

Getting Started September 2007

GCE Mathematics

Edexcel Advanced Subsidiary GCE in Mathematics (8371)
Edexcel Advanced Subsidiary GCE in Further Mathematics (8372)
Edexcel Advanced Subsidiary GCE in Pure Mathematics (8373)
Edexcel Advanced Subsidiary GCE in Further Mathematics
(Additional) (8374)
First examination 2009

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Edexcel GCE e-Spec

Your free e-Spec

This specification comes with a free e-Spec, Edexcel's electronic version of the specification. You will find the e-Spec disc inside the Specification book for this qualification.

Everything you need in one CD

The e-Spec provides a range of useful resources including:

- A Senior Examiner explaining the changes to the new specification
- A customisable student guide to help recruit students
- A course planner to make it easy to plan delivery
- Links to sample assessment materials so you can see what is expected
- Information on the products and services provided by Edexcel to support the specification.

Easy-to-use

Just click on the walkthrough to see how easy and useful the e-Spec is and get more out of this specification today.

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Introduction

This Getting Started book will give you an overview of the new Edexcel GCE in Mathematics and what it means for you and your students.

Key principles

The specification has been developed with the following key principles:

Focus on choice

- 18 units tested fully by written examination
- All units equally weighted, allowing many different combinations of units and greater flexibility
- Choice of pathways leading to full Advanced Subsidiary (AS) and Advanced (A level) GCE in Mathematics, Further Mathematics, Pure Mathematics and Further Mathematics (Additional), so you can choose the most appropriate pathway for your students

Well supported

- Past papers, specimen papers, examiner reports and further support materials available
- A variety of endorsed electronic support material, including Exam Wizard, Topic Tutor and Exam Tutor
- Endorsed textbooks and revision books, as well as information on how to map updated units to current textbooks
- Substantial professional development and training programme

Straightforward assessment

- One written examination per unit
- Each examination paper lasts 1 hour 30 minutes
- Each examination paper has 75 marks
- Calculators can be used for all unit examinations except C1

What's new?

This specification is very similar to the previous specification, but includes new Decision Mathematics unit content and updated Further Pure Mathematics unit content.

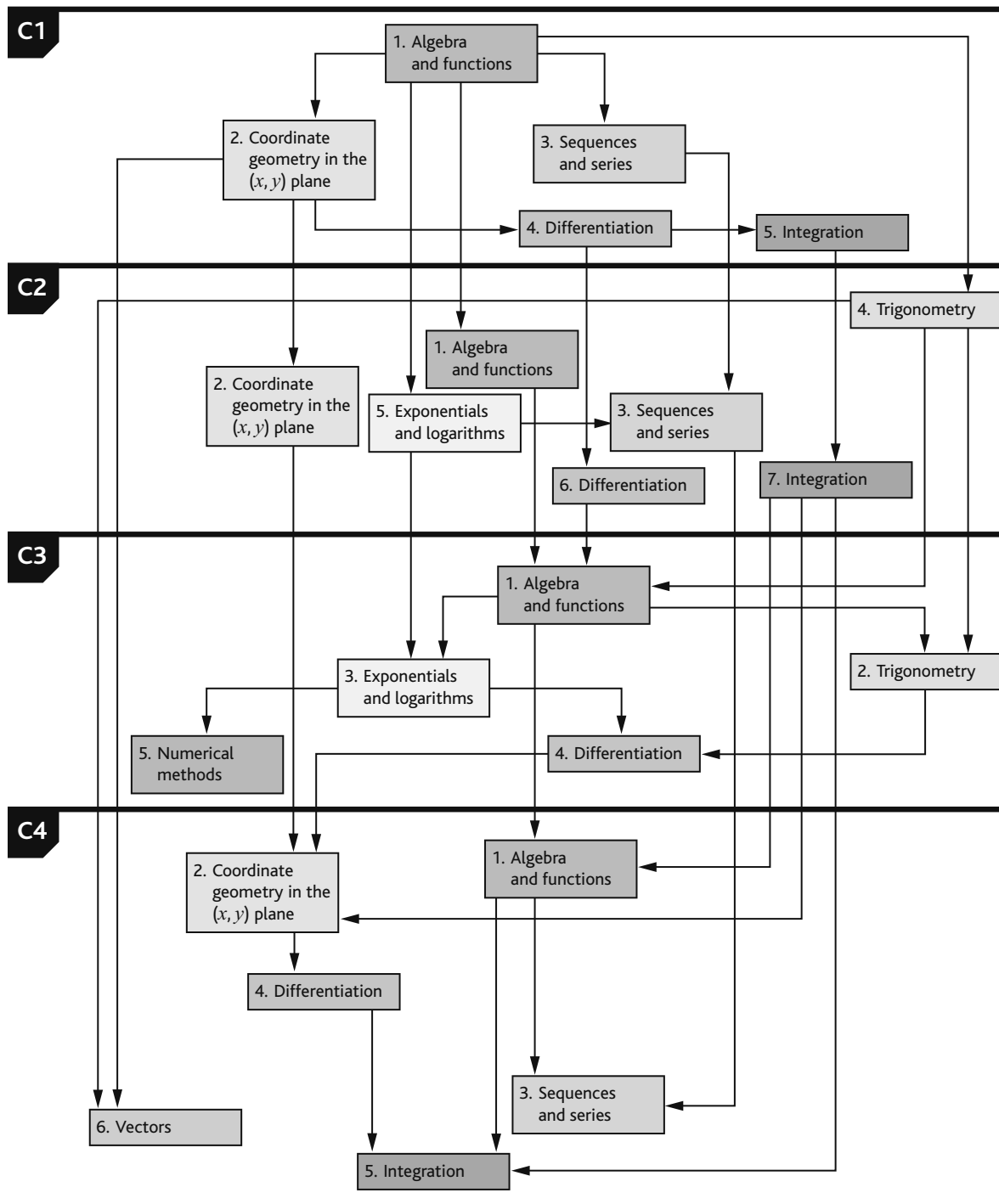
In summary, this specification offers:

- No change to Core, Mechanics or Statistics unit content
- Updated Decision Mathematics 1 and Decision Mathematics 2 units, giving a more balanced approach to the content
- Updated Further Pure Mathematics 1 unit for teaching in the first year of study
- Updated Further Pure Mathematics 2 and Further Pure Mathematics 3 units to offer a coherent curriculum in Further Mathematics

Course overviews

These course overviews have been developed to help you plan the organisation and delivery of the course.

Core mathematics: C1 to C4 progression



Mechanics progression

M1

Facility in algebraic manipulation and the ability to solve linear, quadratic and simultaneous equations as specified in C1 is an essential prerequisite for this unit.

| Paragraph | Description | Prerequisites |
|-----------|---|-------------------------|
| 1 | Modelling | None |
| 2 | Kinematics | C1 Paragraph 1, Algebra |
| 3 | Vectors | C1 Paragraph 1, Algebra |
| 4 | Dynamics Momentum & impulse | C1 Paragraph 1, Algebra |
| 5 | Statics of a particle (Dynamics with $a = 0$) | C1 Paragraph 1, Algebra |
| 6 | Moments | C1 Paragraph 1, Algebra |

M2

Knowledge of the M1 specification and the algebra, trigonometry, differentiation and integration as specified in C1 and C2 are essential for this unit.

| Paragraph | Description | Prerequisites |
|-----------|--|--|
| 1 | Projectiles | M1 Paragraph 3 |
| 1 | Kinematics with variable velocity/acceleration | C1 Paragraph 4, 5 C2 Paragraph 6, 7 |
| 2 | Centres of mass | M1 Paragraph 6 |
| 3 | Work and energy | M1 Paragraph 4, 5 |
| 4 | Collisions Momentum & impulse | M1 Paragraph 4 |
| 5 | Statics of rigid bodies | M1 Paragraph 5, 6 |

M3

Knowledge of the M1 and M2 specifications and the differentiation, integration and differential equations as specified in C1, C2 and C3 are essential for this unit.

| Paragraph | Description | Prerequisites |
|-----------|--|--|
| 2 | Elastic strings and springs | M1 Paragraph 3 |
| 4 | Motion in a circle Horizontal circles Vertical circles | M3 Paragraph 2 |
| 5 | Centres of mass of rigid bodies Statics of rigid bodies | M2 Paragraph 2 M2 Paragraph 5 |
| 1 | Further kinematics | M2 Paragraph 1 Solution of differential equations as specified on C1, C2 and C3 |
| 3 | Variable force Simple Harmonic Motion | M3 Paragraph 1 |

M4

Knowledge of the M1, M2 and M3 specifications and the calculus on FP1 together with

$\int \frac{dx}{a^2 + x^2}$; $\int \frac{dx}{\sqrt{a^2 + x^2}}$ are essential for this unit.

| Paragraph | Description | Prerequisites |
|-----------|--|--|
| 2 | Elastic collisions in 2D | M2 Paragraph 4 |
| 3 | Further particle motion in a straight line | FP1 Paragraph 5, 6 M3 Paragraph 2 |
| 4 | Stability | M2 Paragraph 3 M3 Paragraph 2 C3 Paragraph 4 |
| 1 | Relative motion | M1 Paragraph 2 C4 Paragraph 6 |

M5

Knowledge of the M1, M2, M3 and M4 specifications, the calculus on FP1 and the scalar and vector products are essential for this unit.

| Paragraph | Description | Prerequisites |
|-----------|-----------------------------|---|
| 1 | Application of vectors | FP1 Paragraph 5, 6 M2 Paragraph 3 C3 Paragraph 4 FP3 Paragraph 3 |
| 3 | Moments of inertia | Integration on C1 to 4 |
| 4 | Rotation about a fixed axis | M3 Paragraph 4 M4 Paragraph 3 |
| 2 | Variable mass | M1 Paragraph 4 (Momentum & impulse) FP1 Paragraph 5, 6 |

Statistics progression

S1

| Paragraph | Topic | Prerequisites | Notes |
|-----------|--|--|--|
| 1 | Mathematical modelling | | Can be a starting point but usually better looked at towards the end, as an overview of the content. |
| 2 | Representation and summary of data | Understanding of Σ notation from C1 | Sensible starting point. There are links to GCSE e.g. the diagrams, mean, median etc. Suggest leave coding until Paragraph 5b. |
| 3 | Probability | | A good alternative starting point. Again links to GCSE although style may be different and $P(A B)$ is new. |
| 4a | Correlation | Mean and standard deviation from S1 Paragraph 2 | |
| 4b | Regression | S_{xx} and S_{xy} etc from S1 Paragraph 4a and Paragraph 2 Coordinate geometry of straight line from C1 | |
| 5a | Discrete probability distributions | Probability from S1 Paragraph 3 | |
| 5b | Mean and variance of discrete random variables | Mean and variance from S1 Paragraph 2 | Suggest use $E(aX + b)$ and $\text{Var}(aX + b)$ formula to deal with coding. |
| 5c | Discrete uniform distribution | Arithmetic series ideas from C1 can be useful here. | |
| 6 | Normal distribution | Mean and standard deviation and histograms from S1 Paragraph 2 Probability from S1 Paragraph 3 Total probability = 1 from S1 Paragraph 5 | This topic is often left until the end and candidates find it challenging. |

Possible Routes through S1

1 – 6: Matches the specification order and some text books but by leaving normal distribution to the end, some students may not grasp this content fully.

