Pearson Edexcel
Level 3 Advanced Subsidiary
GCE in Mathematics (8MA0)

Sample Assessment Materials Exemplification
First teaching from September 2017
First certification from June 2018
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About this booklet

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary GCE in Mathematics (8MA0) (first assessment summer 2018).

The booklet provides additional information on all the questions in the Sample Assessment Materials, accredited by Ofqual in 2017. It details the content references and Assessment Objectives being assessed in each question or question part.

How to use this booklet

Callouts have been added to each question in the accredited Sample Assessment Materials. In the callouts, the following information has been presented, as relevant to the question:

- Specification References;
- Assessment Objectives.

Where content references or Assessment Objectives are being assessed across all the parts of a question, these are referred to by a single callout at the end of the question rather than by a callout for each question part.
1. The line $l$ passes through the points $A(3, 1)$ and $B(4, -2)$.

Find an equation for $l$.

**Specification reference (3.1):**
Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)

(Total for Question 1 is 3 marks)

2. The curve $C$ has equation

$$y = 2x^2 - 12 + 16.$$  

Find the gradient of the curve at the point $P(5, 6)$.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

**Specification reference (7.1, 7.2, 7.3):**
Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point $(x, y)$.
Differentiate $x^n$, for rational values of $n$, and related constant multiples, sums and differences.
Apply differentiation to find gradients.

**AO1.1a:** Select and correctly carry out routine procedures: select routine procedures (1 mark)
**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)

(Total for Question 2 is 4 marks)
3. Given that the point $A$ has position vector $3i - 7j$ and the point $B$ has position vector $8i + 3j$.

(a) Find the vector $\overrightarrow{AB}$.

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

**Specification reference (2.2, 9.1, 9.2, 9.3, 9.4):**

- Use and manipulate surds, including rationalising the denominator.
- Use vectors in two dimensions.
- Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.
- Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.
- Understand and use position vectors; calculate the distance between two points represented by position vectors.

**AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)**

(Total for Question 3 is 4 marks)

4. Let $f(x) = 4x^3 - 12x^2 + 2x - 6$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$.

**Specification reference (2.3, 2.6):**

- Solution of quadratic equations, including solving quadratic equations in a function of the unknown.
- Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.

**AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)**

**AO2.1: Construct rigorous mathematical arguments (including proofs) (2 marks)**

**AO2.4: Explain their reasoning (1 mark)**

(Total for Question 4 is 6 marks)
5. Given that \( f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0, \)

show that \( \int_{1}^{\sqrt{2}} f(x) \, dx = 16 + 3\sqrt{2}. \)

**Specification reference (2.1, 2.2, 8.2, 8.3):**
Understand and use the laws of indices for all rational exponents.
Use and manipulate surds, including rationalising the denominator.
Integrate \( x^a \) (excluding \( n = -1 \)) and related sums, differences and constant multiples.
Evaluate definite integrals.

**AO1.1a:** Select and correctly carry out routine procedures: select routine procedures (1 mark)
**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

*(Total for Question 5 is 5 marks)*

6. Prove, from first principles, that the derivative of \( 3x^2 \) is \( 6x. \)

**Specification reference (7.1):**
Differentiation from first principles for small positive integer powers of \( x. \)

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
**AO2.1:** Construct rigorous mathematical arguments (including proofs) (1 mark)
**AO2.5:** Use mathematical language and notation correctly (1 mark)

*(Total for Question 6 is 4 marks)*
7. (a) Find the first 3 terms, in ascending powers of \(x\), of the binomial expansion of \(\left(2 - \frac{x}{2}\right)^7\), giving each term in its simplest form.

(b) Explain how you would use your expansion to give an estimate for the value of 1.995\(^7\).

**Specification reference (4.1):**
Understand and use the binomial expansion of \((a + bx)^n\) for positive integer \(n\); the notations \(n!\) and \(nC_r\) link to binomial probabilities.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)
**AO2.4:** Explain their reasoning (1 mark)

(Total for Question 7 is 5 marks)
8. A triangular lawn is modelled by the triangle $ABC$, shown in Figure 1. The length $AB$ is to be 30 m long.

Given that angle $BAC = 70^\circ$ and angle $ABC = 60^\circ$,

(a) calculate the area of the lawn to 3 significant figures.

(b) Why is your answer unlikely to be accurate to the nearest square metre?

