Pearson Edexcel
Level 3 Advanced Subsidiary
GCE in Mathematics (8MA0)
Pearson Edexcel
Level 3 Advanced
GCE in Mathematics (9MA0)

Content Exemplification

First teaching from September 2017
First certification from June 2018

Contents

About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Advanced GCE in Mathematics (8MA0 & 9MA0) (first assessment summer 2018).

The booklet provides additional information on all the content from the specification accredited by Ofqual in 2017. It details the content references and provides sample questions for each content topic taken from the Sample Assessment Materials.

Pure Mathematics

1. Proof

Structure and calculation

1.1	**Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including:** **Proof by deduction** **Proof by exhaustion** **Disproof by counterexample.** Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).

Sample questions

AS Mathematics SAMs Paper 1 Q10 (1.1, 2.3, 2.5)

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that $0 \leq k < \frac{3}{4}$.

(Total for Question 10 is 4 marks)

AS Mathematics SAMs Paper 1 Q11 (1.1)

(a) Prove that for all positive values of x and y,

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

(2)

(b) Prove by counterexample that this is not true when x and y are both negative.

(1)

(Total for Question 11 is 3 marks)

AS Mathematics SAMs Paper 1 Q16 (1.1, 7.1, 7.2, 7.3)

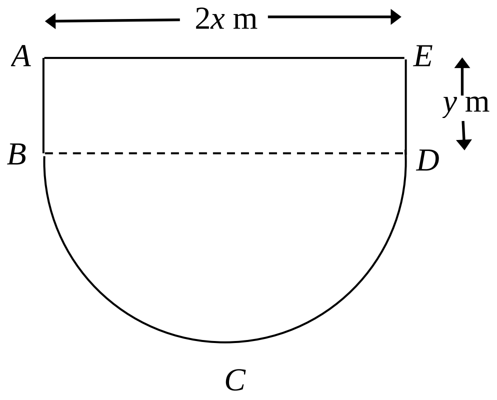

Figure 4

Figure 4 shows the plan view of the design for a swimming pool. The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semi-circular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

(4)

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}.$$

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$.

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

(Total for Question 16 is 10 marks)

A level Mathematics SAMs Paper 1 Q1 (1.1, 7.1, 7.2, 7.3)

The curve C has equation $y = 3x^4 - 8x^3 - 3$.

(a) Find (i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(a) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for Question 1 is 7 marks)

A level Mathematics SAMs Paper 1 Q9 (1.1, 5.4, 5.5, 5.6, 5.8)

(a) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$, $\theta \neq \frac{n\pi}{2}$, $n \in \mathbb{Z}$.

(4)

(b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

A level Mathematics SAMs Paper 1 Q10 (1.1, 5.6, 7.1)

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(Total for Question 10 is 5 marks)

A level Mathematics SAMs Paper 2 Q4 (1.1, 2.8, 6.3)

Given

$$\mathrm{f}(x) = \mathrm{e}^x, \quad x \in \mathbb{R},$$

$$\mathrm{g}(x) = 3\ln x, \quad x > 0, x \in \mathbb{R},$$

(a) find an expression for $\mathrm{gf}(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $\mathrm{gf}(x) = \mathrm{fg}(x)$.

(3)

(Total for Question 4 is 5 marks)

A level Mathematics SAMs Paper 2 Q6 (1.1, 2.3, 2.5)

For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case. The first one has been done for you.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$ When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

A level Mathematics SAMs Paper 2 Q14 (1.1, 2.11, 7.2, 7.3)

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \frac{1000}{r}$.

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

(Total for Question 14 is 9 marks)

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2. Algebra and functions

Structure and calculation

2.1	**Understand and use the laws of indices for all rational exponents.**
2.2	**Use and manipulate surds, including rationalising the denominator.**
2.3	**Work with quadratic functions and their graphs.** **The discriminant of a quadratic function, including the conditions for real and repeated roots.** **Completing the square.** **Solution of quadratic equations including solving quadratic equations in a function of the unknown.**
2.4	**Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.**
2.5	**Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.** **Express solutions through correct use of 'and' and 'or', or through set notation.** **Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically.**
2.6	**Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.** Simplify rational expressions, including by factorising and cancelling, and algebraic division (by linear expressions only).
2.7	**Understand and use graphs of functions; sketch curves defined by simple equations including polynomials.** The modulus of a linear function. **$Y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes)** **Interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations.** **Understand and use proportional relationships and their graphs.**
2.8	Understand and use composite functions; inverse functions and their graphs.
2.9	**Understand the effect of simple transformations on the graph of $y = f(x)$, including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$** and combinations of these transformations.
2.10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).
2.11	Use of functions in modelling, including consideration of limitations and refinements of the models.

Sample questions

AS Mathematics SAMs Paper 1 Q3 (2.1, 2.2, 8.2, 8.3)

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 3 is 4 marks)

AS Mathematics SAMs Paper 1 Q4 (2.3, 2.6)

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$.

(4)

(Total for Question 4 is 6 marks)

AS Mathematics SAMs Paper 1 Q5 (1.1, 2.3, 2.5)

Given that $f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0,$

show that $\int_1^{2\sqrt{2}} f(x)\, dx = 16 + 3\sqrt{2}.$

(Total for Question 5 is 5 marks)

AS Mathematics SAMs Paper 1 Q10 (1.1, 2.3, 2.5)

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that $0 \le k < \frac{3}{4}$.

(Total for Question 10 is 4 marks)

AS Mathematics SAMs Paper 1 Q12 (2.1, 6.5)

A student was asked to give the exact solution to the equation $2^{2x+4} - 9(2^x) = 0$.

The student's attempt is shown below:

$2^{2x+4} - 9(2^x) = 0$

$2^{2x} + 2^4 - 9(2^x) = 0$

Let $2^x = y$

$y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ or $y = 1$

So $x = 3$ or x 0

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(Total for Question 12 is 4 marks)

AS Mathematics SAMs Paper 1 Q13 (2.1, 6.5)

(a) Factorise completely $x^3 + 10x^2 + 25x$.

(2)

(b) Sketch the curve with equation $y = x^3 + 10x^2 + 25x$, showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates (–3, 0) lies on the curve with equation

$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$, where a is a constant.

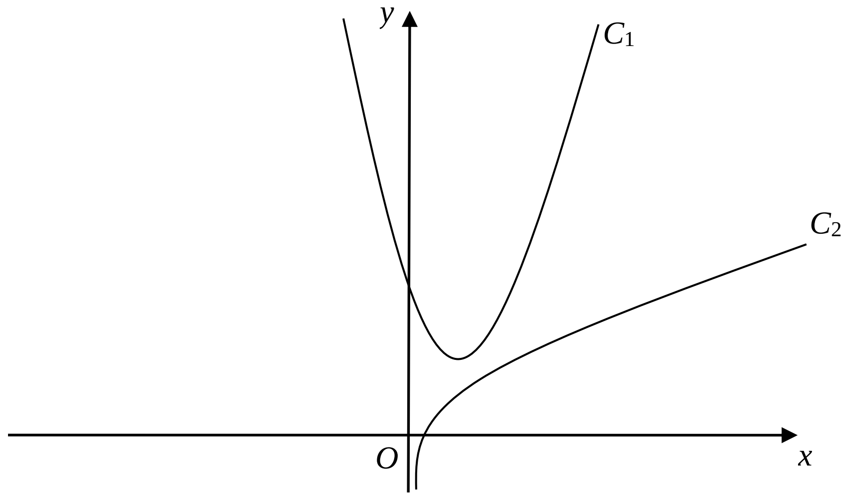
(c) Find the two possible values of a.

(3)

(Total for Question 13 is 7 marks)

AS Mathematics SAMs Paper 1 Q15 (2.4, 6.3, 7.2, 7.3)

Diagram not drawn to scale



The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1.

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$. The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 1 Q3 (2.3, 2.5, 3.2)

A circle C has equation $x^2 + y^2 - 4x + 10y = k$, where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 3 is 4 marks)

A level Mathematics SAMs Paper 1 Q5 (2.6, 3.3)

A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1,$$

where a and b are integers to be found.

(Total for Question 5 is 3 marks)

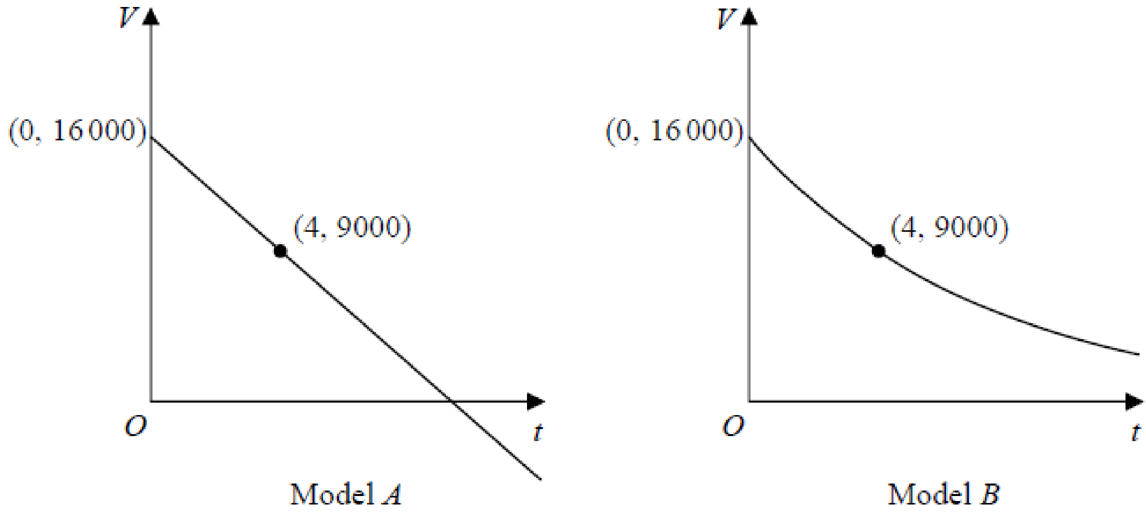
A level Mathematics SAMs Paper 1 Q6 (2.11, 3.1, 6.1, 6.3)

A car dealer wishes to model the value of a certain type of car.

The initial price of the car is £16 000 and the value after 4 years is expected to be £9 000.

In a simple model, the value of the car, £V, depends only on the age of the car, t years.

The diagram below shows the graphs of two possible models over 12 years.



(a) Explain why Model A is an unrealistic model for cars over 10 years of age.

It is given that the equation for Model B is of the form $V = pe^{kt}$, where p and k are constants.

(b) Find the equation for Model B.

Saima wants to know the value of her car when it is 3 years old.

(c) (i) Use Model A to predict the value of Saima's car.

(ii) Use Model B to predict the value of Saima's car.

(2)

(d) Write down one possible refinement of either Model A or Model B.

(5)

(Total for Question 6 is 7 marks)

A level Mathematics SAMs Paper 1 Q8 (2.7, 9.1, 9.2)

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4].

