



Your Starter for 10!

A stone is projected vertically up, from ground level at u (ms^{-1})

Sketch the velocity time diagram, from the moment it is projected to the moment it impacts the ground.

What assumptions have you made when you modelling this to A Level Maths level?

Kinematics

A Deep Dive



Begin with the end in mind! AS Level

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The ball

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Using the

A car mov accelerati these 7 s

Figure 1 s

Using the

(No credit Mechanic

(a) show

(b) find t

(c) find t

(d) Sketc

coord

Two par a light in the string; horizont; accelera

(a) Show

(b) Find

(c) State

After det with the does not

(d) Find

A small b

The ball i

The string attached above the horizontal floor.

The system is released from rest, with the string taut, as shown in Figure 2.

The resistance to the motion of *A* from the rough table is modelled as having constant magnitude 12.7 N. Ball *B* reaches the floor before ball *A* reaches the pulley.

The balls are modelled as particles, the string is modelled as being light and inextensible and the pulley is modelled as being small and smooth.

(a) (i) Write down an equation of motion for *A*.

(ii) Write down an equation of motion for *B*.

(b) Hence find the acceleration of *B*.

(c) Using the model, find the time it takes, from release, for *B* to reach the floor.

A particle *P* moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$

At time $t = 0$, *P* is moving with velocity $4\mathbf{i} \text{ m s}^{-1}$

(a) Find the velocity of *P* at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of *P* relative to a fixed origin *O* is $(\mathbf{i} + \mathbf{j}) \text{ m}$.

(b) Find the position vector of *P* relative to *O* at time $t = 3$ seconds.

(2)

(Total for question = 4 marks)

(3)

(1)

oact

(7)

(4) rks)

(2)

Begin with the end in mind! A Level

At time t with speed v upward vertical gravity.

A particle

At time t

At time t

(a) Find

(a) the

Two particles of a light inextensible string are fixed to a rough horizontal surface.

At time t

A small stone is projected from point O which is 70 m vertically above point N on horizontal ground.

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q , are projected in the vertical plane containing AB .

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB .

Ball Q is projected from B with speed $u \text{ m s}^{-1}$ at angle θ to BA , as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q .

(6)

(b) Find

- (i) the size of angle θ ,
- (ii) the value of u .

(6)

(a) show that L

(6)

(b) find the greatest height of the ball above the horizontal ground NA .

(3)

(6)

(Total 16 marks)



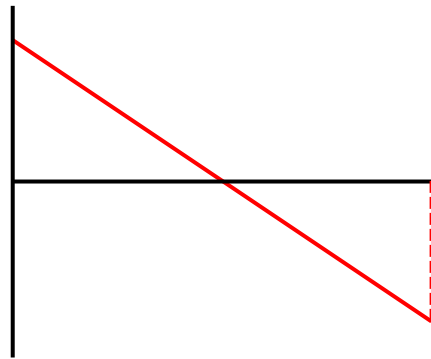
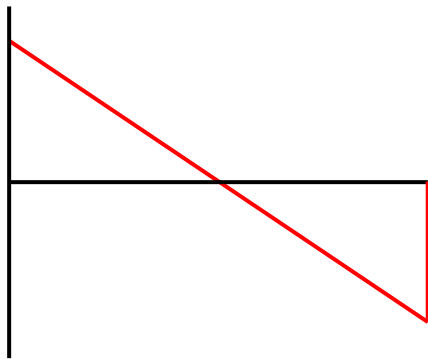
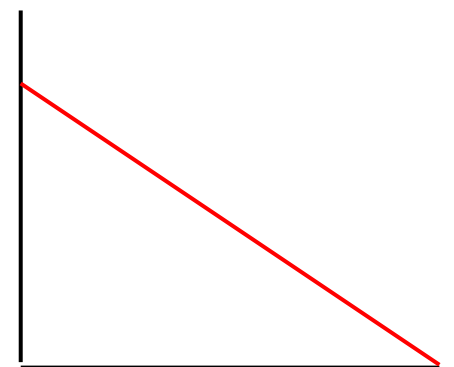
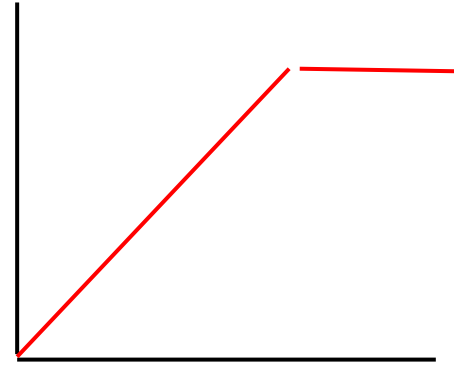
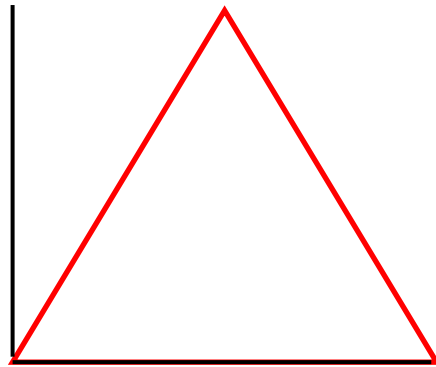
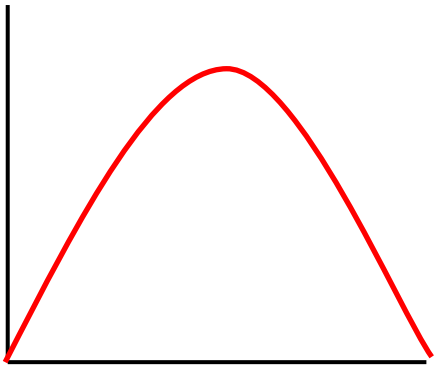
A Fine Point

Statics: Static Equilibrium (vs Dynamic Equilibrium)

Kinematics: the study of motion (velocity or acceleration time graphs, constant acceleration, variable acceleration)

Dynamics: Forces

Velocity Time Diagrams



A Reminder

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Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$a = \frac{v - u}{t}$$

$$Acc = \frac{\text{Change in Vel}}{\text{Time}}$$

A Reminder

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

Final Vel = Initial Vel + Acceleration x Time

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

~~Distance Travelled = Average Speed x Time~~

Displacement = Average Speed x Time

Proving the suvat formulae

A car drives at a constant speed of 13 m s^{-1} for 10 seconds. How far has it travelled?

$$13 \times 10 = 130 \text{ m}$$

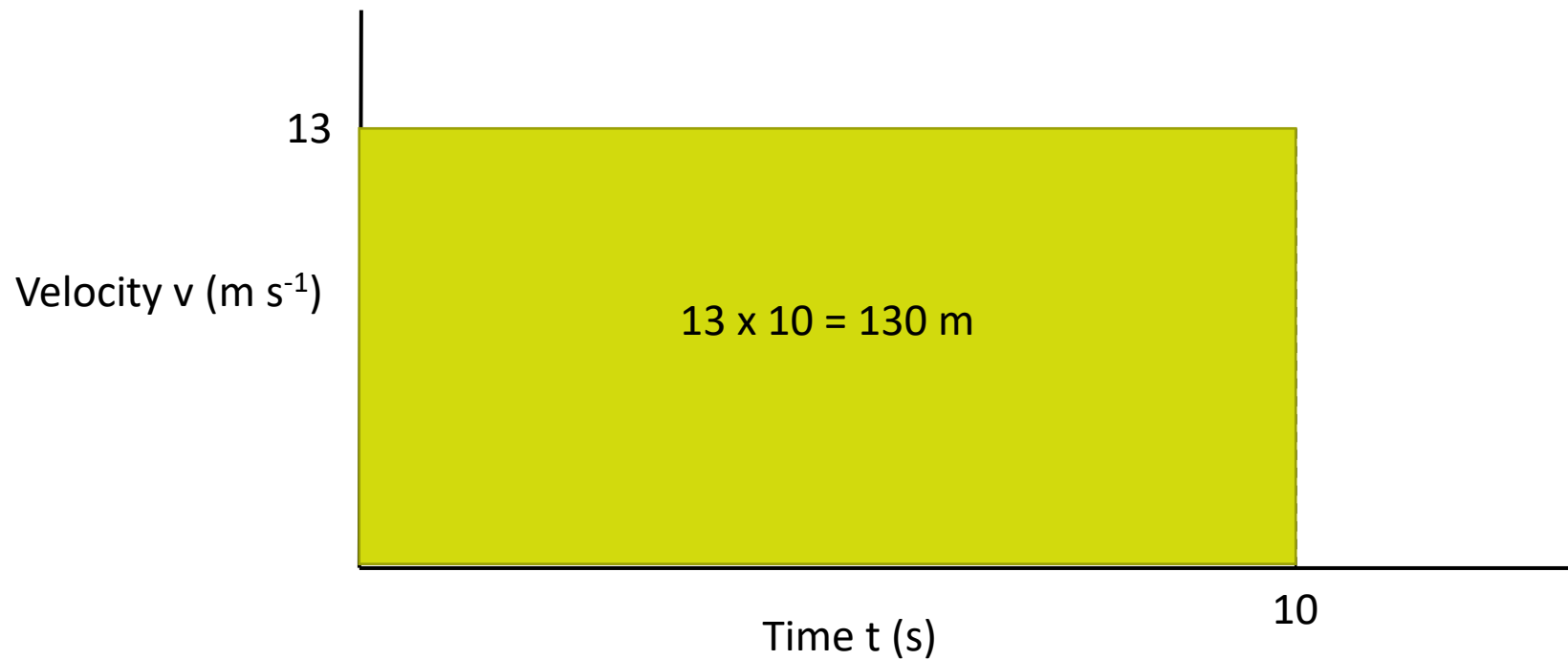


What's wrong with this graph?

