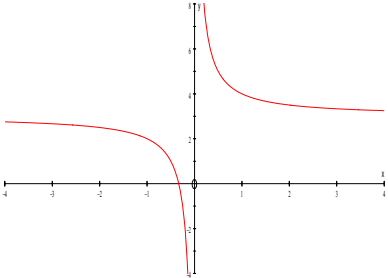
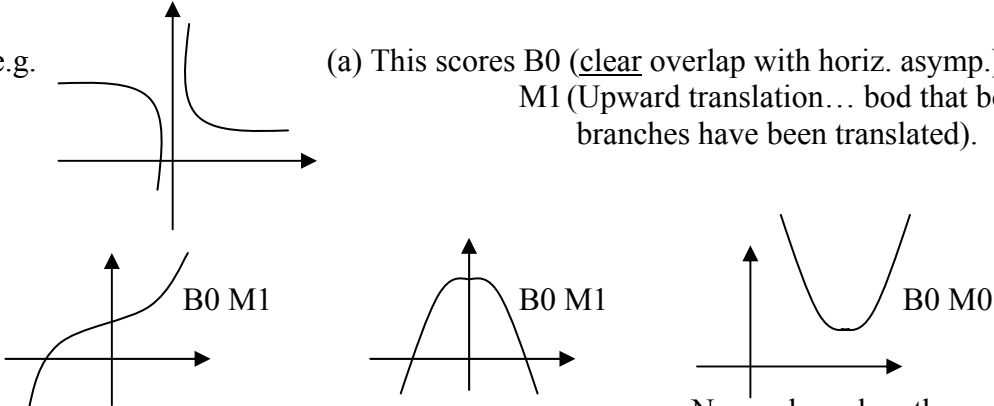


January 2007
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Mark
1.	$4x^3 \rightarrow kx^2$ or $2x^{\frac{1}{2}} \rightarrow kx^{-\frac{1}{2}}$ (k a non-zero constant) $12x^2, + x^{-\frac{1}{2}} \dots\dots, (-1 \rightarrow 0)$	M1 A1, A1, B1 (4) 4
	<p>Accept equivalent alternatives to $x^{-\frac{1}{2}}$, e.g. $\frac{1}{x^{\frac{1}{2}}}$, $\frac{1}{\sqrt{x}}$, $x^{-0.5}$.</p> <p>M1: $4x^3$ ‘differentiated’ to give kx^2, or... $2x^{\frac{1}{2}}$ ‘differentiated’ to give $kx^{-\frac{1}{2}}$ (but not for just $-1 \rightarrow 0$).</p> <p>1st A1: $12x^2$ (Do not allow just $3 \times 4x^2$)</p> <p>2nd A1: $x^{-\frac{1}{2}}$ or equivalent. (Do not allow just $\frac{1}{2} \times 2x^{-\frac{1}{2}}$, but allow $1x^{-\frac{1}{2}}$ or $\frac{2}{2}x^{-\frac{1}{2}}$).</p> <p>B1: -1 differentiated to give zero (or ‘disappearing’). Can be given provided that at least one of the other terms has been changed. Adding an extra term, e.g. $+ C$, is B0.</p>	

Question Number	Scheme	Marks
2.	<p>(a) $6\sqrt{3}$ $(a = 6)$</p> <p>(b) Expanding $(2 - \sqrt{3})^2$ to get 3 or 4 separate terms</p> <p style="margin-left: 40px;">$7, -4\sqrt{3}$ $(b = 7, c = -4)$</p>	<p>B1 (1)</p> <p>M1</p> <p>A1, A1 (3)</p> <p style="text-align: right;">4</p>
	<p>(a) $\pm 6\sqrt{3}$ also scores B1.</p> <p>(b) M1: The 3 or 4 terms may be wrong.</p> <p style="margin-left: 40px;">1st A1 for 7, 2nd A1 for $-4\sqrt{3}$.</p> <p style="margin-left: 40px;">Correct answer $7 - 4\sqrt{3}$ with no working scores all 3 marks.</p> <p style="margin-left: 40px;">$7 + 4\sqrt{3}$ with or without working scores M1 A1 A0.</p> <p style="margin-left: 40px;">Other wrong answers with no working score no marks.</p>	

