Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Core Mathematics 1R (6663_01R)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.
   - bod – benefit of doubt
   - ft – follow through
   - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d... or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - □ or d... The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
General Principles for Core Mathematics Marking
(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

\[(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to } x = \ldots\]

\[(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to } x = \ldots\]

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving \(x^2 + bx + c = 0\):

\[\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \text{ leading to } x = \ldots\]

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. \((x^n \rightarrow x^{n-1})\)

2. Integration

Power of at least one term increased by 1. \((x^n \rightarrow x^{n+1})\)
Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners’ reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners’ reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.
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<tr>
<th>Question Number</th>
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<th>Marks</th>
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</table>
| 1.              | $25x - 9x^3 = x(25 - 9x^2)$  
$25 - 9x^2 = (5+3x)(5-3x)$  
$25x - 9x^3 = x(5 + 3x)(5 - 3x)$ | B1  
M1  
A1 (3) |

**B1** Take out a common factor, usually $x$, to produce $x(25 - 9x^2)$. Accept $(x \pm 0)(25 - 9x^2)$ or $-x(9x^2 - 25)$  
Must be correct.  
Other possible options are $(5 + 3x)(5x - 3x^2)$ or $(5 - 3x)(5x + 3x^2)$

**M1** For factorising their quadratic term, usually $(25 - 9x^2) = (5+3x)(5-3x)$ Accept sign errors  
If $(5 \pm 3x)$ has been taken out as a factor first, this is for an attempt to factorise $(5x \mp 3x^2)$

**A1** cao $(5 + 3x)(5 - 3x)$ or any equivalent with three factors  
e.g. $x(5 + 3x)(-3x + 5)$ or $x(3x-5)(-3x-5)$ etc including $-x(3x+5)(3x-5)$  
isspace if they go on to show that $x = 0$, $\pm \frac{5}{3}$
(a) M1 Dealing with either the ‘cube’ or the ‘square root’ first. A correct answer will imply this mark. Also accept a law of indices approach \(81^\frac{3}{2} = (81^{\frac{1}{2}})^3 = 9^3\) or \(81^\frac{3}{2} = (81^{\frac{1}{2}})^3 = (531441)^{\frac{1}{2}} = 729\)

A1 Cao 729. Accept \((\pm)729\)

(b) M1 For correct use of power 2 on both 4 and the \(x^{-\frac{1}{2}}\) term.

A1 Cao = 16x

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<tr>
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<tr>
<td>2.(a)</td>
<td>(81^\frac{3}{2} = (81^{\frac{1}{2}})^3 = 9^3) or (81^\frac{3}{2} = (81^{\frac{1}{2}})^3 = (531441)^{\frac{1}{2}} = 729)</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(b)</td>
<td>((4x^{-\frac{1}{2}})^2 = 16x^{-\frac{2}{2}}) or (\frac{16}{x}) or equivalent (x^2(4x^{-\frac{1}{2}})^2 = 16x)</td>
<td>M1 A1 (2)</td>
</tr>
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</table>

(4 marks)
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<tr>
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<th>Marks</th>
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<tbody>
<tr>
<td>3.(a)</td>
<td>(a_2 = 4k - 3)</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>(a_3 = 4(4k - 3) - 3)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(\sum_{r=1}^{3} a_r = k + 4k - 3 + 4(4k - 3) - 3 = k + \ldots)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(21k - 18 = 66 \Rightarrow k = \ldots)</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>(k = 4)</td>
<td>A1 (4) (5 marks)</td>
</tr>
</tbody>
</table>

(a) B1 4k - 3 cao

(b) M1 An attempt to find \(a_3\) from iterative formula \(a_3 = 4a_2 - 3\). Condone bracketing errors for the M mark
M1 Attempt to sum their \(a_1\), \(a_2\) and \(a_3\) to get a linear expression in \(k\) (Sum of Arithmetic series is M0)
dM1 Sets their linear expression to 66 and solves to find a value for \(k\). It is dependent upon the previous M mark
A1 cao \(k = 4\)
<table>
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</table>
| 4.(a)           | $y = 2x^5 + \frac{6}{\sqrt{x}}$  
\[ \frac{dy}{dx} = 10x^4 - 3x^{-\frac{3}{2}} \]  
\[ \int 2x^5 + \frac{6}{\sqrt{x}} \, dx \]  
\[ = \frac{x^6}{3} + 12x^\frac{1}{2} + c \] |
| (b)             | $x^n \to x^{n-1}$  
\[ \int 2x^5 + \frac{6}{\sqrt{x}} \, dx \]  
\[ = \frac{x^6}{3} + 12x^\frac{1}{2} + c \] |