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$.

(2)

(Total for Question 8 is 6 marks)

A level Mathematics SAMs Paper 1 Q11 (2.3, 2.11)

An archer shoots an arrow. The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form $A - B(d - C)^2$, where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer. The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

(Total for Question 11 is 9 marks)

A level Mathematics SAMs Paper 1 Q13 (2.2, 3.1, 3.3, 5.3, 5.6, 7.2, 7.3, 7.5)

The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \leq t \leq \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is the normal to C at P.

(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

(Total for Question 13 is 13 marks)

A level Mathematics SAMs Paper 1 Q15 (2.9, 5.4, 7.2, 7.3, 7.4)

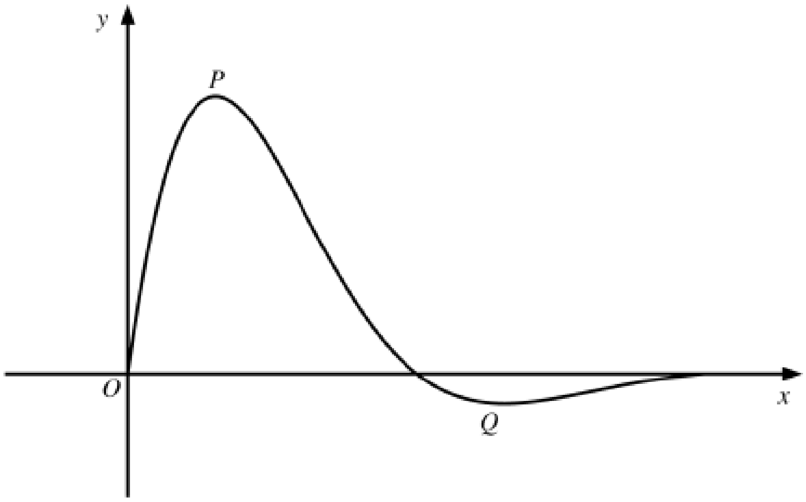


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2x-1}}}, \quad 0 \le x \le \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure 5.

(a) Show that the x-coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{2}$.

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \text{f}(2x)$,

(ii) $y = 3 - 2\text{f}(x)$.

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 2 Q1 (2.6)

$$\text{f}(x) = 2x^3 - 5x^2 + ax + a.$$

Given that $(x + 2)$ is a factor of f (x), find the value of the constant a.

(Total for Question 1 is 3 marks)

A level Mathematics SAMs Paper 2 Q4 (1.1, 2.8, 6.3)

Given

$$f(x) = e^x, \quad x \in \mathbb{R},$$

$$g(x) = 3 \ln x, \qquad x > 0, x \in \mathbb{R},$$

(a) find an expression for gf (x), simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$.

(3)

(Total for Question 4 is 5 marks)

A level Mathematics SAMs Paper 2 Q6 (1.1, 2.3, 2.5)

For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case. The first one has been done for you.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$ When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

A level Mathematics SAMs Paper 2 Q9 (2.3, 8.2, 8.3)

Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A)\ \mathrm{d}x = 2A^2,$$

show that there are exactly two possible values for A.

(Total for question 9 is 5 marks)

A level Mathematics SAMs Paper 2 Q11 (2.5, 2.7, 2.9)

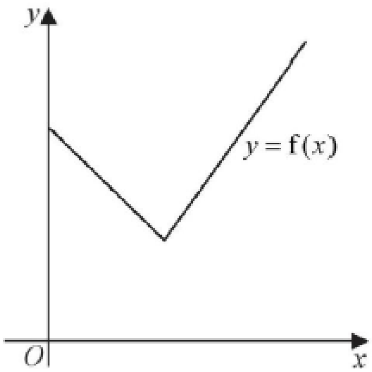


Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geq 0$.

(a) State the range of f.

(1)

(b) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 30$.

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

(Total for Question 11 is 6 marks)

A level Mathematics SAMs Paper 2 Q12 (2.3, 5.3, 5.5, 5.7)

(a) Solve, for $-180° \leq x < 180°$, the equation $3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$, giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3 \sin^2 (2\theta - 30°) + \sin (2\theta - 30°) + 8 = 9 \cos^2 (2\theta - 30°),$$

giving your answer to 2 decimal places.

(2)

(Total for Question 12 is 8 marks)

A level Mathematics SAMs Paper 2 Q13 (5.3, 5.6, 5.9)

(a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

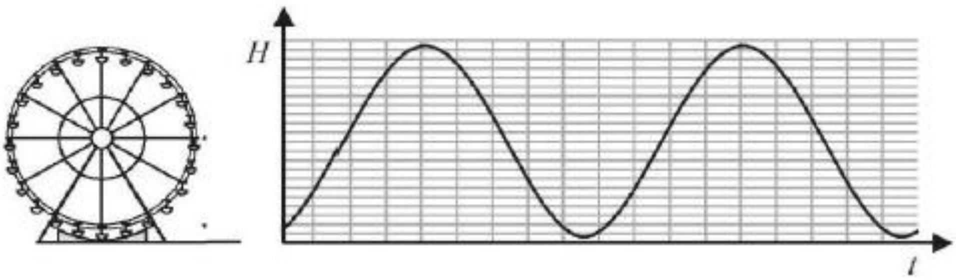


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation $H = a - 10 \cos (80t)° + 3 \sin (80t)°$, where a is a constant. Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model.

(ii) Hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total for Question 13 is 9 marks)

A level Mathematics SAMs Paper 2 Q14 (1.1, 2.11, 7.2, 7.3)

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \dfrac{1000}{r}$.

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

(Total for Question 14 is 9 marks)

A level Mathematics SAMs Paper 2 Q16 (2.1, 6.3, 6.4, 8.6, 8.7, 8.8)

(a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}},$$

where A, B and C are integers to be found.

(3)

(Total for Question 16 is 12 marks)

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3. Coordinate geometry in the (x, y) plane

Structure and calculation

3.1	**Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$;** **Gradient conditions for two straight lines to be parallel or perpendicular.** **Be able to use straight line models in a variety of contexts.**
3.2	**Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$** **Completing the square to find the centre and radius of a circle; use of the following properties:** **● the angle in a semicircle is a right angle** **● the perpendicular from the centre to a chord bisects the chord** **● the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.**
3.3	Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.
3.4	Use parametric equations in modelling in a variety of contexts.

Sample questions

AS Mathematics SAMs Paper 1 Q1 (3.1)

The line l passes through the points A (3, 1) and B (4, – 2).

Find an equation for l.

(Total for Question 1 is 3 marks)

AS Mathematics SAMs Paper 1 Q14 (3.1, 6.4, 6.6, 6.7)

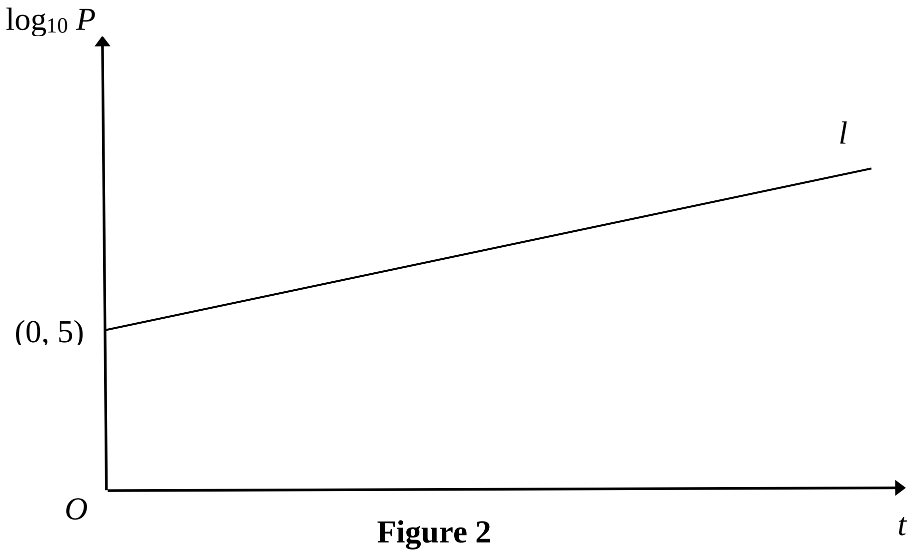


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded. The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model, interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

(Total for Question 14 is 13 marks)

AS Mathematics SAMs Paper 1 Q17 (3.2)

A circle C with centre at (–2, 6) passes through the point (10, 11).

(a) Show that the circle C also passes through the point (10, 1).

(3)

The tangent to the circle C at the point (10, 11) meets the y-axis at the point P and the tangent to the circle C at the point (10, 1) meets the y-axis at the point Q.

(b) Show that the distance PQ is 58, explaining your method clearly.

(7)

(Total for Question 17 is 10 marks)

A level Mathematics SAMs Paper 1 Q3 (2.3, 2.5, 3.2)

A circle C has equation $x^2 + y^2 - 4x + 10y = k$, where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 3 is 4 marks)

A level Mathematics SAMs Paper 1 Q5 (2.6, 3.3)

A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1,$$

where a and b are integers to be found.

(Total for Question 5 is 3 marks)

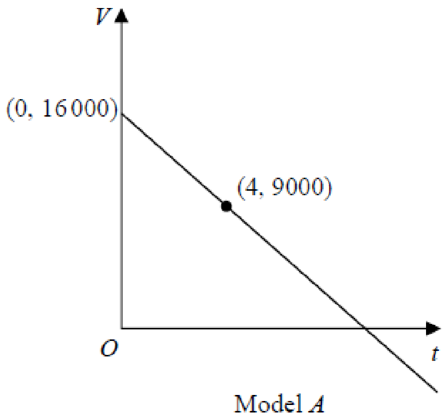
A level Mathematics SAMs Paper 1 Q6 (2.11, 3.1, 6.1, 6.3)

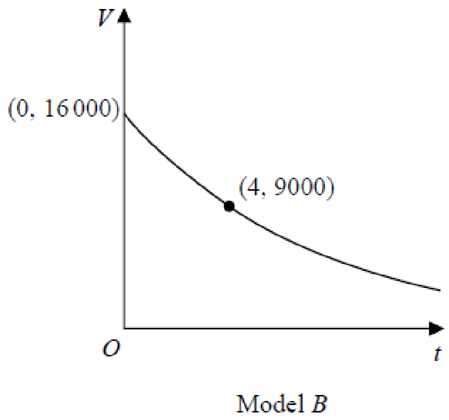
A car dealer wishes to model the value of a certain type of car.

The initial price of the car is £16 000 and the value after 4 years is expected to be £9 000.

In a simple model, the value of the car, £V, depends only on the age of the car, t years.

The diagram below shows the graphs of two possible models over 12 years.





(a) Explain why Model A is an unrealistic model for cars over 10 years of age.

