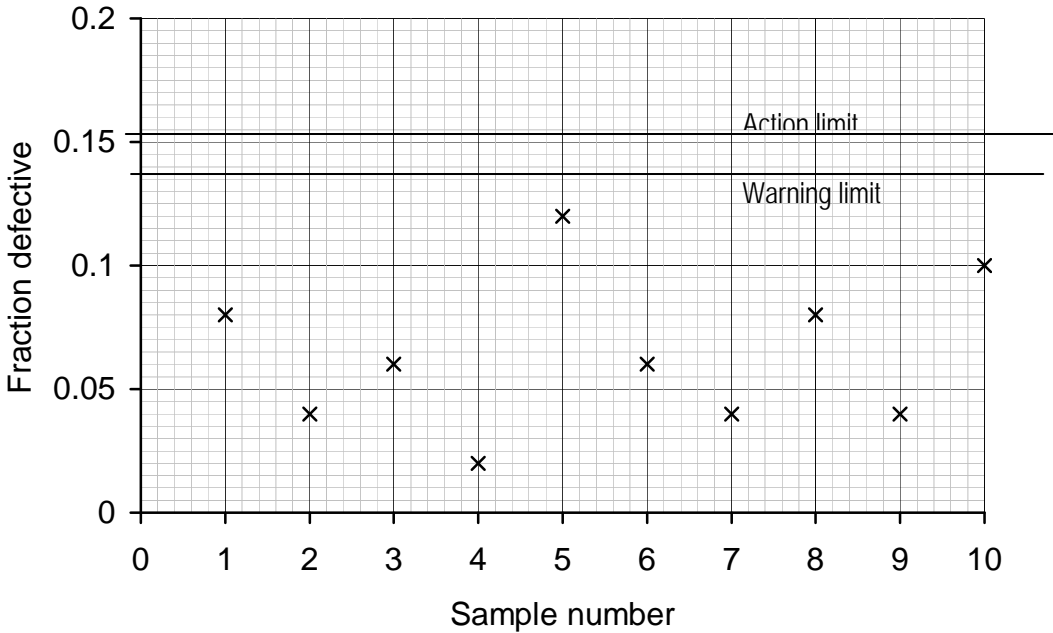


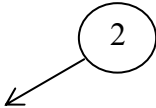
June 2005  
6688 Statistics S6  
Mark Scheme

Question Number	Scheme	Marks																				
<b>1.</b>	$H_0 : \text{Median}_m - \text{Median}_s = 0; H_1 : \text{Median}_m - \text{Median}_s > 0$  8 + & 2 -  $N=10 \quad P(\leq 2 \mid p = 0.5, n = 10) = 0.0547 > 0.05$  $\therefore$ Insufficient evidence to reject $H_0$ . Students do not carry out calculations more accurately when listening to music.	B1  M1 A1  M1 A1  A1√  <b>(6)</b>																				
<b>2.</b>	$H_0 : \text{Median} = 20; H_1 : \text{Median} \neq 20$  <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 10px;"><math>x - \text{median}</math></td> <td style="padding: 2px 10px;">+10</td> <td style="padding: 2px 10px;">-8</td> <td style="padding: 2px 10px;">-5</td> <td style="padding: 2px 10px;">+1</td> <td style="padding: 2px 10px;">+7</td> <td style="padding: 2px 10px;">-3</td> <td style="padding: 2px 10px;">-4</td> <td style="padding: 2px 10px;">-6</td> <td style="padding: 2px 10px;">+2</td> </tr> <tr> <td style="padding: 2px 10px;">Rank</td> <td style="padding: 2px 10px;"><u>9</u></td> <td style="padding: 2px 10px;">8</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;"><u>1</u></td> <td style="padding: 2px 10px;"><u>7</u></td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">4</td> <td style="padding: 2px 10px;">6</td> <td style="padding: 2px 10px;"><u>2</u></td> </tr> </table> $S^+ = 19 \quad (S^- = 26)$  $n = 9 \Rightarrow \text{CR} : S \leq 5$  Since 19 is not in the critical region there is insufficient evidence to reject $H_0$ . The claim is justified on this evidence.	$x - \text{median}$	+10	-8	-5	+1	+7	-3	-4	-6	+2	Rank	<u>9</u>	8	5	<u>1</u>	<u>7</u>	3	4	6	<u>2</u>	B1  M1 M1 A1  A1  B1  A1√  <b>(7)</b>
$x - \text{median}$	+10	-8	-5	+1	+7	-3	-4	-6	+2													
Rank	<u>9</u>	8	5	<u>1</u>	<u>7</u>	3	4	6	<u>2</u>													
<b>3.</b>	$H_0 : \text{Median time of girls} = \text{Median time of boys}$ $H_1 : \text{Median time of girls} \neq \text{Median time of boys}$  $n_1 = 25, n_2 = 25$ $T \approx N\left(\frac{25(25 + 25 + 1)}{2}, \frac{25 \times 25(25 + 25 + 1)}{12}\right) \approx N(637.5, 2656.25)$  $Z = \frac{704 - 637.5}{\sqrt{2656.25}} = 1.29$	both <b>B1</b>  M1 A1 A1  M1 A1  Accept $704 \pm 0.5$  Since 1.29 is not in the critical region ( $z > 1.96$ ) there is no evidence to suggest that boys are quicker at French translation than girls.  A1√  <b>(7)</b>																				