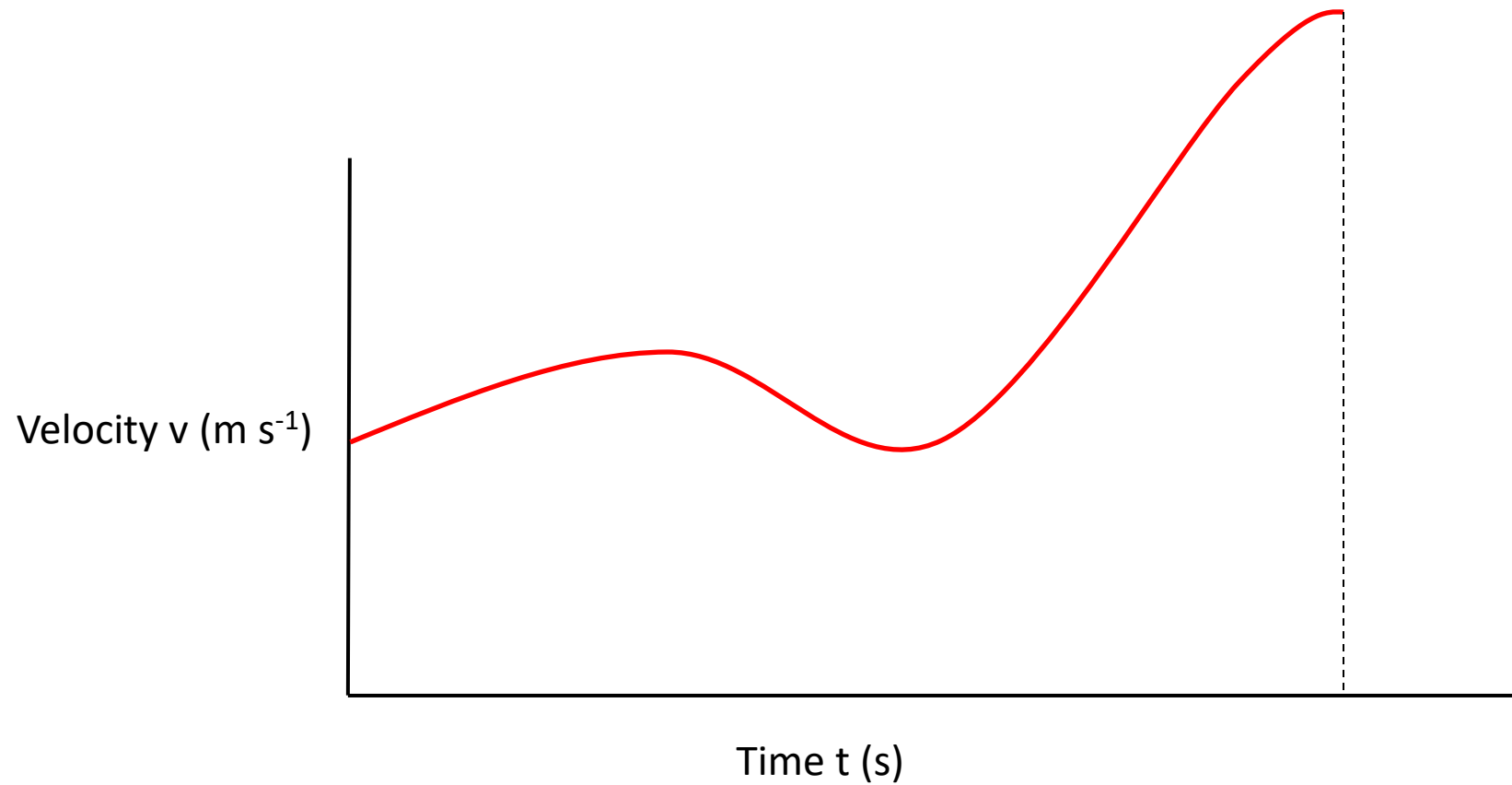
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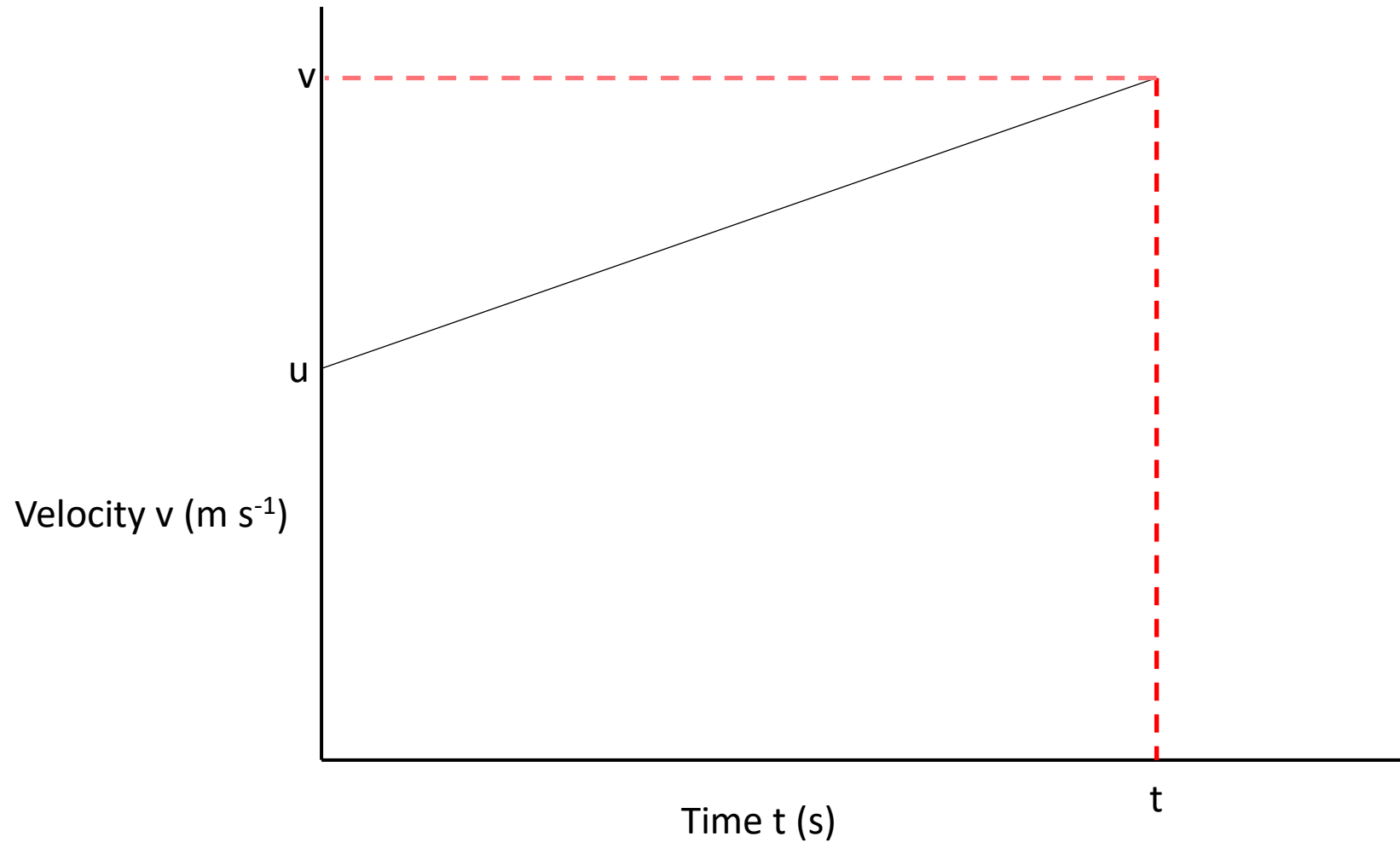
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Proving the suvat formulae



Proving the suvat formulae



Proving the suvat formulae

Kinematics

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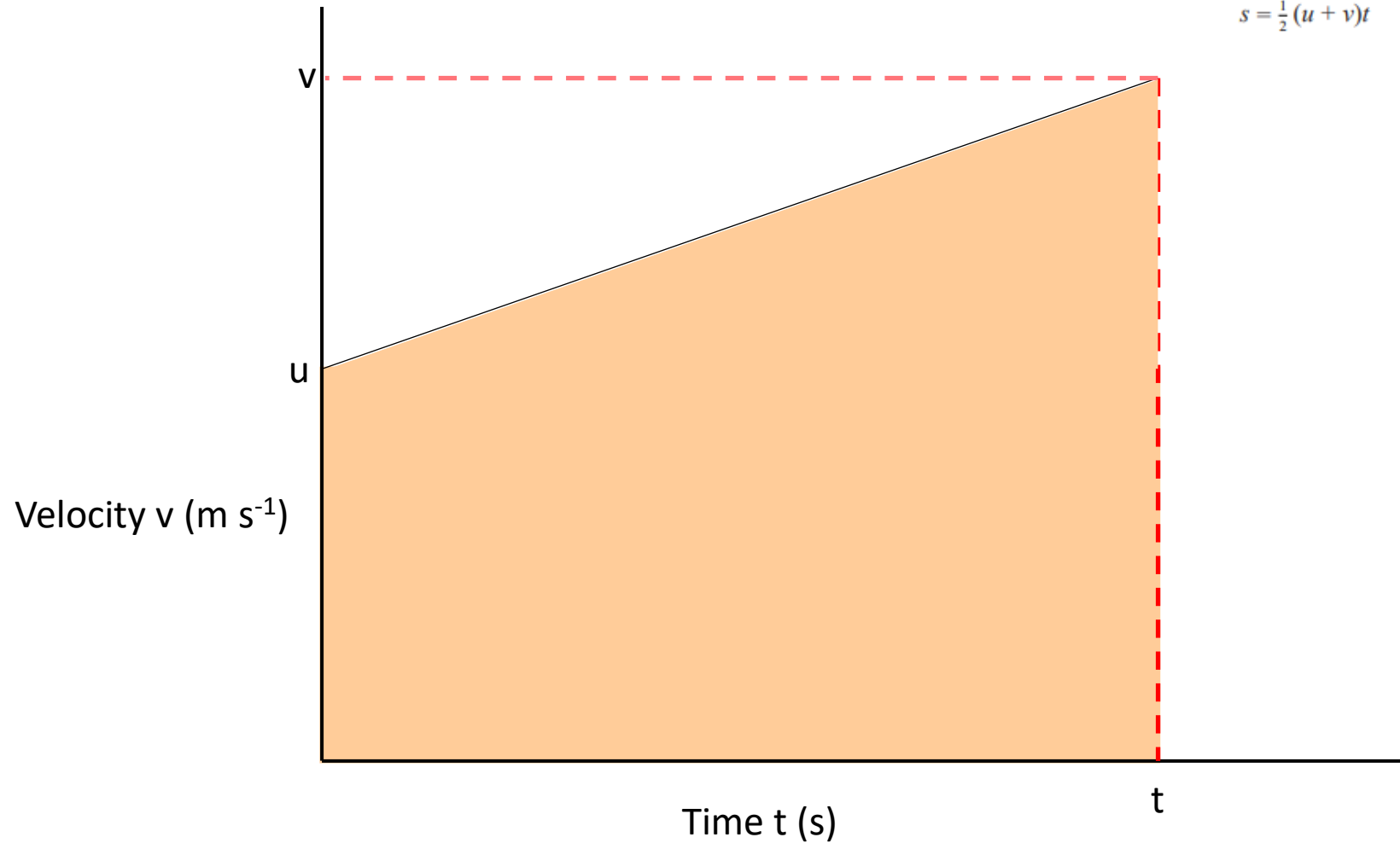
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$$s = \frac{1}{2}(u + v)t$$