It is given that the equation for Model B is of the form $V = pe^{kt}$, where p and k are constants.

(b) Find the equation for Model B.

Saima wants to know the value of her car when it is 3 years old.

(c) (i) Use Model A to predict the value of Saima's car.

(ii) Use Model B to predict the value of Saima's car.

(2)

(d) Write down one possible refinement of either Model A or Model B.

(5)

(Total for Question 6 is 7 marks)

A level Mathematics SAMs Paper 1 Q13 (2.2, 3.1, 3.3, 5.3, 5.6, 7.2, 7.3, 7.5)

The curve C has parametric equations

$$x = 2\cos t,\ \ y = \sqrt{3}\cos 2t, \qquad 0 \le t \le \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is the normal to C at P.

(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

(Total for Question 13 is 13 marks)

A level Mathematics SAMs Paper 2 Q8 (3.1)

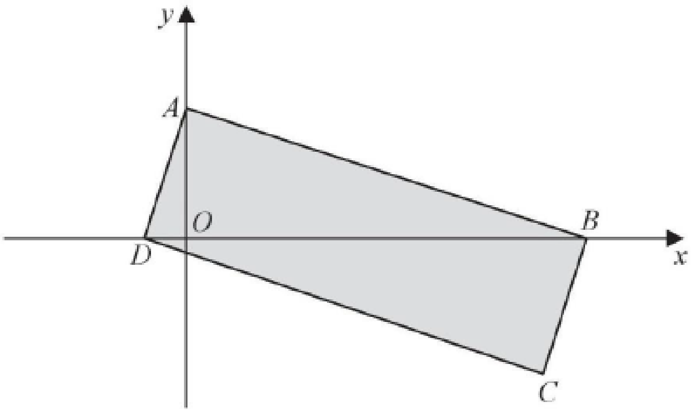


Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$,

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$,

(4)

(b) find the area of the rectangle $ABCD$.

(3)

(Total for Question 8 is 7 marks)

4. Sequences and series

Structure and calculation

4.1	**Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations n! and ${}^nC_r$ link to binomial probabilities.** Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\| < 1$ (proof not required)
4.2	Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$; increasing sequences; decreasing sequences; periodic sequences.
4.3	Understand and use sigma notation for sums of series.
4.4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms.
4.4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms.
4.5	Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation
4.6	Use sequences and series in modelling.

Sample questions

AS Mathematics SAMs Paper 1 Q7 (4.1)

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7.

(1)

(Total for Question 7 is 5 marks)

A level Mathematics SAMs Paper 2 Q7 (4.1)

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

(Total for Question 7 is 5 marks)

A level Mathematics SAMs Paper 2 Q10 (4.5)

In a geometric series the common ratio is r and sum to n terms is S_n.

Given $S_\infty = \frac{8}{7} \times S_6$, show that $1 = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(Total for Question 10 is 4 marks)

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5. Trigonometry

Structure and calculation

5.1	**Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$** Work with radian measure, including use for arc length and area of sector.
5.2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta$, $\cos\theta = 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ Where θ is in radians.
5.3	**Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.** Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof.
5.4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.
5.5	**Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$** **Understand and use $\sin^2\theta + \cos^2\theta = 1$**, $\sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$
5.6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$, understand geometrical proofs of these formulae. Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
5.7	**Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.**
5.8	Construct proofs involving trigonometric functions and identities.
5.9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.

Sample questions

AS Mathematics SAMs Paper 1 Q8 (5.1)

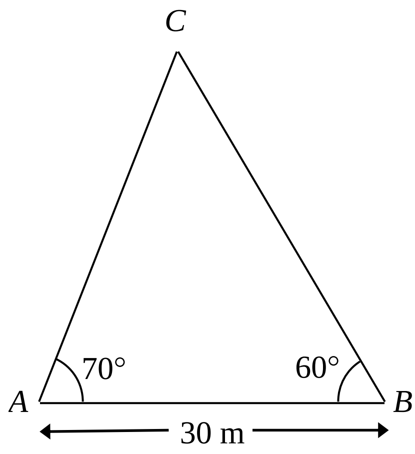


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 70° and angle *ABC* = 60°,

(a) calculate the area of the lawn to 3 significant figures.

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(Total for Question 8 is 5 marks)

AS Mathematics SAMs Paper 1 Q9 (5.3, 5.4)

Solve, for $360° \leq x < 540°$,

$$12 \sin^2 x + 7 \cos x - 13 = 0.$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 9 is 5 marks)

A level Mathematics SAMs Paper 1 Q2 (5.1)

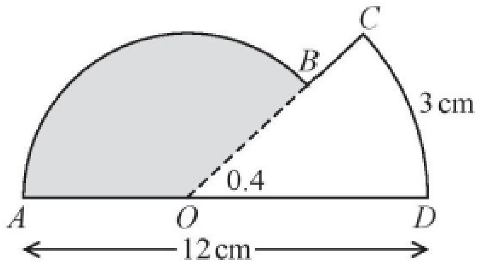


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm, find

(a) the length of *OD*,

(3)

(b) the area of the shaded sector *AOB*.

(2)

(Total for Question 2 is 5 marks)

A level Mathematics SAMs Paper 1 Q7 (5.1, 10.1, 10.2, 10.3)

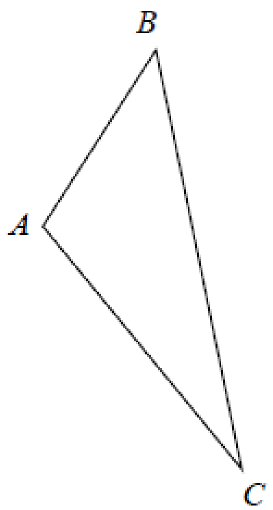


Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$, show that $\angle BAC = 105.9°$ to one decimal place.

(Total for Question 7 is 5 marks)

A level Mathematics SAMs Paper 1 Q9 (1.1, 5.4, 5.5, 5.6, 5.8)

(a) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta,\ \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$.

(4)

(b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

A level Mathematics SAMs Paper 1 Q10 (1.1, 5.6, 7.1)

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(Total for Question 10 is 5 marks)

A level Mathematics SAMs Paper 1 Q13 (2.2, 3.1, 3.3, 5.3, 5.6, 7.2, 7.3, 7.5)

The curve C has parametric equations

$$x = 2\cos t,\ y = \sqrt{3}\cos 2t, \quad 0 \leq t \leq \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is the normal to C at P.

(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

(Total for Question 13 is 13 marks)

A level Mathematics SAMs Paper 1 Q15 (2.9, 5.4, 7.2, 7.3, 7.4)

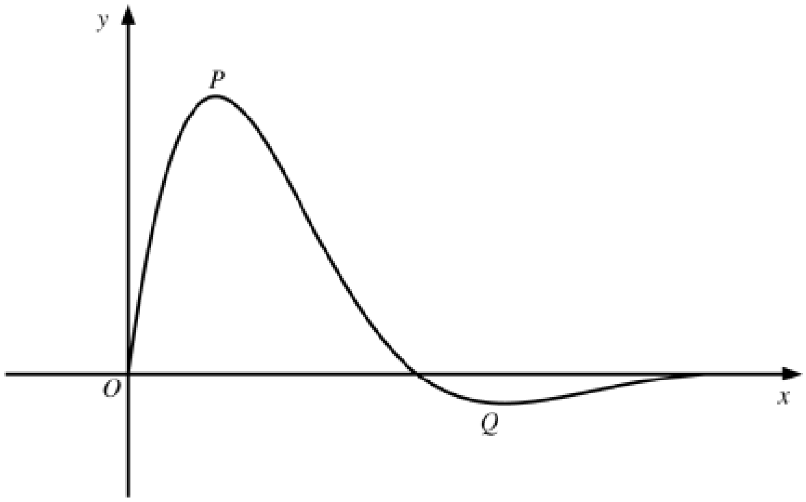


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \le x \le \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure 5.

(a) Show that the x-coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{2}$.

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$,

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 2 Q2 (5.5, 5.7)

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation $\cos\theta = 2\sin\theta$.

The attempts of two of the students are shown below.

Student A	Student B
$\cos\theta = 2\sin\theta$	$\cos\theta = 2\sin\theta$
$\tan\theta = 2$	$\cos^2\theta = 4\sin^2\theta$
$\theta = 63.4°$	$1 - \sin^2\theta = 4\sin^2\theta$
	$\sin^2\theta = \frac{1}{5}$
	$\sin\theta = \pm\frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(a) Identify an error made by student A.

(1)

Student B gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for Question 2 is 3 marks)

A level Mathematics SAMs Paper 2 Q12 (2.3, 5.3, 5.5, 5.7)

(a) Solve, for $-180° \le x < 180°$, the equation $3\sin^2 x + \sin x + 8 = 9\cos^2 x$, giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°),$$

giving your answer to 2 decimal places.

(2)

(Total for Question 12 is 8 marks)

A level Mathematics SAMs Paper 2 Q13 (5.3, 5.6, 5.9)

(a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

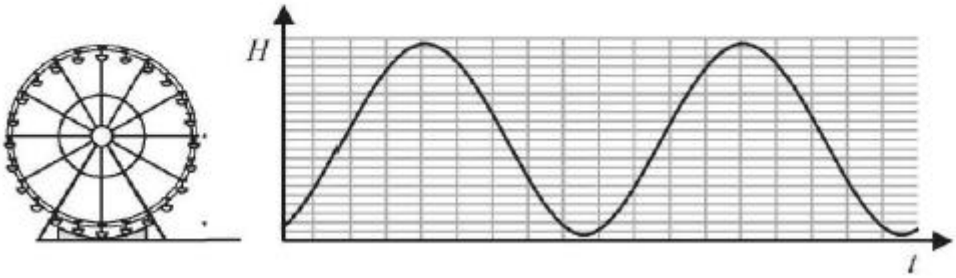


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation $H = a - 10 \cos (80t)° + 3 \sin (80t)°$, where a is a constant. Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model.

(ii) Hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total for Question 13 is 9 marks)

6. Exponentials and logarithms

Structure and calculation

6.1	**Know and use the function a^x and its graph, where a is positive.** **Know and use the function e^x and its graph.**
6.2	**Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.**
6.3	**Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$.** **Know and use the function ln x and its graph.** **Know and use ln x as the inverse function of e^x**
6.4	**Understand and use the laws of logarithms:** $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \frac{x}{y}$ $k \leq \log_a x = \log_a x^k$ **(including, for example, $k = -1$ and $k = -\frac{1}{2}$)**
6.5	**Solve equations of the form $a^x = b$**
6.6	**Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y**
6.7	**Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.**

Sample questions

AS Mathematics SAMs Paper 1 Q12 (2.1, 6.5)

A student was asked to give the exact solution to the equation $2^{2x+4} - 9(2^x) = 0$.

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let } 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x\ 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(Total for Question 12 is 4 marks)

AS Mathematics SAMs Paper 1 Q13 (2.1, 6.5)

(a) Factorise completely $x^3 + 10x^2 + 25x$.