Question Number	Scheme	Marks
3.	<p>(a) </p> <p>Shape of $f(x)$ Moved up \uparrow Asymptotes: $y = 3$ $x = 0$ (Allow “y-axis”) ($y \neq 3$ is B0, $x \neq 0$ is B0).</p> <p>(b) $\frac{1}{x} + 3 = 0$ No variations accepted. $x = -\frac{1}{3}$ (or $-0.33 \dots$) Decimal answer requires at least 2 d.p.</p>	<p>B1 M1 B1 B1 (4)</p> <p>M1 A1 (2)</p> <p>6</p>
	<p>(a) B1: Shape requires both branches and no obvious “overlap” with the asymptotes (see below), but otherwise this mark is awarded generously. The curve may, e.g., bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both horizontal and vertical. M1: Evidence of an upward translation parallel to the y-axis. The shape of the graph can be wrong, but the complete graph (both branches if they have 2 branches) must be translated upwards. This mark can be awarded generously by implication where the graph drawn is an upward translation of another standard curve (but <u>not</u> a straight line). The B marks for asymptote equations are independent of the graph. Ignore extra asymptote equations, if seen.</p> <p>(b) Correct answer with no working scores both marks. The answer may be seen on the sketch in part (a). Ignore any attempts to find an intersection with the y-axis.</p> <p>e.g. </p> <p>(a) This scores B0 (clear overlap with horiz. asymp.) M1 (Upward translation... bod that both branches have been translated).</p> <p>No marks unless the original curve is seen, to show upward translation.</p>	

Question Number	Scheme	Marks
4.	$(x-2)^2 = x^2 - 4x + 4$ or $(y+2)^2 = y^2 + 4y + 4$ M: 3 or 4 terms $(x-2)^2 + x^2 = 10$ or $y^2 + (y+2)^2 = 10$ M: Substitute $2x^2 - 4x - 6 = 0$ or $2y^2 + 4y - 6 = 0$ Correct 3 terms $(x-3)(x+1) = 0, \quad x = \dots$ or $(y+3)(y-1) = 0, \quad y = \dots$ (The above factorisations may also appear as $(2x-6)(x+1)$ or equivalent). $x = 3 \quad x = -1$ or $y = -3 \quad y = 1$ $y = 1 \quad y = -3$ or $x = -1 \quad x = 3$ (Allow equivalent fractions such as: $x = \frac{6}{2}$ for $x = 3$).	M1 M1 A1 M1 A1 M1 A1 (7)
	<p>1st M: ‘Squaring a bracket’, needs 3 or 4 terms, one of which must be an x^2 or y^2 term.</p> <p>2nd M: Substituting to get an equation in one variable (awarded generously).</p> <p>1st A: Accept equivalent forms, e.g. $2x^2 - 4x = 6$.</p> <p>3rd M: Attempting to solve a 3-term quadratic, to get 2 solutions.</p> <p>4th M: Attempting at least one y value (or x value).</p> <p>If y solutions are given as x values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0.</p> <p>Strict “pairing of values” at the end is <u>not</u> required.</p> <p>“Non-algebraic” solutions:</p> <p>No working, and only one correct solution pair found (e.g. $x = 3, y = 1$): M0 M0 A0 M0 A0 M1 A0</p> <p>No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1</p> <p>Both correct solution pairs found, and demonstrated, perhaps in a table of values: Full marks</p> <p><u>Squaring individual terms:</u> e.g.</p> <p>$y^2 = x^2 + 4$ M0</p> <p>$x^2 + 4 + x^2 = 10$ M1 A0 (Eqn. in one variable)</p> <p>$x = \sqrt{3}$ M0 A0 (Not solving 3-term quad.)</p> <p>$y^2 = x^2 + 4 = 7$ $y = \sqrt{7}$ M1 A0 (Attempting one y value)</p>	<p style="text-align: right;">7</p>

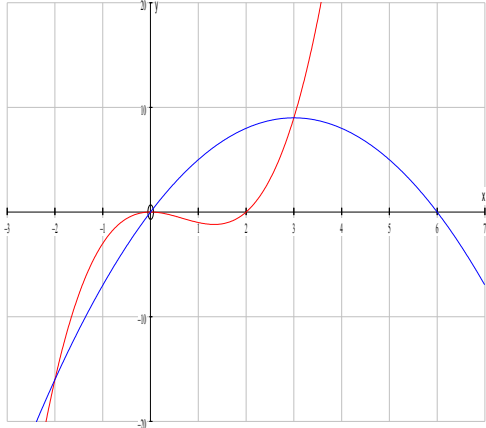
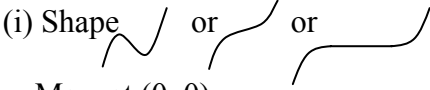