2, 3, 5, 6, 4, 1 : Correlation and regression only depend on the mean and standard deviation material and so can be left until the end. Probability followed by probability distributions doesn't suit everyone's taste.

3, 2, 5, 6, 4, 1: This splits up the probability and brings the normal distribution work a little earlier. There are other possible variations.

S2

| Paragraph | Topic | Prerequisites | Notes |
|-----------|-----------------------------|---|---|
| 1a | Binomial distribution | Discrete probability distributions from S1 Paragraph 5 Binomial theorem from C2 | A good starting point provided work from C2 has been covered. |
| 1b | Poisson distribution | Evaluation of e^x on binomial for approximations from S2 Paragraph 1a S1 Paragraph 5 | |
| 2 | Continuous random variables | Calculus and concept of area under curve and max/min from C1/C2 Parallels with discrete distributions from S1 Paragraph 5 and normal distributions from S1 Paragraph 6 | A reasonable alternative starting point. Depends on later work in C2 though. Max and min is only needed for some mode questions. |
| 3a | Continuous distributions | S2 Paragraph 2 | Rectangular distribution should be compared with discrete uniform distribution. |
| 3b | Normal approximations | Normal distribution from S1 Paragraph 6 and S2 Paragraph 1a and 1b | |
| 4 | Hypothesis testing | Overview of statistics from S1 Paragraph 1 and S2 Paragraph 1a and 1b | This is a key idea and pupils need time to assimilate it and practice the techniques. |

Possible Routes through S2

Most students will have already covered C1, C2 and S1. Some Further Mathematics students may still be covering C1/C2 when they start and the first suggestion is recommended.

1, 4, 2, 3: This introduces hypothesis testing early and only requires the binomial theorem from C2. If C2 is being taught at the same time, it has the advantage of leaving paragraph 2 until later in the course when the C2 calculus has been covered.

1, 2, 3, 4: This follows the order of the specification and some text books and does have the advantage of a break from Binomial and Poisson before using them in hypothesis tests.

2, 3a, 1, 4, 3b: The order of 3a and 1 and of 3b and 4 can be swapped. This makes use of paragraph 2 as an alternative starting point.

Other combinations are possible. If hypothesis testing is covered before normal approximations then care should be taken to avoid questions that require the use of a normal approximation to evaluate a probability in a hypothesis test. The basic idea of hypothesis tests can be covered and most examples require the use of Binomial or Poisson tables.

S3

| Paragraph | Topic | Prerequisites | Notes |
|-----------|---|---|---|
| 1 | Combination of random variables | Normal distribution from S1 Paragraph 6 $E(aX + b)$ and $\text{Var}(aX + b)$ from S1 Paragraph 5 | A good starting point |
| 2 | Sampling | Overview from S1 Paragraph 1 Ideas of sampling frame, population, samples from S2 Paragraph 4 | An alternative starting point. |
| 3 | Estimation, confidence intervals and tests | Hypothesis tests from S2 Paragraph 4 Work on combining random variables from S3 Paragraph 1 | This depends clearly on S3 Paragraph 1 and arguably the work on sampling (S3 Paragraph 2) provides some introduction to the estimation topic. |
| 4 | Goodness of fit tests Contingency table tests | Binomial and Poisson from S2 Paragraph 1 Rectangular distribution from S2 Paragraph 3a Ideas of hypothesis tests from S2 Paragraph 4 Probability for ideas of independence from S1 Paragraph 3 | An alternative starting point. |
| 5 | Spearman's rank correlation coefficient and hypothesis tests for correlation. | Product moment correlation coefficient from S1 Paragraph 4a Ideas of hypothesis test from S2 Paragraph 4 | Another simple starting point. |

Possible Routes through S3

Some knowledge of S2 is required before S3 can be taught, in particular the ideas behind hypothesis testing which are essential. Assuming this has been covered, students can start S3 on any paragraph except 3.

5, 4, 1, 2, 3: Spearman's rank correlation and the hypothesis tests for correlation are possibly the simplest topics in S3 and require little background knowledge. The Chi squared work in paragraph 4 can be introduced without reference to Binomial or Poisson distributions, although these are key examples that they will need to know. But paragraphs 5 and 4 provide suitable and fairly easy starting points for S3. Paragraph 1 needs to come before paragraph 3 but paragraph 2 can be fitted in almost anywhere. This route provides a gentle introduction and covers some key tests that might be useful in other subjects such as biology and geography at the start of the course.

1, 2, 3, 4, 5: This has the advantage of following the order of the specification and some text books, and splits the work on combining random variables with its chief applications in paragraph 3 with the work on sampling which arguably should come before the topics on estimation.

There are other possible variations.



S4

| Paragraph | Topic | Prerequisites | Notes |
|-----------|--|--|--|
| 1 | Quality of tests and Estimators | Combining random variables from S3 Paragraph 1 Estimators from S3 Paragraph 3 Ideas of hypothesis tests from S2 Paragraph 4 Calculus from C1/C2 | Usually the most demanding section in S4. Can be taught independently of the other sections. |
| 2a | One sample t-test and confidence interval | S3 para 3 | Can be taught alongside the parallel test in S3. |
| 2b | One sample test for variance and confidence intervals | Concept of χ^2 distribution from S3 Paragraph 4 | |
| 3a | F-test | Idea of tests for variance from S4 Paragraph 2b | |
| 3b | Test and confidence intervals for difference of two means using t-test | S3 Paragraph 3 S4 Paragraph 2a | This can be taught alongside the parallel test in S3. |
| 3c | Paired t-test | S3 Paragraph 3 S4 Paragraph 2a | This links back to the simple t-test met in S4 Paragraph 2b. |

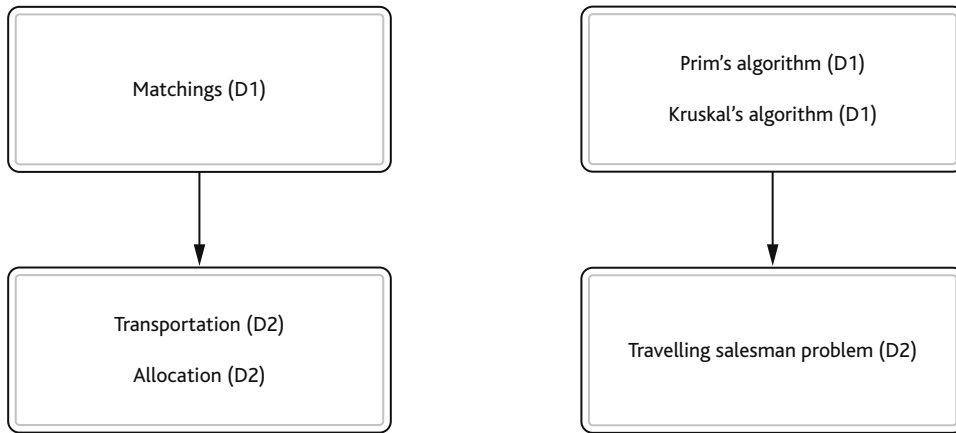
Possible Routes through S4

The course requires a knowledge of paragraph 3 in S3 and parts of S4 can indeed be taught alongside S3. Tests on S3 that assume a knowledge of σ can be taught alongside those from S4 (2a and 3a) that don't require this assumption. Paragraph 1 can be taught at any point in the S4 course.

2, 3, 1: The tests are introduced first and candidates usually find this aspect of the course more straightforward than the work in paragraph 1.

1, 2, 3: This follows the order of the specification and some textbooks and covers the more difficult and theoretical work first.

Decision Mathematics Progression



D1

| Paragraph | Description | Prerequisites |
|-----------|---------------------------------|---------------|
| 1 | Algorithms | None |
| 2 | Prim's and Kruskal's algorithms | None |
| 3 | Route inspection problem | None |
| 4 | Critical path analysis | None |
| 5 | Linear programming | None |
| 6 | Matchings | None |

D2

| Paragraph | Description | Prerequisites |
|-----------|----------------------------------|--|
| 1 | Transportation problems | D1 Paragraph 6 - Matchings |
| 2 | Allocation (assignment) problems | D1 Paragraph 6 - Matchings |
| 3 | Travelling salesman problem | D1 Paragraph 2 - Prim's and Kruskal's algorithms |
| 4 | Further linear programming | None |
| 5 | Game theory | None |
| 6 | Flows in networks | None |
| 7 | Dynamic programming | None |

Most parts of the Decision Mathematics specification can be taught without assuming any prior knowledge; however, some sections in D2 assume knowledge from D1 as shown in the table.



Formulae Booklet

The formulae in this booklet have been arranged according to the unit in which they are first introduced. Thus a candidate sitting a unit may be required to use the formulae that were introduced in a preceding unit (e.g. candidates sitting C3 might be expected to use formulae first introduced in C1 or C2).

It may also be the case that candidates sitting Mechanics and Statistics units need to use formulae introduced in appropriate Core Mathematics units, as outlined in the specification.