(2)

(b) Sketch the curve with equation $y = x^3 + 10x^2 + 25x$, showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates (–3, 0) lies on the curve with equation

$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$, where a is a constant.

(c) Find the two possible values of a.

(3)

(Total for Question 13 is 7 marks)

AS Mathematics SAMs Paper 1 Q14 (3.1, 6.4, 6.6, 6.7)

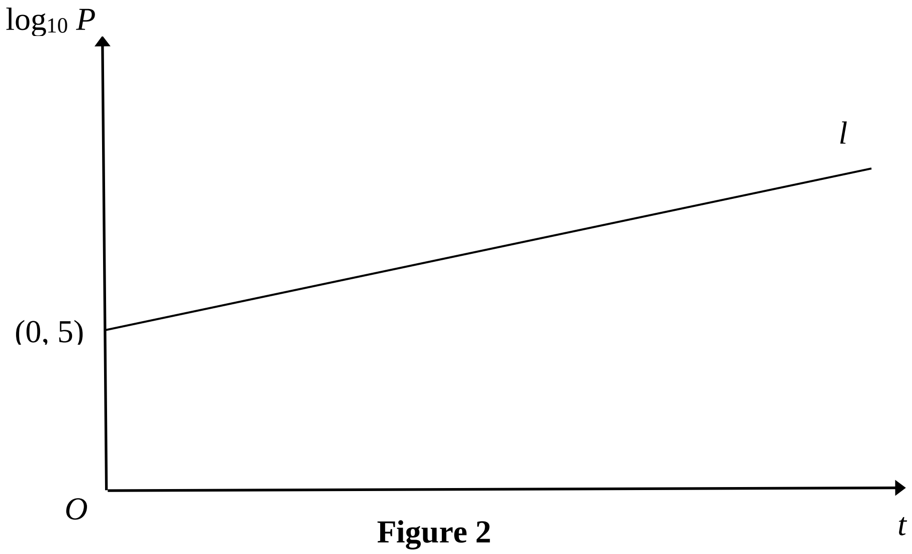


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded. The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model, interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

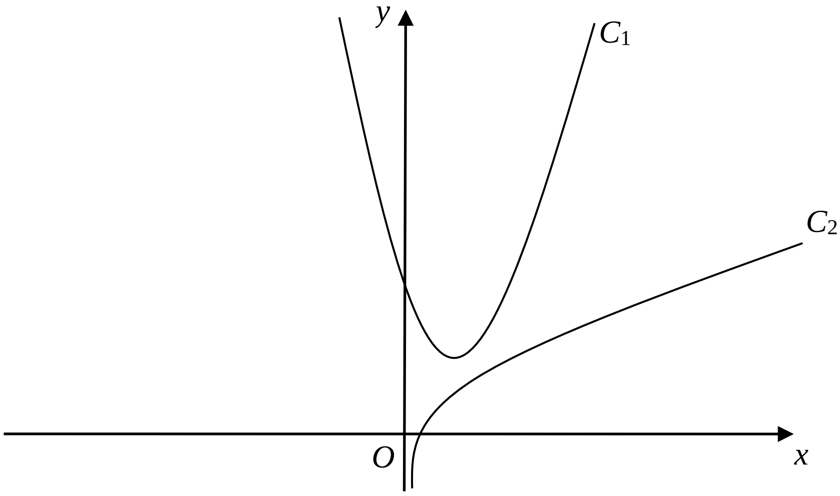
(e) State two reasons why this may not be a realistic population model.

(2)

(Total for Question 14 is 13 marks)

AS Mathematics SAMs Paper 1 Q15 (2.4, 6.3, 7.2, 7.3)

Diagram not
drawn to scale



The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1.

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$. The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 1 Q4 (6.4, 8.2, 8.3)

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t} \, dt = \ln 7,$$

show that $a = \ln k$, where k is a constant to be found.

(Total for Question 4 is 4 marks)

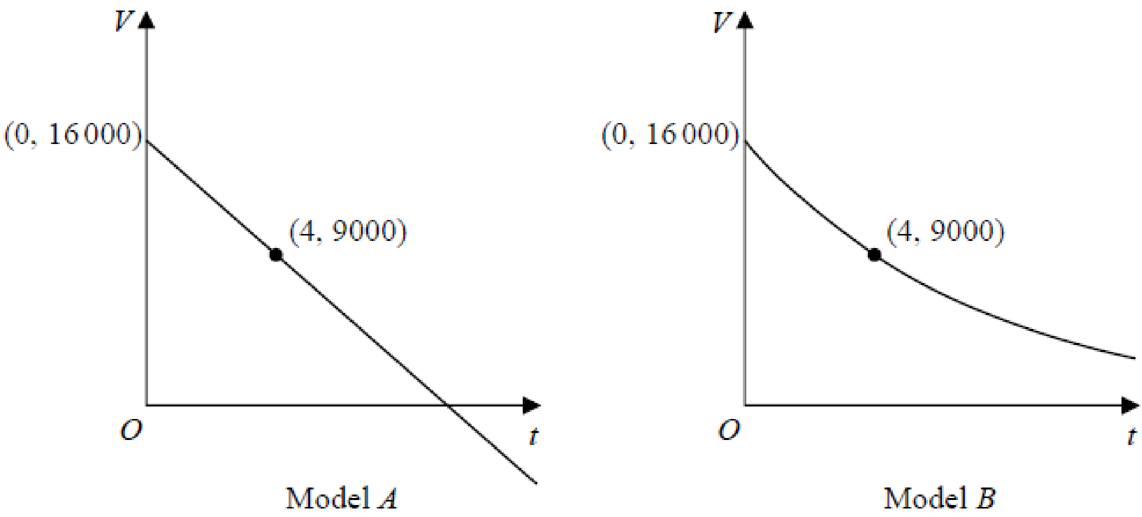
A level Mathematics SAMs Paper 1 Q6 (2.11, 3.1 6.1, 6.3)

A car dealer wishes to model the value of a certain type of car.

The initial price of the car is £16 000 and the value after 4 years is expected to be £9 000.

In a simple model, the value of the car, £V, depends only on the age of the car, t years.

The diagram below shows the graphs of two possible models over 12 years.



(a) Explain why Model A is an unrealistic model for cars over 10 years of age.

It is given that the equation for Model B is of the form $V = pe^{kt}$, where p and k are constants.

(b) Find the equation for Model B.

Saima wants to know the value of her car when it is 3 years old.

(c) (i) Use Model A to predict the value of Saima's car.

(ii) Use Model B to predict the value of Saima's car.

(2)

(d) Write down one possible refinement of either Model A or Model B.

(5)

(Total for Question 6 is 7 marks)

A level Mathematics SAMs Paper 1 Q12 (6.1, 6.3, 6.4, 6.6, 6.7)

In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment, were counted.

N and T are expected to satisfy a relationship of the form $N = aT^b$, where a and b are constants.

(a) Show that this relationship can be expressed in the form $\log_{10} N = m \log_{10} T + c$, giving m and c in terms of the constants a and/or b.

(2)

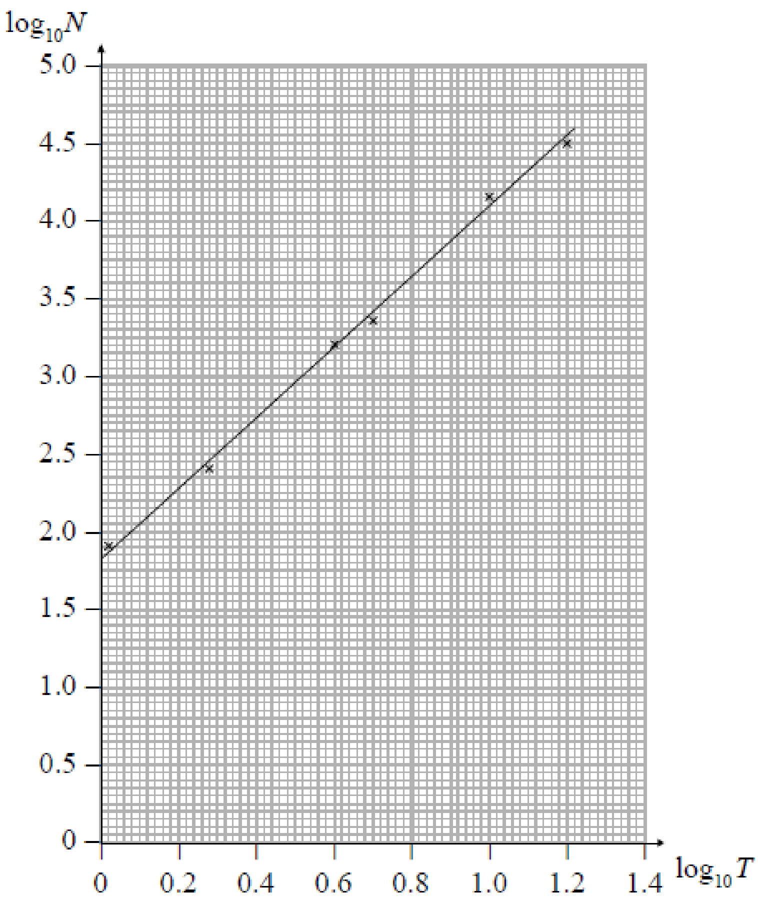


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

(Total for Question 12 is 9 marks)

A level Mathematics SAMs Paper 1 Q14 (6.4, 8.3, 8.5, 9.3)

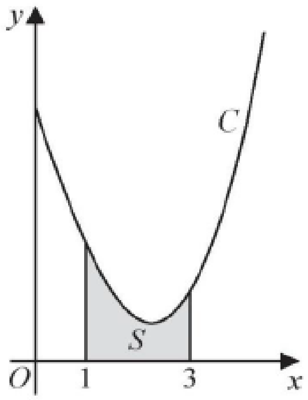


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 14 is 10 marks)

A level Mathematics SAMs Paper 2 Q4 (1.1, 2.8, 6.3)

Given

$$f(x) = e^x, \quad x \in \mathbb{R},$$

$$g(x) = 3 \ln x, \qquad x > 0, x \in \mathbb{R},$$

(a) find an expression for gf (x), simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$.

(3)

(Total for Question 4 is 5 marks)

A level Mathematics SAMs Paper 2 Q5 (6.2, 6.7)

The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}.$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for Question 5 is 4 marks)

A level Mathematics SAMs Paper 2 Q16 (2.1, 6.3, 6.4, 8.6, 8.7, 8.8)

(a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}},$$

where A, B and C are integers to be found.

(3)

(Total for Question 16 is 12 marks)

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7. Differentiation

Structure and calculation

7.1	**Understand and use the derivative of f (x) as the gradient of the tangent to the graph of y = f (x) at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change sketching the gradient function for a given curve second derivatives differentiation from first principles for small positive integer powers of x** and for sin x and cos x
	Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.
7.2	**Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.**
7.3	**Apply differentiation to find gradients, tangents and normal maxima and minima and stationary points.**
	Points on inflection.
	Identify where functions are increasing or decreasing.
7.4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.
7.5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.
7.6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand).

