Paper Reference(s)

6689

Edexcel GCE

Decision Mathematics D1

Advanced Subsidiary

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Nil	D1 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

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You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. *This question should be answered on the sheet provided.*

Five packs of sandwiches have been prepared for lunch, one each of egg, cheese, ham, tuna and salmon. Five people have been invited for lunch and the sandwiches which they like are given in the table.

Mr Large	Egg, Cheese
Mrs Moore	Egg, Tuna, Salmon
Ms Nice	Cheese, Ham
Mr Oliver	Cheese, Tuna, Salmon
Miss Patel	Ham, Tuna, Salmon

(*a*) Draw a bipartite graph to model this situation using the nodes printed in the diagram on the answer sheet.

(1)

The host allocates the egg sandwich to Mr Large, the cheese to Ms Nice, the tuna to Mr Oliver and the salmon to Miss Patel.

(*b*) Indicate this initial matching in a distinctive way on the bipartite graph drawn in the diagram on the answer sheet.

(1)

(*c*) Starting from this matching use the maximum matching algorithm to find a complete matching. Indicate clearly how the algorithm has been applied.

(4)

2. **Figure 1**

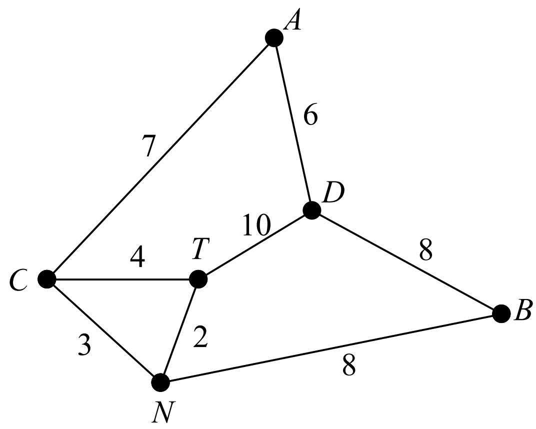

Fig. 1 shows the roads linking villages in an area covered by a district council. The numbers on the edges give the distances, in km, between the villages. After a storm a highways inspector wishes to travel along each road at least once.

(*a*) Use an appropriate algorithm to find the minimum distance she must travel, starting and finishing at *A*.

(5)

(*b*) Write down a possible route which is of minimum length.

(2)

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3. Use the binary search algorithm to locate the name GREGORY in the following list.

1. ARCHER
2. BOWEN
3. COUTTS
4. DENYER
5. EATWELL
6. FULLER
7. GRANT
8. GREGORY
9. LEECH
10. PENNY
11. THOMPSON

(7)

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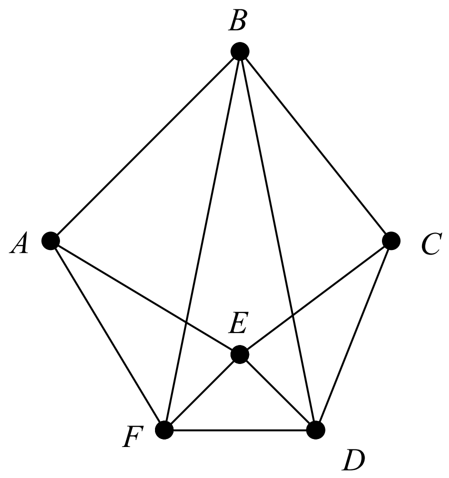
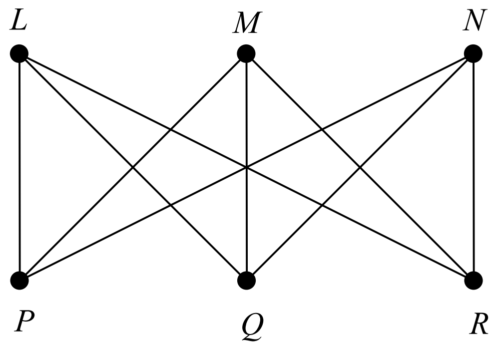
4. **Figure 2** **Figure 3**



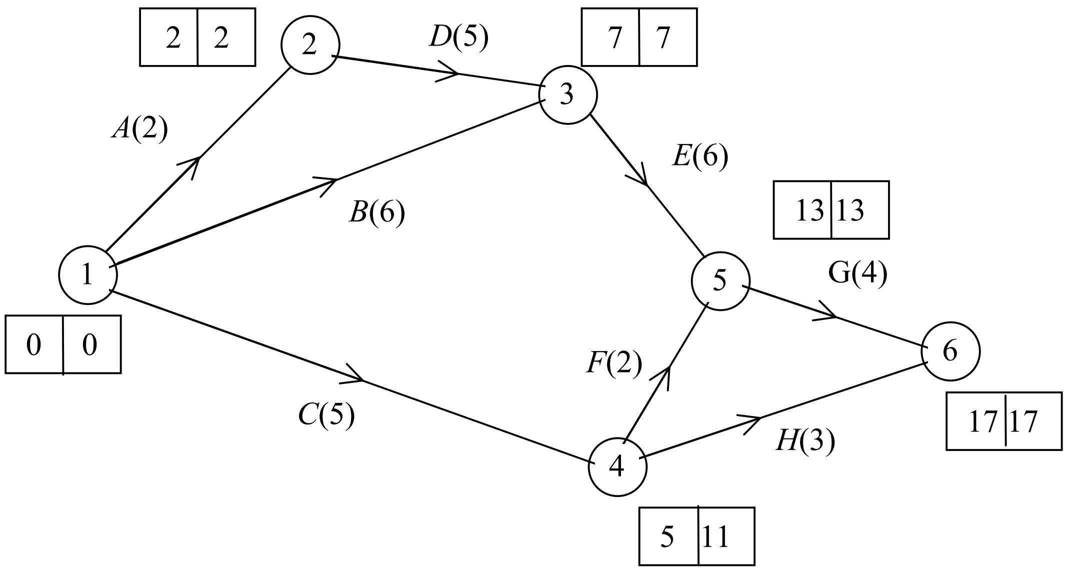


Use the planarity algorithm for graphs to determine which, if either, of the graphs shown in Fig. 2 and Fig. 3 is planar. Make your use of the algorithm clear.

(9)

5. *This question should be answered on the sheet provided.*

Figure 4



A project is modelled by the activity network in Fig. 4. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity. The left box entry at each vertex is the earliest event time and the right box entry is the latest event time.

(*a*) Determine the critical activities and the length of the critical path.

(2)

(*b*) Obtain the total floats for the non-critical activities.