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

$$\text{The trapezium rule: } \int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}, \text{ where } h = \frac{b-a}{n}$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

| $f(x)$ | $f'(x)$ |
|--------------------------|--|
| $\tan kx$ | $k \sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\frac{f(x)}{g(x)}$ | $\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ |



Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

| $f(x)$ | $\int f(x) \, dx$ |
|--------------------------|---|
| $\sec^2 kx$ | $\frac{1}{k} \tan kx$ |
| $\tan x$ | $\ln \sec x $ |
| $\cot x$ | $\ln \sin x $ |
| $\operatorname{cosec} x$ | $-\ln \operatorname{cosec} x + \cot x = \ln \tan(\frac{1}{2}x) $ |
| $\sec x$ | $\ln \sec x + \tan x = \ln \tan(\frac{1}{2}x + \frac{1}{4}\pi) $ |

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Coordinate geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\arctan \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

Conics

| | Parabola | Rectangular Hyperbola |
|-----------------|---------------|----------------------------------|
| Standard Form | $y^2 = 4ax$ | $xy = c^2$ |
| Parametric Form | $(at^2, 2at)$ | $\left(ct, \frac{c}{t} \right)$ |
| Foci | $(a, 0)$ | Not required |
| Directrices | $x = -a$ | Not required |

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2} \int r^2 \, d\theta \quad (\text{polar coordinates})$$

Complex numbers

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^r}{r!} f^{(r)}(a) + \dots$$

$$f(a+x) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \dots + \frac{x^r}{r!} f^{(r)}(a) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

Taylor polynomials

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a) + \text{error}$$

$$f(a+h) = f(a) + h f'(a) + \frac{h^2}{2!} f''(a+\xi) \quad (0 < \xi < h)$$

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \text{error}$$

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(\xi) \quad (a < \xi < x)$$

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of \mathbf{a} in the direction of \mathbf{b} is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector \mathbf{b} is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector \mathbf{b} has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1x + n_2y + n_3z + d = 0 \quad \text{where } d = -\mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points A , B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector \mathbf{a} and parallel to \mathbf{b} and \mathbf{c} has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\operatorname{arcosh} x = \ln \left\{ x + \sqrt{x^2 - 1} \right\} \quad (x \geq 1)$$

$$\operatorname{arsinh} x = \ln \left\{ x + \sqrt{x^2 + 1} \right\}$$

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Conics

| | Ellipse | Parabola | Hyperbola | Rectangular Hyperbola |
|-----------------|---|-----------------|--|----------------------------------|
| Standard Form | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | $y^2 = 4ax$ | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $xy = c^2$ |
| Parametric Form | $(a \cos \theta, b \sin \theta)$ | $(at^2, 2at)$ | $(a \sec \theta, b \tan \theta)$ $(\pm a \cosh \theta, b \sinh \theta)$ | $\left(ct, \frac{c}{t} \right)$ |
| Eccentricity | $e < 1$ $b^2 = a^2(1 - e^2)$ | $e = 1$ | $e > 1$ $b^2 = a^2(e^2 - 1)$ | $e = \sqrt{2}$ |
| Foci | $(\pm ae, 0)$ | $(a, 0)$ | $(\pm ae, 0)$ | $(\pm\sqrt{2}c, \pm\sqrt{2}c)$ |
| Directrices | $x = \pm \frac{a}{e}$ | $x = -a$ | $x = \pm \frac{a}{e}$ | $x + y = \pm\sqrt{2}c$ |
| Asymptotes | none | none | $\frac{x}{a} = \pm \frac{y}{b}$ | $x = 0, y = 0$ |

Differentiation

| $f(x)$ | $f'(x)$ |
|---------------------------|---------------------------|
| $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\arccos x$ | $-\frac{1}{\sqrt{1-x^2}}$ |
| $\arctan x$ | $\frac{1}{1+x^2}$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\operatorname{sech}^2 x$ |
| $\operatorname{arsinh} x$ | $\frac{1}{\sqrt{1+x^2}}$ |
| $\operatorname{arcosh} x$ | $\frac{1}{\sqrt{x^2-1}}$ |
| $\operatorname{artanh} x$ | $\frac{1}{1-x^2}$ |

Integration (+ constant; $a > 0$ where relevant)

| $f(x)$ | $\int f(x) \, dx$ |
|----------------------------|--|
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\ln \cosh x$ |
| $\frac{1}{\sqrt{a^2-x^2}}$ | $\arcsin\left(\frac{x}{a}\right) \quad (x < a)$ |
| $\frac{1}{a^2+x^2}$ | $\frac{1}{a} \arctan\left(\frac{x}{a}\right)$ |
| $\frac{1}{\sqrt{x^2-a^2}}$ | $\operatorname{arcosh}\left(\frac{x}{a}\right) = \ln\{x + \sqrt{x^2-a^2}\} \quad (x > a)$ |
| $\frac{1}{\sqrt{a^2+x^2}}$ | $\operatorname{arsinh}\left(\frac{x}{a}\right) = \ln\{x + \sqrt{x^2+a^2}\}$ |
| $\frac{1}{a^2-x^2}$ | $\frac{1}{2a} \ln\left \frac{a+x}{a-x}\right = \frac{1}{a} \operatorname{artanh}\left(\frac{x}{a}\right) \quad (x < a)$ |
| $\frac{1}{x^2-a^2}$ | $\frac{1}{2a} \ln\left \frac{x-a}{x+a}\right $ |

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

Surface area of revolution

$$S_x = 2\pi \int y ds = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

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TURN OVER FOR MECHANICS & STATISTICS FORMULAE

Mechanics M1

There are no formulae given for M1 in addition to those candidates are expected to know.

Candidates sitting M1 may also require those formulae listed under Core Mathematics C1.

Mechanics M2

Candidates sitting M2 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r , angle at centre 2α : $\frac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r , angle at centre 2α : $\frac{2r \sin \alpha}{3\alpha}$ from centre

Mechanics M3

Candidates sitting M3 may also require those formulae listed under Mechanics M2, and also those formulae listed under Core Mathematics C1–C4.

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of mass

For uniform bodies:

Solid hemisphere, radius r : $\frac{3}{8}r$ from centre

Hemispherical shell, radius r : $\frac{1}{2}r$ from centre

Solid cone or pyramid of height h : $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h : $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Universal law of gravitation

$$\text{Force} = \frac{Gm_1m_2}{d^2}$$

Mechanics M4

There are no formulae given for M4 in addition to those candidates are expected to know.

Candidates sitting M4 may also require those formulae listed under Mechanics M2 and M3, and also those formulae listed under Core Mathematics C1–C4 and Further Pure Mathematics FP1.

Mechanics M5

Candidates sitting M5 may also require those formulae listed under Mechanics M2 and M3, and also those formulae listed under Core Mathematics C1–C4 and Further Pure Mathematics FP1.

Moments of inertia

For uniform bodies of mass m :

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3}m(a^2 + b^2)$

Hoop or cylindrical shell of radius r about axis through centre: mr^2

Hoop of radius r about a diameter: $\frac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis through centre: $\frac{1}{2}mr^2$

Disc of radius r about a diameter: $\frac{1}{4}mr^2$

Solid sphere, radius r , about diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x - y plane)

Moments as vectors

The moment about O of \mathbf{F} acting at \mathbf{r} is $\mathbf{r} \times \mathbf{F}$

Statistics S1

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $P(X = x_i)$

Expectation (mean): $E(X) = \mu = \sum x_i P(X = x_i)$

Variance: $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 P(X = x_i) = \sum x_i^2 P(X = x_i) - \mu^2$

For a function $g(X)$: $E(g(X)) = \sum g(x_i) P(X = x_i)$

Continuous distributions

Standard continuous distribution:

| Distribution of X | P.D.F. | Mean | Variance |
|---------------------------|---|-------|------------|
| Normal $N(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | μ | σ^2 |

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\} \{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right) \left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

THE NORMAL DISTRIBUTION FUNCTION

The function tabulated below is $\Phi(z)$, defined as $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}t^2} dt$.

