Pearson Edexcel
Level 3 Advanced Subsidiary GCE in Mathematics (8MA0)

Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

Sample Assessment Materials Exemplar answers with examiner comments – Statistics and Mechanics

First teaching from September 2017
First certification from June 2018
## Contents

About this booklet  
AS Mathematics – Paper 2 (Statistics)  
AS Mathematics – Paper 2 (Mechanics)  
A level Mathematics – Paper 3 (Statistics)  
A level Mathematics – Paper 3 (Mechanics)
About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Level 3 Advanced GCE in Mathematics specification (8MA0 & 9MA0).

The booklet looks at questions from the AS and A level Sample Assessment Materials, which was used in the trial undertaken in summer 2017. It shows real student responses to questions, and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to all questions from the trial of the Sample Assessment Materials. Following each question you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student Response A

\[
\begin{align*}
(a) & \quad P + 0.05 = 0.25 \quad \therefore \quad P = 0.20 \\
(b) & \quad \theta = 0.15 \\
\end{align*}
\]

\[
\begin{align*}
P(A) + P(T) & = 0.95 \\
P(A \cup T) & = 0.75 \\
\text{So not independent} \\
\end{align*}
\]

\[
\begin{align*}
P(A' \cup C') & = P(A') + P(C') - P(A') \times P(C') \\
& = 0.85 + 0.8 - 0.8 \times 0.65 = 0.93 \\
\end{align*}
\]

Examiner Comments: (a) B1 (b) B1M0A0 (c) B0

Part (a) Correct answer with clear working shown.

Part (b) The value of \( \theta \) is clearly given but then the formulae for independent and mutually exclusive events is confused so no further marks are awarded.

Part (c) Working suggests a misunderstanding of the question asked as this is a B mark, no credit can be given here.

Marks awarded for the question or question parts

Examining commentary on the student response
1. Sara is investigating the variation in daily maximum gust, \( t \) kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used. 

(1)

(b) From your knowledge of the large data set, explain why this process may not generate a sample of size 20.

(1)

The data Sara collected are summarised as follows

\[ n = 20 \quad \sum t = 374 \quad \sum t^2 = 7600 \]

(c) Calculate the standard deviation.

(2)

(Total for Question 1 is 4 marks)
## Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>Systematic (sample) cao</td>
<td>B1</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>In LDS some days have gaps because the data was not recorded</td>
<td>B1</td>
<td>2.4</td>
</tr>
</tbody>
</table>
| **(c)**  | \[ t = \frac{374}{20} = 18.7 \]  
\[ s_t = \sqrt{\frac{7600}{20} - \bar{T}^2} \quad [= \sqrt{30.31}] \]  
\[ = 5.5054\ldots \text{awrt 5.51} \]  
\[ \text{(Accept use of } s_t = \sqrt{\frac{7600 - 20\bar{T}^2}{19}} = 5.6484\ldots) \] | M1 | 1.1a |

**Notes:**

| (b) | B1: A correct explanation |
| (c) | M1: For a correct expression for \( \bar{T} \) and \( \sigma_t \) or \( s_t \). F1 an incorrect evaluation of \( \bar{T} \)  
| A1: For \( \sigma_t = \text{awrt 5.51} \) or \( s_t = \text{awrt 5.65} \) |
Students Response A

Examiner Comments: (a) B0 (b) B0 (c) M1A0

Part (a) The candidate needs to use the correct technical term.

Part (b) No indication of the relevant knowledge of the data in the large data set (some data values being missing).

Part (c) Correct working is seen but the answer to only 2 significant figures is given, need awrt 5.51
In general candidates should always give rounded answers to at least 3 significant figures unless specifically directed otherwise.
Student Response B

(a) Systematic sampling

(b) Some values are recorded as N/A because the data is missing

(c) \[ \sigma = \sqrt{7600 - \left(\frac{374}{20}\right)^2} = \sqrt{7250.31} \]
\[ = 85.14 \]

Examiner Comments: (a) B1 (b) B1 (c) M0A0

Part (a) Correct answer, incorrect spelling is ignored.

Part (b) Correct.

Part (c) An incorrect formula is used. The formula for standard deviation needs to be known.
**Student Response C**

(a) Systematic

(b) You might run out of days

(c) $s_{ee} = 7600 - 374^2 \frac{20}{20} = 606.2$

$$s_{e} = \sqrt{\frac{s_{ee}}{20}} = \sqrt{30.31} = 5.5054...$$

$$= 5.50$$

Examiner Comments: (a) B1 (b) B0 (c) M1A1

Part (a) Correct answer.

Part (b) No indication of relevant knowledge of the large data set is shown.

Part (c) Correct answer calculated but then subsequently the candidate rounds incorrectly. As a value of 5.5054… was given the 5.50 is treated as isw.
Exemplar question 2

2. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists were delayed by roadworks on a stretch of motorway.

<table>
<thead>
<tr>
<th>Delay (minutes)</th>
<th>Number of motorists</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – 6</td>
<td>6</td>
</tr>
<tr>
<td>7 – 8</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10 – 12</td>
<td>45</td>
</tr>
<tr>
<td>13 – 15</td>
<td>9</td>
</tr>
<tr>
<td>16 – 20</td>
<td></td>
</tr>
</tbody>
</table>

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(Total for Question 2 is 5 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>17 + 45 + (\frac{1}{3} \times 9 ) [ = 65 ]</td>
<td>M1</td>
<td>2.2a</td>
</tr>
</tbody>
</table>
|          | \((7 - 8) \ 14 \) or \((16 - 20) \ 5 \)  
[Values may be seen in the table] | M1    | 3.1a |
|          | Percentage of motorists is \( \frac{\text{"65"}}{6 + \text{"14"} + 17 + 45 + 9 + \text{"5"}} \times 100 \) | M1    | 3.1b |
|          | \[ = 67.7\% \] | A1    | 1.1b |

(5 marks)

Notes:

**M1:** For a fully correct expression for the number of motorists in the interval

**M1:** For clear use of frequency density in (4-6) or (13-15) cases to establish the fd scale. Then use of area to find frequency in one of the missing cases

**A1:** For both correct values seen

**M1:** For realising that total is required and attempting a correct expression for %

**A1:** For awrt 67.7%
Examiner Comments: M0M1A1M0A0

This candidate was able to use the properties of the histogram to find the missing frequencies in the table correctly, picking up 2 marks. Whilst the 96 (total number of motorists) is seen in the script it is not used correctly and there is no visible attempt to find the number delayed for between 8.5 and 13.5 minutes so no further marks are scored.
Examiner Comments: M1M1A0M1A0

In this case the candidate uses correct methods and shows good structure in their working picking up all the method marks. The boundary values for each class are shown but an error with the upper boundary for the top group is seen, perhaps an error in reading the scale, which results in the loss of the A marks.
## Student Response C

<table>
<thead>
<tr>
<th>Delay (minutes)</th>
<th>Number of motorists</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – 6</td>
<td>6</td>
</tr>
<tr>
<td>7 – 8</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>10 – 12</td>
<td>45</td>
</tr>
<tr>
<td>13 – 15</td>
<td>9</td>
</tr>
<tr>
<td>16 – 20</td>
<td>5</td>
</tr>
</tbody>
</table>

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

\[
\text{Missing values, Neat } P_{\text{density}}:
\]

- **4-6 group** \( P_{\text{den}} = \frac{6}{3} \times 2 \), scale \( 1 \text{cm} = 1 \text{unit} \).
- **7-8 group** \( P_{\text{den}} = 7 \), area = \( 2 \times 7 = 14 \).
- **16-20 group** \( P_{\text{den}} = 1 \), area = \( 5 \times 1 = 5 \).

Number delayed for 8.5 to 13.5 minutes:

\[
= 17 + 45 + \frac{1}{3} \times 9
\]

\[
\% \text{ delayed} = \frac{17 + 45 + \frac{1}{3} \times 9}{6 + 14 + 17 + 45 + 9 \times 5} \times 100
\]

\[
= \frac{65}{98} = 66 \%
\]

Examiner Comments: M1M1A1M1A0

A strong response with clear well explained working. The candidate finds the frequency density scale and uses it to calculate the missing frequencies and then uses the correct method to find the number of motorists with the required time delay. Working picks up the 3rd method mark but an arithmetic error, total number of motorists is incorrect, means that the final answer is wrong. Even had this been correct giving their final answer to only 2 significant figures would also have cost the last A mark.
Exemplar question 3

3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

- $A$ represents the event that a student takes part in Athletics.
- $T$ represents the event that a student takes part in Tennis.
- $C$ represents the event that a student takes part in Cricket.
- $p$ and $q$ are probabilities.

The probability that a student selected at random takes part in Athletics or Tennis is 0.75.

(a) Find the value of $p$.

(b) State, giving a reason, whether or not the events $A$ and $T$ are statistically independent. Show your working clearly.

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(Total for Question 3 is 5 marks)
## Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3 (a)</strong></td>
<td>( p = [1 - 0.75 - 0.05] = 0.20 )</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>( q = 0.15 )</td>
<td>B1ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>( P(A) = 0.35 \quad P(T) = 0.6 \quad P(A \text{ and } T) = 0.20 )</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>( P(A)^* \times P(T) = 0.21 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since ( 0.20 \neq 0.21 ) therefore ( A ) and ( T ) are not independent</td>
<td>A1</td>
<td>2.4</td>
</tr>
</tbody>
</table>

\[ \text{(3)} \]

| **(c)** | \( P(\text{not } [A \text{ or } C]) = 0.45 \) | B1 | 1.1b |

\[ \text{(1)} \]

\[ \text{(5 marks)} \]

### Notes:

**(a)**

- **B1:** cao for \( p = 0.20 \)

**(b)**

- **B1:** ft for use of their \( p \) and \( P(A \text{ or } T) \) to find \( q \) i.e. \( 0.75 - “p” - 0.40 \) or \( q = 0.15 \)
- **M1:** For the statement of all probabilities required for a suitable test and sight of any appropriate calculations required
- **A1:** All probabilities correct, correct comparison and suitable comment

**(c)**

- **B1:** cao for 0.45
Examiner Comments: (a) B1 (b) B1M0A0 (c) B0

Part (a) Correct answer with clear working shown.

Part (b) The value of q is clearly given but then the formulae for independent and mutually exclusive events is confused so no further marks are awarded.