**Specification reference (5.1):**
Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2} ab \sin C$

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)

**AO2.1:** Construct rigorous mathematical arguments (including proofs) (1 mark)

**AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

**AO3.2b:** Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: where appropriate, evaluation the accuracy and limitations of solutions to problems (1 mark)

(Total for Question 8 is 5 marks)
9. Solve, for $360^\circ \leq x < 540^\circ$,

$$12 \sin^2 x + 7 \cos x - 13 = 0.$$ 

Give your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

**Specification reference (5.3, 5.4):**
Understand and use $\sin^2 \theta + \cos^2 \theta = 1$
Solve simple trigonometric equations in a given interval, including quadratic equations in $\sin$, $\cos$ and $\tan$ and equations involving multiples of the unknown angle.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

**AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

(Total for Question 9 is 5 marks)

10. The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that $0 \leq k \leq \frac{3}{4}$.

**Specification reference (1.1, 2.3, 2.5):**
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: Proof by deduction
The discriminant of a quadratic function, including the conditions for real and repeated roots.
Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (1 mark)

**AO2.1:** Construct rigorous mathematical arguments (including proofs) (1 mark)

**AO2.4:** Explain their reasoning (1 mark)

**AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)

(Total for Question 10 is 4 marks)
11. (a) Prove that for all positive values of \(x\) and \(y\),
\[
\sqrt{xy} \leq \frac{x + y}{2}.
\]
(b) Prove by counterexample that this is not true when \(x\) and \(y\) are both negative.

**Specification reference (1.1):**
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including: Proof by deduction, Proof by exhaustion, Disproof by counter example.

| AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark) |
| AO2.2a: Make deductions and inferences: make deductions (1 mark) |
| AO2.4: Explain their reasoning (1 mark) |

(Total for Question 11 is 3 marks)

12. A student was asked to give the exact solution to the equation \(2^{2x + 4} - 9(2^x) = 0\).

The student’s attempt is shown below:
\[
\begin{align*}
2^{2x + 4} - 9(2^x) &= 0 \\
2^{2x} + 2^4 - 9(2^x) &= 0 \\
\text{Let } 2^x &= y \\
y^2 - 9y + 8 &= 0 \\
(y - 8)(y - 1) &= 0 \\
y &= 8 \text{ or } y = 1 \\
\text{So } x &= 3 \text{ or } x = 0
\end{align*}
\]

(a) Identify the two errors made by the student.

(b) Find the exact solution to the equation.

**Specification reference (2.1, 6.5):**
Understand and use the laws of indices for all rational exponents.
Solve equations of the form \(a^x = b\)

| AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (1 mark) |
| AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark) |
| AO2.3: Assess the validity of mathematical arguments (2 marks) |

(Total for Question 12 is 4 marks)
13. (a) Factorise completely \( x^3 + 10x^2 + 25x \).

(b) Sketch the curve with equation \( y = x^3 + 10x^2 + 25x \), showing the coordinates of the points at which the curve cuts or touches the \( x \)-axis.

The point with coordinates \((-3, 0)\) lies on the curve with equation

\[ y = (x + a)^3 + 10(x + a)^2 + 25(x + a), \]

where \( a \) is a constant.

(c) Find the two possible values of \( a \).

Specification reference (2.1, 6.5):
Understand and use the laws of indices for all rational exponents.
Solve equations of the form \( a^r = b \)

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)
AO3.2a: Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)

(Total for Question 13 is 7 marks)
A town’s population, \( P \), is modelled by the equation \( P = ab^t \), where \( a \) and \( b \) are constants and \( t \) is the number of years since the population was first recorded. The line \( l \) shown in Figure 2 illustrates the linear relationship between \( t \) and \( \log_{10} P \) for the population over a period of 100 years.

The line \( l \) meets the vertical axis at \((0, 5)\) as shown. The gradient of \( l \) is \( \frac{1}{200} \).

(a) Write down an equation for \( l \).

(b) Find the value of \( a \) and the value of \( b \).

(c) With reference to the model, interpret

(i) the value of the constant \( a \),

(ii) the value of the constant \( b \).