| z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ |
|------|-----------|------|-----------|------|-----------|------|-----------|------|-----------|
| 0.00 | 0.5000 | 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 |
| 0.01 | 0.5040 | 0.51 | 0.6950 | 1.01 | 0.8438 | 1.51 | 0.9345 | 2.02 | 0.9783 |
| 0.02 | 0.5080 | 0.52 | 0.6985 | 1.02 | 0.8461 | 1.52 | 0.9357 | 2.04 | 0.9793 |
| 0.03 | 0.5120 | 0.53 | 0.7019 | 1.03 | 0.8485 | 1.53 | 0.9370 | 2.06 | 0.9803 |
| 0.04 | 0.5160 | 0.54 | 0.7054 | 1.04 | 0.8508 | 1.54 | 0.9382 | 2.08 | 0.9812 |
| 0.05 | 0.5199 | 0.55 | 0.7088 | 1.05 | 0.8531 | 1.55 | 0.9394 | 2.10 | 0.9821 |
| 0.06 | 0.5239 | 0.56 | 0.7123 | 1.06 | 0.8554 | 1.56 | 0.9406 | 2.12 | 0.9830 |
| 0.07 | 0.5279 | 0.57 | 0.7157 | 1.07 | 0.8577 | 1.57 | 0.9418 | 2.14 | 0.9838 |
| 0.08 | 0.5319 | 0.58 | 0.7190 | 1.08 | 0.8599 | 1.58 | 0.9429 | 2.16 | 0.9846 |
| 0.09 | 0.5359 | 0.59 | 0.7224 | 1.09 | 0.8621 | 1.59 | 0.9441 | 2.18 | 0.9854 |
| 0.10 | 0.5398 | 0.60 | 0.7257 | 1.10 | 0.8643 | 1.60 | 0.9452 | 2.20 | 0.9861 |
| 0.11 | 0.5438 | 0.61 | 0.7291 | 1.11 | 0.8665 | 1.61 | 0.9463 | 2.22 | 0.9868 |
| 0.12 | 0.5478 | 0.62 | 0.7324 | 1.12 | 0.8686 | 1.62 | 0.9474 | 2.24 | 0.9875 |
| 0.13 | 0.5517 | 0.63 | 0.7357 | 1.13 | 0.8708 | 1.63 | 0.9484 | 2.26 | 0.9881 |
| 0.14 | 0.5557 | 0.64 | 0.7389 | 1.14 | 0.8729 | 1.64 | 0.9495 | 2.28 | 0.9887 |
| 0.15 | 0.5596 | 0.65 | 0.7422 | 1.15 | 0.8749 | 1.65 | 0.9505 | 2.30 | 0.9893 |
| 0.16 | 0.5636 | 0.66 | 0.7454 | 1.16 | 0.8770 | 1.66 | 0.9515 | 2.32 | 0.9898 |
| 0.17 | 0.5675 | 0.67 | 0.7486 | 1.17 | 0.8790 | 1.67 | 0.9525 | 2.34 | 0.9904 |
| 0.18 | 0.5714 | 0.68 | 0.7517 | 1.18 | 0.8810 | 1.68 | 0.9535 | 2.36 | 0.9909 |
| 0.19 | 0.5753 | 0.69 | 0.7549 | 1.19 | 0.8830 | 1.69 | 0.9545 | 2.38 | 0.9913 |
| 0.20 | 0.5793 | 0.70 | 0.7580 | 1.20 | 0.8849 | 1.70 | 0.9554 | 2.40 | 0.9918 |
| 0.21 | 0.5832 | 0.71 | 0.7611 | 1.21 | 0.8869 | 1.71 | 0.9564 | 2.42 | 0.9922 |
| 0.22 | 0.5871 | 0.72 | 0.7642 | 1.22 | 0.8888 | 1.72 | 0.9573 | 2.44 | 0.9927 |
| 0.23 | 0.5910 | 0.73 | 0.7673 | 1.23 | 0.8907 | 1.73 | 0.9582 | 2.46 | 0.9931 |
| 0.24 | 0.5948 | 0.74 | 0.7704 | 1.24 | 0.8925 | 1.74 | 0.9591 | 2.48 | 0.9934 |
| 0.25 | 0.5987 | 0.75 | 0.7734 | 1.25 | 0.8944 | 1.75 | 0.9599 | 2.50 | 0.9938 |
| 0.26 | 0.6026 | 0.76 | 0.7764 | 1.26 | 0.8962 | 1.76 | 0.9608 | 2.55 | 0.9946 |
| 0.27 | 0.6064 | 0.77 | 0.7794 | 1.27 | 0.8980 | 1.77 | 0.9616 | 2.60 | 0.9953 |
| 0.28 | 0.6103 | 0.78 | 0.7823 | 1.28 | 0.8997 | 1.78 | 0.9625 | 2.65 | 0.9960 |
| 0.29 | 0.6141 | 0.79 | 0.7852 | 1.29 | 0.9015 | 1.79 | 0.9633 | 2.70 | 0.9965 |
| 0.30 | 0.6179 | 0.80 | 0.7881 | 1.30 | 0.9032 | 1.80 | 0.9641 | 2.75 | 0.9970 |
| 0.31 | 0.6217 | 0.81 | 0.7910 | 1.31 | 0.9049 | 1.81 | 0.9649 | 2.80 | 0.9974 |
| 0.32 | 0.6255 | 0.82 | 0.7939 | 1.32 | 0.9066 | 1.82 | 0.9656 | 2.85 | 0.9978 |
| 0.33 | 0.6293 | 0.83 | 0.7967 | 1.33 | 0.9082 | 1.83 | 0.9664 | 2.90 | 0.9981 |
| 0.34 | 0.6331 | 0.84 | 0.7995 | 1.34 | 0.9099 | 1.84 | 0.9671 | 2.95 | 0.9984 |
| 0.35 | 0.6368 | 0.85 | 0.8023 | 1.35 | 0.9115 | 1.85 | 0.9678 | 3.00 | 0.9987 |
| 0.36 | 0.6406 | 0.86 | 0.8051 | 1.36 | 0.9131 | 1.86 | 0.9686 | 3.05 | 0.9989 |
| 0.37 | 0.6443 | 0.87 | 0.8078 | 1.37 | 0.9147 | 1.87 | 0.9693 | 3.10 | 0.9990 |
| 0.38 | 0.6480 | 0.88 | 0.8106 | 1.38 | 0.9162 | 1.88 | 0.9699 | 3.15 | 0.9992 |
| 0.39 | 0.6517 | 0.89 | 0.8133 | 1.39 | 0.9177 | 1.89 | 0.9706 | 3.20 | 0.9993 |
| 0.40 | 0.6554 | 0.90 | 0.8159 | 1.40 | 0.9192 | 1.90 | 0.9713 | 3.25 | 0.9994 |
| 0.41 | 0.6591 | 0.91 | 0.8186 | 1.41 | 0.9207 | 1.91 | 0.9719 | 3.30 | 0.9995 |
| 0.42 | 0.6628 | 0.92 | 0.8212 | 1.42 | 0.9222 | 1.92 | 0.9726 | 3.35 | 0.9996 |
| 0.43 | 0.6664 | 0.93 | 0.8238 | 1.43 | 0.9236 | 1.93 | 0.9732 | 3.40 | 0.9997 |
| 0.44 | 0.6700 | 0.94 | 0.8264 | 1.44 | 0.9251 | 1.94 | 0.9738 | 3.50 | 0.9998 |
| 0.45 | 0.6736 | 0.95 | 0.8289 | 1.45 | 0.9265 | 1.95 | 0.9744 | 3.60 | 0.9998 |
| 0.46 | 0.6772 | 0.96 | 0.8315 | 1.46 | 0.9279 | 1.96 | 0.9750 | 3.70 | 0.9999 |
| 0.47 | 0.6808 | 0.97 | 0.8340 | 1.47 | 0.9292 | 1.97 | 0.9756 | 3.80 | 0.9999 |
| 0.48 | 0.6844 | 0.98 | 0.8365 | 1.48 | 0.9306 | 1.98 | 0.9761 | 3.90 | 1.0000 |
| 0.49 | 0.6879 | 0.99 | 0.8389 | 1.49 | 0.9319 | 1.99 | 0.9767 | 4.00 | 1.0000 |
| 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 | | |



PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p ; that is, $P(Z > z) = 1 - \Phi(z) = p$.

| p | z | p | z |
|--------|--------|--------|--------|
| 0.5000 | 0.0000 | 0.0500 | 1.6449 |
| 0.4000 | 0.2533 | 0.0250 | 1.9600 |
| 0.3000 | 0.5244 | 0.0100 | 2.3263 |
| 0.2000 | 0.8416 | 0.0050 | 2.5758 |
| 0.1500 | 1.0364 | 0.0010 | 3.0902 |
| 0.1000 | 1.2816 | 0.0005 | 3.2905 |

Statistics S2

Candidates sitting S2 may also require those formulae listed under Statistics S1, and also those listed under Core Mathematics C1 and C2.

Discrete distributions

Standard discrete distributions:

| Distribution of X | $P(X = x)$ | Mean | Variance |
|-----------------------|-------------------------------------|-----------|-----------|
| Binomial $B(n, p)$ | $\binom{n}{x} p^x (1-p)^{n-x}$ | np | $np(1-p)$ |
| Poisson $Po(\lambda)$ | $e^{-\lambda} \frac{\lambda^x}{x!}$ | λ | λ |

Continuous distributions

For a continuous random variable X having probability density function f

$$\text{Expectation (mean): } E(X) = \mu = \int x f(x) dx$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{For a function } g(X): E(g(X)) = \int g(x) f(x) dx$$

$$\text{Cumulative distribution function: } F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(t) dt$$

Standard continuous distribution:

| Distribution of X | P.D.F. | Mean | Variance |
|-----------------------------------|-----------------|--------------------|-----------------------|
| Uniform (Rectangular) on $[a, b]$ | $\frac{1}{b-a}$ | $\frac{1}{2}(a+b)$ | $\frac{1}{12}(b-a)^2$ |

BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a binomial distribution with index n and parameter p .

| $p =$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n = 5, x = 0$ | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0312 |
| 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
| 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
| 3 | 1.0000 | 0.9995 | 0.9978 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
| 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| $n = 6, x = 0$ | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
| 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
| 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6563 |
| 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.8906 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| $n = 7, x = 0$ | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| 1 | 0.9556 | 0.8503 | 0.7166 | 0.5767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
| 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |
| 3 | 0.9998 | 0.9973 | 0.9879 | 0.9667 | 0.9294 | 0.8740 | 0.8002 | 0.7102 | 0.6083 | 0.5000 |
| 4 | 1.0000 | 0.9998 | 0.9988 | 0.9953 | 0.9871 | 0.9712 | 0.9444 | 0.9037 | 0.8471 | 0.7734 |
| 5 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9910 | 0.9812 | 0.9643 | 0.9375 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9984 | 0.9963 | 0.9922 |
| $n = 8, x = 0$ | 0.6634 | 0.4305 | 0.2725 | 0.1678 | 0.1001 | 0.0576 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
| 1 | 0.9428 | 0.8131 | 0.6572 | 0.5033 | 0.3671 | 0.2553 | 0.1691 | 0.1064 | 0.0632 | 0.0352 |
| 2 | 0.9942 | 0.9619 | 0.8948 | 0.7969 | 0.6785 | 0.5518 | 0.4278 | 0.3154 | 0.2201 | 0.1445 |
| 3 | 0.9996 | 0.9950 | 0.9786 | 0.9437 | 0.8862 | 0.8059 | 0.7064 | 0.5941 | 0.4770 | 0.3633 |
| 4 | 1.0000 | 0.9996 | 0.9971 | 0.9896 | 0.9727 | 0.9420 | 0.8939 | 0.8263 | 0.7396 | 0.6367 |
| 5 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9887 | 0.9747 | 0.9502 | 0.9115 | 0.8555 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9964 | 0.9915 | 0.9819 | 0.9648 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9983 | 0.9961 |
| $n = 9, x = 0$ | 0.6302 | 0.3874 | 0.2316 | 0.1342 | 0.0751 | 0.0404 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
| 1 | 0.9288 | 0.7748 | 0.5995 | 0.4362 | 0.3003 | 0.1960 | 0.1211 | 0.0705 | 0.0385 | 0.0195 |
| 2 | 0.9916 | 0.9470 | 0.8591 | 0.7382 | 0.6007 | 0.4628 | 0.3373 | 0.2318 | 0.1495 | 0.0898 |
| 3 | 0.9994 | 0.9917 | 0.9661 | 0.9144 | 0.8343 | 0.7297 | 0.6089 | 0.4826 | 0.3614 | 0.2539 |
| 4 | 1.0000 | 0.9991 | 0.9944 | 0.9804 | 0.9511 | 0.9012 | 0.8283 | 0.7334 | 0.6214 | 0.5000 |
| 5 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9900 | 0.9747 | 0.9464 | 0.9006 | 0.8342 | 0.7461 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9957 | 0.9888 | 0.9750 | 0.9502 | 0.9102 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9986 | 0.9962 | 0.9909 | 0.9805 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |
| $n = 10, x = 0$ | 0.5987 | 0.3487 | 0.1969 | 0.1074 | 0.0563 | 0.0282 | 0.0135 | 0.0060 | 0.0025 | 0.0010 |
| 1 | 0.9139 | 0.7361 | 0.5443 | 0.3758 | 0.2440 | 0.1493 | 0.0860 | 0.0464 | 0.0233 | 0.0107 |
| 2 | 0.9885 | 0.9298 | 0.8202 | 0.6778 | 0.5256 | 0.3828 | 0.2616 | 0.1673 | 0.0996 | 0.0547 |
| 3 | 0.9990 | 0.9872 | 0.9500 | 0.8791 | 0.7759 | 0.6496 | 0.5138 | 0.3823 | 0.2660 | 0.1719 |
| 4 | 0.9999 | 0.9984 | 0.9901 | 0.9672 | 0.9219 | 0.8497 | 0.7515 | 0.6331 | 0.5044 | 0.3770 |
| 5 | 1.0000 | 0.9999 | 0.9986 | 0.9936 | 0.9803 | 0.9527 | 0.9051 | 0.8338 | 0.7384 | 0.6230 |
| 6 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9965 | 0.9894 | 0.9740 | 0.9452 | 0.8980 | 0.8281 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9952 | 0.9877 | 0.9726 | 0.9453 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9983 | 0.9955 | 0.9893 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 |

