

Publication

June 2005
6685 Statistics S3
Mark Scheme

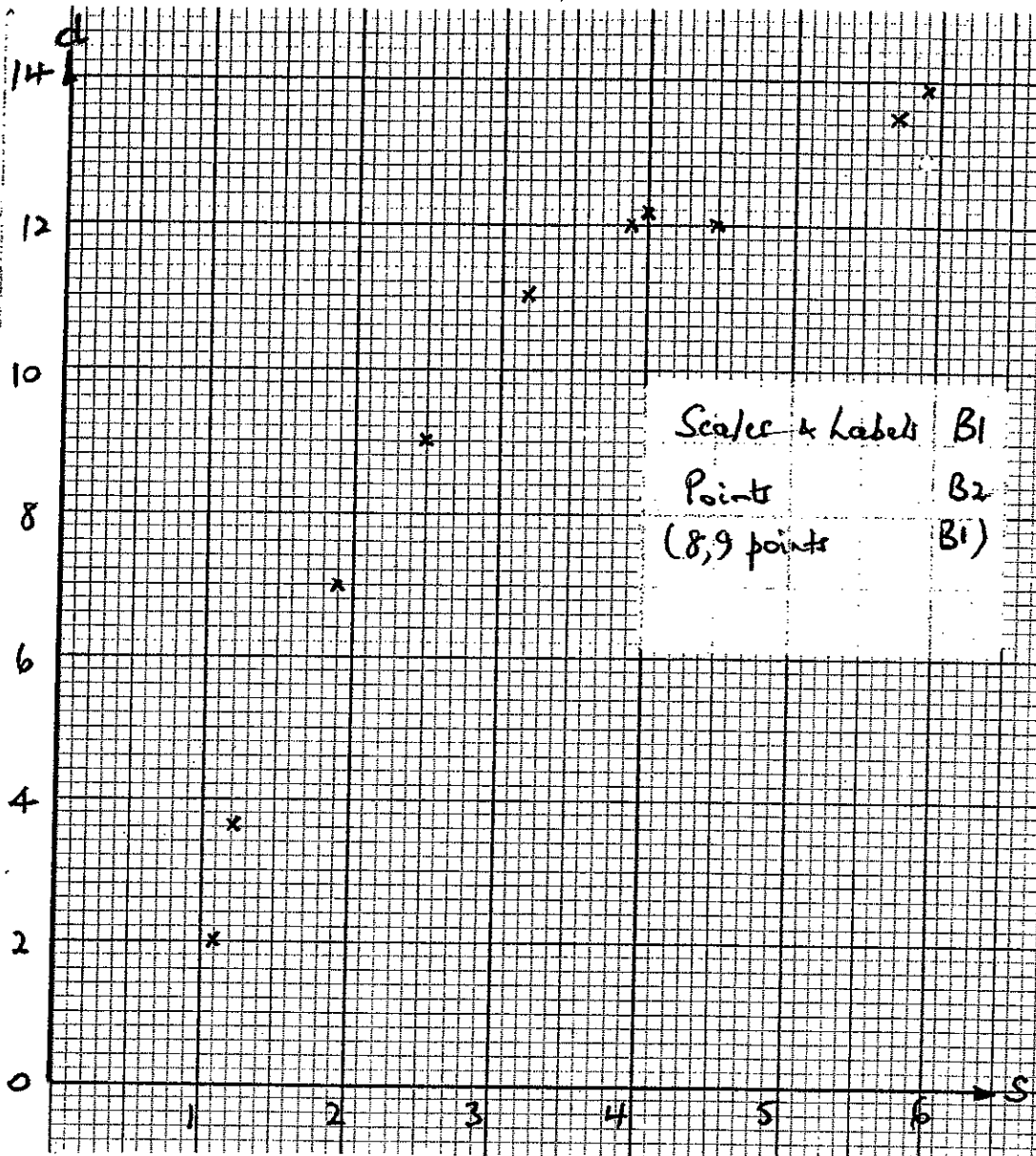
FINAL

Question Number	Scheme	Marks
1.	<p>(a) Population divides into <u>mutually exclusive</u>; groups <u>distinct</u> strata</p> <p>(b) <u>Advantages</u></p> <ul style="list-style-type: none"> - enables fieldwork to be done quickly - costs kept to a minimum - administration is relatively easy <p><u>Disadvantages</u></p> <ul style="list-style-type: none"> - non-random so not possible to estimate sampling error - Subject to possible interviewer bias - non-response not recorded 	<p>B1; B1 (2)</p> <p>Any ONE B1</p> <p>Any ONE B1 (2)</p>
2.	<p>$X \sim N(10, 3^2) \therefore \bar{X} \sim N(10, 9/5)$ can be implied 10; 9/5</p> <p>$P(7 \leq \bar{X} \leq 10) = P\left(\frac{7-10}{\sqrt{9/5}} < Z < 0\right)$ Standardising with 10 & their σ</p> <p>$= P(-2.236 < Z < 0)$ Standardising with 10 & their σ</p> <p>$= \Phi(0) - [1 - \Phi(2.24)]$</p> <p>$= \underline{0.4875}$</p>	<p>B1; B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 (p < 0.5)</p> <p>A1 (6)</p>

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3.	<table border="1"> <thead> <tr> <th></th> <th>No action</th> <th>Remove diseased branches</th> <th>Spray with Chemicals</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Tree died within 1 year</td> <td>10 (7)</td> <td>5 (7)</td> <td>6 (7)</td> <td>21</td> </tr> <tr> <td>Survived 1-4 years</td> <td>5 (7)</td> <td>9 (7)</td> <td>7 (7)</td> <td>21</td> </tr> <tr> <td>Survived > 4 years</td> <td>5 (6)</td> <td>6 (6)</td> <td>7 (6)</td> <td>18</td> </tr> <tr> <td>Totals</td> <td>20</td> <td>20</td> <td>20</td> <td>60</td> </tr> </tbody> </table>					No action	Remove diseased branches	Spray with Chemicals	Total	Tree died within 1 year	10 (7)	5 (7)	6 (7)	21	Survived 1-4 years	5 (7)	9 (7)	7 (7)	21	Survived > 4 years	5 (6)	6 (6)	7 (6)	18	Totals	20	20	20	60	