(a) M1 For $x^n \to x^{n-1}$, i.e. $x^4$ or $x^{-\frac{3}{2}}$ or $\left(\frac{1}{x^{\frac{3}{2}}}\right)$ seen

A1 For $2 \times 5x^4$ or $6 \times -\frac{1}{2}x^{\frac{3}{2}}$ (oe). (Ignore $+c$ for this mark)

A1 For simplified expression $10x^4 - 3x^{\frac{3}{2}}$ or $10x^4 - 3\frac{3}{\sqrt{x^2}}$ o.e. and no $+c$

Apply ISW here and award marks when first seen.

(b) M1 For $x^n \to x^{n-1}$, i.e. $x^6$ or $x^\frac{1}{2}$ or $\left(\sqrt{x}\right)$ seen

Do not award for integrating their answer to part (a)

A1 For either $2 \times \frac{x^6}{6}$ or $6 \times \frac{x^{\frac{1}{2}}}{\sqrt{2}}$ or simplified or unsimplified equivalents

A1 For fully correct and simplified answer with $+c$. 
<table>
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<tr>
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</table>
| **5** Method 1  | \( x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}} \)  
\( \times\sqrt{2} \Rightarrow x\sqrt{16} + 10\sqrt{2} = 6x \)  
\( 4x + 10\sqrt{2} = 6x \Rightarrow 2x = 10\sqrt{2} \Rightarrow x = 5\sqrt{2} \) or \( a = 5 \) and \( b = 2 \) | M1,A1 M1A1 (4) |
| **5** Method 2  | \( x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}} \)  
\( 2\sqrt{2}x + 10 = 3\sqrt{2}x \)  
\( \sqrt{2}x = 10 \Rightarrow x = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \) oe | M1A1 M1,A1 (4) |

**Method 1**

M1 For multiplying both sides by \( \sqrt{2} \) – allow a slip e.g. \( \sqrt{2}x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}} \times \sqrt{2} \) or 
\( \sqrt{2} \times 10 + x\sqrt{8} = \frac{6x}{\sqrt{2}} \times \sqrt{2} \), where one term has an error or the correct \( \sqrt{2}(10 + x\sqrt{8}) = \frac{6x}{\sqrt{2}} \times \sqrt{2} \)

NB \( x\sqrt{8} + 10 = 6x\sqrt{2} \) is M0

A1 A correct equation in \( x \) with no fractional terms. Eg \( x\sqrt{16} + 10\sqrt{2} = 6x \) oe.

M1 An attempt to solve their linear equation in \( x \) to produce an answer of the form \( a\sqrt{2} \) or \( a\sqrt{50} \)

A1 \( 5\sqrt{2} \) oe (accept \( 1\sqrt{50} \))

**Method 2**

M1 For writing \( \sqrt{8} \) as \( 2\sqrt{2} \) or \( \frac{6}{\sqrt{2}} \) as \( 3\sqrt{2} \)

A1 A correct equation in \( x \) with no fractional terms. Eg \( 2\sqrt{2}x + 10 = 3\sqrt{2}x \) or \( x\sqrt{8} + 10 = 3\sqrt{2}x \) oe.

