = \dfrac{50.98 - \mu}{0.5}$

$-50 = -\mu$

$\mu = 50$

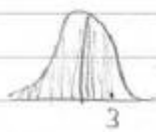
$P(-4 < z < 3)$



$= 0.9987$

$L \sim N(50, 0.5^2)$

$P(49 < L < 50.75)$



$z = \dfrac{49 - 50}{0.25}$

$= -4$

$z = \dfrac{50.75 - 50}{0.25}$

$= 3 \Rightarrow P(z < 3) = 0.9987$

b) $X \sim B(10, 0.9987)$

Prob of a strip being used

$P(X < 4) = {}^{10}C_4 \times (1.3 \times 10^{-3})^4 \times 0.9987^6$

$= 5.951178863 \times 10^{-10}$

c) $Y \sim N(\mu, 0.6^2)$

$X \sim B(15, p)$ $\quad E(X) = 50.4$

$\mu = np$

$50.4 = 15p$

$p = 3.36$

$X \sim B(15, 3.36)$

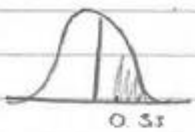
$\alpha = 0.01$

$H_0: p = 3.36$ $\quad H_1: p > 3.36$

$P(X > 50.1) \approx P(Y \geqslant 50.6)$

$z = \dfrac{50.6 - 50.4}{0.6} = 0.\dot{3} \to 0.33333\ldots \to 0.33$

$P(Z \geqslant 0.33)$



$1 - 0.6293$

$= 0.3707$

4/12

Examiner Comments: (a) B1M1A1M0A0 (b) M1A0 (c) B0M0A0A0A0

Part (a) There is a correct probability statement and correct use of standardisation to find μ. The candidate then tries to find the correct probability but uses the variance rather than the standard deviation when standardising and so scores M0. Note as the mean and standard deviation are both now known, candidates should be encouraged to use their calculators to find probabilities from Normal distributions.

Part (b) Whilst the Binomial distribution quoted is that for the number of strips which can be used, the calculation then does use the appropriate distribution for the number of strips which cannot be used (following through on their probability from part (a)) and so the candidate scores M1 but finds P(X = 4) rather than P(X < 4).

Part (c) There is no indication of the correct hypotheses or that the distribution of $\bar{X}$ is being used so no further marks are scored.

Student Response B

(a) Know $P(L > 50.98) = 0.025$

So $\frac{50.98 - \mu}{0.5} = 1.6449$

$\therefore \quad \mu = 50.16$

$\therefore \quad P(49 < L < 50.75) = \underline{0.870829...}$

(b) S = strips that can't be used $\quad S \sim B(10, \cancel{0.87}\ 0.1292)$

$P(S < 4) = P(S \leq 3) = 0.96935..$

$\simeq \underline{0.969}$

(c) $H_0: \mu = 50.1 \qquad H_1: \mu > 50.1$

$P(X > 50.4) = P\left(Z > \frac{50.4 - 50.1}{0.6}\right)$

$z = 0.5$

$0.5 < 2.3263$

so not significant, accept H_0.

6/12

Examiner Comments: (a) B1M1A0M1A1ft (b) M1A0 (c) B1 M0A0A0A0A0

Part (a) There is a correct probability statement and standardisation using a z value > 1 The z value though is incorrect and this leads to an incorrect value of μ. This value is used to find the correct probability and the answer is a correct follow through using their mean.

Part (b) The candidate states and uses a correct binomial distribution with the value of p coming from their answer to part (a). The answer though is incorrect and there is no follow through available for this mark.

Part (c) The hypotheses are started correctly but then N(50.1, 0.6^2) is used rather than the correct normal distribution with standard error of $\frac{0.6}{\sqrt{15}}$. This is an incorrect method and so no further marks are available.

Pearson

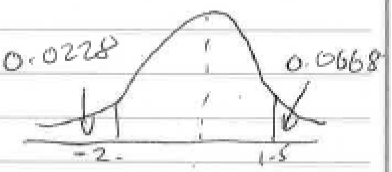
Student Response C

(a) $P(L > 50.98) = 0.025$

$\frac{50.98 - \mu}{0.5} = 1.96$

$\therefore \mu = 50$

$P(49 < L < 50.75)$



$= P(-2 < Z < 1.5) = 1 - (0.0228 + 0.0668)$

$= 0.9104$

(b) $X \sim B(10, 0.0896)$

${}^{10}C_0 (0.0896)^0 (0.9104)^{10} = 0.3911\ldots$

${}^{10}C_1 (0.0896)^1 (0.9104)^9 = 0.38\ldots$

${}^{10}C_2 (0.896)^2 (0.9104)^8 = 0.83\ldots$

${}^{10}C_3\ 0.0896^3\ 0.9104^7 = 0.60$

$\therefore P(X \leq 3) = 0.9913$

(c) $H_0: \mu = 50.1$ $H_1: \mu > 50.1$

$z = \frac{50.4 - 50.1}{0.6/\sqrt{15}} = 1.936\ldots$

$1.936 < 2.58$

Not significant.

10/12

Examiner Comments: (a) B1M1A1M1A1 (b) M1A1 (c) B1M1A1A0A0

Part (a) Fully correct but once the candidate finds the correct value for μ this new specification would expect them to use their calculator to find the probability. This candidate uses the tables which is fine but would take a lot longer.

Part (b) There are only two marks available: one for stating or trying to use a correct binomial distribution and the other for evaluating the probability. Again this candidate works out all 4 binomial probabilities and adds them and since their answer rounds to 0.991 they score both marks.

Part (c) In the final part the hypotheses are correct and the correct distribution is used giving the first 3 marks but the candidate uses an incorrect critical value and does not go on to interpret the conclusion which means the final two marks are lost.

