

4. A spacecraft is travelling in a straight line in deep space where all external forces can be assumed to be negligible. The spacecraft decelerates by ejecting fuel at a constant speed k relative to the spacecraft, in the direction of motion of the spacecraft. At time t , the spacecraft has speed v and mass m .

(a) Show, from first principles, that while the spacecraft is ejecting fuel,

$$\frac{dv}{dm} - \frac{k}{m} = 0 \quad (5)$$

At time $t = 0$, the spacecraft has speed U and mass M .

(b) Find the mass of the spacecraft when it comes to rest. (6)

Given that $m = Me^{-\alpha t^2}$, where α is a positive constant, and that the spacecraft comes to rest at time $t = T$,

(c) find, in terms of U and T only, the distance travelled by the spacecraft in decelerating from speed U to rest. (6)



5. A uniform rod AB , of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L . The axis L is perpendicular to the rod and passes through the point P of the rod, where $AP = \frac{2}{3}a$.

(a) Find the moment of inertia of the rod about L . (3)

The rod is held at rest with B vertically above P and is slightly displaced.

(b) Find the angular speed of the rod when PB makes an angle θ with the upward vertical. (4)

(c) Find the magnitude of the angular acceleration of the rod when PB makes an angle θ with the upward vertical. (3)

(d) Find, in terms of g and a only, the angular speed of the rod when the force acting on the rod at P is perpendicular to the rod. (5)



Question 5 continued

Lined area for writing the answer to Question 5.

Q5

(Total 15 marks)

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6. (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a , about an axis through the centre of the disc and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$. (5)

[You may assume without proof that the moment of inertia of a uniform hoop of mass m and radius r about an axis through its centre and perpendicular to its plane is mr^2 .]

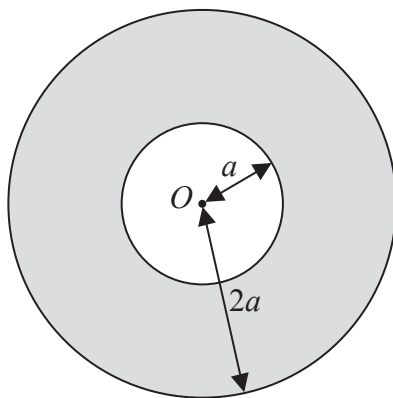


Figure 1

A uniform plane shape S of mass M is formed by removing a uniform circular disc with centre O and radius a from a uniform circular disc with centre O and radius $2a$, as shown in Figure 1. The shape S is free to rotate about a fixed smooth axis L , which passes through O and lies in the plane of the shape.

- (b) Show that the moment of inertia of S about L is $\frac{5}{4}Ma^2$. (4)

The shape S is at rest in a horizontal plane and is free to rotate about the axis L . A particle of mass M falls vertically and strikes S at the point A , where $OA = \frac{3}{2}a$ and OA is perpendicular to L . The particle adheres to S at A . Immediately before the particle strikes S the speed of the particle is u .

- (c) Find, in terms of M and u , the loss in kinetic energy due to the impact. (8)