Sample questions

AS Mathematics SAMs Paper 1 Q2 (7.1, 7.2, 7.3)

The curve C has equation

$$y = 2x^2 – 12 + 16.$$

Find the gradient of the curve at the point P(5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 2 is 4 marks)

AS Mathematics SAMs Paper 1 Q6 (7.1)

Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(Total for Question 6 is 4 marks)

AS Mathematics SAMs Paper 1 Q16 (1.1, 7.1, 7.2, 7.3)

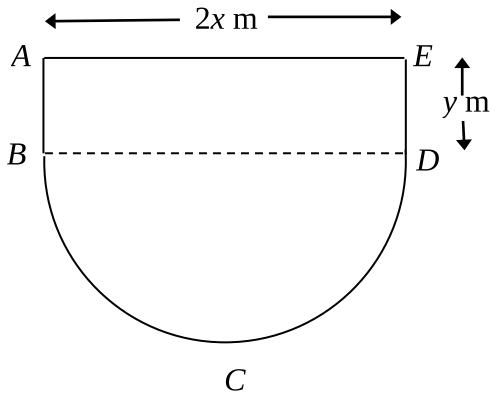


Figure 4

Figure 4 shows the plan view of the design for a swimming pool. The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semi-circular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

(4)

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}.$$

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$.

(2)

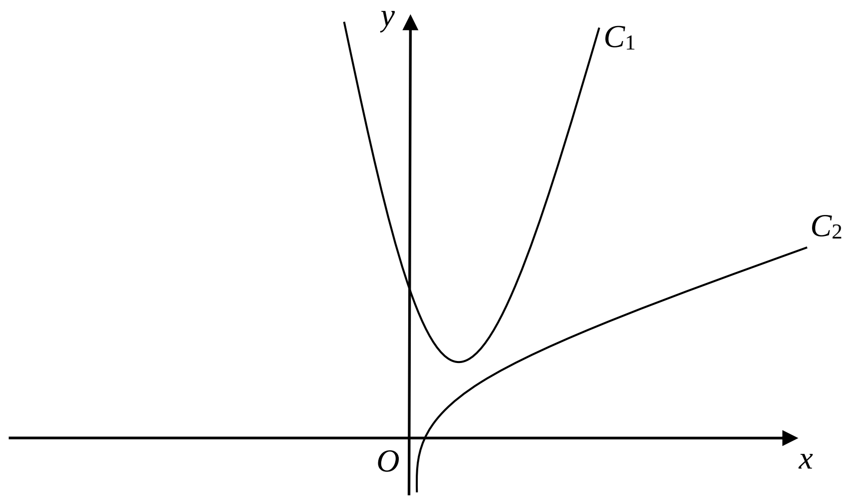
(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

(Total for Question 16 is 10 marks)

AS Mathematics SAMs Paper 1 Q15 (2.4, 6.3, 7.2, 7.3)

Diagram not drawn to scale



The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1.

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$. The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 1 Q1 (1.1, 7.1, 7.2, 7.3)

The curve C has equation $y = 3x^4 - 8x^3 - 3$.

(a) Find (i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(b) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for Question 1 is 7 marks)

A level Mathematics SAMs Paper 1 Q10 (1.1, 5.6, 7.1)

Given that θ is measured in radians, prove, from first principles, that the derivative of sin θ is cos θ

You may assume the formula for sin $(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(Total for Question 10 is 5 marks)

A level Mathematics SAMs Paper 1 Q13 (2.2, 3.1, 3.3, 5.3, 5.6, 7.2, 7.3, 7.5)

The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \leq t \leq \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$.

The line l is the normal to C at P.

(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

(Total for Question 13 is 13 marks)

A level Mathematics SAMs Paper 1 Q15 (2.9, 5.4, 7.2, 7.3, 7.4)

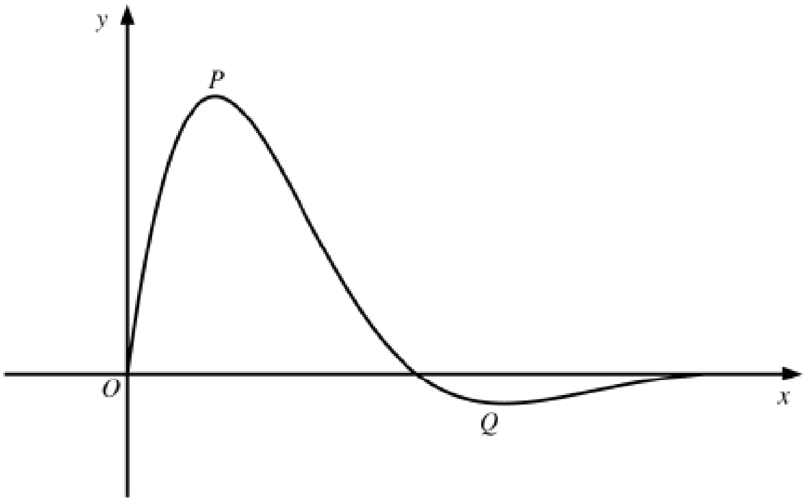


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leq x \leq \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure 5.

(a) Show that the x-coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{2}$.

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \text{f}(2x)$,

(ii) $y = 3 - 2\text{f}(x)$.

(4)

(Total for Question 15 is 8 marks)

A level Mathematics SAMs Paper 2 Q3 (7.2, 7.4)

Given $y = x(2x + 1)^4$, show that $\frac{dy}{dx} = (2x + 1)^n (Ax + B)$, where n, A and B are constants to be found.

(Total for Question 3 is 4 marks)

A level Mathematics SAMs Paper 2 Q14 (1.1, 2.11, 7.2, 7.3)

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \dfrac{1000}{r}$.

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

(Total for Questions 14 is 9 marks)

A level Mathematics SAMs Paper 2 Q15 (3.1, 7.2, 7.3, 8.2, 8.3)

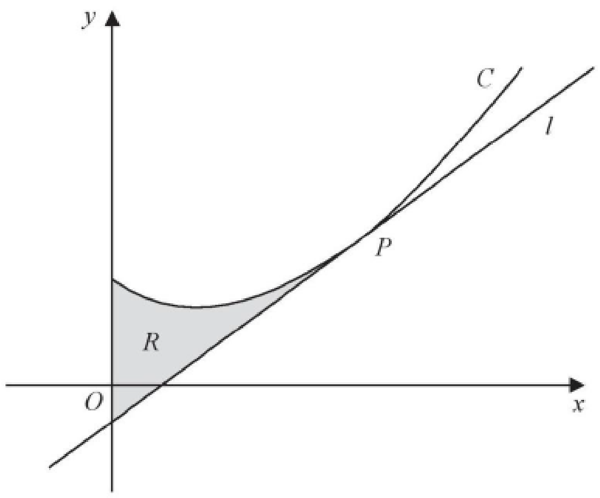


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = 5x^{\frac{3}{2}} - 9x + 11, \quad x \geq 0$.

The point P with coordinates (4, 15) lies on C. The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 15 is 10 marks)

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8. Integration

Structure and calculation

8.1	**Know and use the Fundamental Theorem of Calculus**
8.2	**Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.** Integrate e^{kx}, $\frac{1}{x}$, sin kx ,cos kx and related sums, differences and constant multiples.
8.3	**Evaluate definite integrals; use a definite integral to find the area under a curve** and the area between two curves
8.4	Understand and use integration as the limit of a sum.
8.5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively. (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.)
8.6	Integrate using partial fractions that are linear in the denominator.
8.7	Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor.)
8.8	Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

Sample questions

AS Mathematics SAMs Paper 1 Q3 (2.1, 2.2, 8.2, 8.3)

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 3 is 4 marks)

AS Mathematics SAMs Paper 1 Q5 (2.1, 2.2, 8.2, 8.3)

Given that $f(x) = 2x + 3 + \frac{12}{x^2}$, $x > 0$,

show that $\int_1^{2\sqrt{2}} f(x) \, dx = 16 + 3\sqrt{2}$.

(Total for Question 5 is 5 marks)

A level Mathematics SAMs Paper 1 Q4 (6.4, 8.2, 8.3)

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\, \mathrm{d}t = \ln 7,$$

show that $a = \ln k$, where k is a constant to be found.

(Total for Question 4 is 4 marks)

A level Mathematics SAMs Paper 1 Q14 (6.4, 8.3, 8.5, 9.3)

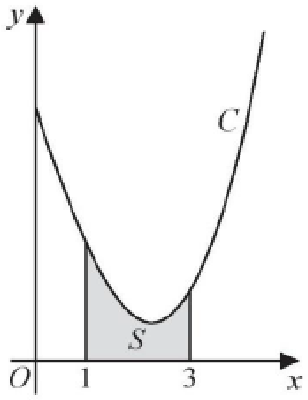

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b}$ + ln c, where a, b and c are integers to be found.

(6)

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 14 is 10 marks)

A **level Mathematics SAMs Paper 2 Q9 (2.3, 8.2, 8.3)**

Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A)\ \mathrm{d}x = 2A^2,$$

show that there are exactly two possible values for A.

(Total for Question 9 is 5 marks)

A level Mathematics SAMs Paper 2 Q15 (3.1, 7.2, 7.3, 8.2, 8.3)

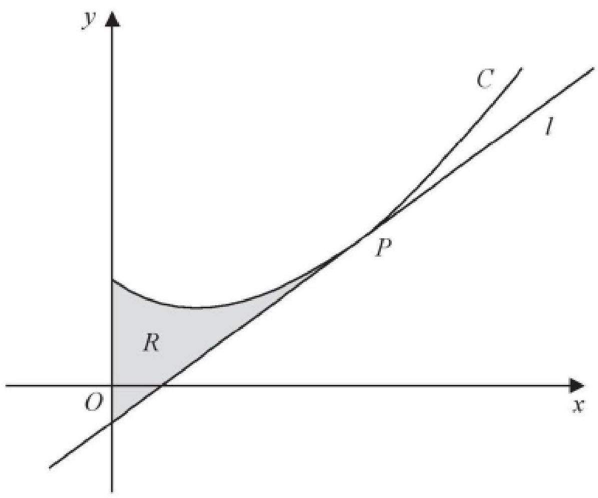

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Figure 4 shows a sketch of the curve C with equation $y = 5x^{\frac{3}{2}} - 9x + 11,\quad x \geq 0.$

The point P with coordinates (4, 15) lies on C. The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 15 is 10 marks)

A level Mathematics SAMs Paper 2 Q16 (2.1, 6.3, 6.4, 8.6, 8.7, 8.8)

(a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\mathrm{e}^{-\frac{1}{2}t}},$$

where A, B and C are integers to be found.