(3)

(*c*) On the grid on the answer sheet, draw a cascade (Gantt) chart showing the information found in parts (*a*) and (*b*).

(4)

Given that each activity requires one worker,

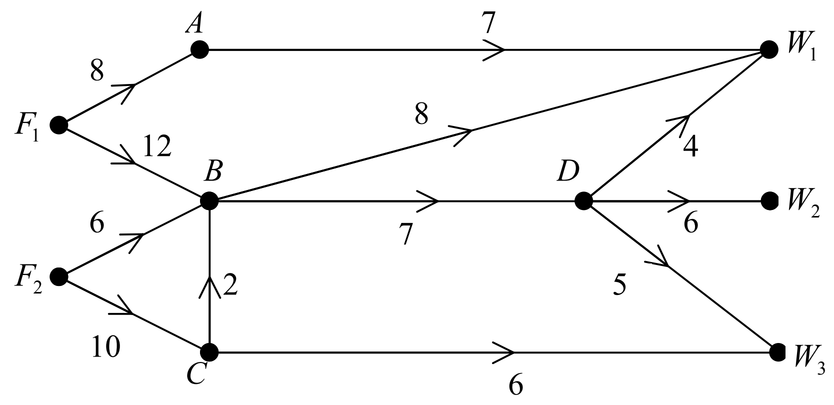
(*d*) draw up a schedule to determine the minimum number of workers required to complete the project in the critical time. State the minimum number of workers.

(3)

6. *This question should be answered on the sheet provided.*

A manufacturing company has two factories F_1 and F_2 and wishes to transport its products to three warehouses W_1, W_2 and W_3. The capacities of the possible routes, in lorry loads per day, are shown in Fig. 5.

Figure 5



(*a*) On the diagram on the answer sheet add a supersource F and a supersink W to obtain a single-source, single-sink capacitated network. State the capacities of the arcs you have added.

(3)

(*b*) Use the labelling procedure to obtain a maximal flow through the network.

(8)

(*c*) Interpret your final flow pattern giving

(i) the number of lorry loads leaving F_1 and F_2,

(ii) the number of lorry loads reaching W_1, W_2 and W_3,

(iii) the number of lorry loads passing through B each day.

(5)

7. The Bonzo Manufacturing Company makes model cars and lorries. Each car sells at a profit of £2.50 and each lorry sells at a profit of £3.00. Three departments: Manufacturing (Dept *A*); Assembly (Dept *B*); Finishing (Dept *C*) are involved in the production of the models. The times, in hours, that the cars and lorries are in each department are shown in the table.

	Car	Lorry
Dept *A*	1.50	3.00
Dept *B*	2.00	1.00
Dept *C*	0.25	0.25

In a given week, 45 hours are available in Department *A*, 35 hours in Department *B* and 5 hours in Department *C*. The manufacturer wishes to maximise his profit £*P* in this week.

Let x be the number of cars made, and y be the number of lorries made.

You may assume that all models made can be sold.

(*a*) Model this situation as a linear programming problem, giving each inequality in its simplest form with integer coefficients.

(5)

(*b*) Display the inequalities on a graph and identify the feasible region.

(4)

(*c*) By testing each vertex in the feasible region, obtain the maximum profit and the corresponding values of x and y.

(7)

(*d*) State which department has unused time and calculate this time.

(2)

END

Sheet for use in answering question 1

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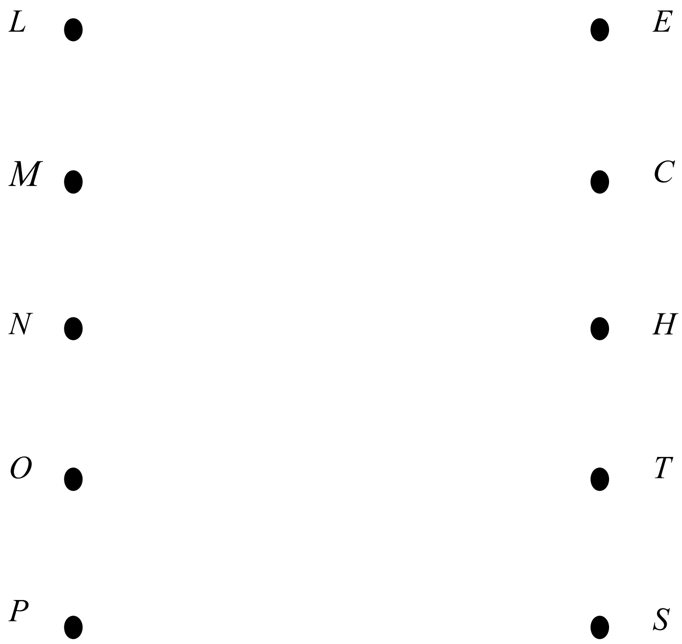
Centre No.	Candidate No.	Surname & Initials (Block Letters)
Please hand this sheet in for marking		

(*a*) & (*b*)



(*c*) ....

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Sheet for use in answering question 5

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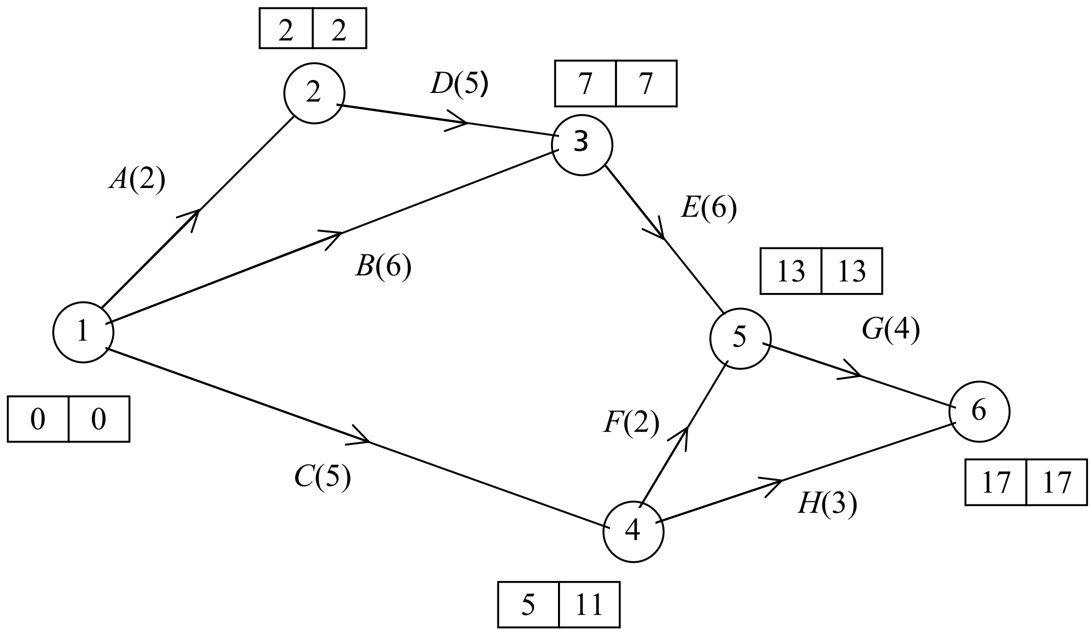
Centre No.	Candidate No.	Surname & Initials (Block Letters)
Please hand this sheet in for marking		



(*a*) Critical activities ……………………………………………………………………..