Question Number	Scheme	Marks
5.	<p><u>Use</u> of $b^2 - 4ac$, perhaps implicit (e.g. in quadratic formula)</p> $(-3)^2 - 4 \times 2 \times -(k+1) < 0 \quad (9 + 8(k+1) < 0)$ $8k < -17 \quad (\text{Manipulate to get } pk < q, \text{ or } pk > q, \text{ or } pk = q)$ $k < -\frac{17}{8} \quad \left(\text{Or equiv : } k < -2\frac{1}{8} \text{ or } k < -2.125 \right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso (4)</p> <p style="text-align: right;">4</p>
	<p>1st M: Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the <u>given</u> quadratic equation. There must <u>not</u> be x terms in the expression, but there must be a k term.</p> <p>1st A: Correct expression (need not be simplified) and correct inequality sign. Allow also $-3^2 - 4 \times 2 \times -(k+1) < 0$.</p> <p>2nd M: Condone sign or bracketing mistakes in manipulation. Not dependent on 1st M, but should not be given for irrelevant work. M0 M1 could be scored: e.g. where $b^2 + 4ac$ is used instead of $b^2 - 4ac$.</p> <p><u>Special cases:</u></p> <ol style="list-style-type: none"> Where there are x terms in the discriminant expression, but then division by x^2 gives an inequality/equation in k. (This could score M0 A0 M1 A1). Use of \leq instead of $<$ loses one A mark only, at first occurrence, so an otherwise correct solution leading to $k \leq -\frac{17}{8}$ scores M1 A0 M1 A1. <p>N.B. Use of $b = 3$ instead of $b = -3$ implies no A marks.</p>	

Question Number	Scheme	Marks
6.	<p>(a) $(4 + 3\sqrt{x})(4 + 3\sqrt{x})$ seen, or a numerical value of k seen, ($k \neq 0$). (The k value need not be explicitly stated... see below). $16 + 24\sqrt{x} + 9x$, or $k = 24$</p> <p>(b) $16 \rightarrow cx$ or $kx^{1/2} \rightarrow cx^{3/2}$ or $9x \rightarrow cx^2$ $\int(16 + 24\sqrt{x} + 9x)dx = 16x + \frac{9x^2}{2} + C, +16x^{3/2}$</p>	<p>M1 A1cso (2)</p> <p>M1 A1, A1ft (3)</p> <p>5</p>
	<p>(a) e.g. $(4 + 3\sqrt{x})(4 + 3\sqrt{x})$ alone scores M1 A0, (but <u>not</u> $(4 + 3\sqrt{x})^2$ alone). e.g $16 + 12\sqrt{x} + 9x$ scores M1 A0. $k = 24$ or $16 + 24\sqrt{x} + 9x$, with no further evidence, scores full marks M1 A1. Correct solution only (cso): any wrong working seen loses the A mark.</p> <p>(b) A1: $16x + \frac{9x^2}{2} + C$. Allow 4.5 or $4\frac{1}{2}$ as equivalent to $\frac{9}{2}$. A1ft: $\frac{2k}{3}x^{3/2}$ (candidate's value of k, or general k). For this final mark, allow for example $\frac{48}{3}$ as equivalent to 16, but do <u>not</u> allow unsimplified "double fractions" such as $\frac{24}{(\frac{3}{2})}$, and do <u>not</u> allow unsimplified "products" such as $\frac{2}{3} \times 24$. A single term is required, e.g. $8x^{3/2} + 8x^{3/2}$ is not enough.</p> <p>An otherwise correct solution with, say, C missing, followed by an incorrect solution including $+ C$ can be awarded full marks (isw, but allowing the C to appear at any stage).</p>	