Exemplar question 4

4. Given that

$$P(A) = 0.35, \ P(B) = 0.45 \ \text{and} \ P(A \cap B) = 0.13$$

find

(a) $P(A' | B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$.

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

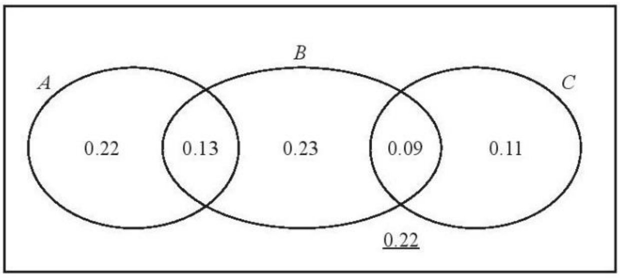
(5)

(d) Find $P([B \cup C]')$

(2)

(Total for Question 4 is 10 marks)

Mark Scheme

Question	Scheme	Marks	AOs
4(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e.	M1	1.1b
	= 0.44	A1	1.1b
		(2)	
			(10 marks)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

B1 for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

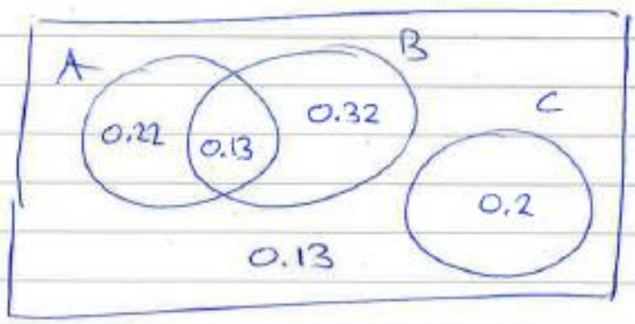
Student Response A

a.) $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.87}{0.55} = \underline{\underline{1.58}}$

b.) $P(A) \times P(B) = 0.35 \times 0.45 \neq 0.13$

$\therefore$ A and B are dependent

c.)



d.) $P([B \cup C]') = 1 - P(B \cup C)$

$= 1 - 0.67$

$= 0.33$

2/10

Examiner Comments: (a) M0A0 (b) B1 (c) B0M0A0M0A0 (d) M1A0

Part (a) The correct probability formula is used and the probability in the denominator is correct however as the candidate gives an answer which is bigger than 1 the method mark is not awarded.

Part (b) Fully correct.

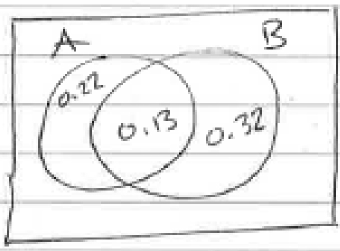
Part (c) The Venn diagram is incorrect as the C circle needs to intersect with B. No marks are awarded as there is no attempt to find P($B \cap C$).

Part (d) Correct method is used but as their diagram is incorrect the answer is also incorrect.

Student Response B

a) $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.33}{0.55} = \underline{\underline{0.6}}$

b)

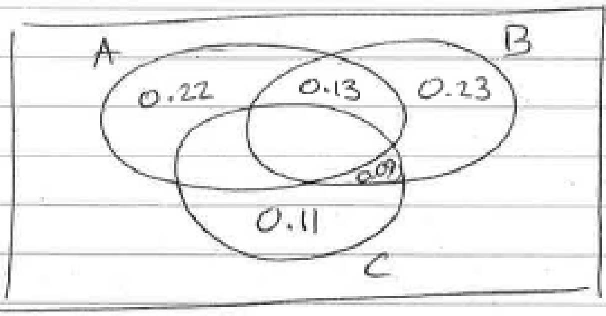


$0.22 \times 0.32 = 0.0704$

$\neq 0.13$

$\therefore$ not independent.

c)



$P(B \cap C) = 0.45 \times 0.20$
$= 0.09$

d) $P([B \cup C]') = P(B' \cup C') = 0.91$

5/12

Examiner Comments: (a) M1A1 (b) B0 (c) B0M1A1M1A0 (d) M0A0

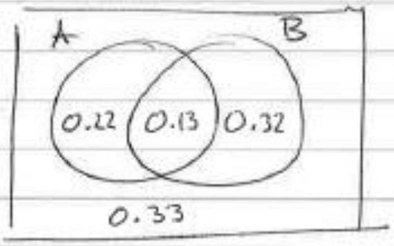
Part (a) All correct.

Part (b) The candidate confuses 0.22 with P(*A*) etc, so the mark is B0 not a correct solution.

Part (c) The three intersecting circles would be acceptable but the candidate needs to give probabilities of 0 is the regions where *A* and *C* intersect. Also $P(A \cup B \cup C)'$ is not given.

Part (d) Incorrect formula is used.

Student Response C

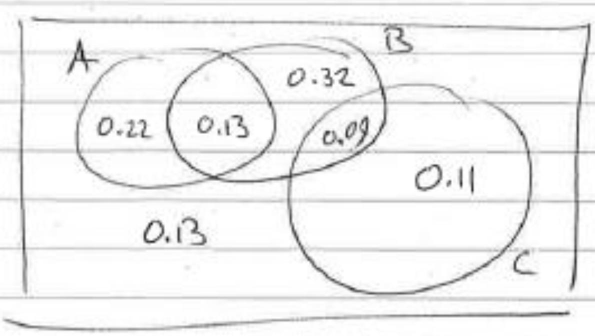
a) 

$P(A'|B') = \frac{0.33}{0.55}$ from diagram

$= 0.6$

b) $P(A'|B') = 0.6$ $\quad P(A') = 1 - 0.35 = 0.65$

$\therefore P(A'|B') \neq P(A')$ $\therefore$ not independent.

c) 

$P(B \cap C) = 0.45 \times 0.2$

$= 0.09$

d) $P([B \cup C]') = 0.22 + 0.13$ from Venn diag

$= 0.35$

7/12

Examiner Comments: (a) M1A1 (b) B1 (c) B1M1A1M0A0 (d) M1A0

Part (a) The candidate appears to use values from a Venn diagram to find the conditional probability, all correct so 2 marks.