Length of critical path ……………………………………………………………..

(*b*) Floats ………………………………………………………………………………

………………………………………………………………………………………….

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………………………………………………………………………………………….

(*c*) 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Sheet for use in answering question 5

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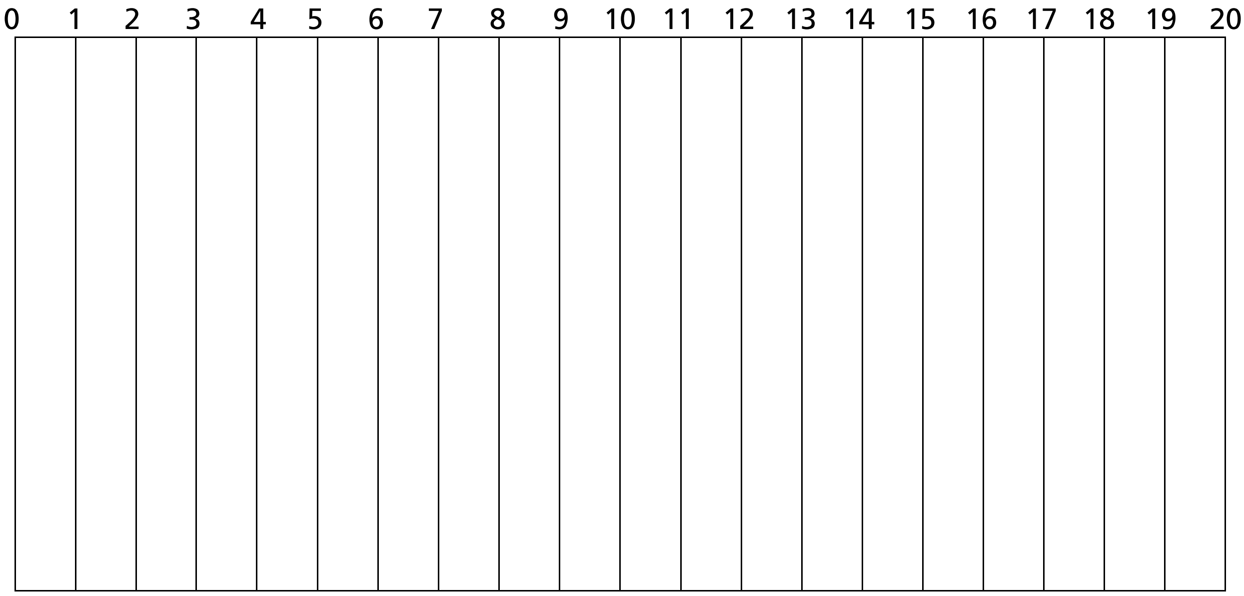
Centre No.	Candidate No.	Surname & Initials (Block Letters)
Please hand this sheet in for marking		

(*d*)



Minimum number of workers ……………………………………………………………….

Sheet for use in answering question 6

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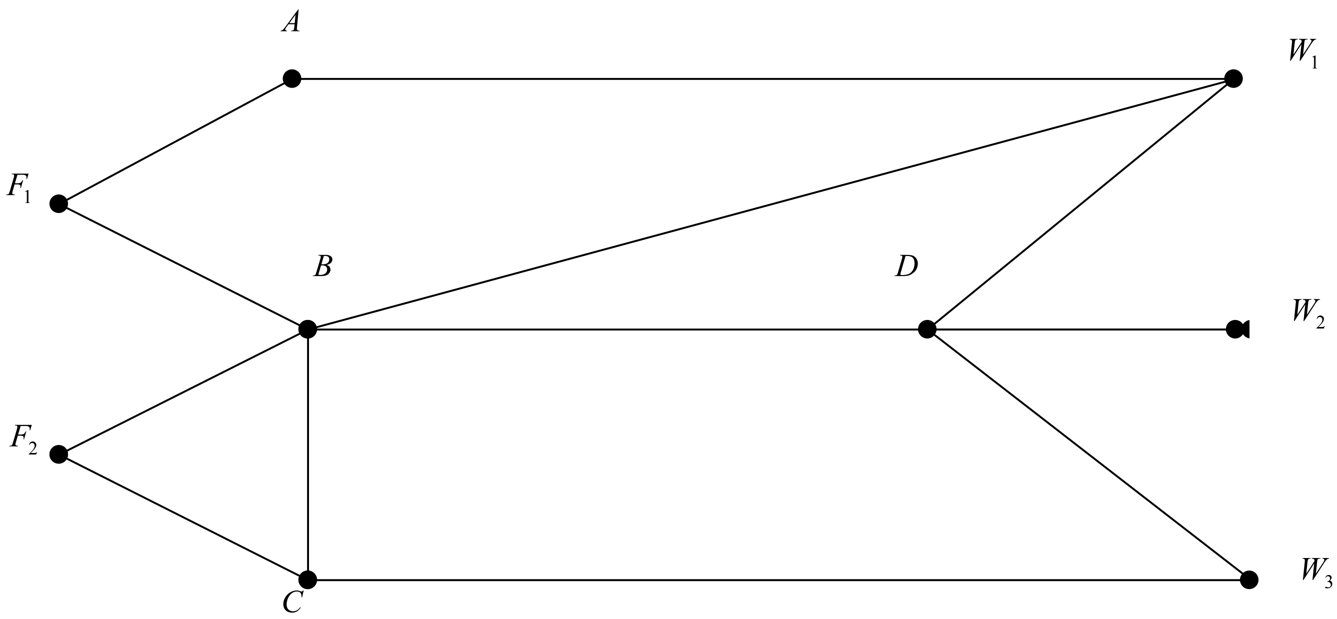
Centre No.	Candidate No.	Surname & Initials (Block Letters)
Please hand this sheet in for marking		

(*a*), (*b*)



Capacity of arcs added

(*c*)(i)……..

(ii) ……….

(iii) ……….