Part (c) Working suggests a misunderstanding of the question asked as this is a B mark, no credit can be given here.
Examiner Comments: (a) B1 (b) B1M0A0 (c) B1

Part (a) Correct answer using the known probability of $A \cup B$

Part (b) B1 mark is awarded as the value of $q$ can be seen, the candidate writes it into the Venn diagram. In the working on independence the correct equation is quoted however the probabilities used are not $P(A)$ and $P(T)$ so M0 given.

Part (c) Correct.
Examiner Comments: (a) B1 (b) B1M1A0 (c) B1

Part (a) A longer method (effectively solving simultaneous equations) is used but achieved the correct answer, B1.

Part (b) B1 mark for the value of $q$ is achieved in part (a) and then the values is used correctly to establish whether $A$ and $T$ are independent, an arithmetic error results in loss of the A mark.

Part (c) Correct.
Exemplar question 4

4. Sara was studying the relationship between rainfall, \( r \) mm, and humidity, \( h \) %, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set. She obtained the following results.

<table>
<thead>
<tr>
<th>( h )</th>
<th>93</th>
<th>86</th>
<th>95</th>
<th>97</th>
<th>86</th>
<th>94</th>
<th>97</th>
<th>87</th>
<th>97</th>
<th>86</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>1.1</td>
<td>0.3</td>
<td>3.7</td>
<td>20.6</td>
<td>0</td>
<td>0</td>
<td>2.4</td>
<td>1.1</td>
<td>0.1</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Sara examined the rainfall figures and found

\[
Q_1 = 0.1 \quad Q_2 = 0.9 \quad Q_3 = 2.4
\]

A value that is more than 1.5 times the interquartile range (IQR) above \( Q_3 \) is called an outlier.

(a) Show that \( r = 20.6 \) is an outlier.

(1)

(b) Give a reason why Sara might

(i) include

(ii) exclude this day’s reading.

(2)

Sara decided to exclude this day’s reading and drew the following scatter diagram for the remaining 10 days’ values of \( r \) and \( h \).

(c) Give an interpretation of the correlation between rainfall and humidity.
The equation of the regression line of $r$ on $h$ for these 10 days is $r = -12.8 + 0.15h$.

(d) Give an interpretation of the gradient of this regression line.

(e) (i) Comment on the suitability of Sara’s sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(Total for Question 4 is 7 marks)

Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td>IQR = 2.3 and $20.6 \geq 2.4 + 1.5 \times 2.3$ (= 5.85) (Compare correct values)</td>
<td>B1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)(i)</strong></td>
<td>e.g. it is a piece of data and we should consider all the data o.e.</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>(ii)</strong></td>
<td>e.g. it is an extreme value and could unduly influence the analysis or it could be a mistake</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>e.g. “as humidity increases rainfall increases”</td>
<td>B1</td>
<td>2.2b</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>e.g. a 10% increase in humidity gives rise to a 1.5 mm increase in rainfall or represents 0.15 mm of rainfall per percentage of humidity</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>(e)(i)</strong></td>
<td>Not a good method since only uses 11 days from one location in one month</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>(ii)</strong></td>
<td>e.g. she should use data from more of the UK locations and more of the months or using a spreadsheet or computer package she could use all of the available UK data</td>
<td>B1</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(7 marks)
**Continued question 4**

**Notes:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For sight of the correct calculation and suitable comparison with 20.6</td>
</tr>
<tr>
<td><strong>(b)(i)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For a suitable reason for including the data point</td>
</tr>
<tr>
<td><strong>(b)(ii)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For a suitable reason for excluding the data point</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For a suitable interpretation of positive correlation mentioning humidity and rainfall</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For a suitable description of the rate: rainfall per percentage of humidity including reference to values</td>
</tr>
<tr>
<td><strong>(e)(i)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For a comment that supports the idea that her sampling method was not a good one</td>
</tr>
<tr>
<td><strong>(e)(ii)</strong></td>
<td></td>
</tr>
<tr>
<td>B1:</td>
<td>For some sensible suggestions that would give a better representation of the data across the UK. Must show some awareness of the fact that LDS has different locations and more months of data available but must be clear they are NOT using any overseas locations</td>
</tr>
</tbody>
</table>

**N.B.**  **B0 for a comment that says use more than one location without specifying that only UK locations are required**
Student Response A

4) a) $\text{IQR} = 2.9 - 0.1 = 2.8$

$$1.5 \times 2.8 = 3.95$$

$$2.4 + 3.45 = 5.85$$

$$20.6 > 5.85 \rightarrow \text{outlier}$$

b) i) because this result was anyway used to calculate $Q_1, Q_2, Q_3$ and the IQR

ii) because its value is far greater than any of the others and is well above the outlier $3.95$ which is the maximum number excluding outliers.

4c) There is a weak positive correlation between humidity and rainfall.

d) the gradient is $0.15$ which shows there is a positive gradient which means there will be a positive weak positive correlation as $0.15$ is a small number.

e) i) It was not suitable as the samples picked may have been too close to each other. There needs to be a more definite spread when picking data, and normal distributions. Some data of the data must have been picked from.

ii) She could have selected every 3rd value in order to have a wider spread of data which is also less bias.
Examiner Comments: (a) B1 (b) B0B1 (c) B0 (d) B0 (e) B1B0

Part (a) Good clear answer showing full working.

Part (b)(i) The argument that all data should be included unless there is a good reason to suspect it is not a valid reading is needed.

Part (b)(ii) Answer is just about sufficient, referring to this value as being ‘well above’ the limit for outliers.

Part (c) The correlation shown in the scatter diagram is described but no interpretation is given as requested - which needs to refer to the context of the question – hence B0.

Part (d) No interpretation in context of the 0.15 value is given. This answer also shows a lack of understanding of the gradient, confusing this with the correlation coefficient.

Part (e) (i) The benefit of the doubt is given as the candidate has the idea of obtaining a wider range of data but ideally would have expanded on this in terms of dates and locations.

Part (e) (ii) The second B1 mark is a tougher mark requiring the candidate to refer to there being more than one UK location available.
Student Response B

(a) \[ IQR = 2.4 - 0.1 = 2.3 \]
\[ 1.5 \times IQR + Q_1 = 1.5 \times 2.3 + 2.4 = 5.85 \]
\[ 5.85 \times 20.6 = 20.6 \text{ is an outlier} \]

(b) i) It is a statistical data which matters this day happened and she cannot just pretend like this never happened.

ii) An effect, e.g., mean.

(c) The correlation between rainfall and humidity is very weak since data is very scattered but it is still clearly shown that it is positive.

(d) As the rainfall increases by 1 mm, the humidity increases by 32%

(e) i) Her sampling frame is small so the predictions and conclusions that come from her calculations aren’t very reliable.

ii) She could use more samples and then code the data to be easier to work with.

Examiner Comments:

(a) B1
(b) B1B1
(c) B0
(d) B0
(e) B1B0

Part (a) Good response showing all required working.
Part (b) Both marks can be awarded, in part (ii) is a little thin but just about enough.
Part (c) Comments on the correlation can be seen in the scatter diagram but no attempt to interpret it is made, ‘interpret’ is the key word in this question.
Part (d) Incorrect answer. This also highlights however that candidates should be aware which is the independent and which the dependent variable.
Part (e) The candidate has the right idea but does not give sufficient detail in the context of the large data set to pick up the second B mark.
Student Response C

 Examiner Comments: (a) B1 (b) B1B0 (c) B1 (d) B0 (e) B1B1

 Part (a) Good response showing all necessary working.

 Part (b)(i) Good response is seen but part (ii) is incorrect as removing the outlier would not significantly influence the quartiles.

 Part (c) Good response which comments on the correlation and then an interpretation in context.

 Part (d) The answer suggests confusion between correlation and the gradient of the regression line but also has the two variables the wrong way around.

 Part (e) Good response referring to the large data set and suggesting an awareness of the data available.
Exemplar question 5

5. The discrete random variable $X \sim B(40, 0.27)$.

(a) Find $P(X \geq 16)$.

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager’s suspicion.

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager’s suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(d) Find the actual significance level of a test based on your critical region from part (c).

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager’s suspicion in the light of this observation.

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(Total for Question 5 is 9 marks)
### Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
</table>
| 5(a) | \[
\begin{align*}
P(X \geq 16) &= 1 - P(X \leq 15) \\
&= 1 - 0.949077... = \text{awrt } 0.0509
\end{align*}
\] | M1 1.1b | A1 1.1b |
| (b) | \[H_0 : p = 0.3 \quad H_1 : p \neq 0.3 \quad (\text{Both correct in terms of } p \text{ or } \pi)\] | B1 2.5 |
| (c) | \[
\begin{align*}
[&Y \sim \text{B}(20, 0.3)] \quad \text{sight of } P(Y \leq 2) = 0.0355 \\
\text{or} \quad P(Y \leq 9) = 0.9520
\end{align*}
\] | M1 2.1 |
|  | Critical region is \{Y \leq 2\} or o.e. | A1 1.1b |
|  | \{Y \geq 10\} or o.e. | A1 1.1b |
| (d) | \[0.0355 + (1 - 0.9520)] = 0.0835 \text{ or } 8.35%\] | B1ft 1.1b |
| (e) | (Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager’s suspicion is supported | B1ft 3.2a |
| (f) | e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid) | B1 3.5a |
| | | (9 marks) |
### Continued question 5

<table>
<thead>
<tr>
<th>(a)</th>
<th><strong>M1:</strong> For dealing with $P(X) \geq 16$ – they need to use cumulative probability function on calculator <strong>A1:</strong> awrt 0.0509 (from calculator)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td><strong>B1:</strong> For both hypothesis in terms of $p$ and $\pi$ and $H_1$ must be 2-tail</td>
</tr>
<tr>
<td>(c)</td>
<td><strong>M1:</strong> For correct use of tables to find probability associated with critical value <strong>A1:</strong> For the correct lower limit of the CR. Do not award for $P(Y \leq 2)$ <strong>A1:</strong> For the correct upper limit</td>
</tr>
<tr>
<td>(d)</td>
<td><strong>B1:</strong> ft on their 0.0355 and (1 – their 0.9520) provided each probability is less than 0.05</td>
</tr>
<tr>
<td>(e)</td>
<td><strong>B1:</strong> ft for a comment that relates 12 to their CR and makes a consistent comment relating this to the manager’s suspicion</td>
</tr>
<tr>
<td>(f)</td>
<td><strong>B1:</strong> For a comment that: gives a suitable reason based on lack of independence or the sample not being random so the binomial model is not valid</td>
</tr>
</tbody>
</table>
Student Response A

\[ X \sim B(40, 0.27) \quad \Phi \left( \frac{16 - 10.8}{\sqrt{7.884}} \right) \]

\[ \frac{16 - 10.8}{\sqrt{7.884}} = 1.85 \]

(b) \[ H_0 : p = 0.3 \quad X \sim B(20, 0.3) \quad \] 
\[ H_1 : p \neq 0.3 \]

(c) \[ 0.05 \quad \phi(x \leq 9) = 0.855 \]
\[ 0.95 \quad \phi(x > 7) = 0.0447 \]
\[ 0.0355 + 0.0443 = 0.0808 \]

(d) \[ \phi(12) = 0.9985 \quad 0.9985 \quad 0.0015 \]
\[ 31 \div 20 = 6 \quad 12 = 6 \text{ (to 1 decimal place)} \]

(f) This is an anomaly so distrust the validity of the model.