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$\frac{RT \times CT}{GT}$ $\frac{6 \times 7}{3 \times 6}$					M1 A1 A1																									
H_0 : Treatment & Survival are independent (not associated) H_1 : Treatment & Survival are not independent (associated)					BI both																									
$\alpha = 0.05$																														
$L = (3-1) \times (3-1) = 4$					BI																									
$CR: \chi^2 > 9.488$					BI ✓																									
$\sum \frac{(O-E)^2}{E} = \frac{9}{7} + \frac{4}{7} + \frac{1}{7} + \frac{4}{7} + \frac{4}{7} + 0 + \frac{1}{6} + 0 + \frac{1}{6}$ $= 3.47619 \dots$					M1 A1 A1																									
<p>Since 3.47619... is NOT in the critical region (ie < 9.488) there is insufficient evidence to reject H_0.</p>																														
<p>There is no evidence of association between treatment and length of survival.</p>					Comparison Conclusion M1 A1 ✓ (11)																									

4

(a)

NB No graph paper \Rightarrow 0/3(b) linear association between s and d

B1 (1)

$$(c) S_{ss} = 141.51 - \frac{33.9^2}{10} = 26.589; S_{dd} = 152.444; S_{sd} = 59.524$$

B1; B1; B1 (3)

$$(d) r = \frac{59.524}{\sqrt{152.444 \times 26.589}}$$

M1

$$= 0.93494\dots$$

AWRT 0.935

A1 (2)

(e) $H_0: \rho = 0$; $H_1: \rho > 0$

Critical Value at 1% = 0.7155

Reject H_0 ; Levels of serum & disease are positively correlated

(f) linear correlation significant ~~but~~ ^{but} scatter diagram looks non-linear.

B1

B1

B1 (3)

B1 (1)

5.