| $p =$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n = 12, x = 0$ | 0.5404 | 0.2824 | 0.1422 | 0.0687 | 0.0317 | 0.0138 | 0.0057 | 0.0022 | 0.0008 | 0.0002 |
| 1 | 0.8816 | 0.6590 | 0.4435 | 0.2749 | 0.1584 | 0.0850 | 0.0424 | 0.0196 | 0.0083 | 0.0032 |
| 2 | 0.9804 | 0.8891 | 0.7358 | 0.5583 | 0.3907 | 0.2528 | 0.1513 | 0.0834 | 0.0421 | 0.0193 |
| 3 | 0.9978 | 0.9744 | 0.9078 | 0.7946 | 0.6488 | 0.4925 | 0.3467 | 0.2253 | 0.1345 | 0.0730 |
| 4 | 0.9998 | 0.9957 | 0.9761 | 0.9274 | 0.8424 | 0.7237 | 0.5833 | 0.4382 | 0.3044 | 0.1938 |
| 5 | 1.0000 | 0.9995 | 0.9954 | 0.9806 | 0.9456 | 0.8822 | 0.7873 | 0.6652 | 0.5269 | 0.3872 |
| 6 | 1.0000 | 0.9999 | 0.9993 | 0.9961 | 0.9857 | 0.9614 | 0.9154 | 0.8418 | 0.7393 | 0.6128 |
| 7 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9972 | 0.9905 | 0.9745 | 0.9427 | 0.8883 | 0.8062 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9983 | 0.9944 | 0.9847 | 0.9644 | 0.9270 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9992 | 0.9972 | 0.9921 | 0.9807 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9968 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| $n = 15, x = 0$ | 0.4633 | 0.2059 | 0.0874 | 0.0352 | 0.0134 | 0.0047 | 0.0016 | 0.0005 | 0.0001 | 0.0000 |
| 1 | 0.8290 | 0.5490 | 0.3186 | 0.1671 | 0.0802 | 0.0353 | 0.0142 | 0.0052 | 0.0017 | 0.0005 |
| 2 | 0.9638 | 0.8159 | 0.6042 | 0.3980 | 0.2361 | 0.1268 | 0.0617 | 0.0271 | 0.0107 | 0.0037 |
| 3 | 0.9945 | 0.9444 | 0.8227 | 0.6482 | 0.4613 | 0.2969 | 0.1727 | 0.0905 | 0.0424 | 0.0176 |
| 4 | 0.9994 | 0.9873 | 0.9383 | 0.8358 | 0.6865 | 0.5155 | 0.3519 | 0.2173 | 0.1204 | 0.0592 |
| 5 | 0.9999 | 0.9978 | 0.9832 | 0.9389 | 0.8516 | 0.7216 | 0.5643 | 0.4032 | 0.2608 | 0.1509 |
| 6 | 1.0000 | 0.9997 | 0.9964 | 0.9819 | 0.9434 | 0.8689 | 0.7548 | 0.6098 | 0.4522 | 0.3036 |
| 7 | 1.0000 | 1.0000 | 0.9994 | 0.9958 | 0.9827 | 0.9500 | 0.8868 | 0.7869 | 0.6535 | 0.5000 |
| 8 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9958 | 0.9848 | 0.9578 | 0.9050 | 0.8182 | 0.6964 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9963 | 0.9876 | 0.9662 | 0.9231 | 0.8491 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9972 | 0.9907 | 0.9745 | 0.9408 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9981 | 0.9937 | 0.9824 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9963 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $n = 20, x = 0$ | 0.3585 | 0.1216 | 0.0388 | 0.0115 | 0.0032 | 0.0008 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.7358 | 0.3917 | 0.1756 | 0.0692 | 0.0243 | 0.0076 | 0.0021 | 0.0005 | 0.0001 | 0.0000 |
| 2 | 0.9245 | 0.6769 | 0.4049 | 0.2061 | 0.0913 | 0.0355 | 0.0121 | 0.0036 | 0.0009 | 0.0002 |
| 3 | 0.9841 | 0.8670 | 0.6477 | 0.4114 | 0.2252 | 0.1071 | 0.0444 | 0.0160 | 0.0049 | 0.0013 |
| 4 | 0.9974 | 0.9568 | 0.8298 | 0.6296 | 0.4148 | 0.2375 | 0.1182 | 0.0510 | 0.0189 | 0.0059 |
| 5 | 0.9997 | 0.9887 | 0.9327 | 0.8042 | 0.6172 | 0.4164 | 0.2454 | 0.1256 | 0.0553 | 0.0207 |
| 6 | 1.0000 | 0.9976 | 0.9781 | 0.9133 | 0.7858 | 0.6080 | 0.4166 | 0.2500 | 0.1299 | 0.0577 |
| 7 | 1.0000 | 0.9996 | 0.9941 | 0.9679 | 0.8982 | 0.7723 | 0.6010 | 0.4159 | 0.2520 | 0.1316 |
| 8 | 1.0000 | 0.9999 | 0.9987 | 0.9900 | 0.9591 | 0.8867 | 0.7624 | 0.5956 | 0.4143 | 0.2517 |
| 9 | 1.0000 | 1.0000 | 0.9998 | 0.9974 | 0.9861 | 0.9520 | 0.8782 | 0.7553 | 0.5914 | 0.4119 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 0.9994 | 0.9961 | 0.9829 | 0.9468 | 0.8725 | 0.7507 | 0.5881 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9949 | 0.9804 | 0.9435 | 0.8692 | 0.7483 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9987 | 0.9940 | 0.9790 | 0.9420 | 0.8684 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9985 | 0.9935 | 0.9786 | 0.9423 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9936 | 0.9793 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9985 | 0.9941 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $p =$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n = 25, x = 0$ | 0.2774 | 0.0718 | 0.0172 | 0.0038 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.6424 | 0.2712 | 0.0931 | 0.0274 | 0.0070 | 0.0016 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 2 | 0.8729 | 0.5371 | 0.2537 | 0.0982 | 0.0321 | 0.0090 | 0.0021 | 0.0004 | 0.0001 | 0.0000 |
| 3 | 0.9659 | 0.7636 | 0.4711 | 0.2340 | 0.0962 | 0.0332 | 0.0097 | 0.0024 | 0.0005 | 0.0001 |
| 4 | 0.9928 | 0.9020 | 0.6821 | 0.4207 | 0.2137 | 0.0905 | 0.0320 | 0.0095 | 0.0023 | 0.0005 |
| 5 | 0.9988 | 0.9666 | 0.8385 | 0.6167 | 0.3783 | 0.1935 | 0.0826 | 0.0294 | 0.0086 | 0.0020 |
| 6 | 0.9998 | 0.9905 | 0.9305 | 0.7800 | 0.5611 | 0.3407 | 0.1734 | 0.0736 | 0.0258 | 0.0073 |
| 7 | 1.0000 | 0.9977 | 0.9745 | 0.8909 | 0.7265 | 0.5118 | 0.3061 | 0.1536 | 0.0639 | 0.0216 |
| 8 | 1.0000 | 0.9995 | 0.9920 | 0.9532 | 0.8506 | 0.6769 | 0.4668 | 0.2735 | 0.1340 | 0.0539 |
| 9 | 1.0000 | 0.9999 | 0.9979 | 0.9827 | 0.9287 | 0.8106 | 0.6303 | 0.4246 | 0.2424 | 0.1148 |
| 10 | 1.0000 | 1.0000 | 0.9995 | 0.9944 | 0.9703 | 0.9022 | 0.7712 | 0.5858 | 0.3843 | 0.2122 |
| 11 | 1.0000 | 1.0000 | 0.9999 | 0.9985 | 0.9893 | 0.9558 | 0.8746 | 0.7323 | 0.5426 | 0.3450 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9966 | 0.9825 | 0.9396 | 0.8462 | 0.6937 | 0.5000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9940 | 0.9745 | 0.9222 | 0.8173 | 0.6550 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9982 | 0.9907 | 0.9656 | 0.9040 | 0.7878 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9971 | 0.9868 | 0.9560 | 0.8852 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9957 | 0.9826 | 0.9461 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9942 | 0.9784 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9927 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9980 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $n = 30, x = 0$ | 0.2146 | 0.0424 | 0.0076 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.5535 | 0.1837 | 0.0480 | 0.0105 | 0.0020 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.8122 | 0.4114 | 0.1514 | 0.0442 | 0.0106 | 0.0021 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9392 | 0.6474 | 0.3217 | 0.1227 | 0.0374 | 0.0093 | 0.0019 | 0.0003 | 0.0000 | 0.0000 |
| 4 | 0.9844 | 0.8245 | 0.5245 | 0.2552 | 0.0979 | 0.0302 | 0.0075 | 0.0015 | 0.0002 | 0.0000 |
| 5 | 0.9967 | 0.9268 | 0.7106 | 0.4275 | 0.2026 | 0.0766 | 0.0233 | 0.0057 | 0.0011 | 0.0002 |
| 6 | 0.9994 | 0.9742 | 0.8474 | 0.6070 | 0.3481 | 0.1595 | 0.0586 | 0.0172 | 0.0040 | 0.0007 |
| 7 | 0.9999 | 0.9922 | 0.9302 | 0.7608 | 0.5143 | 0.2814 | 0.1238 | 0.0435 | 0.0121 | 0.0026 |
| 8 | 1.0000 | 0.9980 | 0.9722 | 0.8713 | 0.6736 | 0.4315 | 0.2247 | 0.0940 | 0.0312 | 0.0081 |
| 9 | 1.0000 | 0.9995 | 0.9903 | 0.9389 | 0.8034 | 0.5888 | 0.3575 | 0.1763 | 0.0694 | 0.0214 |
| 10 | 1.0000 | 0.9999 | 0.9971 | 0.9744 | 0.8943 | 0.7304 | 0.5078 | 0.2915 | 0.1350 | 0.0494 |
| 11 | 1.0000 | 1.0000 | 0.9992 | 0.9905 | 0.9493 | 0.8407 | 0.6548 | 0.4311 | 0.2327 | 0.1002 |
| 12 | 1.0000 | 1.0000 | 0.9998 | 0.9969 | 0.9784 | 0.9155 | 0.7802 | 0.5785 | 0.3592 | 0.1808 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 0.9991 | 0.9918 | 0.9599 | 0.8737 | 0.7145 | 0.5025 | 0.2923 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9973 | 0.9831 | 0.9348 | 0.8246 | 0.6448 | 0.4278 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9936 | 0.9699 | 0.9029 | 0.7691 | 0.5722 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9979 | 0.9876 | 0.9519 | 0.8644 | 0.7077 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9955 | 0.9788 | 0.9286 | 0.8192 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9986 | 0.9917 | 0.9666 | 0.8998 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9971 | 0.9862 | 0.9506 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9950 | 0.9786 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9984 | 0.9919 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9974 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $p =$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n = 40, x = 0$ | 0.1285 | 0.0148 | 0.0015 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.3991 | 0.0805 | 0.0121 | 0.0015 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.6767 | 0.2228 | 0.0486 | 0.0079 | 0.0010 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.8619 | 0.4231 | 0.1302 | 0.0285 | 0.0047 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9520 | 0.6290 | 0.2633 | 0.0759 | 0.0160 | 0.0026 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9861 | 0.7937 | 0.4325 | 0.1613 | 0.0433 | 0.0086 | 0.0013 | 0.0001 | 0.0000 | 0.0000 |
| 6 | 0.9966 | 0.9005 | 0.6067 | 0.2859 | 0.0962 | 0.0238 | 0.0044 | 0.0006 | 0.0001 | 0.0000 |
| 7 | 0.9993 | 0.9581 | 0.7559 | 0.4371 | 0.1820 | 0.0553 | 0.0124 | 0.0021 | 0.0002 | 0.0000 |
| 8 | 0.9999 | 0.9845 | 0.8646 | 0.5931 | 0.2998 | 0.1110 | 0.0303 | 0.0061 | 0.0009 | 0.0001 |
| 9 | 1.0000 | 0.9949 | 0.9328 | 0.7318 | 0.4395 | 0.1959 | 0.0644 | 0.0156 | 0.0027 | 0.0003 |
| 10 | 1.0000 | 0.9985 | 0.9701 | 0.8392 | 0.5839 | 0.3087 | 0.1215 | 0.0352 | 0.0074 | 0.0011 |
| 11 | 1.0000 | 0.9996 | 0.9880 | 0.9125 | 0.7151 | 0.4406 | 0.2053 | 0.0709 | 0.0179 | 0.0032 |
| 12 | 1.0000 | 0.9999 | 0.9957 | 0.9568 | 0.8209 | 0.5772 | 0.3143 | 0.1285 | 0.0386 | 0.0083 |
| 13 | 1.0000 | 1.0000 | 0.9986 | 0.9806 | 0.8968 | 0.7032 | 0.4408 | 0.2112 | 0.0751 | 0.0192 |
| 14 | 1.0000 | 1.0000 | 0.9996 | 0.9921 | 0.9456 | 0.8074 | 0.5721 | 0.3174 | 0.1326 | 0.0403 |
| 15 | 1.0000 | 1.0000 | 0.9999 | 0.9971 | 0.9738 | 0.8849 | 0.6946 | 0.4402 | 0.2142 | 0.0769 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 0.9990 | 0.9884 | 0.9367 | 0.7978 | 0.5681 | 0.3185 | 0.1341 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9953 | 0.9680 | 0.8761 | 0.6885 | 0.4391 | 0.2148 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9983 | 0.9852 | 0.9301 | 0.7911 | 0.5651 | 0.3179 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9994 | 0.9937 | 0.9637 | 0.8702 | 0.6844 | 0.4373 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9976 | 0.9827 | 0.9256 | 0.7870 | 0.5627 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9991 | 0.9925 | 0.9608 | 0.8669 | 0.6821 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9970 | 0.9811 | 0.9233 | 0.7852 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9989 | 0.9917 | 0.9595 | 0.8659 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9966 | 0.9804 | 0.9231 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9988 | 0.9914 | 0.9597 |
| 26 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9966 | 0.9808 |
| 27 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9988 | 0.9917 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9968 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9989 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 |
| 31 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 32 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $p =$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n = 50, x = 0$ | 0.0769 | 0.0052 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.2794 | 0.0338 | 0.0029 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.5405 | 0.1117 | 0.0142 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.7604 | 0.2503 | 0.0460 | 0.0057 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.8964 | 0.4312 | 0.1121 | 0.0185 | 0.0021 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9622 | 0.6161 | 0.2194 | 0.0480 | 0.0070 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.9882 | 0.7702 | 0.3613 | 0.1034 | 0.0194 | 0.0025 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.9968 | 0.8779 | 0.5188 | 0.1904 | 0.0453 | 0.0073 | 0.0008 | 0.0001 | 0.0000 | 0.0000 |
| 8 | 0.9992 | 0.9421 | 0.6681 | 0.3073 | 0.0916 | 0.0183 | 0.0025 | 0.0002 | 0.0000 | 0.0000 |
| 9 | 0.9998 | 0.9755 | 0.7911 | 0.4437 | 0.1637 | 0.0402 | 0.0067 | 0.0008 | 0.0001 | 0.0000 |
| 10 | 1.0000 | 0.9906 | 0.8801 | 0.5836 | 0.2622 | 0.0789 | 0.0160 | 0.0022 | 0.0002 | 0.0000 |
| 11 | 1.0000 | 0.9968 | 0.9372 | 0.7107 | 0.3816 | 0.1390 | 0.0342 | 0.0057 | 0.0006 | 0.0000 |
| 12 | 1.0000 | 0.9990 | 0.9699 | 0.8139 | 0.5110 | 0.2229 | 0.0661 | 0.0133 | 0.0018 | 0.0002 |
| 13 | 1.0000 | 0.9997 | 0.9868 | 0.8894 | 0.6370 | 0.3279 | 0.1163 | 0.0280 | 0.0045 | 0.0005 |
| 14 | 1.0000 | 0.9999 | 0.9947 | 0.9393 | 0.7481 | 0.4468 | 0.1878 | 0.0540 | 0.0104 | 0.0013 |
| 15 | 1.0000 | 1.0000 | 0.9981 | 0.9692 | 0.8369 | 0.5692 | 0.2801 | 0.0955 | 0.0220 | 0.0033 |
| 16 | 1.0000 | 1.0000 | 0.9993 | 0.9856 | 0.9017 | 0.6839 | 0.3889 | 0.1561 | 0.0427 | 0.0077 |
| 17 | 1.0000 | 1.0000 | 0.9998 | 0.9937 | 0.9449 | 0.7822 | 0.5060 | 0.2369 | 0.0765 | 0.0164 |
| 18 | 1.0000 | 1.0000 | 0.9999 | 0.9975 | 0.9713 | 0.8594 | 0.6216 | 0.3356 | 0.1273 | 0.0325 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 0.9991 | 0.9861 | 0.9152 | 0.7264 | 0.4465 | 0.1974 | 0.0595 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9937 | 0.9522 | 0.8139 | 0.5610 | 0.2862 | 0.1013 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9974 | 0.9749 | 0.8813 | 0.6701 | 0.3900 | 0.1611 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9990 | 0.9877 | 0.9290 | 0.7660 | 0.5019 | 0.2399 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9944 | 0.9604 | 0.8438 | 0.6134 | 0.3359 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9976 | 0.9793 | 0.9022 | 0.7160 | 0.4439 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9991 | 0.9900 | 0.9427 | 0.8034 | 0.5561 |
| 26 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9955 | 0.9686 | 0.8721 | 0.6641 |
| 27 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9981 | 0.9840 | 0.9220 | 0.7601 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9993 | 0.9924 | 0.9556 | 0.8389 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9966 | 0.9765 | 0.8987 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9986 | 0.9884 | 0.9405 |
| 31 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9947 | 0.9675 |
| 32 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9978 | 0.9836 |
| 33 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9923 |
| 34 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9967 |
| 35 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9987 |
| 36 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 |
| 37 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 |
| 38 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