Examiner Comments: (a) M0A0 (b) B1 (c) M1A0A0 (d) B1 (e) B0 (f) B0

Part (a) It appears that the candidate tries to use a Normal approximation rather than the Binomial cumulative distribution function. Students should be encouraged to be familiar with the functions available on their calculators.

Part (b) Answer is correct.

Part (c) Starts well correctly quoting the relevant probabilities but then the candidate muddles converting less than or equal to statements into less than or greater than statements. Errors at both ends of the range are seen.

Part (d) Correct.

Part (e) Reference to the critical region compared to the observed value needs to be made.

Part (f) There is a realisation that the scout group may have an impact but there needs to be a statement in context as to why this might be the case.
Examiner Comments:  
(a) M1A0 (b) B1 (c) M1A1A1 (d) B1 (e) B0 (f) B0

Part (a) Attempt to use a valid method for finding $1 - P(X \leq 15)$ is seen but the appropriate calculator function is not used. Candidates should be encouraged to be familiar with the probability distribution functions available on their calculators and to use them whenever possible.

Part (b) Answer is correct.

Part (c) Also correct including clear statement of the relevant probabilities, leading to a correct answer to part (d).

Part (e) Correct idea, however the question does ask students to reference the recent observation when giving their answer: "...in the light of this observation." We would therefore expect a comment that "12 is in the critical region" as well as a comment suggesting that it supports the manager's suspicion or that the proportion of customers buying single tins has increased.

Part (f) The candidate needs to make it clear what impact the group of scouts may have had on the sample and why.
Student Response C

\[ p(x \leq 8) = 0.30 \]

\[ p(x \leq 7) = 0.1021 \]

\[ p(x \leq 6) = 0.0355 \quad \rightarrow \text{closer to } \frac{x}{y}, \]

\[ x = 2 \]

\[ p(x \leq 9) = 0.5930 \quad \rightarrow \text{closer to } \frac{q}{r} \]

\[ p(x \leq 8) = 0.8862 \]

\[ q = 1 \quad r = 10 \]

Critical region:
\[ 0.0355 \]

Actual significance:
\[ (1 - 0.0355) + 0.0355 = 0.9835 \]

\[ 0.357 \]

e) reject \( H_0 \), as \( x \) lies in the critical region \[ \{ x \geq 10 \} \]
and we have sufficient evidence to prove that there is a change in the proportion of campers buying baked beans.

f) Not valid as it is bias.

C: Campers eat baked beans mainly when they are camping.
Examiner Comments: (a) M1A0 (b) B1 (c) M1A1A1 (d) B1 (e) B1 (f) B1

Part (a) This candidate knows what probability to find but the tables are being used rather than the functions on the calculator. Questions will be set that expect candidates to use the cumulative distribution functions on the calculator, the Binomial tables are provided for questions where critical regions need to be found.

Part (b) Answer is correct.

Part (c) Very good answer as the candidate finds the correct critical regions and not only quotes the relevant probabilities but shows in working that they also checked the adjacent values.

Part (d) Answer is correct.

Part (e) Good answer, the result of the hypothesis test are given by referring to the critical region. A conclusion in context is seen.

Part (f) Good answer in the context to provide a relevant comment as to the possible validity.
6. A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is $U$ m s$^{-1}$. The car then accelerates with constant acceleration $a$ m s$^{-2}$ for $T$ seconds. The car travels a distance $D$ metres during these $T$ seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leq t \leq T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics ($svatu$) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(Total for Question 6 is 4 marks)
## Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Using distance = total area under graph (e.g. area of rectangle + triangle or trapezium or rectangle – triangle)</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>e.g. $D = UT + \frac{1}{2} Th$, where $h$ is height of triangle</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Using gradient = acceleration to substitute $h = aT$</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$D = UT + \frac{1}{2} aT^2$ *</td>
<td>A1 *</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(4 marks)

**Notes:**

**M1:** For use of distance = total area to give an equation in $D$, $U$, $T$ and one other variable

**A1:** For a correct equation

**M1:** For using gradient = $a$ to eliminate the other variable to give an equation in $D$, $U$, $T$ and $a$ only

**A1*:** For a correct given answer
Student Response A

6.

A car moves along a straight horizontal road. At time \( t = 0 \), the velocity of the car is \( U \text{ m s}^{-1} \). The car then accelerates with constant acceleration \( a \text{ m s}^{-2} \) for \( T \) seconds. The car travels a distance \( D \) metres during these \( T \) seconds.

Figure 1 shows the velocity-time graph for the motion of the car for \( 0 \leq t \leq T \).

Using the graph, show that \( D = UT + \frac{1}{2} aT^2 \).

(No credit will be given for answers which use any of the kinematics (\( u + v \)) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

\[
\begin{align*}
\text{Area of rectangle} &= UT \\
\text{Area of triangle} &= \left(\frac{1}{2} \times a \times T\right)T \\
\text{Distance} &= \text{area under graph} = \text{triangle + rectangle} \\
&= UT + \frac{1}{2} aT^2
\end{align*}
\]

Examiner Comments:

Given that there is a printed answer, it is essential that sufficient working and explanation is given. The candidate uses the fact that the distance travelled is equal to the total area under the graph to produce an equation but fails to explain where the ‘\( aT \)’ term has come from.
Examiner Comments:

The candidate earns the first M1A1 but then uses a *suvat* formula (see bracket at bottom of question) to obtain the answer and so scores no further marks.
Examiner Comments:
The candidate uses the area under graph = distance travelled and gradient of graph = acceleration to obtain two equations for $v$ and then eliminating to obtain the printed answer. However, the equations and the result should be given in terms of $T$ not $t$, so both A marks have been deducted.
Exemplar question 7

7. A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where \(AB = 22\) m and \(BC = 104\) m. The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(Total for Question 7 is 7 marks)

---

Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(i)(\text{ii})</td>
<td>Using a correct strategy for solving the problem by setting up two equations in (a) and (u) only and solving for either</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Equation in (a) and (u) only</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>(22 = 2u + \frac{1}{2} a 2^2)</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Another equation in (a) and (u) only</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>(126 = 6u + \frac{1}{2} a 6^2)</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>5 m s(^{-2})</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>6 m s(^{-1})</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(7 marks)

Notes:

**M1:** For solving the problem by setting up two equations in \(a\) and \(u\) only and solving for either

**M1:** Use of (one or more) \(suvat\) formulae to produce an equation in \(u\) and \(a\) only

**A1:** For a correct equation

**M1:** Use of (one or more) \(suvat\) formulae to produce another equation in \(u\) and \(a\) only

**A1:** For a correct equation

**A1:** For correct accln 5 m s\(^{-2}\)

**A1:** For correct speed 6 m s\(^{-1}\) (The second of these A marks is an ft mark, following an incorrect value for \(u\) or \(a\), depending on which has been found first)

**N.B.** Do not award the ft mark for absurd answers e.g. \(a > 15\), \(u > 50\)

See alternative on the next page
ALTERNATIVE

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(i)(ii)</td>
<td>Using a correct strategy for solving the problem by obtaining actual speeds at two times and using $a = \frac{\text{change in speed}}{\text{time taken}}$.</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Actual speed at $t = 1 = \text{Average speed over interval}$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{22}{2} = 11$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Actual speed at $t = 4 = \text{Average speed over interval}$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$\frac{104}{4} = 26$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$5 \text{ m s}^{-2}$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$6 \text{ m s}^{-1}$</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
</tbody>
</table>

(7 marks)

Notes:

M1: For solving the problem by obtaining two actual speeds and use of $a = \frac{(v - u)}{t}$

M1: Use of speed at half-time = average speed over interval to produce a speed at $t = 1$

A1: For a correct speed

M1: Use of speed at half-time = average speed over interval to produce a speed at $t = 4$

A1: For a correct speed

A1: For correct accln $5 \text{ m s}^{-2}$

A1: ft for correct speed $6 \text{ m s}^{-1}$ (This is an ft mark, following an incorrect value of $a$)

N.B. Do not award the ft mark for absurd answers e.g. $a > 15$, $u > 50$
Examiner Comments:

The candidate uses the same letter \((u)\) to represent two different speeds, namely the speed at \(A\) and the speed at \(B\). Consequently, only the M1A1 for one equation can be awarded. Note that the A1 ft can only be earned if the first M mark is earned (i.e. if two equations in two unknowns have been correctly obtained) which is not the case here.
Examiner Comments:

The candidate uses a slightly different method from that on the mark scheme by first setting up two equations in $a$ and '$y$', where $y$ is defined as the speed at $B$, then solving for $a$ and using $v = u + at$ to find $u$. The candidate has a correct equation for the motion from $B$ to $C$ but makes a slip in the $A$ to $B$ equation and loses an A mark. The equations are then solved simultaneously (M1) but the value of $a$ is incorrect (A0). The value of $u$ found is a correct follow through value (A1ft).
Student Response C

Examiner Comments:
The candidate uses the same method as in the previous response but this time all working is correct and full marks are achieved.
**Exemplar question 8**

8. A bird leaves its nest at time \( t = 0 \) for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, \( s \) metres, of the bird from its nest at time \( t \) seconds is given by

\[
s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leq t \leq 10.
\]

(a) Explain the restriction \( 0 \leq t \leq 10 \).