Part (b) An alternative property of independent events is used. All correct.

Part (c) Correct structure is seen but an error is made with the probabilities losing 2 marks.

Part (d) Correct method is seen but the candidate uses values from the incorrect Venn diagram and so the final answer is not correct.

Exemplar question 5

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

(Total for Question 5 is 9 marks)

Mark Scheme

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[S = no. of seeds out of 24 that germinate, $S \sim B(24, 0.55)$]		
	T = no. of trays with at least 15 germinating. $T \sim B(10, p)$	M1	3.3
	$p = P(S \geq 15) = 0.299126...$	A1	1.1b
	So $P(T \geq 5) = 0.1487\ldots$ awrt **<u>0.149</u>**	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geq 149.5) = P\left(Z \geq \frac{149.5-132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\ldots$ awrt **<u>0.0116</u>**	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			(9 marks)

Notes:

(a)
B1: cao

(b)
M1: for selection of an appropriate model for T
1st A1: for a correct value of the parameter p (accept 0.3 or better)
2nd A1: for awrt 0.149

(c)
B1: both correct conditions

(d)
B1: for correct normal distribution
M1: for correct use of continuity correction
A1: cso

(e)
B1: correct statement

Response A (low)

Student Response A

(a) Not possible to carry out a long time consumming

(b) $10 \times 24 = 240$

half way $= 240 \times \frac{1}{2} = 120$

$X \sim B(24, 0.55)$

$P(X \geq 12)$

(c)

(d) $\mu = (0.55 \times 240) = 132$

$\sigma^2 = (240 \times 0.55) \times (0.45)$

$= 59.4$

$\therefore X \sim N(132, 59.4)$

continuity correction

$P(X \geq 149.5)$

$\dfrac{149.5 - 132}{\sqrt{59.4}} = 2.2706\ldots$

z value:

2.27

$P(X \geq 2.27) = 0.9887$

2/9

Examiner Comments: (a) B0 (b) M0A0A0 (c) B0 (d) B1M1A0 (e) B0

Part (a) No mention that the seeds would be destroyed in the process. Just saying it would be "time consuming" is not sufficient.

Part (b) The distribution for *S*, the number of seeds in a tray that germinate are identified, but the the distribution for the variable *T*, the number of trays with at least 15 seeds germinating is not found. The method requires that they identify this distribution and so no marks are scored. Candidates sometimes find it difficult to identify the correct random variables and defining these as in the mark scheme may be helpful to them.

Part (c) No attempt. All that was required is a statement about *n* being large and *p* close to 0.5.

Part (d) A correct normal distribution is given and a correct use of a continuity correction is seen to obtain a correct *z* value. The final probability is not correct as they forgot to subtract their tables value from 1. Candidates should be able to find these probabilities using a calculator on this new specification.

Student Response B

a) biased

b) $X \sim B(24, 0.55)$

$P(X \geq 15) = 1 - P(X \leq 14)$ 13

use answer

~~P(x~~

$X \sim B(10, p)$

c) n large $p \approx 0.5$

d) $X \sim B(240, 0.55)$

$Y \sim N(132, \sqrt{59.4}^2)$

$P(X \geq 150) = P(Y \geq 149.5)$

$\frac{149.5 - 132}{\sqrt{59.4}} = 2.27$

$1 - 0.9884$

$= 0.0116$

5/9

Examiner Comments: (a) B0 (b) M1A0A0 (c) B1 (d) B1M1A1 (e) B0

Part (a) There is no mention of the seeds being destroyed.

Part (b) there is sight of a B(10, *p*) distribution and an intention to use P(*X* …15) from *X* ~ B(24, 0.55) so a correct method is being attempted. Candidates are expected to be able to find cumulative binomial probabilities using their calculators on this specification, this candidate seems to have faltered over this step.

Part (c) Answer is correct.

Part (d) Answer is fully correct. Calculator could have been used to evaluate the normal probability.

Part (e) No attempt.

Student Response C

(a) There would be no seeds left to sell.

(b) X = no of trays out of 10 $\sim B(10, p)$

$p = 1 - 0.7008732\ldots = 0.299126\ldots$

$P(X \geqslant 5) = 0.1487\ldots \approx 0.149$

(c) large n, $p \approx 0.5$

(d) $G \sim B(240, 0.55)$ $G \approx N(132, 59.4)$

$P(G \geqslant 150.5) = 0.008189\ldots$

(e) probability is small so proportion probably is different from 0.55.

8/9

Examiner Comments: (a) B1 (b) M1A1A1 (c) B1 (d) B1M1A0 (e) B1

Part (a) The student mentions the idea that the seeds would be destroyed and scores the mark.

Part (b) B(10, p) is used and found the correct probability so full marks are scored.

Part (c) Answer is correct.

Part (d) A correct normal distribution is identified and the student attempts to use a continuity correction but 150.5 instead of 149.5 is used. Answer is a correct follow through from the error but is not correct to obtain the mark.

In the final part they have commented that the proportion is probably different because the probability in part (e) is small.