Question Number	Scheme	Marks
7.	<p>(a) $3x^2 \rightarrow cx^3$ or $-6 \rightarrow cx$ or $-8x^{-2} \rightarrow cx^{-1}$</p> $f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1} \quad (+C) \quad \left(x^3 - 6x + \frac{8}{x} \right)$ <p>Substitute $x = 2$ <u>and</u> $y = 1$ into a 'changed function' to form an equation in C.</p> $1 = 8 - 12 + 4 + C \quad C = 1$ <p>(b) $3 \times 2^2 - 6 - \frac{8}{2^2}$</p> $= 4$ <p>Eqn. of tangent: $y - 1 = 4(x - 2)$</p> $y = 4x - 7 \quad (\text{Must be in this form})$	<p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1cso (5)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>9</p>
	<p>(a) First 2 A marks: $+C$ is not required, and coefficients need <u>not</u> be simplified, but powers must be simplified.</p> <p>All 3 terms correct: A1 A1 Two terms correct: A1 A0 Only one term correct: A0 A0</p> <p>Allow the M1 A1 for finding C to be scored either in part (a) or in part (b).</p> <p>(b) 1st M: Substituting $x = 2$ into $3x^2 - 6 - \frac{8}{x^2}$ (must be this function). 2nd M: Awarded generously for attempting the equation of a straight line through (2, 1) or (1, 2) with any value of m, however found. 2nd M: Alternative is to use (2, 1) or (1, 2) in $y = mx + c$ to <u>find a value</u> for c.</p> <p>If calculation for the gradient value is seen in part (a), it must be <u>used</u> in part (b) to score the first M1 A1 in (b).</p> <p><u>Using (1, 2) instead of (2, 1)</u>: Loses the 2nd method mark in (a). Gains the 2nd method mark in (b).</p>	

Question Number	Scheme	Marks
8.	<p>(a) $4x \rightarrow k$ or $3x^{3/2} \rightarrow kx^{1/2}$ or $-2x^2 \rightarrow kx$</p> $\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$ <p>(b) For $x = 4$, $y = (4 \times 4) + (3 \times 4\sqrt{4}) - (2 \times 16) = 16 + 24 - 32 = 8$ (*)</p> <p>(c) $\frac{dy}{dx} = 4 + 9 - 16 = -3$ M: Evaluate their $\frac{dy}{dx}$ at $x = 4$</p> <p>Gradient of normal = $\frac{1}{3}$</p> <p>Equation of normal: $y - 8 = \frac{1}{3}(x - 4)$, $3y = x + 20$ (*)</p> <p>(d) $y = 0$: $x = \dots (-20)$ and use $(x_2 - x_1)^2 + (y_2 - y_1)^2$</p> $PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from $(k, 0)$ <p>May also be scored with $(-24)^2$ and $(-8)^2$.</p> $= 8\sqrt{10}$	<p>M1</p> <p>A1 A1 (3)</p> <p>B1 (1)</p> <p>M1</p> <p>A1ft</p> <p>M1, A1 (4)</p> <p>M1</p> <p>A1ft</p> <p>A1 (3)</p> <p>11</p>
	<p>(a) For the 2 A marks coefficients need <u>not</u> be simplified, but powers must be simplified. For example, $\frac{3}{2} \times 3x^{1/2}$ is acceptable.</p> <p>All 3 terms correct: A1 A1</p> <p>Two terms correct: A1 A0</p> <p>Only one term correct: A0 A0</p> <p>(b) There must be some evidence of the “24” value.</p> <p>(c) In this part, beware ‘working backwards’ from the given answer.</p> <p>A1ft: Follow through is just from the candidate’s <u>value</u> of $\frac{dy}{dx}$.</p> <p>2nd M: Is not given if an m value appears “from nowhere”.</p> <p>2nd M: Must be an attempt at a <u>normal</u> equation, not a tangent.</p> <p>2nd M: Alternative is to use $(4, 8)$ in $y = mx + c$ to <u>find a value</u> for c.</p> <p>(d) M: Using the normal equation to attempt coordinates of Q, (even if using $x = 0$ instead of $y = 0$), <u>and</u> using Pythagoras to attempt PQ or PQ^2.</p> <p>Follow through from $(k, 0)$, but <u>not</u> from $(0, k)$...</p> <p>A common wrong answer is to use $x = 0$ to give $\frac{20}{3}$. This scores M1 A0 A0.</p> <p>For final answer, accept other simplifications of $\sqrt{640}$, e.g. $2\sqrt{160}$ or $4\sqrt{40}$.</p>	