Paper Reference(s)

6690

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
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1. A coach company has 20 coaches. At the end of a given week, 8 coaches are at depot *A*, 5 coaches are at depot *B* and 7 coaches are at depot *C*. At the beginning of the next week, 4 of these coaches are required at depot *D*, 10 of them at depot *E* and 6 of them at depot *F*. The table below shows the distances, in miles, between the relevant depots.

	D	*E*	*F*
A	40	70	25
B	20	40	10
C	35	85	15

The company needs to move the coaches between depots at the weekend. The total mileage covered is to be a minimum. Formulate this information as a Linear Programming Problem.

(a) State clearly your decision variables.

(1)

(b) Write down the objective function in terms of your decision variables.

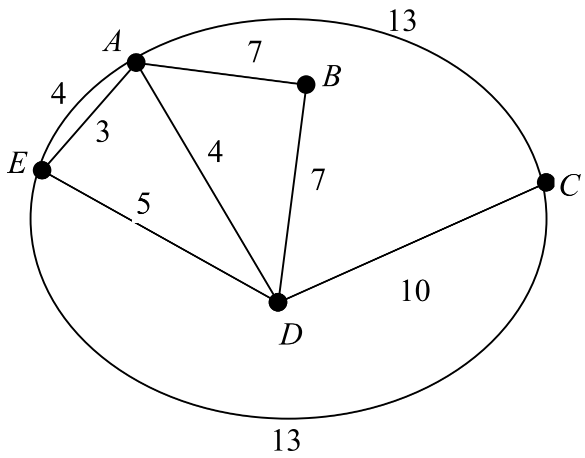
(1)

(*c*) Write down the constraints, explaining what each constraint represents.

(5)

2.

Figure 1



The network in Fig. 1 shows a number of hostels in a national park and the possible paths joining them. The numbers on the edges give the lengths, in km, of the paths.

(a) Draw a complete network showing the shortest distances between the hostels.
(You may do this by inspection. The application of an algorithm is not required.)

(2)

(b) Use the nearest neighbour algorithm on the complete network to obtain an upper bound to the length of a tour in this network which starts at A and visits each hostel exactly once.

(3)

(c) Interpret your result in part (b) in terms of the original network.

(2)

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3. A two-person zero-sum game is represented by the following payoff matrix for player *A*.

Given that the game does not have a stable solution, find the best mixed strategy for each player and the value of the game.

		B I	*B* II
A	I	4	–2
	II	–5	6

(10)

4.

Factory \ Warehouse	W_1	W_2	W_3	Availabilities
F_1	7	8	6	4
F_2	9	2	4	3
F_3	5	6	3	8
Requirements	2	9	4	

A manufacturer has 3 factories F_1, F_2, F_3 and 3 warehouses W_1, W_2, W_3. The table shows the cost C_{ij}, in appropriate units, of sending one unit of product from factory F_i to warehouse W_j. Also shown in the table are the number of units available at each factory F_i and the number of units required at each warehouse W_j. The total number of units available is equal to the total number of units required.

(*a*) Use the North-West Corner rule to obtain a possible pattern of distribution and find its cost.

(5)

(*b*) Calculate shadow costs R_i and K_j for this pattern and hence obtain improvement indices I_{ij} for each route.

(6)

(*c*) Using your answer to part (*b*), explain why the pattern is optimal.

(1)

5. *This question should be answered on the sheet provided.*

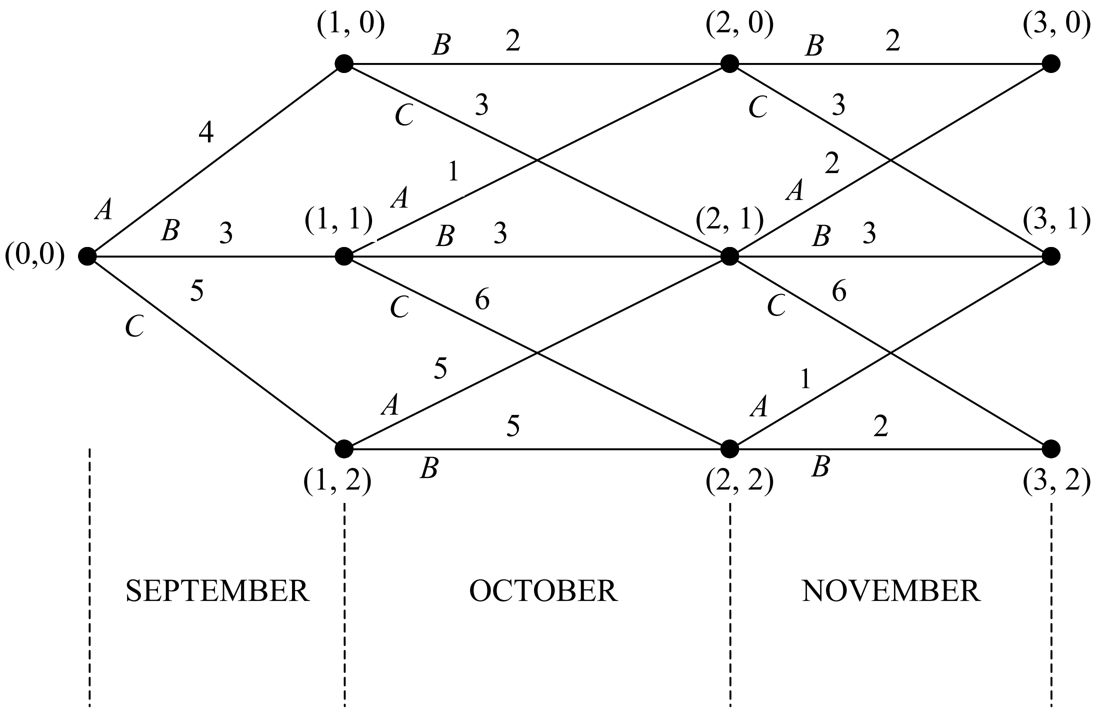
At the beginning of each month an advertising manager must choose one of 3 adverts:

A: use the previous advert;

B: use the current advert;

C: run a new advert.

The possible choices are shown in the network below together with (stage, state) variables at the vertices and the expected profits, in thousands of pounds, on the arcs.



The manager wants to maximise her profits for the 3 month period.

(*a*) Complete the table on the answer sheet.

(9)

(*b*) Hence obtain the sequence of decisions she should make to obtain the maximum profit. State the maximum profit.

(3)

6. A large room in a hotel is to be prepared for a wedding reception. The tasks that need to be carried out are:

I clean the room,

II arrange the tables and chairs,

III set the places,

IV arrange the decorations.