M1 An attempt to solve their linear equation in \( x \) to produce an answer of the form \( a\sqrt{2} \) or \( a\sqrt{50} \)

\( \sqrt{2}x = 10 \Rightarrow x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \)

or \( \sqrt{2}x = 10 \Rightarrow 2x^2 = 100 \Rightarrow x^2 = 50 \Rightarrow x = \sqrt{50} \) or \( 5\sqrt{2} \)

A1 \( 5\sqrt{2} \) oe Accept \( 1\sqrt{50} \)
<table>
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</table>
| 6(a).           | $P = 20x + 6$ o.e  
$20x + 6 > 40 \Rightarrow x > \frac{5}{2}$ | B1  
M1  
A1* (3) |
| (b)             | $A = 2x(2x + 1) + 2x(6x + 3) = 16x^2 + 8x$  
$16x^2 + 8x - 120 < 0$  
Try to solve their $2x^2 + x - 15 = 0$ e.g. $(2x - 5)(x + 3) = 0$ so $x =$  
Choose inside region  
$-3 < x < \frac{5}{2}$ or $0 < x < \frac{5}{2}$ (as $x$ is a length)  
$1.7 < x < \frac{5}{2}$ | B1  
M1  
A1 (5) |
| (c )            | $51.7 < x$ | B1cao (1) |

(a) B1 Correct expression for perimeter but may not be simplified so accept  
$2x + 1 + 2x + 4x + 2 + 2x + 6x + 3 + 4x$ or $2(10x + 3)$ or any equivalent  
M1: Set $P > 40$ with their linear expression for $P$ (this may not be correct but should be a sum of sides) and manipulate to get $x >$  
A1* cao $x > 1.7$. This is a given answer, there must not be any errors, but accept $1.7 < x$

(b) Marks parts (b) and (c) together  
B1 Writes a correct statement in $x$ for the area. It need not be simplified. You may isw  
Amongst numerous possibilities are.  
$2x(2x + 1) + 2x(6x + 3)$, $16x^2 + 8x$, $4x(6x + 3) - 2x(4x + 2)$, $4x(2x + 1) + 2x(4x + 2)$  
M1 Sets their quadratic expression < 120 and collects on one side of the inequality  
M1 For an attempt to solve a 3 Term quadratic equation producing two solutions by factorising, formula or completion of the square with the usual rules (see notes)  
M1 For choosing the ‘inside’ region. Can follow through from their critical values – must be stated – not just a table or a graph. Can also be implied by $0 < x < upper value$  
A1 $-3 < x < \frac{5}{2}$. Accept $x > -3$ and $x < 2.5$ or $(-3, 2.5)$  
As $x$ is a width, accept $0 < x < \frac{5}{2}$. Also accept $\frac{10}{4}$ or 2.5 instead of $\frac{5}{2}$. ≤ would be M1A0  
Allow this final answer to part (b) to appear as answer to part (c) This would score final M1A1 in (b) then B0 in (c)

(c) B1cao $1.7 < x < \frac{5}{2}$. Must be correct. [This does not imply final M1 in (b)]
Method 1

\[ y - 2 = -\frac{3}{4}(x + 1) \text{ or } y + 4 = \frac{3}{4}(x - 7) \text{ or } y = \text{their}^{\prime} - \frac{3}{4}x + c \]

\[ \Rightarrow \pm(4y + 3x - 5) = 0 \]

Method 2

\[ y - 2 = \frac{x - x_1}{x_2 - x_1} \text{ or } y - y_1 = \frac{x - x_1}{x_2 - x_1} \]

\[ \Rightarrow \pm \frac{y - y_1}{x - x_1} = 1 \]

Method 3: Substitute \(x = -1, y = 2\) and \(x = 7, y = -4\) into \(ax + by + c = 0\)

\[ -a + 2b + c = 0 \quad \text{and} \quad 7a - 4b + c = 0 \]

Solve to obtain \(a = 3, \ b = 4\) and \(c = -5\) or multiple of these numbers

**Alternative for (b)**

Attempt Pythagoras:

\[ (p + 4)^2 + 9^2 + (6^2 + 8^2) = (p - 2)^2 + 17^2 \]

So \(p^2 + 8p + 16 + 81 + 36 + 64 = p^2 - 4p + 4 + 289 \Rightarrow p = \ldots \)

\[ p = 8 \]

**(c)**

Either \((y =) p + 6\) or \(2 + p + 4\)

Or use 2 perpendicular line equations through \(L\) and \(N\) and solve for \(y\)

\[ (y = 14) \]

(a) M1 Uses the gradient formula with points \(L\) and \(M\) i.e. quote \(gradient = \frac{y_1 - y_2}{x_1 - x_2}\) and attempt to substitute correct numbers. Formula may be implied by the correct \(\frac{2 - (-4)}{-1 - 7}\) or equivalent.