(d) Find

(i) the population predicted by the model when \( t = 100 \), giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(e) State two reasons why this may not be a realistic population model.
Specification reference (3.1, 6.4, 6.6, 6.7):
Understand and use the laws of indices for all rational exponents.
Solve equations of the form \( a^x = b \)
Understand and use the laws of logarithms:
\[
\log_a x + \log_a y = \log_a (xy); \log a^x - \log ay = \log a \left( \frac{x}{y} \right); k\log a x = \log a x^k
\]
(including, for example \( k = -1 \) and \( k = -\frac{1}{2} \))
Use logarithmic graphs to estimate parameters in relationships of the form \( y = ax^n \) and \( y = kb^x \), given data for \( x \) and \( y \)
Understand and use exponential growth and decay; use in modelling (examples may include the use of \( e \) in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (6 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)
AO3.4: Use mathematical models (4 mark)
AO3.5b: Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: recognise the limitations of models (2 marks)

(Total for Question 14 is 13 marks)
The curve $C_1$, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P \left( \frac{1}{2}, 2 \right)$ lies on $C_1$.

The curve $C_2$, also shown in Figure 3, has equation $y = \frac{1}{x} + \ln (2x)$. The normal to $C_1$ at the point $P$ meets $C_2$ at the point $Q$.

Find the exact coordinates of $Q$.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

Specification reference (2.4, 6.3, 7.2, 7.3):
Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.
Know and use $\ln x$ as the inverse function of $e^x$.
Differentiate $x^n$, for rational values of $n$, and related constant multiples, sums and differences.
Apply differentiation to find gradients, tangents and normal.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (6 marks)
AO3.1a: Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (2 marks)

(Total for Question 15 is 8 marks)
Figure 4 shows the plan view of the design for a swimming pool. The shape of this pool $ABDEA$ consists of a rectangular section $ABDE$ joined to a semi-circular section $BCD$ as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250$ m$^2$,

(a) show that the perimeter, $P$ metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}.$$ 

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$.

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

**Specification reference (1.1, 7.1, 7.2, 7.3):**

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion.

Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point $(x, y)$; the gradient of the tangent as a limit; interpretation as a rate of change.

Differentiate $x^n$, for rational values of $n$, and related constant multiples, sums and differences.

Apply differentiation to find gradients, tangents and normal, maxima and minima and stationary points.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

**AO2.1:** Construct rigorous mathematical arguments (including proofs) (2 marks)

**AO2.4:** Explain their reasoning (1 mark)

**AO3.2a:** Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)

**AO3.4:** Use mathematical models (1 mark)

**(Total for Question 16 is 10 marks)**
17. A circle $C$ with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle $C$ also passes through the point $(10, 1)$.

The tangent to the circle $C$ at the point $(10, 11)$ meets the $y$-axis at the point $P$ and the tangent to the circle $C$ at the point $(10, 1)$ meets the $y$-axis at the point $Q$.

(b) Show that the distance $PQ$ is 58, explaining your method clearly.
1. Sara is investigating the variation in daily maximum gust, $t$ kn, for Camborne in June and July 1987.

She used the large data set to select a sample of size 20 from the June and July data for 1987. Sara selected the first value using a random number from 1 to 4 and then selected every third value after that.

(a) State the sampling technique Sara used.

(b) From your knowledge of the large data set, explain why this process may not generate a sample of size 20.

The data Sara collected are summarised as follows

$$n = 20 \quad \sum t = 374 \quad \sum t^2 = 7600$$

(c) Calculate the standard deviation.

**Specification reference (1.1, 2.3):**

Understand and use the terms ‘population’ and ‘sample’.

Use samples to make informal inferences about the population.

Understand and use sampling techniques, including simple random sampling and opportunity sampling.

Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.

Interpret measures of central tendency and variation, extending to standard deviation.

Be able to calculate standard deviation, including from summary statistics.

**AO1.1a:** Select and correctly carry out routine procedures: select routine procedures (1 mark)

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (1 mark)

**AO1.2:** Accurately recall facts, terminology and definitions (1 mark)

**AO2.4:** Explain their reasoning (1 mark)

(Total for Question 1 is 4 marks)
2. The partially completed histogram and the partially completed table show the time, to the nearest minute, that a random sample of motorists were delayed by roadworks on a stretch of motorway.

<table>
<thead>
<tr>
<th>Delay (minutes)</th>
<th>Number of motorists</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – 6</td>
<td>6</td>
</tr>
<tr>
<td>7 – 8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>10 – 12</td>
<td>45</td>
</tr>
<tr>
<td>13 – 15</td>
<td>9</td>
</tr>
<tr>
<td>16 – 20</td>
<td></td>
</tr>
</tbody>
</table>

Estimate the percentage of these motorists who were delayed by the roadworks for between 8.5 and 13.5 minutes.