(3)

(Total for Question 16 is 12 marks)

9. Numerical methods

Structure and calculation

9.1	Locate roots of f $(x) = 0$ by considering changes of sign of f (x) in an interval of x on which f (x) is sufficiently well behaved. Understand how change of sign methods can fail.
9.2	Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams.
9.3	Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$ Understand how such methods can fail.
9.4	Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.
9.5	Use numerical methods to solve problems in context.

Sample questions

A level Mathematics SAMs Paper 1 Q8 (2.7, 9.1, 9.2)

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4].

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

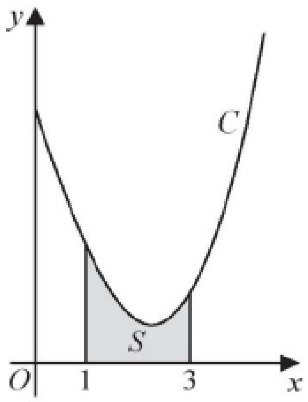
(2)

(c) Show that α is the only root of $f(x) = 0$.

(2)

(Total for Question 8 is 6 marks)

A level Mathematics SAMs Paper 1 Q14 (6.4, 8.3, 8.5, 9.3)



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$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$.

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 14 is 10 marks)

10. Vectors

Structure and calculation

10.1	**Use vectors in two dimensions** and in three dimensions
10.2	**Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.**
10.3	**Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.**
10.4	**Understand and use position vectors; calculate the distance between two points represented by position vectors.**
10.5	**Use vectors to solve problems in pure mathematics and in context, (including forces).**

Sample questions

AS Mathematics SAMs Paper 1 Q3 (2.1, 2.2, 8.2, 8.3)

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 3 is 4 marks)

A level Mathematics SAMs Paper 1 Q7 (5.1, 10.1, 10.2, 10.3)

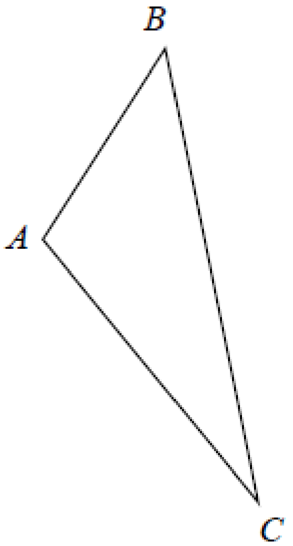


Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$, show that $\angle BAC = 105.9°$ to one decimal place.

(Total for Question 7 is 5 marks)

Applied Mathematics

1. Statistical sampling

Structure and calculation

1.1	**Understand and use the terms 'population' and 'sample'.** **Use samples to make informal inferences about the population.** **Understand and use sampling techniques, including simple random sampling and opportunity sampling.** **Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.**

Sample questions

AS Mathematics SAMs Paper 2 Q1 (1.1)

Sara is investigating the variation in daily maximum gust, t kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(1)

(b) From your knowledge of the large data set, explain why this process may not generate a sample of size 20.

(1)

The data Sara collected are summarised as follows

$$n = 20 \qquad \sum t = 374 \qquad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

(2)

(Total for Question 1 is 4 marks)

AS Mathematics SAMs Paper 2 Q4 (1.1, 2.2, 2.4)

Sara was studying the relationship between rainfall, r mm, and humidity, *h* %, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(1)

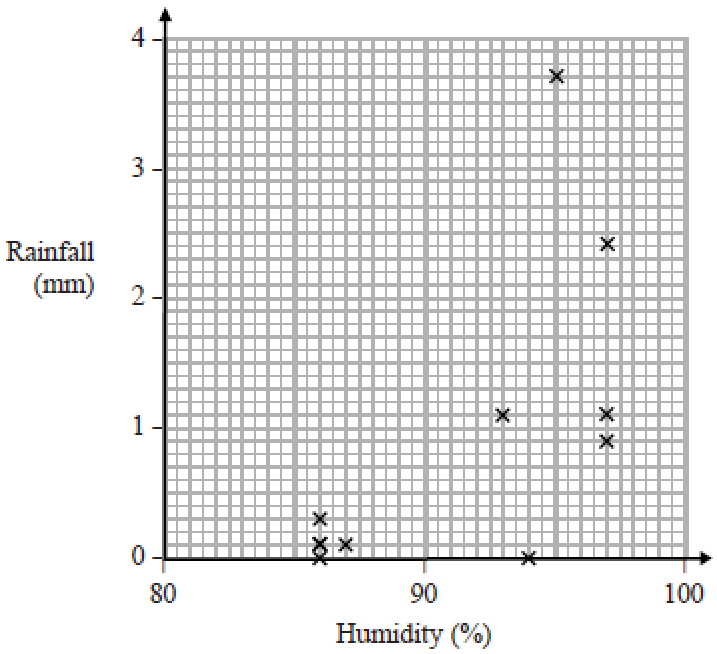
(b) Give a reason why Sara might (i) include

(ii) exclude

this day's reading.

(2)

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of *r* and *h*.



(c) Give an interpretation of the correlation between rainfall and humidity.

(1)

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$.

(d) Give an interpretation of the gradient of this regression line.

(1)

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(2)

(Total for Question 4 is 7 marks)

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2. Data presentation and interpretation

Structure and calculation

2.1	**Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency.** **Connect to probability distributions.**
2.2	**Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded).** **Understand informal interpretation of correlation.** **Understand that correlation does not imply causation.**
2.3	**Interpret measures of central tendency and variation, extending to standard deviation.** **Be able to calculate standard deviation, including from summary statistics.**
2.4	**Recognise and interpret possible outliers in data sets and statistical diagrams.** **Select or critique data presentation techniques in the context of a statistical problem.** **Be able to clean data, including dealing with missing data, errors and outliers.**

Sample questions

AS Mathematics SAMs Paper 2 Q1 (1.1)

Sara is investigating the variation in daily maximum gust, t kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(b) From your knowledge of the large data set, explain why this process may not generate a sample of size 20.

The data Sara collected are summarised as follows

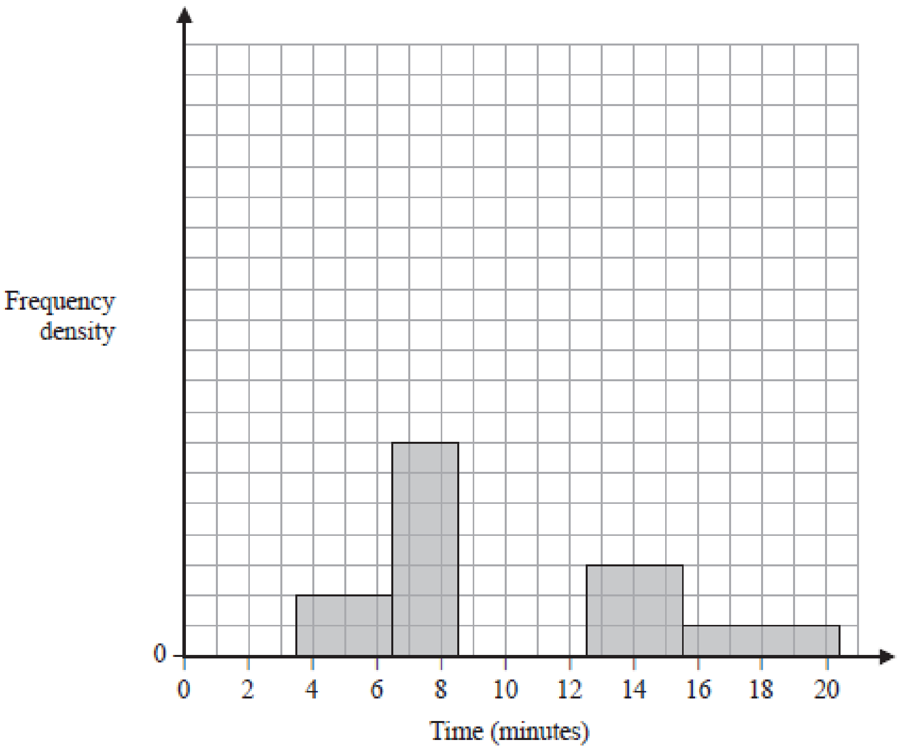
$$n = 20 \qquad \sum t = 374 \qquad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

(Total for Question 1 is 4 marks)

AS Mathematics SAMs Paper 2 Q2 (2.1)

The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists were delayed by roadworks on a stretch of motorway.



Delay (minutes)	Number of motorists
4 – 6	6
7 –8	
9	17
10 – 12	45
13 – 15	9
16 – 20	

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

(Total for Question 2 is 5 marks)

AS Mathematics SAMs Paper 2 Q4 (1.1, 2.2, 2.4)

Sara was studying the relationship between rainfall, r mm, and humidity, h %, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

h	93	86	95	97	86	94	97	97	87	97	86
r	1.1	0.3	3.7	20.6	0	0	2.4	1.1	0.1	0.9	0.1

Sara examined the rainfall figures and found

$$Q_1 = 0.1 \qquad Q_2 = 0.9 \qquad Q_3 = 2.4$$

A value that is more than 1.5 times the interquartile range (IQR) above Q_3 is called an outlier.

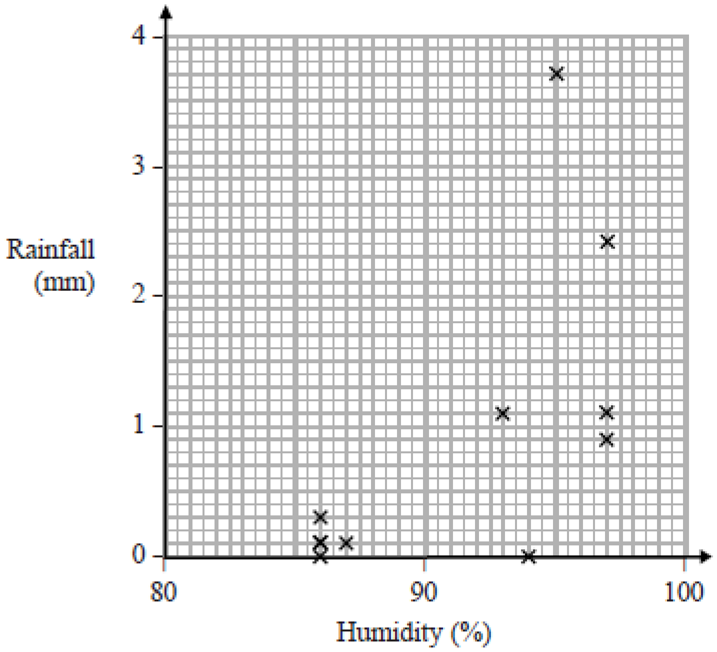
(a) Show that $r = 20.6$ is an outlier.

(b) Give a reason why Sara might (i) include

(ii) exclude

this day's reading.

Sara decided to exclude this day's reading and drew the following scatter diagram for the remaining 10 days' values of r and h.