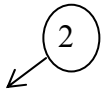
<p><b>4.</b></p>	$C.F. = \frac{(48.13 + 43.13 + 46.50 + 53.65)^2}{24} = \frac{(191.41)^2}{24}$ $\therefore SST = 1543.9043 - \frac{(191.41)^2}{24} = 17.3298$ $SSA = \frac{1}{6} \{48.13^2 + 43.13^2 + 46.50^2 + 53.65^2\} - \frac{(191.41^2)}{24} = 9.6365$ <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="border-top: 1px solid black; border-bottom: 1px solid black;">Source</th> <th style="border-top: 1px solid black; border-bottom: 1px solid black;">df</th> <th style="border-top: 1px solid black; border-bottom: 1px solid black;">s.s</th> <th style="border-top: 1px solid black; border-bottom: 1px solid black;">mss</th> <th style="border-top: 1px solid black; border-bottom: 1px solid black;">Ratio</th> </tr> </thead> <tbody> <tr> <td style="border-bottom: 1px solid black;">Between areas</td> <td style="border-bottom: 1px solid black;">3</td> <td style="border-bottom: 1px solid black;">9.6365</td> <td style="border-bottom: 1px solid black;">3.2122</td> <td style="border-bottom: 1px solid black;">8.35</td> </tr> <tr> <td style="border-bottom: 1px solid black;">Residual</td> <td style="border-bottom: 1px solid black;">20</td> <td style="border-bottom: 1px solid black;">7.6933</td> <td style="border-bottom: 1px solid black;">0.3847</td> <td style="border-bottom: 1px solid black;"></td> </tr> <tr> <td>Total</td> <td>23</td> <td>17.3298</td> <td></td> <td></td> </tr> </tbody> </table> <p><math>H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 ; H_1 : \text{Not all means are equal}</math></p> <p>(Assume <math>\alpha = 0.05</math>) <math>F_{3,20} = 3.10</math> (4.94 for 1%)</p> <p>Since 8.35 is in the critical region there is evidence that there is a difference in the mean yields between areas.</p>	Source	df	s.s	mss	Ratio	Between areas	3	9.6365	3.2122	8.35	Residual	20	7.6933	0.3847		Total	23	17.3298			<p>M1 A1</p> <p>M1 A1</p> <p>df Residual Ratio</p> <p>B1 B1 M1 A1</p> <p>B1</p> <p>B1</p> <p>A1√ (11)</p>
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<p><b>5. (a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> <p><b>(d)</b></p> <p><b>(e)</b></p>	$\hat{p} = \frac{32}{10 \times 50} = 0.064$ $UWL = 0.064 + 1.96 \times \sqrt{\frac{0.064 \times 0.936}{59}} = 0.1318\dots$ $UAL = 0.064 + 2.5758 \times \sqrt{\frac{0.064 \times 0.936}{50}} = 0.153156\dots$ <p>Graph (Limits and scales)</p> <p>Target value is zero; Company not concerned if <math>\hat{p}</math> tends to zero.</p> <p>Graph (Points)</p> <p>All points below warning limit so production is in control.</p>	<p>B1 (1)</p> <p>M1 B1 A1</p> <p>B1 A1 (5)</p> <p>B2 (2)</p> <p>B1 B1 (2)</p> <p>B2 (2)</p> <p>B1 (1)</p>																				