A level Mathematics – Paper 3 (Mechanics)

Exemplar question 6

6. At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}\,.$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

(Total for Question 6 is 6 marks)

Mark scheme

Question	Scheme	Marks	AOs
6	Integrate $\mathbf{a}$ w.r.t. time	M1	1.1a
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of $\mathbf{C}$)	A1	1.1b
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\,\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1**ft**	1.1b
			(6 marks)

Notes:

1st M1: for integrating $\mathbf{a}$ w.r.t. time (powers of t increasing by 1)
1st A1: for a correct $\mathbf{v}$ expression without $\mathbf{C}$
2nd A1: for a correct $\mathbf{v}$ expression including $\mathbf{C}$
2nd M1: for putting $t = 4$ into their $\mathbf{v}$ expression
3rd M1: for finding magnitude of their $\mathbf{v}$
3rd A1: **ft** for 100 m s^{-1}, follow through on an incorrect $\mathbf{v}$

Student Response A

$\int 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$

$\left(\frac{5}{2}t^2 + c\right)\mathbf{i} \quad -\left(10t^{\frac{3}{2}} + c\right)\mathbf{j}$

$\frac{5}{2}t^2 + c = 20\mathbf{i}$

$c = 20$

$\left(\frac{5}{2}t^2 + 20\right)\mathbf{i} + \left(20 - 16t^{\frac{3}{2}}\right)\mathbf{j}$

$60\mathbf{i} + -118\mathbf{j}$

132.4 m/s

3/6

Examiner Comments:

The candidate integrates and obtains a correct expression but then obtains an incorrect constant *vector* so loses the second A mark. The value $t = 4$ is substituted (this is easily seen by the examiner so the M mark is awarded but to be safe, candidates should explain what they are doing, so here, write ‘when $t = 4$’). The candidate does not show the method used to find the speed and so loses the final M mark and also the final A mark.

Student Response B

$$v = \frac{5t^2}{2}i - 3\cancel{0t} \frac{15t^{\frac{3}{2}}}{\frac{3}{2}}j + 20i$$

$$v = \left(\frac{5t^2}{2} + 20\right)i + \left(-10t^{\frac{3}{2}}\right)j$$

$$\underline{60i - 80j}$$

4/6

Examiner Comments:

The candidate obtains the correct velocity vector when $t = 4$, but forgets to find the speed so scores the first 4 marks only.

Student Response C

$v = \int a\,dt = \left(\frac{5}{2}t^2 + C\right)i - \left(10t^{\frac{3}{2}} + D\right)j$

$\therefore$ when $t = 0$,

$C = 20$ & $D = 0$.

$\therefore v = \left(\frac{5}{2}t^2 + 20\right)i - 10t^{\frac{3}{2}}j$

$\therefore$ when $t = 4$

$v = 60i + 80j$

$\therefore$ speed $= |v| = \sqrt{60^2 + 80^2} = \underline{\underline{100\ ms^{-1}}}$

5/6

Examiner Comments:

The candidate fortuitously obtains a correct final answer, having made a sign error earlier when putting $t = 4$, and so loses the final A mark. Note that the final A mark is follow through on the expression for v in terms of t and not on the numerical value (as here).

Exemplar question 7

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(Total for Question 7 is 8 marks)

Mark scheme

Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 **ft**	2.2a
		(2)	
			(8 marks)

Notes:

(a)

B1: for $R = mg\cos\alpha$

1st M1: for resolving parallel to the plane

1st A1: for a correct equation

2nd M1: for use of $F = \mu R$

3rd M1: for eliminating F and R to give a value for μ

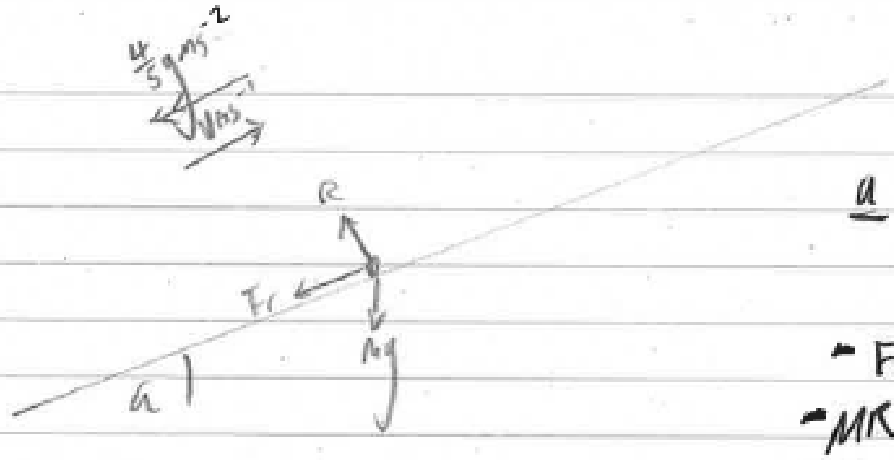
2nd A1: for $\mu = \frac{1}{4}$

(b)

M1: comparing size of limiting friction with weight component down the plane

A1ft: for an appropriate conclusion from their values

Student Response A



$\underline{a}$ $Fr = \mu R$

$F = ma$

$-Fr = \frac{4}{5}gm$

$-\mu R = \frac{4}{5}gm$

$-\mu mg\cos\alpha = \frac{4}{5}gm$

$-\mu\cos\alpha = \frac{4}{5}$

$\mu = \frac{4/5}{\cos\alpha} = 1$

$R\ (\nwarrow): R = mg\cos\alpha$

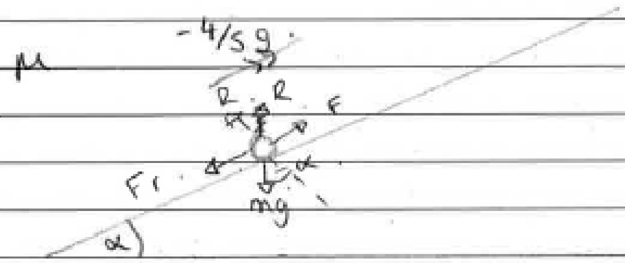
3/8

Examiner Comments:

Part (a) The candidate scores the B mark for a correct expansion for the reaction but omits the component of the weight when resolving parallel to the plane and so loses the first M mark and the dependent A mark. The next two marks are scored but the final A mark is lost for an incorrect answer.

