### January 2006
#### 6686 Statistics S4
#### Mark Scheme

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (a)</td>
<td>[ \bar{x} = 49.6; \bar{x}^2 = 362.36 ] [ s^2 = \frac{1}{6} (362.36 - 49.6^2) = 1.810952 \ldots \text{ approx } 1.82 ]</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(b)</td>
<td>[ t = \left( \frac{6 \times 1.810952}{12.592} \right) = 6.67 \ldots ]</td>
<td>B1 B1 A1 A1 (5)</td>
</tr>
<tr>
<td>(c)</td>
<td>[ 0.9^2 &lt; 0.866, \text{ so trend does not support } \beta = 0.9 \text{ at } 99 % \text{ confidence level} ]</td>
<td>B1 (1) T=14.8</td>
</tr>
<tr>
<td>2 (a)(i)</td>
<td>Type I: ( H_0 ) rejected when true</td>
<td>B1</td>
</tr>
<tr>
<td>(b)(i)</td>
<td>( \hat{p} = \frac{7.5}{50} = 0.15 )</td>
<td>B1 (2)</td>
</tr>
<tr>
<td>(b)(ii)</td>
<td>( p = 0.15 ), ( H_1: p &lt; 0.15 ) ( x = 2 ) in ( n = 50 ) so ( H_0 ) is rejected. ( \text{ new machine has reduced number of faulty socks} )</td>
<td>B1 (2) M1 A1 (3)</td>
</tr>
<tr>
<td>(c)</td>
<td>( P(\text{Type I error}) = P(X \leq 3</td>
<td>p = 0.15) = 0.0460 )</td>
</tr>
<tr>
<td>(d)</td>
<td>( P(\text{Type II error}) = P(X &gt; 4</td>
<td>p = 0.1) = 0.0250 )</td>
</tr>
<tr>
<td>(e)</td>
<td>Critical region changes to ( x \leq 2 ). ( H_0 ) still rejected.</td>
<td>B1 (1) T=13</td>
</tr>
</tbody>
</table>

---

6688/01 Statistics S4
January 2006 Advanced Subsidiary/Advanced Level in GCE Mathematics
### 3(a) (i) \[ E \left( \frac{1}{2} \bar{X} + \frac{2}{3} \bar{Y} \right) = \frac{1}{3} E \left( \frac{X_1 + X_2 + X_3}{3} \right) + \frac{2}{3} E \left( \frac{Y_1 + Y_2 + Y_3 + Y_4}{4} \right) \]
\[ = \frac{1}{3} \mu + \frac{2}{3} \mu \]
\[ = \mu \text{ Therefore unbiased estimator} \]

(ii) \[ E \left( \frac{5\bar{X} + 4\bar{Y}}{9} \right) = \frac{1}{9} (5E(X) + 4E(Y)) \]
\[ = \frac{1}{9} (5\mu + 4\mu) \]
\[ = \mu \text{ Therefore unbiased estimator} \]

(b) \[ \text{Var}(\bar{X}) = \frac{\sigma^2}{3}, \text{Var}(\bar{Y}) = \frac{\sigma^2}{4} \]
\[ \text{Var}(\bar{Y}_n) = \frac{1}{4} \left( \frac{\sigma^2}{3} + 4 \cdot \frac{\sigma^2}{4} \right) = \frac{4\sigma^2}{27} \]

(c) \[ \frac{4}{27} \sigma^2 < \frac{37}{24} \sigma^2 \text{ so } \hat{\mu}_n. \]

### 4(a) Size of test \[ P(X > 4 \mid \lambda = 2) \]
\[ = 1 - P(X \leq 4 \mid \lambda = 2) = 1 - 0.8153 \]
\[ = 0.1847 \text{ a.s.} \]

(b) \[ s = 1 - 0.6286 = 0.3712 \approx 0.37 (2\text{d}f) \]
\[ s = 1 - 0.2851 = 0.7149 = 0.71 (2\text{d}f) \]

(c) When \( \lambda = 4 \), power = 0.27 < 0.5
Probability of coming to wrong conclusion is less than
Probability of coming to wrong conclusion.
Not suitable.

TOTAL 7

TOTAL 6
5(a) \[ d = 0 - c, \quad 2, -4, 18, 16, 0, 15, 9 \]
\[
\bar{d} = \frac{56}{7} = 8
\]
\[
\overline{d^2} = \frac{906 - 7 \times 8^2}{6} = 76\frac{1}{2}
\]
\[ H_0: \mu_d = 0, \quad H_1: \mu_d \neq 0 \]
\[
t = \frac{8}{\sqrt{76\frac{1}{2}}} = 2.4226 \ldots \quad \text{(work 2.42)}
\]
\[ t_{c}(2.5\%) = 2.447 \]
Insufficient evidence to reject \( H_0 \).
No evidence of a difference between the mean amount of corrosion on coated and uncoated pipes.

(b) (i) Differences are normally distributed
(ii) Values do not appear to be normally distributed

(c) \[ t_{c}(5\%) = 1.943 \]. There is evidence to reject \( H_0 \).
There is evidence to suggest that there is a greater corrosion on coated pipes.

6 (a) \[ \sigma = \frac{1}{14} \left( 274050 - \frac{(2340)^2}{20} \right) = 14.2 \]
\[ \sigma_0 = \frac{1}{36} \left( 645252 - \frac{(4884)^2}{37} \right) = 16.5 \]
\[ s_1 = \sqrt{\frac{19\times 142 + 36 \times 16.5}{55}} = 3.963 \ldots \]
Mean outside = \[ \frac{2340}{20} = 117 \], Mean inside = 132
Confidence limit = \[ (132 - 117) + \frac{2.045 \times 3.963}{\sqrt{20}} \]
\[ = (12.8, 17.2) \]
(c) \( \sigma \) lies outside confidence interval. The means are different.

TOTAL 13

TOTAL 12
### Question 7

(a) \[ S_A^2 = 5.11, \quad S_B^2 = 5.14 \]

\[ H_0: \sigma_A^2 = \sigma_B^2, \quad H_1: \sigma_A^2 \neq \sigma_B^2 \]

**Critical Value** \( F_{6, 8} = 3.58 \)

\[
\frac{S_B^2}{S_A^2} = 1.0042112 \ldots \quad \text{approx. 1.01} \]

No evidence to reject \( H_0 \). The variances are equal.

(b) \[ S_p^2 = \frac{8 \times 5.14 + 6 \times 5.11}{9 + 7 - 2} = 5.1247 \quad \text{approx. 5.12} \]

\[ m_A = 14.11 \ldots, \quad m_B = 11.857 \ldots \]

\[ H_0: m_A = m_B, \quad H_1: m_A > m_B \]

**Critical Value** \( t_{14}(5\%) = 1.761 \)

\[
t = \frac{14.11 \ldots - 11.857 \ldots}{\sqrt{5.1247 \left(\frac{1}{9} + \frac{1}{7}\right)}} = 1.9757 \ldots \quad \text{approx. 1.98} \]

There is evidence to reject \( H_0 \). Mean have taken from school A is greater than school B.

(c) Equal variances are a condition for the test in part (b).

(d) Groups with equal ability.

**Total**: 16