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

*(Total for Question 8 is 9 marks)*
### Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8(a)</strong></td>
<td>Substitution of both $t = 0$ and $t = 10$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$s = 0$ for both $t = 0$ and $t = 10$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Explanation ($s &gt; 0$ for $0 &lt; t &lt; 10$) since $s = \frac{1}{10} t^2 (t - 10)^2$</td>
<td>A1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>($3$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Differentiate displacement $s$ w.r.t. $t$ to give velocity, $v$</td>
<td>M1</td>
<td>1.1a</td>
</tr>
<tr>
<td></td>
<td>$v = \frac{1}{10} (4t^3 - 60t^2 + 200t)$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Interpretation of ‘rest’ to give</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$v = \frac{1}{10} (4t^3 - 60t^2 + 200t) = \frac{2}{5} t(t - 5)(t - 10) = 0 \Rightarrow t = 0, 5, 10$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Select $t = 5$ and substitute their $t = 5$ into $s$</td>
<td>M1</td>
<td>1.1a</td>
</tr>
<tr>
<td></td>
<td>Distance = 62.5 m</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>($6$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)
- **M1:** For substituting $t = 0$ and $t = 10$ into $s$ expression
- **A1:** For noting that $s = 0$ at both times
- **A1:** Since $s$ is a perfect square, $s > 0$ for all other $t$-values

(b)
- **M1:** For differentiating $s$ w.r.t. $t$ to give $v$ (powers of $t$ reducing by 1)
- **A1:** For a correct $v$ expression in any form
- **M1:** For equating $v$ to 0 and factorising
- **A1:** For correct $t$ values
- **M1:** For substituting their intermediate $t$ value into $s$
- **A1:** ft following an incorrect $t$-value
Examiner Comments:

Part (a) To earn the M mark the candidate needs to substitute both $t = 0$ and $t = 10$ into the $s$ expression.

Part (b) The first 4 marks are earned but the candidate forgets to substitute in their $t = 5$ value to find the $s$ value at that time.
Student Response B

8a) It must be greater than or equal to 0 because if \( s < 0 \), 'negative time' cannot exist. It must be less than 10 (or equal to) because at \( t = 10 \), \( s = 0 \); therefore it has returned from the rest and has finished its journey, then

\[
V \times = \frac{ds}{dt} = \frac{7}{5} t^3 - 6t^2 + 20t
\]

\[
\therefore \text{When at rest, } V = 0
\]

\[
\frac{7}{5} t^3 - 6t^2 + 20t = 0
\]

\[
\frac{7}{5} t^3 - 6t^2 + 20 = 0
\]

\[
2t^3 - 30t + 100 = 0
\]

\[
(t - 5)(t - 10) = 0
\]

\[
\therefore t = 5
\]

Examiner Comments:

Part (s) The candidate does not do enough to earn the M mark (needs to explicitly show that \( s = 0 \) at both \( t = 0 \) and \( t = 10 \)).

Part (b) Here the candidate, after a false start, earns all the marks.
Student Response C

(a) \[ s = \frac{1}{10} \left(0^4 - 20(0)^3 + 100(0)^2\right) = 0 \]
when \( t = 0 \), bird leaves the nest, displacement \( s = 0 \),
and when \( t = 10 \), bird returns to the nest, displacement \( t = 0 \), hence \( 0 \leq t \leq 10 \) is the time of its flight.

(b) \[ \frac{ds}{dt} = \frac{4}{10} t^3 - 6 t^2 + 70 t \]
\[ = \frac{2}{5} t^3 - 6 t^2 + 70 t \]
\[ \therefore \text{when } u = 0 \text{, width becomes } 0 \]
\[ \frac{2}{5} t^3 - 6 t^2 + 70 t = 0 \]
\[ \frac{2}{5} t \left( t^2 - 15 t + 50 \right) = 0 \]
\[ \frac{2}{5} t \left( t - 5 \right) \left( t - 10 \right) = 0 \]
\[ \therefore \text{when } t = 5, \text{ at width becomes } 0 \]
\[ s = \frac{1}{10} \left(5^4 - 20(5)^3 + 100(5)^2\right) \]
\[ = \frac{1}{10} \left(625\right) \]
\[ s = 62.5 \text{ m} \]

Examiner Comments:

Part (a) The candidate earns the M mark and the first A mark. The key point is that \( s \) is the distance, which is non-negative by definition, of the bird from the nest and so, to earn the final A mark, the candidate needs to show that \( s \) is always positive for \( t \) values between 0 and 10.

Part (b) Full marks earned here for a neat and concise solution.
9.

A small ball $A$ of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley $P$ which is fixed at the edge of the table. The other end of the string is attached to a small ball $B$ of mass 1.5 kg hanging freely, vertically below $P$ and with $B$ at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of $A$ from the rough table is modelled as having constant magnitude 12.7 N. Ball $B$ reaches the floor before ball $A$ reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible, the pulley is modelled as being small and smooth and the acceleration due to gravity, $g$, is modelled as being $9.8 \text{ m s}^{-2}$

(a) (i) Write down an equation of motion for $A$.

(ii) Write down an equation of motion for $B$. (4)

(b) Hence find the acceleration of $B$. (2)

(c) Using the model, find the time it takes, from release, for $B$ to reach the floor. (2)

(d) Suggest two improvements that could be made in the model. (2)

(Total for Question 9 is 10 marks)
## Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>9(a) (i)</td>
<td>Equation of motion for A</td>
<td>M1</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>$T - 12.7 = 2.5a$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(ii)</td>
<td>Equation of motion for B</td>
<td>M1</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>$1.5g - T = 1.5a$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Solving two equations for a</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(b)</td>
<td>$a = 0.5$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(c)</td>
<td>$1 = \frac{1}{2} \times 0.5 \ t^2$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$t = 2$ seconds</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td>(d)</td>
<td>Valid improvement, see below in notes</td>
<td>B1</td>
<td>3.5c</td>
</tr>
<tr>
<td></td>
<td>Valid improvement, see below in notes</td>
<td>B1</td>
<td>3.5c</td>
</tr>
</tbody>
</table>

(10 marks)
### Continued question 9

**Notes:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)(i)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>For resolving horizontally for $A$</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>For a correct equation</td>
</tr>
<tr>
<td><strong>(a)(ii)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>For resolving vertically for $B$</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>For a correct equation</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>For complete correct strategy for solving the problem, setting up <strong>two</strong> equations in $a$, and then solving them for $a$</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>For $a = 0.5$</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>For a complete method (which could involve use of more than one suvat formula) to give an equation in $t$ only</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>ft from their $a$ to get time in seconds</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td></td>
</tr>
</tbody>
</table>
| **B1, B1** | For any two of  
  e.g. Include the dimensions of the ball in the model so that the distance it falls changes  
  e.g. Include the dimensions of the pulley in the model so string not parallel to table  
  e.g. Include a variable resistance in the model instead of taking it to be constant  
  e.g. Include a more accurate value for $g$ in the model  |
Examiner Comments:

Part (a)(i) The candidate has included an extra friction force in the equation of motion and so loses the M mark (incorrect number of terms) and therefore the dependent A mark also.

Part (a)(ii) Full marks here.

Part (b) The candidate does not solve for \( a \), so no marks awarded.
Student Response B

Examiner Comments:

Part (a) Full marks for the two equations of motion.

Part (b) The candidate solves the two equations to obtain a correct value of $a$.

Part (c) No valid attempts.
Student Response C

Examiner Comments:

Parts (a), (b) and (c) are all fully correct.

Part (d) The candidate suggests two valid improvements (air resistance on balls and mass of string) and so both B marks are scored.
Exemplar question 1

1. The number of hours of sunshine each day, \( y \), for the month of July at Heathrow are summarised in the table below.

<table>
<thead>
<tr>
<th>Hours</th>
<th>( 0 \leq y &lt; 5 )</th>
<th>( 5 \leq y &lt; 8 )</th>
<th>( 8 \leq y &lt; 11 )</th>
<th>( 11 \leq y &lt; 12 )</th>
<th>( 12 \leq y &lt; 14 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>6</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

A histogram was drawn to represent these data. The \( 8 \leq y < 11 \) group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the \( 0 \leq y < 5 \) group.  

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures.

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectively.

Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas’ belief.

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by \( N(6.6, 3.7^2) \).

(e) Use Helen’s model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen’s model.

(Total for Question 1 is 13 marks)
### Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>Area = (8 \times 1.5 = 12) cm(^2) <strong>Frequency</strong> = 8 so 1 cm(^2) = (\frac{2}{3}) <strong>hour</strong> (o.e.)</td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td>Frequency of 12 corresponds to area of 18 so height = (18 \div 2.5 = 7.2) (cm)</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td><strong>Width</strong> = (5 \times 0.5 = 2.5) (cm)</td>
<td>B1cao</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>[\bar{y} = \frac{205.5}{31} = \text{awrt 6.63}]</td>
<td>B1cao</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>[\sigma_y = \sqrt{\frac{1785.25}{31} - \bar{y}^2} = \sqrt{13.644641} = \text{awrt 3.69}]</td>
<td>M1</td>
<td>1.1a</td>
</tr>
<tr>
<td></td>
<td>allow [s = \sqrt{\frac{1785.25 - 31\bar{y}^2}{30}} = \text{awrt 3.75}]</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Hurn is South of Heathrow so does not support his belief</td>
<td>A1</td>
<td>2.2b</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>(\bar{x} + \sigma \approx 10.3) so number of days is e.g. (\frac{(11 - &quot;10.3&quot;)}{3} \times 8 + 5)</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>= 6.86 so <strong>7 days</strong></td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(e)</strong></td>
<td>([H = \text{no. of hours}]\ \ P(H &gt; 10.3)) or (P(Z &gt; 1) = [0.15865...])</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Predict (31 \times 0.15865... = 4.9\ or \ 5\ days)</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td><strong>(f)</strong></td>
<td>(5 or ) 4.9 days &lt; (7 or) 6.9 days so model may <strong>not</strong> be suitable</td>
<td>B1</td>
<td>3.5a</td>
</tr>
</tbody>
</table>

*(13 marks)*
<table>
<thead>
<tr>
<th>Question 1 continued</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes:</td>
</tr>
<tr>
<td>(a) M1: for clear attempt to relate the area to frequency. Can also award if their height ( \times ) their width = 18 A1: for height = 7.2 (cm)</td>
</tr>
<tr>
<td>(b) M1: for a correct expression for ( \sigma ) or ( s ), can fit their value for mean A1: awrt 3.69 (allow ( s = 3.75 ))</td>
</tr>
<tr>
<td>(c) M1: for a suitable comparison of standard deviations to comment on reliability. A1: for stating Hurn is south of Heathrow and a correct conclusion</td>
</tr>
<tr>
<td>(d) M1: for a correct expression – fit their ( \bar{x} + \sigma \approx 10.3 ) A1: for 7 days but accept 6 (rounding down) following a correct expression</td>
</tr>
<tr>
<td>(e) M1: for a correct probability attempted A1: for a correct prediction</td>
</tr>
<tr>
<td>(f) B1: for a suitable comparison and a compatible conclusion</td>
</tr>
</tbody>
</table>
Examiner Comments: (a) M0A0B1 (b) B0M0A0 (c) M1A0 (d) M0A0 (e) M0A0 (f) B0

Part (a) The candidate has a correct value for the width but the height is incorrect. The height of the 8~11 class is the same as the frequency and the candidate just assumes that the same would be true in the case of the 0~5 class but of course this ignores the principle of histograms that the area is proportional to the frequency.