Scales & labels B1, Limits B1



Points B2



6 (a)	Randomised Block Design	B1 (1)																									
(b)	$SSO = \frac{1}{4} \{302^2 + 297^2 + 287^2\} - \frac{886^2}{12} = 29.17$ $SSM = \frac{1}{3} \{179^2 + 250^2 + 271^2 + 186^2\} - \frac{886^2}{12} = 2109.67$ <table border="1" style="width: 100%; border-collapse: collapse; margin: 10px 0;"> <thead> <tr> <th style="text-align: left;">Source</th> <th style="text-align: center;">df</th> <th style="text-align: center;">s.s.</th> <th style="text-align: center;">MSS</th> <th style="text-align: center;">Ratio</th> </tr> </thead> <tbody> <tr> <td>Operators</td> <td style="text-align: center;">2</td> <td style="text-align: center;">29.17</td> <td style="text-align: center;">14.59</td> <td style="text-align: center;">0.75</td> </tr> <tr> <td>Machines</td> <td style="text-align: center;">3</td> <td style="text-align: center;">2109.67</td> <td style="text-align: center;">703.22</td> <td style="text-align: center;">36.12</td> </tr> <tr> <td>Residual</td> <td style="text-align: center;">6</td> <td style="text-align: center;">116.83</td> <td style="text-align: center;">19.47</td> <td></td> </tr> <tr> <td>Total</td> <td style="text-align: center;">11</td> <td style="text-align: center;">2255.67</td> <td></td> <td></td> </tr> </tbody> </table> <p style="margin-left: 20px;">(i) <math>H_0 : \mu_A = \mu_B = \mu_C = \mu_D</math>; <math>H_1</math> : Not all means are equal</p> <p style="margin-left: 20px;"><math>\alpha = 0.05</math> (say) <math>F_6^3(0.05) = 4.76</math> (<math>F_6^3(0.01) = 9.78</math>)</p> <p style="margin-left: 20px;">Since 36.12 is in the critical region there is evidence of differences between machines</p> <p style="margin-left: 20px;">(ii) <math>H_0 : \mu_1 = \mu_2 = \mu_3</math>; <math>H_1</math> : Not all means are equal</p> <p style="margin-left: 20px;"><math>\alpha = 0.05</math> (say) <math>F_6^2(0.05) = 5.14</math> (<math>F_6^2(0.01) = 10.90</math>)</p> <p style="margin-left: 20px;">Since 0.75 is not in the critical region there is insufficient evidence to reject <math>H_0</math>. There are no differences in the mean quality of the operators.</p>	Source	df	s.s.	MSS	Ratio	Operators	2	29.17	14.59	0.75	Machines	3	2109.67	703.22	36.12	Residual	6	116.83	19.47		Total	11	2255.67			B1 B1 B1 df Residual Ratios B1 B1 M1 A1 A1 B1 B1 B1 A1√ B1 B1 A1√ <b>(14)</b>
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7. (a)	$S_{xy} = 6493576 - \frac{14061 \times 9297}{20} = -42679.85$ $\therefore \hat{\beta} = \frac{-42679.85}{13998.95} = -3.048789\dots$ $\therefore \hat{\alpha} = \frac{9297}{20} - (-3.048789\dots) \times \frac{14061}{20} = 2608.303167\dots$ $\therefore y = 2608.30 - 3.05x$	B1 -3.05 M1 A1 2608.30 M1 A1 (5)
(b)	$RSS = 164592.55 - \frac{(-42679.85)^2}{13998.95} = 34470.67686$ <p><math>\therefore</math> 95% CI is given by</p> $-3.048789\dots \pm 2.101 \times \sqrt{\frac{34470.67686}{18 \times 13998.95}}$ <p>i.e. -3.83 &amp; -2.27</p>	M1 A1 M1 B1 A1 A1 (6)
(c)	-3.00 is in the CI $\therefore$ Assumption is justified.	B1 B1 (2)
(d)	$\varepsilon_i \sim N(0, \sigma^2)$ Plot residuals $(y - \hat{y})$ against $x$ Residuals randomly scattered about $x$ axis $\Rightarrow$ model justified	B1 B1 B1 (3)