Proving the suvat formulae

A car travelling at 10 m s^{-1} accelerates for 3 seconds at 5 m s^{-2} .
What is the car's final velocity?

$$v = 10 + 3 \times 5 = 25 \text{ m s}^{-2}$$

$$v = u + at$$

How is acceleration represented on the graph?

Kinematics

For motion in a straight line with constant acceleration:

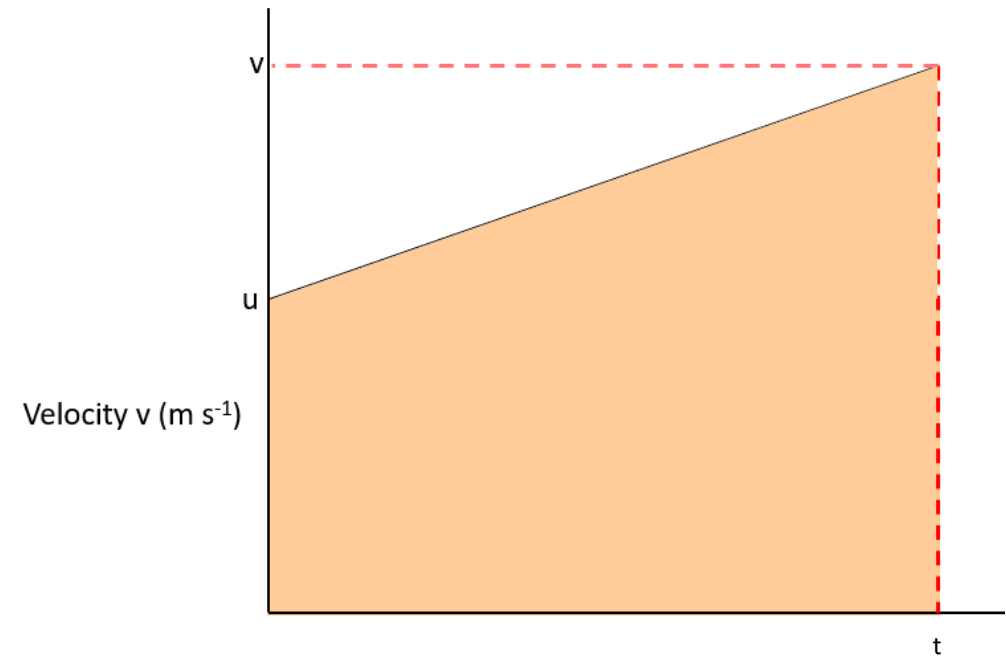
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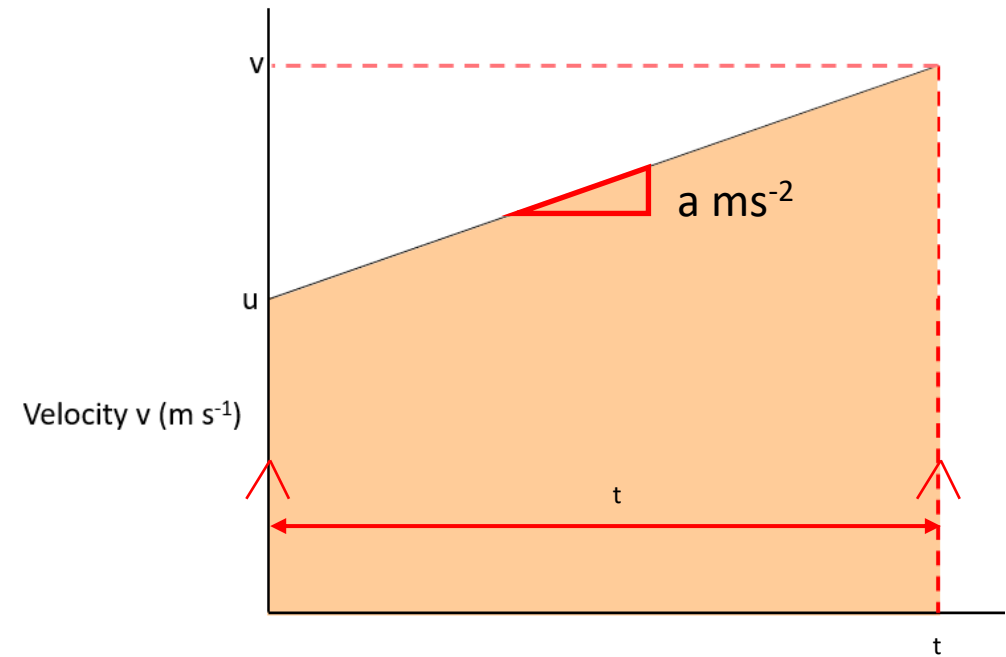
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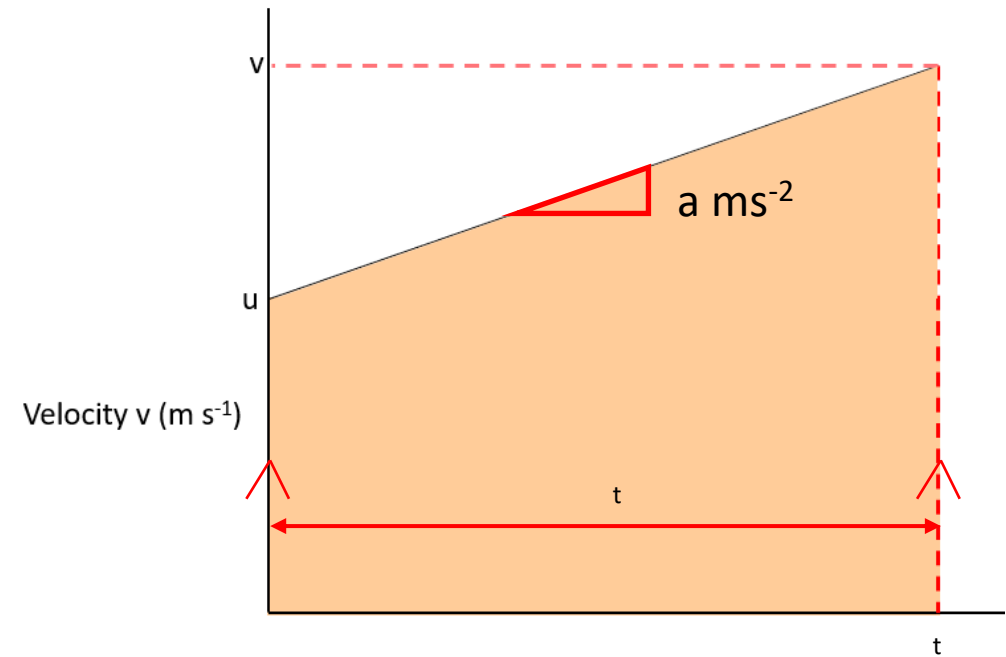
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Proving the suvat formulae

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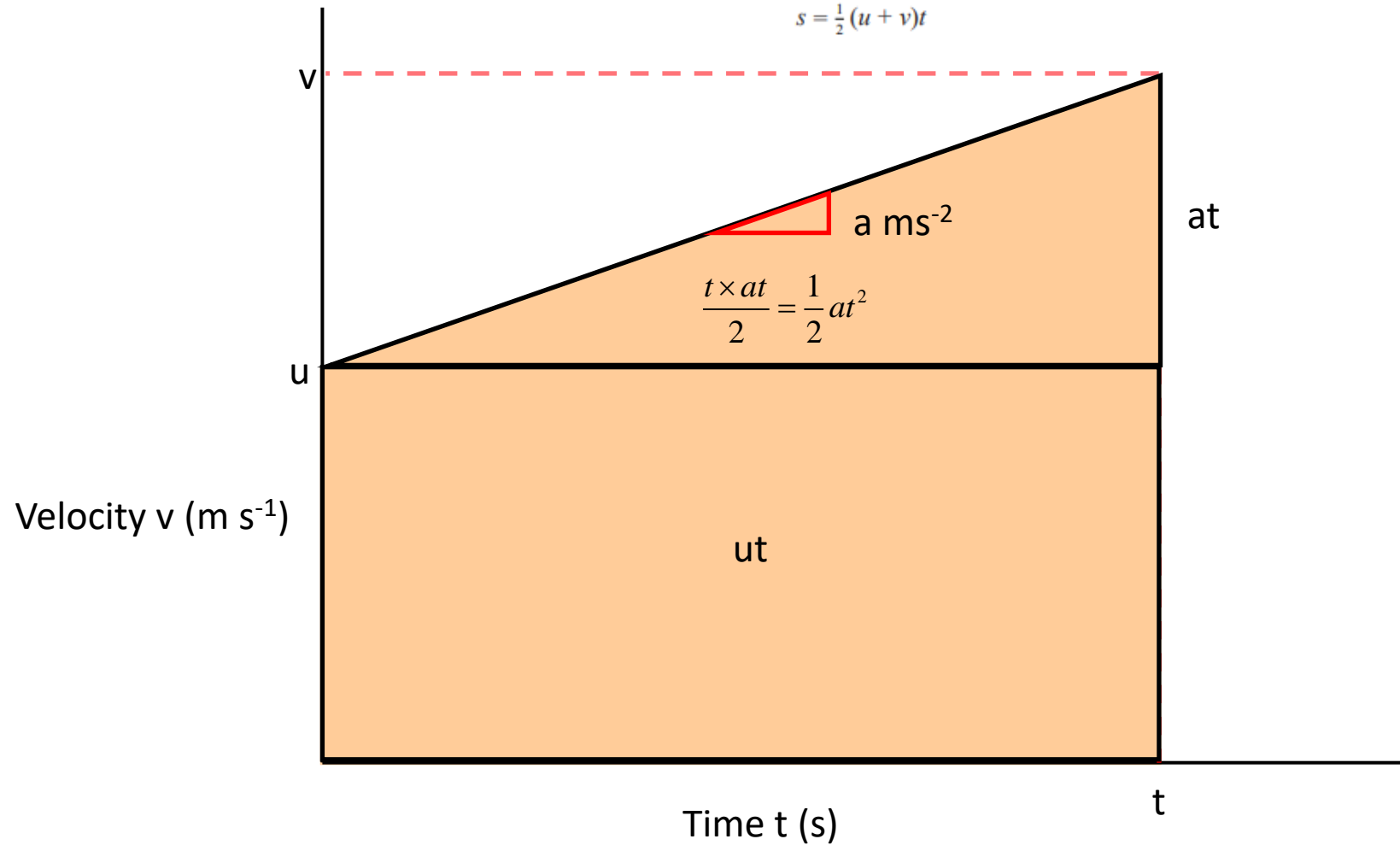
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$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$v - u = at$$