(c) Give an interpretation of the correlation between rainfall and humidity.

The equation of the regression line of r on h for these 10 days is $r = -12.8 + 0.15h$.

(d) Give an interpretation of the gradient of this regression line.

(e) (i) Comment on the suitability of Sara's sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

(Total for Question 4 is 7 marks)

A level Mathematics SAMs Paper 3 Q1 (2.1, 2.3, 2.4, 4.2, 4.3)

The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \le y < 5$	$5 \le y < 8$	$8 \le y < 11$	$11 \le y < 12$	$12 \le y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \le y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \le y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

(Total for Question 1 is 13 marks)

A level Mathematics SAMs Paper 3 Q2 (2.2, 5.1)

A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$.

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C.

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days, a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$.

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

(Total for Question 2 is 6 marks)

3. Probability

Structure and calculation

3.1	**Understand and use mutually exclusive and independent events when calculating probabilities.** **Link to discrete and continuous distributions.**
3.2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables. Understand and use the conditional probability formula $\mathrm{P}(A/B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
3.3	Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.

Sample questions

AS Mathematics SAMs Paper 2 Q3 (3.1)

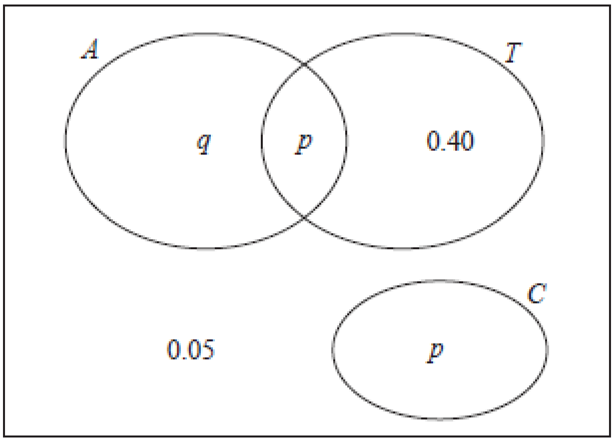
The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.

T represents the event that a student takes part in Tennis.

C represents the event that a student takes part in Cricket.

p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75.

(a) Find the value of p.

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(Total for Question 3 is 5 marks)

A level Mathematics SAMs Paper 3 Q4 (3.1, 3.2)

Given that

$$P(A) = 0.35, \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$.

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

(Total for Question 4 is 10 marks)

4. Statistical distributions

Structure and calculation

4.1	**Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.**
4.2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution. Link to histograms, mean, standard deviation, points of inflection and the binomial distribution.
4.3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate.

Sample questions

AS Mathematics SAMs Paper 2 Q5 (3.1)

The discrete random variable $X \sim B(40, 0.27)$.

(a) Find $P(X \geq 16)$.

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(d) Find the actual significance level of a test based on your critical region from part (c).

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(Total for Question 5 is 9 marks)

A level Mathematics SAMs Paper 3 Q1 (2.1, 2.3, 2.4, 4.2, 4.3)

The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leq y < 5$	$5 \leq y < 8$	$8 \leq y < 11$	$11 \leq y < 12$	$12 \leq y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leq y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leq y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow. Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

(Total for Question 1 is 13 marks)

A level Mathematics SAMs Paper 3 Q3 (4.1, 4.2, 4.3, 5.3)

A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.
(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips cannot be used.
(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm.

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm.

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm.
(5)

(Total for Question 3 is 12 marks)

A level Mathematics SAMs Paper 3 Q5 (4.1, 4.2, 5.2)

A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.
(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.
(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.
(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.
(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%
(1)

(Total for Question 5 is 9 marks)

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5. Statistical hypothesis testing

Structure and calculation

5.1	**Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value;** extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded).
5.2	**Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.**
5.3	**Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.**
5.4	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context.

Sample questions

AS Mathematics SAMs Paper 2 Q5 (3.1)

The discrete random variable $X \sim B(40, 0.27)$.

(a) Find $P(X \geq 16)$.

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager’s suspicion.

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(d) Find the actual significance level of a test based on your critical region from part (c).

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager’s suspicion in the light of this observation.

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(Total for Question 5 is 9 marks)

A level Mathematics SAMs Paper 3 Q2 (2.2, 5.1)

A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$.

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C.

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days, a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$.

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

(Total for Question 2 is 6 marks)

A level Mathematics SAMs Paper 3 Q3 (4.1, 4.2, 4.3, 5.3)

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Strips with length either less than 49 cm or greater than 50.75 cm cannot be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal can be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips cannot be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm.

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm.

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm.

(5)

(Total for Question 3 is 12 marks)

A level Mathematics SAMs Paper 3 Q5 (4.1, 4.2, 5.2)

A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

(Total for Question 5 is 9 marks)

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6. Quantities and units in mechanics

Structure and calculation

6.1	**Understand and use fundamental quantities and units in the S.I. system: length, time, mass.** **Understand and use derived quantities and units: velocity, acceleration, force, weight,** moment**.**

Sample questions

AS Mathematics SAMs Paper 2 Q6 (6.1, 7.1, 7.3)

Unless otherwise indicated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

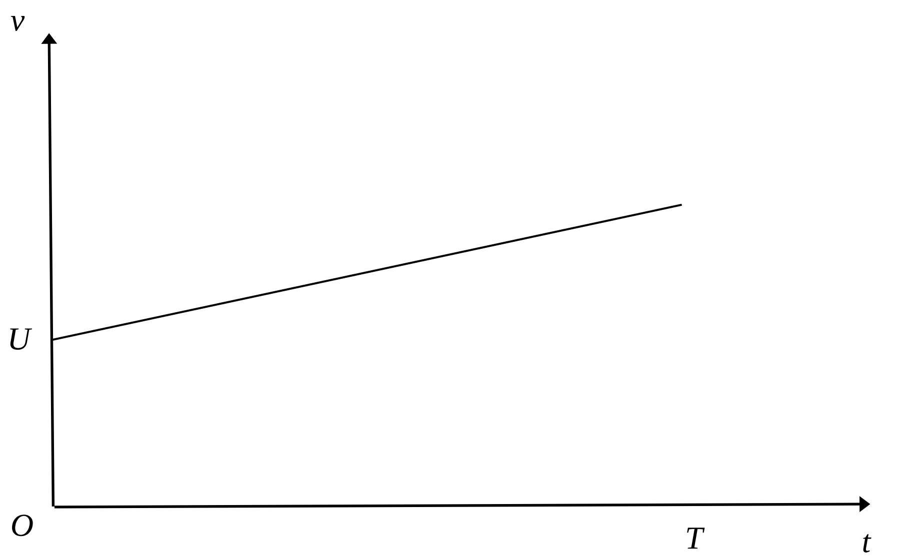

Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \le t \le T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(Total for Question 6 is 4 marks)

AS Mathematics SAMs Paper 2 Q7 (6.1, 7.1, 7.4)

A car is moving along a straight horizontal road with constant acceleration. There are three points *A*, *B* and *C*, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from *A* to *B* and 4 s to travel from *B* to *C*. Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes *A*.

(Total for Question 7 is 7 marks)

AS Mathematics SAMs Paper 2 Q8 (6.1, 7.1, 7.4, AS Mathematics Pure Mathematics 2.6)

A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \tfrac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leq t \leq 10.$$

(a) Explain the restriction $0 \leq t \leq 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for Question 8 is 9 marks)

AS Mathematics SAMs Paper 2 Q9 (6.1, 7.1, 7.3, 8.1, 8.2, 8.4)

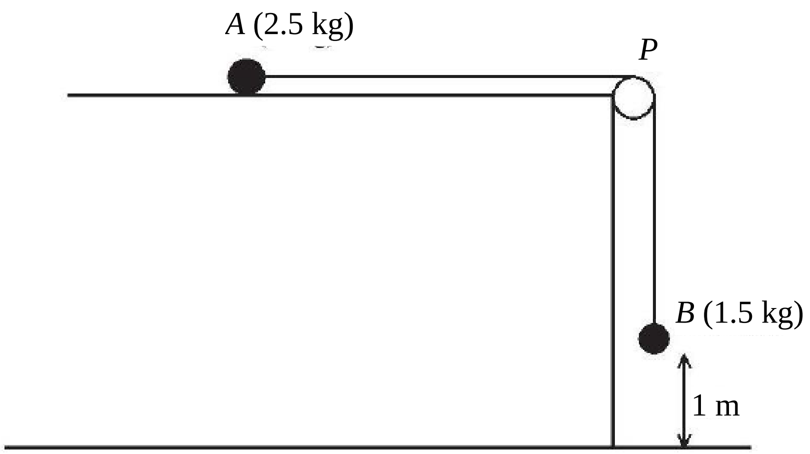

Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth and the acceleration due to gravity, g, is being modelled as being 9.8 m s^{-2}.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)

(Total for Question 9 is 10 marks)

A level Mathematics SAMs Paper 3 Q6 (6.1, 7.1, 7.4)

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}\,.$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

(Total for Question 6 is 6 marks)

A level Mathematics SAMs Paper 3 Q7 (6.1, 7.1, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6)

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(Total for Question 7 is 8 marks)

A level Mathematics SAMs Paper 3 Q8 (6.1, 7.1, 7.3)

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for Question 8 is 10 marks)

A level Mathematics SAMs Paper 3 Q9 (6.1, 8.4, 8.5, 8.6, 9.1)

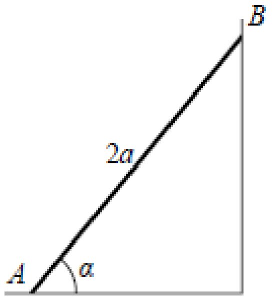

Figure 1

A uniform ladder *AB*, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.
The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 9 is 13 marks)

A level Mathematics SAMs Paper 3 Q10 (6.1, 7.1, 7.5)

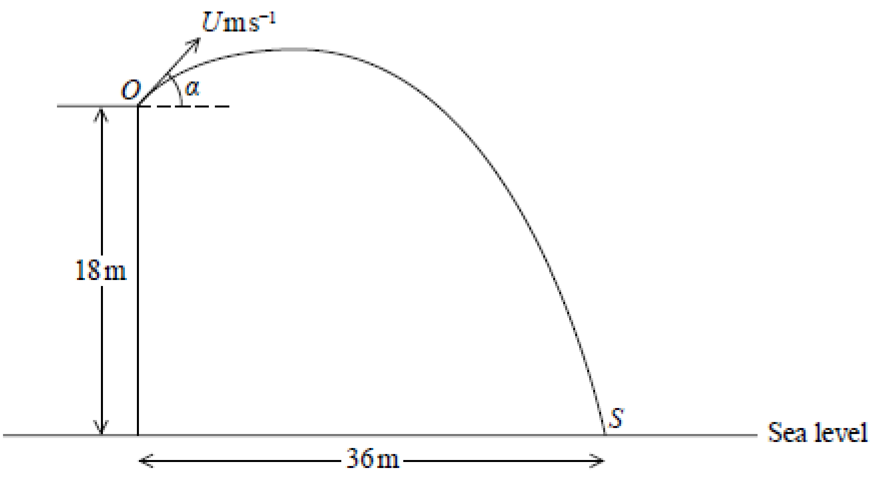

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with g = 10 m s^{-1}.