Part (b) The mean is incorrect (the midpoint for 8~11 class was given as 9 not 9.5) and $\sum f^2x$ rather than $\sum f \cdot x^2$ is used when finding the standard deviation.

Part (c) The candidate comments on the difference between the standard deviations meaning greater variability but the answer is incomplete because they do not mention that Hurn is south of Heathrow and that is the reason that Thomas’ belief is not supported.

Parts (d) and (e) Calculations seen represents hours of sunshine but the question asks about number of days.

No attempt at part (f).
Student Response B

Question 1 continued

\[ X^2 = (2.5 - 12 + 6.5 \times 6 + 9.5 \times 8 + 11.5 \times 14 + 13 \times 6)^2 = 31 = 6.628322 \]

\[ \bar{x} = \frac{6.628322}{11} = 0.6 \]

\[ \text{Var}(X) = \frac{(x - \bar{x})^2}{n} = 5.585 \text{, } \text{Sd}(X) = \sqrt{5.585} = 2.36 \text{ (3dp)} \]

\[ \hat{\theta} = \text{Var}^{1/2} \text{, } \theta = (\text{Var}^{1/2})^2 = 5.585 \]

\[ \sigma^2 = (\text{Var}^{1/2})^2 = 5.585 \]

\[ \sigma = \text{Var}^{1/2} = \sqrt{5.585} = 2.36 \text{ (3dp)} \]

b) I'm not good at geography but like my seal island smaller than his so my data spread is smaller and my mean is bigger so that means the weather here more sunny days and if the temperature more on the south than nor is.

(Total for Question 1 is 13 marks)
Examiner Comments: (a) M0A0B1 (b) B1M1A1 (c) M1A0 (d) M0A0 (e) M1A1 (f) B0

Part (a) The width is correct but there is no clear working for the height and an answer of 12 is incorrect.

Part (b) Fully correct. There is a clear expression given for the standard deviation which meant that, had the mean been incorrect, a method mark could have been awarded.

Part (c) The candidate comments that they don’t know whether Hurn is north or south of Heathrow. It is a requirement of the specification that candidates are familiar with the large data set and so they should know from this that Hurn is south of Heathrow. The candidate uses the standard deviations to comment on the variability and so scores one mark.

Part (d) An attempt to interpolate is seen but the class boundary is incorrect and so the expression is incorrect. This approach works up from the bottom of the class interval and to answer the question one would then need to find 31 – (18 + their 𝑥) which some candidates did not appreciate.

Part (e) Fully correct.

In the final part we would expect to see a comparison of their answers to parts (d) and (e) to support the conclusion. In this case a comment that “the number of days in parts (d) and (e) are similar” followed by the statement that Helen’s model is therefore suitable would have been sufficient for the mark but the student’s response here was not sufficient.
Student Response C

a) \( y \leq 11 \):

- **Class width = 3**
  - Frequency density = \( \frac{8}{3} \)

- **Width = 1.5 cm**
  - Height: \( \frac{8}{3} \) f.d.
  - \( 0.5 \text{ cm} = \text{width } 1 \)
  - \( 3 \text{ cm} = \text{1 f.d.} \)

Question 1 continued

b) \( 0 \leq y \leq 5 \):

- **Class width = 5**
  - Frequency density = \( \frac{12}{5} \)

- **Width = 0.5 cm x 5**
  - Height = \( 3 \text{ cm} \times \frac{12}{5} \)
  - \( = 7.2 \text{ cm} \)

\[ \text{mean} = \frac{\sum fx}{\sum f} = \frac{(12 \times 1.5) + (6 \times 6.5) + (8 \times 9.5) + (3 \times 11.5) + (2 \times 13)}{12 + 6 + 8 + 3 + 2} \]

\[ = \frac{705.5}{31} \]

\[ = 6.62 \text{ (3 s.f.)} \]

\[ \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2} \]

\[ \sum fx^2 = (12 \times 1.5^2) + (6 \times 6.5^2) + (8 \times 9.5^2) + (3 \times 11.5^2) + (2 \times 13^2) \]

\[ = 1785.25 \]

\[ \sigma = \frac{\sqrt{1785.25} - (705.5)^2}{31} = 3.69 \text{ (3 s.f.)} \]

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean</th>
<th>S.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heighton</td>
<td>6.62</td>
<td>3.69</td>
</tr>
<tr>
<td>Hurn</td>
<td>5.98</td>
<td>4.12</td>
</tr>
</tbody>
</table>

- Larger S.D. so results more spread out. Thomas wrong as Hurn more South than Heathrow.

\[ 6.62 + 3.69 = 10.31 \]

\[ \frac{18.13 + 3.2}{2} = 6.81 \text{ days} \] (Total for Question 1 is 13 marks)
Examiner Comments: (a) M1A1B1 (b) B1M1A1 (c) M1A1 (d) M1A1 (e) M1A0 (f) B0

Parts (a) and (b) Fully correct.

Part (c) Correct full response that compares standard deviations and mentions that Thomas’ belief is not supported because Hurn is south of Heathrow.

Part (d) There is a correct interpolation and the candidate uses the correct tail to reach 6.8 days.

Part (e) Correct calculation for the probability is seen and score the method mark but the candidate do not go on to multiply this by 31 to obtain an estimate of the number of days so lose the accuracy mark.

Part (f) In the final part there is no comparison of the number of days between parts (d) and (e) so no mark is scored.
Exemplar question 2

2. A meteorologist believes that there is a relationship between the daily mean windspeed, \(w\) \(\text{k}\text{n}\), and the daily mean temperature, \(t\) °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

<table>
<thead>
<tr>
<th>(t)</th>
<th>13.3</th>
<th>16.2</th>
<th>15.7</th>
<th>16.6</th>
<th>16.3</th>
<th>16.4</th>
<th>19.3</th>
<th>17.1</th>
<th>13.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w)</td>
<td>7</td>
<td>11</td>
<td>8</td>
<td>11</td>
<td>13</td>
<td>8</td>
<td>15</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained \(r = 0.609\).

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C.

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days, a location from the large data set gave \(t = 27.2\) and \(w = 3.5\).

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

(Total for Question 2 is 6 marks)
Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>e.g. It requires extrapolation so will be unreliable (o.e.)</td>
<td>B1 1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>(b)</td>
<td>e.g. Linear association between $w$ and $t$</td>
<td>B1 1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>(c)</td>
<td>$H_0$: $\rho = 0$  $H_1$: $\rho &gt; 0$  Critical value 0.5822  Reject $H_0$  There is evidence that the product moment correlation coefficient is greater than 0</td>
<td>B1 2.5  M1 1.1a  A1 2.2b</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>Higher $t$ suggests overseas and not Perth…lower wind speed so perhaps not close to the sea so suggest <strong>Beijing</strong></td>
<td>B1 2.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

(6 marks)

Notes:

(a)  
**B1:** for a correct statement (unreliable) with a suitable reason

(b)  
**B1:** for a correct statement

(c)  
**B1:** for both hypotheses in terms of $\rho$  
**M1:** for selecting a suitable 5% critical value compatible with their $H_1$  
**A1:** for a correct conclusion stated

(d)  
**B1:** for suggesting Beijing with some supporting reason based on $t$ or $w$  
Allow Jacksonville with a reason based just on higher $t$
Examiner Comments: (a) B1 (b) B0 (c) B0M0A0 (d) B0

Part (a) The candidate mentions that extrapolation is involved which is sufficient to score the mark.

Part (b) No mention that the product moment correlation coefficient is a measure of the strength of the linear relationship between the variables and so no mark is scored.

Part (c) The candidate states the hypotheses but uses $r$ rather than the population parameter $\rho$ and so the 1st mark is lost. For tests involving correlation the critical values are given in the book of tables. This student attempts to use a normal distribution to carry out the test which is an incorrect method so no further marks are scored in this part.

There was no attempt at part (d).
Examiner Comments: (a) B1 (b) B0 (c) B1M1A0 (d) B0

Part (a) The candidate mentions that 24°C is outside the range of the data, which is equivalent to mentioning extrapolation.

Part (b) There is a mention of the strength of the relationship but no mention that it is a linear relationship.

Part (c) The hypotheses are correctly written in terms of \( \rho \) and the critical value is stated correctly. The candidate states they would reject \( H_0 \) but no explanation what this means is given. We would expect a comment specifying that there is evidence that the correlation between the variables is greater than zero.

Part (d) In the final part the candidate uses a comparison of temperatures to state that the location is outside of the UK but their knowledge of the large data set is not used to suggest a specific location.
Student Response C

(a) Extrapolation beyond range of data

(b) Measures the strength of fit of $w$ & $t$ to a straight line

(c) Let $\rho$ be the population parameter

Testing $H_0: \rho = 0$ against $H_1: \rho > 0$ ($t$-test)

From tables, critical value is 0.6000 for $n=9.5$%

Test statistic $t = 0.609 > 0.6000$; in critical region $\Rightarrow$ sufficient evidence to reject $H_0$

$\Rightarrow$ evidence that population parameter is greater than 0

(d) The mean temperature is very high & the mean windspeed is low

Could be an intertropical location or large data set possibly Jacksonville.

Examiner Comments: (a) B1 (b) B1 (c) B1M1A0 (d) B1

Parts (a) and (b) The candidate mentions extrapolation and “the strength of fit … to a straight line” which conveys the idea of the strength of the linear relationship and scores the mark.

Part (c) The hypotheses are written correctly but the critical value quoted is from the Spearman’s table. Care should be taken when using these tables to use the correct columns. We would allow this for the method mark but, even though the conclusion is stated correctly the accuracy mark cannot be given since the critical value is incorrect.