Part (b) No attempt.

Student Response B



$\tan\alpha = 3/4$
$\sin\alpha = 3/5$
$\cos\alpha = 4/5$

a) $R = mg\cos\alpha$

$Fr = mg\sin\alpha + 4/5\, mg$

$F = ma$
$= m(-4/5\, g)$
$= -4/5\, mg$

$Fr = \mu R$

$\mu = \dfrac{Fr}{R}$

$\mu = \dfrac{mg\sin\alpha - 4/5\, mg}{mg\cos\alpha}$

$= \dfrac{\sin\alpha - 4/5}{\cos\alpha}$

$= \dfrac{3/5 - 4/5}{4/5} = -1/4 = 0.25$

b) The particle will not stay at A because the frictional force and weight acting on the particle would be larger than the normal reaction.

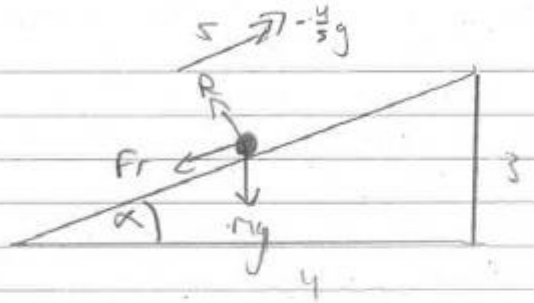
4/8

Examiner Comments:

Part (a) The candidate makes a sign error when resolving parallel to the plane and so loses an A mark for an incorrect equation and also the final A mark for an incorrect answer.

Part (b) Nothing scored.

Student Response C



$Fr = \mu R$ $\quad$ Res$(\nwarrow)$ $R = mg\cos\alpha = \frac{4}{5}mg$

$F = ma$ $\quad$ $Fr + mg\sin\alpha = m\left(\frac{4}{5}g\right)$

$Fr + \frac{3}{5}gm = \frac{4}{5}mg$

$Fr = \frac{1}{5}mg$

$\mu = \frac{Fr}{R} = \frac{1}{5}mg \div \frac{8}{5}g = \frac{1}{8}m$

$Fr\ max = \frac{1}{5}mg$

Weight $= mg\sin\alpha$ down the slope

$= \frac{3}{5}mg$

$\frac{3}{5}mg > \frac{1}{5}mg$ ∴ weight down slope > maximum friction ∴

it will not stay at A

7/8

Examiner Comments:

Part (a) All correct until the candidate makes a transcription error and loses the final A mark.

Part (b) A correct explanation, with working to back it up, and full marks scored.

Exemplar question 8

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for Question 8 is 10 marks)

Mark scheme

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \times 0.7\,t^2 = 0.6\,t - \frac{1}{2} \times 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 **ft**	1.1b
		(3)	
		(10 marks)	

Notes:

(a)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for **r** in terms of t

(c)

M1: for equating the **i** and **j** components of their **r**

A1ft: for a correct equation following their **r**

A1: for $t = 1.5$

(d)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t

M1: for equating the **i** and **j** components of their **v**

A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Student Response A

a)

$10.5\mathbf{i} - 0.9\mathbf{j} - 0.6\mathbf{j}$

$= 10.5\mathbf{i} - 1.5\mathbf{j}$

$(10\mathbf{i} - 1.5\mathbf{j}) \div 15 = 0.7\mathbf{i} - 0.1\mathbf{j}$

b) $(0.7\mathbf{i} - 0.1\mathbf{j})t + 0.6\mathbf{j}$

$\mathbf{r} = 0.7t\,\mathbf{i} + (0.6 - 0.1t)\mathbf{j}$

c) $0.7t > 0$

$t > 0$

$0.6 - 0.1t > 0$

$0.1t < 0.6$

$t < 6$

$0 < t < 6$

d) $0.7t > 0$

$t > 0$

$0.6 - 0.1t > 0$

$0.1t > 0.6$

$t > 6$

$0 < t < 6$

2/10

Examiner Comments:

Part (a) Correct.

Part (b) Nothing scored as candidate is finding **v** not **r**.

Part (c) Incorrect method.

Part (d) Incorrect method.

Student Response B

a) at $t=0$ $v_i = 0$ $v_j =$ something $|v| = 0.6$ ∴ $v = 0.6j$

at $t=15$ $v = 10.5i - 0.9j$

$$a = \frac{v-u}{t} = \frac{10.5i - 0.9j - 0.6j}{15} = \frac{10.5i - 1.5j}{15}$$

$a = 0.7i - 0.1j \text{ ms}^{-2}$

b) for t: $v = u + at$ ∴ $v = 0.6j + t(0.7i - 0.1j)$

$v = 0.7ti + (0.6 - 0.1t)j$

$r = r_0 + vt$ ∴ $r = 0 + t(0.7ti + (0.6 - 0.1t)j)$

$r = 0.7t^2 i + (0.6t - 0.1t^2)j$

c)

NE along i coefficient of r = j coefficient of r

∴ $0.7t^2 = 0.6t - 0.1t^2$

d) moving north east along i of v = j of v

∴ $0.7t = 0.6 - 0.1t$

$0.8t = 0.6$ $t = \frac{3}{4}$

7/8

Examiner Comments:

Part (a) Correct.

Part (b) The candidate uses **v** = **u** + **a***t* correctly but then assumes constant velocity to find **r** and so scores no marks.