POISSON CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a Poisson distribution with parameter λ .

| $\lambda =$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.6065 | 0.3679 | 0.2231 | 0.1353 | 0.0821 | 0.0498 | 0.0302 | 0.0183 | 0.0111 | 0.0067 |
| 1 | 0.9098 | 0.7358 | 0.5578 | 0.4060 | 0.2873 | 0.1991 | 0.1359 | 0.0916 | 0.0611 | 0.0404 |
| 2 | 0.9856 | 0.9197 | 0.8088 | 0.6767 | 0.5438 | 0.4232 | 0.3208 | 0.2381 | 0.1736 | 0.1247 |
| 3 | 0.9982 | 0.9810 | 0.9344 | 0.8571 | 0.7576 | 0.6472 | 0.5366 | 0.4335 | 0.3423 | 0.2650 |
| 4 | 0.9998 | 0.9963 | 0.9814 | 0.9473 | 0.8912 | 0.8153 | 0.7254 | 0.6288 | 0.5321 | 0.4405 |
| 5 | 1.0000 | 0.9994 | 0.9955 | 0.9834 | 0.9580 | 0.9161 | 0.8576 | 0.7851 | 0.7029 | 0.6160 |
| 6 | 1.0000 | 0.9999 | 0.9991 | 0.9955 | 0.9858 | 0.9665 | 0.9347 | 0.8893 | 0.8311 | 0.7622 |
| 7 | 1.0000 | 1.0000 | 0.9998 | 0.9989 | 0.9958 | 0.9881 | 0.9733 | 0.9489 | 0.9134 | 0.8666 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9989 | 0.9962 | 0.9901 | 0.9786 | 0.9597 | 0.9319 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9989 | 0.9967 | 0.9919 | 0.9829 | 0.9682 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9972 | 0.9933 | 0.9863 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9976 | 0.9945 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9993 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\lambda =$ | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 |
| $x = 0$ | 0.0041 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 1 | 0.0266 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 | 0.0012 | 0.0008 | 0.0005 |
| 2 | 0.0884 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 | 0.0062 | 0.0042 | 0.0028 |
| 3 | 0.2017 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 | 0.0212 | 0.0149 | 0.0103 |
| 4 | 0.3575 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 | 0.0550 | 0.0403 | 0.0293 |
| 5 | 0.5289 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 | 0.1157 | 0.0885 | 0.0671 |
| 6 | 0.6860 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 | 0.2068 | 0.1649 | 0.1301 |
| 7 | 0.8095 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 | 0.3239 | 0.2687 | 0.2202 |
| 8 | 0.8944 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 | 0.4557 | 0.3918 | 0.3328 |
| 9 | 0.9462 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 | 0.5874 | 0.5218 | 0.4579 |
| 10 | 0.9747 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 | 0.7060 | 0.6453 | 0.5830 |
| 11 | 0.9890 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 | 0.8030 | 0.7520 | 0.6968 |
| 12 | 0.9955 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 | 0.8758 | 0.8364 | 0.7916 |
| 13 | 0.9983 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 | 0.9261 | 0.8981 | 0.8645 |
| 14 | 0.9994 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 | 0.9585 | 0.9400 | 0.9165 |
| 15 | 0.9998 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 | 0.9780 | 0.9665 | 0.9513 |
| 16 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 | 0.9889 | 0.9823 | 0.9730 |
| 17 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 | 0.9947 | 0.9911 | 0.9857 |
| 18 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 | 0.9976 | 0.9957 | 0.9928 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9989 | 0.9980 | 0.9965 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9991 | 0.9984 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9993 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 |

Statistics S3

Candidates sitting S3 may also require those formulae listed under Statistics S1 and S2.