**Specification reference (2.1):**
Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency. Connect to probability distributions.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (2 marks)
**AO2.2a:** Make deductions and inferences; make deductions (1 mark)
**AO3.1a:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in mathematical contexts into mathematical processes (1 mark)
**AO3.1b:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (1 mark)

(Total for Question 2 is 5 marks)
3. The Venn diagram shows the probabilities for students at a college taking part in various sports.

\( A \) represents the event that a student takes part in Athletics.
\( T \) represents the event that a student takes part in Tennis.
\( C \) represents the event that a student takes part in Cricket.
\( p \) and \( q \) are probabilities.

![Venn Diagram]

The probability that a student selected at random takes part in Athletics or Tennis is 0.75.

(a) Find the value of \( p \).

(b) State, giving a reason, whether or not the events \( A \) and \( T \) are statistically independent. Show your working clearly.

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

**Specification reference (3.1):**
Understand and use mutually exclusive and independent events when calculating probabilities.
Link to discrete and continuous distributions.

**AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)**
**AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)**
**AO2.4: Explain their reasoning (1 mark)**

(Total for Question 3 is 5 marks)
4. Sara was studying the relationship between rainfall, $r$ mm, and humidity, $h \%$, in the UK. She takes a random sample of 11 days from May 1987 for Leuchars from the large data set.

She obtained the following results.

<table>
<thead>
<tr>
<th>$h$</th>
<th>93</th>
<th>86</th>
<th>95</th>
<th>97</th>
<th>86</th>
<th>94</th>
<th>97</th>
<th>97</th>
<th>87</th>
<th>97</th>
<th>86</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>1.1</td>
<td>0.3</td>
<td>3.7</td>
<td>20.6</td>
<td>0</td>
<td>0</td>
<td>2.4</td>
<td>1.1</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Sara examined the rainfall figures and found

\[ Q_1 = 0.1 \quad Q_2 = 0.9 \quad Q_3 = 2.4 \]

A value that is more than 1.5 times the interquartile range (IQR) above $Q_3$ is called an outlier.

(a) Show that $r = 20.6$ is an outlier.

(b) Give a reason why Sara might (i) include (ii) exclude this day’s reading.

Sara decided to exclude this day’s reading and drew the following scatter diagram for the remaining 10 days’ values of $r$ and $h$.

(c) Give an interpretation of the correlation between rainfall and humidity.

The equation of the regression line of $r$ on $h$ for these 10 days is $r = -12.8 + 0.15h$.

(d) Give an interpretation of the gradient of this regression line.
(e) (i) Comment on the suitability of Sara’s sampling method for this study.

(ii) Suggest how Sara could make better use of the large data set for her study.

**Specification reference (1.1, 2.2, 2.4):**

- Understand and use the terms ‘population’ and ‘sample’.
- Use samples to make informal inferences about the population.
- Understand and use sampling techniques, including simple random sampling and opportunity sampling.
- Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.
- Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded).
- Understand informal interpretation of correlation.
- Understand that correlation does not imply causation.
- Recognise and interpret possible outliers in data sets and statistical diagrams.
- Select or critique data presentation techniques in the context of a statistical problem.
- Be able to clean data, including dealing with missing data, errors and outliers.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (1 mark)

**AO2.2b:** Make deductions and inferences: make inferences (1 mark)

**AO2.4:** Explain their reasoning (4 marks)

**AO3.4:** Use mathematical models (1 mark)

(Total for Question 4 is 7 marks)
5. The discrete random variable $X \sim B(40, 0.27)$.

(a) Find $P(X \geq 16)$.

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager’s suspicion.

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(d) Find the actual significance level of a test based on your critical region from part (c).

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager’s suspicion in the light of this observation.

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

**Specification reference (4.1, 5.1, 5.2):**

Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution.

Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value.

Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context.

AO1.1b: Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)
AO2.1: Construct rigorous mathematical arguments (including proofs) (1 mark)
AO2.5: Uses mathematical language and notation correctly (1 mark)
AO3.2a: Interpret solutions to problems in their original context, and, where appropriate evaluate their accuracy and limitations: interpret solutions to problems in their original context (1 mark)
AO3.5a: Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: evaluate the outcomes of modelling in context (1 mark)

(Total for Question 5 is 9 marks)
Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6.

A car moves along a straight horizontal road. At time $t = 0$, the velocity of the car is $U \text{ m s}^{-1}$. The car then accelerates with constant acceleration $a \text{ m s}^{-2}$ for $T$ seconds. The car travels a distance $D$ metres during these $T$ seconds.