A1 Any correct single fraction gradient i.e \(\frac{6}{-8}\) or equivalent

M1 Uses their gradient with either \((-1, 2)\) or \((7, -4)\) to form a linear equation

Eg \(y - 2 = \text{their}^{\prime} - \frac{3}{4}(x + 1) \) or \(y + 4 = \text{their}^{\prime} - \frac{3}{4}(x - 7) \) or \(y = \text{their}^{\prime} - \frac{3}{4}x + c\) then find a value for \(c\) by substituting \((-1, 2)\) or \((7, -4)\) in the correct way( not interchanging \(x\) and \(y\))

A1 Accept \(\pm k(4y + 3x - 5) = 0\) with \(k\) an integer (This implies previous M1)

(b) M1 Attempts to use \(gradient \ LM \times gradient \ MN = -1\) i.e. \(-\frac{3}{4} \times \frac{p + 4}{16 - 7} = -1\) (allow sign errors)

Or Attempts Pythagoras correct way round (allow sign errors)

M1 An attempt to solve their linear equation in \(p^{\prime}\). A1 cao \(p = 8\)

(c) M1 For using their numerical value of \(p\) and adding 6. This may be done by any complete method (vectors, drawing, perpendicular straight line equations through \(L\) and \(N\)) or by no method. Assuming \(x = 7\) is M0

A1 Accept 14 for both marks as long as no incorrect working seen (Ignore left hand side – allow \(k\)). If there is wrong working resulting fortuitously in 14 give M0A0. Allow (8, 14) as the answer.
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<tr>
<td>8.</td>
<td>$\frac{dy}{dx} = 6x^{\frac{1}{2}} + x\sqrt{x}$</td>
<td>(7 marks)</td>
</tr>
<tr>
<td></td>
<td>$x\sqrt{x} = x^{\frac{3}{2}}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$x^{n} \rightarrow x^{n+1}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$y = 6 \frac{1}{2}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{5} + c$</td>
<td>A1, A1</td>
</tr>
<tr>
<td>Use $x = 4, y = 37$ to give equation in $c$,</td>
<td>$37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^{5} + c$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$(y) = 12x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $\frac{x^{\frac{5}{2}}}{5} + c$ oe in the subsequent work.

M1 $x^{n} \rightarrow x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both.

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{1}x^{\frac{1}{2}}$ or $\frac{x^{\frac{5}{2}}}{5} + \frac{c}{2}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see $\frac{6}{1}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{5}$ or a simplified correct version

M1 Substitute $x = 4, y = 37$ to produce an equation in $c$.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{3}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{3}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^{\frac{5}{2}}} + \frac{1}{5}$
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<tr>
<td><strong>9.(a)</strong></td>
<td><img src="image" alt="U shaped parabola" /></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>U shaped parabola – symmetric about y axis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Graph passes through (0, 8)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Shape and position for L</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Both (-\frac{k}{3}, 0) and ((0, k))</td>
<td>A1</td>
</tr>
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</table>

**9(b)**

Allow marks **even if** on the **same** diagram

Method 1:

Equate curves \(\frac{1}{3}x^2 + 8 = 3x + k\) and proceed to collect terms on one side

\[
\frac{1}{3}x^2 - 3x + (8 - k)
\]

Method 1a

Uses “\(b^2 = 4ac\)”

\[
9 = 4\times\frac{1}{3}\times(8-k) \Rightarrow k = ..
\]

Method 1b

Attempt \(\frac{1}{3}(x - \frac{9}{2})^2 - \lambda + 8 - k\)

Deduce that \(k = 8 - \lambda\)

\[
k = \frac{5}{4} \text{ o.e.}
\]

Method 2:

Attempts to set \(\frac{dy}{dx} = 3\)

\[
\frac{2}{3}x = 3 \Rightarrow x = 4.5
\]