Expectation algebra

For independent random variables X and Y

$$E(XY) = E(X)E(Y), \quad \text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Sampling distributions

For a random sample X_1, X_2, \dots, X_n of n independent observations from a distribution having mean μ and variance σ^2

$$\bar{X} \text{ is an unbiased estimator of } \mu, \text{ with } \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

$$S^2 \text{ is an unbiased estimator of } \sigma^2, \text{ where } S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$$

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

Correlation and regression

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_v^2$

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

The values in the table are those which a random variable with the χ^2 distribution on ν degrees of freedom exceeds with the probability shown.

| ν | 0.995 | 0.990 | 0.975 | 0.950 | 0.900 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.705 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.580 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.558 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.088 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

These tables concern tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

| Product Moment Coefficient | | | | | Sample Level | Spearman's Coefficient | | |
|----------------------------|--------|-------------|--------|--------|--------------|------------------------|-------------|--------|
| 0.10 | 0.05 | Level 0.025 | 0.01 | 0.005 | | 0.05 | Level 0.025 | 0.01 |
| 0.8000 | 0.9000 | 0.9500 | 0.9800 | 0.9900 | 4 | 1.0000 | - | - |
| 0.6870 | 0.8054 | 0.8783 | 0.9343 | 0.9587 | 5 | 0.9000 | 1.0000 | 1.0000 |
| 0.6084 | 0.7293 | 0.8114 | 0.8822 | 0.9172 | 6 | 0.8286 | 0.8857 | 0.9429 |
| 0.5509 | 0.6694 | 0.7545 | 0.8329 | 0.8745 | 7 | 0.7143 | 0.7857 | 0.8929 |
| 0.5067 | 0.6215 | 0.7067 | 0.7887 | 0.8343 | 8 | 0.6429 | 0.7381 | 0.8333 |
| 0.4716 | 0.5822 | 0.6664 | 0.7498 | 0.7977 | 9 | 0.6000 | 0.7000 | 0.7833 |
| 0.4428 | 0.5494 | 0.6319 | 0.7155 | 0.7646 | 10 | 0.5636 | 0.6485 | 0.7455 |
| 0.4187 | 0.5214 | 0.6021 | 0.6851 | 0.7348 | 11 | 0.5364 | 0.6182 | 0.7091 |
| 0.3981 | 0.4973 | 0.5760 | 0.6581 | 0.7079 | 12 | 0.5035 | 0.5874 | 0.6783 |
| 0.3802 | 0.4762 | 0.5529 | 0.6339 | 0.6835 | 13 | 0.4835 | 0.5604 | 0.6484 |
| 0.3646 | 0.4575 | 0.5324 | 0.6120 | 0.6614 | 14 | 0.4637 | 0.5385 | 0.6264 |
| 0.3507 | 0.4409 | 0.5140 | 0.5923 | 0.6411 | 15 | 0.4464 | 0.5214 | 0.6036 |
| 0.3383 | 0.4259 | 0.4973 | 0.5742 | 0.6226 | 16 | 0.4294 | 0.5029 | 0.5824 |
| 0.3271 | 0.4124 | 0.4821 | 0.5577 | 0.6055 | 17 | 0.4142 | 0.4877 | 0.5662 |
| 0.3170 | 0.4000 | 0.4683 | 0.5425 | 0.5897 | 18 | 0.4014 | 0.4716 | 0.5501 |
| 0.3077 | 0.3887 | 0.4555 | 0.5285 | 0.5751 | 19 | 0.3912 | 0.4596 | 0.5351 |
| 0.2992 | 0.3783 | 0.4438 | 0.5155 | 0.5614 | 20 | 0.3805 | 0.4466 | 0.5218 |
| 0.2914 | 0.3687 | 0.4329 | 0.5034 | 0.5487 | 21 | 0.3701 | 0.4364 | 0.5091 |
| 0.2841 | 0.3598 | 0.4227 | 0.4921 | 0.5368 | 22 | 0.3608 | 0.4252 | 0.4975 |
| 0.2774 | 0.3515 | 0.4133 | 0.4815 | 0.5256 | 23 | 0.3528 | 0.4160 | 0.4862 |
| 0.2711 | 0.3438 | 0.4044 | 0.4716 | 0.5151 | 24 | 0.3443 | 0.4070 | 0.4757 |
| 0.2653 | 0.3365 | 0.3961 | 0.4622 | 0.5052 | 25 | 0.3369 | 0.3977 | 0.4662 |
| 0.2598 | 0.3297 | 0.3882 | 0.4534 | 0.4958 | 26 | 0.3306 | 0.3901 | 0.4571 |
| 0.2546 | 0.3233 | 0.3809 | 0.4451 | 0.4869 | 27 | 0.3242 | 0.3828 | 0.4487 |
| 0.2497 | 0.3172 | 0.3739 | 0.4372 | 0.4785 | 28 | 0.3180 | 0.3755 | 0.4401 |
| 0.2451 | 0.3115 | 0.3673 | 0.4297 | 0.4705 | 29 | 0.3118 | 0.3685 | 0.4325 |
| 0.2407 | 0.3061 | 0.3610 | 0.4226 | 0.4629 | 30 | 0.3063 | 0.3624 | 0.4251 |
| 0.2070 | 0.2638 | 0.3120 | 0.3665 | 0.4026 | 40 | 0.2640 | 0.3128 | 0.3681 |
| 0.1843 | 0.2353 | 0.2787 | 0.3281 | 0.3610 | 50 | 0.2353 | 0.2791 | 0.3293 |
| 0.1678 | 0.2144 | 0.2542 | 0.2997 | 0.3301 | 60 | 0.2144 | 0.2545 | 0.3005 |
| 0.1550 | 0.1982 | 0.2352 | 0.2776 | 0.3060 | 70 | 0.1982 | 0.2354 | 0.2782 |
| 0.1448 | 0.1852 | 0.2199 | 0.2597 | 0.2864 | 80 | 0.1852 | 0.2201 | 0.2602 |
| 0.1364 | 0.1745 | 0.2072 | 0.2449 | 0.2702 | 90 | 0.1745 | 0.2074 | 0.2453 |
| 0.1292 | 0.1654 | 0.1966 | 0.2324 | 0.2565 | 100 | 0.1654 | 0.1967 | 0.2327 |

RANDOM NUMBERS

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 86 13 | 84 10 | 07 30 | 39 05 | 97 96 | 88 07 | 37 26 | 04 89 | 13 48 | 19 20 |
| 60 78 | 48 12 | 99 47 | 09 46 | 91 33 | 17 21 | 03 94 | 79 00 | 08 50 | 40 16 |
| 78 48 | 06 37 | 82 26 | 01 06 | 64 65 | 94 41 | 17 26 | 74 66 | 61 93 | 24 97 |
| 80 56 | 90 79 | 66 94 | 18 40 | 97 79 | 93 20 | 41 51 | 25 04 | 20 71 | 76 04 |
| 99 09 | 39 25 | 66 31 | 70 56 | 30 15 | 52 17 | 87 55 | 31 11 | 10 68 | 98 23 |
| 56 32 | 32 72 | 91 65 | 97 36 | 56 61 | 12 79 | 95 17 | 57 16 | 53 58 | 96 36 |
| 66 02 | 49 93 | 97 44 | 99 15 | 56 86 | 80 57 | 11 78 | 40 23 | 58 40 | 86 14 |
| 31 77 | 53 94 | 05 93 | 56 14 | 71 23 | 60 46 | 05 33 | 23 72 | 93 10 | 81 23 |
| 98 79 | 72 43 | 14 76 | 54 77 | 66 29 | 84 09 | 88 56 | 75 86 | 41 67 | 04 42 |
| 50 97 | 92 15 | 10 01 | 57 01 | 87 33 | 73 17 | 70 18 | 40 21 | 24 20 | 66 62 |
| 90 51 | 94 50 | 12 48 | 88 95 | 09 34 | 09 30 | 22 27 | 25 56 | 40 76 | 01 59 |
| 31 99 | 52 24 | 13 43 | 27 88 | 11 39 | 41 65 | 00 84 | 13 06 | 31 79 | 74 97 |
| 22 96 | 23 34 | 46 12 | 67 11 | 48 06 | 99 24 | 14 83 | 78 37 | 65 73 | 39 47 |
| 06 84 | 55 41 | 27 06 | 74 59 | 14 29 | 20 14 | 45 75 | 31 16 | 05 41 | 22 96 |
| 08 64 | 89 30 | 25 25 | 71 35 | 33 31 | 04 56 | 12 67 | 03 74 | 07 16 | 49 32 |
| 86 87 | 62 43 | 15 11 | 76 49 | 79 13 | 78 80 | 93 89 | 09 57 | 07 14 | 40 74 |
| 94 44 | 97 13 | 77 04 | 35 02 | 12 76 | 60 91 | 93 40 | 81 06 | 85 85 | 72 84 |
| 63 25 | 55 14 | 66 47 | 99 90 | 02 90 | 83 43 | 16 01 | 19 69 | 11 78 | 87 16 |
| 11 22 | 83 98 | 15 21 | 18 57 | 53 42 | 91 91 | 26 52 | 89 13 | 86 00 | 47 61 |
| 01 70 | 10 83 | 94 71 | 13 67 | 11 12 | 36 54 | 53 32 | 90 43 | 79 01 | 95 15 |

Statistics S4

Candidates sitting S4 may also require those formulae listed under Statistics S1, S2 and S3.