The tasks need to be completed consecutively and the room must be prepared in the *least possible time*. The tasks are to be assigned to four teams of workers A, B, C and D. Each team must carry out only one task. The table below shows the times, in minutes, that each team takes to carry out each task.

	A	B	C	D
I	17	24	19	18
II	12	23	16	15
III	16	24	21	18
IV	12	24	18	14

(*a*) Use the Hungarian algorithm to determine which team should be assigned to each task. You must make your method clear and show

(i) the state of the table after each stage in the algorithm,

(ii) the final allocation.

(11)

(*b*) Obtain the minimum total time taken for the room to be prepared.

(2)

7. *This question should be answered on the sheet provided*

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	A	*B*	*C*	*D*	*E*	*F*	*G*
A	-	103	89	42	54	143	153
B	103	-	60	98	56	99	59
C	89	60	-	65	38	58	77
D	42	98	65	-	45	111	139
E	54	56	38	45	-	95	100
F	143	99	58	111	95	-	75
G	153	59	77	139	100	75	-

A computer supplier has outlets in seven cities *A*, *B*, *C*, *D*, *E*, *F* and *G*. The table shows the distances, in km, between the seven cities. Joan lives in city *A* and has to visit each city to advise on displays. She wishes to plan a route, starting and finishing at *A,* visiting each city once and covering a minimum distance.

(*a*) Use Prim's algorithm to obtain a minimum spanning tree for the network and draw this tree. Start with *A* and state the order in which the vertices are added to your tree.

(5)

Given that the network representing this problem is complete and satisfies the triangle inequality,

(b) determine an initial upper bound for the length of the route travelled by Joan.

(2)

(c) Starting from your initial upper bound for the length of the route and using an appropriate method, find an upper bound which is less than 430 km.

(3)

(*d*) By deleting city *A*, determine a lower bound for the length of Joan's route.

(4)

END

Sheet for use in answering question 5

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Centre No.	Candidate No.	Surname & Initials (Block Letters)
Hand this sheet in for marking		

Stage	State	Action	Cost	Value
2	0	*B* *C*		
	1	*A* *B* *C*		
	2	*A* *B*		
1	0	*B* *C*		
	1	*A* *B* *C*		
	2	*A* *B*		
0	0	*A* *B* *C*		

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Sheet for use in answering question 7

6690 Decision Mathematics D2 Specimen Paper

Centre No.	Candidate No.	Surname & Initials (Block Letters)
hand this sheet in for marking		

(*a*)

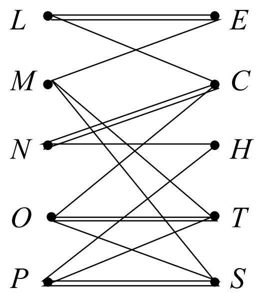
	A	*B*	*C*	*D*	*E*	*F*	*G*
A	-	103	89	42	54	143	153
B	103	-	60	98	56	99	59
C	89	60	-	65	38	58	77
D	42	98	65	-	45	111	139
E	54	56	38	45	-	95	100
F	143	99	58	111	95	-	75
G	153	59	77	139	100	75	-

Minimum spanning tree.

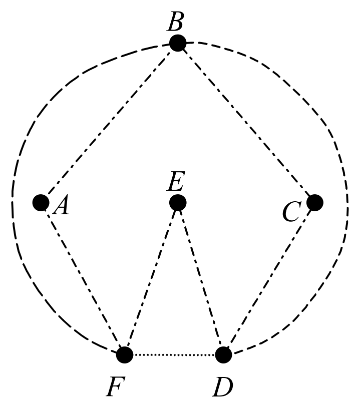
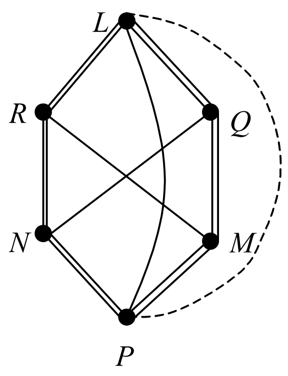
(*b*) Initial upper bound.

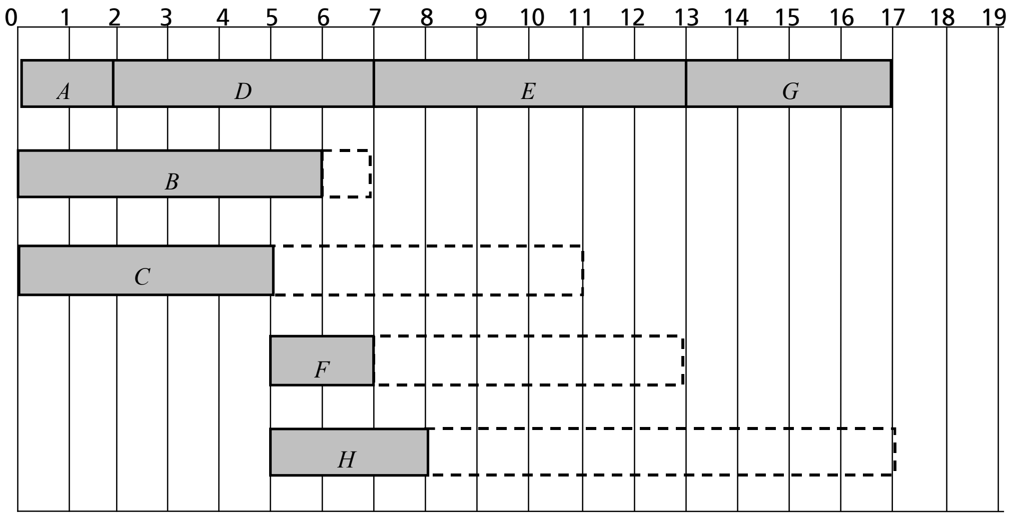
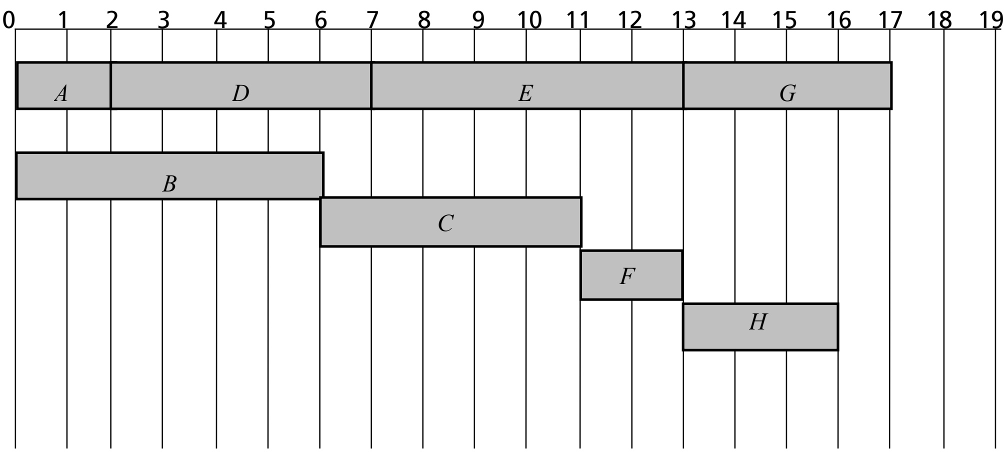
(*c*) Improved upper bound.

