edexcel

January 2006
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\Sigma x = 49.6$; $\Sigma x^2 = 362.36$	
	$s^2 = \frac{1}{6}\left(362.36 - \frac{49.6^2}{7}\right) = 1.8180952...$ awrt 1.82	M1 A1 (2)
(b)	$CI = \left(\frac{6 \times 1.818..}{12.592}, \frac{6 \times 1.818..}{1.635}\right)$	M1
	12.592, 1.635	B1 B1
	$= (0.866..., 6.67...)$ awrt (0.866, 6.67)	A1 A1 (5)
(c)	$0.9^2 < 0.866$, interval does not support $\sigma = 0.9$ as out of range.	B1 (1)
		TOTAL 8
2 (a)(i)	Type I: Ho rejected when true	B1
(ii)	Type II: Ho accepted when false	B1 (2)
(b)(i)	$p = \frac{7.5}{50} = 0.15$	B1
	cr $X \leq 3$	B1 (2)
(ii)	$H_0: p = 0.15$, $H_1: p < 0.15$ both	B1
	$x = 2$ in cr $X \leq 3$ so Ho is rejected	M1
	The new machine has reduced the mean number of faulty sacks	A1 (3)
(c)	$P(\text{Type I error}) = P(X \leq 3 \mid p = 0.15) = 0.0460$	M1 A1 (2)
(d)	$P(\text{Faulty}) = \frac{5}{50} = 0.1$	B1
	$P(\text{Type II error}) = P(X \geq 4 \mid p = 0.1) = 1 - 0.2503$	M1
	$= 0.7497$ awrt 0.750	A1 (3)
(e)	Critical region changes to $X \leq 2$. Ho still rejected.	B1 (1)
		TOTAL 13

3(a) (i)	$E\left(\frac{1}{3}\bar{X}+\frac{2}{3}\bar{Y}\right)=\frac{1}{3}E\left(\frac{X_1+X_2+X_3}{3}\right)+\frac{2}{3}E\left(\frac{Y_1+Y_2+Y_3+Y_4}{4}\right)$	M1
	$=\frac{1}{3}\mu+\frac{2}{3}\mu$	
	$=\mu$ therefore unbiased estimator	A1 (2)
(ii)	$E\left(\frac{5\bar{X}+4\bar{Y}}{9}\right)=\frac{1}{9}(5E(\bar{X})+4E(\bar{Y}))$	M1
	$=\frac{1}{9}(5\mu+4\mu)$	
	$=\mu$ therefore unbiased estimator	A1 (2)
(b)	$\mathrm{Var}(\bar{X})=\frac{\sigma^2}{3}$, $\mathrm{Var}(\bar{Y})=\frac{\sigma^2}{4}$	
	$\mathrm{Var}(\hat{\mu}_1)=\frac{1}{9}\cdot\frac{\sigma^2}{3}+\frac{4}{9}\cdot\frac{\sigma^2}{4}=\frac{4\sigma^2}{27}$	M1A1 (2)
(c)	$\frac{4}{27}\sigma^2<\frac{37}{243}\sigma^2$ so use $\hat{\mu}_1$.	B1 (1)
		TOTAL 7
4(a)	Size of test $=P(X>4 \mid \lambda=3)$	M1
	$=1-P(X\le 4 \mid \lambda=3)=1-0.8153$	A1
	$=0.1847$ awrt 0.185	A1 (3)
(b)	$r=1-0.6288=0.3712=0.37$ (2dp)	B1
	$s=1-0.2851=0.7149=0.71$ (2dp)	B1 (2)
(c)	When $\lambda=4$, power $=0.27<0.5$	
	Probability of coming to correct conclusion is less than probability of coming to wrong conclusion.	B1
	Not suitable.	(1)
		TOTAL 6

5(a)	$d = U - C$, 2, −4, 18, 16, 0, 15, 9	M1
	$\bar{d} = \frac{56}{7} = 8$	B1
	$s_d^2 = \frac{906 - 7 \times 8^2}{6} = 76\frac{1}{3}$	M1A1
	$H_0: \mu_d = 0$, $H_1: \mu_d \neq 0$	B1
	$t = \frac{8}{\sqrt{\frac{76.\dot{3}}{7}}} = 2.42260\ldots$ awrt 2.42	M1A1
	$t_6(2.5\%) = 2.447$	B1
	Insufficient evidence to reject H_0. No evidence of a difference between the mean amount of corrosion on coated and uncoated pipes.	A1 (9)
(b) (i)	Differences are normally distributed	B1 (1)
(ii)	Values do not appear to be normally distributed	B1 (1)
(c)	$t_6(5\%) = 1.943$. There is evidence to reject H_0.	B1
	There is evidence to suggest that there is a greater corrosion on coated pipes.	B1 (2)
		TOTAL 13
6 (a)	$\sigma_I = \frac{1}{19}\left(274050 - \frac{(2340)^2}{20}\right) = 14.2$	M1
	$\sigma_o = \frac{1}{36}\left(645282 - \frac{(4884)^2}{37}\right) = 16.5$ *AG both	A1 (2)
(b)	$s_p = \sqrt{\frac{19 \times 14.2 + 36 \times 16.5}{55}} = \sqrt{15.705} = 3.963\ldots$	M1A1
	Mean outside $= \frac{2340}{20} = 117$, Mean Inside $= 132$	B1 B1
	confidence limits $= (132 - 117) \pm 2.004 \times 3.963\ldots\sqrt{\frac{1}{20} + \frac{1}{37}}$	M1A1ft
	$= (12.8, 17.2)$	A1A1 (8)
(c)	0 lies outside confidence interval. The means are different.	B1 B1 (2)
		TOTAL 12

7(a)	$s_A^2 = 5.11$, $s_B^2 = 5.14$	B1 B1
	$H_0: \sigma_A^2 = \sigma_B^2$, $H_1: \sigma_A^2 \neq \sigma_B^2$	B1
	Critical Value $F_{6,8} = 3.58$	B1
	$\dfrac{s_B^2}{s_A^2} = 1.0062112\ldots$ awrt 1.01	M1 A1
	No evidence to reject H_0. The variances are equal.	A1 (7)
(b)	$s_p^2 = \dfrac{8 \times 5.14 + 6 \times 5.11}{9+7-2} = 5.1247$ awrt 5.12	M1 A1
	$\mu_A = 14.11\ldots$, $\mu_B = 11.857\ldots$	
	$H_0: \mu_A = \mu_B$, $H_1: \mu_A > \mu_B$	B1
	Critical value $t_{14}(5\%) = 1.761$	B1
	$t = \dfrac{14.11\ldots - 11.857\ldots}{\sqrt{5.1247\ldots\left(\frac{1}{9}+\frac{1}{7}\right)}} = 1.9757\ldots$ awrt 1.98	M1 A1
	There is evidence to reject H_0. Mean time taken from school A is greater than school B.	A1 (7)
(c)	Equal variances are a condition for the test in part (b)	B1 (1)
(d)	Groups not equal ability	B1 (1)
		TOTAL 16