In the final part the candidate comments on the temperature and windspeed and selects Jacksonville which is an acceptable answer in the mark scheme.
Exemplar question 3

3. A machine cuts strips of metal to length $L$ cm, where $L$ is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.  

(b) Find the probability fewer than 4 of these strips cannot be used.

A second machine cuts strips of metal of length $X$ cm, where $X$ is normally distributed with standard deviation 0.6 cm.

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm.

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm.

(Total for Question 3 is 12 marks)
# Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q3(a)</strong></td>
<td><img src="image" alt="Normal Distribution" /></td>
</tr>
</tbody>
</table>

\[
P (L > 50.98) = 0.025\]

\[
\therefore \quad \frac{50.98 - \mu}{0.5} = 1.96
\]

\[
\therefore \quad \mu = 50
\]

\[
P(49 < L < 50.75) = 0.9104… \quad \text{awrt} \ 0.910
\]

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(b)</strong></td>
<td>(S = \text{number of strips that cannot be used so } S \sim B(10, 0.090))</td>
</tr>
</tbody>
</table>

\[
P(S \leq 3) = 0.991166… \quad \text{awrt} \ 0.991
\]

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(c)</strong></td>
<td>(H_0 : \mu = 50.1 \quad H_1 : \mu &gt; 50.1)</td>
</tr>
</tbody>
</table>

\[
\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right) \quad \text{and} \quad \bar{X} > 50.4
\]

\[
P(\bar{X} > 50.4) = 0.0264
\]

\[p = 0.0264 > 0.01 \quad \text{or} \quad z = 1.936… < 2.3263 \quad \text{and not significant}
\]

There is insufficient evidence that the mean length of strips is greater than 50.1

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
</table>

(12 marks)
### Question 3 continued

<table>
<thead>
<tr>
<th>Notes:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong></td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; <strong>M1:</strong> for standardizing with $\mu$ and 0.5 and setting equal to a $z$ value ($</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>M1:</strong> for attempting the correct probability for strips that can be used</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>A1</strong>: awrt 0.910 (allow ft of their $\mu$)</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
</tr>
<tr>
<td><strong>M1:</strong> for identifying a suitable binomial distribution</td>
</tr>
<tr>
<td><strong>A1:</strong> awrt 0.991 (from calculator)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
</tr>
<tr>
<td><strong>B1:</strong> hypotheses stated correctly</td>
</tr>
<tr>
<td><strong>M1:</strong> for selecting a correct model (stated or implied)</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; <strong>A1:</strong> for use of the correct model to find $p = $ awrt 0.0264 (allow $z = $ awrt 1.94)</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; <strong>A1:</strong> for a correct calculation, comparison and correct statement</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; <strong>A1:</strong> for a correct conclusion in context mentioning “mean length” and 50.1</td>
</tr>
</tbody>
</table>
Student Response A

a) $H_0: (\mu, 0.95)$

$P(2 > 50.98) = 0.015$

$z = \frac{50.98 - \mu}{0.95}$

$1.9600 = 50.98 - \mu$

$\mu = 50$

$b)\ x = 8 (10, 0.9987)$

$P(x < 4) = 10 \times (1.3 \times 10^{-3})^4 \times 0.9987^6$

$= 5.75117883 \times 10^{-10}$
Examiner Comments: (a) B1M1A1M0A0 (b) M1A0 (c) B0M0A0A0A0

Part (a) There is a correct probability statement and correct use of standardisation to find $\mu$. The candidate then tries to find the correct probability but uses the variance rather than the standard deviation when standardising and so scores M0. Note as the mean and standard deviation are both now known, candidates should be encouraged to use their calculators to find probabilities from Normal distributions.

Part (b) Whilst the Binomial distribution quoted is that for the number of strips which can be used, the calculation then does use the appropriate distribution for the number of strips which cannot be used (following through on their probability from part (a)) and so the candidate scores M1 but finds $P( X = 4)$ rather than $P( X < 4)$.

Part (c) There is no indication of the correct hypotheses or that the distribution of $\bar{X}$ is being used so no further marks are scored.
Examiner Comments: (a) B1M1A0M1A1ft (b) M1A0 (c) B1 M0A0A0A0A0

Part (a) There is a correct probability statement and standardisation using a $z$ value $> 1$. The $z$ value though is incorrect and this leads to an incorrect value of $\mu$. This value is used to find the correct probability and the answer is a correct follow through using their mean.

Part (b) The candidate states and uses a correct binomial distribution with the value of $p$ coming from their answer to part (a). The answer though is incorrect and there is no follow through available for this mark.

Part (c) The hypotheses are started correctly but then $N(50.1, 0.6^2)$ is used rather than the correct normal distribution with standard error of $\frac{0.6}{\sqrt{15}}$. This is an incorrect method and so no further marks are available.
Student Response C

(a) \( \Pr(X > 50.98) = 0.025 \)

\[ 50.98 - \mu = 1.96 \]

\[ \mu = 50 \]

\[ \Pr(49 < X < 50.75) = \Pr(-2 < z < 1.5) = 1 - (0.0228 + 0.0668) \]

\[ = 0.9104 \]

(b) \( X \sim \text{B}(10, 0.0896) \)

\[ 10C_0 (0.0896)^0 (0.9104)^{10} = 0.2911 \]

\[ 10C_1 (0.0896)^1 (0.9104)^9 = 0.28 \]

\[ 10C_2 (0.0896)^2 (0.9104)^8 = 0.03 \]

\[ 10C_3 (0.0896)^3 (0.9104)^7 = 0.60 \]

\[ \Pr(X \leq 3) = 0.9913 \]

(c) \( H_0: \mu = 50.1 \quad H_1: \mu > 50.1 \)

\[ z = \frac{50.4 - 50.1}{0.6/\sqrt{15}} \]

\[ = 1.936 \]

\[ 1.936 < 2.58 \]

Not significant.
Examiner Comments: (a) B1M1A1M1A1 (b) M1A1 (c) B1M1A1A0A0

Part (a) Fully correct but once the candidate finds the correct value for \( \mu \) this new specification would expect them to use their calculator to find the probability. This candidate uses the tables which is fine but would take a lot longer.

Part (b) There are only two marks available: one for stating or trying to use a correct binomial distribution and the other for evaluating the probability. Again this candidate works out all 4 binomial probabilities and adds them and since their answer rounds to 0.991 they score both marks.

Part (c) In the final part the hypotheses are correct and the correct distribution is used giving the first 3 marks but the candidate uses an incorrect critical value and does not go on to interpret the conclusion which means the final two marks are lost.
Exemplar question 4

4. Given that

\[ P(A) = 0.35, \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13 \]

find

(a) \( P(A'|B') \) \hspace{2cm} (2)

(b) Explain why the events A and B are not independent. \hspace{2cm} (1)

The event C has \( P(C) = 0.20 \).

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region. \hspace{2cm} (5)

(d) Find \( P( [B \cup C]' ) \) \hspace{2cm} (2)

(Total for Question 4 is 10 marks)
# Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td>( P(A'</td>
<td>B') = \frac{P(A' \cap B')}{P(B')} ) or ( \frac{0.33}{0.55} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{3}{5} ) or 0.6</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>e.g. ( P(A) \times P(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq P(A \cap B) = 0.13 = \frac{52}{400} ) or ( P(A'</td>
<td>B') = 0.6 \neq P(A') = 0.65 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
<td>B1</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
<td>3.1a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td><strong>(d)</strong></td>
<td>( P(B \cup C)' = 0.22 + 0.22 ) or ( 1 - [0.56] ) or ( 1 - [0.13 + 0.23 + 0.09 + 0.11] ) o.e.</td>
<td>M1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 0.44 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10 marks)</td>
</tr>
</tbody>
</table>

**Notes:**

(a) M1: for a correct ratio of probabilities formula and at least one correct value.  
A1: a correct answer

(b) B1: for a fully correct explanation: correct probabilities and correct comparisons.

(c) B1: for box with \( B \) intersecting \( A \) and \( C \) but \( C \) not intersecting \( A \). (Or accept three intersecting circles, but with zeros entered for \( A \cap C \) and \( A \cap B \cap C \)) No box is B0

M1: for method for finding \( P(B \cap C) \)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

(d) M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao
Examiner Comments: (a) M0A0 (b) B1 (c) B0M0A0M0A0 (d) M1A0

Part (a) The correct probability formula is used and the probability in the denominator is correct however as the candidate gives an answer which is bigger than 1 the method mark is not awarded.

Part (b) Fully correct.

Part (c) The Venn diagram is incorrect as the C circle needs to intersect with B. No marks are awarded as there is no attempt to find \( P(B \cap C) \).

Part (d) Correct method is used but as their diagram is incorrect the answer is also incorrect.
Examiner Comments: (a) M1A1 (b) B0 (c) B0M1A1M1A0 (d) M0A0

Part (a) All correct.

Part (b) The candidate confuses 0.22 with P(A) etc, so the mark is B0 not a correct solution.

Part (c) The three intersecting circles would be acceptable but the candidate needs to give probabilities of 0 is the regions where A and C intersect. Also P(A \cup B \cup C)’ is not given.

Part (d) Incorrect formula is used.
Examiner Comments: (a) M1A1 (b) B1 (c) B1M1A1M0A0 (d) M1A0

Part (a) The candidate appears to use values from a Venn diagram to find the conditional probability, all correct so 2 marks.

Part (b) An alternative property of independent events is used. All correct.

Part (c) Correct structure is seen but an error is made with the probabilities losing 2 marks.

Part (d) Correct method is seen but the candidate uses values from the incorrect Venn diagram and so the final answer is not correct.
Exemplar question 5

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company’s claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company’s claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(e) Using your answer to part (d), comment on whether or not the proportion of the company’s pea seeds that germinate is different from the company’s claim of 55%

(Total for Question 5 is 9 marks)
### Question 5

#### Part (a)

The seeds would be destroyed in the process so they would have none to sell.

- **Marks:** B1 2.4

#### Part (b)

[S = no. of seeds out of 24 that germinate,  \( S \sim B(24, 0.55) \)]

\[ T = \text{no. of trays with at least 15 germinating}. \quad T \sim B(10, p) \]

- **Marks:** B1 3.3

\[ p = P(S \geq 15) = 0.299126... \]

- **Marks:** A1 1.1b

So \( P(T \geq 5) = 0.1487... \) awrt 0.149

- **Marks:** A1 1.1b

#### Part (c)

\( n \) is large and \( p \) close to 0.5

- **Marks:** B1 1.2

#### Part (d)

\( X \sim N(132, 59.4) \)

\[
P(X \geq 149.5) = P\left(Z \geq \frac{149.5-132}{\sqrt{59.4}}\right)
\]

- **Marks:** B1 3.4

\[
= 0.01158... \quad \text{awrt 0.0116}
\]

- **Marks:** A1cso 1.1b

#### Part (e)

E.g. The probability is very small therefore there is evidence that the company’s claim is incorrect.