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

(Total for Question 10 is 13 marks)

7. Kinematics

Structure and calculation

7.1	**Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.**
7.2	**Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.**
7.3	**Understand, use and derive the formulae for constant acceleration for motion in a straight line.** Extend to 2 dimensions using vectors.
7.4	**Use calculus in kinematics for motion in a straight line:** $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$, $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}$, $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$, $\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$ Extend to 2 dimensions using vectors.
7.5	Model motion under gravity in a vertical plane using vectors; projectiles.

Sample questions

AS Mathematics SAMs Paper 2 Q6 (6.1, 7.1, 7.3)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

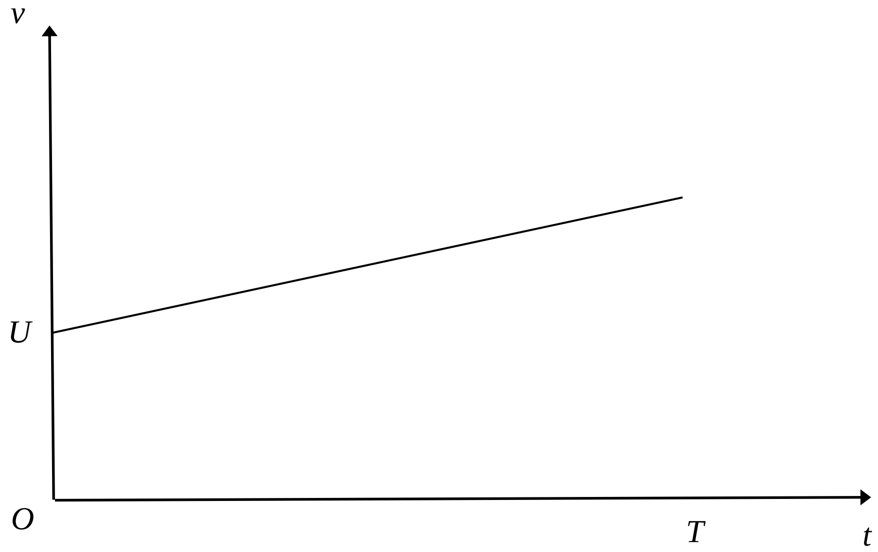

Figure 1

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is U m s^{-1}. The car then accelerates with constant acceleration a m s^{-2} for T seconds. The car travels a distance D metres during these T seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leq t \leq T$.

Using the graph, show that $D = UT + \frac{1}{2}aT^2$.

(No credit will be given for answers which use any of the kinematics (*suvat*) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

(Total for Question 6 is 4 marks)

AS Mathematics SAMs Paper 2 Q7 (6.1, 7.1, 7.4)

A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from A to B and 4 s to travel from B to C. Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

(Total for Question 7 is 7 marks)

AS Mathematics SAMs Paper 2 Q8 (6.1, 7.1, 7.4, AS Mathematics Pure Mathematics 2.6)

A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leq t \leq 10.$$

(a) Explain the restriction $0 \leq t \leq 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for Question 8 is 9 marks)

AS Mathematics SAMs Paper 2 Q9 (6.1, 7.1, 7.3, 8.1, 8.2, 8.4)

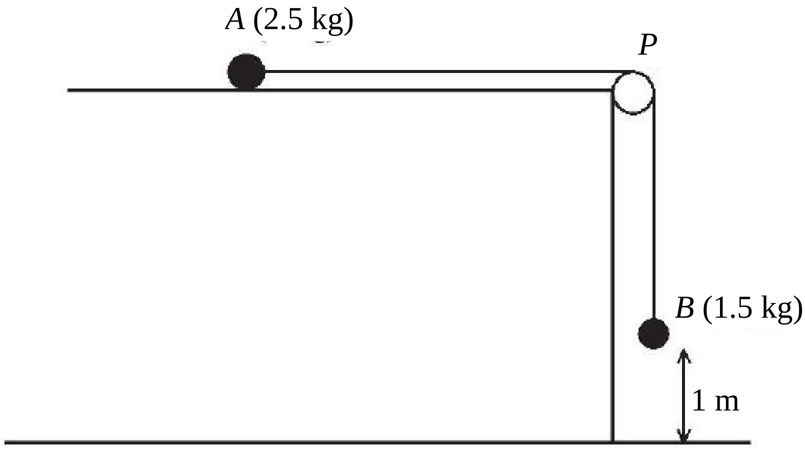

Figure 2

A small ball A of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley P which is fixed at the edge of the table. The other end of the string is attached to a small ball B of mass 1.5 kg hanging freely, vertically below P and with B at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of A from the rough table is modelled as having constant magnitude 12.7 N. Ball B reaches the floor before ball A reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence find the acceleration of B.

(2)

(c) Using the model, find the time it takes, from release, for B to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)

(Total for Question 9 is 10 marks)

A level Mathematics SAMs Paper 3 Q6 (6.1, 7.1, 7.4)

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}\,.$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

(Total for Question 6 is 6 marks)

A level Mathematics SAMs Paper 3 Q7 (6.1, 7.1, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6)

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(Total for Question 7 is 8 marks)

A level Mathematics SAMs Paper 3 Q8 (6.1, 7.1, 7.3)

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for Question 8 is 10 marks)

A level Mathematics SAMs Paper 3 Q10 (6.1, 7.1, 7.5)

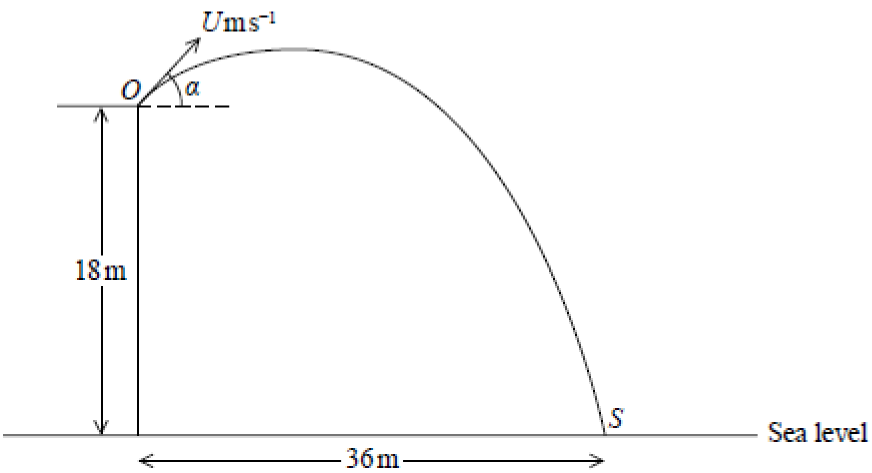

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where tan $\alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with g = 10 m s^{-1}.

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

(Total for Question 10 is 13 marks)

8. Kinematics Forces and Newton's laws

Structure and calculation

8.1	**Understand the concept of a force; understand and use Newton's first law.**
8.2	**Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors);** extend to situations where forces need to be resolved (restricted to 2 dimensions).
8.3	**Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy.**
8.4	**Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles;** resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces**.**
8.5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane.
8.6	Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.

Sample questions

AS Mathematics SAMs Paper 2 Q9 (6.1, 7.1, 7.3, 8.1, 8.2, 8.4)

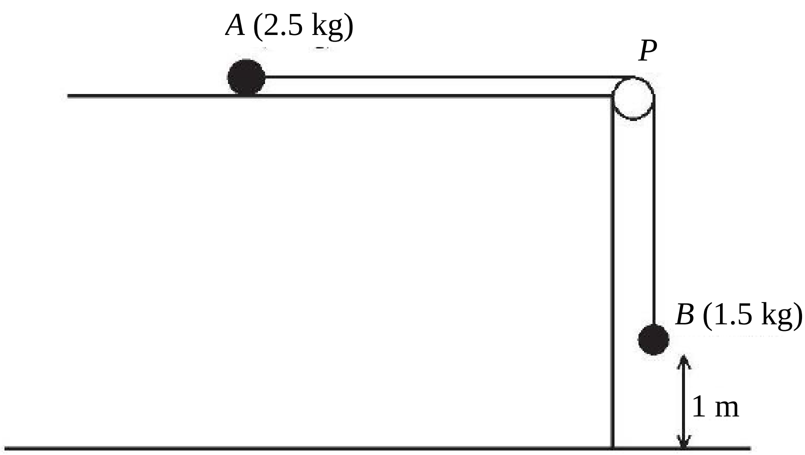

Figure 2

A small ball *A* of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley *P* which is fixed at the edge of the table. The other end of the string is attached to a small ball *B* of mass 1.5 kg hanging freely, vertically below *P* and with *B* at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of *A* from the rough table is modelled as having constant magnitude 12.7 N. Ball *B* reaches the floor before ball *A* reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for *A*.

(ii) Write down an equation of motion for *B*.

(4)

(b) Hence find the acceleration of *B*.

(2)

(c) Using the model, find the time it takes, from release, for *B* to reach the floor.

(2)

(d) Suggest two improvements that could be made in the model.

(2)

(Total for Question 9 is 10 marks)

A level Mathematics SAMs Paper 3 Q7 (6.1, 7.1, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6)

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point *A* on the plane.

(b) Determine whether the particle will remain at *A*, carefully justifying your answer.

(2)

(Total for Question 7 is 8 marks)

A level Mathematics SAMs Paper 3 Q9 (6.1, 8.4, 8.5, 8.6, 9.1)

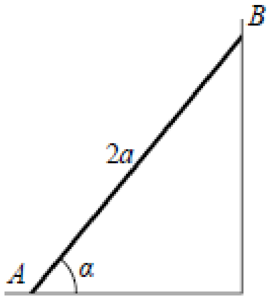

Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.
The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 9 is 13 marks)

9. Moments

Structure and calculation

9.1	Understand and use moments in simple static contexts.

Sample questions

A level Mathematics SAMs Paper 3 Q9 (6.1, 8.4, 8.5, 8.6, 9.1)

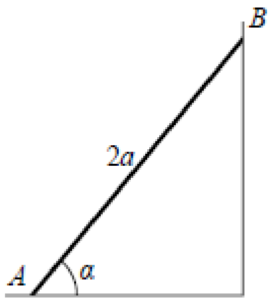

Figure 1

A uniform ladder *AB*, of length 2*a* and weight *W*, has its end *A* on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end *B* of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight 7*W* stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude *P* to the ladder at *A*, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at *B* has magnitude 3*W*.

(5)

(b) Find, in terms of *W*, the range of possible values of *P* for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 9 is 13 marks)