Part (c) Correct method used to produce a correct follow through equation but the second A mark is lost for an incorrect answer.

Part (d) Correct.

Student Response C

a) $a = \frac{v-u}{t}$; At $t=15$, $v = 10.5i - 0.9j$

At $t=0$, $u = 0.6j$

$\therefore a = \frac{10.5i - 0.9j - 0.6j}{15-0}$

$a = \frac{10.5i - 1.5j}{15}$

$a = 0.7i - 0.1j$ (ms^{-2})

b) $s: r$

$u: 0.6j$

$v: v$

$a: 0.7i - 0.1j$

$t: t$

$s = ut + \frac{1}{2}at^2$

$r = (0.6t)j + \frac{1}{2}(0.7i - 0.1j)t^2$

$r = (0.35t^2)i + (0.6t - 0.05t^2)j$

c) When NE; $0.35t^2 = 0.6t + 0.05t^2$

$0.3t^2 = 0.6t$

$3t^2 = 6t$

$t^2 = 2t$

$t^2 - 2t = 0$

$t(t-2) = 0$

$\therefore t = 2$

d) When moving NE;

$s: r$

$u: 6j$

$v: v$

$a: 0.7i - 0.1j$

$t: t$

$v = u + at$

$v = 6j + t(0.7i - 0.1j)$

$v = (0.7t)i + (6 - 0.1t)j$

$\therefore$ When moving NE; $0.7t = 6 - 0.1t$

$0.8t = 6$

$t = 7.5$

8/8

Examiner Comments:

Part (a) Correct.

Part (b) Correct.

Part (c) M mark earned for an *attempt* at a correct method but there is a sign error so loses both A marks are lost.

Part (d) Both M marks earned and also Aft mark for a correct follow through answer.

Exemplar question 9

9.

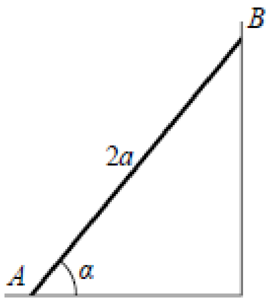


Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.
The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 9 is 13 marks)

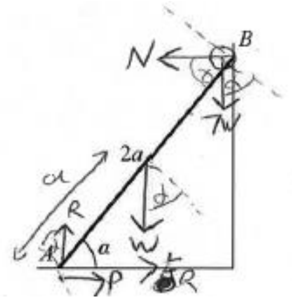
Mark scheme

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 9 continued
Notes:
(a) **1st M1:** for producing an equation in S, W and α only **1st A1:** for an equation that is correct, or which has one error or omission **2nd A1:** for a fully correct equation **2nd M1:** for use of $\tan\alpha = \frac{5}{2}$ to obtain S in terms of W only **3rd A1*:** for given answer $S = 3W$ correctly obtained
(b) **B1:** for $R = 8W$ **1st M1:** for use of $F = \frac{1}{4}R$ **2nd M1:** for either $P = (3W + \text{their } F)$ or $P = (3W - \text{their } F)$ **1st A1:** for a correct max or min value for a correct range for P **2nd A1:** for a correct range for P
(c) **1st M1:** for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder **2nd M1:** for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder **3rd M1:** for concluding that this increases the limiting friction at A

Student Response A

9.



(3)

$\tan\alpha = \frac{5}{2}$ $\sin\alpha = \frac{5}{\sqrt{29}}$ $\cos\alpha = \frac{2}{\sqrt{29}}$

a) $R(\rightarrow)$ $N = P + \frac{1}{4}R$

$R(\uparrow)$ $R = 8W$ $\therefore N = P + 2W$

MOM(B) $\cancel{a} \times W\cos\alpha + 2\cancel{a}(P+\frac{1}{4}R)\sin\alpha = 2\cancel{a} \times R\cos\alpha$

$\frac{2}{\sqrt{29}}W + \frac{10}{\sqrt{29}}(P + \frac{1}{4}\times 8W) = \frac{4}{\sqrt{29}}\times 8W$

$\times\sqrt{29}$

$2W + 10(P+2W) = 32W$

$2W + 10P + 20W = 32W$ $10P + 22W = 32W$

$10P = 10W$

$P = W$

$N = P + 2W$ $\therefore N = W + 2W$ $\therefore N = 3W$

b) $N - P \leq 2W$ ($\frac{1}{4}R$)

mom(A) $\cancel{a} \times W\cos\alpha + 2\cancel{a}\, 7W\cos\alpha = 2\cancel{a} \times N\sin\alpha$

$\frac{2}{\sqrt{29}}W + \frac{28}{\sqrt{29}}W = \frac{10}{\sqrt{29}}N$ $\times\sqrt{29}$

$10N = 30W$ $N = 3W$

$\therefore 3W - P \leq 2W$ $P \geq W$

6/13

Examiner Comments:

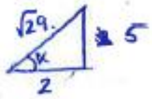
Part (a) Candidate incorrectly assumes limiting equilibrium and is awarded only the two M marks.

Part (b) The first 4 marks are scored for a correct working leading to correct lower limit for *P*.

Part (c) No attempt.

Student Response B

9.



$\sin\alpha = \frac{5}{\sqrt{29}}$

$\cos\alpha = \frac{2}{\sqrt{29}}$

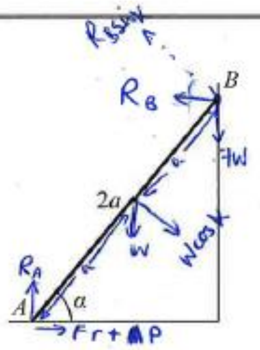


Figure 1

(a) Moments from A.