Kinematics

For motion in a straight line with constant acceleration:

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$$s = ut + \frac{1}{2}at^2$$

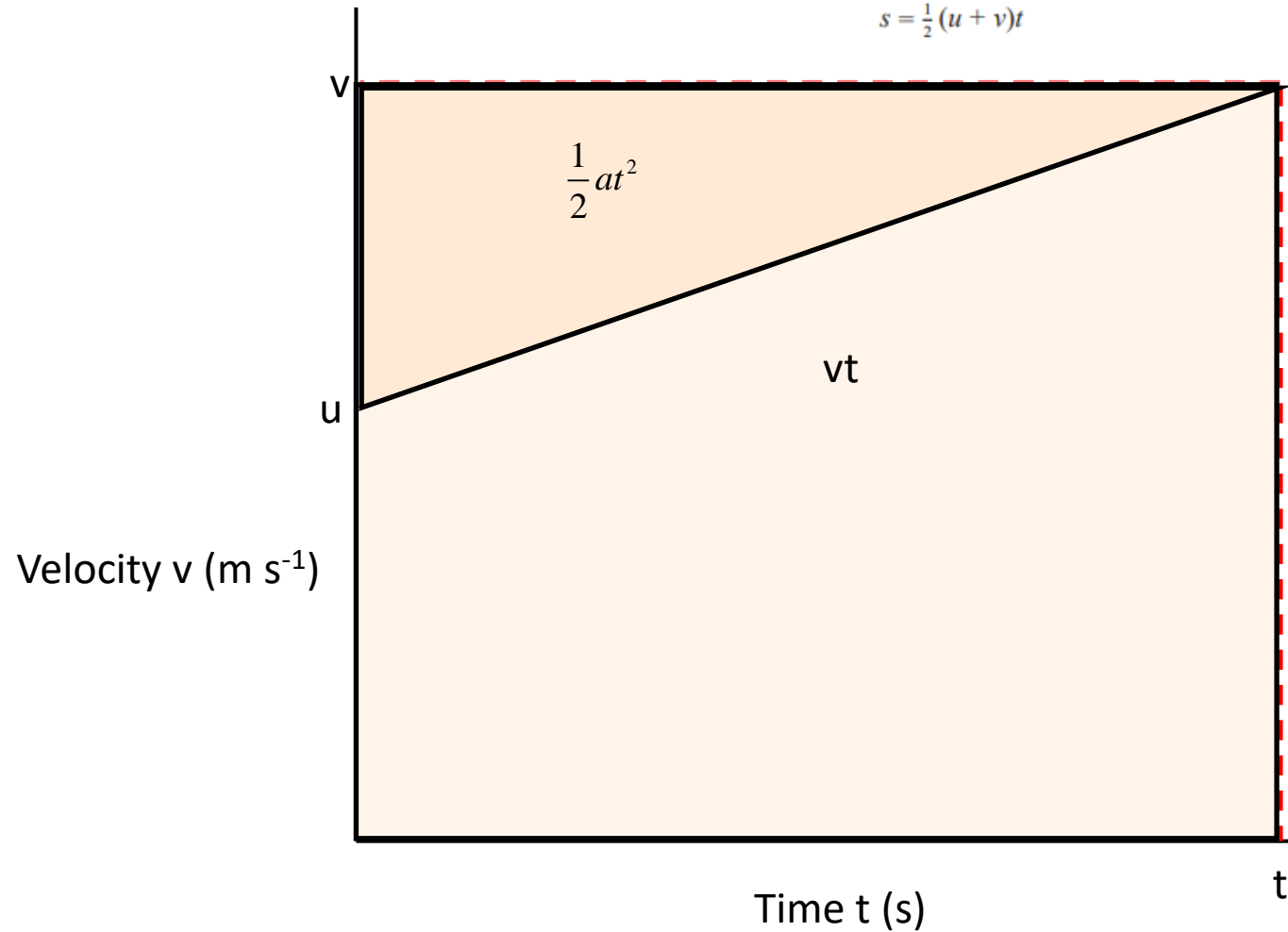
$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Proving the suvat formulae

$$s = vt - \frac{1}{2}at^2$$



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$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Proving the suvat formulae

A stunt driver starts his car from rest at the base of a 100 m long ramp. At the end of the ramp he must be travelling at 40 m s^{-1} in order to jump the gap. At what rate must he accelerate?

$$s = \frac{1}{2}(u + v)t$$

$$100 = \frac{1}{2}(0 + 40)t$$

$$100 = 20t$$

$$\frac{100}{20} = t$$

$$t = 5s$$

$$v = u + at$$

$$40 = 0 + a \times 5$$

$$\frac{40}{5} = a$$

$$a = 8 \text{ ms}^{-2}$$

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Proving the suvat formulae

$$s = \frac{1}{2}(u + v)t \quad v = u + at$$

$$\frac{2s}{u+v} = t \quad \textcircled{1} \quad \frac{v-u}{a} = t \quad \textcircled{2}$$

Sub $\textcircled{1}$ in $\textcircled{2}$ for t

$$\frac{2s}{\cancel{u+v}} = \frac{v-u}{a} \times (u+v)$$

$$a \times 2s = \frac{(v-u)(u+v)}{\cancel{a}}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

Calculus to derive suvat

What is the formula of the line on the graph?

Think in terms of $y = mx + c$

$$v = u + at$$
$$s = \int v dt$$
$$s = \int u + at dt$$
$$s = ut + \frac{1}{2}at^2 + C?$$

Kinematics

For motion in a straight line with constant acceleration:

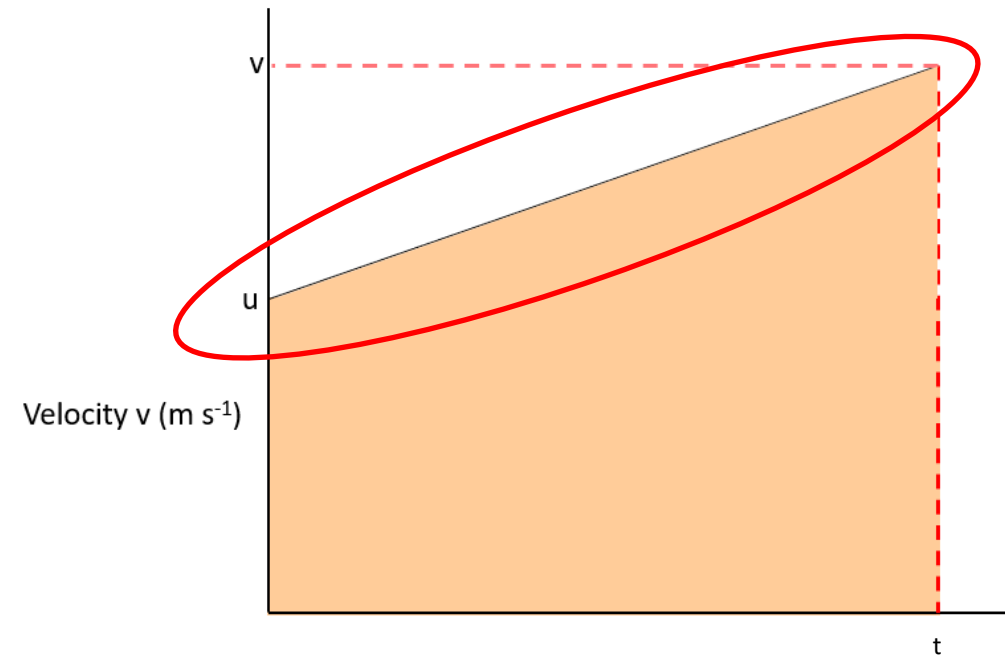
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$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$



Horizontal Motion

A car is moving along a straight horizontal road with constant acceleration.

There are three points A, B and C, in that order, on the road, where $AB = 22 \text{ m}$ and $BC = 104 \text{ m}$.

The car takes 2 s to travel from A to B and 4 s to travel from B to C.