H_0 : Poisson distribution is a suitable model both

H_1 : Poisson distribution is not a suitable model

$$\hat{\lambda} = \frac{(0 \times 99) + (1 \times 65) + \dots + (4 \times 2)}{200} = \frac{153}{200} = 0.765$$

Using $P(X=x) = \frac{0.765^x e^{-0.765}}{x!}$ where X represents the number of restarts given $200 \times P(X=x)$

X	Observed Frequency	Expected Frequency
0	99	93.06678...
1	65	71.19604... 0,1,2
2	22	27.23250...
3	12	6.94428... 8.50468
≥ 4	2 } 14	1.56040...

$\chi^2 = 4 - 1 - 1 = 2$; CR: $\chi^2 > 5.991$ from Poisson
 $\chi^2 = 4 - 1 = 3$ CR: $\chi^2 > 7.815$ from Poisson (0.765)
 $\sum \frac{(O-E)^2}{E} = 5.47368...$ OR $\sum (O-E)/E$

5.47 is not in the critical region.

Number of computer failures per day can be modelled by a Poisson distribution

B1

M1 A1

M1

A1, A1
(-1e.e.)

A1

B1; B1✓

M1

A1

A1✓ (12)

6.	<p>(a) Let X represent repair time</p> <p>$\therefore \sum x = 1435 \quad \therefore \bar{x} = \frac{1435}{5} = \underline{287}$</p> <p>$\sum x^2 = 442575 \quad \therefore s^2 = \frac{1}{4} \left\{ 442575 - \frac{1435^2}{5} \right\}$</p> <p>$= \underline{7682.5}$</p> <p>(b) $P(\mu - \bar{x} < 20) = 0.95$</p> <p>$\therefore \frac{20}{\sigma/\sqrt{n}} = 1.96$</p> <p>$\therefore n = \frac{1.96^2 \sigma^2}{20^2} = \frac{1.96^2 \times 100^2}{400} = \underline{96.04}$</p> <p>$\therefore \underline{\text{Sample size } (\geq) 97 \text{ required}}$</p>	<p>BI</p> <p>MIAI</p> <p>AI (4)</p> <p>MIAI MI</p> <p>BI</p> <p>MIAI AI</p> <p>MI</p> <p>AI</p> <p>AI (6)</p>
7.	<p>Let $W = C_1 - C_2$ is $W = C_1 + C_2 \Rightarrow$ MIA or MI only</p> <p>(a) $\therefore W \sim N(0, 16)$ Normal</p> <p>$\therefore P(W > 6) = 2P(W > 6)$ $0; 16$</p> <p>$= 2 \times P\left(Z > \frac{6-0}{\sqrt{16}}\right)$</p> <p>is $W = C - L$ treat as MR $\text{Prob} = 0.4346$</p> <p>$= 2 \times P(Z > 1.5)$ Standardizing, their σ</p> <p>$= 2 \times (1 - 0.9332) = \underline{0.1336}$</p> <p>(b) Let $W = C - L$</p> <p>$\therefore W \sim N(5, 25)$ $5; 25$</p> <p>$P(W > 0) = P\left(Z > \frac{0-5}{\sqrt{25}}\right)$</p> <p>$= P(Z < 1)$</p> <p>$= \underline{0.8413}$</p>	<p>MI</p> <p>AI; MI</p> <p>MI</p> <p>MI</p> <p>AI (6)</p> <p>BI; BI</p> <p>MIAI</p> <p>MI ($p > 0.5$)</p> <p>AI (6)</p>

(g) Let $W = C_1 + \dots + C_{24} + B$

$\therefore E(W) = 24 \times 350 + 100 = \underline{8500}$

$Var(W) = 24 \times 8 + 2^2 = \underline{196}$

$P(8510 \leq W \leq 8520) = P\left(\frac{8510 - 8500}{\sqrt{196}} \leq Z \leq \frac{8520 - 8500}{\sqrt{196}}\right)$

$= P(0.714 \leq Z \leq 1.428) \text{ AwRT}$

$= 0.9236 - 0.7611$

$= \underline{0.1625} \qquad 0.161 - 0.163$

BI

BI

M1

A1/A1

A1 (6)

(d) All random variables are independent.

BI (1)

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13/06/05