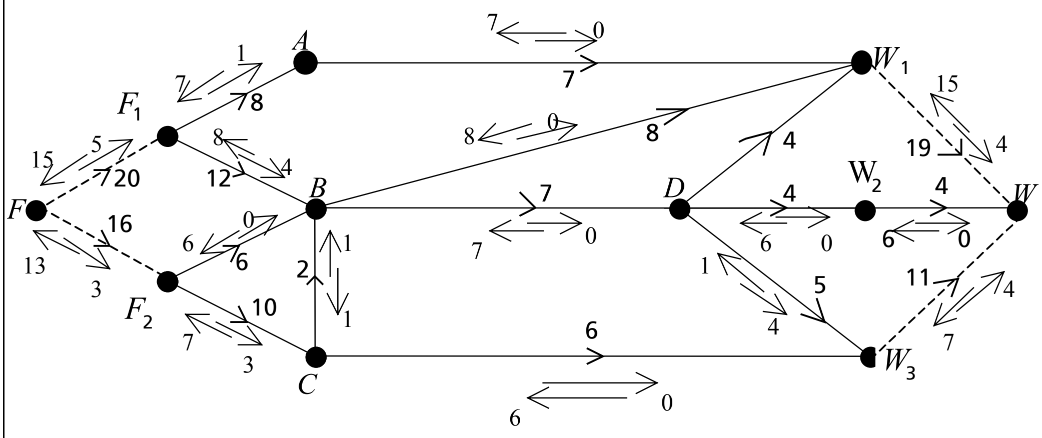
(*d*) Lower bound.

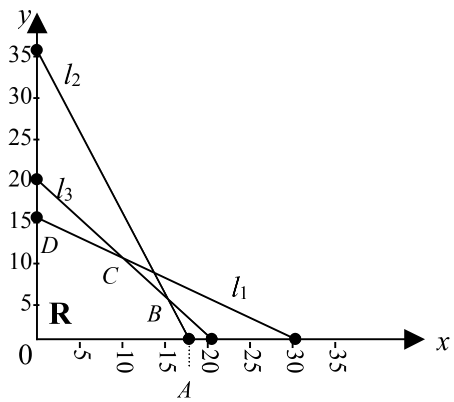
Question number	Scheme	Marks
1. (*a*)		
	Bipartite graph	B1 (1)
(*b*)	Initial matching	B1 (1)
(*c*)	Alternating path	
	$M - E = L - C = N - H$ (breakthrough)	M1 A1
	(Changing status) $M = E - L = C - N = H$	
	Complete Matching: Nice—Ham, Oliver—Tuna, Patel—Salmon, Moore—Egg, Large—Cheese	M1 A1 (4)
		(6 marks)
2. (*a*)	Vertices of odd valency $C(3)$, $D(3)$, $T(3)$, $N(3)$	M1
	Possible pairings (i) C & D and T & N	
	$13 + 2 = 15$	A1
	(ii) C & T and D & N	
	$4 + 12 = 16$	A1
	(iii) C & N and D & T	
	$3 + 10 = 13$	A1
	(iii) is min. So repeat CN & DT	M1
	Min distance $(7 + 6 + 4 + 10 + 8 + 8 + 2 + 3) + 13 = 61$ km	(5)
(*b*)	Possible route $ADTDBNTCNCA$	M1 A1 (2)
		(7 marks)

Question number	Scheme	Marks
3.	As there are 11 names in list, middle location is [(11 + 1)/2] = 6, i.e. FULLER	M1 A1
	GREGORY must occur after FULLER if at all, so list reduces to:	
	7 GRANT	
	8 GREGORY	
	9 LEECH	
	10 PENNY	
	11 THOMPSON	A1
	Middle location now [(11 + 7)/2] = 9, i.e. LEECH	
	GREGORY must occur before LEECH if at all, so list reduces to	
	7 GRANT	
	8 GREGORY	M1 A1
	Middle location now [(8 + 7)/2] = 8, i.e. GREGORY	
	The name GREGORY has been found at position 8	M1 A1 (7)
		(7 marks)

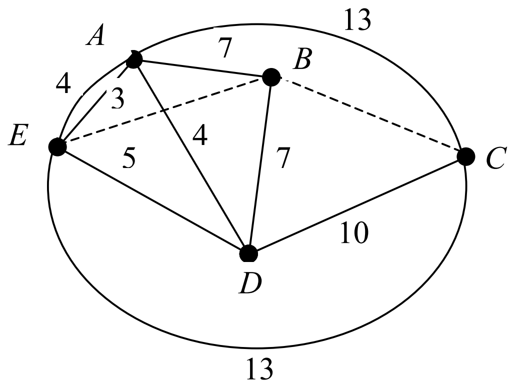
Question number	Scheme	Marks
4. (i)	Identify Hamiltonian circuit Leave *FD* outside Move *BD* outside	
	 Move *BF* outside	B1
	Now no intersections and so graph is planar	M1 A1 A1
	Redraw graph	M1
	 Identify Hamiltonian circuit by the double line	B1
	Move *LP* outside Leave *RM* inside *NQ* crosses *RM* if inside and *LP* if outside $\therefore$ Non planar	M1 A1 A1 (9)
		(9 marks)

Question number	Scheme	Marks
5. (*a*)	Critical activities *ADEG*	
	Length of critical path 17 days	M1 A1 (2)
(*b*)	Floats *B*: 7 – 0 – 6=1	
	C: 11 – 0 – 5=6	
	F: 13 – 5 – 2=6	
	H: 17 – 5 – 3=9	M1 A1 A1 (3)
(*c*)		M1 A1 A1 A1 (4)
(*d*)		M1 A1
	Using its float of 6 activity *C* can be started when *B* is completed. Activity *F* also has a float of 6 and so can start when *C* has finished and can be completed without delaying the project. We only need to use 8 of the float on *H* to start *H* when *F* has finished. Therefore only 2 workers are required to complete project in given time.	A1 (3)
		(12 marks)