Find

- the acceleration of the car,
- the speed of the car at the instant it passes A.

$$\begin{aligned} AB \quad s &= 22 \text{ m} \\ u &= u \\ v &= - \\ a &= a \\ t &= 2 \text{ s} \end{aligned}$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 22 &= 2u + \frac{1}{2}a \times 2^2 \\ \underline{22} &= \underline{2u + 2a} \end{aligned}$$

~~$$\begin{aligned} BC \quad s &= 104 \text{ m} \\ u &= - \\ v &= - \\ a &= a \\ t &= 4 \text{ s} \end{aligned}$$~~

$$\begin{aligned} AC \quad s &= 126 \text{ m} \\ u &= u \\ v &= - \\ a &= a \\ t &= 6 \text{ s} \end{aligned} \quad \begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 126 &= 6u + \frac{1}{2}a \times 6^2 \\ \underline{126} &= \underline{6u + 18a} \end{aligned}$$

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Vertical Motion

A person throws a ball, modelled as a particle, is thrown at 15 ms^{-1} upward. The ball leaves his hand 1.5 m above the ground.

- What is the highest point, above the ground, that the ball reaches?
- For how long is the ball 5 m , above the ground?

a)

$$s = 5$$

$$u = 15 \text{ ms}^{-1}$$

$$v = 0$$

$$a = -9.8$$

$$t = -$$

$$v^2 = u^2 + 2as$$

$$0 = 15^2 + 2(-9.8)s$$

$$s = \frac{-225}{-19.6} = 11.47 \text{ m}$$

$$\text{Above ground} = 11.47 + 1.5 = \cancel{12.97} \text{ m}$$

$$= 13.0 \text{ m (to 3 s.f.)}$$

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Vertical Motion

A person throws a ball, modelled as a particle, is thrown at 15 ms^{-1} upward. The ball leaves his hand 1.5 m above the ground.

The person misses the catch.

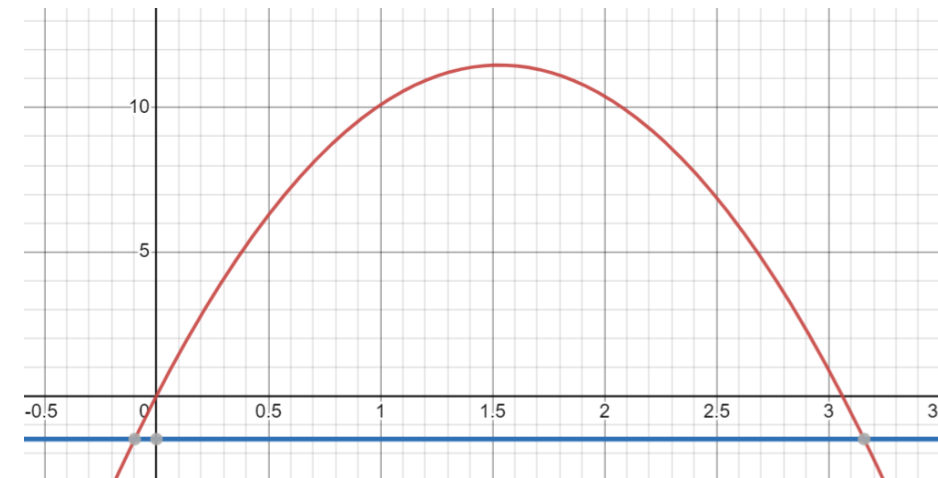
c) How long, after the launch does it hit the ground?

d) With what speed does it first hit the ground?

$$\begin{array}{l}
 c) \quad s = -1.5 \text{ m} \\
 u = 15 \text{ ms}^{-1} \\
 \uparrow \\
 v = \\
 a = -9.8 \text{ ms}^{-2} \\
 \text{+ve} \\
 t = t
 \end{array}$$

$$\begin{array}{l}
 s = ut + \frac{1}{2}at^2 \\
 -1.5 = 15t + \frac{1}{2} \times (-9.8)t^2 \\
 0 = -4.9t^2 + 15t + 1.5 \\
 t = 3.16 \text{ s}, \quad -0.10 \text{ s}
 \end{array}$$

$$\begin{array}{l}
 y = -4.9t^2 + 15t + 1.5 \\
 y = -1.5
 \end{array}$$



For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

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$$v^2 = u^2 + 2as$$

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c) How long, after the launch does it hit the ground?

d) With what speed does it first hit the ground?

c)

$$s = +1.5 \text{ m}$$

$$u = -15 \text{ ms}^{-1}$$

$$v =$$

$$a = +9.8 \text{ ms}^{-2}$$

$$t = t$$

\downarrow
+ve

$$s = ut + \frac{1}{2}at^2$$

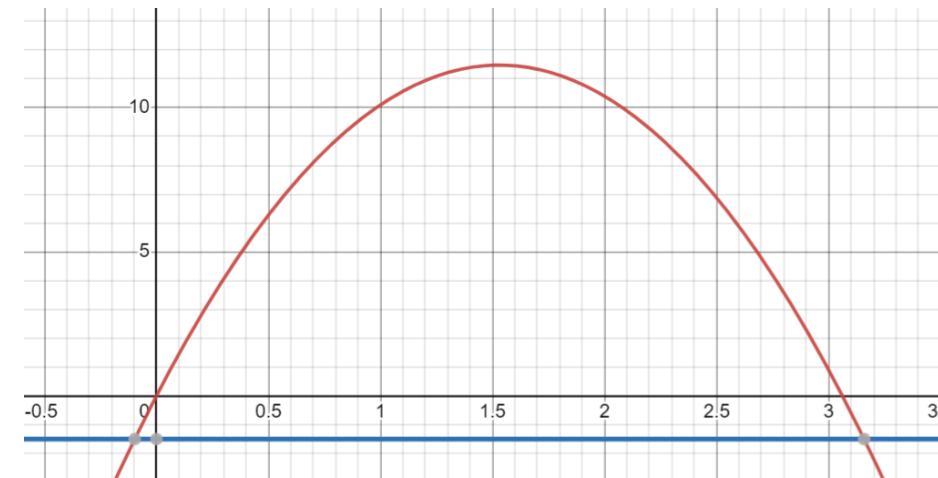
$$+1.5 = -15t + \frac{1}{2} \times (+9.8)t^2$$

$$0 = +4.9t^2 - 15t - 1.5$$

$$t = 3.16 \text{ s}, -0.10 \text{ s}$$

$$y = -4.9t^2 + 15t + 1.5$$

$$y = -1.5$$



For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

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d)

$$s = \text{---}$$

$$u = 15 \text{ ms}^{-1}$$

$$v = v$$

$$a = -9.8$$

$$t = 3.16 \text{ s}$$

↑
+ve

$$v = u + at$$

$$v = 15 + (-9.8) \times 3.16$$

$$v = -15.9 \text{ ms}^{-1}$$

Speed = 15.9 ms^{-1}

d)

$$s = -1.5$$

$$u = 15 \text{ ms}^{-1}$$

$$v = v$$

$$a = -9.8$$

$$t = \text{---}$$

↑
+ve

$$v^2 = u^2 + 2as$$

$$v^2 = 15^2 + 2(-9.8)(-1.5)$$

$$v^2 = 254.4$$

$$v = 15.9 \text{ ms}^{-1}$$

Vertical Motion

Object A is projected vertically upwards at 50ms^{-1} from the base of a tower 130 m high.

At the same instant object B is dropped from the top of the tower.

At what height, above the ground, do they collide?

A

$$s = s_A$$
$$u = 50\text{ms}^{-1}$$
$$v = \text{---}$$
$$a = -9.8\text{ms}^{-2}$$
$$t = t$$

+ve ↑

$$s = ut + \frac{1}{2}at^2$$
$$s_A = 50t + \frac{1}{2}(-9.8)t^2$$
$$s_A = 50t - 4.9t^2$$

B

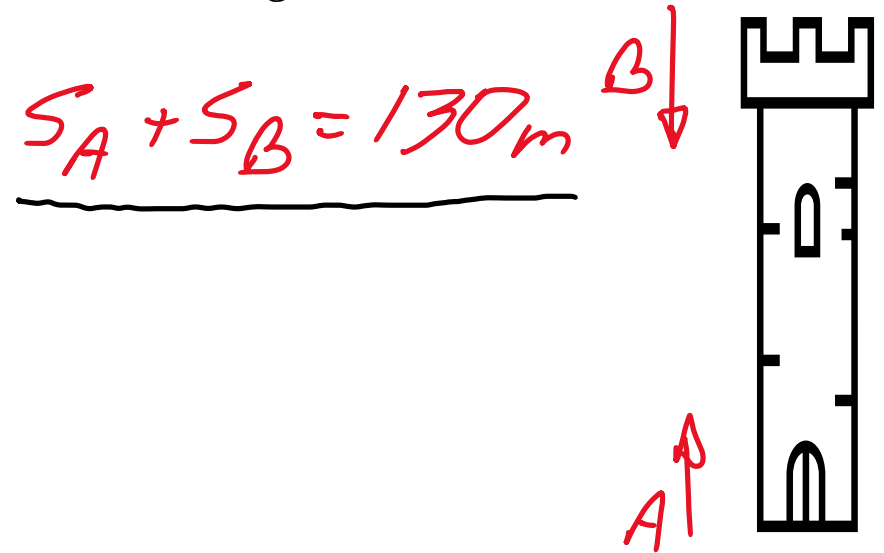
$$s = s_B$$
$$u = 0$$
$$v = \text{---}$$
$$a = 9.8\text{ms}^{-2}$$
$$t = t$$

+ve ↓

$$s = ut + \frac{1}{2}at^2$$
$$s_B = 0t + \frac{1}{2}9.8t^2$$
$$s_B = 4.9t^2$$

$$50t - 4.9t^2 + 4.9t^2 = 130$$

$$50t = 130 \Rightarrow t = \frac{130}{50} = 2.6\text{s}$$



An Interesting Question!

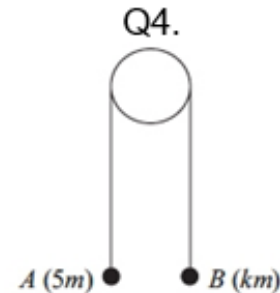


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(b) Find the value of k .

(3)

(c) State how you have used the information that the pulley is smooth.

(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.