Question Number	Scheme	Marks
9.	<p>(a) Recognising arithmetic series with first term 4 and common difference 3. (If not scored here, this mark may be given if seen elsewhere in the solution). $a + (n - 1)d = 4 + 3(n - 1) \quad (= 3n + 1)$</p> <p>(b) $S_n = \frac{n}{2} \{2a + (n - 1)d\} = \frac{10}{2} \{8 + (10 - 1) \times 3\}, = 175,$</p> <p>(c) $S_k < 1750: \frac{k}{2} \{8 + 3(k - 1)\} < 1750$ (or $S_{k+1} > 1750: \frac{k+1}{2} \{8 + 3k\} > 1750$) $3k^2 + 5k - 3500 < 0$ (or $3k^2 + 11k - 3492 > 0$) (Allow equivalent 3-term versions such as $3k^2 + 5k = 3500$). $(3k - 100)(k + 35) < 0$ Requires use of correct inequality throughout.(*)</p> <p>(d) $\frac{100}{3}$ or equiv. <u>seen</u> (or $\frac{97}{3}$), $k = 33$ (and no other values)</p>	<p>B1 M1 A1 (3)</p> <p>M1 A1, A1 (3)</p> <p>M1 M1 A1 A1cso (4) M1, A1 (2)</p> <p>12</p>
	<p>(a) B1: Usually identified by $a = 4$ and $d = 3$. M1: Attempted use of term formula for arithmetic series, or... answer in the form $(3n + \text{constant})$, where the constant is a non-zero value. Answer for (a) does not require simplification, and a correct answer without working scores all 3 marks.</p> <p>(b) M1: Use of correct sum formula with $n = 9, 10$ or 11. A1: Correct, perhaps unsimplified, numerical version. A1: 175 <u>Alternative:</u> (Listing and summing terms). M1: Summing 9, 10 or 11 terms. (At least 1st, 2nd and last terms must be seen). A1: Correct terms (perhaps implied by last term 31). A1: 175 <u>Alternative:</u> (Listing all sums) M1: Listing 9, 10 or 11 sums. (At least 4, 7,, "last"). A1: Correct sums, correct finishing value 175. A1: 175 <u>Alternative:</u> (Using last term). M1: Using $S_n = \frac{n}{2}(a + l)$ with T_9, T_{10} or T_{11} as the last term. A1: Correct numerical version $\frac{10}{2}(4 + 31)$. A1: 175 Correct answer with <u>no</u> working scores 1 mark: 1,0,0.</p> <p>(c) For the first 3 marks, allow <u>any inequality sign</u>, or <u>equals</u>. 1st M: Use of correct sum formula to form inequality or equation in k, with the 1750. 2nd M: (Dependent on 1st M). Form 3-term quadratic in k. 1st A: Correct 3 terms. Allow credit for part (c) if valid work is seen in part (d).</p> <p>(d) Allow both marks for $k = 33$ seen without working. Working for part (d) must be seen in part (d), not part (c).</p>	

Question Number	Scheme	Marks
10.	<p>(a) </p> <p>(i) Shape  or  or </p> <p>Max. at (0, 0). (2, 0), (or 2 shown on x-axis).</p> <p>(ii) Shape  (It need not go below x-axis) Through origin. (6, 0), (or 6 shown on x-axis).</p> <p>(b) $x^2(x - 2) = x(6 - x)$ $x^3 - x^2 - 6x = 0$ Expand to form 3-term cubic (or 3-term quadratic if divided by x), with all terms on one side. The “= 0” may be implied. $x(x - 3)(x + 2) = 0$ $x = \dots$ Factor x (or divide by x), and solve quadratic. $x = 3$ and $x = -2$ $x = -2$: $y = -16$ Attempt y value for a non-zero x value by substituting back into $x^2(x - 2)$ or $x(6 - x)$. $x = 3$: $y = 9$ Both y values are needed for A1. (-2, -16) and (3, 9) (0, 0) This can just be written down. Ignore any ‘method’ shown. (But must be seen in part (b)).</p>	<p>B1 B1 B1 (3) B1 B1 B1 (3) M1 M1 M1 A1 M1 A1 B1 (7) 13</p>
	<p>(a) (i) For the third ‘shape’ shown above, where a section of the graph coincides with the x-axis, the B1 for (2, 0) can still be awarded if the 2 is shown on the x-axis. For the final B1 in (i), and similarly for (6, 0) in (ii): There must be a sketch. If, for example (2, 0) is written <u>separately</u> from the sketch, the sketch must not clearly contradict this. If (0, 2) instead of (2, 0) is shown <u>on the sketch</u>, allow the mark. Ignore extra intersections with the x-axis. (ii) 2nd B is dependent on 1st B. Separate sketches can score all marks.</p> <p>(b) Note the dependence of the first three M marks. A common wrong solution is (-2, 0), (3, 0), (0, 0), which scores M0 A0 B1 as the last 3 marks. A solution using <u>no</u> algebra (e.g. trial and error), can score up to 3 marks: M0 M0 M0 A0 M1 A1 B1. (The final A1 requires both y values). Also, if the cubic is found but not solved algebraically, up to 5 marks: M1 M1 M0 A0 M1 A1 B1. (The final A1 requires both y values).</p>	