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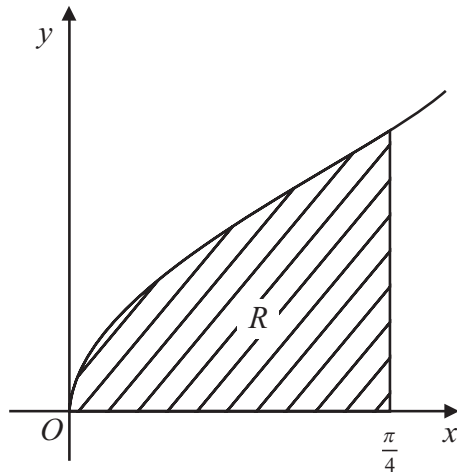


Figure 1

Figure 1 shows part of the curve with equation  $y = \sqrt{(\tan x)}$ . The finite region  $R$ , which is bounded by the curve, the  $x$ -axis and the line  $x = \frac{\pi}{4}$ , is shown shaded in Figure 1.

- (a) Given that  $y = \sqrt{(\tan x)}$ , complete the table with the values of  $y$  corresponding to  $x = \frac{\pi}{16}$ ,  $\frac{\pi}{8}$  and  $\frac{3\pi}{16}$ , giving your answers to 5 decimal places.

$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
$y$	0				1

(3)

- (b) Use the trapezium rule with all the values of  $y$  in the completed table to obtain an estimate for the area of the shaded region  $R$ , giving your answer to 4 decimal places.

(4)

The region  $R$  is rotated through  $2\pi$  radians around the  $x$ -axis to generate a solid of revolution.

- (c) Use integration to find an exact value for the volume of the solid generated.

(4)

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8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where  $P$  is the population,  $t$  is the time measured in days and  $k$  is a positive constant.

Given that the initial population is  $P_0$ ,

- (a) solve the differential equation, giving  $P$  in terms of  $P_0$ ,  $k$  and  $t$ . (4)

Given also that  $k = 2.5$ ,

- (b) find the time taken, to the nearest minute, for the population to reach  $2P_0$ . (3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where  $P$  is the population,  $t$  is the time measured in days and  $\lambda$  is a positive constant.

Given, again, that the initial population is  $P_0$  and that time is measured in days,

- (c) solve the second differential equation, giving  $P$  in terms of  $P_0$ ,  $\lambda$  and  $t$ . (4)

Given also that  $\lambda = 2.5$ ,

- (d) find the time taken, to the nearest minute, for the population to reach  $2P_0$  for the first time, using the improved model. (3)

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