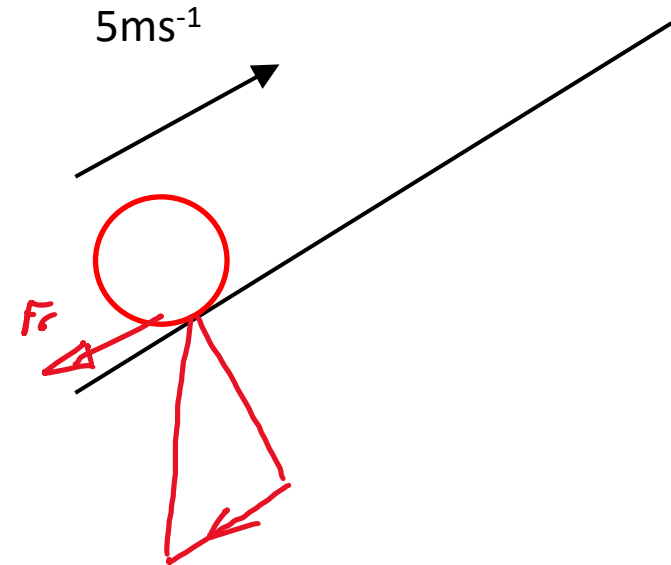
(7)

Other misconceptions

An object, of mass 3 kg is projected up a slope, of 30° at a velocity of 5 ms^{-1}

The coefficient of friction between the object, and the slope is 0.1.

Find the time taken for the object to return to its original position



\nearrow
+ve

$$s = 0$$

$$u = 3 \text{ ms}^{-1}$$

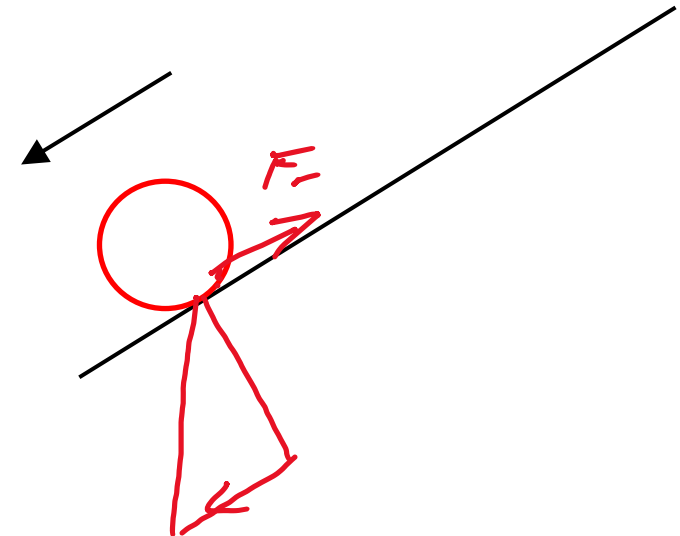
$$v = -$$

$$a = -5.75 \text{ ms}^{-2}$$

$$t = t$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = \dots$$



Projectiles

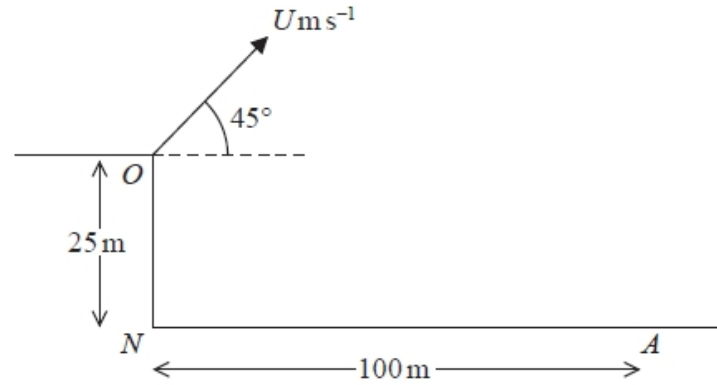


Figure 2

A small ball is projected with speed $U\text{ m s}^{-1}$ from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A , where $AN = 100\text{ m}$, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

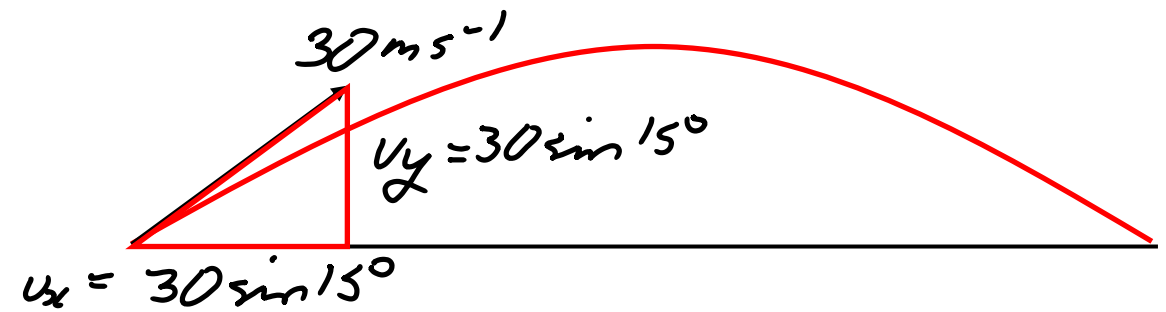
(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA .

(3)

Projectiles



An object is projected at 30 ms^{-1} , at an angle of 15° to a horizontal surface. How far does it travel?

\uparrow
 +ve

$$s = 0$$

$$u = 30 \sin 15^\circ \text{ ms}^{-1} \quad s = ut + \frac{1}{2} at^2$$

$$v = -$$

$$a = -9.8 \text{ ms}^{-2} \quad 0 = 30 \sin 15^\circ t + \frac{1}{2} (-9.8) t^2$$

$$t = t \quad 0 = -4.9 t^2 + 30 \sin 15^\circ t$$

$$t = 1.58, 0 \text{ s}$$

\rightarrow
 +ve

$$s = s_x$$

$$u = 30 \cos 15^\circ \text{ ms}^{-1} \quad s = ut + \frac{1}{2} at^2$$

$$v = -$$

$$a = 0 \quad s_x = 30 \cos 15^\circ \times 1.58$$

$$t = 1.58 \quad s_x = 45.9 \text{ m (to 3sf)}$$

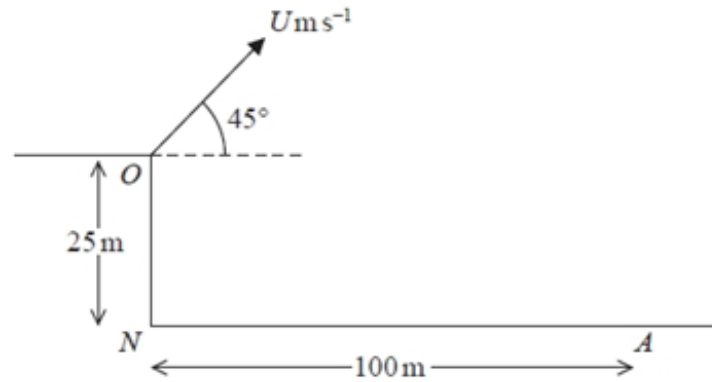


Figure 2

A small ball is projected with speed $U\text{ m s}^{-1}$ from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A , where $AN = 100\text{ m}$, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA .

(3)

(a) show that $U = 28$

(b) find the greatest height of the ball above the horizontal ground NA.

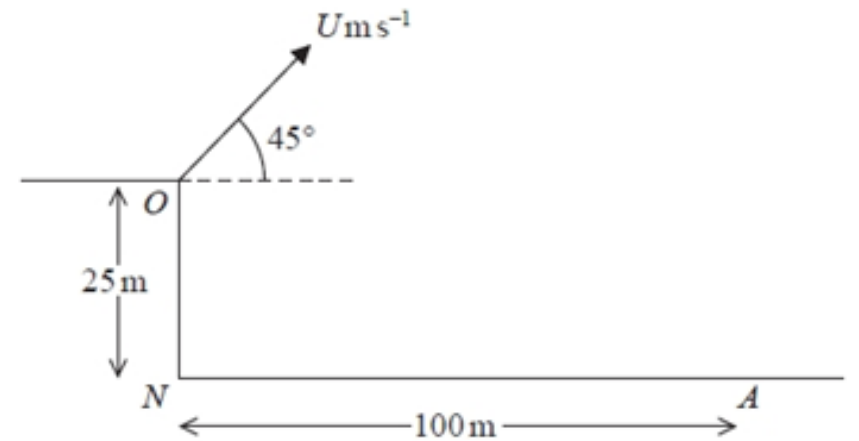


Figure 2

$$\begin{aligned}
 s &= 100 \text{ m} \\
 \rightarrow u &= u \cos 45^\circ \text{ m s}^{-1} \\
 +ve \quad v &= - \\
 u &= 0 \\
 t &= t
 \end{aligned}$$

$$s = ut + \frac{1}{2}at^2$$

$$100 = u \cos 45^\circ \times t$$

$$100 = \frac{\sqrt{2}}{2} u \times t$$

$$t = \frac{200}{u\sqrt{2}}$$

$$\begin{aligned}
 s &= -25 \text{ m} \\
 \uparrow u &= u \sin 45^\circ \text{ m s}^{-1} \\
 v &= - \\
 +ve \quad a &= -9.8 \text{ m s}^{-2} \\
 t &= t
 \end{aligned}$$

$$s = ut + \frac{1}{2}at^2$$

$$-25 = u \sin 45^\circ t + \frac{1}{2}(-9.8)t^2$$

$$-25 = u \frac{\sqrt{2}}{2} t - 4.9 t^2$$

$$-25 = u \frac{\sqrt{2}}{2} \times \frac{200}{u\sqrt{2}} - 4.9 \left(\frac{200}{u\sqrt{2}} \right)^2$$

$$-25 = 100 - \frac{98000}{u^2} \Rightarrow$$

$$\frac{98000}{u^2} = 125$$

$$u = \sqrt{\frac{98000}{125}} = 28 \text{ m s}^{-1}$$

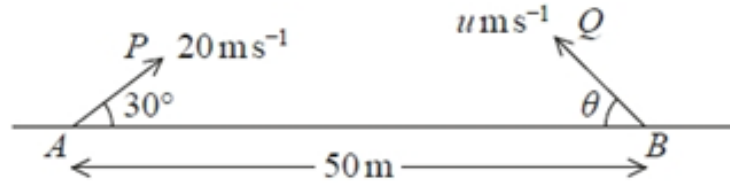


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q , are projected in the vertical plane containing AB .