- **Marks:** B1 2.2b

---

### Notes:

- **(a)**
  - **B1:** cao

- **(b)**
  - **M1:** for selection of an appropriate model for \( T \)
  - **1st A1:** for a correct value of the parameter \( p \) (accept 0.3 or better)
  - **2nd A1:** for awrt 0.149

- **(c)**
  - **B1:** both correct conditions

- **(d)**
  - **B1:** for correct normal distribution
  - **M1:** for correct use of continuity correction
  - **A1:** cso

- **(e)**
  - **B1:** correct statement
Examiner Comments: (a) B0 (b) M0A0A0 (c) B0 (d) B1M1A0 (e) B0

Part (a) No mention that the seeds would be destroyed in the process. Just saying it would be “time consuming” is not sufficient.

Part (b) The distribution for $S$, the number of seeds in a tray that germinate are identified, but the the distribution for the variable $T$, the number of trays with at least 15 seeds germinating is not found. The method requires that they identify this distribution and so no marks are scored. Candidates sometimes find it difficult to identify the correct random variables and defining these as in the mark scheme may be helpful to them.

Part (c) No attempt. All that was required is a statement about $n$ being large and $p$ close to 0.5.

Part (d) A correct normal distribution is given and a correct use of a continuity correction is seen to obtain a correct $z$ value. The final probability is not correct as they forgot to subtract their tables value from 1. Candidates should be able to find these probabilities using a calculator on this new specification.
Examiner Comments: (a) B0  (b) M1A0A0  (c) B1  (d) B1M1A1  (e) B0

Part (a) There is no mention of the seeds being destroyed.

Part (b) there is sight of a $B(10, p)$ distribution and an intention to use $P(X \geq 15)$ from $X \sim B(24, 0.55)$ so a correct method is being attempted. Candidates are expected to be able to find cumulative binomial probabilities using their calculators on this specification, this candidate seems to have faltered over this step.

Part (c) Answer is correct.

Part (d) Answer is fully correct. Calculator could have been used to evaluate the normal probability.

Part (e) No attempt.
Examiner Comments: (a) B1 (b) M1A1A1 (c) B1 (d) B1M1A0 (e) B1

Part (a) The student mentions the idea that the seeds would be destroyed and scores the mark.
Part (b) B(10, p) is used and found the correct probability so full marks are scored.
Part (c) Answer is correct.
Part (d) A correct normal distribution is identified and the student attempts to use a continuity correction but 150.5 instead of 149.5 is used. Answer is a correct follow through from the error but is not correct to obtain the mark.
In the final part they have commented that the proportion is probably different because the probability in part (e) is small.
Exemplar question 6

6. At time \( t \) seconds, where \( t \geq 0 \), a particle \( P \) moves so that its acceleration \( a \) m s\(^{-2}\) is given by

\[
a = 5t \mathbf{i} - 15t^2 \mathbf{j}.
\]

When \( t = 0 \), the velocity of \( P \) is \( 20 \mathbf{i} \) m s\(^{-1}\).

Find the speed of \( P \) when \( t = 4 \).

(Total for Question 6 is 6 marks)

Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Integrate ( a ) w.r.t. time</td>
<td>M1</td>
<td>1.1a</td>
</tr>
</tbody>
</table>
|          | \[
|          | \mathbf{v} = \frac{5t^2}{2} \mathbf{i} - 10t^2 \mathbf{j} + C \quad \text{(allow omission of } C)\]
|          | A1 | 1.1b |
|          | \[
|          | \mathbf{v} = \frac{5t^2}{2} \mathbf{i} - 10t^2 \mathbf{j} + 20 \mathbf{i} \quad \text{(allow omission of } C)\]
|          | A1 | 1.1b |
|          | When \( t = 4 \), \( \mathbf{v} = 60 \mathbf{i} - 80 \mathbf{j} \) | M1 | 1.1b |
|          | Attempt to find magnitude: \( \sqrt{(60^2 + 80^2)} \) | M1 | 3.1a |
|          | Speed = 100 m s\(^{-1}\) | A1 ft | 1.1b |

(6 marks)

Notes:

1st \( M1 \): for integrating \( a \) w.r.t. time (powers of \( t \) increasing by 1)
1st \( A1 \): for a correct \( \mathbf{v} \) expression without \( C \)
2nd \( A1 \): for a correct \( \mathbf{v} \) expression including \( C \)
2nd \( M1 \): for putting \( t = 4 \) into their \( \mathbf{v} \) expression
3rd \( M1 \): for finding magnitude of their \( \mathbf{v} \)
3rd \( A1 \): \( \text{ft} \) for 100 m s\(^{-1}\), follow through on an incorrect \( \mathbf{v} \)
Examiner Comments:

The candidate integrates and obtains a correct expression but then obtains an incorrect constant vector so loses the second A mark. The value $t = 4$ is substituted (this is easily seen by the examiner so the M mark is awarded but to be safe, candidates should explain what they are doing, so here, write 'when $t = 4$'). The candidate does not show the method used to find the speed and so loses the final M mark and also the final A mark.
Student Response B

\[ v = \frac{5t^2}{2} i - 30t^\frac{3}{2} \left( \frac{3}{2} \right) + 20i \]
\[ v = \left( \frac{5t^2}{2} + 20 \right) i + \left( -10t^\frac{3}{2} \right) j \]
\[ 60i - 80j \]

Examiner Comments:
The candidate obtains the correct velocity vector when \( t = 4 \), but forgets to find the speed so scores the first 4 marks only.
Examiner Comments:

The candidate fortuitously obtains a correct final answer, having made a sign error earlier when putting \( t = 4 \), and so loses the final A mark. Note that the final A mark is follow through on the expression for \( v \) in terms of \( t \) and not on the numerical value (as here).
Exemplar question 7

7. A rough plane is inclined to the horizontal at an angle $\alpha$, where $\tan \alpha = \frac{3}{4}$.

A particle of mass $m$ is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\mu$.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of $\mu$. (6)

The particle comes to rest at the point $A$ on the plane.

(b) Determine whether the particle will remain at $A$, carefully justifying your answer. (2)

(Total for Question 7 is 8 marks)
Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(a)</td>
<td>$R = mg\cos\alpha$</td>
<td>B1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Resolve parallel to the plane</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>$- F - mgsin\alpha = -0.8mg$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$F = \mu R$</td>
<td>M1</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Produce an equation in $\mu$ only and solve for $\mu$</td>
<td>M1</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>$\mu = \frac{1}{4}$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Compare $\mu mg\cos\alpha$ with $mgsin\alpha$</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Deduce an appropriate conclusion</td>
<td>A1 ft</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8 marks)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

(a)

B1: for $R = mg\cos\alpha$

1st M1: for resolving parallel to the plane

1st A1: for a correct equation

2nd M1: for use of $F = \mu R$

3rd M1: for eliminating $F$ and $R$ to give a value for $\mu$

2nd A1: for $\mu = \frac{1}{4}$

(b)

M1: comparing size of limiting friction with weight component down the plane

A1 ft: for an appropriate conclusion from their values
Examiner Comments:

Part (a) The candidate scores the B mark for a correct expansion for the reaction but omits the component of the weight when resolving parallel to the plane and so loses the first M mark and the dependent A mark. The next two marks are scored but the final A mark is lost for an incorrect answer.

Part (b) No attempt.
Student Response B

Examiner Comments:

Part (a) The candidate makes a sign error when resolving parallel to the plane and so loses an A mark for an incorrect equation and also the final A mark for an incorrect answer.

Part (b) Nothing scored.
Examiner Comments:

Part (a) All correct until the candidate makes a transcription error and loses the final A mark.

Part (b) A correct explanation, with working to back it up, and full marks scored.
8. [In this question \( \mathbf{i} \) and \( \mathbf{j} \) are horizontal unit vectors due east and due north respectively.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time \( t = 0 \), the boat is at the fixed point \( O \) and is moving due north with speed 0.6 m \( \text{s}^{-1} \).

Relative to \( O \), the position vector of the boat at time \( t \) seconds is \( \mathbf{r} \) metres.

At time \( t = 15 \), the velocity of the boat is \( (10.5 \mathbf{i} - 0.9 \mathbf{j}) \) m \( \text{s}^{-1} \).

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is \( (0.7 \mathbf{i} - 0.1 \mathbf{j}) \) m \( \text{s}^{-2} \).

(b) Find \( \mathbf{r} \) in terms of \( t \).

(c) Find the value of \( t \) when the boat is north-east of \( O \).

(d) Find the value of \( t \) when the boat is moving in a north-east direction.

(Total for Question 8 is 10 marks)
Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8(a)</strong></td>
<td>Use of ( \mathbf{v} = \mathbf{u} + \mathbf{a} t ) : ((10.5 \mathbf{i} - 0.9 \mathbf{j}) = 0.6 \mathbf{j} + 15 \mathbf{a} )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( \mathbf{a} = (0.7 \mathbf{i} - 0.1 \mathbf{j}) \text{ m} s^{-2} ) Given answer</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Use of ( \mathbf{r} = \mathbf{u} t + \frac{1}{2} \mathbf{a} t^2 )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( \mathbf{r} = 0.6 \mathbf{j} t + \frac{1}{2} (0.7 \mathbf{i} - 0.1 \mathbf{j}) t^2 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Equating the ( \mathbf{i} ) and ( \mathbf{j} ) components of ( \mathbf{r} )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{2} 0.7 t^2 = 0.6 t - \frac{1}{2} 0.1 t^2 )</td>
<td>A1ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>( t = 1.5 )</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>Use of ( \mathbf{v} = \mathbf{u} + \mathbf{a} t ) : ( \mathbf{v} = 0.6 \mathbf{j} + (0.7 \mathbf{i} - 0.1 \mathbf{j}) t )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>Equating the ( \mathbf{i} ) and ( \mathbf{j} ) components of ( \mathbf{v} )</td>
<td>M1</td>
<td>3.1b</td>
</tr>
<tr>
<td></td>
<td>( t = 0.75 )</td>
<td>A1 ft</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(10 marks)</strong></td>
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</tr>
</tbody>
</table>

**Notes:**

(a)
- **M1:** for use of \( \mathbf{v} = \mathbf{u} + \mathbf{a} t \)
- **A1:** for given answer correctly obtained

(b)
- **M1:** for use of \( \mathbf{r} = \mathbf{u} t + \frac{1}{2} \mathbf{a} t^2 \)
- **A1:** for a correct expression for \( \mathbf{r} \) in terms of \( t \)

(c)
- **M1:** for equating the \( \mathbf{i} \) and \( \mathbf{j} \) components of their \( \mathbf{r} \)
- **A1ft:** for a correct equation following their \( \mathbf{r} \)
- **A1:** for \( t = 1.5 \)

(d)
- **M1:** for use of \( \mathbf{v} = \mathbf{u} + \mathbf{a} t \) for a general \( t \)
- **M1:** for equating the \( \mathbf{i} \) and \( \mathbf{j} \) components of their \( \mathbf{v} \)
- **A1ft:** for \( t = 0.75 \), or a correct follow through answer from an incorrect equation
Student Response A

Part (a)
Correct.