Method 2a

Substitutes \(x = "4.5"\) into

\[
y = \frac{1}{3}x^2 + 8 \Rightarrow y = ....(14.75)
\]

Substitutes both their \(x\) and \(y\) into

\[
y = 3x + k \text{ to find } k
\]

\[
k = 1.25 \text{ o.e.}
\]

Method 2b

Substitutes \(x = "4.5"\) into

\[
\frac{1}{3}x^2 + 8 = 3x + k
\]

Finds \(k =\)

\[
k = \text{dM1}
\]

\[
\text{A1}
\]

(5 marks)
(a) B1 Shape for C. \[ \cup \] Approximately Symmetrical about the \( y \) axis

B1 Coordinates of (0, 8) There must be a graph.
Accept graph crossing positive \( y \) axis with only 8 marked. Accept (8,0) if given on \( y \) axis.

M1 Shape for \( L \). A straight line with positive gradient and positive intercept

A1 Coordinates of (0, \( k \)) and (-\( k/3 \), 0) or \( k \) marked on \( y \) axis, and –\( k/3 \) marked on \( x \) axis or even Accept (\( k \), 0) on \( y \) axis and (0, -\( k/3 \)) on \( x \) axis

(b) Either

Methods 1

M1 Equate curves \( \frac{1}{3}x^2 + 8 = 3x + k \) and proceed to collect \( x \) terms on one side and \((8 – k)\) terms together on the same side or on the other side

A1 Achieves an expression that leads to the point of intersection e.g \( \frac{1}{3}x^2 - 3x + (8 – k) \)

Method 1a

dM1 (depends on previous M mark) Uses the fact that \( b^2 = 4ac \) or \( 'b^2 - 4ac' = 0 \) is true
dM1 (depends on previous M mark) Solves their \( b^2 = 4ac \), leading to \( k = . . . \)

A1 cso \( k = \frac{5}{4} \) Accept equivalents like 1.25 etc.

Method 1b

dM1 (depends on previous M mark) Uses completion of the square as shown in scheme
dM1 (depends on previous M mark) Uses \( k = 8 - \lambda \)

A1 cso \( k = \frac{5}{4} \) Accept equivalents like 1.25 etc.

Methods 2

M1 Equate \( \frac{dy}{dx} = 3 \) Not given just for derivative

A1 Solves to get \( x = 4.5 \)

Method 2a

M1 Substitutes their 4.5 into equation for \( C \) to give \( y \) coordinate

A1 cso \( k = \frac{5}{4} \) Accept equivalents like 1.25 etc.

Method 2b

M1 Substitutes their 4.5 into \( \frac{1}{3}x^2 + 8 = 3x + k \)

A1 cso \( k = \frac{5}{4} \) Accept equivalents like 1.25 etc.
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<td>10(a).</td>
<td>Attempts to use (a + (n-1)d) with (a=A) and &quot;d&quot;=(d+1) and (n=14) (A+13(d+1) = A+13d+13)</td>
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| (b)             | Calculates time for Yi on Day 14 = \((A-13)+13(2d-1)\)  
Calculates time for Yi on Day 14 = \((A-13)+13(2d-1)\)  
Sets times equal \(A+13d+13 = (A-13)+13(2d-1) \Rightarrow d = \ldots\) \(d = 3\) | M1 M1 A1 cso |
| (c)             | Uses \(\frac{n}{2}\{2A+(n-1)(D)\}\) with \(n=14\), and with \(D=d\) or \(d+1\) \(\frac{n}{2}\{2A+13\times(d+1)^{'}\} = 784 \Rightarrow A = \ldots\) \(A = 30\) | dM1 A1 |

(a) M1 Attempts to use \(a + (n-1)d\) with \(a=A\) and \(d=d+1\) AND \(n=14\)
A1* cao This is a given answer and there is an expectation that the intermediate answer is seen and that all work is correct with correct brackets.
The expressions \(A+13(d+1)\) and \(A+13d+13\) should be seen.