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB .

Ball Q is projected from B with speed $u \text{ m s}^{-1}$ at angle θ to BA , as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q .

(6)

(b) Find

- (i) the size of angle θ ,
- (ii) the value of u .

(6)

General Equation of a Projectile

$$\begin{aligned}
 s &= y \\
 u &= u \sin \theta \\
 v &= \\
 a &= -g \\
 t &= t
 \end{aligned}$$

$$\begin{aligned}
 s &= ut + \frac{1}{2}at^2 \\
 y &= u \sin \theta t - \frac{g}{2}t^2
 \end{aligned}$$

$$\begin{aligned}
 s &= x \\
 u &= u \cos \theta \\
 v &= \\
 a &= 0 \\
 t &= t
 \end{aligned}$$

$$\begin{aligned}
 s &= ut + \frac{1}{2}at^2 \\
 x &= u \cos \theta \times t \\
 t &= \frac{x}{u \cos \theta}
 \end{aligned}$$

Initial Velocity $u \text{ ms}^{-1}$

Angle to the horizontal θ°



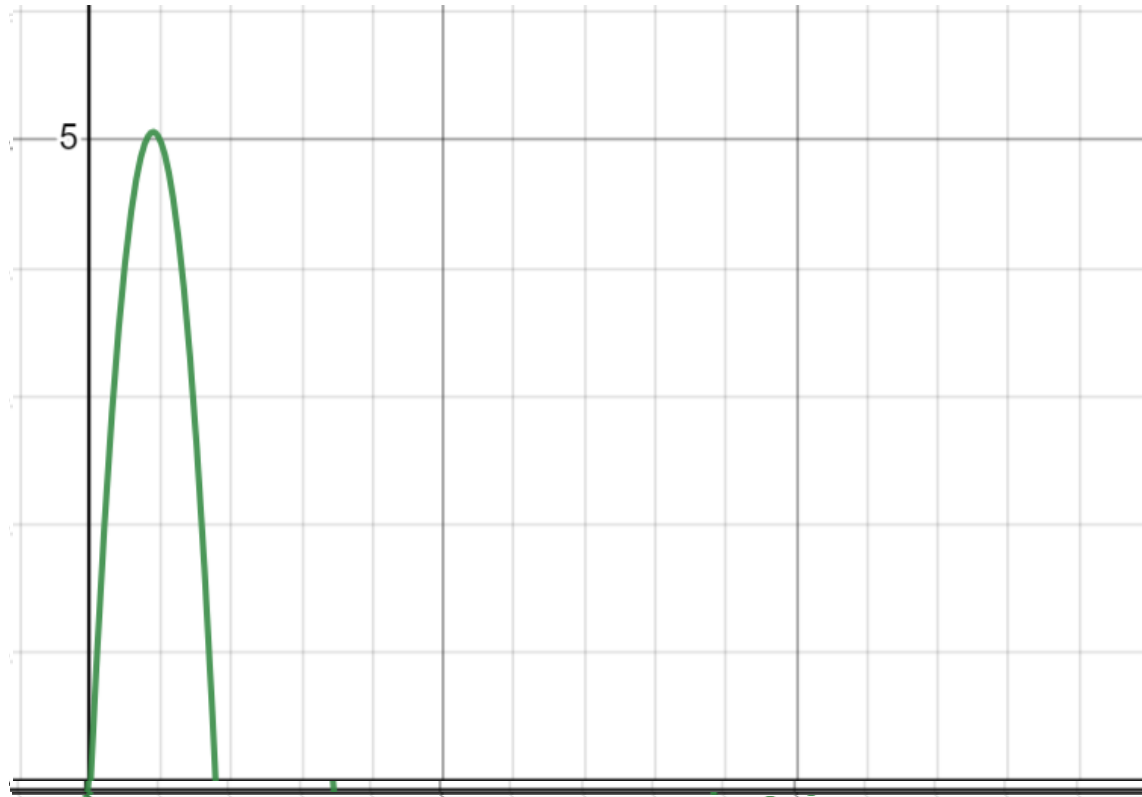
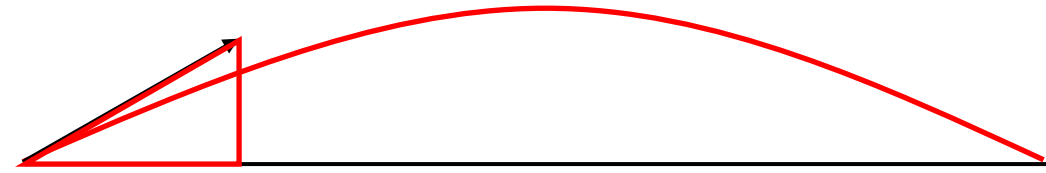
$$y = u \sin \theta \left(\frac{x}{u \cos \theta} \right) - \frac{g}{2} \left(\frac{x}{u \cos \theta} \right)^2$$

$$y = x \tan \theta - \frac{g}{2} \frac{x^2}{u^2} \sec^2 \theta$$

$$y = x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$

General Equation of a Projectile

$$y = x \tan \theta - \frac{g x^2}{2u^2} (1 + \tan^2 \theta)$$



Use of Vectors

A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i} \text{ m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j}) \text{ m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

$$\begin{aligned} s &= - \\ u &= 4\mathbf{i} \text{ m s}^{-1} \\ v &= v \\ a &= 2\mathbf{i} - 3\mathbf{j} \text{ m s}^{-2} \\ t &= 2 \end{aligned}$$

$$v = u + at$$

$$v = 4\mathbf{i} + (2\mathbf{i} - 3\mathbf{j}) \times 2$$

$$v = 8\mathbf{i} - 6\mathbf{j} \text{ m s}^{-1}$$

$$\begin{aligned} s &= - \\ u &= \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{ m s}^{-1} \\ v &= - \\ a &= \begin{pmatrix} 2 \\ -3 \end{pmatrix} \text{ m s}^{-2} \\ t &= 2 \end{aligned}$$

$$v = u + at$$

$$v = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \times 2$$

$$v = \begin{pmatrix} 8 \\ -6 \end{pmatrix} = 8\mathbf{i} - 6\mathbf{j} \text{ m s}^{-1}$$

Calculus

A particle P is moving in a plane. At time t seconds, P is moving with velocity \mathbf{v} m s⁻¹, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$

(2)

(b) the acceleration of P when $t = 4$

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$

(5)

$$\begin{aligned} \text{b) } \mathbf{a} &= 2\mathbf{i} - 6t\mathbf{j} \\ t &= 4 \\ \mathbf{a} &= 2\mathbf{i} - 6 \times 4\mathbf{j} \\ &= 2\mathbf{i} - 24\mathbf{j} \text{ m s}^{-2} \end{aligned}$$

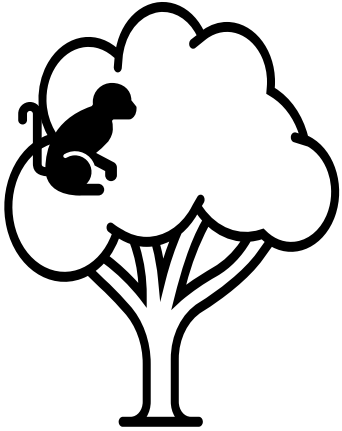
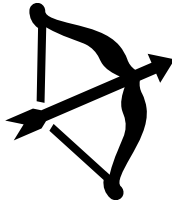
$$\begin{aligned} \text{c) } \mathbf{s} &= \int \mathbf{v} dt = \int (2t\mathbf{i} - 3t^2\mathbf{j}) dt \\ &= \frac{2t^2}{2}\mathbf{i} - \frac{3t^3}{3}\mathbf{j} + \mathbf{C} \\ &= t^2\mathbf{i} - t^3\mathbf{j} + \cancel{\mathbf{C}}\mathbf{i} + \mathbf{C}\mathbf{j} \end{aligned}$$

When $t = 1$

$$\begin{aligned} -4\mathbf{i} + \mathbf{j} &= 1^2\mathbf{i} - 1^3\mathbf{j} + \mathbf{C}\mathbf{i} + \mathbf{C}\mathbf{j} \\ -5\mathbf{i} + 2\mathbf{j} &= \mathbf{C}\mathbf{i} + \mathbf{C}\mathbf{j} \Rightarrow \dots \end{aligned}$$



Stretch!!



Stretch!!

You drop a stone down a well and it takes 5 s before you hear it hit the water?

Using a simple model, how deep is the well?

Do you think your answer is an under, or over, estimate

Sound travels at 340 ms^{-1}

Calculate the depth of the well.

$$T = T_{\text{STONE}} + T_{\text{SOUND}}$$

Marking Guidance

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ OR integration: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$, with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i} + \mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2 \right]$ OR: find \mathbf{v} at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $(\mathbf{i} + \mathbf{j}) + \left[\frac{1}{2} [4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3 \right]$	M1	3.1a

Mechanics Teaching Guide

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Teaching Mechanics Guide

Dropping a ball bearing into a water t

Dropping a ball from a balcony (link to

Software for analysing motion

[Tracker Video Analysis and Modelling](#)

Equate i components

$$\vec{OS} = (-4 + 3t)\mathbf{i} + (2 + 3t)\mathbf{j}$$

The correct process is equating components.

$$\therefore -4 + 3t = 6 - 2t$$

$$\therefore 3t + 2t = 6 + 4$$

$$\therefore 5t = 10$$

$$\therefore t = 2$$

Equate j components

$$2 + 3t = 1 + 3.5t$$

$$\therefore 2 - 1 = 3.5t - 3t$$

$$\therefore 1 = 0.5t$$

$$\therefore \underline{2} = t$$

The two ships are at the point $2\mathbf{i} + 8\mathbf{j}$ at the same time and therefore the two ships meet.