$(W\cos\alpha)\,a = 2a\,(7W\cos\alpha)$

$\cancel{W \tfrac{2}{\sqrt{29}}\, a = 2a\,(7W \tfrac{2}{\sqrt{29}})}$

$(W\cos\alpha)\,a + (7W\cos\alpha)\,2a = 2a\,(R_B\sin\alpha)$

$\frac{2}{\sqrt{29}}W + \frac{28\sqrt{29}}{\sqrt{29}}W = 2R_B \times \frac{5}{\sqrt{29}}$

$\frac{30\sqrt{29}}{29}W = 2R_B\left(\frac{5}{\sqrt{29}}\right)$ $\Big\}$ $R_B = 3$

(b) $Fr + P = 3W$

$Fr = R_A \times \frac{1}{4}$

(B) $(W\cos\alpha)\,a + (Fr+P)\sin\alpha = R_A\cos\alpha$

$\frac{2}{\sqrt{29}}W + 3W \times \frac{5}{\sqrt{29}} \times 2 = \frac{2}{\sqrt{29}}R_A \times 2$

$\cancel{R_A = 8}$

$\frac{W}{8} = R_A$ $\qquad W + 3W \times \frac{5}{2} \times 2 = 2R_A$

$16W = 2R_A$

$8W = R_A$

$W = \frac{R_A}{8}$

$8W = R_A$

$Fr = 2W$

$2W + P = 3W$

$P = W$

$P \geq W$

(c) the reaction force at point (A) will be larger ~~and~~, hence, the anticlockwise moment from (B) will be larger, If the moments are unbalanced there will be a resultant turning motion, resulting in a more stable laddle.

8/13

Examiner Comments :

Part (a) Correct method used, after deleting first attempt at moments equation, but candidate makes a slip when stating the answer (omits W) so loses final A mark.

Part (b) The first 4 marks are scored for a correct working leading to a correct lower limit for P.

Part (c) No valid comments.

Student Response C

9.

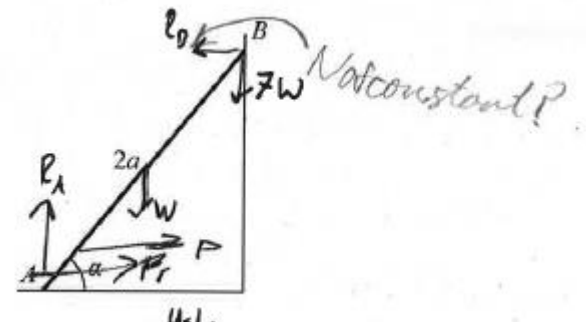


Figure 1 $\mu = \frac{1}{4}$

(3)

a) about A: $R_p \sin\alpha\, 2a = 7W \times 2a\cos\alpha + a \times W\cos\alpha$ $\tan\alpha = \frac{5}{2}$, $\sin\alpha = \frac{5}{\sqrt{29}}$, $\cos\alpha = \frac{2}{\sqrt{29}}$

$$2R_p \frac{5}{\sqrt{29}} = 14W \times \frac{2}{\sqrt{29}} + W \times \frac{2}{\sqrt{29}}$$

$$R_p = \frac{15W \times \frac{2}{\sqrt{29}}}{\frac{5}{\sqrt{29}} \times 2}$$

$$= \frac{30W}{10} = 3W$$

b) $\cancel{P \leq 3W}$ FR + P

$Fr\,max = \frac{1}{4} \times 8W$

$= 2W$ $R_p + Fr\,max = P$
$= 5$

$3W - 2W = 1W$

$\therefore P \geq W$

$W \leq P \leq 5W$

c) Increases the normal reaction force at B, increasing Fr. This stops the ladder from slipping back

11/13

Examiner Comments:

Part (a) Fully correct.

Part (b) Not the best explanation but full marks awarded for a correct range for *P*.

Part (c) Candidate makes one valid comment.

Exemplar question 10

10.

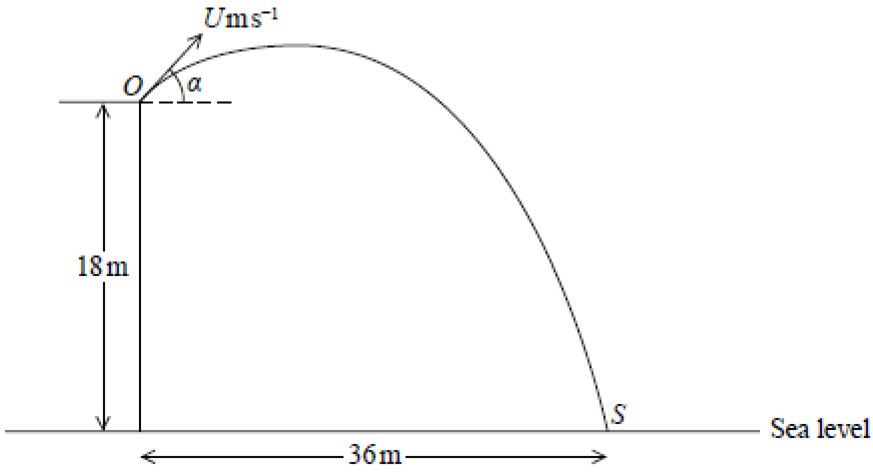


Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with g = 10 m s^{-1}.