Part (b)
Nothing scored as candidate is finding \( v \) not \( r \).

Part (c)
Incorrect method.

Part (d)
Incorrect method.
Examiner Comments:

Part (a) Correct.

Part (b) The candidate uses $\mathbf{v} = \mathbf{u} + \mathbf{at}$ correctly but then assumes constant velocity to find $\mathbf{r}$ and so scores no marks.

Part (c) Correct method used to produce a correct follow through equation but the second A mark is lost for an incorrect answer.

Part (d) Correct.
Examiner Comments:

Part (a) Correct.

Part (b) Correct.

Part (c) M mark earned for an attempt at a correct method but there is a sign error so loses both A marks are lost.

Part (d) Both M marks earned and also Aft mark for a correct follow through answer.
Exemplar question 9

9. A uniform ladder $AB$, of length $2a$ and weight $W$, has its end $A$ on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end $B$ of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.

To stop the ladder from slipping, the builder’s assistant applies a horizontal force of magnitude $P$ to the ladder at $A$, towards the wall.

The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle $\alpha$ with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at $B$ has magnitude $3W$.  

(b) Find, in terms of $W$, the range of possible values of $P$ for which the ladder remains in equilibrium.

(c) Explain briefly how this helps to stop the ladder from slipping.

(Total for Question 9 is 13 marks)
## Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9(a)</strong></td>
<td>Take moments about A (or any other complete method to produce an equation in $S$, $W$ and $\alpha$ only)</td>
<td>M1</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>$Wa \cos \alpha + 7W2a \cos \alpha = S \cdot 2a \sin \alpha$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>Use of $\tan \alpha = \frac{5}{2}$ to obtain $S$</td>
<td>M1</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>$S = 3W \ast$</td>
<td>A1*</td>
<td>2.2a</td>
</tr>
<tr>
<td></td>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$R = 8W$</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$F = \frac{1}{4} R \ (= 2W)$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{MAX}} = 3W + F \ \text{or} \ P_{\text{MIN}} = 3W - F$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$P_{\text{MAX}} = 5W \ \text{or} \ P_{\text{MIN}} = W$</td>
<td>A1</td>
<td>1.1b</td>
</tr>
<tr>
<td></td>
<td>$W \leq P \leq 5W$</td>
<td>A1</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>M(A) shows that the reaction on the ladder at $B$ is unchanged</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>also $R$ increases (resolving vertically)</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>which increases max $F$ available</td>
<td>M1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td><strong>(3)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**(13 marks)**
### Question 9 continued

#### Notes:

(a)
- **1st M1:** for producing an equation in S, W and \( \alpha \) only  
- **1st A1:** for an equation that is correct, or which has one error or omission  
- **2nd A1:** for a fully correct equation  
- **2nd M1:** for use of \( \tan \alpha = \frac{S}{2} \) to obtain \( S \) in terms of \( W \) only  
- **3rd A1*: for given answer \( S = 3W \) correctly obtained

(b)
- **B1:** for \( R = 8W \)  
- **1st M1:** for use of \( F = \frac{1}{4} R \)  
- **2nd M1:** for either \( P = (3W + \text{their } F) \) or \( P = (3W - \text{their } F) \)  
- **1st A1:** for a correct max or min value for a correct range for \( P \)  
- **2nd A1:** for a correct range for \( P \)

(c)
- **1st M1:** for showing, by taking moments about \( A \), that the reaction at \( B \) is unchanged by the builder’s assistant standing on the bottom of the ladder  
- **2nd M1:** for showing, by resolving vertically, that \( R \) increases as a result of the builder’s assistant standing on the bottom of the ladder  
- **3rd M1:** for concluding that this increases the limiting friction at \( A \)
Student Response A

105

6/13
Examiner Comments:
Part (a) Candidate incorrectly assumes limiting equilibrium and is awarded only the two M marks.
Part (b) The first 4 marks are scored for a correct working leading to correct lower limit for $P$.
Part (c) No attempt.
A level Mathematics – Paper 3 (Applied – Mechanics)

Student Response B

\[ \sin \theta = \frac{5}{12} \]
\[ \cos \theta = \frac{2}{\sqrt{129}} \]

(a) Moment from A:

\[ (W \cos \theta) a = 2a (7W \cos \theta) \]
\[ \alpha \cos \theta = 2a (7W \cos \theta) \]

\[ (W \cos \theta) a + (W \cos \theta) 2a = 2a (R_B \sin \theta) \]
\[ 2a W + \frac{28W}{\sqrt{129}} \frac{\sqrt{129}}{2} = 2R_B \frac{\sqrt{129}}{2} \]

\[ \frac{30\sqrt{129}}{2} W = 2R_B \left( \frac{5}{\sqrt{129}} \right) \]

\[ R_B = 3 \]

(b) \[ F_r + P = 3W \]
\[ F_r = R_A \times \frac{1}{4} \]

\[ R_A \times \frac{1}{4} = 3W \]
\[ R_A = 12W \]

\[ W_B = 8R_A \]
\[ W + 3W \times \frac{1}{4} \times 2 = 2R_A \]
\[ W + 3W \times \frac{1}{4} \times 2 = 2R_A \]
\[ 8W = R_A \]

\[ W = \frac{R_A}{8} \]

\[ P = \frac{W}{2} \]
\[ P = 2W \]

\[ 2W + P = 3W \]
\[ P = W \]
Examiner Comments:

Part (a) Correct method used, after deleting first attempt at moments equation, but candidate makes a slip when stating the answer (omits $W$) so loses final A mark.

Part (b) The first 4 marks are scored for a correct working leading to a correct lower limit for $P$.

Part (c) No valid comments.
Examiner Comments:

Part (a) Fully correct.

Part (b) Not the best explanation but full marks awarded for a correct range for $P$.

Part (c) Candidate makes one valid comment.
Exemplar question 10

A boy throws a stone with speed $U \text{ m s}^{-1}$ from a point $O$ at the top of a vertical cliff. The point $O$ is 18 m above sea level.

The stone is thrown at an angle $\alpha$ above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point $S$ which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.

The stone is modelled as a particle moving freely under gravity with $g = 10 \text{ m s}^{-1}$.

Find

(a) the value of $U$, \hspace{1cm} (6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures. \hspace{1cm} (5)

(c) Suggest two improvements that could be made to the model. \hspace{1cm} (2)

(Total for Question 10 is 13 marks)
### Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
<th>AOs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10(a)</strong></td>
<td>Using the model and horizontal motion: $s = ut$</td>
<td>M1</td>
<td>3.4</td>
</tr>
<tr>
<td>36 = $U\cos\alpha$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$</td>
<td>M1</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>-18 = $U\sin\alpha - \frac{1}{2}gt^2$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>Correct strategy for solving the problem by setting up two equations in $t$ and $U$ and solving for $U$</td>
<td>M1</td>
<td>3.1b</td>
<td></td>
</tr>
<tr>
<td>$U = 15$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Using the model and horizontal motion: $U\cos\alpha$ (12)</td>
<td>B1</td>
<td>3.4</td>
</tr>
<tr>
<td>Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$</td>
<td>M1</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>$v = 15$</td>
<td>A1</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{12^2 + 15^2}$</td>
<td>M1</td>
<td>3.1b</td>
<td></td>
</tr>
<tr>
<td>$\sqrt{369} = 19 \text{ m s}^{-1}$ (2sf)</td>
<td>A1 ft</td>
<td>1.1b</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Possible improvement (see below in notes)</td>
<td>B1</td>
<td>3.5c</td>
</tr>
<tr>
<td>Possible improvement (see below in notes)</td>
<td>B1</td>
<td>3.5c</td>
<td></td>
</tr>
<tr>
<td>(13 marks)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Question 10 continued

**Notes:**

(a)  
1st M1: for use of $s = ut$ horizontally  
1st A1: for a correct equation  
2nd M1: for use of $s = ut + \frac{1}{2} at^2$ vertically  
2nd A1: for a correct equation  
3rd M1: for correct strategy (need both equations)  
2nd A1: for $U = 15$

(b)  
B1: for $U\cos\alpha$ used as horizontal velocity component  
1st M1: for attempt to find vertical component  
1st A1: for 15  
2nd M1: for correct strategy (need both components)  
2nd A1ft: for 19 m s$^{-1}$ (2sf) following through on incorrect component(s)

(c)  
B1, B1: for any two of  
   - e.g. Include air resistance in the model of the motion  
   - e.g. Use a more accurate value for $g$ in the model of the motion  
   - e.g. Include wind effects in the model of the motion  
   - e.g. Include the dimensions of the stone in the model of the motion
Examiner Comments:

Part (a) Correct equation for horizontal motion but a sign error in the equation for vertical motion leads to loss of two A marks.

Part (b) Only the first M mark only for attempt to find vertical component of velocity. A sign error leads to an incorrect answer and there is no attempt to use the horizontal component to find the speed.

Part (c) No attempt.
Examiner Comments:

Part (a) Correct equation for horizontal motion but a sign error in the equation for vertical motion leads to loss of two A marks.

Part (b) As in the previous response, M mark only for attempt to find vertical component of velocity.

Part (c) Two valid improvements to the model suggested.
Examiner Comments:

Part (a) Fully correct.

Part (b) Method correct but first A mark lost due to arithmetical slip (2 × 7.2 = 14.2) but scores second. The second A mark is scored for a correct follow through answer.

Part (c) Full marks.