Alternatively, to determine whether the two ships meet we could calculate the position vector of one ship relative to the other ship \vec{SP} or \vec{PS} (it doesn't matter which) and use Pythagoras' theorem to find its length in order to find out whether the distance between the two ships can be zero. The previous method is far simpler in the case where two objects do not meet. This method is more difficult but very important for your students. Its application to this question would look like:

The position vector of ship P relative to ship S is:

$$\vec{SP} = \vec{OP} - \vec{OS}$$

$$\vec{SP} = (6 - 2t)\mathbf{i} + (1 + 3.5t)\mathbf{j} - [(-4 + 3t)\mathbf{i} + (2 + 3t)\mathbf{j}]$$

$$\therefore \vec{SP} = (10 - 5t)\mathbf{i} + (-1 + 0.5t)\mathbf{j}$$

Annotations:

- Important notation to preface their work and indicate they are equating i or j components. They could start with either.
- It is not necessary to include i or j here.
- It's important to have a final concluding statement
- Students will often need reminding which vector to subtract. A useful way to remember this is final - start, which is true for any change in maths.
- Many students forget the extra brackets, commonly leading to sign errors in their manipulation

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Exemplars



Pearson Edexcel
Level 3 Advanced GCE in Mathematics

May/June 2022 Exemplar
9MA0-32 A level Mathematics
Paper 32: Mechanics
Senior Examiner's feedback on student

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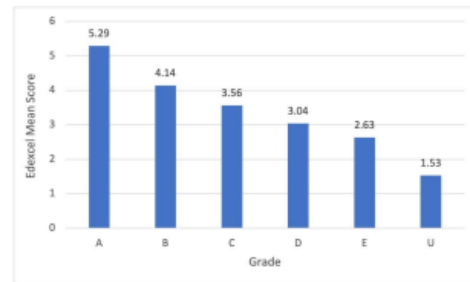
Question: [1](#) [2](#) [3](#) [4](#)

Question 1 - Examiner Comments

Candidates who had not read the question and used g as 9.8 or 9.81 were few and far between. Part (a) was generally well done with most candidates using the figures 10, 10 and 1.8 in a correct constant acceleration formula to obtain the given ' $U = 8$ '. Occasionally there was a sign error and although the correct answer was quoted, it did not actually follow from the working. A few used 0 as opposed to 10 for the final velocity. Some considered the motion 'up' and 'down' separately and used the distances to successfully derive the value of ' U '. The most common approach in part (b) was to write down a quadratic equation in t and to solve it using the quadratic formula. There were occasional sign errors in the equation, and some were either unable to deal with the quadratic or misquoted the formula. Nevertheless, a significant number did successfully find the two values of t and gave the correct one as their answer. The alternative approach of 'up' and 'down' separately was seen, but often only one of the times was calculated correctly. The other most common method being use of $v = u + at$ but a significant number made a sign error in their equation and 0.2 was a very common wrong answer. In part (c), correct refinements seemed to be in the majority but only just. A more accurate value of gravity (9.8) was the most common correct response, but wind, spin and dimensions of the stone were also seen. Many reasoned that due to air resistance the acceleration was smaller making the velocity smaller. Very few considered how changing the value of a in the equation they used would affect their answer for U . Weight/mass and force were the most common incorrect answers, often being given as an incorrect extra answer to what would have been a correct one, and as a result losing the B mark. In the final part, correct answers were in the minority, with most candidates failing to appreciate that if there was air resistance, the stone would have to be projected faster in order to still hit the ground at 10 ms^{-1} .

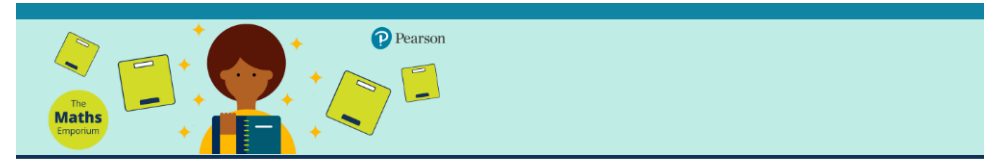
Question 1 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade:						
			ALL	A	B	C	D	E	U
3.64	7	52	3.64	5.29	4.14	3.56	3.04	2.63	1.53



Q1

- [?](#)
- [✓](#)
- [☰](#)
- [📊](#)
- [📄](#)
- [A](#)
- [B](#)
- [C](#)



Maths Emporium > GCE AS/A level Mathematics

Category: GCE AS/A level Mathematics

8MA0 and 9MA0 GCE AS/A level Mathematics documents for current specification, for first teaching September 2017 and first assessment summer 2018.

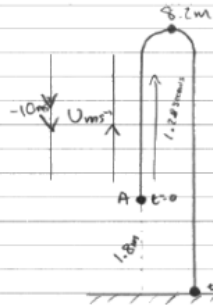
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Exemplars

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Question: 1 2 3 4

Question 1 - Response A



Part (a)

M0: Incorrect method to produce an equation in U only.

A0: Follows M0.

A0: Follows M0.

Part (b)

M0: Incorrect method to produce an equation in T only. Seems to look at time up and time down but distances used are not justified.

A0: Follows M0.

Part (c)

B0: Incorrect refinement. Any mention of mass is B0.

Part (d)

B1: Correct explanation.

a) S 8.2
 U U
 V 0
 A -10
 T —

$$v^2 = u^2 + 2as$$

$$0 = u^2 + 2(-10)1$$

$$-u^2 = -164$$

$$u^2 = 164$$

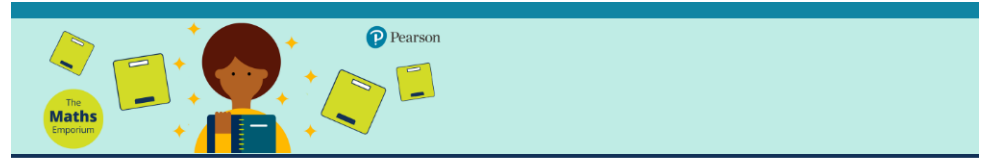
$$u = \sqrt{164}$$

$$u = 12.8 \text{ (3s)}$$

b) S 8.2 $v = u + at$ S 10 $v = u + at$
 U 12.8 $0 = 12.8 + (-10)(T)$ U 0 $0 = u + at$
 V 0 $-12.8 = -10T$ V 0 $10 = 0t + \frac{1}{2}(-10)(t)^2$
 A -10 $+1.28 = T$ A +10 $t = 1.414$
 T T T T $t = 1.41$

$$1.28 + 1.41 = 2.69 \text{ seconds}$$

$$T = 2.69$$



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8MA0 and 9MA0 GCE AS/A level Mathematics documents for current specification, for first teaching September 2017 and first assessment summer 2018.

01 Specifications	02 Teacher Guidance	03 Schemes of Work and Mappings	04 Specimen and SAMs	05 Mock Papers	06 Past Papers and Mark Schemes NEW	07 Examiner Reports and Exemplars
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15 Professional Development Materials						

Scheme of Work

**7a. Graphical representation of velocity, acceleration and displacement
(7.1) (7.2)**

Teaching time
4 hours

USE OF TECHNOLOGY

Calculator Activities:

Velocity-Time Graphs

<https://education.casio.co.uk/resources/leaflet/pearson-displacement-trapezium/>

GeoGebra:

Explore how the area of the trapezium changes as the value of T changes

<https://ggbm.at/VXVu8zEm>

**7b. Motion in a straight line under constant acceleration; *suvat*
formulae for constant acceleration; Vertical motion under gravity (7.3)
(8.3)**

Teaching time
6 hours

USE OF TECHNOLOGY

Calculator Teaching Resources:

Constant Acceleration Equations (Includes: 1st exercise is 9.4. 2nd exercise has some 9.5. Covers sections 9.4 Equations of motion 2 and 9.5 Vertical motion under gravity)

https://education.casio.co.uk/resources/leaflet/constant-acceleration-equations?utm_source=ext_pearson&utm_medium=referral&utm_campaign=resources_pearson&utm_content=partner&utm_term=graphic_calculator

OBJECTIVES

By the end of the sub-unit, students should:

- recognise when it is appropriate to use the *suvat* formulae for constant acceleration;
- be able to solve kinematics problems using constant acceleration formulae;
- be able to solve problems involving vertical motion under gravity.

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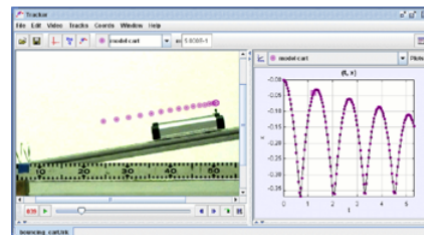
Windows 11 users, having Microsoft Defender issues? See [Installer Help](#) [Change Log](#) [Discussion Forum](#)

Tip: save your work as a [Tracker Project](#). Easy to build and share. Easy to browse in the [Library Browser](#).

What is Tracker?

Tracker is a free video analysis and modeling tool built on the [Open Source Physics](#) (OSP) Java framework. It is designed to be used in physics education.

Tracker **video modeling** is a powerful way to combine videos with computer modeling. For more information see [Particle Model Help](#) or AAPT Summer Meeting posters [Video Modeling](#) (2008) and [Video Modeling with Tracker](#) (2009).



Tracker Features

Tracking:

- Manual and automated object tracking with position, velocity and acceleration overlays and data.
- Center of mass tracks.
- Interactive graphical vectors and vector sums.
- RGB line profiles at any angle, time-dependent RGB regions.

Modeling:

- Model Builder creates kinematic and dynamic models of point mass particles and two-body systems.
- External models animate and overlay multi-point data from separate modeling programs such as spreadsheets and [Easy Java Simulations](#).
- Model overlays are automatically synchronized and scaled to the video for direct visual comparison with the real world.

Video:

- Free Xuggle video engine plays and records most formats (mov/avi/flv/mp4/wmv etc) on



Pearson