N.B. If brackets are missing and formula is not stated
   e.g. \(A+13d+1\)\( \Rightarrow A+13d+13\) or \(A+(13)d+1 \Rightarrow A+13d+13\) then this is M0A0

If formula is quoted and \(a = A\) and \(d = d + 1\) is quoted or implied, then M1 A0 may be given
So \(a + (n-1)d\) followed by \(A+(13)d+1= A+13d+13\) achieves M1A0

(b) M1 States a time for Yi on Day 14 = \((A-13)+13(2d-1)\)
M1 Sets their time for Yi, equal to \(A+13d+13\) and uses this equation to proceed to \(d = \ldots\)
A1 cso \(d=3\) Needs both M marks and must be simplified to \(3\) (not 39/13)
   [NB Setting each of the times separately equal to 0 leads to \(d = 3\) – this will gain M0A0]

(c) M1 Uses the sum formula \(\frac{n}{2}\{2A+(n-1)(D)\}\) with \(n=14\) and \(D = d+1\) or allow \(D = d\) (usually 4 or 3)

NB May use \(\frac{n}{2}\{A+(A+13D)\}\) with \(n=14\) and and \(D = d+1\) or allow \(D = d\)
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A1 cao \(A = 30\)

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<td>11.(a)</td>
<td>Substitutes $x = 2$ into $y = 20 - 4x - \frac{18}{x^2}$ and gets 3</td>
<td>B1</td>
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<td>Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal $(-2)$</td>
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<td>Method 1 States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their $(2, 3)$ to deduce that $y = -2x + 7$</td>
<td>dM1</td>
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<td>Method 2 Or: Check that $(2, 3)$ lies on the line $y = -2x + 7$ Deduce equation of normal as it has the same gradient and passes through a common point</td>
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<td></td>
<td>(6)</td>
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<tr>
<td>(b)</td>
<td>Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$</td>
<td>M1 A1</td>
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<td>Or put $y = 20 - 4\left(\frac{7 - y}{2}\right) - \frac{18}{\left(\frac{7 - y}{2}\right)}$ to give $y^2 - y - 6 = 0$</td>
<td>dM1</td>
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<td>$(2x - 9)(x - 2) = 0$ so $x = \frac{9}{2}$ or $(y - 3)(y + 2) = 0$ so $y = \frac{9}{2}$, $y = -2$</td>
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(a) B1 Substitutes $x = 2$ into expression for $y$ and gets 3 cao  (must be in part (a) and **must use curve equation – not line equation** This must be seen to be substituted.

M1 For an attempt to differentiate the negative power with $x^{-1} \rightarrow x^{-2}$.

A1 Correct expression for $\frac{dy}{dx} = -4 + \frac{18}{x^2}$, accept equivalents

dM1 Dependent on **first** M1 Substitutes $x = 2$ into their derivative to obtain a numerical gradient and find negative reciprocal or states that $-2 \times \frac{1}{2} = -1$

(Method 1)

dM1 Dependent on **first** M1 Finds equation of line using changed gradient (not their $\frac{1}{2}$ but -1/2, 2 or -2) e.g. $y - 3 = -2(x - 2)$ or $y = -2x + c$ and use of (2, “3”) to find $c = \ldots$

A1* CSO. This is a given answer $y = -2x + 7$ obtained with no errors seen and equation should be stated

(Method 2)- checking given answer

dM1 Uses given equation of line and checks that (2, 3) lies on the line

A1* CSO. This is a given answer $y = -2x + 7$ so statement that normal and line **have the same gradient** and **pass through the same point** must be stated

(b) M1 Equate the **two given** expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms But putting for example $20x - 4x^2 - 18 = -2x + 7$ is M0 here

A1 Correct 3TQ = 0 (need = 0 for A mark) $2x^2 - 13x + 18 = 0$

dM1 Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).

A1 $x = \frac{9}{2}$ or $y = -2$ (allow second answers for this mark so ignore $x = 2$ or $y = 3$)

A1 Correct solution only so both $x = \frac{9}{2}, y = -2$ or $\left(\frac{9}{2}, -2\right)$

If $x = 2, y = 3$ is included as an answer and point $B$ is not identified then last mark is A0 Answer only – with no working – send to review. The question stated “use algebra”