Question number	Scheme	Marks
6. (*a*)	Add F & W	
	Capacities are $FF_1 \geq 20,\ FF_2 \geq 16$	
	$W_1W \geq 19,\ W_2W \geq 6,\ W_3W \geq 11$	M1 A1 A1 (3)
(*b*)		M1 A4
	In this pattern no further flow into W_1 possible, or into D, or into W_3. Suggests flow is maximal.	M1
	A flow value 28 is shown on above diagram. This flow is maximal as there is a cut consisting of arcs AW_1(7), BW_1 (8), BD (7) and CW_3 (6) of capacity 28. [Or there is a partition of the vertices $\{FF_1F_2ABC\}$ and $\{W_1W_2W_3DW\}$]	M1 A1 (8)
(*c*)	From maximal flow pattern	
	(i) Number of lorry loads leaving F_1 is 8 + 7 = 15	
	Number of lorry loads leaving F_2 is 6 + 7 = 13	M1 A1
	(ii) Reaching W_1 15 lorry loads	
	Reaching W_2 6 lorry loads	
	Reaching W_3 7 lorry loads	M1 A1
	(iii) 8+6+1 or 8+7 = 15 lorry loads	B1 (5)
		(16 marks)

Question number	Scheme	Marks
7. (*a*)	Objective Max $P = 2.5x + 3.0y$	B1
	Dept A $1.5x + 3y \le 45; 3x + 6y \le 90$	B1
	Dept B $2x + y \le 35$	B1
	Dept C $0.25x + 0.25y \le 5; x + y \le 20$	B1
	$x \ge 0 \quad y \ge 0$	B1 (5)
(*b*)	l_1 $3x + 6y = 90$ through $(0, 15)(30, 0)$	B1
	l_2 $2x + y = 35$ through $(0, 35)\left(17\frac{1}{2}, 0\right)$	B1
	l_3 $x + y = 20$ through $(0, 20)(20, 0)$	B1
		
	Feasible region	B1 (4)
(*c*)	Vertices O is (0, 0)	
	A is $\left(17\frac{1}{2}, 0\right)$, D is $(0, 15)$	B1
	B intersection of $\left.\begin{matrix} 2x + y = 35 \\ x + y = 20 \end{matrix}\right\} \begin{matrix} x = 15 \\ y = 5 \end{matrix}$	
	C intersection of $\left.\begin{matrix} 3x + 6y = 90 \\ x + y = 20 \end{matrix}\right\} \begin{matrix} y = 10 \\ x = 10 \end{matrix}$	M1 A1 A1
	$P_o = 0,\ P_A = 43.75,\ P_B = 52.5$	
	$P_C = 55,\ P_D = 45$	M1 A1
	Max value P is £55 at $x = 10$, $y = 10$	A1 (7)
(*d*)	l_1 & l_3 intersect at C and so are tight. Dept B (l_2) therefore has spare time. $35 - 2(10) - 10 = 5$ hrs.	M1 A1 (2)
		(18 marks)

Question number		Scheme	Marks	
1.	(*a*)	x_{11} no. of coaches from *A* to *D*		
		x_{12} no. of coaches from *A* to *E*		
		x_{13} no. of coaches from *A* to *F*		
		x_{21} no. of coaches from *B* to *D*		
		x_{22} no. of coaches from *B* to *E*		
		x_{23} no. of coaches from *B* to *F*		
		x_{31} no. of coaches from *C* to *D*		
		x_{32} no. of coaches from *C* to *E*		
		x_{33} no. of coaches from *C* to *F*	B1	(1)
	(*b*)	Minimise $z = 40\,x_{11} + 70x_{12} + 25x_{13}$		
		$+ 20\,x_{21} + 40x_{22} + 10x_{23}$		
		$+ 35\,x_{31} + 85x_{32} + 15x_{33}$	B1	(1)
	(*c*)	Depot *A* $x_{11} + x_{12} + x_{13} = 8$ (no. of coaches at *A*)		
		Depot *B* $x_{21} + x_{22} + x_{23} = 5$ (no. of coaches at *B*)		
		Depot *C* $x_{31} + x_{32} + x_{33} = 7$ (no. of coaches at *C*)	M1 A1	
		Depot *D* $x_{11} + x_{21} + x_{31} = 4$ (no. required at *D*)		
		Depot *E* $x_{21} + x_{22} + x_{32} = 10$ (no. required at *E*)		
		Depot *F* $x_{31} + x_{32} + x_{33} = 6$ (no. required at *F*)	M1 A1	
		Reference to number of coaches at *A*, *B* and *C* = number of coaches at *D*, *E* and *F*	B1	(5)

Question number	Scheme	Marks
2. (*a*)		
	BC: 17	M1
	EB: 10	A1 (2)
(*b*)	*AE*(3), *ED*(5), *DB*(7), *BC*(17)	M1 A1
	Complete with edge *CA*(13)	
	Total length 45 km	A1 (3)
(*c*)	Tour in original is *A E D B D C A*	
	Since *BC* is not in original network and shortest distance is *BD* plus *DC*	M1 A1 (2)
		(7 marks)

Question number	Scheme	Marks
3.	Suppose *A* chooses I with probability p	
	A chooses II with probability $(1-p)$	
	Expected gain if *B* chooses I $4p-5(1-p)$	
	II $-2p+6(1-p)$	M1 A1
	Optimal value when	
	$4p-5(1-p)=-2p+6(1-p)$	
	$p=\frac{11}{17},\ 1-p=\frac{6}{17}$	
	Play I, $\frac{11}{17}$ of time and II, $\frac{6}{17}$ of time	M1 A1
	Suppose *B* chooses I with probability q	
	B chooses II with probability $(1-q)$	
	Expected loss if *A* chooses I $4q-2(1-q)$	
	II $-5q+6(1-q)$	M1 A1
	Optimal value when	
	$4q-2(1-q)=-5q+6(1-q)$	
	$q=\frac{8}{17},\ 1-q=\frac{9}{17}$	M1 A1
	Play I, $\frac{8}{17}$ of time and II, $\frac{9}{17}$ of time	
	Value of game $=9p-5(=4p-5(1-p))=\frac{14}{17}$ gain to player *A*	M1 A1 (10)
	[or $6-11q=-5q+6(1-q)$] $=\frac{14}{17}$ loss to Player *B*]	
		(10 marks)

Question number	Scheme	Marks
4. (*a*)		