Sampling distributions

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \quad (\text{also valid in matched-pairs situations})$$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{S_x^2 / \sigma_x^2}{S_y^2 / \sigma_y^2} \sim F_{n_x-1, n_y-1}$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}} \sim t_{n_x+n_y-2} \quad \text{where} \quad S_p^2 = \frac{(n_x-1)S_x^2 + (n_y-1)S_y^2}{n_x + n_y - 2}$$

PERCENTAGE POINTS OF STUDENT'S t DISTRIBUTION

The values in the table are those which a random variable with Student's t distribution on ν degrees of freedom exceeds with the probability shown.

| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
|-------|-------|-------|--------|--------|--------|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 |
| 36 | 1.306 | 1.688 | 2.028 | 2.435 | 2.719 |
| 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 55 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 90 | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| 110 | 1.289 | 1.659 | 1.982 | 2.361 | 2.621 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |

PERCENTAGE POINTS OF THE F DISTRIBUTION

The values in the table are those which a random variable with the F distribution on ν_1 and ν_2 degrees of freedom exceeds with probability 0.05 or 0.01.

| Probability | ν_2/ν_1 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 24 | ∞ |
|-------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| 0.05 | 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 238.9 | 241.9 | 243.9 | 249.1 | 254.3 |
| | 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.37 | 19.40 | 19.41 | 19.46 | 19.50 |
| | 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.85 | 8.79 | 8.74 | 8.64 | 8.53 |
| | 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.04 | 5.96 | 5.91 | 5.77 | 5.63 |
| | 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.82 | 4.74 | 4.68 | 4.53 | 4.37 |
| | 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.15 | 4.06 | 4.00 | 3.84 | 3.67 |
| | 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.73 | 3.64 | 3.57 | 3.41 | 3.23 |
| | 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.44 | 3.35 | 3.28 | 3.12 | 2.93 |
| | 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.23 | 3.14 | 3.07 | 2.90 | 2.71 |
| | 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.07 | 2.98 | 2.91 | 2.74 | 2.54 |
| | 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 2.95 | 2.85 | 2.79 | 2.61 | 2.40 |
| | 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.85 | 2.75 | 2.69 | 2.51 | 2.30 |
| | 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.70 | 2.60 | 2.53 | 2.35 | 2.13 |
| | 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.59 | 2.49 | 2.42 | 2.24 | 2.01 |
| | 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.51 | 2.41 | 2.34 | 2.15 | 1.92 |
| | 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.45 | 2.35 | 2.28 | 2.08 | 1.84 |
| | 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.34 | 2.24 | 2.16 | 1.96 | 1.71 |
| | 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.27 | 2.16 | 2.09 | 1.89 | 1.62 |
| | 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.18 | 2.08 | 2.00 | 1.79 | 1.51 |
| | 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.10 | 1.99 | 1.92 | 1.70 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.02 | 1.91 | 1.83 | 1.61 | 1.25 | |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 1.94 | 1.83 | 1.75 | 1.52 | 1.00 | |
| 0.01 | 1 | 4052. | 5000. | 5403. | 5625. | 5764. | 5859. | 5982. | 6056. | 6106. | 6235. | 6366. |
| | 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.37 | 99.40 | 99.42 | 99.46 | 99.50 |
| | 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.49 | 27.23 | 27.05 | 26.60 | 26.13 |
| | 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.80 | 14.55 | 14.37 | 13.93 | 13.45 |
| | 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.29 | 10.05 | 9.89 | 9.47 | 9.02 |
| | 6 | 13.70 | 10.90 | 9.78 | 9.15 | 8.75 | 8.47 | 8.10 | 7.87 | 7.72 | 7.31 | 6.88 |
| | 7 | 12.20 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.84 | 6.62 | 6.47 | 6.07 | 5.65 |
| | 8 | 11.30 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.03 | 5.81 | 5.67 | 5.28 | 4.86 |
| | 9 | 10.60 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.47 | 5.26 | 5.11 | 4.73 | 4.31 |
| | 10 | 10.00 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.06 | 4.85 | 4.71 | 4.33 | 3.91 |
| | 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.74 | 4.54 | 4.40 | 4.02 | 3.60 |
| | 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.50 | 4.30 | 4.16 | 3.78 | 3.36 |
| | 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.70 | 4.46 | 4.14 | 3.94 | 3.80 | 3.43 | 3.00 |
| | 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 3.89 | 3.69 | 3.55 | 3.18 | 2.75 |
| | 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.71 | 3.51 | 3.37 | 3.00 | 2.57 |
| | 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.56 | 3.37 | 3.23 | 2.86 | 2.42 |
| | 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.86 | 3.63 | 3.32 | 3.13 | 2.99 | 2.62 | 2.17 |
| | 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.17 | 2.98 | 2.84 | 2.47 | 2.01 |
| | 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 2.99 | 2.80 | 2.66 | 2.29 | 1.80 |
| | 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.82 | 2.63 | 2.50 | 2.12 | 1.60 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.66 | 2.47 | 2.34 | 1.95 | 1.38 | |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.51 | 2.32 | 2.18 | 1.79 | 1.00 | |

If an *upper* percentage point of the F distribution on ν_1 and ν_2 degrees of freedom is f , then the corresponding *lower* percentage point of the F distribution on ν_2 and ν_1 degrees of freedom is $1/f$.

Student Guide

What do I need to know, or be able to do, before taking this course?

This course is suitable for students who have achieved at least a grade C at Higher tier in GCSE Mathematics.

What will I learn?

Mathematics at AS and Advanced GCE is a course worth studying not only as a supporting subject for the physical and social sciences, but in its own right. It is challenging but interesting. It builds on work you will have met at GCSE, but also involves new ideas produced by some of the greatest minds of the last millennium.

While studying mathematics you will be expected to:

- use mathematical skills and knowledge to solve problems
- solve problems by using mathematical arguments and logic. You will also have to understand and demonstrate what is meant by proof in mathematics
- simplify real-life situations so that you can use mathematics to show what is happening and what might happen in different circumstances
- use the mathematics that you learn to solve problems that are given to you in a real-life context
- use calculator technology and other resources (such as formulae booklets or statistical tables) effectively and appropriately; understand calculator limitations and when it is inappropriate to use such technology.



Mathematics is divided into four branches:

Pure Mathematics C1, C2, C3, C4 (covering the A level Pure Core content) and FP1, FP2, FP3 (covers the Further Mathematics Pure content)

When studying pure mathematics at AS and A2 level you will be extending your knowledge of such topics as algebra and trigonometry as well as learning some brand new ideas such as calculus. While many of the ideas you will meet in pure mathematics are interesting in their own right, they also serve as an important foundation for other branches of mathematics, especially mechanics and statistics.

Mechanics (M1, M2, M3, M4, M5)

Mechanics deals with the action of forces on objects. It is therefore concerned with many everyday situations, e.g. the motion of cars, the flight of a cricket ball through the air, the stresses in bridges, the motion of the earth around the sun. Such problems have to be simplified or modelled to make them capable of solution using relatively simple mathematics. The study of one or more of the Mechanics units will enable you to use the mathematical techniques which you learn in the Core units to help you to produce solutions to these problems. Many of the ideas you will meet in the course form an almost essential introduction to such important modern fields of study such as cybernetics, robotics, bio-mechanics and sports science, as well as the more traditional areas of engineering and physics.

Statistics (S1, S2, S3, S4)

When you study statistics you will learn how to analyse and summarise numerical data in order to arrive at conclusions about it. You will extend the range of probability problems that you looked at in GCSE using the new mathematical techniques learnt in the pure mathematics units. Many of the ideas in this part of the course have applications in a wide range of other fields, from assessing what your car insurance is going to cost to how likely it is that the Earth will be hit by a comet in the next few years. Many of the techniques are used in sciences and social sciences. Even if you are not going on to study or work in these fields, in today's society we are bombarded with information (or data) and the statistics units will give you useful tools for looking at this information critically and efficiently.

Decision Mathematics (D1, D2)

In decision mathematics you will learn how to solve problems involving networks, systems, planning and resource allocation. You will study a range of methods, or algorithms, which enable such problems to be tackled. The ideas have many important applications in such different problems as the design of circuits on microchips to the scheduling of tasks required to build a new supermarket.

Is this the right subject for me?

Mathematics is rather different from many other subjects. An essential part of mathematical study is the challenge of analysing and solving a problem and the satisfaction and confidence gained from achieving a 'correct' answer. If you choose mathematics you will not have to write essays, but you will need to be able to communicate well in written work to explain your solutions.

Mathematics is not about learning facts. You will not achieve success by just reading a textbook or by producing and revising from detailed notes... you actually need to 'do' mathematics.

How will I be assessed?

This will depend on your choice of units of study. For AS Level you will take 3 units and for a full A Level you will take a further 3 units. For Further Mathematics at AS, you will take 9 units in total, and for Further Mathematics A level, 12 units in total. Each unit is tested by a 1½ hour written examination and the units are equally weighted.

What can I do after I've completed the course?

An AS in mathematics is very valuable as a supporting subject to many courses at Advanced GCE and degree level, especially in the sciences and geography, psychology, sociology and medical courses.

Advanced GCE mathematics is a much sought-after qualification for entry to a wide variety of full-time courses in higher education. There are also many areas of employment that see a Mathematics Advanced GCE as an important qualification and it is often a requirement for the vocational qualifications related to these areas.

Higher Education courses or careers that either require Advanced GCE mathematics or are strongly related include:

- economics
- medicine
- architecture
- engineering
- accountancy
- teaching
- psychology
- physics
- computing
- information and communication technology.

If you wanted to continue your study of mathematics after Advanced GCE you could follow a course in mathematics at degree level or even continue further as a postgraduate and get involved in mathematical research.

People entering today's most lucrative industries such as IT, banking and the stock market need to be confident using mathematics on a daily basis. To be sure of this, many employers still look for a traditional mathematics A-level qualification. Researchers at the London School of Economics have recently found that people who have studied mathematics can expect to earn up to 11% more than their colleagues, even in the same job!

Even in areas where pure mathematics isn't required, other mathematics skills learned at AS and A level, such as logical thinking, problem solving and statistical analysis, are often very desirable in the workplace. Mathematics is the new lingua franca of commerce, business and even journalism.

Next steps!

Find out more about the course by talking to your mathematics teachers or by visiting the Edexcel website, www.edexcel.org.uk.

Edexcel, a Pearson company, is the UK's largest awarding body, offering academic and vocational qualifications and testing to more than 25,000 schools, colleges, employers and other places of learning in the UK and in over 100 countries worldwide. Qualifications include GCSE, AS and A Level, NVQ and our BTEC suite of vocational qualifications from entry level to BTEC Higher National Diplomas, recognised by employers and higher education institutions worldwide.

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