

Question Number	Scheme	Marks
<p>1 (a)</p> <p>Alternative</p>	$\left(1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2}(-2x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3}(-2x)^3 + \dots\right)$ $= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \dots$ <p>May use McLaurin $f(0)=1$ and $f'(0)=1$ to obtain 1st two terms $1 + x$ Differentiates two further times and uses formula with correct factorials to give</p> $\frac{3}{2}x^2 + \frac{5}{2}x^3$ <p>(b) $(100 - 200x)^{-\frac{1}{2}} = 100^{-\frac{1}{2}}(1 - 2x)^{-\frac{1}{2}}$. So series is $\frac{1}{10}$(previous series)</p>	<p>M1 (corr bin coeffs) M1 (powers of $-2x$)</p> <p>A1, A1</p> <p>(4)</p> <p>M1 A1 M1</p> <p>A1</p> <p>(4)</p> <p>M1A1 ft</p> <p>(2)</p>
<p>2</p>	<p>Uses $f(2) = 0$ to give $16 - 4 + 2a + b = 0$</p> <p>Uses $f(-1) = 6$ to give $-2 - 1 - a + b = 6$</p> <p>Solves simultaneous equations to give $a = -7$, and $b = 2$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>(7)</p>
<p>3 (a)</p> <p>Alternative</p> <p>(b)</p>	<p>Uses circle equation $(x-4)^2 + (y-3)^2 = (\sqrt{5})^2$</p> <p>Multiplies out to give $x^2 - 8x + 16 + y^2 - 6y + 9 = 5$ and thus $x^2 + y^2 - 8x - 6y + 20 = 0$ (*)</p> <p>Or states equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$ and so $g = -4$ and $f = -3$</p> <p>Uses $g^2 + f^2 - c = r^2$ to give $c = 3^2 + 4^2 - \sqrt{5}^2$, i.e. $c = 20$</p> $x^2 + y^2 - 8x - 6y + 20 = 0$ <p>$y = 2x$ meets the circle when $x^2 + (2x)^2 - 8x - 6(2x) + 20 = 0$</p> $5x^2 - 20x + 20 = 0$ <p>Solves and substitutes to obtain $x = 2$ and $y = 4$. Coordinates are $(2, 4)$</p> <p>Or Implicit differentiation attempt, $2x + 2y \frac{dy}{dx} - 8 - 6 \frac{dy}{dx} = 0$</p> <p>Uses $y = 2x$ and $\frac{dy}{dx} = 2$ to give $10x - 20 = 0$.</p> <p>Thus $x = 2$ and $y = 4$</p>	<p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>A1 M1 A1</p> <p>(4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>(4)</p>

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4.(a)	$f'(x) = (x^2 + 1) \times \frac{1}{x} + \ln x \times 2x$ $f'(e) = (e^2 + 1) \times \frac{1}{e} + 2e = 3e + \frac{1}{e}$	M1 A1 M1 A1 (4)
(b)	$\left(\frac{x^3}{3} + x\right) \ln x - \int \left(\frac{x^3}{3} + x\right) \frac{1}{x} dx$ $= \left(\frac{x^3}{3} + x\right) \ln x - \int \left(\frac{x^2}{3} + 1\right) dx$ $= \left[\left(\frac{x^3}{3} + x\right) \ln x - \left(\frac{x^3}{9} + x\right) \right]_1^e$ $= \frac{2}{9}e^3 + \frac{10}{9}$	M1 A1 A1 M1 A1 (5)
5. (a)	$\frac{9+4x^2}{9-4x^2} = -1 + \frac{18}{(3+2x)(3-2x)}, \text{ so } A = -1$ <p>Uses $18 = B(3-2x) + C(3+2x)$ and attempts to find B and C</p> <p>$B = 3$ and $C = 3$</p> <p>Or</p> <p>Uses $9 + 4x^2 = A(9 - 4x^2) + B(3 - 2x) + C(3 + 2x)$ and attempts to find A, B and C</p> <p>$A = -1$, $B = 3$ and $C = 3$</p>	B1 M1 A1 A1 (4) M1 A1, A1, A1 (4)
(b)	<p>Obtains $Ax + \frac{B}{2} \ln(3+2x) - \frac{C}{2} \ln(3-2x)$</p> <p>Substitutes limits and subtracts to give $2A + \frac{B}{2} \ln(5) - \frac{C}{2} \ln\left(\frac{1}{5}\right)$</p> $= -2 + 3\ln 5 \quad \text{or} \quad -2 + \ln 125$	M1 A1 M1 A1ft A1 (5)

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8(a)	$\frac{dx}{dt} = -3a \sin 3t, \quad \frac{dy}{dt} = a \cos t \quad \text{therefore} \quad \frac{dy}{dx} = \frac{\cos t}{-3 \sin 3t}$ <p>When $x = 0, t = \frac{\pi}{6}$</p> <p>Gradient is $-\frac{\sqrt{3}}{6}$</p> <p>Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$</p>	<p>M1 A1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>(6)</p>
(b)	<p>Area beneath curve is $\int a \sin t(-3a \sin 3t)dt$</p> $= -\frac{3a^2}{2} \int (\cos 2t - \cos 4t)dt$ $= -\frac{3a^2}{2} [\frac{1}{2} \sin 2t - \frac{1}{4} \sin 4t]$ <p>Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$</p> <p>Area of triangle beneath tangent is $\frac{1}{2} \times \frac{a}{2} \times \sqrt{3}a = \frac{\sqrt{3}a^2}{4}$</p> <p>Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$</p>	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1</p> <p>(9)</p>
N.B.	<p>The integration of the product of two sines is worth 3 marks (lines 2 and 3 of scheme to part (b))</p> <p>If they use parts</p> $\int \sin t \sin 3t dt = -\cos t \sin 3t + \int 3 \cos 3t \cos t dt$ $= -\cos t \sin 3t + 3 \cos 3t \sin t + \int 9 \sin 3t \sin t dt$ $8I = \cos t \sin 3t - 3 \cos 3t \sin t$	<p>M1</p> <p>M1 A1</p>