	W_1	W_2	W_3	Available
F_1	2	2		4
F_2		3		3
F_3		4	4	8
Require	2	9	4	

M1 A1 A1

Cost $2 \times 7 + 2 \times 8 + 3 \times 2 + 4 \times 6 + 4 \times 3$

$= 14 + 16 + 6 + 24 + 12 = 72$ — M1 A1 (5)

(*b*) For occupied cells $R_i + K_j = C_{ij}$ gives

$(1,1)\, R_1 + K_1 = 7; (1,2)\, R_1 + K_2 = 8; (2,2)\, R_2 + K_2 = 2$

$(3,2)\, R_3 + K_2 = 6; (3,3)\, R_3 + K_3 = 3$ — M1 A1

Taking $R_1 = 0$ we obtain $K_1 = 7,\ K_2 = 8,\ R_2 = -6,\ R_3 = -2,\ K_3 = 5$

	$K_1 = 7$	$K_2 = 8$	$K_3 = 5$
$R_1 = 0$	(0)	(0)	6
$R_2 = -6$	9	(0)	4
$R_3 = -2$	5	(0)	(0)

M1 A1

Improvement indices $I_{ij} = C_{ij} - R_i - K_j$

$I_{13} = 6 - 5 - 0 = 1$

$I_{21} = 9 - 7 - (-6) = 8$

$I_{23} = 4 - 5 - (-6) = 5$

$I_{31} = 5 - 7 - (-2) = 0$ — M1 A1 (6)

(*c*) No negative improvement indices and so given solution is optimal and gives minimum cost. If there were a negative I_{ij} then using this route would reduce cost. — M1 (1)

(12 marks)

Question number	Scheme	Marks
5. (a)		

Stage	State	Action	Cost	Value	Marks
2	0	*B*	2	2	
		C	3	3 ←	
	1	*A*	2	2	
		B	3	3	
		C	6	6 ←	
	2	*A*	1	1	
		B	2	2 ←	M1 A1 A1
1	0	*B*	2	2 + 3 = 5	
		C	3	3 + 6 = 9 ←	M1 A1
	1	*A*	1	1 + 3 = 4	
		B	3	3 + 6 = 9 ←	
		C	6	6 + 2 = 8	A1
	2	*A*	5	5 + 6 =11 ←	
		B	5	5 + 2 = 7	
					A1
0	0	*A*	4	4 + 9 = 13	
		B	3	3 + 9 = 12	
		C	5	5 + 11=16 ←	M1 A1 (9)

(b)	Hence maximum profit is 16	B1
	Tracing back through calculations the optimal strategy is *C A C*	M1 A1 (3)
		(12 marks)

Question number	Scheme	Marks
6. (a)	Row minimum	
	17 24 19 18 ⋮ 17	
	12 23 16 15 ⋮ 12	
	16 24 21 18 ⋮ 16	
	12 24 18 14 ⋮ 12	
	Reducing rows gives: 0 7 2 1	
	0 11 4 3	
	0 8 5 2	
	0 12 6 2	
	Column minimum 0 7 2 1	M1 A1
	Reducing columns gives: 0 0 0 0	
	0 4 2 2	
	0 1 3 1	
	0 5 4 1	M1 A1
	No assignment possible as zeroes can all be covered by 2 lines (2<4)	B1
	Minimum uncovered element is 1	
	Applying algorithm gives:	
	1 0 0* 0	
	0* 3 1 1	
	0 0* 2 0	
	0 4 3 0*	M1 A1, A1
	Now requires 4 lines to cover all zeroes so assignment now possible	B1
	(1, 3) - only zero in column 3	
	(3, 2) - row 1 already used and now only zero in *C*2	
	(4, 4) - only remaining possibility in *C*4	
	(2, 1) - must then be used	
	I – *C*, II – *A*, III – *B*, IV – *D*	M1 A1 (11)
(b)	Cost of this assignment	
	19 + 12 + 24 + 14 = 69 minutes	M1 A1 (2)
		(13 marks)

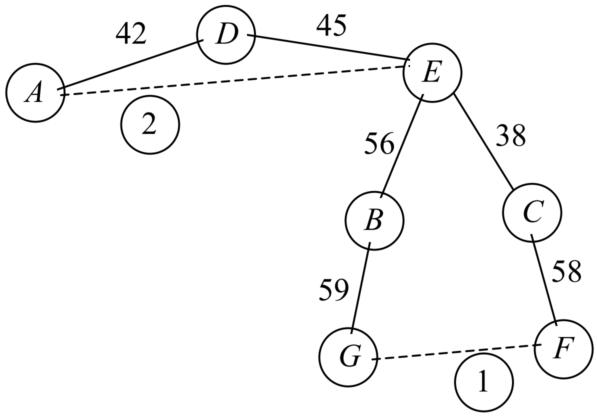
Question number	Scheme	Marks

7. (a)

	A (1)	*B* (5)	*C* (4)	*D* (2)	*E* (3)	*F* (6)	*G* (7)
	-	103	89	42	54	143	153
B	103	-	60	98	(56)	99	59
C	89	60	-	65	(38)	58	77
D	(42)	98	65	-	45	111	139
E	54	56	38	(45)	-	95	100
F	143	99	(58)	111	95	-	75
G	153	(59)	77	139	100	75	-

M1 A1 A1

Vertices added in order *ADECBFG*



M1 A1 (5)

(b) Upper bound = 2 (weight of M.S.T.)

= 2(42 + 45 + 38 + 58 + 56 + 59)

= 2(298) = 596 M1 A1 (2)

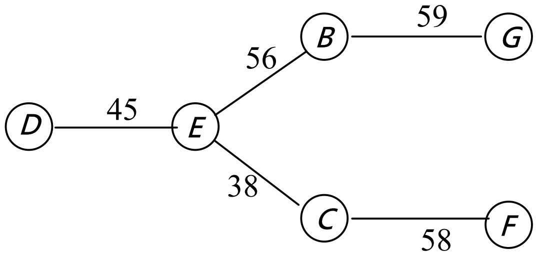
(c) Short cut 1 replaces *FCEBG* by *FG* saving (58 + 38 + 56 + 59) – 75 = 136

Now upper bound is 460 M1 A1

Short cut 2 replaces *EDA* by *EA* saving (45 + 42) – 54 = 33

Now upper bound is 427 A1 (3)

continued over…

7.	(d)	If *A* is removed then M.S.T. of remaining network is 	M1 A1
		Lower bound is obtained by adding weights of edges *AD*(42) and *AE*(54) (edges of least weight at *A*)	
		So lower bounds is 256 + 42 + 54 = 352	M1 A1 (4)
			(14 marks)