Figure 1 shows the velocity-time graph for the motion of the car for $0 \leq t \leq T$.

Using the graph, show that $D = UT + \frac{1}{2} aT^2$.

(No credit will be given for answers which use any of the kinematics ($suvat$) formulae listed under Mechanics in the AS Mathematics section of the formulae booklet.)

**Specification reference (6.1, 7.1, 7.3):**
Understand and use fundamental quantities and units in the S.I. system: length, time, mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight.
Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.
Understand, use and derive the formulae for constant acceleration for motion in a straight line.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (3 marks)
**AO2.1:** Construct rigorous mathematical arguments (including proofs) (1 mark)

(Total for Question 6 is 4 marks)
7. A car is moving along a straight horizontal road with constant acceleration. There are three points A, B and C, in that order, on the road, where \( AB = 22 \text{ m} \) and \( BC = 104 \text{ m} \). The car takes 2 s to travel from A to B and 4 s to travel from B to C. Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes A.

**Specification reference (6.1, 7.1, 7.4):**

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight.

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Use calculus in kinematics for motion in a straight line: \( v = \frac{dr}{dt} \), \( a = \frac{dv}{dt} = \frac{d^2r}{dt^2} \), \( r = \int v \, dt \), \( v = \int a \, dt \)

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (4 marks)

**AO3.1b:** Translate problems in mathematical and non-mathematical contexts into mathematical processes: translate problems in non-mathematical contexts into mathematical processes (3 marks)

(Total for Question 7 is 7 marks)
8. A bird leaves its nest at time \( t = 0 \) for a short flight along a straight line. The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, \( s \) metres, of the bird from its nest at time \( t \) seconds is given by

\[
s = \frac{1}{10} (t^4 - 20t^3 + 100t^2), \quad \text{where } 0 \leq t \leq 10.
\]

(a) Explain the restriction \( 0 \leq t \leq 10 \)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

**Specification reference (6.1, 7.1, 7.4, AS Mathematics Pure Mathematics 2.6):**

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Understand and use derived quantities and units: velocity, acceleration, force, weight.

Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.

Use calculus in kinematics for motion in a straight line: \( v = \frac{dr}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}, \quad r = \int vdt, \quad v = \int adt \)

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.

**AO1.1a:** Select and correctly carry out routine procedures: select routine procedures (2 marks)

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)

**AO2.1:** Construct rigorous mathematical arguments (including proofs) (1 mark)

**AO2.4:** Explain their reasoning (1 mark)

(Total for Question 8 is 9 marks)
9. A small ball $A$ of mass 2.5 kg is held at rest on a rough horizontal table.

The ball is attached to one end of a string.

The string passes over a pulley $P$ which is fixed at the edge of the table. The other end of the string is attached to a small ball $B$ of mass 1.5 kg hanging freely, vertically below $P$ and with $B$ at a height of 1 m above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of $A$ from the rough table is modelled as having constant magnitude 12.7 N. Ball $B$ reaches the floor before ball $A$ reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for $A$.

(ii) Write down an equation of motion for $B$.

(b) Hence find the acceleration of $B$.

(c) Using the model, find the time it takes, from release, for $B$ to reach the floor.

It was found that it actually took 2.3 seconds for ball $B$ to reach the floor.
(d) Using this information,

(i) comment on the appropriateness of using the model to find the time it takes ball $B$ to reach the floor, justifying your answer.

(ii) suggest one improvement that could be made in the model.

**Specification reference (6.1, 7.1, 7.3, 8.1, 8.2, 8.4):**

Understand and use fundamental quantities and units in the S.I. system: length, time, mass.
Understand and use derived quantities and units: velocity, acceleration, force, weight.
Understand, use and derive the formulae for constant acceleration for motion in a straight line.
Understand the concept of a force; understand and use Newton’s first law.
Understand and use Newton’s second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).
Understand and use Newton’s third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles.

**AO1.1b:** Select and correctly carry out routine procedures: correctly carry out routine procedures (5 marks)
**AO3.3:** Translate situations in context into mathematical models (2 marks)
**AO3.4:** Use mathematical models (1 mark)
**AO3.5a:** Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: evaluate the outcomes of modelling in context (1 mark)
**AO3.5c:** Evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them: where appropriate, explain how to refine models (1 mark)

(Total for Question 9 is 10 marks)