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

(Total for Question 10 is 13 marks)

Mark scheme

Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s $^{-1}$ (2sf)	A1 **ft**	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 10 continued
Notes:
(a) **1st M1:** for use of $s = ut$ horizontally **1st A1:** for a correct equation **2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1:** for a correct equation **3rd M1:** for correct strategy (need both equations) **2nd A1:** for $U = 15$
(b) **B1:** for $U\cos\alpha$ used as horizontal velocity component **1st M1:** for attempt to find vertical component **1st A1:** for 15 **2nd M1:** for correct strategy (need both components) **2nd A1ft:** for 19 m s^{-1} (2sf) following through on incorrect component(s)
(c) **B1, B1:** for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion

Student Response A

a. Horizontal:

$36 = Vt\cos a$, $V = \frac{36}{t\cos a}$

Vertical:

$18 = Ut\sin a + {-5t^2} \rightarrow \cancel{36 = 2Ut\sin a + 10t^2}$

$\Rightarrow \cancel{Vt\cos a = 2Ut\sin a + 10t^2}$ $\quad 18 = \frac{36t\sin a}{t\cos a} + {-5t^2}$

$18 = 36\tan a + 5t^2$, $-5t^2 = 18 - 36\tan a$, $t^2 = -1.8$, $t = \frac{3\sqrt{5}}{5}$

$\therefore\ V = \frac{36}{\frac{4}{5}t} = \frac{36}{\left(\frac{12\sqrt{5}}{25}\right)} = 15\sqrt{5}\ \text{ms}^{-1}$

b. $S = 7.2$, $U = 15\sqrt{5}\sin a$, $V = V$, $a = -10$, $t = x$

$v^2 = u^2 + 2as$

$v^2 = 405 + -288$, $V = 3\sqrt{13}$

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Examiner Comments:

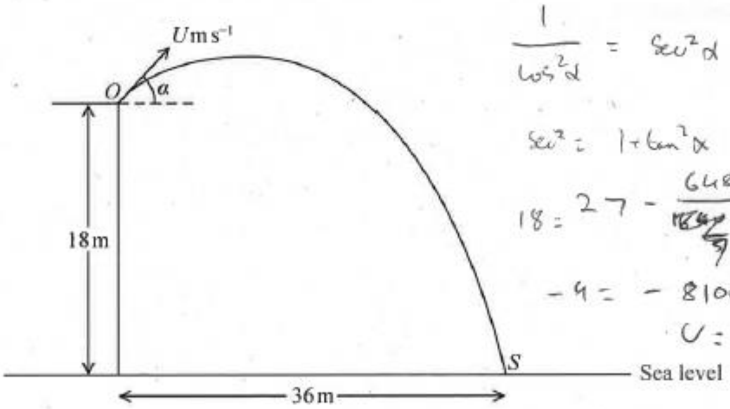
Part (a) Correct equation for horizontal motion but a sign error in the equation for vertical motion leads to loss of two A marks.

Part (b) Only the first M mark only for attempt to find vertical component of velocity. A sign error leads to an incorrect answer and there is no attempt to use the horizontal component to find the speed.

Part (c) No attempt.

Student Response B

10.



$(5\times2)^2$

5^2

$\frac{1}{\cos^2\alpha} = \sec^2\alpha$

$\sec^2 = 1+\tan^2\alpha$

$18 = 27 - \frac{6480}{\cancel{1844}\ 9}$

$-9 = -8100U$

$U =$

(2)

$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$s = 18$

$u = U\sin\alpha$

$v =$

$a = -10$

$t = t$

$U\cos\alpha = \frac{36}{t}$

$t = \frac{36}{U\cos\alpha}$

$18 = U\sin\alpha t - 5t^2$

$18 = \frac{U\sin\alpha\ 36}{U\cos\alpha} - 5\left(\frac{36}{U\cos\alpha}\right)^2$

$18 = \tan\alpha\ 36 - \frac{6480}{\cancel{U\cos\alpha}}$

$18 = 27 - \frac{6480}{(U\cos\alpha)^2}$

$\cancel{6480}$

$6480 = 9(U\cos\alpha)^2$

$(U\cos\alpha)^2 = 720$

$U\cos\alpha = 12\sqrt{5}$

$U = \frac{12\sqrt{5}}{\frac{4}{5}}$

$U = 15\sqrt{5}$

$= 33.5$

Question 10 continued

7.2

18

$\cancel{18}$ 10.8

b) $s = 7.2$

$u = 15\sqrt{5}\sin\alpha$

$v = \cancel{\ }$

$a = -10$

$t =$

$v^2 = 405 - 144$

$v = 3\sqrt{29}$

$v = 16.2$

c) Take into account air resistance

Make gravity 9.8 m/s^2 instead 10

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Examiner Comments:

Part (a) Correct equation for horizontal motion but a sign error in the equation for vertical motion leads to loss of two A marks.

Part (b) As in the previous response, M mark only for attempt to find vertical component of velocity.

Part (c) Two valid improvements to the model suggested.

Student Response C

(2)

$\cos\alpha = \frac{4}{5}$ $\sin\alpha = \frac{3}{5}$

a) $S = -18$ $u = U \times \frac{3}{5}$ $V =$ $a = -10$ $t = ?$

$-18 = \frac{3}{5}Ut - 5t^2$

$36 = U\frac{4}{5} \times t$

$t = \frac{45}{U}$

$\cancel{-18 = 27U - 10125U^2}$

$\cancel{100980 \quad 18 \quad U^2 = \sqrt{\frac{18}{100972}}}$

$-18 = 27 - \frac{10125}{U^2}$

$45U^2 = 10125$

$U^2 = 225$

$U = 15 \text{ ms}^{-1}$

b) $S = -7.2$ $U = 9$ $V = ?$ $a = -10$ $t =$

$V = \sqrt{81 - 14.2 \times -10} = \sqrt{223}$

$\text{Speed} = \sqrt{223 + 144} = \sqrt{367} = 19 \text{ ms}^{-1}$

b) Model air resistance on particle

Use a more accurate value for g i.e 9.8 or 9.81 ms^{-2}

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Examiner Comments:

Part (a) Fully correct.

Part (b) Method correct but first A mark lost due to arithmetical slip (2 × 7.2 = 14.2) but scores second The second A mark is scored for a correct follow through answer.

Part (c) Full marks.